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Strong decays of baryons and missing resonances

R. Bijker

Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México, AP 70-543, 04510 México DF, México

J. Ferretti[*]

Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México, AP 70-543, 04510 México DF, México and Dipartimento di Fisica and INFN, "Sapienza" Università di Roma, P.le Aldo Moro 5, I-00185 Roma, Italy

G. Galatà

Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México, 04510 México DF, México

H. García-Tecocoatzi

Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México, 04510 México DF, Mexico and INFN, Sezione di Genova and Dipartimento di Fisica, Università di Genova, via Dodecaneso 33, 16146 Genova, Italy

E. Santopinto[†]

INFN, Sezione di Genova, via Dodecaneso 33, 16146 Genova, Italy

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We provide results for the open-flavor strong decays of strange and nonstrange baryons into a baryon-vector/pseudoscalar meson pair. The decay amplitudes are computed in the $^3P_0$ pair-creation model, where $s\bar{s}$ pair-creation suppression is included for the first time in the baryon sector, in combination with the $U(7)$ and hypercentral models. The effects of this $s\bar{s}$ suppression mechanism cannot be reabsorbed in a redefinition of the model parameters or in a different choice of the $^3P_0$ model vertex factor. Our results for the decay amplitudes are compared with the existing experimental data and previous $^3P_0$ and elementary meson emission model calculations. In this respect, we show that distinct quark models differ in the number of missing resonances they predict and also in the quantum numbers of states. Therefore, future experimental results will be important in order to disentangle different models of baryon structure. Finally, in the appendixes, we provide some details of our calculations, including the derivation of all relevant flavor couplings with strangeness suppression. This derivation may be helpful to calculate the open-flavor decay amplitudes starting from other models of baryons.

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I. INTRODUCTION

It is well known that the baryon spectrum is very rich and much more complex than the meson spectrum. Nevertheless, at the moment, the number of known light-quark mesons is much larger than the number of known baryon resonances [1]. This problem has to do with the difficulty of identifying those high-lying baryon resonances that are only weakly coupled to the $N\pi$ channel [2,3] and thus cannot be seen in elastic $N\pi$ scattering experiments. Since the experimental observations of baryon resonances mainly come from reactions in which the pion is present either in the incoming channel, such as $N\pi \to N\pi$, or in the outgoing one, such as $N\gamma \to N\pi$, it would not be surprising if some baryon resonances, very weakly coupled to the single pion, were missing from experimental results. These baryons may decay mainly into two pion channels ($N\pi\pi$) or into channels such as $N\eta$, $N\eta'$, $N\omega$, and $K^+\Lambda$, where the final-state meson is different from the pion [3]. Although interesting results were provided by CB-ELSA [4], TAPS [5], GRAAL [6], SAPHIR [7], and CLAS [8], theoretical calculations of strong, electromagnetic, and weak decays of baryons may still help experimentalists in their search for those resonances that are still unknown.

The QCD mechanism behind the Okubo-Zweig-Iizuka-allowed strong decays [9] is still not clear. For this reason, several phenomenological models have been developed in

[*]Present address: Dipartimento di Fisica and INFN, "Sapienza" Università di Roma, P.le Aldo Moro 5, I-00185 Roma, Italy.
[†]Corresponding author.
santopinto@ge.infn.it

order to carry out this type of study, including pair-creation models [10–17], elementary meson emission models [18–23], and effective Lagrangian approaches (for example, see Ref. [24]). Attempts at modeling strong decays within the quark model (QM) formalism date from Micu's suggestion [10] that hadron decays proceed through $q\bar{q}$ pair production with vacuum quantum numbers, i.e., $J^{PC} = 0^{++}$. Since the $q\bar{q}$ pair corresponds to a $^3P_0$ quark-antiquark state, this model is known as the $^3P_0$ pair-creation model [10,11,16]. A few years after its introduction, Le Yaouanc *et al.* used the $^3P_0$ model to compute meson and baryon open-flavor strong decays [11] and also evaluated the strong decay widths of $\psi(3770)$, $\psi(4040)$, and $\psi(4415)$ charmonium states [25]. The $^3P_0$ model, extensively applied to the decays of light mesons and baryons [26], has recently been applied to heavy meson strong decays in the charmonium [27–29], bottomonium [30,31], open-charm [32–35], and open-bottom [34] sectors. In the 1990s, Capstick and Roberts calculated the $N\pi$ and the strange decays of nonstrange baryons [3] by using relativized quark model wave functions for baryons and mesons. The baryon and meson spectra were predicted within the relativized QMs of Refs. [21,36]. It is also worthwhile to cite Ref. [37], where the authors computed the open-flavor strong decays of Ξ baryons up to the $N = 2$ shell in a chiral quark model.

In this paper, we present our results for the open-flavor strong decay widths of light baryons (i.e., made up of u, d, s valence quarks) into a baryon-pseudoscalar meson pair and a baryon-vector meson pair. The widths are computed within a modified version of the $^3P_0$ pair-creation model where, for the first time in the baryon case, a strange quark pair suppression mechanism has been taken into account, analogous to what was done in the meson sector to suppress unphysical heavy quark pair creation [28–31,38]. The effects of this mechanism, which breaks the SU(3) symmetry, cannot be reabsorbed in a redefinition of the model parameters or in a different choice of the $^3P_0$ model vertex factor.

In the next section, we briefly mention the models for the baryon spectrum and structure that we have used for the calculation of the strong decays: the $U(7)$ algebraic model [39,40] by Bijker *et al.* and the hypercentral model (hQM) [41] developed by Giannini and Santopinto. In Sec. III, we review the $^3P_0$ model for the two-body decay of a baryon into a baryon and a meson, including a discussion of the phase space factor. The results for the strong decays are presented in Sec. IV. Finally, we present a summary and conclusions. Some details of the calculation of $^3P_0$ matrix elements are presented in the appendixes. Of particular interest is Appendix E, where the flavor couplings with $SU(3)$ breaking induced by the presence of the strange pair suppression mechanism have been derived explicitly for the first time.

II. STRANGE AND NONSTRANGE BARYON SPECTRA

A. $U(7)$ algebraic model

The baryon spectrum is computed by means of algebraic methods introduced by Bijker *et al.* [39,40]. The algebraic structure of the model consists of combining the symmetry of the internal spin-flavor-color part, $SU_{\rm sf}(6) \otimes SU_{\rm c}(3)$, with that of the spatial part, $U(7)$ into

$$U(7) \otimes SU_{\rm sf}(6) \otimes SU_{\rm c}(3). \tag{1}$$

The $U(7)$ model was introduced [39] to describe the relative motion of the three constituent parts of the baryon. The general idea is to introduce a so-called spectrum-generating algebra $U(k+1)$ for quantum systems characterized by k degrees of freedom. For baryons, there are the $k = 6$ relevant degrees of freedom of the two relative Jacobi vectors

$$\vec{\rho} = \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2),$$
$$\vec{\lambda} = \frac{1}{\sqrt{6}}(\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3), \tag{2}$$

and their canonically conjugate momenta, $\vec{p}_\rho = (\vec{p}_1 - \vec{p}_2)/\sqrt{2}$ and $\vec{p}_\lambda = (\vec{p}_1 + \vec{p}_2 - 2\vec{p}_3)/\sqrt{6}$. The $U(7)$ model is based on a bosonic quantization that consists of introducing two vector boson operators, $b_\rho^\dagger$ and $b_\lambda^\dagger$, associated to the Jacobi vectors, and an additional auxiliary scalar boson, $s^\dagger$. The scalar boson does not represent an independent degree of freedom, but it is added under the restriction that the total number of bosons N is conserved. The model space consists of harmonic oscillator shells with $n = 0, 1, \ldots, N$.

The baryon mass formula is written as the sum of three terms,

$$\hat{M}^2 = M_0^2 + \hat{M}^2_{\rm space} + \hat{M}^2_{\rm sf}, \tag{3}$$

where M_0^2 is a constant, $\hat{M}^2_{\rm space}$ is a function of the spatial degrees of freedom, and $\hat{M}^2_{\rm sf}$ depends on the internal degrees of freedom. The spin-flavor part is treated in the same way as in Ref. [40] in terms of a generalized Gürsey-Radicati formula [42], which in turn is a generalization of the Gell-Mann-Okubo mass formula [43,44]

$$\hat{M}^2_{\rm sf} = a(\hat{C}_2(SU_{\rm sf}(6)) - 45) + b(\hat{C}_2(SU_{\rm f}(3)) - 9) + c\left(\hat{C}_2(SU_{\rm s}(2)) - \frac{3}{4}\right) + d(\hat{C}_1(U_{\rm Y}(1)) - 1) + e(\hat{C}_2(U_{\rm Y}(1)) - 1) + f\left(\hat{C}_2(SU_{\rm I}(2)) - \frac{3}{4}\right). \tag{4}$$

The operators $\hat{C}_1(G)$ and $\hat{C}_2(G)$ correspond to the linear and quadratic Casimir invariants of the relevant groups for the internal degrees of freedom. The values of the coefficients M_0^2, a, b, c, d, e, and f are taken from [40].

Since the space-spin-flavor wave function is symmetric under permutation group S_3 of the three identical constituents, the permutation symmetry of the spatial wave function has to be the same as that of the spin-flavor part. Thus, the spatial part of the mass operator $\hat{M}^2_{\rm space}$ has to be invariant under the S_3 permutation symmetry. The dependence of the mass spectrum on the spatial degrees of freedom is given by

$$\hat{M}^2_{\rm space} = \hat{M}^2_{\rm vib} + \hat{M}^2_{\rm rot}. \tag{5}$$

In Refs. [23,40], strong decays of baryons were studied in the collective string model, which is a special case of $U(7)$ in which the radial excitations are interpreted as rotations and vibrations of an oblate top, in combination with the elementary emission model for the strong decays. Here, we do the same, but calculate the decays in the ${}^3P_0$ pair-creation model [10,11,16]. The rotational part of the operator (5) is written in the same form as in [40]

$$\hat{M}^2_{\rm rot} = \alpha\sqrt{\hat{L}\cdot\hat{L} + \frac{1}{4}}, \tag{6}$$

with eigenvalues

$$M^2_{\rm rot} = \alpha\left(L + \frac{1}{2}\right). \tag{7}$$

In this way one gets linear Regge trajectories with a slope α, as required by the phenomenology [45]. The spectrum of the vibrational part is given by [40]

$$\hat{M}^2_{\rm vib} = M^2_{\rm vib} = \kappa_1 v_1 + \kappa_2 v_2, \tag{8}$$

where $v_1 = n_u$ and $v_2 = n_v + n_w$ are the vibrational quantum numbers, corresponding to the symmetric stretching vibration along the direction of the strings (breathing mode) and two degenerate bending vibrations of the strings. The spectrum consists of a series of vibrational excitations characterized by the labels (v_1, v_2), and a tower of rotational excitations built on top of each vibration. κ_1 and κ_2 are free parameters fitted to the data.

The spectra calculated for the nonstrange and strange baryons are shown in Tables III–XVI, in the column labeled as Mass (MeV). The baryon wave functions are denoted in the standard form as

$$|{}^{2S+1}\dim\{SU_f(3)\}_J[\dim\{SU_{sf}(6)\}, L_i^P]\rangle, \tag{9}$$

where S and J are the spin and total angular momentum $\vec{J} = \vec{L} + \vec{S}$. As an example, in this notation the nucleon and delta wave functions are given by $|{}^2 8_{1/2}[56, 0_1^+]\rangle$ and $|{}^4 10_{3/2}[56, 0_1^+]\rangle$, respectively.

B. Hypercentral model

The starting point of the hQM is the assumption that quark interaction is hypercentral; i.e., it only depends on the hyper-radius x [41,46],

$$V_{3q}(\vec{\rho}, \vec{\lambda}) = V(x), \tag{10}$$

with $x = \sqrt{\vec{\rho}^2 + \vec{\lambda}^2}$ [47]. Thus, the spatial part of the three-quark wave function, $\psi_{\rm space}$, is factorized as

$$\psi_{\rm space} = \psi_{3q}(\vec{\rho}, \vec{\lambda}) = \psi_{\gamma\nu}(x) Y_{[\gamma] l_\rho l_\lambda}(\Omega_\rho, \Omega_\lambda, \xi), \tag{11}$$

where the hyper-radial wave function, $\psi_{\gamma\nu}(x)$, is labeled by the grand angular quantum number γ and the number of nodes ν. $Y_{[\gamma] l_\rho l_\lambda}(\Omega_\rho, \Omega_\lambda, \xi)$ are the hyperspherical harmonics, with angles $\Omega_\rho = (\theta_\rho, \phi_\rho)$, $\Omega_\lambda = (\theta_\lambda, \phi_\lambda)$ and hyperangle $\xi = \arctan\frac{\rho}{\lambda}$ [47]. The dynamics is contained in $\psi_{\gamma\nu}(x)$, which is a solution of the hyper-radial equation

$$\left[\frac{d^2}{dx^2} + \frac{5}{x}\frac{d}{dx} - \frac{\gamma(\gamma+4)}{x^2}\right]\psi_{\gamma\nu}(x) = -2m[E - V_{3q}(x)]\psi_{\gamma\nu}(x). \tag{12}$$

In the hQM, the quark interaction has the form [41,46]

$$V(x) = -\frac{\tau}{x} + \alpha x, \tag{13}$$

where τ and α are free parameters fitted to the reproduction of the experimental data. Equation (13) can be seen as the hypercentral approximation of a Cornell-type quark interaction [12], whose form can be reproduced by lattice QCD calculations [48]. Now, to introduce splittings within the SU(6) multiplets, an SU(6)-breaking term must be added. In the case of the hQM, such violation of the SU(6) symmetry is provided by the hyperfine interaction [49,50]. The complete hQM Hamiltonian is then [41,46]

$$H_{\rm hQM} = 3m + \frac{\vec{p}_\rho^2}{2m} + \frac{\vec{p}_\lambda^2}{2m} - \frac{\tau}{x} + \alpha x + H_{\rm hyp}, \tag{14}$$

where $\vec{p}_\rho$ and $\vec{p}_\lambda$ are the momenta conjugated to the Jacobi coordinates $\vec{\rho}$ and $\vec{\lambda}$. In addition to τ and α, there are two more free parameters in the hQM: the constituent quark mass, m, and the strength of the hyperfine interaction. The former is taken, as usual, as $1/3$ of the nucleon mass. The latter, as in the case of τ and α, is fitted in [41] to the reproduction of the *** and **** resonances reported in the PDG [1]. The hQM has an approximate $O(7)$ symmetry [41].

C. Extension to strange baryons

In Ref. [51], the hQM was extended to strange baryons. Specifically, the authors considered a Hamiltonian whose SU(6) invariant part is the same as in the hypercentral model [41], while the SU(6) symmetry is broken by a Gürsey-Radicati-inspired interaction [42].

