SU(2) low energy quark effective couplings in weak external magnetic field

Fábio L. Braghin

Instituto de Física, Federal University of Goias, Av. Esperanca, s/n, 74690-900 Goiânia, Goiás, Brazil

(Received 31 May 2016; published 21 October 2016)

In this work corrections to the usual flavor SU(2) Nambu-Jona-Lasinio (NJL) coupling due to a weak external magnetic field are calculated by considering quark polarization in a (dressed) gluon exchange mechanism for quark interactions. The quark field is split into two components: one that condenses and another one that is a background field for interacting quarks, the former being integrated out. The resulting determinant is expanded for relatively large quark mass and small magnetic field $(eB_0/M^{*2}) < 1$ by resolving magnetic-field-dependent low energy quark effective interactions. Besides the corrections for the NJL and vector NJL effective couplings, different B_0-dependent effective couplings that break isospin and chiral symmetry emerge.

DOI: 10.1103/PhysRevD.94.074030

I. INTRODUCTION

Quark interactions involve a large variety of different effects and mechanisms. To identify their particular roles in observables and to establish a realistic hierarchy among all of them for each of the processes under their conditions within the complexity of quantum chromodynamics (QCD) is a difficult task. High energy density (temperature and baryon density) systems are known to be suitable to test quark (and gluon) dynamics, from relativistic (heavy) ion collisions to several dense stars. Magnetic fields are also expected to be sizable in such systems [1–3] and, actually, they are expected to produce a large variety of effects not only in such high energy systems but also in the vacuum by means of phenomena such as the magnetic catalysis and the inverse effect at finite temperatures, for example Refs. [2,4–9], to produce changes in the CP violation phase transition [10], the emergence of superconducting vacuum [11] or chiral asymmetry or imbalance and the chiral magnetic effect [12–14] among others. In particular, it has been argued that finite temperature inverse magnetic catalysis may be traced back to the chirality imbalance [15]. In the core of dense stars (magnetars) and in the early Universe magnetic fields are expected to be of the order of $eB_0 \sim 10^{15}$ G and in noncentral relativistic heavy ion collisions they may reach $eB_0 \sim 10^{18}$ G $\sim m_\pi^2$ or 0.04–0.3 GeV^2 from RHIC to LHC [1,3,16], even if within a short time interval. More recently it has been envisaged that one of the most emblematic quark-quark effective interactions, the Nambu-Jona-Lasinio (NJL) coupling [17,18], might receive a magnetic field contribution due to the QCD coupling constant dependence on B_0 [19–24], being that for strong B_0 an explicit form for the corrected running coupling constant has been derived [2,4]. Anisotropic contributions for the NJL coupling have also been found [19,25]. Although the usual benchmark for the investigation of low energy effects of quark dynamics in a magnetic field, including dynamical chiral symmetry breaking, is the NJL model, other hadron models can also be considered and compared [26,27]. Besides that, it has been shown that vector NJL interaction provides meaningful corrections for quark dynamics and strong interactions phase diagram [28–30]. If quark NJL and vector NJL interactions receive corrections due to magnetic fields, they might produce relevant effects in quark dynamics favoring or not chiral imbalance or vector condensation.

Even before the establishment of quantum electrodynamics, vacuum fluctuations for the electromagnetic field had already been calculated with the Euler-Heisenberg action [31]. With QED several approaches have been employed to derive effective actions or Hamiltonians for higher order contributions of the electromagnetic field firstly in the absence and then in the presence of matter; a few examples are given in Refs. [32–34]. For several strongly interacting systems where magnetic fields are sizable and relevant it becomes interesting to investigate the vacuum polarization effects in the presence of magnetic fields. In this work, effective quark-quark interactions are calculated in the presence of constant weak magnetic field from vacuum polarization effects. The one-loop background field method for quarks, as employed in Refs. [35–37], will be considered in the presence of a constant weak magnetic field.

