

# Yukawa sector for lepton flavor violating in $h \rightarrow \mu\tau$ and $CP$ violation in $h \rightarrow \tau\tau$

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The Higgs boson discovered at the LHC opened a new chapter for particle physics. Its properties need to be studied in detail to distinguish a purely standard model (SM) Higgs boson from one of many scalars in an enlarged Higgs sector. The CMS Collaboration has reported a possible signal for lepton flavor violation in  $h \rightarrow \mu\tau$ , which if confirmed, implies that the Higgs sector is larger than in the SM. New physics responsible for this type of decay may, in general, also introduce other observable effects such as charge-parity ( $CP$ ) violation in  $h \rightarrow \tau\tau$ . We study two types of models that single out the third generation and can induce large  $h \rightarrow \mu\tau$  rates with different consequences for  $CP$  violation in  $h \rightarrow \tau\tau$ . Predictions for the size of the  $CP$  violating couplings require knowledge of the lepton Yukawa matrices and we discuss this in the context of two different textures considering all existing constraints.

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## I. INTRODUCTION

The Higgs boson discovered at the LHC has opened a new chapter for particle physics. During the current phase of LHC running, the Higgs couplings need to be studied precisely in order to distinguish a standard model (SM) Higgs boson from a scalar forming part of an enlarged Higgs sector (beyond the SM). Yukawa interactions provide channels to probe Higgs properties in a very direct way. In this work we concentrate on Higgs boson decays into a charged lepton pair, such as  $\tau$  and  $\mu$ , which can be studied at the LHC or future colliders such as FCC, ILC and CEPC. Within the SM the lepton couplings to the Higgs boson are uniquely determined by their mass, the Yukawa Lagrangian being given by

$$\mathcal{L}_Y = -y_{ij}\bar{\ell}_{Li}e_{Rj}\phi + \text{H.c.} \quad (1)$$

Here  $\ell_{Li}$  is the left-handed SM lepton doublet,  $e_{Rj}$  the right-handed lepton singlet,  $\phi$  is the scalar Higgs doublet and  $i, j = 1, 2, 3$  are generation indices. The fields  $\ell_{Li}$ ,  $e_{Ri}$  and  $\phi$  transform under the SM gauge group  $SU(3)_C \times SU(2)_L \times U(1)_Y$  as  $(1, 2, -1)$ ,  $(1, 1, -2)$  and  $(1, 2, 1)$ , respectively.

The leptons acquire a mass when electroweak symmetry is broken and the Higgs field develops a vacuum

expectation value (vev)  $\langle\phi\rangle = v/\sqrt{2}$ ,  $v \approx 246$  GeV, in which case Eq. (1) becomes

$$\mathcal{L}_Y = -\left(1 + \frac{h}{v}\right)\frac{y_{ij}v}{\sqrt{2}}\bar{\ell}_{Li}e_{Rj} + \text{H.c.} \quad (2)$$

The Yukawa interaction in the lepton mass eigenstate basis is obtained from Eq. (2) with a biunitary transformation  $(S_e^\dagger(vy/\sqrt{2})T_e)_{ij} = m_i\delta_{ij}$ . In this basis the Higgs-lepton couplings, given by  $g_{h\ell_i\ell_j} = m_i\delta_{ij}/v$ , are proportional to the lepton masses, flavor diagonal, real and therefore  $CP$  conserving.

The CMS Collaboration has reported a possible signal for lepton flavor violation (LFV) in the process  $h \rightarrow \mu\tau$  ( $\mu\tau = \mu\bar{\tau} + \tau\bar{\mu}$ ). If confirmed, this implies that the Higgs sector must have new flavor changing neutral current (FCNC) interactions, not present in the SM. LFV in Higgs decays has been discussed recently using the effective Lagrangian [1] framework by a number of authors [2–8], in the context of two Higgs doublet models [9,10], and in models in which it occurs at one-loop [8,11]. When new physics introduces a flavor violating coupling of the form  $h\bar{e}_ie_j$ , in general, it also brings in a  $CP$  violating component into the flavor diagonal sector. These are parametrized as

$$g_{he_i e_i} = -\frac{h}{v}m_i\bar{e}_i(r_{e_i} + i\tilde{r}_{e_i}\gamma_5)e_i, \quad (3)$$

and to separate the SM contribution, it is usual to write  $r_{e_i} = 1 + \epsilon_{e_i}$ . It is well known that the simultaneous

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existence of scalar and pseudoscalar couplings in Eq. (3) induces a  $CP$  violating spin-spin correlation that can in principle be measured for tau leptons and muons, which have weak decays that analyze their polarization [12]. In the standard treatment of this problem, one can define a density matrix  $R$  for the production of polarized tau leptons with polarization described by a unit polarization vector  $\mathbf{n}_{\tau(\bar{\tau})}$  in the  $\tau(\bar{\tau})$ -rest frame. With the amplitude in Eq. (3) the  $CP$  violating part of the density matrix is given by

$$R_{CP} = -N\beta_{\tau}\text{Re}(r_{\tau}\bar{r}_{\tau})\vec{p}_{\tau} \cdot (\mathbf{n}_{\tau} \times \mathbf{n}_{\bar{\tau}}), \quad (4)$$

where  $N$  is a normalization constant,  $\vec{p}_{\tau}$  is the three-momentum direction of the tau lepton, and  $\beta_{\tau} = \sqrt{1 - 4m_{\tau}^2/m_h^2}$ .

The existence of  $CP$  violation in the Higgs interaction may have far reaching implications for why our Universe is dominated by matter over antimatter [baryon asymmetric universe (BAU)]. In the SM  $CP$  violation resides only in the charged current interaction of  $W$  bosons with fermions and is known to be too small to solve the BAU problem. Searches for  $CP$  violation in Higgs interactions are therefore an important topic in particle physics even if the mechanism by which new physics gives rise to LFV and CP violation (CPV) interactions of Higgs with fermions is not understood.

In a recent paper we have discussed the connection between LFV and  $CP$  violation in a model independent manner using the language of effective Lagrangians [6]. In this paper we extend that work as follows. In Sec. II we present explicit models illustrating the ingredients required for CPV to occur in models with LFV. In Sec. III we review existing constraints emphasizing the importance of possible textures in the *corrections* to the Yukawa couplings. We have considered two cases: democratic and hierarchical corrections which lead to substantially different interpretations of the current experimental constraints. In Sec. IV we tabulate  $CP$ -odd observables sensitive to these SM extensions that are suitable for future linear colliders and use our constraints to estimate the allowed size of these asymmetries. Finally in Sec. V we present our conclusions.