The complete Hamiltonian is given by

$$H = H_0 + H_{\rm GR}, \tag{15}$$

where

$$H_0 = 3m + \frac{p_\lambda^2}{2m} + \frac{p_\rho^2}{2m} + V(x) \tag{16}$$

and

$$V(x) = -\frac{\tau}{x} + \alpha x. \tag{17}$$

The Gürsey-Radicati-inspired interaction, $H_{\rm GR}$, which describes the splittings within the SU(6) baryon multiplets [51], is written in terms of Casimir operators as

$$\begin{aligned} H_{\rm GR} = {}& A\hat{C}_2(SU_{\rm sf}(6)) + B\hat{C}_2(SU_{\rm f}(3)) \\ &+ C\hat{C}_2(SU_{\rm s}(2)) + D\hat{C}_1(U_{\rm Y}(1)) \\ &+ E\left(\hat{C}_2(SU_{\rm I}(2)) - \frac{1}{4}\hat{C}_1^2(U_{\rm Y}(1))\right), \end{aligned} \tag{18}$$

where A, B, C, D, and E are free parameters fitted to the data with the values reported in Ref. [51].

III. $^3P_0$ PAIR-CREATION MODEL

The $^3P_0$ pair-creation model is an effective model to compute open-flavor strong decays [10,11,16]. Here, a hadron decay takes place in its rest frame and proceeds via the creation of an additional $q\bar{q}$ pair. The quark-antiquark pair is created with the quantum numbers of the vacuum, i.e., $J^{PC} = 0^{++}$ (see Fig. 1), and the decay amplitude can be expressed as [10,11,17,27,28,30,31]

$$\Gamma_{A\to BC} = \Phi_{A\to BC}(q_0)\sum_\ell |\langle BCq_0\ell J|T^\dagger|A\rangle|^2. \tag{19}$$

In this paper, we focus on the open-flavor strong decays of light baryons (i.e., made up of u, d, s quarks) in the $^3P_0$ model. We assume harmonic oscillator wave functions, depending on a single oscillator parameter α_b for the baryons and α_c for the mesons. The final state is characterized by the relative orbital angular momentum ℓ between B and C and a total angular momentum $\vec{J} = \vec{J}_b + \vec{J}_c + \vec{\ell}$.

A. Phase space factor

The coefficient $\Phi_{A\to BC}(q_0)$ is the phase space factor for the decay. We show three possible prescriptions. The first in the nonrelativistic expression,

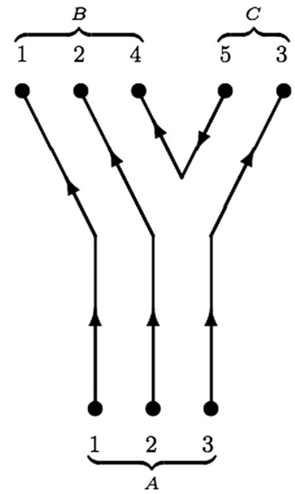

FIG. 1. The $^3P_0$ pair-creation model of hadron vertices. The $q\bar{q}$ pair (45) is created in a $^3P_0$ flavor-color singlet. A is the initial-state baryon; B and C are the final baryon and meson states, respectively.

$$\Phi_{A\to BC}(q_0) = 2\pi q_0 \frac{M_b M_c}{M_a}, \tag{20}$$

depending on the relative momentum q_0 between B and C and on the masses of the two intermediate-state hadrons, M_b and M_c. The second option is the standard relativistic form,

$$\Phi_{A\to BC}(q_0) = 2\pi q_0 \frac{E_b(q_0)E_c(q_0)}{M_a}, \tag{21}$$

depending on q_0 and on the energies of the two intermediate-state hadrons, $E_b = \sqrt{M_b^2 + q_0^2}$ and $E_c = \sqrt{M_c^2 + q_0^2}$. The third possibility is to use an effective phase space factor [3,15],

$$\Phi_{A\to BC}(q_0) = 2\pi q_0 \frac{\tilde{M}_b \tilde{M}_c}{M_a}, \tag{22}$$

where $\tilde{M}_b$ and $\tilde{M}_c$ are the effective baryon and meson masses, respectively, evaluated by means of a spin-independent interaction (see Table I). According to Ref. [15], this is valid in the weak-binding limit, where ρ and π are degenerate and $\tilde{m}_\pi = 5.1 m_\pi$.

In the case of heavy baryons and mesons, whose internal dynamics is almost nonrelativistic and the hyperfine interactions are relatively small, the three types of phase space factors provide almost the same results.

B. Transition operator

The transition operator of the $^3P_0$ model is given by [28,30,31]

TABLE I. Effective meson and baryon masses, $\tilde{M}$ [see Eq. (22)], from Refs. [3,15].

State	$\tilde{M}$ (GeV)
N	1.10
Δ	1.10
π	0.72
ρ	0.72
η	0.85
ω	0.85

$$T^\dagger = -3\gamma_0 \int d\vec{p}_4 d\vec{p}_5 \delta(\vec{p}_4 + \vec{p}_5) C_{45} F_{45} V(\vec{p}_4 - \vec{p}_5) \times [\chi_{45} \times \mathcal{Y}_1(\vec{p}_4 - \vec{p}_5)]_0^{(0)} b_4^\dagger(\vec{p}_4) d_5^\dagger(\vec{p}_5). \quad (23)$$

Here, γ_0 is the pair-creation strength, and $b_4^\dagger(\vec{p}_4)$ and $d_5^\dagger(\vec{p}_5)$ are the creation operators for a quark and an antiquark with momenta $\vec{p}_4$ and $\vec{p}_5$, respectively. The $q\bar{q}$ pair is characterized by a color-singlet wave function C_{45}, a flavor-singlet wave function F_{45}, a spin-triplet wave function χ_{45} with spin $S = 1$, and a solid spherical harmonic $\mathcal{Y}_1(\vec{p}_4 - \vec{p}_5)$, since the quark and antiquark are in a relative P-wave. $V(\vec{p}_4 - \vec{p}_5) = e^{-\alpha_d^2(\vec{p}_4 - \vec{p}_5)^2/8}$ is the pair-creation vertex or quark form factor. If one does not consider the quark form factor, i.e., $\alpha_d = 0$, the vertex reduces to a constant ($V = 1$); see Appendix A.

In our calculations, we introduce a strange quark-pair suppression mechanism in the baryon sector analogously to what was done in charmonium to suppress unphysical heavy quark pair creation [28,30,31,38]. The mechanism consists of introducing a reduction factor m_n/m_q in the pair-creation operator that suppresses the creation of strange quark pairs ($q = s$) by a factor of m_n/m_s with respect to the nonstrange quark pairs ($q = u$ and $q = d$). This particular choice for the pair-creation strength breaks the $SU(3)$ flavor symmetry and suppresses the creation of $s\bar{s}$ pairs. Its effect cannot be reabsorbed in a redefinition of the model parameters or in a different choice of the $^3P_0$ model vertex factor.

In Appendix D we present the derivation of the transition amplitudes in the $^3P_0$ pair-creation model using the Jacobi coordinates of Eq. (2) and including the effects of a Gaussian smearing of the pair-creation vertex. In Appendix E we present a derivation of the flavor coupling coefficients including the strangeness suppression mechanism, which is valid for both pseudoscalar and vector mesons.

C. Mixing between $N(1535)S_{11}$ and $N(1650)S_{11}$

To better reproduce the experimental data, we introduce a mixing between $N(1535)S_{11}$ and $N(1650)S_{11}$ resonances,

$$|N(1535)S_{11}\rangle = |^2 8_{1/2}\rangle \cos\theta + |^4 8_{1/2}\rangle \sin\theta,$$
$$|N(1650)S_{11}\rangle = -|^2 8_{1/2}\rangle \sin\theta + |^4 8_{1/2}\rangle \cos\theta, \quad (24)$$

where $\theta = 38°$ is the mixing angle. This was done in Refs. [40,52], to correct the disagreement between experimental and theoretical results for the helicity amplitudes of the $N(1535)S_{11}$ and $N(1650)S_{11}$ resonances. In Ref. [53] this problem was solved with a mixture of pseudoscalar meson-baryon and vector meson-baryon in a coupled-channel scheme.

IV. OPEN-FLAVOR STRONG DECAYS: RESULTS AND DISCUSSION

In this section, we present the results of our calculations of the open-flavor strong decays of nonstrange baryons and hyperons into baryon-pseudoscalar and baryon-vector mesons, using the $U(7)$ model (see Sec. IV A) and hQM model (see Sec. IV B). The decay amplitudes are computed in the $^3P_0$ model of Refs. [10,11] and Sec. III.

A. Open-flavor strong decays calculated by using the $U(7)$ model

Here, we show our results for the open-flavor decays by using the $U(7)$ algebraic model of Sec. II A and Refs. [39,40].

The strong decay widths are computed in the $^3P_0$ model using Eqs. (19), (D7), and (D12), by considering two possible choices for the phase space factor: the standard relativistic form of Eq. (21) and the effective phase space factor of Eq. (22). The results obtained with the relativistic phase space factor (RPSF) and the model parameters of the second column of Table II are reported in Tables III–XVI; those obtained with the effective phase space factor (EPSF) and the model parameters of the third column of Table II are reported in Tables III–IV. The $^3P_0$ model parameters of Table II are fitted to a sample of 9 transitions, as discussed in Appendix A. In our calculations, whenever available we use the experimental values for the masses of the decaying

TABLE II. $^3P_0$ model parameter values used in the calculations, in combination with the relativistic phase space factor of Eq. (21) (column 2) and the effective phase space factor of Eq. (22) (column 3). The parameter values are obtained in a fit to the experimental strong decay widths; see Appendix A and Table XVIII, last column. The values of the constituent quark masses m_n ($n = u$, d) and m_s are used in the pair-creation operator of Eq. (D10). α_b is the harmonic oscillator parameter of baryons A and B, α_c that of meson C, and α_d is the quark form factor parameter.

Parameter	Rel. PSF	Eff. PSF
γ_0	14.3	13.2
α_b	2.99 GeV^{-1}	2.69 GeV^{-1}
α_c	2.38 GeV^{-1}	2.02 GeV^{-1}
α_d	0.52 GeV^{-1}	0.82 GeV^{-1}
m_n	0.33 GeV	
m_s	0.55 GeV	

TABLE III. Strong decay widths of three- and four-star nucleon resonances (in MeV), calculated with the $U(7)$ algebraic model and the hypercentral QM. For the $U(7)$ model the calculation is done for the relativistic and the effective phase space factors of Eqs. (21) and (22), respectively, in combination with the parameters of Table II (RPSF and EPSF). For the hypercentral QM we present the results for the relativistic phase space factor (RPSF) in combination with the parameters of Table XVII. The experimental values are taken from Ref. [1]. Decay channels labeled by three dots are below threshold or forbidden by selection rules. The symbols (S) and (D) stand for S- and D-wave decays, respectively.

Model	Resonance	Status	Mass (MeV)	$N\pi$	$N\eta$	ΣK	ΛK	$\Delta\pi$	$N\rho$	$N\omega$	
	$N(1440)P_{11}$	****	1430–1470	110–338	0–5			22–101			Exp.
$U(7)$	$^28_{1/2}[56,0_2^+]$		1444	85	…	…	…	13	…	…	RPSF
$U(7)$	$^28_{1/2}[56,0_2^+]$		1444	108	…			22			EPSF
hQM	$^28_{1/2}[56,0_2^+]$		1550	105	…	…	…	12	…	…	RPSF
	$N(1520)D_{13}$	****	1515–1530	102	0			342			Exp.
$U(7)$	$^28_{3/2}[70,1_1^-]$		1563	134	0	…	…	207	…	…	RPSF
$U(7)$	$^28_{3/2}[70,1_1^-]$		1563	102	0			342			EPSF
hQM	$^28_{3/2}[70,1_1^-]$		1525	111	0	…	…	206	…	…	RPSF
	$N(1535)S_{11}$	****	1520–1555	44–96	40–91			<2			Exp.
$U(7)$	$^28_{1/2}[70,1_1^-]$		1563	63	75	…	…	16	…	…	RPSF
$U(7)$	$^28_{1/2}[70,1_1^-]$		1563	106	86			14			EPSF
hQM	$^28_{1/2}[70,1_1^-]$		1525	84	50	…	…	6	…	…	RPSF
	$N(1650)S_{11}$	****	1640–1680	60–162	6–27		4–20	0–45			Exp.
$U(7)$	$^48_{1/2}[70,1_1^-]$		1683	41	72	…	0	18	…	…	RPSF
$U(7)$	$^48_{1/2}[70,1_1^-]$		1683	71	83			15			EPSF
hQM	$^28_{1/2}[70,1_2^-]$		1574	51	29	…	0	4	…	…	RPSF
	$N(1675)D_{15}$	****	1670–1685	46–74	0–2		<2	65–99			Exp.
$U(7)$	$^48_{5/2}[70,1_1^-]$		1683	47	11	…	0	108	…	…	RPSF
$U(7)$	$^48_{5/2}[70,1_1^-]$		1683	29	7			79			EPSF
hQM	$^48_{5/2}[70,1_1^-]$		1579	41	9	…	…	85	…	…	RPSF
	$N(1680)F_{15}$	****	1675–1690	78–98	0–1			6–21			Exp.
$U(7)$	$^28_{5/2}[56,2_1^+]$		1737	121	1	…	0	100	…	…	RPSF
$U(7)$	$^28_{5/2}[56,2_1^+]$		1737	63	0			99			EPSF
hQM	$^28_{5/2}[56,2_1^+]$		1798	91	0	0	0	92	…	…	RPSF
	$N(1700)D_{13}$	***	1650–1750	7–43	0–3		<8	10–225 (S) <50 (D)			Exp.
$U(7)$	$^48_{3/2}[70,1_1^-]$		1683	9	3	…	0	561	…	…	RPSF
$U(7)$	$^48_{3/2}[70,1_1^-]$		1683	5	2			657			EPSF
hQM	$^28_{3/2}[70,1_2^-]$		1606	0	0	0	0	0	…	…	RPSF
	$N(1710)P_{11}$	***	1680–1740	3–50	5–75		3–63	8–100	3–63		Exp.
$U(7)$	$^28_{1/2}[70,0_1^+]$		1683	5	9	0	3	56	…	…	RPSF
$U(7)$	$^28_{1/2}[70,0_1^+]$		1683	11	9			58			EPSF
hQM	$^28_{1/2}[70,0_1^+]$		1808	18	12	0	14.1	70	…	…	RPSF
	$N(1720)P_{13}$	****	1650–1750	12–56	5–20		2–60	90–360	105–340		Exp.
$U(7)$	$^28_{3/2}[56,2_1^+]$		1737	111	7	0	14	36	5	0	RPSF
$U(7)$	$^28_{3/2}[56,2_1^+]$		1737	123	7			28			EPSF
hQM	$^28_{3/2}[56,2_1^+]$		1797	141	8	0	12	30	77	5	RPSF
	$N(1875)D_{13}$	***	1820–1920	3–70	0–22	0–4		48–192 (S) 11–86 (D)	0–38	22–90	Exp. Exp.
$U(7)$	$^48_{3/2}[70,2_1^-]$		1975	0	0	0	0	0	0	0	RPSF
$U(7)$	$^48_{3/2}[70,2_1^-]$		1975	0	0			0			EPSF
hQM	$^48_{3/2}[70,1_1^-]$		1899	14	8	2	0	560	80	82	RPSF
	$N(1900)P_{13}$	***	1875–1935	20–37	24–44	6–26	0–37			75–120	Exp.
$U(7)$	$^28_{3/2}[70,2_1^+]$		1874	11	12	1	13	63	64	24	RPSF
$U(7)$	$^28_{3/2}[70,2_1^+]$		1874	17	11			33			EPSF
hQM	$^28_{3/2}[70,2_1^+]$		1853	15	12	1	13	70	53	23	RPSF

TABLE IV. As Table III, but for Δ resonances. The symbols (*S*), (*P*), (*D*), and (*F*) stand for *S*-, *P*-, *D*-, and *F*-wave decays, respectively.

