Maximal CP violation in minimal seesaw model

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In the minimal seesaw model, we derive required constraints on Dirac neutrino masses inducing maximal CP violation in neutrino oscillations. If the maximal atmospheric neutrino mixing is further assumed, Dirac neutrino masses are uniquely determined to respect μ-τ flavored CP symmetry for neutrinos.

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I. INTRODUCTION

Neutrino oscillations have been theoretically predicted [1] and experimentally observed as atmospheric, solar, accelerator, and reactor neutrino oscillations for more than a decade [2–7]. Extensive analyses of the current experimental data on neutrino oscillations seem to suggest the presence of the Dirac CP violation in neutrino physics [8]. The Dirac CP violation is described by the CP-violating Dirac phase δ_{CP}, which turns out to lie in the 1σ-region of $\delta_{CP}/\pi = 1.13$–1.64 for the normal mass hierarchy (NH) or of $\delta_{CP}/\pi = 1.07$–1.67 for the inverted mass hierarchy (IH) [9]. There is another type of CP violation called Majorana CP violation. The relevant CP-violating phases are the Dirac phase and the Majorana phase [10], which enter into the Pentecorvo-Maki-Nakagawa-Sakata mixing matrix $U_{\rm PNMS}$ [1] that converts the mass eigenstates of neutrinos $\nu_{1,2,3}$ into the flavor neutrinos $\nu_{e,\mu,\tau}$. Denoting the atmospheric neutrino mixing angle by θ_{23}, the solar neutrino mixing angle by θ_{12}, and the reactor neutrino mixing angle by θ_{13}, the standard parametrization of $U_{\rm PNMS}$ is given by the Particle Data Group (PDG) [11] to be $U_{\rm PDG} = U_\nu^{\rm PDG} K^{\rm PDG}$,

$$U_\nu^{\rm PDG} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix},$$
$$K^{\rm PDG} = \begin{pmatrix} e^{i\phi_1/2} & 0 & 0 \\ 0 & e^{i\phi_2/2} & 0 \\ 0 & 0 & e^{i\phi_3/2} \end{pmatrix}, \tag{1}$$

for $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$ and similarly $t_{ij} = \tan\theta_{ij}$ $(i, j = 1, 2, 3)$, where $\phi_{1,2,3}$ stand for the Majorana phases, from which two independent combinations become the CP-violating Majorana phases.

It is interesting to note that the experimentally allowed region of δ_{CP} includes $\delta_{CP} = 3\pi/2$ indicating maximal CP violation. From the theoretical point of view, δ_{CP} arises from phases of flavor neutrino masses to be denoted by M_{ij} $(i, j = e, \mu, \tau)$. We have been advocating the following useful relation among δ_{CP} and M_{ij} [12,13]:

$$\frac{M_{\tau\tau} - M_{\mu\mu}}{2}\sin 2\theta_{23} - M_{\mu\tau}\cos 2\theta_{23} = \tan\theta_{13}(M_{e\mu}\cos\theta_{23} - M_{e\tau}\sin\theta_{23})e^{-i\delta_{CP}}, \tag{2}$$

which is used to express θ_{23} in terms of M_{ij}. The maximal CP violation can be induced if

$$M_{\tau\tau} - M_{\mu\mu} = \text{imaginary}, \tag{3}$$

as well as

$$M_{\mu\tau} = \text{imaginary}, \quad M_{e\mu}\cos\theta_{23} - M_{e\tau}\sin\theta_{23} = \text{real}, \tag{4}$$

for $\cos 2\theta_{23} \neq 0$, or

$$M_{e\mu} - \sigma M_{e\tau} = \text{real}, \tag{5}$$

for $\cos 2\theta_{23} = 0$ indicating the maximal atmospheric neutrino mixing, where $\sigma = \pm 1$ takes care of the sign of $\sin\theta_{23}$.

From the recent result of the Planck [14], the upper limit of the neutrino masses is given by $\sum m_\nu \leq 0.17$ eV. On the other hand, the neutrino oscillation experiments measure $\Delta m^2_{31} = m_3^2 - m_1^2$ and $\Delta m^2_{32} = m_3^2 - m_2^2$. Choosing $\Delta m^2_{31} = 2.46 \times 10^{-3}$ eV^2($\sim m_3^2$) for NH with $m_3^2 \gg m_2^2 \gg m_1^2$ and $\Delta m^2_{32} = -2.45 \times 10^{-3}$ eV^2($\sim -m_2^2$) for IH with $m_2^2 > m_1^2 \gg m_3^2$ [8], we obtain that the heaviest

