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Multiplicity distributions in e^+e^- collisions using Weibull distribution

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The two-parameter Weibull function is used to describe the charged particle multiplicity distribution in e^+e^- collisions at the highest available energy measured by the TASSO and ALEPH experiments. The Weibull distribution has wide applications in naturally evolving processes based on fragmentation and sequential branching. The Weibull model describes the multiplicity distribution very well, as particle production processes involve QCD parton fragmentation. The effective energy model of particle production was verified using Weibull parameters and the same was used to predict the multiplicity distribution in e^+e^- collisions at future collider energies.

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I. INTRODUCTION

The charged particle multiplicity distribution is one of the most basic measurements performed in high energy leptonic collision experiments. This particular measurement provides an insight to the multiparticle production mechanism. The formulation of this multiparticle production is a complex task. So, one has to rely on model studies that are based on quantum chromodynamics as well as a "soft" physics that has a significant contribution towards particle production.

Several model studies based on the perturbative quantum chromodynamics (pQCD) approach as well as the semiclassical approach have been done to understand this complex task [1–4]. Most of the theoretical models that have a pQCD contribution based on parton fragmentation and sequential branching have successfully explained the measured data from LEP experiments [5–7]. The particle production in such models has an iterative branching process in which the initial quarks produced in e^+e^- collisions radiate gluons, which in turn branch into a cascade of partons until the virtualities become negligible to allow further branching [8]. This is followed by hadronization.

The multiplicity distribution follows a Poisson distribution if the final states of particles are produced independently. In experiments with higher center of mass energies and different rapidity ranges, it was seen that the shape of the multiplicity distribution deviates from the Poissonian shape. The negative binomial distribution (NBD) [9,10], which has two parameters, namely, k (measures deviation from Poisson distribution) and $\langle n\rangle$ (average number of particles), successfully described the particle multiplicity both in $pp(p\bar{p})$ [9] and e^+e^- [11] collision systems. However, the NBD failed to provide a good description of the multiplicity distributions at higher energies and this deviation was also observed in LEP experiments [5–8,12]. In view of this, several other distributions such as the modified negative binomial distribution (MNBD) [13] and log-normal [14] distributions emerged and successfully described the data. A nice description of multiplicity distributions and various approaches can be found in Refs. [15–20].

Since the hadron multiplicities in e^+e^- collisions can be understood as an outcome of a broad class of branching processes, Weibull distribution [21–23] can be used to describe the distribution of produced charged particles. This distribution has successfully described the multiplicity distribution in $pp(p\bar{p})$ collisions for a broad range of center of mass energies and pseudorapidity intervals [23]. As e^+e^- collisions are more fundamental and less complex than the $pp(p\bar{p})$ collisions and a sequential branching is one of the major processes for particle production, it will be interesting to see whether the Weibull function describes the measured multiplicity distributions in leptonic collisions for a broad range of energies.

In the present paper, the Weibull function is used to describe the multiplicity distribution in e^+e^- collisions measured by the TASSO [24] and ALEPH [25] experiments at PETRA and LEP energies, respectively. Although there are several statistical models that describe the charged particle distribution in e^+e^- collisions, the idea of this present paper is to test the applicability of the Weibull distribution in such collision systems and interpret the mechanism of particle production in terms of its parameters.

II. WEIBULL DISTRIBUTION

Many evolving systems are found to show a skewed behavior. These kinds of systems are well described by a power law as well as log-normal distribution and Weibull distribution. A detailed review of its applications is given in Refs. [22,23].

The Weibull distribution for a random variable n is expressed as

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$$P(n,\lambda,k)=\frac{k}{\lambda}\left(\frac{n}{\lambda}\right)^{k-1}e^{-(n/\lambda)k}. \tag{1}$$

Here k is known as the shape parameter and λ is known as the scale parameter of the distribution. The mean of the distribution is expressed as

$$\langle n\rangle=\lambda\Gamma\left(1+\frac{1}{k}\right). \tag{2}$$

Weibull distribution can be used to describe the multiplicity distribution obtained from the hadronic or leptonic collisions at high energy, as the underlying mechanism is based on initial parton fragmentation and successive branching.