Model	Resonance	Status	Mass (MeV)	$N\pi$	ΣK	$\Delta\pi$	$\Delta\eta$	$\Sigma^* K$	$N\rho$	
	$\Delta(1232)P_{33}$	****	1230–1234	114–120						Exp.
$U(7)$	$^4 10_{3/2}[56, 0_1^+]$		1246	71	…	…	…	…	…	RPSF
$U(7)$	$^4 10_{3/2}[56, 0_1^+]$		1246	115		…	…			EPSF
hQM	$^4 10_{3/2}[56, 0_1^+]$		1240	63	…	…	…	…	…	RPSF
	$\Delta(1600)P_{33}$	***	1550–1700	22–105		88–294			<88	Exp.
$U(7)$	$^4 10_{3/2}[56, 0_2^+]$		1660	17	…	65	…	…	…	RPSF
$U(7)$	$^4 10_{3/2}[56, 0_2^+]$		1660	24		74	…			EPSF
hQM	$^4 10_{3/2}[56, 0_2^+]$		1727	31	…	69	…	…	…	RPSF
	$\Delta(1620)S_{31}$	****	1615–1675	26–45		39–90			9–38	Exp.
$U(7)$	$^2 10_{1/2}[70, 1_1^-]$		1649	5	…	76	…	…	…	RPSF
$U(7)$	$^2 10_{1/2}[70, 1_1^-]$		1649	10		61	…			EPSF
hQM	$^2 10_{1/2}[70, 1_1^-]$		1584	9	…	59	…	…	…	RPSF
	$\Delta(1700)D_{33}$		1670–1770	20–80		50–200 (*S*) 10–60 (*D*)	6–28		48–165	Exp.
$U(7)$	$^2 10_{3/2}[70, 1_1^-]$		1649	46	…	311	…	…	…	RPSF
$U(7)$	$^2 10_{3/2}[70, 1_1^-]$		1649	27		343	…			EPSF
hQM	$^2 10_{3/2}[70, 1_1^-]$		1584	40	…	333	…	…	…	RPSF
	$\Delta(1905)F_{35}$	****	1855–1910	24–60		<100			>162	Exp.
$U(7)$	$^4 10_{5/2}[56, 2_1^+]$		1921	31	1	188	19	0	99	RPSF
$U(7)$	$^4 10_{5/2}[56, 2_1^+]$		1921	14		139	14			EPSF
hQM	$^4 10_{5/2}[56, 2_1^+]$		1844	26	0	182	15	…	88	RPSF
	$\Delta(1910)P_{31}$	****	1860–1910	33–102	9–48	70–299				Exp.
$U(7)$	$^4 10_{1/2}[56, 2_1^+]$		1921	26	38	32	4	…	64	RPSF
$U(7)$	$^4 10_{1/2}[56, 2_1^+]$		1921	39		27	3			EPSF
hQM	$^4 10_{1/2}[56, 2_1^+]$		1871	49	38	34	4	…	60	RPSF
	$\Delta(1920)P_{33}$	***	1900–1970	9–60	3–7	18–102 (*P*) 45–195 (*F*)	13–69		0	Exp.
$U(7)$	$^4 10_{3/2}[56, 2_1^+]$		1921	7	23	132	22	5	105	RPSF
$U(7)$	$^4 10_{3/2}[56, 2_1^+]$		1921	14		96	15			EPSF
hQM	$^4 10_{3/2}[56, 2_1^+]$		1856	17	22	137	20	…	102	RPSF
	$\Delta(1930)D_{35}$	***	1920–1970	11–75						Exp.
$U(7)$	$^2 10_{5/2}[70, 2_1^-]$		1946	0	0	0	0	0	0	RPSF
$U(7)$	$^2 10_{5/2}[70, 2_1^-]$		1946	0		0	0			EPSF
	$\Delta(1950)F_{37}$	****	1940–1960	82–151	1–2	47–101			<34	Exp.
$U(7)$	$^4 10_{7/2}[56, 2_1^+]$		1921	172	5	92	1	0	22	RPSF
$U(7)$	$^4 10_{7/2}[56, 2_1^+]$		1921	72		40	1			EPSF
hQM	$^4 10_{7/2}[56, 2_1^+]$		1851	146	3	70	1	…	16	RPSF

resonances from the PDG [1]; otherwise, we use the theoretical predictions of Sec. II A.

B. Open-flavor strong decays calculated by using the hQM

Below, we provide results for the open-flavor decay widths of strange and nonstrange baryons into light baryons plus pseudoscalar or vector mesons in the $^3P_0$ model formalism of Sec. III, using the hQM results of Refs. [41,46,51]. For the results, see Tables III–IX. The decays are calculated with the new values of the $^3P_0$ model parameters of Table XVII, which we fitted to a sample of nine transitions: $\Delta \to N\pi$, $N(1520) \to N\pi$, $N(1535) \to N\pi$, $N(1650) \to N\pi$, $N(1680) \to N\pi$, $N(1720) \to N\pi$, $\Delta(1905) \to N\pi$, $\Delta(1910) \to N\pi$, and $\Delta(1920) \to N\pi$.

Although we use the same decay model as in Sec. IV A, there are some differences with respect to the previous case. (1) As in Sec. IV A, for *** and **** states we use the experimental values of the masses, but the quantum number assignments are provided by the hQM and do not always coincide with those of the $U(7)$ model, since the two models are different. For example, this happens for $N(1650)S_{11}$, $N(1700)D_{13}$, and $N(1875)D_{13}$. Thus, in these cases, we expect to obtain quite different results for the decay widths. (2) The hQM and $U(7)$ models predict

TABLE V. As Table III, but for Σ and Σ^* resonances.

Model	Baryon	Status	Mass (MeV)	$N\bar{K}$	$\Sigma\pi$	$\Lambda\pi$	$\Sigma\eta$	ΞK	$\Delta\bar{K}$	$\Sigma^*\pi$	$N\bar{K}^*$	$\Sigma\rho$	$\Lambda\rho$	$\Sigma\omega$	
	$\Sigma(1660)P_{11}$	***	1630–1690	4–60	Seen	Seen									Exp.
$U(7)$	$^2 8_{1/2}[56,0_2^+]$		1604	3	38	14	…	…	…	7	…	…	…	…	RPSF
hQM	$^2 8_{1/2}[56,0_2^+]$		1704	4	45	18	…	…	…	6	…	…	…	…	RPSF
	$\Sigma(1670)D_{13}$	****	1665–1685	3–10	12–48	2–12									Exp.
$U(7)$	$^4 8_{3/2}[70,1_1^-]$		1711	5	78	8	…	…	…	36	…	…	…	…	RPSF
hQM	$^2 8_{3/2}[70,1_1^-]$		1799	4	62	7	…	…	…	29	…	…	…	…	RPSF
	$\Sigma(1750)S_{11}$	***	1730–1800	6–64	<13	Seen	9–88								Exp.
$U(7)$	$^4 8_{1/2}[70,1_1^-]$		1711	3	109	3	28	…	…	9	…	…	…	…	RPSF
hQM	$^2 8_{1/2}[70,1_1^-]$		1799	5	151	6	27	…	…	7	…	…	…	…	RPSF
	$\Sigma(1775)D_{15}$	****	1770–1780	39–58	2–7	15–27				8–16					Exp.
$U(7)$	$^4 8_{5/2}[70,1_1^-]$		1822	101	17	38	0	…	4	11	…	…	…	…	RPSF
hQM	$^4 8_{5/2}[70,1_1^-]$		1914	79	13	30	0	…	2	7	…	…	…	…	RPSF
	$\Sigma(1915)F_{15}$	****	1900–1935	4–24	Seen	Seen				<8					Exp.
$U(7)$	$^2 8_{5/2}[56,2_1^+]$		1872	6	58	33	2	1	96	23	6	…	2	…	RPSF
hQM	$^2 8_{5/2}[56,2_1^+]$		1906	4	44	26	1	0	88	22	5	…	2	…	RPSF
	$\Sigma(1940)D_{13}$	***	1900–1950	<60	Seen	Seen			Seen	Seen	Seen				Exp.
$U(7)$	$^2 8_{3/2}[56,1_1^-]$		1974	0	0	0	0	0	0	0	0	…	0	…	RPSF
hQM	$^4 8_{3/2}[70,1_1^-]$		1914	31	6	11	0	0	554	99	251	…	0	…	RPSF
	$\Sigma^*(1385)P_{13}$	****	1383–1385		30–32	4–5									Exp.
$U(7)$	$^4 10_{3/2}[56,0_1^+]$		1382	…	3	27	…	…	…	…	…	…	…	…	RPSF
hQM	$^4 10_{3/2}[56,0_1^+]$		1372	…	3	24	…	…	…	…	…	…	…	…	RPSF
	$\Sigma^*(2030)F_{17}$	****	2025–2040	26–46	8–20	26–46		<4	15–40	8–30					Exp.
$U(7)$	$^4 10_{7/2}[56,2_1^+]$		2012	54	37	75	8	1	30	37	7	0	4	0	RPSF
hQM	$^4 10_{7/2}[56,2_1^+]$		2085	44	30	62	6	1	22	27	5	0	2	0	RPSF

the existence of a few missing states below the energy of 2.1 GeV. The masses, quantum numbers, and also the quantity of missing states in the two previous models are different. Information on missing states is important to the experimentalists in their search for new baryon resonances.

C. Comparison with other QM calculations

The quality of our results is comparable to that of Refs. [3,54]. Capstick and Roberts (CR) studied the strong decays of nonstrange baryons, nucleon and delta resonances, by using Capstick and Isgur's relativized baryon model [36] to describe the masses of unknown resonances, harmonic oscillator wave functions and an effective phase space [3,54]; they did not calculate the open-flavor decays of strange baryons. Our study is more complete, as it includes many more decay channels (such as, for example, decays into Ξ + meson) and a detailed analysis of the decays of strange baryons. Both calculations, ours and Capstick and Roberts's [3,54], are performed within the $^3P_0$ model, though there are some differences. The main one is in the pair-creation mechanism: in Refs. [3,54] it does not depend on the flavor of the created $q\bar{q}$ pair, while in the present case the strange quark pair-creation is suppressed with respect to the nonstrange pairs. The effects of this strangeness-suppression mechanism cannot be reabsorbed in a redefinition of the model parameters or in a different choice of the $^3P_0$ model vertex factor. Whether $s\bar{s}$ pair-creation has to be suppressed or not may be evaluated by comparing theoretical predictions and experimental data for these particular decay channels. Unfortunately, we think that the large uncertainties on experimental decay widths into channels due to $s\bar{s}$ pair-creation do not permit a strong definitive conclusion to be drawn. Nevertheless, it is worthwhile to observe that the introduction of a strangeness suppression mechanism may be justified by experimental results concerning the electroproduction ratios of ΛK^+, $\Sigma^* K$, $p\pi^0$, and $n\pi^+$ baryon meson-states from N^*'s [55].

In our paper, we also compared results obtained with relativistic or effective phase space factors. Our conclusion is that the results for the amplitudes calculated by using one phase space or the other have a similar quality. Thus, we think that it is preferable to use a relativistic phase space, in order to reduce the number of unnecessary parametrizations. Finally, we can say that both studies are characterized by the same problems [a few results are far from data, like $\Gamma_{N(1700)D_{13}\to\Delta\pi}$] and strengths (a great number of data are fitted with a few parameters). Similar problems can be found in previous works, for instance, CR obtained for the decay widths $N(1535)S_{11} \to N\pi$, $N(1700)D_{13} \to \Delta\pi$, and $N(1720)P_{13} \to N\rho$, the following theoretical results 216 MeV, 778 MeV, and 11 MeV, respectively, which

TABLE VI. As Table III, but for Λ and Λ^* resonances.