The departure point of the present work is the global color model (GCM) obtained by considering gluon exchange corrected by gluon interactions and its non-Abelian character; i.e. it can be a realistic gluon propagator. It is given by [38–40]

$$S_{\rm eff}[\bar\psi,\psi] = \int_x \left[\bar\psi(i\partial\!\!\!/ - M)\psi - \frac{g^2}{2}\int_y j^b_\mu(x)(R^{\mu\nu}_{bc})^{-1}(x-y)j^c_\nu(y)\right], \quad (1)$$

where the color quark current is $j^\mu_a = \bar\psi\lambda_a\gamma^\mu\psi$, the sums in color, flavor and Dirac indices are implicit, and the kernel

$(R_{bc}^{\mu\nu})^{-1}$ is the gluon propagator. Non-Abelian gluon interactions can be considered to dress the gluon exchange by considering a nonperturbative (realistic) gluon propagator that, together with the quark-gluon vertex, will be assumed to provide the strength for dynamical chiral symmetry breaking (DChSB). Several different effects are known to contribute to the strength of the quark-quark interaction above [41–43]. Therefore this calculation presents, in this sense, a similar level of approximation to the rainbow ladder approximation for the Schwinger Dyson equations that yield DChSB [38–41,44–46]. The quark-gluon vertex was shown to depend on B_0 [22] and this will not be considered in the present work. This model will be coupled to the electromagnetic field via the quark minimal coupling. To investigate the flavor structure of the model, one performs a Fierz transformation from which a NJL coupling emerges in the local limit, besides other structures. This work is organized as follows. In the next section the Fierz transformation of this GCM interaction coupled (minimally) to a constant magnetic field is presented and the quark field is integrated out in the presence of background quark. In the following section the determinant is expanded for small magnetic field with respect to the quark effective mass, i.e. $(eB_0) \ll M^{*2}$. Several simple ratios between the effective couplings in the limit of large quark effective mass are obtained. In the final section a summary and discussion are presented.

II. QUARK COMPONENTS AND LIGHT MESON FIELDS

The generating functional to be considered is the following:

$$Z[\xi,\bar{\xi}] = N\int \mathcal{D}[\bar{\psi},\psi]e^{i\int(\mathcal{L}+\bar{\psi}J+\bar{J}\psi)},$$

where the Lagrangian density for the minimal electromagnetic coupling (for a background electromagnetic field) to the global color model can be written as

$$\mathcal{L} = \bar{\psi}(i\gamma\cdot D - m)\psi - \frac{g^2}{2}\int_y \bar{\psi}(x)\gamma_\mu\lambda^b\psi(x)R_{bc}^{\mu\nu}(x-y)\bar{\psi}(y)\gamma_\nu\lambda^c\psi(y), \quad (2)$$

where $a, b\ldots = 1,\ldots(N_c^2-1)$ stand for color in the adjoint representation and $i,j,k = 0,\ldots(N_f^2-1)$ will be used for SU(2) flavor indices, the sums in color, flavor and Dirac indices are implicit, and the covariant quark derivative is $D = D_\mu = \partial_\mu\delta_{ij} - ieQ_{ij}A_\mu$ for the diagonal matrix $\hat{Q} = \mathrm{diag}(2/3,-1/3)$. In several gauges, the gluon kernel can be written in terms of the transversal and longitudinal components as $R_{ab}^{\mu\nu}(x-y) = \delta_{ab}[(g^{\mu\nu} - \frac{\partial^\mu\partial^\nu}{\partial^2})R_T(x-y) + \frac{\partial^\mu\partial^\nu}{\partial^2}R_L(x-y)]$. The infrared regime of the gluon propagator exhibits a nontrivial behavior that is often parameterized in terms of an effective gluon mass [47]. This will be discussed further in Sec. IV.