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neutrino mass, either m_3 or m_2, is approximately estimated to be 0.05 eV. Why neutrinos are so light is a puzzling question to be solved. One of the promising theoretical ideas is the one based on the seesaw mechanism [15], which utilizes right-handed neutrinos. The right-handed neutrinos can provide Dirac masses for flavor neutrinos and light flavor neutrinos can be generated if the right-handed neutrinos are very heavy. Furthermore, CP violation in the early Universe is able to be induced by the heavy right-handed neutrinos via the Dirac mass terms supplemented by the Higgs scalar. If the heavy right-handed neutrinos come in two families, all of CP-violating phases associated with the Dirac neutrino masses can be converted into the CP-violating Dirac and Majorana phases associated with the light flavor neutrino masses. The model with two extra heavy right-handed neutrinos is called the minimal seesaw model [16]. If the seesaw mechanism is the right answer to give tiny neutrino masses, our relation Eq. (2) is also described by more fundamental quantities, namely the Dirac neutrino masses.

In this article, within the framework of the minimal seesaw model, we argue how Dirac neutrino masses are constrained so as to induce maximal CP violation and simultaneously to induce maximal atmospheric neutrino mixing as well [17,18]. In Sec. II, we introduce six Dirac neutrino masses associated with two extra heavy right-handed neutrinos. Three relations determining three neutrino mixing angles such as Eq. (2) are expressed in terms of these six Dirac neutrino masses and the CP-violating Dirac phase, which are used to find constraints to induce maximal CP violation. To obtain simple and useful relations in the minimal seesaw model, we choose one combination of Dirac neutrino masses to vanish, which includes texture one zero. The detailed discussions to reach various constraints on Dirac neutrino masses are presented in the Appendix. In Sec. III, we derive necessary constraints on the Dirac neutrino masses to induce maximal CP violation. Finally, further assuming maximal atmospheric neutrino mixing, we determine six Dirac neutrino masses to be real or imaginary. The final section, Sec. IV, is devoted to summary and discussions, which include a preliminary argument on the creation of the baryon number of the Universe via the leptogenesis based on our constraints on the Dirac neutrino masses.

II. DIRAC MASSES AND DIRAC CP VIOLATION

The minimal seesaw model contains two extra right-handed neutrinos. We understand that a 2×2 heavy neutrino mass matrix M_R and a charged lepton mass matrix are transformed into diagonal and real ones. After the heavy right-handed neutrinos are decoupled, the minimal seesaw mechanism generates a symmetric 3×3 light neutrino mass matrix M_ν containing M_{ij} as elements to yield $M_\nu = -m_D M_R^{-1} m_D^T$, where m_D is a 3×2 Dirac neutrino mass matrix. We parametrize M_R by

$$M_R = \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix} \quad (M_1 < M_2), \tag{6}$$

and m_D by

$$m_D = \begin{pmatrix} \sqrt{M_1}a_1 & \sqrt{M_2}b_1 \\ \sqrt{M_1}a_2 & \sqrt{M_2}b_2 \\ \sqrt{M_1}a_3 & \sqrt{M_2}b_3 \end{pmatrix}, \tag{7}$$

which result in

$$M_\nu = \begin{pmatrix} M_{ee} & M_{e\mu} & M_{e\tau} \\ M_{e\mu} & M_{\mu\mu} & M_{\mu\tau} \\ M_{e\tau} & M_{\mu\tau} & M_{\tau\tau} \end{pmatrix} = -\begin{pmatrix} a_1^2+b_1^2 & a_1a_2+b_1b_2 & a_1a_3+b_1b_3 \\ a_1a_2+b_1b_2 & a_2^2+b_2^2 & a_2a_3+b_2b_3 \\ a_1a_3+b_1b_3 & a_2a_3+b_2b_3 & a_3^2+b_3^2 \end{pmatrix}, \tag{8}$$

where the minus sign in front of the mass matrix is discarded for the later discussions. One of the masses of $\nu_{1,2,3}$ is required to vanish owing to $\det(M_\nu)=0$.