Model	Baryon	Status	Mass (MeV)	$N\bar{K}$	$\Sigma\pi$	$\Lambda\eta$	ΞK	$\Sigma^*\pi$	$N\bar{K}^*$	$\Sigma\rho$	$\Lambda\omega$	
	$\Lambda(1600)P_{01}$	***	1560–1700	8–75	5–150							Exp.
$U(7)$	$^28_{1/2}[56,0_2^+]$		1577	79	40	…	…	19	…	…	…	RPSF
hQM	$^28_{1/2}[56,0_2^+]$		1627	93	46	…	…	18	…	…	…	RPSF
	$\Lambda(1670)S_{01}$	****	1660–1680	5–15	6–28	3–13						Exp.
$U(7)$	$^28_{1/2}[70,1_1^-]$		1686	217	33	9	…	7	…	…	…	RPSF
hQM	$^28_{1/2}[70,1_1^-]$		1722	267	39	0	…	5	…	…	…	RPSF
	$\Lambda(1690)D_{03}$	****	1685–1690	10–21	10–28							Exp.
$U(7)$	$^28_{3/2}[70,1_1^-]$		1686	150	16	0	…	168	…	…	…	RPSF
hQM	$^28_{3/2}[70,1_1^-]$		1722	119	23	0	…	168	…	…	…	RPSF
	$\Lambda(1800)S_{01}$	***	1720–1850	50–160	Seen			Seen				Exp.
$U(7)$	$^48_{1/2}[70,1_1^-]$		1799	0	67	85	…	13	…	…	…	RPSF
hQM	$^48_{1/2}[70,1_1^-]$		1837	0	98	90	…	10	…	…	…	RPSF
	$\Lambda(1810)P_{01}$	***	1750–1850	10–125	5–100			Seen				Exp.
$U(7)$	$^28_{1/2}[70,0_1^+]$		1799	16	4	2	…	40	…	…	…	RPSF
hQM	$^28_{1/2}[56,0_3^+]$		1973	0	0	0	…	0	…	…	…	RPSF
	$\Lambda(1820)F_{05}$	****	1815–1825	39–59	6–13			4–9				Exp.
$U(7)$	$^28_{5/2}[56,2_1^+]$		1849	78	31	1	0	73	…	…	…	RPSF
hQM	$^28_{5/2}[56,2_1^+]$		1829	57	22	0	0	66	…	…	…	RPSF
	$\Lambda(1830)D_{05}$	****	1810–1830	2–11	21–83			>9				Exp.
$U(7)$	$^48_{5/2}[70,1_1^-]$		1799	0	99	9	0	82	…	…	…	RPSF
hQM	$^48_{5/2}[70,1_1^-]$		1837	0	84	7	0	64	…	…	…	RPSF
	$\Lambda(1890)P_{03}$	****	1850–1910	12–70	2–20			Seen				Exp.
$U(7)$	$^28_{3/2}[56,2_1^+]$		1849	96	69	31	2	30	28	…	…	RPSF
hQM	$^28_{3/2}[56,2_1^+]$		1829	120	79	11	1	24	27	…	…	RPSF
	$\Lambda(2110)F_{05}$	****	2090–2140	8–63	15–100			Seen				Exp.
$U(7)$	$^48_{5/2}[70,2_1^+]$		2074	0	14	3	1	136	0	38	16	RPSF
hQM	$^28_{5/2}[70,2_1^+]$		1995	167	20	0	3	69	15	79	25	RPSF
	$\Lambda^*(1405)S_{01}$	****	1402–1410		48–52							Exp.
$U(7)$	$^21_{1/2}[70,1_1^-]$		1641	…	230	…	…	…	…	…	…	RPSF
hQM	$^21_{1/2}[70,1_1^-]$		1658	…	222	…	…	…	…	…	…	RPSF
	$\Lambda^*(1520)D_{03}$	****	1518–1520	6–8	6–7			1				Exp.
$U(7)$	$^21_{3/2}[70,1_1^-]$		1641	10	17	…	…	…	…	…	…	RPSF
hQM	$^21_{3/2}[70,1_1^-]$		1658	8	13	…	…	…	…	…	…	RPSF

are to be compared with the experimental data 44–96 MeV, 10–225 MeV, and 105–340 MeV, respectively.

We can also compare our results with those of Refs. [23,40]. Bijker, Iachello, and Leviatan (BIL) [23,40] computed the open-flavor decay amplitudes within a modified version of the elementary meson emission model (EME), with two parameters, and used the $U(7)$ algebraic model to calculate the baryon spectrum. The

TABLE VII. As Table III, but for Ξ and Ξ^* resonances.

Model	Baryon	Status	Mass (MeV)	$\Sigma\bar{K}$	$\Lambda\bar{K}$	$\Xi\pi$	$\Xi^*\pi$	
	$\Xi(1690)S_{11}$	***	1680–1700					Exp.
$U(7)$	$^28_{1/2}[70,1_1^-]$		1828	58	85	14	0	RPSF
hQM	$^28_{1/2}[70,1_1^-]$		1938	55	86	15	0	RPSF
	$\Xi(1820)D_{13}$	***	1818–1828	2–18	3–12	0–8	2–18	Exp.
$U(7)$	$^28_{3/2}[70,1_1^-]$		1828	38	26	6	55	RPSF
hQM	$^28_{3/2}[70,1_1^-]$		1938	29	21	5	55	RPSF
	$\Xi^*(1530)P_{13}$	****	1531–1532			9–10		Exp.
$U(7)$	$^410_{3/2}[56,0_1^+]$		1524	…	…	11	…	RPSF
hQM	$^410_{3/2}[56,0_1^+]$		1511	…	…	9	…	RPSF

TABLE VIII. Strong decay widths of missing nucleon resonances (in MeV) calculated in the $U(7)$ model of Sec. II A and Refs. [39,40] (top) and the hypercentral QM of Sec. II B and Refs. [41,46] (bottom). The calculations are carried out using the model parameters of Table II (second column) and Table XVII, respectively, in combination with the relativistic phase space factor of Eq. (21). Tentative assignments of one and two star resonances are labeled by ‡.

N	Mass (MeV)	$N\pi$	$N\eta$	ΣK	ΛK	$\Delta\pi$	$\Sigma^* K$	$N\rho$	$N\omega$	ΣK^*	ΛK^*	$\Delta\rho$
						$U(7)$ Model						
${}^2 8_J[20, 1_1^+]$	1713	0	0	0	0	0	…	…	…	…	…	…
${}^4 8_{3/2}[70, 0_1^+]$	1796	0	3	5	0	65	…	7	7	…	…	…
${}^2 8_{5/2}[70, 2_1^+]$	1874‡	106	10	0	3	79	…	161	8	…	…	…
${}^2 8_J[70, 2_1^-]$	1874	0	0	0	0	0	…	0	0	…	…	…
${}^4 8_{1/2}[70, 2_1^+]$	1975‡	1	8	23	0	19	1	9	9	…	…	…
${}^4 8_{3/2}[70, 2_1^+]$	1975‡	0	4	11	0	109	5	14	14	…	…	…
${}^4 8_{5/2}[70, 2_1^+]$	1975‡	6	3	1	0	176	6	16	16	…	…	…
${}^4 8_{7/2}[70, 2_1^+]$	1975‡	25	13	4	0	99	0	5	4	…	…	…
${}^4 8_J[70, 2_1^-]$	1975‡	0	0	0	0	0	0	0	0	…	…	…
${}^2 8_{1/2}[56, 1_1^-]$	2094	5	1	1	5	3	2	48	6	2	2	14
${}^2 8_{3/2}[56, 1_1^-]$	2094‡	27	0	0	1	23	1	53	11	0	2	13
${}^2 8_{1/2}[70, 1_2^-]$	1829‡	42	7	0	1	38	…	0	0	…	…	…
${}^4 8_{1/2}[70, 1_2^-]$	1933	8	12	3	0	0	0	0	0	…	…	…
${}^4 8_{3/2}[70, 1_2^-]$	1933	0	0	3	0	0	0	0	0	…	…	…
${}^4 8_{5/2}[70, 1_2^-]$	1933	0	2	5	0	1	0	0	0	…	…	…
						hQM						
${}^4 8_{3/2}[70, 2_1^+]$	1835	4	8	7	0	97	…	8	7	…	…	…
${}^2 8_{1/2}[20, 1_1^+]$	1836	0	0	0	0	0	…	0	0	…	…	…
${}^2 8_{3/2}[20, 1_1^+]$	1836	0	0	0	0	0	…	0	0	…	…	…
${}^4 8_{1/2}[70, 2_1^+]$	1839‡	8	16	15	0	27	…	6	5	…	…	…
${}^4 8_{7/2}[70, 2_1^+]$	1840‡	12	4	0	0	25	…	0	0	…	…	…
${}^4 8_{5/2}[70, 2_1^+]$	1844‡	3	1	0	0	137	…	9	8	…	…	…
${}^4 8_{5/2}[70, 2_1^+]$	1851‡	3	1	0	0	137	…	9	9	…	…	…
${}^4 8_{3/2}[70, 0_1^+]$	1863‡	0	4	22	0	83	…	12	12	…	…	…
${}^4 8_{1/2}[70, 1_1^-]$	1887‡	0	22	119	0	87	…	32	32	…	…	…
${}^4 8_{1/2}[70, 1_2^-]$	1937	0	0	0	0	0	…	0	0	…	…	…
${}^4 8_{5/2}[70, 1_2^-]$	1942‡	0	0	0	0	0	0	0	0	…	…	…
${}^2 8_{1/2}[56, 0_3^+]$	1943‡	0	0	0	0	0	0	0	0	…	…	…
${}^4 8_{3/2}[70, 1_2^-]$	1969	0	0	0	0	0	0	0	0	…	…	…

TABLE IX. As Table VIII, but for missing Δ resonances.

Δ	Mass (MeV)	$N\pi$	ΣK	$\Delta\pi$	$\Delta\eta$	$\Sigma^* K$	$N\rho$
			$U(7)$ Model				
${}^2 10_{1/2}[70, 0_1^+]$	1764‡	0	1	70	…	…	23
${}^2 10_{3/2}[70, 2_1^-]$	1946	0	0	0	0	0	0
${}^2 10_{3/2}[70, 2_1^+]$	1947	1	3	106	5	1	80
${}^2 10_{5/2}[70, 2_1^+]$	1947‡	18	1	107	18	4	32
${}^2 10_{1/2}[70, 1_2^-]$	1904‡	0	0	0	0	0	0
${}^2 10_{3/2}[70, 1_2^-]$	1904	0	0	0	0	0	0
			hQM				
${}^2 10_{1/2}[70, 0_1^+]$	1832‡	0	2	89	7	…	57
${}^2 10_{3/2}[70, 2_1^+]$	1843	4	1	43	1	…	51
${}^2 10_{1/2}[70, 1_2^-]$	1947‡	0	0	1	0	0	0
${}^2 10_{3/2}[70, 1_2^-]$	1947‡	0	0	1	0	0	0
${}^2 10_{5/2}[70, 2_1^+]$	1859‡	10	0	97	7	…	13
${}^4 10_{3/2}[56, 0_3^+]$	2103	0	0	0	0	0	0

TABLE X. As Table VIII, but for missing Σ resonances.

Σ	Mass (MeV)	$N\bar{K}$	$\Sigma\pi$	$\Lambda\pi$	$\Sigma\eta$	ΞK	$\Delta\bar{K}$	$\Sigma^*\pi$	$\Sigma^*\eta$	$\Xi^* K$	$N\bar{K}^*$	$\Sigma\rho$	$\Lambda\rho$	$\Sigma\omega$	$\Delta\bar{K}^*$
							$U(7)$ Model								
$^4 8_{1/2}[70, 1_1^-]$	1822	24	11	6	20	10	5	4	…	…	…	…	…	…	…
$^4 8_{3/2}[70, 1_1^-]$	1822	22	4	8	0	0	602	98	…	…	…	…	…	…	…
$^2 8_{1/2}[70, 0_1^+]$	1822‡	1	20	1	1	0	33	9	…	…	…	…	…	…	…
$^2 8_J[20, 1_1^+]$	1849‡	0	0	0	0	0	0	0	…	…	0	…	…	…	…
$^2 8_{3/2}[56, 2_1^+]$	1872	4	62	19	23	12	17	6	…	…	12	…	…	…	…
$^4 8_{3/2}[70, 0_1^+]$	1926	0	0	0	5	1	65	12	…	…	25	…	3	…	…
$^2 8_{3/2}[70, 2_1^+]$	1999	1	31	1	7	14	28	8	1	…	26	11	6	1	…
$^2 8_{5/2}[70, 2_1^+]$	1999	4	76	6	1	1	60	13	4	…	7	13	21	3	…
$^2 8_{5/2}[70, 2_1^-]$	1999	0	0	0	0	0	0	0	0	…	0	0	0	0	…
$^4 8_{1/2}[70, 2_1^+]$	2095	4	2	1	3	4	18	4	4	1	23	5	9	7	…
$^4 8_{3/2}[70, 2_1^+]$	2095	2	1	1	2	2	84	18	13	2	39	7	14	11	…
$^4 8_{5/2}[70, 2_1^+]$	2095‡	15	3	5	1	0	128	29	18	3	45	8	15	11	…
$^4 8_{7/2}[70, 2_1^+]$	2095	69	13	24	2	1	54	15	1	0	17	0	3	1	…
$^4 8_J[70, 2_1^-]$	2095	0	0	0	0	0	0	0	0	0	0	0	0	0	…
$^2 8_{1/2}[70, 1_2^-]$	1957‡	0	0	0	0	0	0	0	0	…	0	…	0	…	…
$^2 8_{3/2}[70, 1_2^-]$	1957	0	0	0	0	0	0	0	0	…	0	…	0	…	…
$^4 8_{1/2}[70, 1_2^-]$	2055	0	0	0	0	0	0	0	0	…	0	0	0	0	…
$^4 8_{3/2}[70, 1_2^-]$	2055	0	0	0	0	0	1	0	0	…	0	0	0	0	…
$^4 8_{5/2}[70, 1_2^-]$	2055	0	0	0	0	0	0	0	0	…	0	0	0	0	…
							hQM								
$^2 8_{3/2}[56, 2_1^+]$	1906	4	69	22	19	20	20	7	…	…	21	…	…	…	…
$^4 8_{1/2}[70, 1_1^-]$	1914	9	7	3	17	22	22	9	…	…	100	…	…	…	…
$^2 8_{1/2}[56, 0_3^+]$	2050†	0	0	0	0	0	0	0	…	…	0	…	…	…	…
$^2 8_{3/2}[70, 2_1^+]$	2072	1	31	1	7	16	39	11	…	…	29	…	…	…	…
$^2 8_{5/2}[70, 2_1^+]$	2072	5	89	7	3	3	66	15	…	…	11	…	…	…	…
$^2 8_{1/2}[70, 1_2^-]$	2149	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$^4 8_{1/2}[70, 2_1^+]$	2187	3	2	1	3	4	16	3	5	1	21	6	9	2	16
$^4 8_{3/2}[70, 2_1^+]$	2187	1	1	1	1	2	91	21	17	5	40	10	15	3	18
$^4 8_{5/2}[70, 2_1^+]$	2187	19	3	6	1	0	147	34	23	6	52	10	18	3	49
$^4 8_{7/2}[70, 2_1^+]$	2187	83	15	29	2	1	83	21	3	0	30	2	7	1	159
$^2 8_J[20, 1_1^+]$	2238	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$^4 8_J[70, 1_2^-]$	2263	0	0	0	0	0	0	0	0	0	0	0	0	0	0

EME is an effective model, in which the decay occurs by the emission of a meson from the decaying hadron. Even though the EME and $^3P_0$ models share some features, there are important differences. For example, the internal quark dynamics is invisible to the EME vertex, because it does not depend on the meson internal wave function, which brings a nonlocal character to the $^3P_0$ matrix elements. Moreover, as in the CR case, BIL did not consider a suppression of $s\bar{s}$ pair creation, though it has been shown that this mechanism can be beneficial in meson strong decays [28,30,31]. It can also be shown that it is impossible to get a correspondence between EME and $^3P_0$ models, unless the factor in front of the recoil term in the EME is taken as equal to $k_0/2m = 1$. However, this factor has been taken as a free parameter in BIL (and also in other EME studies, Refs. [20,23,40]) and by fit it came out to be equal to 0.04, which means that the recoil term is practically absent in BIL. This fact, in combination with the nonlocal character of the $^3P_0$ model, the quark form factor, and flavor suppression, explains the differences in the two model predictions. A more detailed explanation is contained in Appendix F. Finally, we can say that the general quality of our results is comparable to those of Refs. [23,40]. Both of them reproduce the general trend of the data, but, in some cases, they show a few large disagreements with experiments. For example, the results of Refs. [23,40] do not agree with the data in the case of the decays $N(1720)P_{13} \to \Lambda K$ and $N(1440)P_{11} \to \Delta\pi$, where they get null amplitudes, while ours do agree. On the contrary, our results for other channels, like $N(1520)D_{13} \to \Delta\pi$ and $\Lambda(1670)S_{01} \to N\bar{K}$, do not agree with the data,

TABLE XI. As Table VIII, but for missing Σ^* resonances.