## II. MODELS WITH FCNC AND $CP$ VIOLATION

### IN $h \rightarrow \ell_i \bar{\ell}_i$

In the SM, when diagonalizing the mass terms, the Yukawa couplings are also diagonalized, so there are no FCNC nor  $CP$  violation. However, the existence of the flavor changing couplings  $h \rightarrow \ell_i \bar{\ell}_j$  does not necessarily imply  $CP$  violating couplings as well. In more complicated models where the Yukawa couplings have off-diagonal entries which allow  $h \rightarrow \mu\tau$  to occur, the diagonal entries may still be real implying no  $CP$  violation of the type in Eq. (3). A simple way to obtain a  $CP$  violating interaction of the type in Eq. (3) is to mix the scalar and pseudoscalar components in the Higgs potential via spontaneous or

explicit  $CP$  violation [13]. Conversely, it is also possible to have  $CP$  violation without FCNC in multi-Higgs doublet models, like the Weinberg model of spontaneous  $CP$  violation [14] which cannot induce  $h \rightarrow \mu\tau$ . Type-III two Higgs doublet models [15], on the other hand, are able to accommodate both effects.

We now discuss two models that are both motivated by treating the third generation differently from the first two generations to reduce the hierarchy problem in the Yukawa sector. We first look at the  $SU(2)_l \times SU(2)_h \times U(1)_Y$  model known as ‘‘top-flavor’’ which provides a concrete example where the simplest scalar sector generates flavor changing couplings  $h \rightarrow \ell_i \bar{\ell}_j$  but no  $CP$  violation. Next we consider the nonuniversal left-right model  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  which generates both LFV and CPV Higgs couplings with the simplest scalar sector.

### A. The $SU(2)_l \times SU(2)_h \times U(1)_Y$ model

The  $SU(2)_l \times SU(2)_h \times U(1)_Y$  model treats the first two and the third generations differently, by assuming that the usual  $SU(2)_L$  for the first two generations is replaced by  $SU(2)_l$  and for the third generation it is replaced by  $SU(2)_h$ . The left-handed quark doublets  $Q_L$ , the right-handed quark singlets  $U_R$  and  $D_R$ , the left-handed lepton doublets  $L_L$ , and the right-handed charged leptons  $E_R$  transform under the gauge group as

$$\begin{aligned} Q_L^{1,2} &: (3, 2, 1, 1/3), & Q_L^3 &: (3, 1, 2, 1/3), \\ U_R^{1,2,3} &: (3, 1, 1, 4/3), & D_R^{1,2,3} &: (3, 1, 1, -2/3), \\ L_L^{1,2} &: (1, 2, 1, -1), & L_L^3 &: (1, 1, 2, -1), \\ E_R^{1,2,3} &: (1, 1, 1, -2), \end{aligned} \quad (5)$$

where the numbers in each bracket are the quantum numbers of the corresponding field under  $SU(3)_C$ ,  $SU(2)_l$ ,  $SU(2)_h$  and  $U(1)_Y$ , respectively. The superscript on each field labels the generation of the fermion. The model and most of its associated phenomenology have been described in the literature before [16–19], here we concentrate on the scalar sector which will be responsible for the effects we want.

Symmetry breaking of  $SU(2)_l \times SU(2)_h$  down to the usual  $SU(2)_L$  is achieved by the vev  $u$ , of order  $\mathcal{O}(\text{TeV})$ , of a bi-doublet scalar  $\eta: (1, 2, 2, 0)$ . The fermion masses are provided by the subsequent symmetry breaking achieved by two Higgs doublets  $\Phi_1: (1, 2, 1, 1)$  and  $\Phi_2: (1, 1, 2, 1)$  with respective vevs  $v_{1,2}$  such that  $v_1^2 + v_2^2 = v^2$ .  $\Phi_1$  and  $\Phi_2$  only couple to the first two and the third left-handed fermions, respectively. In general this extension of the SM produces FCNC at tree level by exchanging physical neutral Higgs scalars. It also produces FCNC due to the exchange of  $Z$  and  $Z'$  as discussed in the literature but this effect will not concern us here. The Yukawa Lagrangian, including leptons, is given by

$$\begin{aligned}\mathcal{L}_Y &= f_{ij}^u \bar{u}_{iR} \tilde{\Phi}_1^\dagger Q_{jL} + g_{i3}^u \bar{u}_{iR} \tilde{\Phi}_2^\dagger Q_{3L} + f_{ij}^d \bar{d}_{iR} \Phi_1^\dagger Q_{jL} \\ &+ g_{i3}^d \bar{d}_{iR} \Phi_2^\dagger Q_{3L} + f_{ij}^e \bar{E}_{iR} \Phi_1^\dagger L_{jL} + g_{i3}^e \bar{E}_{iR} \Phi_2^\dagger L_{3L} \\ &+ \text{H.c.},\end{aligned}\quad (6)$$

where  $\tilde{\Phi} = i\sigma_2 \Phi$ . In the above,  $j$  takes values of 1 and 2, and  $i$  takes values of 1, 2, and 3. Depending on whether neutrinos are Dirac or Majorana particles, neutrino masses can be generated by introducing right-handed neutrinos  $\nu_R$  to give neutrino Dirac masses. If one also allows  $\nu_R$  to have a Majorana mass, then the type-I seesaw mechanism is used to give neutrino masses. Since  $\Phi_1$  and  $\Phi_2$  give masses to the first two and the third generations,  $v_1$  should be much smaller than  $v_2$  so that the hierarchy in Yukawa couplings can be reduced.

It is convenient to work in a rotated basis for the scalar doublets  $\Psi_{1,2}$  where only one Higgs boson develops a nonzero vev. With  $\tan\beta = v_1/v_2$ , this is

$$\begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}.\quad (7)$$

In this basis, we have

$$\Psi_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v+h+iG^0) \end{pmatrix}, \quad \Psi_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(H^0+iA^0) \end{pmatrix},\quad (8)$$

where  $G^+$  and  $G^0$  are the Goldstone bosons.

One can write the neutral Higgs boson couplings to charged leptons as

$$\mathcal{L}_Y = -\bar{e}_L \left( M^e \left( 1 + \frac{h}{v} \right) + (\lambda_1^e - \lambda_2^e)(H^0 - iA^0) \right) e_R + \text{H.c.}$$

where

$$\begin{aligned}M^e &= \frac{1}{\sqrt{2}}(v_1\lambda_1^e + v_2\lambda_2^e) = \frac{v}{\sqrt{2}}(s_\beta\lambda_1^e + c_\beta\lambda_2^e), \\ \lambda_1^e &= \begin{pmatrix} f_{11}^{e*} & f_{21}^{e*} & f_{31}^{e*} \\ f_{12}^{e*} & f_{22}^{e*} & f_{32}^{e*} \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2^e = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ g_{13}^{e*} & g_{23}^{e*} & g_{33}^{e*} \end{pmatrix}.\end{aligned}\quad (9)$$

Note that the structure of the model with two vevs, of which  $v_1$  enters the first two diagonal elements of  $\lambda_{1,2}^e$  and  $v_2$  enters the third one allows one to significantly reduce the hierarchy in  $f_{11}^e, f_{22}^e$  and  $g_{33}^e$  as compared to the SM case by selecting  $v_2 \gg v_1$ . However, since  $v_1$  contributes to both the first and the second generation masses, a (reduced) hierarchical structure in  $f_{ij}^e$  and  $g_{ij}^e$  is still needed.