The useful relation Eq. (2) expressed in terms of M_{ij} is converted into

$$a_+a_- + b_+b_- = -t_{13}(a_1a_- + b_1b_-)e^{-i\delta_{CP}}, \tag{9}$$

where $a_+ = s_{23}a_2 + c_{23}a_3$, $b_+ = s_{23}b_2 + c_{23}b_3$, $a_- = c_{23}a_2 - s_{23}a_3$ and $b_- = c_{23}b_2 - s_{23}b_3$. There are two more similar relations to Eq. (2) that determine $\theta_{12,13}$ for given M_{ij} [13] and these two relations give rise to

$$\sin 2\theta_{12}\left[\frac{c_{13}^2(a_1^2+b_1^2) - s_{13}^2(a_+^2+b_+^2)e^{2i\delta_{CP}}}{\cos 2\theta_{13}} - (a_-^2+b_-^2)\right] = -2\cos 2\theta_{12}\frac{a_1a_- + b_1b_-}{c_{13}}, \tag{10}$$

$$\sin 2\theta_{13}[(a_+^2+b_+^2)e^{i\delta_{CP}} - (a_1^2+b_1^2)e^{-i\delta_{CP}}] = 2\cos 2\theta_{13}(a_1a_+ + b_1b_+). \tag{11}$$

Similarly, neutrino masses accompanied by Majorana phases are calculated to be

$$m_1e^{-i\phi_1} = a_-^2 + b_-^2 - \frac{a_1a_- + b_1b_-}{t_{12}c_{13}},$$
$$m_2e^{-i\phi_2} = a_-^2 + b_-^2 + \frac{t_{12}}{c_{13}}(a_1a_- + b_1b_-),$$
$$m_3e^{-i\phi_3} = \frac{c_{13}^2(a_+^2+b_+^2) - s_{13}^2(a_1^2+b_1^2)e^{-2i\delta_{CP}}}{\cos 2\theta_{13}}. \tag{12}$$

These three relations, Eqs. (9)–(11), can be cast into more compact forms since one of the three neutrino masses turns out be 0 owing to $\det(M_\nu)=0$. For NH, we have $m_1 = 0$ leading to

$$a_-^2 + b_-^2 = \frac{1}{t_{12}c_{13}}(a_1 a_- + b_1 b_-), \tag{13}$$

and obtain that

$$a_+a_- + b_+b_- = -t_{13}(a_1a_- + b_1b_-)e^{-i\delta_{CP}}, \tag{14}$$

$$a_1^2 + b_1^2 - t_{12}^2(a_-^2 + b_-^2) = t_{13}(a_1a_+ + b_1b_+)e^{i\delta_{CP}}, \tag{15}$$

$$c_{13}^2(a_1^2+b_1^2) - s_{13}^2(a_+^2+b_+^2)e^{2i\delta_{CP}} = (c_{13}^2 - s_{13}^2)t_{12}^2(a_-^2+b_-^2), \tag{16}$$

and

$$m_2 e^{-i\phi_2} = \frac{1}{c_{12}^2}(a_-^2 + b_-^2),$$
$$m_3 e^{-i\phi_3} = \frac{1}{c_{13}^2}[a_+^2 + b_+^2 - s_{13}^2 t_{12}^2(a_-^2 + b_-^2)e^{-2i\delta_{CP}}]. \tag{17}$$

For IH, we have $m_3 = 0$ leading to

$$a_+^2 + b_+^2 = t_{13}^2(a_1^2 + b_1^2)e^{-2i\delta_{CP}}, \tag{18}$$

and obtain that

$$a_+a_- + b_+b_- = -t_{13}(a_1a_- + b_1b_-)e^{-i\delta_{CP}}, \tag{19}$$

$$\sin 2\theta_{12}\left(\frac{a_1^2+b_1^2}{c_{13}^2} - (a_-^2+b_-^2)\right) = -2\cos 2\theta_{12}\frac{a_1a_- + b_1b_-}{c_{13}}, \tag{20}$$

$$a_1a_+ + b_1b_+ = -t_{13}(a_1^2 + b_1^2)e^{-i\delta_{CP}}, \tag{21}$$

and

$$m_1 e^{-i\phi_1} = a_-^2 + b_-^2 - \frac{a_1a_- + b_1b_-}{t_{12}c_{13}},$$
$$m_2 e^{-i\phi_2} = a_-^2 + b_-^2 + \frac{t_{12}}{c_{13}}(a_1a_- + b_1b_-). \tag{22}$$

We obtain simple solutions to these equations for $a_{1,+,-}$ and $b_{1,+,-}$ and choose several plausible sets of the solutions, which are consistent with the hierarchical condition of $m_3^2 \gg m_2^2$ requiring that $|a_+^2 + b_+^2|^2 \gg |a_-^2 + b_-^2|^2$ for NH or $m_1^2 \approx m_2^2$ requiring that $|a_-^2 + b_-^2|^2 \gg |a_1a_- + b_1b_-|^2$ for IH. As stated in the introduction, we choose one combination of Dirac neutrino masses to vanish, which includes texture one zero. The discussions on our choices of the solutions are presented in the Appendix, from which we can summarize our results as follows: For NH,