Σ	Mass (MeV)	$N\bar{K}$	$\Sigma\pi$	$\Lambda\pi$	$\Sigma\eta$	ΞK	$\Delta\bar{K}$	$\Sigma^*\pi$	$\Sigma^*\eta$	$\Xi^* K$	$N\bar{K}^*$	$\Sigma\rho$	$\Lambda\rho$	$\Sigma\omega$	$\Delta\bar{K}^*$
							$U(7)$ Model								
$^{2}10_{1/2}[70,1_1^-]$	1755	4	5	4	11	…	1	30	…	…	…	…	…	…	…
$^{2}10_{3/2}[70,1_1^-]$	1755	9	6	14	0	…	181	165	…	…	…	…	…	…	…
$^{2}10_{1/2}[70,0_1^+]$	1863	0	1	0	1	0	45	39	…	…	5	…	…	…	…
$^{4}10_{1/2}[56,2_1^+]$	2012	12	18	16	35	14	21	17	0	…	24	6	28	76	…
$^{4}10_{3/2}[56,2_1^+]$	2012	6	9	8	18	7	79	69	1	…	35	9	40	106	…
$^{4}10_{5/2}[56,2_1^+]$	2012	11	7	15	1	0	112	101	1	…	37	9	41	106	…
$^{2}10_{3/2}[70,2_1^+]$	$2037^{\ddagger}$	1	1	1	2	1	38	42	0	0	28	12	36	315	…
$^{2}10_{5/2}[70,2_1^+]$	2037	5	4	7	1	0	63	56	1	0	10	2	9	62	…
$^{2}10_{J}[70,2_1^-]$	2037	0	0	0	0	0	0	0	0	0	0	0	0	0	…
$^{4}10_{3/2}[56,0_2^+]$	$1765^{\ddagger}$	8	8	9	1	…	13	38	…	…	…	…	…	…	…
$^{2}10_{J}[70,1_2^-]$	1996	0	0	0	0	0	0	0	0	…	0	0	0	0	…
							hQM								
$^{4}10_{3/2}[56,0_2^+]$	1883	6	9	8	11	2	68	63	…	…	14	…	…	…	…
$^{4}10_{3/2}[56,2_1^+]$	2085	6	9	9	18	8	86	76	2	…	39	25	53	19	…
$^{2}10_{3/2}[70,2_1^+]$	2136	0	1	1	2	1	61	63	0	2	29	25	43	20	1
$^{2}10_{5/2}[70,2_1^+]$	2136	7	5	9	2	0	70	63	1	10	17	7	18	5	4
$^{2}10_{J}[70,1_2^-]$	2212	0	0	0	0	0	0	0	0	0	0	0	0	0	0

TABLE XII. As Table VIII, but for missing Λ resonances.

Λ	Mass (MeV)	$N\bar{K}$	$\Sigma\pi$	$\Lambda\eta$	ΞK	$\Sigma^*\pi$	$\Xi^* K$	$N\bar{K}^*$	$\Sigma\rho$	$\Lambda\omega$
					$U(7)$ Model					
$^{4}8_{3/2}[70,1_1^-]$	1799	0	15	1	…	447	…	…	…	…
$^{2}8_{J}[20,1_1^+]$	1826	0	0	0	0	0	…	…	…	…
$^{4}8_{3/2}[70,0_1^+]$	1904	0	3	4	2	54	…	0	…	0
$^{2}8_{3/2}[70,2_1^+]$	1978	27	6	4	10	31	…	56	1	4
$^{2}8_{5/2}[70,2_1^+]$	1978	109	12	2	1	58	…	123	3	16
$^{2}8_{J}[70,2_1^-]$	1978	0	0	0	0	0	…	0	0	0
$^{4}8_{J}[70,2_1^-]$	2074	0	0	0	0	0	0	0	0	0
$^{4}8_{1/2}[70,2_1^+]$	2075	0	13	11	12	17	1	0	20	9
$^{4}8_{3/2}[70,2_1^+]$	2075	0	6	6	6	82	4	0	28	13
$^{4}8_{7/2}[70,2_1^+]$	2075	0	51	10	2	57	0	0	1	2
$^{2}8_{J}[70,1_2^-]$	1936	0	0	0	0	0	…	0	…	0
$^{4}8_{1/2}[70,1_2^-]$	2034	0	0	0	0	0	0	0	0	0
$^{4}8_{3/2}[70,1_2^-]$	2034	0	0	0	0	1	0	0	0	0
$^{4}8_{5/2}[70,1_2^-]$	2034	0	0	0	0	0	0	0	0	0
					hQM					
$^{4}8_{3/2}[70,1_1^-]$	1837	0	15	2	…	477	…	…	…	…
$^{2}8_{3/2}[70,2_1^+]$	1995	38	8	0	10	29	…	55	2	4
$^{2}8_{1/2}[70,1_2^-]$	2072	0	0	0	0	0	…	0	…	0
$^{2}8_{3/2}[70,1_2^-]$	2072	0	0	0	0	0	0	0	0	0
$^{4}8_{3/2}[70,0_1^+]$	2110	0	0	1	4	35	11	0	41	8
$^{4}8_{1/2}[70,2_1^+]$	2110	0	18	13	12	35	2	0	23	9
$^{4}8_{3/2}[70,2_1^+]$	2110	0	10	6	2	87	7	0	33	14
$^{4}8_{7/2}[70,2_1^+]$	2110	0	50	10	2	19	0	0	2	2
$^{2}8_{J}[20,1_1^+]$	2160	0	0	0	0	0	0	0	0	0
$^{4}8_{1/2}[70,1_2^-]$	2186	0	0	0	0	0	0	0	0	0
$^{4}8_{3/2}[70,1_2^-]$	2186	0	0	0	0	1	0	0	0	0
$^{4}8_{5/2}[70,1_2^-]$	2186	0	0	0	0	0	0	0	0	0

TABLE XIII. As Table VIII, but for missing Λ^* resonances.

Λ	Mass (MeV)	$N\bar{K}$	$\Sigma\pi$	$\Lambda\eta$	ΞK	$N\bar{K}^*$	$\Sigma\rho$	$\Lambda\omega$
				$U(7)$ Model				
$^2 1_{1/2}[70, 0_1^+]$	1756	29	44	14	…	…	…	…
$^4 1_J[20, 1_1^+]$	1891	0	0	0	0	0	…	…
$^2 1_{3/2}[70, 2_1^+]$	1939	35	66	36	17	39	…	6
$^2 1_{5/2}[70, 2_1^+]$	1939	88	85	10	0	94	…	15
$^2 1_J[70, 2_1^-]$	1939	0	0	0	0	0	…	0
$^2 1_J[70, 1_2^-]$	1896	0	0	0	0	0	…	0
				hQM				
$^2 1_{1/2}[70, 1_2^-]$	2008	0	1	0	0	1	…	0
$^2 1_{3/2}[70, 1_2^-]$	2008	0	0	0	0	0	…	0

TABLE XIV. As Table VIII, but for missing Ξ resonances.

Ξ	Mass (MeV)	$\Sigma\bar{K}$	$\Lambda\bar{K}$	$\Xi\pi$	$\Xi\eta$	$\Sigma^*\bar{K}$	$\Xi^*\pi$	$\Lambda\bar{K}^*$	$\Sigma\bar{K}^*$	$\Xi\rho$	$\Xi\omega$
					$U(7)$ Model						
$^2 8_{1/2}[70, 0_1^+]$	1932	36	6	1	11	7	13	…	…	…	…
$^4 8_{1/2}[70, 1_1^-]$	1932	43	20	69	0	0	4	…	…	…	…
$^4 8_{3/2}[70, 1_1^-]$	1932	4	7	22	0	216	152	…	…	…	…
$^4 8_{5/2}[70, 1_1^-]$	1932	23	39	132	0	2	19	…	…	…	…
$^2 8_J[20, 1_1^+]$	1957	0	0	0	0	0	0	…	…	…	…
$^2 8_{3/2}[56, 2_1^+]$	1979	198	7	6	47	4	7	…	…	…	…
$^2 8_{5/2}[56, 2_1^+]$	1979	59	5	4	1	20	27	…	…	…	…
$^4 8_{3/2}[70, 0_1^+]$	2031	2	1	3	0	24	19	2	…	…	…
$^2 8_{1/2}[56, 0_2^+]$	1727	26	4	3	…	…	2	…	…	…	…
					hQM						
$^2 8_{1/2}[56, 0_2^+]$	1843	125	6	5	…	…	15	…	…	…	…
$^4 8_{3/2}[70, 1_1^-]$	2053	8	11	37	0	223	154	…	…	…	…
$^2 8_{1/2}[56, 0_3^+]$	2190	0	0	0	0	0	0	0	0	0	0
$^2 8_{1/2}[70, 1_2^-]$	2288	0	0	0	0	0	0	0	0	0	0
$^2 8_{3/2}[70, 1_2^-]$	2288	0	0	0	0	0	0	0	0	0	0
$^4 8_{1/2}[70, 2_1^+]$	2327	3	1	8	1	6	5	8	10	40	1
$^4 8_{3/2}[70, 2_1^+]$	2327	2	1	4	0	35	32	16	16	62	1
$^4 8_{5/2}[70, 2_1^+]$	2327	6	7	24	0	57	53	20	18	69	1
$^4 8_{7/2}[70, 2_1^+]$	2327	26	33	108	1	33	33	11	5	16	0
$^2 8_J[20, 1_1^+]$	2377	0	0	0	0	0	0	0	0	0	0
$^4 8_J[70, 1_2^-]$	2403	0	0	0	0	0	0	0	0	0	0

TABLE XV. As Table VIII, but for missing Ξ^* resonances.

Ξ	Mass (MeV)	$\Sigma\bar{K}$	$\Lambda\bar{K}$	$\Xi\pi$	$\Xi\eta$	$\Sigma^*\bar{K}$	$\Xi^*\pi$	$\Lambda\bar{K}^*$	$\Sigma\bar{K}^*$	$\Xi\rho$	$\Xi\omega$
					$U(7)$ Model						
$^2 10_{1/2}[70, 1_1^-]$	1869	17	10	7	7	…	7	…	…	…	…
$^2 10_{3/2}[70, 1_1^-]$	1869	5	10	9	0	…	61	…	…	…	…
$^2 10_{1/2}[70, 0_1^+]$	1971	2	1	1	2	51	14	…	…	…	…
$^4 10_{3/2}[56, 0_2^+]$	1878	19	16	13	1	…	12	…	…	…	…
					hQM						
$^4 10_{3/2}[56, 0_2^+]$	2022	19	12	13	12	…	24	…	…	…	…
$^4 10_{1/2}[56, 2_1^+]$	2225	33	19	23	35	33	7	40	34	31	16
$^4 10_{3/2}[56, 2_1^+]$	2225	17	10	12	17	133	29	61	50	44	23
$^4 10_{5/2}[56, 2_1^+]$	2225	14	19	16	3	194	44	66	51	45	24
$^4 10_{7/2}[56, 2_1^+]$	2225	64	87	70	13	56	16	13	3	2	1
$^2 10_{1/2}[70, 1_2^-]$	2352	0	0	0	0	0	0	0	0	0	0
$^2 10_{3/2}[70, 1_2^-]$	2352	0	0	0	0	1	0	0	0	0	0
$^4 10_{3/2}[56, 0_3^+]$	2369	0	0	0	0	0	0	0	0	0	0

TABLE XVI. As Table VIII, but for missing Ω resonances.

Ω	Mass (MeV)	$\Xi\bar{K}$	$\Xi^*\bar{K}$	$\Omega\eta$	$\Xi\bar{K}^*$
		$U(7)$ Model			
$^2 10_{1/2}[70,1_1^-]$	1989	68	…	…	…
$^2 10_{3/2}[70,1_1^-]$	1989	20	…	…	…
$^2 10_{1/2}[70,0_1^+]$	2085	8	32	…	…
$^4 10_{3/2}[56,0_2^+]$	1998	79	…	…	…
		hQM			
$^2 10_{1/2}[70,1_1^-]$	2142	26	48	…	…
$^2 10_{3/2}[70,1_1^-]$	2142	68	403	…	…
$^4 10_{3/2}[56,0_2^+]$	2162	68	102	…	…
$^4 10_{1/2}[56,2_2^+]$	2364	109	34	27	155
$^4 10_{3/2}[56,2_2^+]$	2364	55	137	88	225
$^4 10_{5/2}[56,2_2^+]$	2364	69	199	117	234
$^4 10_{7/2}[56,2_2^+]$	2364	308	58	4	23
$^2 10_{1/2}[70,1_2^-]$	2492	0	0	0	0
$^2 10_{3/2}[70,1_2^-]$	2492	0	1	0	0
$^4 10_{3/2}[56,0_3^+]$	2508	0	0	0	0

TABLE XVII. Parameter values used in the calculations, in combination with the relativistic phase space factor of Eq. (21). The parameter values are fitted to a sample of nine transitions: $\Delta \to N\pi$, $N(1520) \to N\pi$, $N(1535) \to N\pi$, $N(1650) \to N\pi$, $N(1680) \to N\pi$, $N(1720) \to N\pi$, $\Delta(1905) \to N\pi$, $\Delta(1910) \to N\pi$, and $\Delta(1920) \to N\pi$. The quantum number assignments for the decaying states are now taken from the hQM results of Refs. [41,46] and Table III.

Parameter	Value
γ_0	13.319
α_b	2.758
α_c	2.454
α_d	0
m_n	0.33
m_s	0.55

being much larger, while those of Refs. [23,40] do agree or they are closer.