The Yukawa Lagrangian in the fermion mass eigenstate basis becomes

$$\mathcal{L}_Y = -\bar{e}_L \left( \hat{M}^e \left( 1 + \frac{h}{v} \right) + \lambda^e (H^0 - iA^0) \right) e_R + \text{H.c.}\quad (10)$$

where  $M^e = S_e \hat{M}^e T_e^\dagger$  with  $S_e$  and  $T_e$  being unitary matrices and  $\hat{M}^e$  the lepton mass eigenstate matrix.  $\lambda^e$  is given by

$$\begin{aligned}\lambda^e &= S_e^\dagger (\lambda_1^e - \lambda_2^e) T_e \\ &= -\frac{\sqrt{2}}{vc_\beta} \hat{M}^e + \left( 1 + \frac{s_\beta}{c_\beta} \right) S_e^\dagger \lambda_1^e T_e \\ &= \frac{\sqrt{2}}{vs_\beta} \hat{M}^e - \left( 1 + \frac{c_\beta}{s_\beta} \right) S_e^\dagger \lambda_2^e T_e.\end{aligned}\quad (11)$$

The scalar  $h$  is approximately the SM Higgs-like particle, but it is not yet a mass eigenstate of the Higgs potential because in general,  $h$  and  $H$  mix with each other. On the other hand, the Higgs potential for this model is constructed with the fields  $\eta$ ,  $\Phi_1$ , and  $\Phi_2$  which are the only ones needed for symmetry breaking, and does not have mixing between the  $A^0$  and the  $h$  or  $H^0$  states [18]. The scalar mass eigenstates  $h^{m1, m2}$  can then be written in terms of  $h$  and  $H$  with a mixing angle  $\alpha$  as

$$\begin{pmatrix} h \\ H \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} h^{m1} \\ h^{m2} \end{pmatrix}.\quad (12)$$

If we now identify  $h^{m1}$  with the 125 GeV state observed by the LHC collider, the Yukawa coupling between charged leptons and  $h^{m1}$  takes the form

$$\mathcal{L}_{hee} = -\bar{e}_L \left( \frac{\hat{M}^e}{v} \cos\alpha + \lambda^e \sin\alpha \right) e_R h^{m1} + \text{H.c.}\quad (13)$$

Inspecting the above equation, one sees that the 23 and 32 entries are nonzero in general, and thus allow  $h \rightarrow \mu\tau$  to occur. Naively, one may also expect that the 33 entry which contributes to  $h \rightarrow \tau\bar{\tau}$  can be complex indicating a  $CP$  violating coupling of the type in Eq. (3). This is, however, not true. When diagonalizing the mass matrix above, the phase of the 33 entry in  $\lambda^e$  is automatically removed leading to a  $CP$  conserving  $h\tau\bar{\tau}$  coupling. To prove this, it is sufficient to show that in the mass eigenstate basis, the 33 entry of  $(S_e^\dagger \lambda_2^e T_e)_{33}$  is real.

From  $M^e = S_e \hat{M}^e T_e^\dagger$ , we have  $(M^e T_e)_{33} = (S_e \hat{M}^e)_{33}$  which leads to

$$c_\beta \frac{v}{\sqrt{2}} (T_{e13} g_{13}^{e*} + T_{e23} g_{23}^{e*} + T_{e33} g_{33}^{e*}) = m_\tau S_{e33}.\quad (14)$$

At the same time, expanding  $(S_e^\dagger \lambda_2^e T_e)_{33}$ , we obtain

$$\begin{aligned} (S_e^\dagger \lambda_2^e T_e)_{33} &= (T_{e13} g_{13}^{e*} + T_{e23} g_{23}^{e*} + T_{e33} g_{33}^{e*}) S_{e33}^* \\ &= \frac{\sqrt{2}}{vc\beta} m_\tau |S_{e33}|^2. \end{aligned} \quad (15)$$

Since  $m_\tau$  is normalized to be real, so is  $(S_e^\dagger \lambda_2^e T_e)_{33}$ .

To also have  $CP$  violation in  $h \rightarrow \tau\tau$  decay, one needs to modify the Yukawa structure of the model in such a way that the couplings responsible for flavor changing  $h \rightarrow e_i \bar{e}_j$  decays cannot be written in the form given in Eq. (11). This can be achieved by introducing one more Higgs doublet transforming as either  $(1,2,1,1)$  or  $(1,1,2,1)$ . The additional fields introduce additional couplings in the Yukawa and Higgs potentials which allow the mixing of  $A^0$  with  $h$  and  $H^0$ , for example. They can also allow the resulting  $h\tau\tau$  coupling to be complex from the structure of the Yukawa couplings alone. We will not pursue this avenue here, but instead we provide a different model with the latter feature, the nonuniversal left-right model, in the next subsection.

### B. The nonuniversal $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model

The gauge group of the nonuniversal left-right model is  $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ . The quantum numbers for the first two and the third generations are chosen to be different in such a way that right-handed interactions are enhanced for third generation fermions and suppressed for the first two generations. This is motivated by the large top-quark mass, the possible anomalies that have been observed in  $t$ ,  $b$  and  $\tau$  couplings [20–23], and the stringent constraints that exist on the couplings of the lighter fermions. The left-handed quark doublets  $Q_L$ , the right-handed quark singlets  $U_R$  and  $D_R$ , the left-handed lepton doublets  $L_L$ , and the right-handed charged leptons  $E_R$  transform under the original gauge group as

$$\begin{aligned} Q_L^{1,2,3} &: (3, 2, 1, 1/3), & Q_R^3 &: (3, 1, 2, 1/3), \\ U_R^{1,2} &: (3, 1, 1, 4/3), & D_R^{1,2} &: (3, 1, 1, -2/3), \\ L_L^{1,2,3} &: (1, 2, 1, -1), & L_R^3 &: (1, 1, 2, -1), \\ E_R^{1,2} &: (1, 1, 1, -2), & \nu_R^{1,2} &: (1, 1, 1, 0). \end{aligned} \quad (16)$$

The model and many aspects of its phenomenology have been discussed before in the literature [24–28]. Here we concentrate on the relevant scalar-lepton interactions. There are three scalar fields affecting Yukawa couplings which we list below together with their transformation properties under the gauge group,

$$\begin{aligned} H_L &= \begin{pmatrix} \frac{1}{\sqrt{2}}(v_L + h_L + iA_L) \\ h_L^- \end{pmatrix} : (1, 2, 1, -1), \\ H_R &= \begin{pmatrix} \frac{1}{\sqrt{2}}(v_R + h_R + iA_R) \\ h_R^- \end{pmatrix} : (1, 1, 2, -1), \\ \phi &= \begin{pmatrix} \frac{1}{\sqrt{2}}(v_1 + h_1 + ia_1) & h_2^+ \\ h_1^- & \frac{1}{\sqrt{2}}(v_2 + h_2 + ia_2) \end{pmatrix} \\ &: (1, 2, 2, 0). \end{aligned} \quad (17)$$