(1) in the case of $a_1 = 0$, $a_{+,-}$ and $b_{1,+,-}$ should satisfy $a_- = -s_{13}a_+e^{i\delta_{CP}}/t_{12}$ and $b_1 = t_{12}b_-/c_{13} + t_{13}b_+ e^{i\delta_{CP}}$ as well as $a_+a_- = -(b_+ + t_{13}b_1e^{-i\delta_{CP}})b_-$ and $a_-^2 + b_-^2 = b_1b_-/t_{12}c_{13}$;
(2) in the case of $b_1 = 0$, $a_{1,+,-}$ and $b_{+,-}$ should satisfy relations in the case of $a_1 = 0$ with the interchange of $a \leftrightarrow b$;
(3) in the case of $a_- = 0$, $a_{1,+}$ and $b_{1,+,-}$ should satisfy $a_1 = t_{13}a_+e^{i\delta_{CP}}$, $b_- = b_1/t_{12}c_{13}$ and $b_+ = -t_{13}b_1e^{-i\delta_{CP}}$;
(4) in the case of $b_- = 0$, $a_{1,+,-}$ and $b_{1,+}$ should satisfy relations in the case of $a_- = 0$ with the interchange of $a \leftrightarrow b$;
(5) in the cases of $a_+ = 0$ and $b_+ = 0$, no simple linear expressions arise.

And, for IH, we find that

(1) in the case of $a_+ = -t_{13}a_1e^{-i\delta_{CP}}$, a_- and $b_{1,+,-}$ should satisfy $b_+ = -t_{13}b_1e^{-i\delta_{CP}}$;
(2) in the case of $a_1 = 0$, $a_{-,+}$ and $b_{1,+,-}$ should satisfy $a_+ = 0$ and $b_+ = -t_{13}b_1e^{-i\delta_{CP}}$;
(3) in the case of $b_1 = 0$, $a_{1,+,-}$ and $b_{+,-}$ should satisfy relations in the case of $a_1 = 0$ with the interchange of $a \leftrightarrow b$;
(4) the case of $a_+ = 0$ ($b_+ = 0$) is identical to case 2 (case 3);
(5) the case of $a_- = 0$ ($b_- = 0$) is included in case 1 or 2 (case 1 or 3) as an additional requirement.

The case of 5 for NH is not further discussed because it does not supply any useful linear relations with respect to $a_{1,+,-}$ and $b_{1,+,-}$ and the cases of 4 and 5 for IH are irrelevant.

III. MAXIMAL *CP* VIOLATION

In this section, we find appropriate conditions on $a_{1,2,3}$ and $b_{1,2,3}$, which are similar to Eqs. (3) and (4), to induce maximal *CP* violation. From the discussions in Sec. II, we find several such candidates in both NH and IH. We choose the phase to be $e^{-i\delta_{CP}}$ appearing in the equations in much the same way as in Eq. (2). The results are summarized in Table I for NH and Table II for IH that show which Dirac neutrino masses are real or imaginary.

TABLE I. Constraints for NH to induce maximal *CP* violation.

Case	Relevant constraint for $\delta_{CP} = \pm\pi/2$	Real	Imaginary
1	$a_-e^{-i\delta_{CP}} = -s_{13}a_+/t_{12}$	a_-	a_+
2	$b_-e^{-i\delta_{CP}} = -s_{13}b_+/t_{12}$	b_-	b_+
3	$a_1e^{-i\delta_{CP}} = t_{13}a_+$, $b_+ = -t_{13}b_1e^{-i\delta_{CP}}$	a_1, b_1	a_+, b_+
4	$b_1e^{-i\delta_{CP}} = t_{13}b_+$, $a_+ = -t_{13}a_1e^{-i\delta_{CP}}$		

TABLE II. Constraints for IH to induce maximal *CP* violation.

Case	Relevant constraint for $\delta_{CP} = \pm\pi/2$	Real	Imaginary
1	$a_+ = -t_{13}a_1e^{-i\delta_{CP}}$, $b_+ = -t_{13}b_1e^{-i\delta_{CP}}$	a_1, b_1	a_+, b_+
2	$b_+ = -t_{13}b_1e^{-i\delta_{CP}}$	b_1	b_+
3	$a_+ = -t_{13}a_1e^{-i\delta_{CP}}$	a_1	a_+

In these tables, the real or imaginary Dirac neutrino masses give maximal CP violation through the relevant constraint(s).