There was also an attempt to improve the study of the strong decays using covariant calculations with relativistic constituent quark models (rCQM) by Melde *et al.* [56,57]. The authors computed the transitions for all π, η, and K strong decay modes of several well-established nonstrange and strange baryon states, using the so-called point-form spectator model (PFSM), whose nonrelativistic limit is the classic EME; they did not calculate decay amplitudes into baryon-vector meson pairs. Unfortunately, these results still cannot provide a satisfactory explanation of the experimental decay widths and, in general, underestimate the available experimental data. Finally, it is worthwhile to cite the results of a dynamical coupled-channel study of the $\pi N \to \pi\pi N$ reactions of Ref. [58], where the authors provided a comprehensive analysis of world data of πN, γN, and $N(e,e')$ reactions, including a coupled-channel model for meson production reactions considering all possible final states.

D. Exotic states and alternative decay modes

As widely discussed in the literature, there are several baryons (mesons) whose nature may not be a pure three-quark (quark-antiquark) one. Some well-known examples are the $X(3872)$ [28,29,38,59–62] and the $D_{s0}^*(2317)$ and $D_{s1}(2460)$ [63,64] mesons. Some alternative hypotheses (hadron-hadron molecules, hybrids, tetra/pentaquarks, and so on) might explain why naïve quark models fail to reproduce some of the main properties of these resonances, including mass and decay modes.

Let us focus on $\Lambda^*(1405)$. By combining the $U(7)$ and $^3P_0$ models, we get a mass of 1641 MeV and a $\Sigma\pi$ amplitude of 230 MeV; these numbers can be compared with experimental results, namely, 1402–1410 MeV (mass) and 48–52 MeV ($\Sigma\pi$ amplitude) [1]. Such a strong deviation between theoretical predictions and data might be explained if we interpret the $\Lambda^*(1405)$ as a baryon-meson molecular state (for example, see Refs. [65,66]). Given this, our predictions would refer to the lowest-lying qqq state with the same quantum numbers as $\Lambda^*(1405)$.

$\Delta(1930)D_{35}$ represents another failure of our $^3P_0$/QM predictions, as we get a null $N\pi$ width, while it should be in the range 11–75 MeV [1]. Nevertheless, unlike the $\Lambda^*(1405)$ case, here we obtain a prediction for the mass that is compatible with the experimental data [1]. Possible explanations of this deviation from the data include the eventuality that the $\Delta(1930)D_{35}$ is a $\Delta\rho$ bound state [67] or simply the fact that the $N\pi$ decay may proceed in a different way. It would thus be worthwhile to investigate whether the inclusion of baryon-meson higher Fock components in $\Delta(1930)D_{35}$'s wave function via the unquenched quark model (UQM) formalism [68] may help to solve this problem. Similar issues also occur in the $\Sigma(1750)$ and $\Sigma(1940)$ cases. See Refs. [69–71].

Another interesting case of departure from experimental data is that of the $\Delta(1700)D_{33}$ $\Delta\pi$ width [1], for which we predict a larger value. On the contrary, in the molecular picture this decay mode is dynamically suppressed. It would be worthwhile to investigate this case in the UQM formalism and see if the introduction of continuum components, also determining a renormalization of the $\Delta(1700)$ wave function, may improve the quality of our result. It would also be interesting to calculate the couplings for the $\Delta\eta$ virtual channel, which is relevant to several reactions. This coupling has been evaluated in the molecular model in Refs. [72–74].

In addition to a calculation of the decay amplitudes within the UQM, it would also be interesting to investigate baryon-meson-meson decays. A possible way to do that is via the formalism of quasi-two-body decays, discussed in Refs. [3,15,29]. In quasi-two-body decays, the decay of a baryon A into a baryon B and mesons C_1 and C_2 proceeds as $A \to B^* C_1 \to B C_1 C_2$, where B^* is a baryon resonance. Alternately, one may also decide to use the coupled-channel approach. Of particular interest is the $N(1710)D_{13} \to N\pi\pi$ decay mode, which is the main one of $N(1710)$ (40–90%); it is worthwhile to note that this is larger than the $N\pi$ width, even if the latter has more phase space for the decay. For example, see Refs. [75].

V. SUMMARY AND CONCLUSION

We computed the open-flavor strong decays of light baryons (i.e., made up of u, d, and s valence quarks) into baryon-pseudoscalar and baryon-vector mesons using a modified version of the $^3P_0$ pair-creation model [10,11], in which we considered a flavor-dependent pair-creation strength to suppress the contributions from heavier $q\bar{q}$ pairs, like $s\bar{s}$ with respect to $u\bar{u}$ ($d\bar{d}$).

The baryon models, which we used in our study to get predictions for missing or higher-lying states, were the $U(7)$ [40] and hypercentral [41,46] models. The possibility of using two models to extract the baryon spectrum makes it possible to give two different points of view, especially in the study of the energy region above 1.8–2 GeV and the related problem of the missing resonances. Indeed, an important difference between the $U(7)$ and hQM models is in the number of missing states that they predict and also in the quantum number predictions for some *** and **** states, like the $N(1875)D_{13}$. In a subsequent paper [76], the present results will be extended up to an energy region (2.5 GeV) which will be tested by forthcoming experiments at the JLab.

It is worthwhile to enumerate some of the difficulties and problems connected to this type of calculation. One problem is related to the difficulty of assigning quantum numbers to resonances within a QM. Sometimes, this can generate strong conflicts between theoretical results and experimental data. For example, this is the case of the $N(1875)D_{13}$, whose decay amplitudes change significantly whether we use the $^3P_0$ model in combination with the $U(7)$ or hQM models. See Table III. Another problem has to do with the large quantity of decay thresholds, sometimes lying at similar energies or almost overlapping with one another. Thus, we think that a more complete study would require the introduction of continuum coupling effects (i.e., higher Fock components) in the baryon wave functions. In some cases, the presence of a threshold can deeply influence the quark structure of a hadron, close in energy, as in the well-known case of the $X(3872)$ meson [28,29,60,61]. In this respect, it is worthwhile to cite the results of the EBAC project, developed by Matsuyama *et al.* [77]. This is a dynamical coupled-channel model for investigating the nucleon resonances in the meson production reactions induced by pions and photons. See also the interesting coupled-channel model results of the Bonn-Jülich group of Refs. [78]. A similar formalism, which would make it possible to include meson cloud effects in baryon and meson open- and hidden-flavor decays, will be the subject of a subsequent paper [76].

Finally, one of the most important points is related to the problem of the missing resonances. Is this a matter of degrees of freedom? In this case, the use of other types of models, characterized by a smaller number of effective degrees of freedom, such as the quark-diquark one, could, at least partially, solve the problem [79]. Otherwise, does it have to do with the coupling of these missing states with other types of decay channels, which are more difficult to observe? This is still an open question. Thus, we think that it would be worthwhile to compare the results for spectrum and decays of a three-quark QM to those of other type of models, such as the quark-diquark one. Moreover, in the case of higher-lying states, it would also be interesting to compare the predictions of three-quark models to those for hybrid baryons, where baryons are described as bound states of three constituent quarks and a constituent gluon [80].

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APPENDIX A: PAIR-CREATION VERTEX

Analogously to what was done in Ref. [81], we can study different forms for the pair-creation vertex, to improve the description of the experimental data. The determination of the best vertex results from a χ^2 analysis based on a sample of nine transitions: $\Delta \to N\pi$, $N(1520) \to N\pi$, $N(1535) \to N\pi$, $N(1650) \to N\pi$, $N(1680) \to N\pi$, $N(1720) \to N\pi$, $\Delta(1905) \to N\pi$, $\Delta(1910) \to N\pi$, and $\Delta(1920) \to N\pi$. The different forms we consider are given by

$$
\begin{aligned}
V_1(2p) &= e^{-\alpha_d^2 p^2/2} \\
V_2(2p) &= (1 + \gamma_1 p^2) e^{-\alpha_d^2 p^2/2} \\
V_3(2p) &= 1 + \gamma_1 e^{-\alpha_d^2 p^2/2} \\
V_4(2p) &= 1 + (\gamma_1 + \gamma_2 p^2) e^{-\alpha_d^2 p^2/2}
\end{aligned} \tag{A1}
$$

where $p^2 = (\vec{p}_4 - \vec{p}_5)^2/4$. As observed in Ref. [81], the forms containing a p_0 parameter, such as $V \propto e^{-\alpha_d^2 (p-p_0)^2}$ or $1/[(p-p_0)^2 + B]$, present a bump around $p - p_0$, and thus do not show the expected decreasing behavior. Thus, we do not include them in our analysis. The quality of the description of the experimental data provided by the four vertices is equivalent (see Table XVIII). Thus, we choose

TABLE XVIII. Comparison of the results obtained with different vertex functions, fitted to a selected number of experimental strong decays [1]. Columns 2–5 show the theoretical open-flavor decay widths, calculated with the vertices V_i of Eq. (A1) in combination with the effective phase space factor of Eq. (22).

Channel	V_1	V_2	V_3	V_4	Exp. (MeV)
$\Delta(1232) \to N\pi$	115	118	116	120	114–120
$N(1520) \to N\pi$	102	98	101	98	55–81
$N(1535) \to N\pi$	106	108	102	107	44–96
$N(1650) \to N\pi$	71	72	68	72	60–162
$N(1680) \to N\pi$	63	55	60	50	78–98
$N(1720) \to N\pi$	123	114	114	118	12–56
$\Delta(1905) \to N\pi$	14	14	14	14	24–60
$\Delta(1910) \to N\pi$	39	42	38	43	33–102
$\Delta(1920) \to N\pi$	14	16	14	16	9–60

the vertex with the smallest number of free parameters, the first vertex with

$$V(\vec{p}_4 - \vec{p}_5) = e^{-\alpha_d^2(\vec{p}_4-\vec{p}_5)^2/8}. \tag{A2}$$

This is the one used in the calculations of Sec. IV and for the analytic derivation of the $^3P_0$ amplitudes of Appendix D. The reader may object that we did not consider in our analysis the simplest choice for the pair-creation vertex, namely, $V = 1$. Actually, this particular choice is a special case of $V_1(2p)$, i.e., when $\alpha_d = 0$.

APPENDIX B: SPIN WAVE FUNCTIONS

In the following, we list the conventions used for the spin wave functions [20]:

$$\begin{aligned} S = 1/2\colon\ |\chi^{\rho}_{1/2}\rangle &= \frac{1}{\sqrt{2}}(|\uparrow\downarrow\uparrow\rangle - |\downarrow\uparrow\uparrow\rangle),\\ |\chi^{\lambda}_{1/2}\rangle &= \frac{1}{\sqrt{6}}(2|\uparrow\uparrow\downarrow\rangle - |\uparrow\downarrow\uparrow\rangle - |\downarrow\uparrow\uparrow\rangle),\\ S = 3/2\colon\ |\chi^{S}_{3/2}\rangle &= |\uparrow\uparrow\uparrow\rangle. \end{aligned} \tag{B1}$$

We only show the state with the largest component of the projection $M_S = S$. The other states are obtained by applying the lowering operator in spin space.

APPENDIX C: FLAVOR WAVE FUNCTIONS

The meson and baryon states are written according to the usual prescriptions. Below, we list the conventions used for the flavor wave functions of mesons and baryons.

1. Mesons

Since the mixing angle $\theta_{\eta\eta'}$ between η and η' is small, we take $\theta_{\eta\eta'} = 0$. Thus, we identify $\eta = \eta_8$ and $\eta' = \eta_1$.

(i) *The octet mesons*

$$\begin{aligned} |\pi^+\rangle &= -|u\bar{d}\rangle\\ |\pi^0\rangle &= \frac{1}{\sqrt{2}}[|u\bar{u}\rangle - |d\bar{d}\rangle]\\ |\pi^-\rangle &= |d\bar{u}\rangle\\ |K^+\rangle &= -|u\bar{s}\rangle\\ |K^-\rangle &= |s\bar{u}\rangle\\ |K^0\rangle &= -|d\bar{s}\rangle\\ |\bar{K}^0\rangle &= -|s\bar{d}\rangle\\ |\eta\rangle &= \frac{1}{\sqrt{6}}[|u\bar{u}\rangle + |d\bar{d}\rangle - 2|s\bar{s}\rangle] \end{aligned} \tag{C1}$$

(ii) *The singlet mesons*

$$|\eta'\rangle = \frac{1}{\sqrt{3}}[|u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle] \tag{C2}$$

2. Baryons

For the baryon flavor wave functions, $|(p,q),I,M_I,Y\rangle$, we adopt the convention of Ref. [82]. We only show the highest charge state $M_I = I$ with $Q = I + Y/2$. The other charge states are obtained by applying the lowering operator in isospin space.

(i) *The octet baryons*

$$\begin{aligned} \left|(1,1),\frac{1}{2},\frac{1}{2},1\right\rangle\colon\ \phi_\rho(p) &= \frac{1}{\sqrt{2}}[|udu\rangle - |duu\rangle]\\ \colon\ \phi_\lambda(p) &= \frac{1}{\sqrt{6}}[2|uud\rangle - |udu\rangle - |duu\rangle] \end{aligned} \tag{C3}$$

$$\begin{aligned} |(1,1),1,1,0\rangle\colon\ \phi_\rho(\Sigma^+) &= \frac{1}{\sqrt{2}}[|suu\rangle - |usu\rangle]\\ \colon\ \phi_\lambda(\Sigma^+) &= \frac{1}{\sqrt{6}}[|suu\rangle + |usu\rangle - 2|uus\rangle] \end{aligned} \tag{C4}$$

$$\begin{aligned} |(1,1),0,0,0\rangle\colon\ \phi_\rho(\Lambda) &= \frac{1}{\sqrt{12}}[2|uds\rangle - 2|dus\rangle - |dsu\rangle + |sdu\rangle - |sud\rangle + |usd\rangle]\\ \colon\ \phi_\lambda(\Lambda) &= \frac{1}{2}[-|dsu\rangle - |sdu\rangle + |sud\rangle + |usd\rangle] \end{aligned} \tag{C5}$$

$$\left|(1,1),\frac{1}{2},\frac{1}{2},-1\right\rangle:\ \phi_\rho(\Xi^0)=\frac{1}{\sqrt{2}}[|sus\rangle-|uss\rangle]$$
$$:\ \phi_\lambda(\Xi^0)=\frac{1}{\sqrt{6}}[2|ssu\rangle-|sus\rangle-|uss\rangle] \quad \text{(C6)}$$