The Yukawa couplings that can be constructed with these fields are

$$\begin{aligned} \mathcal{L}_Y &= -(\bar{Q}_L^{1,2,3} \lambda_L^u H_L U_R^{1,2} + \bar{Q}_L^{1,2,3} \lambda_L^d \tilde{H}_L D_R^{1,2} \\ &\quad + \bar{Q}_L^{1,2,3} (\lambda^q \phi + \tilde{\lambda}^q \tilde{\phi}) Q_R^3) + \\ &\quad - (\bar{L}_L^{1,2,3} \lambda_L^e H_L \nu_R^{1,2} + \bar{L}_L^{1,2,3} \lambda_L^e \tilde{H}_L E_R^{1,2} \\ &\quad + \bar{L}_L^{1,2,3} (\lambda^l \phi + \tilde{\lambda}^l \tilde{\phi}) L_R^3) + \text{H.c.}, \end{aligned} \quad (18)$$

where  $\tilde{H}_L = -i\sigma_2 H_L^*$  and  $\tilde{\phi} = \sigma_2 \phi^* \sigma_2$ .

As in the previous example, the Higgs potential in this model does not allow mixing between the scalars and pseudoscalars, therefore the 125 GeV Higgs boson will be a linear combination of  $h_L$ ,  $h_1$  and  $h_2$ . To find the Yukawa coupling of the 125 GeV Higgs boson to the charged leptons, one needs to understand how  $h_{L,1,2}$  couple to the charged leptons in the basis where the neutrino mass matrix has been diagonalized. One can write the lepton Yukawa couplings as follows:

$$\begin{aligned} \mathcal{L}_Y &= -\frac{1}{\sqrt{2}} \bar{e}_L [\lambda_L^e (v_L + h_L) + \tilde{\lambda}^l (v_1 + h_1) \\ &\quad + \lambda^l (v_2 + h_2)] e_R + \text{H.c.}, \end{aligned} \quad (19)$$

and from this read the charged lepton mass matrix,

$$\begin{aligned} M^e &= \frac{1}{\sqrt{2}} (\lambda_L^e v_L + \tilde{\lambda}^l v_1 + \lambda^l v_2), \\ \lambda_L^e &= \begin{pmatrix} f_{11}^l & f_{12}^l & 0 \\ f_{21}^l & f_{22}^l & 0 \\ f_{31}^l & f_{32}^l & 0 \end{pmatrix}, & \tilde{\lambda}^l &= \begin{pmatrix} 0 & 0 & \tilde{g}_{13}^l \\ 0 & 0 & \tilde{g}_{23}^l \\ 0 & 0 & \tilde{g}_{33}^l \end{pmatrix}, \\ \lambda^l &= \begin{pmatrix} 0 & 0 & g_{13}^l \\ 0 & 0 & g_{23}^l \\ 0 & 0 & g_{33}^l \end{pmatrix}. \end{aligned} \quad (20)$$

It is convenient to work in a basis where only one Higgs has nonzero vev  $v = (v_L^2 + v_1^2 + v_2^2)^{1/2}$ . To do so we define



$$\begin{pmatrix} h_L \\ h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} v_L/v & 0 & v'/v \\ v_1/v & v_2/v' & -v_L v_1/v'v \\ v_2/v & -v_1/v' & -v_L v_2/v'v \end{pmatrix} \begin{pmatrix} \tilde{h} \\ H_1 \\ H_2 \end{pmatrix}, \quad (21)$$

where  $v' = (v_1^2 + v_2^2)^{1/2}$ .

Assuming  $S_e$  and  $T_e$  diagonalize the charged lepton mass matrix,  $S_e^\dagger M^e T_e = \hat{M}^e$ , we find that the Yukawa couplings in the charged lepton mass eigenstate basis is

$$\mathcal{L}_{Y_e} = -\bar{e}_L \left( \hat{M}^e \left( 1 + \frac{\tilde{h}}{v} \right) + \lambda_1^e H_1 + \lambda_2^e H_2 \right) e_R + \text{H.c.} \quad (22)$$

where

$$\begin{aligned} \lambda_1^e &= \frac{S_e^\dagger (\tilde{\lambda}^l v_2 - \lambda^l v_1) T_e}{\sqrt{2} v'}, \\ \lambda_2^e &= \frac{S_e^\dagger (\lambda_L^e v' - \tilde{\lambda}^l \frac{v_1 v_L}{v'} - \lambda^l \frac{v_2 v_L}{v'}) T_e}{\sqrt{2} v}. \end{aligned} \quad (23)$$

The Higgs mass eigenstates can now be written as linear combinations of  $\tilde{h}$ ,  $H_1$ ,  $H_2$  as  $h^{m_i} = V^{ih} \tilde{h} + V^{i1} H_1 + V^{i2} H_2$  in terms of an orthogonal matrix  $V^{ij}$ . Further identifying the lightest mass eigenstate  $h^{m_1} = h$  with the 125 GeV Higgs boson, we have

$$\mathcal{L}_{h e_i e_j} = - \left( \frac{\hat{M}^e}{v} V^{1h} + \lambda_1^e V^{11} + \lambda_2^e V^{12} \right)_{ij} \bar{e}_{L_i} e_{R_j} h. \quad (24)$$

One can write  $(\frac{\hat{M}^e}{v} V^{1h} + \lambda_1^e V^{11} + \lambda_2^e V^{12})_{ij} = (m_i/v) \delta_{ij} + (S_e^\dagger g^e T_e)_{ij}$ , so that  $(S_e^\dagger g^e T_e)_{ij}$  reflects the FCNC flavor structure of the  $h$  Higgs interaction with leptons.

The normalized tau couplings  $\epsilon_\tau$  and  $\tilde{r}_\tau$  defined in Eq. (3) are then

$$\begin{aligned} \epsilon_\tau &= V_{33}^{1h} - 1 + \text{Re}((\lambda_1^e)_{33} V^{11} + (\lambda_2^e)_{33} V^{12}) \frac{v}{m_\tau}, \\ \tilde{r}_\tau &= \text{Im}((\lambda_1^e)_{33} V^{11} + (\lambda_2^e)_{33} V^{12}) \frac{v}{m_\tau}. \end{aligned} \quad (25)$$

Equation (24) is similar to Eq. (13), but this time there are two terms which are nondiagonal. This difference is sufficient to reach opposite conclusions to the previous model: in the mass eigenstate basis  $h$  can decay to  $\mu\tau$  and at the same time the Yukawa coupling for  $h$  to  $\tau\bar{\tau}$  can be complex leading to  $CP$  violating coupling of the type in Eq. (3). This model naturally has nonzero values for  $r_\tau$  and  $\tilde{r}_\tau$  simultaneously.