If the atmospheric neutrino mixing is maximal as well, a_+ and a_- turn out to be $a_+ = (\sigma a_2 + a_3)/\sqrt{2}$ and $a_- = \sigma(\sigma a_2 - a_3)/\sqrt{2}$. Therefore, it can be observed that the relation of $a_3 = -\sigma a_2{}^*$ as long as $a_+ \neq 0$ and $a_- \neq 0$ ensures the appearance of the imaginary a_+ in all focused cases requiring a_- to be real and similarly for $b_{+,-}$. This constraint on $a_{2,3}$ (or $b_{2,3}$) is equivalent to Eqs. (3) and (5). In terms of $a_{1,+,-}$ and $b_{1,+,-}$, Eqs. (3) and (5) can be expressed as $\mathrm{Re}(a_+a_- + b_+b_-) = 0$ and $\mathrm{Im}(a_1a_- + b_1b_-) = 0$.

So far, we have assumed that one of $a_{1,+,-}$ and $b_{1,+,-}$ vanishes but a more general conclusion can be obtained without making any assumptions. It is known that the relations of $M_{e\tau} = -\sigma M^*_{e\mu}$ and $M_{\tau\tau} = M^*_{\mu\mu}$ supplemented by $M_{ee,\mu\tau} = \text{real}$ lead to maximal CP violation as well as maximal atmospheric neutrino mixing [17,19]. In our point of view, it is understood that these relations serve as specific solutions to Eqs. (3) and (5) [12]. In terms of $a_{1,+,-}$ and $b_{1,+,-}$, the solution consists of $a_3 = -\sigma a_2{}^*$ and $b_3 = -\sigma b_2{}^*$ supplemented by $a_1 = \text{real}$ and $b_1 = \text{real}$. The Dirac neutrino masses are uniquely determined to be

$$m_D = \begin{pmatrix} \sqrt{M_1}a_1 & \sqrt{M_2}b_1 \\ \sqrt{M_1}a_2 & \sqrt{M_2}b_2 \\ \sqrt{M_1}(-\sigma a_2^*) & \sqrt{M_2}(-\sigma b_2^*) \end{pmatrix}, \tag{23}$$

where a_1 and b_1 are real. As in Ref. [19,20], if a unitary matrix S is defined to be

$$S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -\sigma \\ 0 & -\sigma & 0 \end{pmatrix}, \tag{24}$$

on the $(\nu_e, \nu_\mu, \nu_\tau)$ basis, which provides $\mu-\tau$ flavored CP symmetry for the flavor neutrinos [20] subjected to the interchange of ν_μ and ν_τ, it is found that m_D of Eq. (23) satisfies that $S^T m_D = m_D^*$ as expected.

IV. SUMMARY AND DISCUSSIONS

We are able to derive the useful and simple relations to induce maximal CP violation, which dictate that $a_+ = \text{imaginary}$ and $a_1 = \text{real}$ and/or $b_+v = \text{imaginary}$ and $b_1 = \text{real}$ for both NH and IH. For NH, either $a_+ = \text{imaginary}$ and $a_- = \text{real}$ or $b_+ = \text{imaginary}$ and $b_- = \text{real}$ also arises. These relations are limited to hold in specific textures where at least one of $a_{1,+,-}$ and $b_{1,+,-}$ vanishes. If the atmospheric neutrino mixing is also maximal, we have obtained $a_3 = -\sigma a_2{}^*$, $b_3 = -\sigma b_2{}^*$, $a_1 = \text{real}$ and $b_1 = \text{real}$ applicable to more general textures. These relations turn out to be equivalent to the familiar relations of $M_{e\tau} = -\sigma M^*_{e\mu}$, $M_{\tau\tau} = M^*_{\mu\mu}$, $M_{ee} = \text{real}$ and $M_{\mu\tau} = \text{real}$ for flavor neutrinos.

Our findings about various relations among the Dirac masses giving the maximal CP violation for flavor neutrinos become useful when neutrino physics is affected by phases of the Dirac masses. The immediate such example is to apply our method to the process of the creation of the baryon number of the Universe via the leptogenesis. In fact, the result indicates that the CP-violating Majorana phases for the leptogenesis come from $a_{2,3}$ and $b_{2,3}$ [see Eq. (29)] although there is no Majorana CP violation for flavor neutrinos if the above relations are satisfied.