(ii) *The decuplet baryons*

$$\left|(3,0),\frac{3}{2},\frac{3}{2},1\right\rangle:\ \phi_S(\Delta^{++})=|uuu\rangle \quad \text{(C7)}$$

$$|(3,0),1,1,0\rangle:\phi_S(\Sigma^+)=\frac{1}{\sqrt{3}}[|suu\rangle+|usu\rangle+|uus\rangle] \quad \text{(C8)}$$

$$\left|(3,0),\frac{1}{2},\frac{1}{2},-1\right\rangle:\ \phi_S(\Xi^0)=\frac{1}{\sqrt{3}}[|ssu\rangle+|sus\rangle+|uss\rangle] \quad \text{(C9)}$$

$$|(3,0),0,0,-2\rangle:\ \phi_S(\Omega^-)=|sss\rangle \quad \text{(C10)}$$

(iii) *The singlet baryons*

$$|(0,0),0,0,0\rangle:\ \phi_A(\Lambda)=\frac{1}{\sqrt{6}}[|uds\rangle-|dus\rangle+|dsu\rangle-|sdu\rangle+|sud\rangle-|usd\rangle] \quad \text{(C11)}$$

APPENDIX D: $^3P_0$ AMPLITUDES: GENERAL EXPRESSION

The color, spin, and spatial parts of the $^3P_0$ amplitude $M_{A\to BC}(q_0)=\langle BCq_0\ell J|T^\dagger|A\rangle$, excluding the flavor couplings, were derived in a harmonic oscillator basis by Roberts and Silvestre-Brac (RSB) [16]. They did not consider a quark form factor ($\alpha_d=0$) and used the following momenta,

$$\begin{aligned}
\vec{p}_\rho&=\frac{1}{2}(\vec{p}_1-\vec{p}_2),\\
\vec{p}_\lambda&=\frac{1}{3}(\vec{p}_1+\vec{p}_2-2\vec{p}_4),\\
\vec{q}_c&=\frac{1}{2}(\vec{p}_3-\vec{p}_5),\\
\vec{q}&=\frac{1}{2}(\vec{K}_b-\vec{K}_c),\\
\vec{P}_{cm}&=\vec{K}_b+\vec{K}_c,
\end{aligned} \quad \text{(D1)}$$

with conjugate coordinates

$$\begin{aligned}
\vec{\rho}&=\vec{r}_1-\vec{r}_2,\\
\vec{\lambda}&=\frac{1}{2}(\vec{r}_1+\vec{r}_2-2\vec{r}_4),\\
\vec{r}_c&=\vec{r}_3-\vec{r}_5,\\
\vec{r}&=\frac{1}{3}\vec{R}_b-\frac{1}{2}\vec{R}_c,\\
\vec{R}_{cm}&=\frac{1}{6}(\vec{r}_1+\vec{r}_2+\vec{r}_4)+\frac{1}{4}(\vec{r}_3+\vec{r}_5).
\end{aligned} \quad \text{(D2)}$$

On the contrary, here we use the standard Jacobi coordinates and conjugate momenta for the initial baryon A, final state baryon B, and meson C [83,84],

$$\begin{aligned}
\vec{\rho}&=\frac{1}{\sqrt{2}}(\vec{r}_1-\vec{r}_2),\\
\vec{\lambda}&=\frac{1}{\sqrt{6}}(\vec{r}_1+\vec{r}_2-2\vec{r}_4),\\
\vec{r}_c&=\vec{r}_3-\vec{r}_5,\\
\vec{r}&=\vec{R}_b-\vec{R}_c,\\
\vec{R}_{cm}&=\frac{m_b\vec{R}_b+m_c\vec{R}_c}{m_b+m_c},
\end{aligned} \quad \text{(D3)}$$

where m_i and p_i are the mass and momentum of the quark i (see Fig. 1), and $m_b=m_1+m_2+m_4$ and $m_c=m_3+m_5$ are the masses of the hadrons B and C. Their center-of-mass coordinates are defined as

$$\begin{aligned}
\vec{R}_b&=\frac{1}{3}(\vec{r}_1+\vec{r}_2+\vec{r}_4),\\
\vec{R}_c&=\frac{1}{2}(\vec{r}_3+\vec{r}_5).
\end{aligned} \quad \text{(D4)}$$

The conjugate momenta are given by

$$\begin{aligned}
\vec{p}_\rho&=\frac{1}{\sqrt{2}}(\vec{p}_1-\vec{p}_2),\\
\vec{p}_\lambda&=\frac{1}{\sqrt{6}}(\vec{p}_1+\vec{p}_2-2\vec{p}_4),\\
\vec{q}_c&=\frac{1}{2}(\vec{p}_3-\vec{p}_5),\\
\vec{q}&=\frac{m_c\vec{K}_b-m_b\vec{K}_c}{m_b+m_c},\\
\vec{P}_{cm}&=\vec{K}_b+\vec{K}_c,
\end{aligned} \quad \text{(D5)}$$

and

$$\begin{aligned}
\vec{K}_b&=\vec{p}_1+\vec{p}_2+\vec{p}_4,\\
\vec{K}_c&=\vec{p}_3+\vec{p}_5.
\end{aligned} \quad \text{(D6)}$$

The final result is [83,84]

$$M_{A\to BC}(q_0)=6\gamma_0\theta_{A\to BC}\epsilon(l_{\lambda_b},l_c,L_{bc},l,l_{\lambda_a},L,q_0), \quad \text{(D7)}$$

where the angular momenta of the baryons A and B, L_a and L_b, are the sum of the ρ and λ oscillator contributions, l_ρ and l_λ, and the term $\theta_{A\to BC}$ contains the dependence on the color-spin-flavor part

$$\theta_{A\to BC} = \frac{\mathcal{F}_{A\to BC}}{3\sqrt{3}}(-1)^{l+l_{\lambda_a}}\sum_{L_{bc}S_{bc}}(-1)^{S_a-S_{bc}+L_{bc}} \times \begin{bmatrix} J_\rho & \frac{1}{2} & S_b \\ \frac{1}{2} & \frac{1}{2} & S_c \\ S_a & 1 & S_{bc} \end{bmatrix}\begin{bmatrix} S_b & l_{\lambda_b} & J_b \\ S_c & l_c & J_c \\ S_{bc} & L_{bc} & J_{bc} \end{bmatrix}\sum_L \hat{L}^2 \times \begin{Bmatrix} S_a & l_{\lambda_a} & J_a \\ L & S_{bc} & 1 \end{Bmatrix}\begin{Bmatrix} S_{bc} & L_{bc} & J_{bc} \\ l & J_a & L \end{Bmatrix}. \quad \text{(D8)}$$

The coefficient $\mathcal{F}_{A\to BC}$ contains the flavor couplings and is defined as

$$\mathcal{F}_{A\to BC} = \langle \phi_B\phi_C|\phi_0\phi_A\rangle. \quad \text{(D9)}$$

Here, ϕ_0 denotes the flavor wave function of the created quark-antiquark pair

$$|\phi_0\rangle = \frac{1}{\sqrt{2+(m_n/m_s)^2}}\left[|u\bar{u}\rangle + |d\bar{d}\rangle + \frac{m_n}{m_s}|s\bar{s}\rangle\right], \quad \text{(D10)}$$

which, in the limit of equal quark masses, reduces to the usual expression for a flavor singlet. The coefficients in square brackets are proportional to 9-j coefficients

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \hat{c}\,\hat{f}\,\hat{g}\,\hat{h}\,\hat{i}\begin{Bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{Bmatrix}, \quad \text{(D11)}$$

where $\hat{\ell} = \sqrt{2\ell+1}$,

Finally, $\epsilon(l_{\lambda_b}, l_c, L_{bc}, l, l_{\lambda_a}, L, q_0)$ represents the spatial contribution [83,84]

$$\epsilon(l_{\lambda_b}, l_c, L_{bc}, l, l_{\lambda_a}, L, q_0) = \mathcal{J}\mathcal{N}_{n_{\lambda_a}l_{\lambda_a}}(\alpha_b)\mathcal{N}^*_{n_{\lambda_b}l_{\lambda_b}}(\alpha_b)\mathcal{N}^*_{n_c l_c}(\alpha_c)(-1)^{L_{bc}}\frac{\exp(-F^2q_0^2)}{2G^{l_{\lambda_a}+l_{\lambda_b}+l_c+4}}\sum_{l_1,l_2,l_3,l_4} C^{l_{\lambda_b}}_{l_1}C^{l_c}_{l_2} \times C^1_{l_3}C^{l_{\lambda_a}}_{l_4}\left(x-\sqrt{\frac{2}{3}}\right)^{l_1}\left(\frac{1}{2}-\sqrt{\frac{2}{3}}x\right)^{l_2}\left(-\sqrt{\frac{2}{3}}\right)^{l'_2}\left(\sqrt{\frac{2}{3}}x-1\right)^{l_3}\left(\sqrt{\frac{2}{3}}\right)^{l'_3}x^{l_4} \times \sum_{l_{12},l_5,l_6,l_7,l_8}(-1)^{l_{12}+l_6}\frac{\hat{l}_5}{\hat{L}}\begin{bmatrix} l_1 & l'_1 & l_{\lambda_b} \\ l_2 & l'_2 & l_c \\ l_{12} & l_6 & L_{bc} \end{bmatrix}\begin{bmatrix} l_3 & l'_3 & 1 \\ l_4 & l'_4 & l_{\lambda_a} \\ l_7 & l_8 & L \end{bmatrix}\begin{Bmatrix} l & l_{12} & l_5 \\ l_6 & L & L_{bc} \end{Bmatrix}B^{l_{12}}_{l_1l_2}B^{l_5}_{ll_{12}}B^{l_6}_{l'_1l'_2}B^{l_7}_{l_3l_4}B^{l_8}_{l'_3l'_4} \times \sum_{\lambda,\mu,\nu}D_{\lambda\mu\nu}(x)I_\nu(l_5,l_6,l_7,l_8;L)\left(\frac{l'_1+l'_2+l'_3+l'_4+2\mu+\nu+1}{2}\right)!\frac{q_0^{l_1+l_2+l_3+l_4+2\lambda+\nu}}{G^{2\mu+\nu-l_1-l_2-l_3-l_4}}, \quad \text{(D12)}$$

with

$$C^L_l = \sqrt{\frac{4\pi(2L+1)!}{(2l+1)![2(L-l)+1]!}}$$
$$B^l_{l_1,l_2} = \frac{(-1)^l}{\sqrt{4\pi}}\hat{l}_1\,\hat{l}_2\begin{pmatrix} l_1 & l_2 & l \\ 0 & 0 & 0 \end{pmatrix}. \quad \text{(D13)}$$

Here $l'_1 = l_{\lambda_b} - l_1$, $l'_2 = l_c - l_2$, $l'_3 = 1 - l_3$, $l'_4 = l_{\lambda_a} - l_4$, and $\hat{l} = \sqrt{2l+1}$. $\mathcal{J} = 1/3\sqrt{3}$ is the Jacobian for the change of momenta $\{\vec{p}_1, \vec{p}_2, \vec{p}_3, \vec{p}_4, \vec{p}_5\} \to \{\vec{p}_\rho, \vec{p}_\lambda, \vec{q}_c, \vec{q}, \vec{P}_{cm}\}$ [see Eqs. (D5)–(D6)]. $\mathcal{N}_{n_{\lambda_a}l_{\lambda_a}}(\alpha_b)$, $\mathcal{N}_{n_{\lambda_b}l_{\lambda_b}}(\alpha_b)$, and $\mathcal{N}_{n_c l_c}(\alpha_c)$, where

$$\mathcal{N}_{n,L}(\alpha) = \sqrt{\frac{2n!}{\Gamma(n+L+3/2)}}\alpha^{-L-\frac{3}{2}}, \quad \text{(D14)}$$

are the normalization coefficients of the harmonic oscillator wave functions of the baryons A and B,

$$\Phi_{nLM}(\vec{p}_\lambda, \vec{p}_\rho) = \sum_m \langle l_\rho, m; l_\lambda, M-m|LM\rangle \times \mathcal{N}_{n_\rho,l_\rho}(\alpha_b)L^{l_\rho+1/2}_{n_\rho}(p_\rho^2/\alpha_b^2)e^{-p_\rho^2/2\alpha_b^2}\mathcal{Y}_{l_\rho m}(\vec{p}_\rho) \times \mathcal{N}_{n_\lambda,l_\lambda}(\alpha_b)L^{l_\lambda+1/2}_{n_\lambda}(p_\lambda^2/\alpha_b^2)e^{-p_\lambda^2/2\alpha_b^2}\mathcal{Y}_{l_\lambda M-m}(\vec{p}_\lambda), \quad \text{(D15)}$$

and meson C,

$$\Phi_{n_c l_c m_c}(\vec{q}_c) = \mathcal{N}_{n_c l_c}(\alpha_c)L^{l_c+1/2}_{n_c}(q_c^2/\alpha_c^2)e^{-q_c^2/2\alpha_c^2}\mathcal{Y}_{l_c m_c}(\vec{q}_c), \quad \text{(D16)}$$

where n is the number of nodes in the harmonic oscillator wave function, $L^{L+1/2}_n(\alpha p^2)$ is a Laguerre polynomial, and $\mathcal{Y}_{LM}(\vec{p})$ is a solid spherical harmonic. The remaining coefficients are given by

$$\begin{aligned} G^2 &= \alpha_b^2 + \frac{1}{3}\alpha_{\mathrm{c}}^2 + \frac{1}{3}\alpha_{\mathrm{d}}^2, \\ x &= \frac{2\alpha_{\mathrm{b}}^2 + \alpha_{\mathrm{c}}^2 + 2\alpha_{\mathrm{d}}^2}{2\sqrt{6}G^2}, \\ F^2 &= \frac{\alpha_{\mathrm{b}}^2(12\alpha_{\mathrm{b}}^2 + 5\alpha_{\mathrm{c}}^2) + \alpha_{\mathrm{d}}^2(20\alpha_{\mathrm{b}}^2 + 3\alpha_{\mathrm{c}}^2)}{24(3\alpha_{\mathrm{b}}^2 + \alpha_{\mathrm{c}}^2 + \alpha_{\mathrm{d}}^2)}. \end{aligned} \tag{D17}$$

The present results for the $^3P_0$ amplitudes were obtained in a consistent way using the same Jacobi coordinates for the baryon wave functions and the $^3P_0$ matrix elements. We observe that the coefficients of Eqs. (D17) also depend on the parameter α_{d} of the Gaussian quark form factor, $V(\vec{p}_4 - \vec{p}_5) = e^{-\alpha_{\mathrm{d}}^2(\vec{p}_4-\vec{p}_5)^2/8}$. The case $V = 1$ is a subcase of the previous one.