### III. EXISTING CONSTRAINTS AND $CP$ VIOLATION IN $h \rightarrow \tau\tau$

One might think that the hierarchical structure of the lepton mass matrix is already encoded in the (presumably dominant) SM contribution to the Yukawa couplings. In this case the flavor structure of the corrections,  $(S_e^\dagger g^e T_e)_{ij}$  could be democratic. Furthermore, within the models we are discussing we can choose appropriate values for  $v_{L,1,2}$  to reduce the hierarchical structure of the Yukawa couplings making democratic  $\lambda_{1,2}^e$  matrices plausible. We would write in this case,

$$(S_e^\dagger g^e T_e)_{ij} \sim \lambda_{1,2}^e \sim \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad (26)$$

and the diagonal elements would satisfy

$$\frac{(\epsilon_i + i\tilde{r}_i)}{(\epsilon_j + i\tilde{r}_j)} \sim \frac{m_j}{m_i}. \quad (27)$$

However, this needs not be the case. For example, as mentioned before, one still needs to split the first and second generations and hierarchical  $\lambda_{1,2}^e$  matrices may still be needed. In this case we could write following Refs. [4,29]

$$(S_e^\dagger g^e T_e)_{ij} \sim \lambda_{1,2}^e \sim \begin{pmatrix} m_e & \sqrt{m_e m_\mu} & \sqrt{m_e m_\tau} \\ \sqrt{m_e m_\mu} & m_\mu & \sqrt{m_\mu m_\tau} \\ \sqrt{m_e m_\tau} & \sqrt{m_\mu m_\tau} & m_\tau \end{pmatrix}, \quad (28)$$

and this time the diagonal elements would satisfy

$$\frac{(\epsilon_i + i\tilde{r}_i)}{(\epsilon_j + i\tilde{r}_j)} \sim 1. \quad (29)$$

We will consider the above two cases as benchmarks for discussion in the remaining of the paper.

#### A. Constraints on Yukawa couplings and $h \rightarrow \mu\tau$

To explain the CMS data for  $h \rightarrow \mu\tau$ , it is necessary to have nonzero  $g_{h\mu\tau}$  and  $g_{h\tau\mu}$  in the expression

$$\begin{aligned} \mathcal{L}_{\mu\tau} &= -(g_{h\mu\tau} \bar{\mu}_L \tau_R + g_{h\tau\mu} \bar{\tau}_L \mu_R) h - (g_{h\mu\tau}^* \bar{\tau}_R \mu_L + g_{h\tau\mu}^* \bar{\mu}_R \tau_L) h \\ &= - \left( \frac{g_{h\mu\tau} + g_{h\tau\mu}^*}{2} \bar{\mu} \tau + \frac{g_{h\mu\tau} - g_{h\tau\mu}^*}{2} \bar{\mu} \gamma_5 \tau \right) h \\ &\quad - \left( \frac{g_{h\tau\mu} + g_{h\mu\tau}^*}{2} \bar{\tau} \mu + \frac{g_{h\tau\mu} - g_{h\mu\tau}^*}{2} \bar{\tau} \gamma_5 \mu \right) h. \end{aligned} \quad (30)$$

Including loop effects,  $g_{h\mu\tau}$  may have a nonzero absorptive part which leads to a rate difference between  $h \rightarrow \bar{\mu}\tau$

and  $h \rightarrow \bar{\tau}\mu$ . However, if the absorptive parts are small, the rate for  $h \rightarrow \bar{\mu}\tau$  and  $h \rightarrow \bar{\tau}\mu$  will be approximately equal.

These couplings have been recently constrained in connection with the CMS first report [30]

$$B(h \rightarrow \mu\tau) = (0.84_{-0.37}^{+0.39})\%. \quad (31)$$

When the absorptive parts in  $g_{hij}$  are neglected, one obtains

$$\sqrt{g_{h\tau\mu}^2 + g_{h\mu\tau}^2} < 3.6 \times 10^{-3}. \quad (32)$$

Very recently, CMS has updated Eq. (31) to [31]

$$B(h \rightarrow \mu\tau) = (-0.76_{-0.84}^{+0.81})\%, \quad (33)$$

which leads to a 95% C.L. limit

$$\sqrt{g_{h\tau\mu}^2 + g_{h\mu\tau}^2} < 3.16 \times 10^{-3}. \quad (34)$$

This latter number thus implies for the two benchmark flavor structures in Eqs. (26) and (28)

(i) democratic

$$\sqrt{|\epsilon_\tau|^2 + |\tilde{r}_\tau|^2} \leq \frac{1}{\sqrt{2}} 3.16 \times 10^{-3} \frac{v}{m_\tau} \quad (35)$$

(ii) hierarchical

$$\sqrt{|\epsilon_\tau|^2 + |\tilde{r}_\tau|^2} \leq \frac{1}{\sqrt{2}} 3.16 \times 10^{-3} \sqrt{\frac{m_\tau}{m_\mu}} \frac{v}{m_\tau}. \quad (36)$$

Additional constraints can be obtained from the measured rates  $h \rightarrow \tau\tau$  and  $h \rightarrow \mu\mu$  as reported in the ATLAS-CMS combination of results from run 1 assuming that there is no new physics. These are [32]

$$\begin{aligned} |\kappa_i|^2 &\equiv \frac{\Gamma(h \rightarrow \ell_i \ell_i)}{\Gamma(h \rightarrow \ell_i \ell_i)_{\text{SM}}} \\ \kappa_\tau &= \sqrt{(1 + \epsilon_\tau)^2 + \tilde{r}_\tau^2} = 0.90_{-0.16}^{+0.14} \\ \kappa_\mu &= \sqrt{(1 + \epsilon_\mu)^2 + \tilde{r}_\mu^2} = 0.2_{-0.2}^{+1.2}. \end{aligned} \quad (37)$$

There is also a constraint on  $h \rightarrow e^+e^-$  at 95% C.L. [33]

$$\kappa_e = \sqrt{(1 + \epsilon_e)^2 + \tilde{r}_e^2} \leq 611. \quad (38)$$

Very recently, ATLAS has presented a new limit on Higgs decaying to muon pairs, limiting the signal strength  $\mu_S < 3.5$  at 95% C.L. when combining runs 1 and 2 [34].

This new number does not yet improve the constraint in Eq. (37) which was obtained assuming no new physics is present.