To see how the baryon-photon ratio in the Universe via the leptogenesis scenario can be predicted by the use of our requirement on the Dirac neutrino masses for the maximal CP violation, we provide a preliminary result as a viable example. First of all, we summarize the recipes, which are known as follows [21,22].

(i) The CP asymmetry parameters from the decay of the lightest right-handed neutrino N_1 (we assume $M_1 \ll M_2$) is obtained from

$$\epsilon_i = -\frac{3M_1}{16\pi v^2}\frac{\mathrm{Im}[a_i^* b_i(a_1^*b_1 + a_2^*b_2 + a_3^*b_3)]}{|a_1|^2 + |a_2|^2 + |a_3|^2}, \tag{25}$$

where $i = e, \mu, \tau = 1, 2, 3$ and $v \simeq 174$ GeV.

(ii) The baryon number in the comoving volume is calculated to be

$$Y_B \simeq -\frac{12}{37g_*}\left[(\epsilon_e + \epsilon_\mu)\eta\left(\frac{417}{589}(|a_1|^2 + |a_2|^2)\right) + \epsilon_\tau\eta\left(\frac{390}{589}|a_3|^2\right)\right], \tag{26}$$

for $10^9 \leq M_1[\mathrm{GeV}] \leq 10^{12}$ where washout effect on ϵ_i in the expanding Universe is controlled by

$$\eta(x) = \left(\frac{8.25\times10^{-3}\ \mathrm{eV}}{x} + \left(\frac{x}{2\times10^{-4}\ \mathrm{eV}}\right)^{1.16}\right)^{-1}, \tag{27}$$

and g_* denotes the effective number of relativistic degree of freedom. We take $g_* = 106.75$.

(iii) The baryon-photon ratio η_B is estimated to be $\eta_B = 7.04 Y_B$.

Next, we estimate the baryon-photon ratio by assuming the maximal CP violation and the maximal atmospheric neutrino mixing in the neutrino sector: e.g., $a_1 = \text{real}$, $b_1 = \text{real}$, $a_3 = -\sigma a_2^*$ and $b_3 = -\sigma b_2^*$. In this case, there are only two independent phases $\arg(a_2)$ and $\arg(b_2)$. The CP asymmetry parameter ϵ_i is obtained as

$$\epsilon_e = 0,$$
$$\epsilon_\mu = -\frac{3M_1}{16\pi v^2}\frac{(|a_1||b_1| + 2\mathrm{Re}[a_2^* b_2])\mathrm{Im}[a_2^* b_2]}{|a_1|^2 + 2|a_2|^2} = -\frac{3M_1}{16\pi v^2}\frac{(|a_1||b_1| + 2|a_2||b_2|\cos\Delta)|a_2||b_2|\sin\Delta}{|a_1|^2 + 2|a_2|^2},$$
$$\epsilon_\tau = -\epsilon_\mu, \quad (28)$$

where

$$\Delta = \arg(b_2) - \arg(a_2). \quad (29)$$

From Eq. (28), as we expected, the phase difference Δ has a crucial role in the baryon asymmetry generation in the Universe and $\Delta \neq n\pi (n = 0, \pm 1, \pm 2 \cdots)$ is required for nonvanishing baryon-photon ratio.

To confirm results of our discussions more concretely, we estimate the CP asymmetry parameters shown in Eq. (28) with the horizontal equality in the Dirac mass matrix [23]. There are the following three cases of the horizontal equality for elements denoted by X:

$$\mathrm{I:}\begin{pmatrix} X & X \\ * & * \\ * & * \end{pmatrix}, \quad \mathrm{II:}\begin{pmatrix} * & * \\ X & X \\ * & * \end{pmatrix}, \quad \mathrm{III:}\begin{pmatrix} * & * \\ * & * \\ X & X \end{pmatrix}, \quad (30)$$

where the mark "$*$" denotes a nonvanishing element. The vertical equality is also discussed [24]. In case II and case III, we obtain $\Delta = 0$. Case I only survives for the maximal CP violation as well as the maximal atmospheric neutrino mixing for nonvanishing baryon-photon ratio. The phenomenological consequences with the horizontal equality have been obtained by numerical calculations. In this paper, we show the clear constraint on the models with horizontal equality by exact analytical expressions. This is an advantage of our research.