III. RESULTS AND DISCUSSIONS

The multiplicity distributions measured in the e^+e^- collisions by the TASSO [24] and ALEPH [25] experiments are fitted with the Weibull function. The parameters of Weibull distribution are studied for two extreme rapidity intervals ($|y| < 2.0$ and $|y| < 0.5$) at various $\sqrt{s}$ (14 GeV, 22 GeV, 34.8 GeV, 43.6 GeV, and 91.2 GeV).

Figures 1 and 2 show the charged particle multiplicity distribution for $|y| < 0.5$ and $|y| < 2.0$, respectively, fitted with Weibull distribution. The distributions corresponding to different $\sqrt{s}$ are shown by different markers and are scaled by a suitable factor (see Figs. 1 and 2) for visual clarity. The solid lines are the Weibull fits to the data points. It can be observed from both the figures that the Weibull distribution describes the data very nicely. The fitting is performed using the χ^2 minimization method. The Weibull parameters k and λ, along with the χ^2/NDF and $\langle n\rangle$, are listed in Tables I and II for $|y| < 0.5$ and $|y| < 2.0$, respectively.

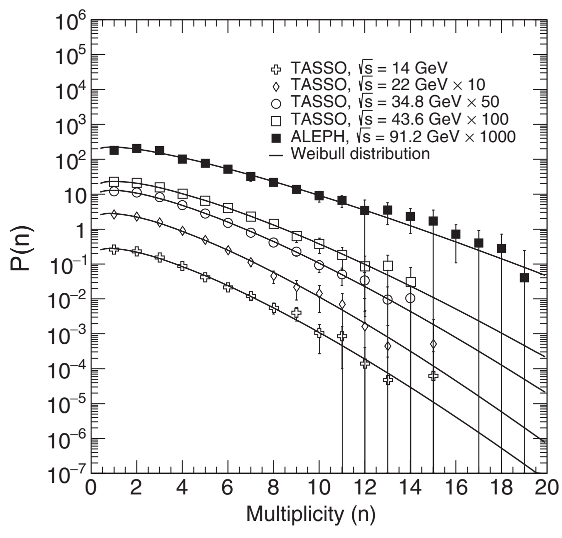

FIG. 1. Multiplicity distribution for $|y| < 0.5$ measured at $\sqrt{s}=14$, 22, 34.8, and 43.6 GeV by the TASSO Collaboration [24] and $\sqrt{s}=91$ GeV by the ALEPH Collaboration [25]. The solid line represents the Weibull fit to the data points. The data points for a given energy are appropriately scaled for better visibility.

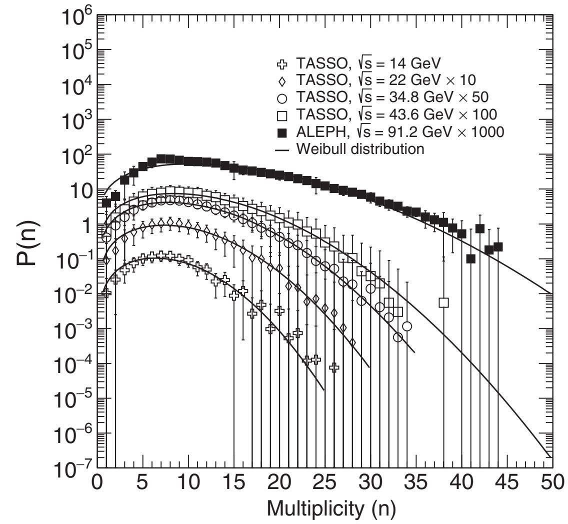

FIG. 2. Multiplicity distribution for $|y| < 2$ measured at $\sqrt{s}=14$, 22, 34.8, and 43.6 GeV by the TASSO Collaboration [24] and $\sqrt{s}=91.2$ GeV by the ALEPH Collaboration [25]. The solid line represents the Weibull fit to the data points. The data points for a given energy are appropriately scaled for better visibility.