To make possible a more detailed investigation of the different flavor channels of quark interactions a Fierz transformation [18,48] can be done next. Then, for the quark interaction above,

$$\Omega \equiv \frac{g^2}{2}\bar{\psi}(x)\gamma_\mu\lambda^b\psi(x)R_{bc}^{\mu\nu}(x-y)\bar{\psi}(y)\gamma_\nu\lambda^c\psi(y),$$

the Fierz transformed $\mathcal{F}(\Omega)$ color singlet expression is given by

$$\begin{aligned}\mathcal{F}(\Omega) = \alpha g^2\{&[j_S(x,y)j_S(y,x) \\ &+ j_P^i(x,y)j_P^i(y,x)]R(x-y) \\ &-\frac{1}{2}[j_{V,\mu}^i(x,y)j_{V,\nu}^i(y,x) \\ &- j_{\mu A}^i(x,y)j_{\nu A}^i(y,x)]\bar{R}^{\mu\nu}(x-y)\},\end{aligned}$$

where $\alpha = 8/9$ for SU(2) flavor, and the following bilocal quark bilinears $[j_i^q(x,y) = \bar{\psi}(x)\Gamma^q\psi(y)$ for operators Γ_q where $q = s,\ p,\ v,\ a]$ were defined:

$$\begin{aligned} j^S(x,y) &= \bar{\psi}(x)\psi(y), \\ j_i^P(x,y) &= \bar{\psi}(x)\sigma_i i\gamma_5\psi(y), \\ j_i^{V,\mu}(x,y) &= \bar{\psi}(x)\gamma^\mu\sigma_i\psi(y), \\ j_i^{\mu,A}(x,y) &= \bar{\psi}(x)i\gamma_5\gamma^\mu\sigma_i\psi(y). \end{aligned} \quad (3)$$

In these expressions the following kernels were used:

$$\begin{aligned} R(x-y) \equiv R &= 3R_T(x-y) + R_L(x-y), \\ \bar{R}^{\mu\nu}(x-y) \equiv \bar{R}^{\mu\nu} &= g^{\mu\nu}(R_T(x-y) + R_L(x-y)) \\ &\quad + 2\frac{\partial^\mu\partial^\nu}{\partial^2}(R_T(x-y) - R_L(x-y)). \end{aligned} \quad (4)$$

The long-wavelength or local limit of the scalar and pseudoscalar interactions yields the NJL coupling with $G \sim \frac{g^2}{\Lambda_{qcd}^2}$ or $G \sim \frac{g^2}{M_G^2}$ for massless and massive gluons [35,49,50].

The quark field will be split according to the background field method (BFM) [51,52]. One component is considered to be a (constituent quark) background field (ψ_1), and the sea quark field (ψ_2) will be integrated out. This splitting of the field can be made by means of the bilinears $\bar{\psi}\Gamma\psi$, where Γ stands for Dirac, color or flavor operators, such that the resulting determinant corresponds basically to the one-loop BFM results. The splitting can be written as [35,36]

$$\bar{\psi}\Gamma^q\psi \to (\bar{\psi}\Gamma^q\psi)_2 + (\bar{\psi}\Gamma^q\psi)_1, \quad (5)$$

where $(\bar{\psi}\psi)_2$ will be integrated out, being possible that it composes light mesons and the scalar condensate and the component $(\bar{\psi}\psi)_1$ stands for the background field that yields baryon constituent quarks. This separation preserves chiral symmetry, and it may not correspond to a simple mode separation of low and high energies which might be a very restrictive assumption since pions and constituent quarks might be composed by quarks with similar energy modes (fully or in part). Therefore it seems the criterion might not involve a separation scale and, at the end, the shape of the results should be basically the same. The shift of bilinear may also be suitable for envisaging quark-antiquark states which are the most important states for the very low energy QCD, i.e. below the nucleon mass scale. The effective interaction Ω is split accordingly and terms with mixed bilinear of ψ_1 and ψ_2 can be written such that the quadratic part of bilinear $\bar{\psi}_2\psi_2$ will be suitable to be integrated out. The interaction Ω_2 deserves some more attention and it can be handled in two ways: (i) By resorting to a weak field approximation $\Omega_2 \ll \Omega_1$ which yields directly the one-loop BFM that might receive corrections by a perturbative expansion which incorporates Ω_2 [51]; (ii) by making use of the auxiliary field method according to which a set of auxiliary fields is introduced by means of unitary functional integrals multiplying the generating functional [36,38,39,53,54]. Auxiliary fields (AFs) allow one to incorporate properly DChSB with the formation of the scalar quark condensate which endows quarks with a large effective mass. Therefore the use of the AFs improves the one-loop background field method as usually implemented. Auxiliary fields are introduced by multiplying the generating functional by the following normalized Gaussian integrals:

$$
\begin{aligned}
1 = N \int D[S]D[P_i] e^{-\frac{i}{2}t_2^2 \int_{x,y} R\alpha[(S-gj^S_{(2)})^2+(P_i-gj^P_{i,(2)})^2]} \\
\times \int D[V^i_\mu] \int D[\bar{A}^i_\mu] e^{-\frac{i}{4}t_2^2 \int_{x,y} \bar{R}^{\mu\nu}\alpha[(V^i_\mu - gj^{i,(2)}_{V,\mu})(V^i_\nu - gj^{i,(2)}_{V,\nu})]} \\
\times e^{-\frac{i}{4}t_2^2 \int_{x,y} \bar{R}^{\mu\nu}\alpha[(\bar{A}^i_\mu - gj^{i,(2)A}_\mu)(\bar{A}^i_\nu - gj^{i,(2)A}_\nu)]}.
\end{aligned}
\tag{6}
$$

In these expressions the bilocal AFs are $S(x,y)$, $P_i(x,y)$, $V^i_\mu(x,y)$ and $\bar{A}^i_\mu(x,y)$ and they have been shifted by quark currents such as to cancel out the fourth-order interactions Ω_2. These shifts have unit Jacobian and they generate a coupling to quarks. The nonlocality of these auxiliary fields gives rise to form factors which nevertheless can produce punctual meson fields by expanding in an infinite basis of local fields. Finally it is also possible to consider the long-wavelength limit by keeping only the lowest energy states and by simply considering the local limit for structureless light mesons [36]. The resulting effective action for quarks (ψ_1 and ψ_2) interacting with auxiliary fields (quark-antiquark mesons) is quadratic in ψ_2 requiring a typical Gaussian integration. The resulting determinant can be written, by considering the identity $\det A = \exp \mathrm{Tr} \ln(A)$, as

$$
\begin{aligned}
S_{\mathrm{eff}} = i\mathrm{Tr}\ln\{S_0{}^{-1}(x-y) + \Xi(x-y) - \alpha g^2 \bar{R}^{\mu\nu}(x-y)\gamma_\mu\sigma_i[(\bar{\psi}(y)\gamma_\nu\sigma_i\psi(x))_1 + i\gamma_5(\bar{\psi}(y)i\gamma_5\gamma_\nu\sigma_i\psi(x))_1] \\
+ 2R(x-y)\alpha g^2[(\bar{\psi}(y)\psi(x))_1 + i\gamma_5\sigma_i(\bar{\psi}(y)i\gamma_5\sigma_i\psi(x))_1]\} - \frac{1}{2}\int_{x,y}\left\{R[S^2+P_i^2] + \frac{1}{2}\bar{R}^{\mu\nu}[V^i_\mu V^i_\nu + \bar{A}^i_\mu\bar{A}^i_\nu]\right\} \\
- \int_x \bar{\psi}_1(x)(i\gamma_\mu D^\mu - m)\psi_1(x) - \frac{g^2}{2}\int_{x,y} j^{a,(1)}_\mu(x) R^{\mu\nu}_{ab}(x-y) j^{b,(1)}_\nu(y),
\end{aligned}
\tag{7}
$$