We show a numerical example of the baryon-photon ratio in case I of the horizontal equality requiring $\sqrt{M_1}a_1 = \sqrt{M_2}b_1$ for the maximal CP violation and the maximal atmospheric neutrino mixing. The effective mass of the neutrinoless double beta decay is estimated as $M_{ee} = (1 + M_1/M_2)a_1^2$. For the sake of simplicity, we assume $|a_2| = |b_2|$ and $\Delta = \pi/2$. The CP asymmetry parameter ϵ_μ is

$$\epsilon_\mu = -\frac{3M_1}{16\pi v^2}\sqrt{\frac{M_1}{M_2}}\frac{|M_{ee}||a_2|^2}{|M_{ee}| + 2(1 + M_1/M_2)|a_2|^2}, \quad (31)$$

and we obtain

$$\eta_B = 6.1 \times 10^{-10}, \quad (32)$$

for $M_1 = 9.7 \times 10^{11}$ GeV, $M_2 = 100 M_1$, $|M_{ee}| = 0.069$ eV and $|a_2| = 0.063$ eV, which is consistent with the observed value of $\eta_B = (6.02\text{–}6.18) \times 10^{-10}$ [25]. More general analysis is found elsewhere [26].

APPENDIX: USEFUL CONSTRAINTS

In this appendix, we describe how to obtain various constraints on $a_{1,+,-}$ and $b_{1,+,-}$ as solutions to the equations, Eqs. (13)–(16) for NH and Eqs. (18)–(21) for IH. We use constraints on $a_{1,+,-}$ as initial conditions to find our solutions, which can be transformed into other solutions based on those on $b_{1,+,-}$ by the interchange of $a \leftrightarrow b$. The initial setup for $a_{1,+,-}$, where one combination of Dirac neutrino masses vanishes, turns out to be given by $a_1 = 0$, $a_+ = 0$, $a_- = 0$, or $a_+ + t_{13}a_1 e^{-i\delta_{CP}} = 0$. For NH,

(1) $a_1 = 0$: From Eq. (13), $a_-^2 = -(b_- - \frac{1}{t_{12}c_{13}}b_1)b_-$ is required to have $m_1 = 0$. From Eq. (14) for θ_{23} and Eq. (15) for θ_{12}, we, respectively, obtain $a_+ a_- = -(b_+ + t_{13}e^{-i\delta_{CP}}b_1)b_-$ and $b_1 = \frac{t_{12}}{c_{13}}b_- + t_{13}b_+ e^{i\delta_{CP}}$, which turn out to satisfy (16) for θ_{13}. Inserting the expression of b_1 into those of a_-^2 and $a_+ a_-$, finally, gives a simpler relation $a_- = -\frac{s_{13}}{t_{12}}a_+ e^{i\delta_{CP}}$. We obtain that $b_1 = \frac{t_{12}}{c_{13}}b_- + t_{13}b_+ e^{i\delta_{CP}}$ and $a_- = -\frac{s_{13}}{t_{12}}a_+ e^{i\delta_{CP}}$ as useful relations together with $a_- = 0$.

(2) $a_+ = 0$: It is readily recognized that no simple linear relations are deduced from the equations and $a_{1,-}$ and $b_{1,+,-}$ should satisfy $a_-^2 + b_-^2 = \frac{1}{t_{12}c_{13}}(a_1 a_- + b_1 b_-)$ from Eq. (13), $b_+ b_- = -t_{13}e^{-i\delta_{CP}}(a_1 a_- + b_1 b_-)$ from Eq. (14), $a_1^2 + b_1^2 - t_{12}^2(a_-^2 + b_-^2) = t_{13}b_1 b_+ e^{i\delta_{CP}}$ from Eq. (15) and $c_{13}^2(a_1^2 + b_1^2) - s_{13}^2 e^{2i\delta_{CP}}b_+^2 = (c_{13}^2 - s_{13}^2)t_{12}^2(a_-^2 + b_-^2)$ from Eq. (16).

(3) $a_- = 0$: From Eq. (13), $b_- = \frac{1}{t_{12}c_{13}}b_1$ is required to have $m_1 = 0$. From Eqs. (14) and (15), we, respectively, obtain $b_+ = -t_{13}b_1 e^{-i\delta_{CP}}$ and $a_1 = t_{13}a_+ e^{i\delta_{CP}}$, which turn out to satisfy (16) for θ_{13}. We obtain that $b_+ + t_{13}b_1 e^{-i\delta_{CP}} = 0$, $b_- = \frac{1}{t_{12}c_{13}}b_1$ and $a_1 = t_{13}a_+ e^{i\delta_{CP}}$ together with $a_1 = 0$.