The Weibull parameters k and λ are studied as a function of $\sqrt{s}$ and rapidity and are shown in Figs. 3 and 4, respectively. The parameters k and λ are parametrized as a function of $\sqrt{s}$ with a power law of the form $A\times\sqrt{s}^B$. The parametrized values are given in Tables III and IV.

It is observed that as a function of $\sqrt{s}$, the value of λ shows a slight increase within uncertainties for $|y| < 0.5$, whereas for $|y| < 2.0$, λ increases significantly with increasing $\sqrt{s}$. As the parameter λ is associated with mean multiplicity [23], it is straightforward to see its increase with collision energies. In this particular result, it can be seen that the value of λ is ~4–5 times higher for $|y| < 2.0$ in comparison to $|y| < 0.5$. Since the increase of λ is not very significant for $|y| < 0.5$, one can attribute the increase for $|y| < 2.0$ to the contribution coming from soft processes in the forward rapidity region. The value of k does not vary

TABLE I. Weibull parameters obtained from multiplicity distribution for $|y| < 0.5$ measured at $\sqrt{s}=14$, 22, 34.8, and 43.6 GeV by the TASSO Collaboration [24] and at $\sqrt{s}=91$ GeV by the ALEPH Collaboration [25].

$\sqrt{s}$ (GeV)	k	λ	$\langle n\rangle$	χ^2/NDF
14.00	1.33 ± 0.06	2.42 ± 0.06	2.22 ± 0.05	0.38
22.00	1.36 ± 0.07	2.47 ± 0.06	2.26 ± 0.06	0.13
34.80	1.34 ± 0.03	2.66 ± 0.04	2.45 ± 0.03	0.70
43.60	1.35 ± 0.06	2.95 ± 0.11	2.71 ± 0.08	0.07
91.2	1.25 ± 0.07	3.31 ± 0.17	3.08 ± 0.11	0.39

TABLE II. Weibull parameters obtained from multiplicity distribution for $|y| < 2$ measured at $\sqrt{s} = 14$, 22, 34.8, and 43.6 GeV by the TASSO Collaboration [24] and at $\sqrt{s} = 91$ GeV by the ALEPH Collaboration [25].

$\sqrt{s}$ (GeV)	k	λ	$\langle n \rangle$	χ^2/NDF
14.00	2.20 ± 0.08	8.39 ± 0.37	7.43 ± 0.33	0.19
22.00	2.18 ± 0.07	9.70 ± 0.52	8.59 ± 0.46	0.07
34.80	2.10 ± 0.04	10.62 ± 0.33	9.41 ± 0.29	0.14
43.60	2.05 ± 0.18	11.59 ± 0.64	10.27 ± 0.56	0.02
91.00	1.96 ± 0.04	14.97 ± 0.22	13.74 ± 0.19	1.51

significantly with the center of mass energy for the same rapidity intervals, indicating that the dynamics associated with the fragmentation process in e^+e^- collisions is very similar for the given range of energies. However, the value of k is higher for the larger rapidity interval and similar behavior was observed in hadronic collisions [23]. This can be related to probing a softer region where the produced partons merge with very soft gluons to form hadrons, usually the large mass resonances that eventually decay.

It has been observed that the average multihadronic final states in different interacting systems show a dependence on $\sqrt{s}$. The observed dependence disappears if the "effective energy" is used to characterize the interacting system rather than the center of mass energy [26–28]. Taking into account the effective energy scenario, the energy available for particle production in $pp(p\bar{p})$ collisions is the energy of the single interacting quark pair. As a result, about one third of the entire nucleon energy is only available for particle production in such collisions. However, in e^+e^- collisions, the annihilation process utilizes the entire available collision energy for the production of final state particles.