The two flavor structure benchmarks then imply at 95% C.L., using the notation for mass ratios  $x_\mu = m_\tau/m_\mu$  and  $x_e = m_\tau/m_e$ , that

(i) democratic

$$\begin{aligned} 0.645 &\leq \sqrt{|1 + \epsilon_\tau|^2 + |\tilde{r}_\tau|^2} \leq 1.174 \\ \sqrt{|1 + x_\mu \epsilon_\tau|^2 + |x_\mu \tilde{r}_\tau|^2} &\leq 2.55 \\ \sqrt{|1 + x_e \epsilon_\tau|^2 + |x_e \tilde{r}_\tau|^2} &\leq 611 \end{aligned} \quad (39)$$

(ii) hierarchical

$$\begin{aligned} 0.645 &\leq \sqrt{|1 + \epsilon_\tau|^2 + |\tilde{r}_\tau|^2} \leq 1.174 \\ \sqrt{|1 + \epsilon_\tau|^2 + |\tilde{r}_\tau|^2} &\leq 2.55 \\ \sqrt{|1 + \epsilon_\tau|^2 + |\tilde{r}_\tau|^2} &\leq 611. \end{aligned} \quad (40)$$

For the LFV violating coupling, there is also a constraint from  $\tau \rightarrow \mu\gamma$ . From  $Br(\tau \rightarrow \mu\gamma)_{\text{exp}} < 4.4 \times 10^{-8}$  [35], the allowed range encompasses  $2.0 \times 10^{-3} < \sqrt{g_{h\tau\mu}^2 + g_{h\mu\tau}^2} < 3.3 \times 10^{-3}$  [36–38] and yields a weaker constraint than the CMS result quoted above.

With some relatively weak constraints on  $r_\tau$  and  $\tilde{r}_\tau$ , one may wonder whether a large  $\tau$  edm  $d_\tau$  can be generated. We have checked this possibility and found that since the one-loop contribution to  $d_\tau$  from Eq. (3) is proportional to  $m_\tau^3(r_\tau \tilde{r}_\tau)/16\pi^2 v^2 m_h^2$ , the current upper limit  $d_\tau < 10^{-17}$  em does not constrain  $r_\tau \tilde{r}_\tau$  significantly. The two-loop Barr-Zee diagram contribution to the electron edm [39] can be more important, and in conjunction with the best current limit [35] implies that  $\tilde{r}_\tau \leq 2.35$ . In addition to this bound being subject to additional assumptions about possible cancellations between different contributions to the electron edm, it is not competitive with other constraints shown in Fig. 1.

Our numerical results are summarized in Fig. 1. The panel on the left corresponds to the democratic flavor scenario. We see in this case that the most restrictive bounds arise from the limits on  $h \rightarrow \mu\mu$  and  $h \rightarrow ee$ . This is due to the much smaller SM Yukawa couplings for electrons and muons relative to tau leptons, which significantly enhance the effects of democratic absolute deviations from the SM in the relative couplings probed by experiment. We see in this case that the maximum allowed value of the ratio that quantifies  $CP$  violation is

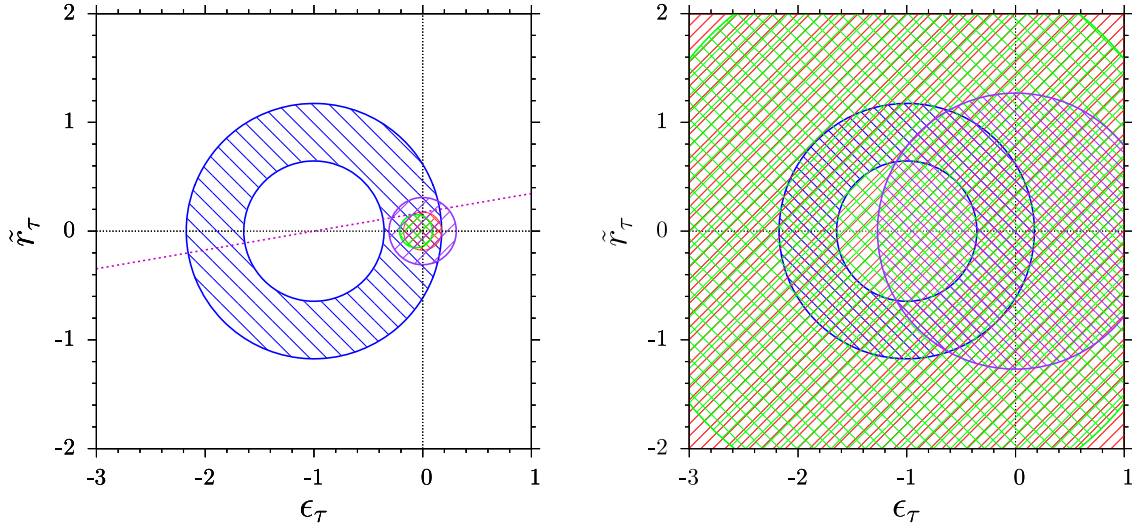


FIG. 1. Region of parameter space allowed by the constraints discussed in the text at the 95% C.L. The blue region is from the  $h \rightarrow \tau\tau$  rate, the green region from the  $h \rightarrow \mu\mu$  limit, the red region is from the  $h \rightarrow ee$  limit and the purple region is from the CMS  $h \rightarrow \tau\mu$  upper bound. The left panel corresponds to the democratic flavor scenario and allows a maximum value  $r_\tau \tilde{r}_\tau / (r_\tau^2 + \tilde{r}_\tau^2) \approx 0.15$  shown by the red dotted line. The right panel corresponds to the hierarchical flavor scenario and still permits a maximum value  $r_\tau \tilde{r}_\tau / (r_\tau^2 + \tilde{r}_\tau^2) = 0.5$ .

$$\left| \frac{r_\tau \tilde{r}_\tau}{r_\tau^2 + \tilde{r}_\tau^2} \right| \lesssim 0.15 \quad (41)$$

shown by the red dotted line.

The panel on right corresponds to the hierarchical flavor scenario. In this case the most restrictive constraints are those from  $h \rightarrow \tau\tau$ . This case does not yet constrain  $CP$  violation, allowing the theoretical maximum possible value for the relevant ratio

$$\left| \frac{r_\tau \tilde{r}_\tau}{r_\tau^2 + \tilde{r}_\tau^2} \right| \leq 0.50. \quad (42)$$

Constraints that can be placed on these couplings in future colliders have been recently investigated in Ref. [40], and in the next section we introduce some possible  $CP$ -odd asymmetries for that purpose.

#### IV. $CP$ VIOLATION IN $h \rightarrow \tau\bar{\tau}$ AT FUTURE $e^+e^-$ COLLIDERS

As discussed above, the couplings  $r$  and  $\tilde{r}$  in Eq. (3) give rise to a  $CP$  violating spin-spin correlation as in Eq. (4). The polarizations of  $\tau$  and  $\bar{\tau}$  can be extracted in principle by studying the angular distributions of their decay. In this section we will study the relative sensitivity of the different tau-lepton decay modes to  $CP$  violation at a more theoretical level by comparing the  $T$ -odd correlations for each case. Experimental study of these correlations requires the reconstruction of the Higgs rest frame, which in the di-tau mode, is not possible at LHC. They are thus better suited for study at an  $e^+e^-$  collider. For example, an ILC or CPEC running at 250 GeV would produce the Higgs through the

$e^+e^- \rightarrow Zh$  reaction and modes that reconstruct the  $Z$  completely (such as the di-muon mode) will allow full reconstruction of the Higgs rest frame [41–43].