For the special case of ground-state baryons and pseudoscalar mesons, the orbital angular momenta vanish $l_{\lambda_a} = l_{\lambda_b} = l_c = L_{bc} = 0$ and therefore $J_a = S_a$, $J_b = S_b$, $J_c = S_c = 0$, and $J_{bc} = S_{bc} = J_b$. Due to the conservation of angular momentum and parity, the relative orbital angular momentum between the baryon B and the meson C is equal to $l = 1 = L$. As a result, the general expression for the $^3P_0$ transition amplitude simplifies considerably. The color-spin-flavor part $\theta_{A\to BC}$ reduces to

$$\begin{aligned} \theta_{A\to BC} &= -\frac{1}{3}\sqrt{\frac{2J_b+1}{2}}(-1)^{J_\rho+J_a-\frac{1}{2}} \\ &\quad \times \begin{Bmatrix} J_a & 1 & J_b \\ \frac{1}{2} & J_\rho & \frac{1}{2} \end{Bmatrix} \mathcal{F}_{A\to BC}, \end{aligned} \tag{D18}$$

where J_ρ is the total angular momentum of the first two quarks and $\mathcal{F}_{A\to BC}$ are the flavor matrix elements of the $A \to BC$ transition [16]. The spatial contribution simplifies to

$$\epsilon(q_0) = -\frac{1}{2}\left(\frac{9\alpha_b^4\alpha_c^2}{\pi}\right)^{3/4} \frac{(4\alpha_b^2 + \alpha_c^2)q_0 \mathrm{e}^{-F^2 q_0^2}}{(3\alpha_b^2 + \alpha_c^2 + \alpha_d^2)^{5/2}}. \tag{D19}$$

APPENDIX E: FLAVOR COUPLINGS

In the following, we give the flavor coefficients $\mathcal{F}_{A\to BC}$. These expressions are valid for both pseudoscalar and vector mesons.

(i) $A_8 \to B_8 + C_8$ *couplings*

For octet baryons the flavor wave function has two components, labeled ρ and λ, both of which give rise to the flavor coupling coefficient

$$\begin{aligned} \mathcal{F}^\rho_{A\to BC} &= \langle \phi_\rho(B)\phi(C)|\phi_0\phi_\rho(A)\rangle, \\ \mathcal{F}^\lambda_{A\to BC} &= \langle \phi_\lambda(B)\phi(C)|\phi_0\phi_\lambda(A)\rangle, \end{aligned} \tag{E1}$$

where we introduced the superscripts ρ and λ to distinguish the two contributions. In this case, the flavor couplings are given by

$$\begin{aligned} \begin{pmatrix} N \\ \Sigma \\ \Lambda \\ \Xi \end{pmatrix} &\to \begin{pmatrix} N\pi & N\eta_8 & \Sigma K & \Lambda K & \\ N\bar{K} & \Sigma\pi & \Lambda\pi & \Sigma\eta_8 & \Xi K \\ N\bar{K} & \Sigma\pi & \Lambda\eta_8 & \Xi K & \\ \Sigma\bar{K} & \Lambda\bar{K} & \Xi\pi & \Xi\eta_8 & \end{pmatrix} \\ &= \mathcal{N}\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{3\sqrt{2}} & 0 & -\frac{\sqrt{2}}{3}\frac{m_n}{m_s} & \\ 0 & \frac{1}{\sqrt{3}} & \frac{1}{3\sqrt{2}} & \frac{1}{3\sqrt{2}} & -\frac{1}{\sqrt{3}}\frac{m_n}{m_s} \\ \frac{2}{3} & -\frac{1}{\sqrt{6}} & \frac{1-4\frac{m_n}{m_s}}{9\sqrt{2}} & \frac{1}{3}\frac{m_n}{m_s} & \\ \frac{1}{\sqrt{2}} & \frac{1}{3\sqrt{2}} & 0 & -\frac{\sqrt{2}}{3}\frac{m_n}{m_s} & \end{pmatrix} \end{aligned} \tag{E2}$$

for the ρ component and by

$$\mathcal{N}\begin{pmatrix} -\frac{1}{3\sqrt{2}} & \frac{1}{3\sqrt{2}} & \frac{\sqrt{2}}{3}\frac{m_n}{m_s} & 0 & \\ \frac{2}{3\sqrt{3}} & \frac{1}{3\sqrt{3}} & -\frac{1}{3\sqrt{2}} & \frac{1-4\frac{m_n}{m_s}}{9\sqrt{2}} & \frac{1}{3\sqrt{3}}\frac{m_n}{m_s} \\ 0 & \frac{1}{\sqrt{6}} & \frac{1}{3\sqrt{2}} & \frac{1}{3}\frac{m_n}{m_s} & \\ -\frac{1}{3\sqrt{2}} & \frac{1}{3\sqrt{2}} & \frac{\sqrt{2}}{3} & \frac{\sqrt{2}(1-\frac{m_n}{m_s})}{9} & \end{pmatrix} \tag{E3}$$

for the λ component. The normalization coefficient is given by

$$\mathcal{N} = \sqrt{\frac{3}{2 + (\frac{m_n}{m_s})^2}}. \tag{E4}$$

(ii) $A_8 \to B_8 + C_1$ *couplings*

The flavor coefficients for octet baryons in combination with a singlet meson are given by

$$\begin{pmatrix} N \\ \Sigma \\ \Lambda \\ \Xi \end{pmatrix} \to \begin{pmatrix} N\eta_1 \\ \Sigma\eta_1 \\ \Lambda\eta_1 \\ \Xi\eta_1 \end{pmatrix} = \mathcal{N}\begin{pmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1+2\frac{m_n}{m_s}}{9} \\ \frac{1}{3}\frac{m_n}{m_s} \end{pmatrix} \tag{E5}$$

for the ρ component and by

$$\mathcal{N}\begin{pmatrix} \frac{1}{3} \\ \frac{1+2\frac{m_n}{m_s}}{9} \\ \frac{1}{3} \\ \frac{2+\frac{m_n}{m_s}}{9} \end{pmatrix} \tag{E6}$$

for the λ component.

(iii) $A_8 \to B_{10} + C_8$ *couplings*

In this case the final baryon belongs to the decuplet that only couples to the λ component of the initial octet baryon:

$$\mathcal{F}_{A\to BC} = \langle \phi_S(B)\phi(C)|\phi_0\phi_\lambda(A)\rangle. \tag{E7}$$

The flavor coefficients are given by

$$\begin{pmatrix} N \\ \Sigma \\ \Lambda \\ \Xi \end{pmatrix} \to \begin{pmatrix} & \Delta\pi & \Sigma^* K & \\ \Delta\bar{K} & \Sigma^*\pi & \Sigma^*\eta_8 & \Xi^* K \\ & \Sigma^*\pi & \Xi^* K & \\ \Sigma^*\bar{K} & \Xi^*\pi & \Xi^*\eta_8 & \Omega K \end{pmatrix}$$

$$= \mathcal{N} \begin{pmatrix} & \frac{2}{3} & -\frac{1}{3}\frac{m_n}{m_s} & \\ -\frac{2\sqrt{2}}{3\sqrt{3}} & \frac{\sqrt{2}}{3\sqrt{3}} & \frac{1+2\frac{m_n}{m_s}}{9} & -\frac{\sqrt{2}}{3\sqrt{3}}\frac{m_n}{m_s} \\ & \frac{1}{\sqrt{3}} & -\frac{\sqrt{2}}{3}\frac{m_n}{m_s} & \\ -\frac{1}{3} & \frac{1}{3} & \frac{1+2\frac{m_n}{m_s}}{9} & -\frac{\sqrt{2}}{3}\frac{m_n}{m_s} \end{pmatrix}. \tag{E8}$$

(iv) $A_{10} \to B_8 + C_8$ *couplings*

As in the previous case, the decuplet baryon only couples to the λ component of the octet baryon:

$$\mathcal{F}_{A\to BC} = \langle \phi_\lambda(B)\phi(C)|\phi_0\phi_S(A)\rangle. \tag{E9}$$

The flavor coefficients are given by

$$\begin{pmatrix} \Delta \\ \Sigma^* \\ \Xi^* \\ \Omega \end{pmatrix} \to \begin{pmatrix} & N\pi & \Sigma K & & \\ N\bar{K} & \Sigma\pi & \Lambda\pi & \Sigma\eta_8 & \Xi K \\ \Sigma\bar{K} & \Lambda\bar{K} & \Xi\pi & \Xi\eta_8 & \\ & & \Xi\bar{K} & & \end{pmatrix}$$

$$= \mathcal{N} \begin{pmatrix} & -\frac{\sqrt{2}}{3} & \frac{\sqrt{2}}{3}\frac{m_n}{m_s} & & \\ -\frac{\sqrt{2}}{3\sqrt{3}} & \frac{\sqrt{2}}{3\sqrt{3}} & -\frac{1}{3} & \frac{1+2\frac{m_n}{m_s}}{9} & \frac{\sqrt{2}}{3\sqrt{3}}\frac{m_n}{m_s} \\ \frac{1}{3} & -\frac{1}{3} & \frac{1}{3} & \frac{1+2\frac{m_n}{m_s}}{9} & \\ & & \frac{2}{9} & & \end{pmatrix}. \tag{E10}$$

(v) $A_{10} \to B_{10} + C_8$ *couplings*

The flavor coefficients for decuplet baryons in combination with a octet meson,

$$\mathcal{F}_{A\to BC} = \langle \phi_S(B)\phi(C)|\phi_0\phi_S(A)\rangle, \tag{E11}$$

are given by

$$\begin{pmatrix} \Delta \\ \Sigma^* \\ \Xi^* \\ \Omega \end{pmatrix} \to \begin{pmatrix} \Delta\pi & \Delta\eta_8 & \Sigma^* K & \\ \Delta\bar{K} & \Sigma^*\pi & \Sigma^*\eta_8 & \Xi^* K \\ \Sigma^*\bar{K} & \Xi^*\pi & \Xi^*\eta_8 & \Omega K \\ & \Xi^*\bar{K} & \Omega\eta_8 & \end{pmatrix}$$

$$= \mathcal{N} \begin{pmatrix} \frac{\sqrt{5}}{3\sqrt{2}} & \frac{1}{3\sqrt{2}} & -\frac{1}{3}\frac{m_n}{m_s} & \\ \frac{2}{3\sqrt{3}} & \frac{2}{3\sqrt{3}} & -\frac{\sqrt{2}(\frac{m_n}{m_s}-1)}{9} & -\frac{2}{3\sqrt{3}}\frac{m_n}{m_s} \\ \frac{\sqrt{2}}{3} & \frac{1}{3\sqrt{2}} & -\frac{(4\frac{m_n}{m_s}-1)}{9\sqrt{2}} & -\frac{1}{3}\frac{m_n}{m_s} \\ & \frac{\sqrt{2}}{3} & -\frac{\sqrt{2}}{3}\frac{m_n}{m_s} & \end{pmatrix}. \tag{E12}$$

(vi) $A_{10} \to B_{10} + C_1$ *couplings*

The flavor coefficients for decuplet baryons in combination with a singlet meson are given by

$$\begin{pmatrix} \Delta \\ \Sigma^* \\ \Xi^* \\ \Omega \end{pmatrix} \to \begin{pmatrix} \Delta\eta_1 \\ \Sigma^*\eta_1 \\ \Xi^*\eta_1 \\ \Omega\eta_1 \end{pmatrix} = \mathcal{N} \begin{pmatrix} \frac{1}{3} \\ \frac{2+\frac{m_n}{m_s}}{9} \\ \frac{1+2\frac{m_n}{m_s}}{9} \\ \frac{1}{3}\frac{m_n}{m_s} \end{pmatrix}. \tag{E13}$$

APPENDIX F: COMPARISON BETWEEN ELEMENTARY MESON EMISSION AND $^3P_0$ MODELS

In the following, we shall compare the elementary emission (EME) and $^3P_0$ models. In the EME model, the decay proceeds via the emission of a meson in terms of an elementary quantum. On the contrary, in the $^3P_0$ model the decay process is described in terms of the creation of an additional quark-antiquark pair.

In order to do a comparison between EME and $^3P_0$ models, we consider the simplest form for the EME transition operator [85],

$$\mathcal{H}_s = \frac{g}{(2\pi)^{3/2}(2k_0)^{1/2}} X^c[(\vec{\sigma}\cdot\vec{k})\mathrm{e}^{-i\vec{k}\cdot\vec{r}} \tag{F1}$$

$$+ \frac{k_0}{2m}\vec{\sigma}\cdot(\vec{p}\mathrm{e}^{-i\vec{k}\cdot\vec{r}} + \mathrm{e}^{-i\vec{k}\cdot\vec{r}}\vec{p})], \tag{F2}$$

where $g = G_{\mathrm{cqq}}/2m$ is the ratio between the meson-emission strength (emission of a meson C by a quark) and the quark mass X^c is the flavor operator related to the emission of meson C. On the other hand, the $^3P_0$ operator in matrix form is

$$(\langle C|T|0\rangle)_{i_3' i_3} = \sum_{i_5} \Psi^*_{C i_3 i_5} P_{i_3' i_5} = PC^\dagger, \tag{F3}$$

where T is the $^3P_0$ operator,

$$T^{\dagger} = -3\int d\vec{p}_4 d\vec{p}_5 \delta(\vec{p}_4+\vec{p}_5) C_{45} F_{45} \times [\chi_{45} \times \mathcal{Y}_1(\vec{p}_4-\vec{p}_5)]_0^{(0)} b_4^{\dagger}(\vec{p}_4) d_5^{\dagger}(\vec{p}_5). \quad \text{(F4)}$$

In Ref. [85], it is shown that Eq. (F3) can also be written as

$$PC^{\dagger} = \frac{\gamma}{2(6\pi)^{1/2}} e^{-i\vec{k}_c\cdot\vec{r}} X^c [\vec{\sigma}\cdot(\vec{k}_c-\vec{p})]\psi_C(2\vec{p}-\vec{k}_c). \quad \text{(F5)}$$

If we compare Eqs. (F2) and (F5), we can observe that in the $^3P_0$ model there are direct and recoil terms with equal weights; on the contrary, the recoil term in the EME has a different weight factor $k_0/2m$. To have a perfect correspondence between EME and $^3P_0$ models, this factor in the recoil term has to be taken as equal to $k_0/2m = 1$. However, this factor has been taken as a free parameter in BIL [23,40] (and also in several other studies, like [20,23,40]) and, by fitting, it turned out to be equal to 0.04, which means that the recoil term is practically absent in BIL.

It is worthwhile to note that in the calculation of open-flavor strong decays for $L_b = L_c = 0$, the final BC state is characterized by a relative angular momentum $\ell = L_a \pm 1$, where L_a is the angular momentum of the initial state, A. In Ref. [85], it is shown that in the EME, the partial wave $\ell = L_a + 1$ is essentially given by the direct term with a small correction from the recoil one, while $\ell = L_a - 1$ receives its essential contribution from the recoil term. Thus, the presence of the factor $k_0/2m$ in the EME operator can give rise to different results with respect to the $^3P_0$ model.

Another important difference is due to the wave function factor $\psi_C(2\vec{p}-\vec{k}_c)$ in Eq. (F5), related to the composite structure of the meson C, which gives rise to a nonlocal character of the operator.

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