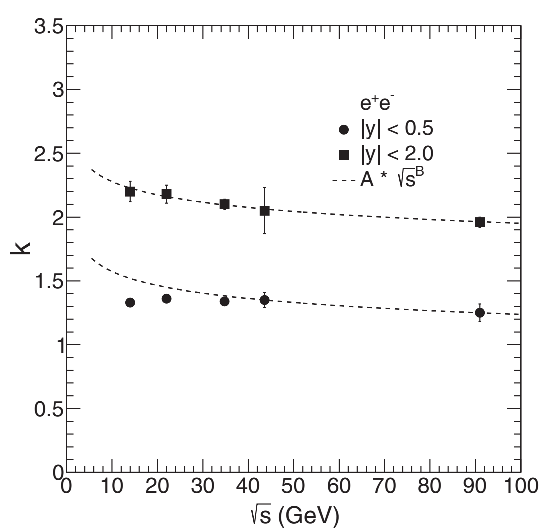

FIG. 3. The variation of k as a function of $\sqrt{s}$ for $|y| < 0.5$ and $|y| < 2.0$. The variation is parameterized with the power law of the form $\mathrm{A} \times \sqrt{s}^B$, shown by the dashed line

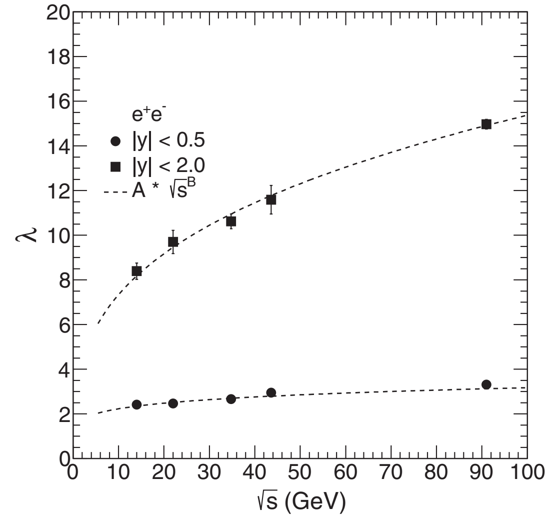

FIG. 4. The variation of λ as a function of $\sqrt{s}$ for $|y| < 0.5$ and $|y| < 2.0$. The variation is parameterized with the power law of the form $\mathrm{A} \times \sqrt{s}^B$, shown by the dashed line.

Thus, one expects to observe a good agreement on charged particle multiplicity distributions between e^+e^- collisions and $pp(p\bar{p})$ collisions when the center of mass energy of the later is three times that of e^+e^- collisions. It will be noteworthy to see whether the multiplicity distribution in $pp(p\bar{p})$ collisions can be obtained from the measured multiplicity distributions in e^+e^- collisions and vice versa using the Weibull parameters.

The parameters k and λ for e^+e^- collisions are studied as a function of $\sqrt{s}$. In order to verify the effective energy scenario, data from the UA5 Collaboration for $p\bar{p}$ collisions in 200 GeV are used [29]. The values of k and λ for e^+e^- collisions are interpolated from Figs. 3 and 4, respectively, for the center of mass energy 66.66 GeV (which is one third of 200 GeV). Figure 5 compares the multiplicity distribution in e^+e^- collisions at 66.66 GeV using the Weibull parametrization with the measured multiplicity distribution in $p\bar{p}$ collisions at 200 GeV. One can observe an excellent agreement between the

TABLE III. The values of parameters obtained for k as a function of $\sqrt{s}$.

Parameters	$\lvert y\rvert < 0.5$	$\lvert y\rvert < 2.0$
A	1.46 ± 0.176	2.66 ± 0.187
B	-0.067 ± 0.034	-0.067 ± 0.0186

TABLE IV. The values of parameters obtained for λ as a function of $\sqrt{s}$.