For IH, the combined use of Eqs. (18) and (21) yields $a_1 a_+ + b_1 b_+ = -t_{13}(a_1^2 + b_1^2)e^{-i\delta_{CP}}$ for Eq. (21) giving

$$(a_+ + t_{13}a_1 e^{-i\delta_{CP}})a_1 + (b_+ + t_{13}b_1 e^{-i\delta_{CP}})b_1 = 0, \quad (A1)$$

by which Eq. (18) is further reduced to

$$(a_+ + t_{13}a_1 e^{-i\delta_{CP}})a_+ + (b_+ + t_{13}b_1 e^{-i\delta_{CP}})b_+ = 0. \quad (A2)$$

Similarly, Eq. (19) leads to

$$(a_+ + t_{13}a_1 e^{-i\delta_{CP}})a_- + (b_+ + t_{13}b_1 e^{-i\delta_{CP}})b_- = 0. \quad (A3)$$

Considering Eqs. (A1)–(A3), we find the following cases:

(1) $\frac{b_+ + t_{13}b_1 e^{-i\delta_{CP}}}{a_+ + t_{13}a_1 e^{-i\delta_{CP}}} = -\frac{a_1}{b_1} = -\frac{a_+}{b_+} = -\frac{a_-}{b_-}$ for $a_+ + t_{13}a_1 e^{-i\delta_{CP}} \neq 0$, $b_+ + t_{13}b_1 e^{-i\delta_{CP}} \neq 0$ as well as

$a_{1,+,-} \neq 0$ and $b_{1,+,-} \neq 0$: Eq. (A3) with $a_+ = b_+ a_1/b_1$ yields either $b_+ + t_{13} b_1 e^{-i\delta_{CP}} = 0$, which is not allowed by the initial conditions, or $a_1 a_- + b_1 b_- = 0$ giving $|m_1| = |m_2|$ from Eq. (22), which contradicts the fact that $|m_1| < |m_2|$. This case cannot provide a solution.

(2) $a_+ + t_{13} a_1 e^{-i\delta_{CP}} = 0$: It is readily found that $b_+ + t_{13} b_1 e^{-i\delta_{CP}} = 0$ is the solution. We obtain that $a_+ + t_{13} a_1 e^{-i\delta_{CP}} = 0$ and $b_+ + t_{13} b_1 e^{-i\delta_{CP}} = 0$.

(3) $a_1 = 0$: $(b_+ + t_{13} b_1 e^{-i\delta_{CP}}) b_1 = 0$ is required and $b_+ + t_{13} b_1 e^{-i\delta_{CP}} = 0$ is the solution because $b_1 = 0$ gives $|m_1| = |m_2|$. The remaining conditions from Eqs. (A1)–(A3) are fulfilled by $a_+ = 0$. For Eq. (20), $\sin 2\theta_{12}(b_1^2 - c_{13}^2(a_-^2 + b_-^2)) = -2\cos 2\theta_{12} c_{13} b_1 b_-$ should be satisfied. We obtain that $b_+ + t_{13} b_1 e^{-i\delta_{CP}} = 0$ and $a_1 = a_+ = 0$.

(4) $a_+ = 0$: Either $b_+ + t_{13} b_1 e^{-i\delta_{CP}} = 0$ or $b_+ = 0$ is the solution. If $b_+ = 0$, Eq. (A3) yields $a_1 a_- + b_1 b_- = 0$, which results in $|m_1| = |m_2|$ from Eq. (22). For $b_+ + t_{13} b_1 e^{-i\delta_{CP}} = 0$, $a_1 = 0$ is derived. We obtain that $b_+ + t_{13} b_1 e^{-i\delta_{CP}} = 0$ and $a_1 = a_+ = 0$.

(5) $a_- = 0$: $(b_+ + t_{13} b_1 e^{-i\delta_{CP}}) b_- = 0$ is required and $b_+ + t_{13} b_1 e^{-i\delta_{CP}} = 0$ is the solution because $b_- = 0$ gives $|m_1| = |m_2|$ from Eq. (22). The remaining conditions are fulfilled by either $a_+ + t_{13} a_1 e^{-i\delta_{CP}} = 0$ or $a_1 = a_+ = 0$. We obtain that $a_+ + t_{13} a_1 e^{-i\delta_{CP}} = 0$, $b_+ + t_{13} b_1 e^{-i\delta_{CP}} = 0$ and $a_- = 0$ or that $b_+ + t_{13} b_1 e^{-i\delta_{CP}} = 0$ and $a_1 = a_+ = a_- = 0$.

All of the cases for IH are not independent. For instance, case 5 is included in case 2 or in case 3 both with the additional condition of $a_- = 0$.

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