The simplest mode to consider is the two body decay already discussed in Refs. [6,12]

$$\tau^- \rightarrow \pi^- \nu_\tau, \quad \tau^+ \rightarrow \pi^+ \bar{\nu}_\tau. \quad (43)$$

Denoting by  $\vec{p}_{\pi^\pm}$  the three-momenta of the pions in the Higgs rest frame, Eq. (4) generates the  $T$ -odd correlation

$$\mathcal{O}_\pi = \vec{p}_\tau \cdot (\vec{p}_{\pi^+} \times \vec{p}_{\pi^-}). \quad (44)$$

This can be measured, for example, by the integrated counting asymmetry

$$A_\pi = \frac{N(\mathcal{O}_\pi > 0) - N(\mathcal{O}_\pi < 0)}{N(\mathcal{O}_\pi > 0) + N(\mathcal{O}_\pi < 0)} = \frac{\pi}{4} \beta_\tau \frac{(r_\tau \tilde{r}_\tau)}{\beta_\tau^2 r_\tau^2 + \tilde{r}_\tau^2}, \quad (45)$$

as has been known for a long time [12]. In conjunction with the constraints from Eqs. (41) and (42) this implies that the largest  $CP$ -odd asymmetry that is still allowed is 11% for the democratic scenario and 40% for the hierarchical scenario.

The asymmetry for the leptonic three-body decay  $\tau^\pm \rightarrow \ell^\pm \nu \bar{\nu}$  can also be calculated analytically. In this case it is simplest to directly construct the triple product correlation between final particle momenta using the methods of Ref. [44] to compute the relevant density matrices and obtain the Lorentz invariant form of the  $CP$  violating matrix element squared,

$$|\mathcal{M}_{C/P}| = -\frac{32\pi^2 r_\tau \tilde{r}_\tau}{\Gamma_\tau^2} (4\sqrt{2}G_F)^2 \delta(p_{\tau^+}^2 - m_\tau^2) \delta(p_{\tau^-}^2 - m_\tau^2) \mathcal{O}$$

$$\mathcal{O} = \epsilon^{\mu\nu\alpha\beta} p_{\tau^-}^\mu p_{\tau^+}^\nu p_{\nu\ell}^\alpha p_{\bar{\nu}\ell}^\beta. \quad (46)$$

The delta functions reveal that we have used the narrow-width approximation for the denominator of the tau-lepton propagators, but we have kept all spin correlations, and  $\mathcal{O}$  is the Lorentz invariant form of the raw triple product that occurs in this decay. The total decay width for this channel is given by

$$\Gamma = \frac{\beta_\tau}{8\pi m_H} m_\tau^2 \left( \frac{m_H^2}{v^2} \right) (\beta_\tau^2 |r_\tau|^2 + |\tilde{r}_\tau|^2) Br(\tau \rightarrow \mu + \nu's)^2. \quad (47)$$

To measure the  $CP$ -odd correlation we would use an integrated counting asymmetry

$$A = \frac{N_{\text{ev}}(\mathcal{O} > 0) - N_{\text{ev}}(\mathcal{O} < 0)}{N_{\text{ev}}(\mathcal{O} > 0) + N_{\text{ev}}(\mathcal{O} < 0)}. \quad (48)$$

In the limit  $m_\tau \ll m_H$ ,  $\beta_\tau \rightarrow 1$  and  $m_\ell \ll m_\tau$  it is possible to compute this analytically by integrating over the six-body phase space as sketched in Ref. [45], resulting in

$$A = -\frac{\pi}{4} \frac{r_\tau \tilde{r}_\tau}{|r_\tau|^2 + |\tilde{r}_\tau|^2}. \quad (49)$$

Of course this is just the raw asymmetry as the neutrino momenta cannot be measured. It represents the largest possible asymmetry in this mode as there are dilution factors when the triple product is projected onto observable momenta. This part of the calculation is better done numerically and to this aim we implemented the Lagrangian of Eq. (3) in FEYNRULES [46,47] to generate the Universal Feynrules Output (UFO) file, then feeding this UFO file into MG5\_aMC@NLO [48] in combination with TAUDECAY [49] package which performs the hadronic decays of the tau lepton. A suitable  $T$ -odd correlation for the leptonic decay mode is

$$\mathcal{O}_\ell = \vec{p}_\tau \cdot (\vec{p}_{\ell^+} \times \vec{p}_{\ell^-}), \quad (50)$$

where now  $\vec{p}_{\ell^\pm}$  denotes the three-momenta of the charged lepton in the Higgs rest frame, and can be measured with the integrated counting asymmetry

$$A_\ell = \frac{\pi}{36} \frac{r_\tau \tilde{r}_\tau}{|r_\tau|^2 + |\tilde{r}_\tau|^2}. \quad (51)$$

For more than one pion in the decay of  $\tau$ 's, we have carried out a similar analysis with results summarized in Tables I and II. For all cases in the Tables, we simulated the Higgs boson decay in its rest frame with 200000 events with no kinematic cuts for a sufficient number of values  $r_\tau$ ,  $\tilde{r}_\tau$  to obtain a good fit to the asymmetry. For modes with more than one pion we measured different  $T$ -odd correlations using the different pion momenta available, but in all

TABLE I. Semileptonic modes with tau jet producing the largest asymmetry and their respective coefficients  $c_i$  for Eq. (53).

	Mode	Jets	$c_i$
1	$(\tau^- \rightarrow \nu_\tau \mu^- \bar{\nu}_\mu), (\tau^+ \rightarrow \bar{\nu}_\tau \pi^+)$	$j = \pi^+$	-0.27
2	$(\tau^- \rightarrow \nu_\tau \mu^- \bar{\nu}_\mu), (\tau^+ \rightarrow \bar{\nu}_\tau \pi^+ \pi^0)$	$j = \pi^+ + \pi^0$	-0.11
3	$(\tau^- \rightarrow \nu_\tau \mu^- \bar{\nu}_\mu), (\tau^+ \rightarrow \bar{\nu}_\tau \pi^+ \pi^0 \pi^0)$	$j = \pi^+ + \pi^0 + \pi^0$	-0.017
4	$(\tau^- \rightarrow \nu_\tau \mu^- \bar{\nu}_\mu), (\tau^+ \rightarrow \bar{\nu}_\tau \pi^+ \pi^+ \pi^-)$	$j = \pi^+ + \pi^+ + \pi^-$	0.0005

TABLE II. Double hadronic tau decays with tau jets producing the largest asymmetry and their respective coefficients  $c_i$  for Eq. (53).