Parameters	$\lvert y\rvert < 0.5$	$\lvert y\rvert < 2.0$
A	1.57 ± 0.132	3.504 ± 0.292
B	0.153 ± 0.025	0.321 ± 0.0200

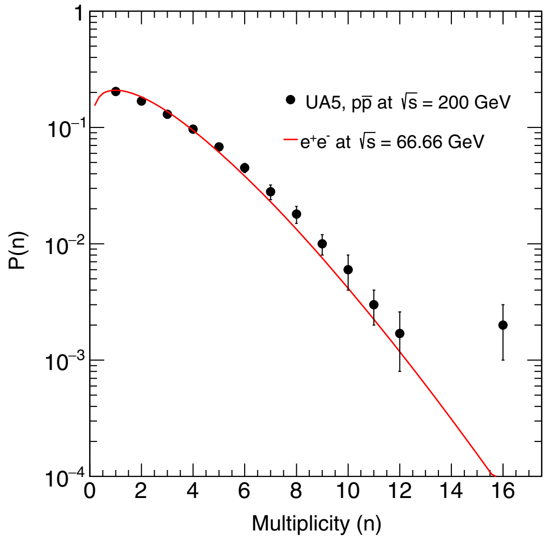

FIG. 5. Comparison of the multiplicity distribution for 200 GeV $p\bar{p}$ data [29] (solid markers) and the multiplicity distribution obtained using Weibull parameters for e^+e^- collisions (solid line) at $\sqrt{s} = 66.66$ GeV, in accordance with the effective energy approach [26].

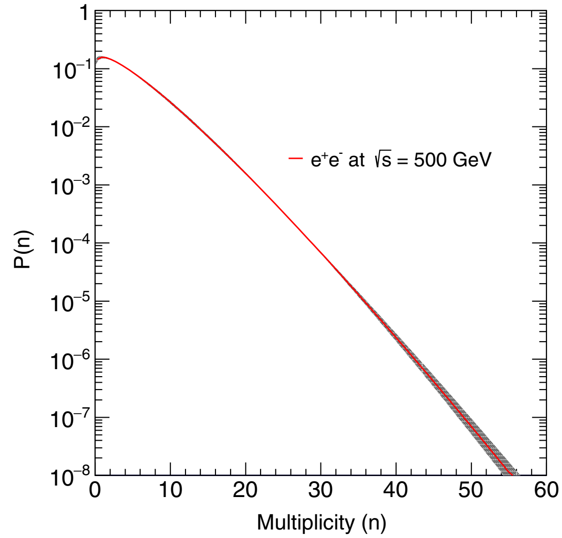

FIG. 6. The charged particle multiplicity distribution in e^+e^- collisions for $|\eta| < 0.5$ at $\sqrt{s} = 500$ GeV as predicted by Weibull parametrization. The shaded band around the solid line shows the associated systematic errors.

two collision systems favoring the effective energy model of particle production. This remarkable agreement can be used to predict the multiplicity distribution for e^+e^- collisions at 500 GeV [30], as it should be similar to the multiplicity distribution of $p + p$ collisions at 1500 GeV. The multiplicity distribution for $p + p$ collisions can be obtained by interpolating the Weibull parameters at 1500 GeV from Ref. [23]. The k and λ values obtained for $\sqrt{s} = 1500$ GeV are 1.17 ± 0.02 and 4.84 ± 0.17, respectively. The resultant distribution is shown in Fig. 6.

IV. CONCLUSION

The Weibull distribution provides an excellent description of the multiplicity distributions in e^+e^- collisions at a broad range of energies for two extreme rapidity intervals. The parameters of the distribution were studied as a function of the collision energies for two different rapidity intervals. The λ parameter shows a slight increase with collision energy, while the k parameter does not vary significantly with energy. This study suggests that most of the particles are produced via the soft processes in the forward rapidity region. Furthermore, the effective energy model was also verified and was used to predict the multiplicity distributions in e^+e^- collisions at ILC energies. Thus, the wide applicability of the Weibull model as an effective model to describe the particle production in different collision systems has been demonstrated.

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