	Mode	Jets	$c_i$
1	$(\tau^- \rightarrow \nu_\tau \pi^-)(\tau^+ \rightarrow \bar{\nu}_\tau \pi^+)$	$j_1 = \pi^-, j_2 = \pi^+$	0.79
2	$(\tau^- \rightarrow \nu_\tau \pi^-), (\tau^+ \rightarrow \bar{\nu}_\tau \pi^+ \pi^0)$	$j_1 = \pi^-, j_2 = \pi^+ + \pi^0$	0.33
3	$(\tau^- \rightarrow \nu_\tau \pi^- \pi^0), (\tau^+ \rightarrow \bar{\nu}_\tau \pi^+ \pi^0)$	$j_1 = \pi^- + \pi^0, j_2 = \pi^+ + \pi^0$	0.13
4	$(\tau^- \rightarrow \nu_\tau \pi^-), (\tau^+ \rightarrow \bar{\nu}_\tau \pi^+ \pi^0 \pi^0)$	$j_1 = \pi^-, j_2 = \pi^+ + \pi^0 + \pi^0$	0.06
5	$(\tau^- \rightarrow \nu_\tau \pi^-), (\tau^+ \rightarrow \bar{\nu}_\tau \pi^+ \pi^+ \pi^-)$	$j_1 = \pi^-, j_2 = \pi^+ + \pi^+ + \pi^-$	0.06
6	$(\tau^- \rightarrow \nu_\tau \pi^- \pi^0), (\tau^+ \rightarrow \bar{\nu}_\tau \pi^+ \pi^0 \pi^0)$	$j_1 = \pi^- + \pi^0, j_2 = \pi^+ + \pi^0 + \pi^0$	0.02
7	$(\tau^- \rightarrow \nu_\tau \pi^- \pi^0), (\tau^+ \rightarrow \bar{\nu}_\tau \pi^+ \pi^+ \pi^-)$	$j_1 = \pi^- + \pi^0, j_2 = \pi^+ + \pi^+ + \pi^-$	0.02
8	$(\tau^- \rightarrow \nu_\tau \pi^- \pi^0 \pi^0), (\tau^+ \rightarrow \bar{\nu}_\tau \pi^+ \pi^0 \pi^0)$	$j_1 = \pi^- + \pi^0 + \pi^0, j_2 = \pi^+ + \pi^0 + \pi^0$	0.004
9	$(\tau^- \rightarrow \nu_\tau \pi^- \pi^0 \pi^0), (\tau^+ \rightarrow \bar{\nu}_\tau \pi^+ \pi^+ \pi^-)$	$j_1 = \pi^- + \pi^0 + \pi^0, j_2 = \pi^+ + \pi^+ + \pi^-$	0.003
10	$(\tau^- \rightarrow \nu_\tau \pi^- \pi^+ \pi^-), (\tau^+ \rightarrow \bar{\nu}_\tau \pi^+ \pi^+ \pi^-)$	$j_1 = \pi^- + \pi^+ + \pi^-, j_2 = \pi^+ + \pi^+ + \pi^-$	0.003



cases studied found that the largest sensitivity was obtained by using a “tau-jet” momenta defined as the sum of all the pion momenta in the corresponding decay.

Table I shows the semileptonic modes  $h \rightarrow \tau^+ \tau^- \rightarrow \tau_\ell \tau_h$ , noting that at the level of our study electrons are indistinguishable from muons. We write for each mode a  $T$ -odd operator

$$\mathcal{O}_i = \vec{p}_{\tau^-} \cdot (\vec{p}_\ell \times \vec{p}_j) \quad (52)$$

and construct a corresponding integrated asymmetry

$$A_i = c_i \frac{r_\tau \tilde{r}_\tau}{|r_\tau|^2 + |\tilde{r}_\tau|^2} \quad (53)$$

where the coefficient  $c_i$  is estimated numerically as described above and tabulated in the fourth column. The table shows only leptonic decays on the  $\tau^-$  side, but we also checked that the conjugated modes have the same asymmetries. If used on charge specific modes as the ones on the table, the asymmetries are  $T$  odd but not  $CP$  odd. True  $CP$ -odd observables are constructed as in Eq. (52) where leptons (and corresponding hadronic modes) and antileptons are included in the sum.

The table indicates that the one and two pion modes have the largest asymmetries by far, so that one loses sensitivity by including higher multiplicity modes in the tau jet. Of course, the higher multiplicity may actually facilitate the experimental reconstruction of the events or the asymmetries so a full study is needed to reach definitive conclusions.

Table II shows the modes with two hadronic tau decays  $h \rightarrow \tau^+ \tau^- \rightarrow \tau_h \tau_h$  covering one, two and three pion modes. As with the semileptonic case we studied several possibilities for the definition of the tau jet, and found the largest asymmetries for the ones shown in the table. We write for each mode a  $T$ -odd operator

$$\mathcal{O}_i = \vec{p}_{\tau^-} \cdot (\vec{p}_{j1} \times \vec{p}_{j2}) \quad (54)$$

and construct a corresponding integrated asymmetry equation (53).

As with the semileptonic case, we have not listed all the conjugate modes. If the counting asymmetry is constructed for a particular (not self-conjugate) mode, the result is  $T$  odd but not necessarily  $CP$  odd. However, if sums over conjugate modes are considered, then any nonzero asymmetry signals  $CP$  violation. We find here also that the most sensitive modes are those with only one or two pions.

## V. CONCLUSIONS

In the SM, the Higgs boson decays to fermions are flavor diagonal and conserve  $CP$ . We have argued generically that if one goes BSM to allow LFV decays of the Higgs, such as the one suggested by a recent CMS result, one also introduces  $CP$  violation. We have constructed two specific multi-Higgs models in which the 125 GeV Higgs can have LFV decays and argued that only one of them exhibits  $CP$  violation as well. These two examples illustrate the different ingredients that are needed for both effects to appear BSM.

A channel where it is in principle possible to study  $CP$  violation is  $h \rightarrow \tau\tau$ , but this is very hard to do at LHC. We have studied the relative sensitivity of different tau-lepton decay modes to  $CP$  violating couplings for possible application at future  $e^+e^-$  colliders.

The correlation between LFV and CPV couplings depends on the details of the flavor sector BSM and we have considered two benchmark scenarios. In the first one, the lepton flavor sector has a dominant hierarchical structure that produces the charged lepton masses, but the deviations from this are democratic. We found that in this case the tightest constraint on possible new physics arises from bounds on  $h \rightarrow \mu\mu$  and  $h \rightarrow ee$ . Within factors of 2, this constraint is consistent with the upper bound on LFV from CMS, and allows for a  $CP$  violating asymmetry as large as 11%.

In the second benchmark scenario we assumed the corrections to the SM lepton flavor sector are also hierarchical as in the Fritzsche ansatz. In this case the tightest constraints on new physics arise from  $h \rightarrow \tau\tau$ . Within factors of 2 they are consistent with the upper bound on LFV from CMS, and they allow for a  $CP$  violating asymmetry as large as 40%.

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