where Tr stands for traces of discrete internal indices and integration of spacetime coordinates, the inverse Fierz transformation was done for the ψ_1 interaction that is written in the last line, and where $S_0^{-1} = (i\not{D} - m)$, being $\not{D} = \gamma^\mu(\partial_\mu\delta_{ij} - ieQ_{ij}A_\mu)$. Ξ stands for the auxiliary fields coupling to sea quarks. Vector and axial auxiliary fields yield heavier excitations and may be neglected for the low energy regime. The bilocal AFs can be expanded in a basis of local meson excitations. However, this work is concerned with the effects of weak magnetic field in the low energy regime of quark effective interactions and the local limit of these composite fields might be adopted because the only leading effect of the AFs is to produce the large quark effective mass due to DChSB. The quark coupling to the local scalar and pseudoscalar fields, in the absence of the heavier vector states, can be written as

$$
\Xi(x,y) = g\alpha F_{0,0}(x-y) R\left[S\left(\frac{x+y}{2}\right) + P_i\left(\frac{x+y}{2}\right) i\gamma_5\sigma_i\right],
\tag{8}
$$

where, due to the structureless mesons approximation, it will be considered $z = (x+y)/2 = x$. Then it reduces to

$$
\Xi(x,y) \simeq F[s(x) + p_i(x)\gamma_5\sigma_i]\delta(x-y),
\tag{9}
$$

where F is the pion decay constant that allows for the canonical definition of the pion field as $\pi_i = F p_i$. The saddle point equations for expression (7) yield the usual gap equations; by denoting the auxiliary fields $\phi_q = S(x,y)$, $P_i(x,y)$, $V^\mu_i(x,y)$ and the axial field $\bar{A}^\mu_i(x,y)$ these equations are

$$
\frac{\partial S_{\mathrm{eff}}}{\partial \phi_q} = 0.
\tag{10}
$$

These equations for the NJL model and GCM have been analyzed in many works, under external B or not, for the vacuum or at finite temperatures or quark densities, including in the complete form which corresponds to Dyson Schwinger equations in the rainbow ladder approximation.

The only possible nontrivial solution might exist for the scalar field since the ground state is scalar. It yields a correction to the quark mass, as the leading effect, and therefore only the limit of local auxiliary field S is needed from here on. The magnetic field is known to increase the effective mass in the magnetic catalysis effect, for example in Refs. [2,5,6,55]. By considering solutions for which the quark-gluon coupling of the model is sufficiently strong to generate DChSB, as shown in Sec. IV, it yields a correction to the quark effective mass (M^*) such that the quark kernel in expression (7) receives a correction, being written as

$$S_0^{-1} = (i\not{D} - M^*). \tag{11}$$

At this point it is worth noticing that an estimate of the effect of the AFs on the eventual quark-quark effective interactions can be obtained by expanding the quark determinant above in powers of the AFs. However it is seen that the effects of AFs on the quark-quark effective interactions only will appear at least in the third order of the expansion to produce, for example, terms of the following form: $\phi_q(\bar{\psi}\Gamma_q\psi)^2$. These terms are of higher order in S_0^n and consequently numerically smaller in the large quark mass limit. Alternatively, if these auxiliary fields are kept as a whole and afterwards eliminated, for example being integrated out approximately when expanding the determinant in a steepest descent approximation, their contribution to the photon-quark interaction would be again of higher order. Therefore the AFs can be neglected by keeping the nontrivial value of the scalar field that endows quarks with an effective mass. Results will be precisely those from one-loop BFM with the corrected quark effective mass.

The determinant can be rewritten as

$$I_{\rm det} = {\rm Tr}\ln\left(S_0^{-1} + \sum_q a_q\Gamma_q j_q\right) = \frac{1}{2}{\rm Tr}\ln\left[\left(S_0^{-1} + \sum_q a_q\Gamma_q j_q\right)\left(\bar{S}_0^{-1} + \left(\sum\nolimits_q \bar{a}_q\Gamma_q j_q\right)^*\right)\right], \tag{12}$$

where $\bar{S}_0^{-1} = (i\not{D} + M^*)$; for $q = s, p, v, a$ and $\bar{a}_s = -a_s$, $\bar{a}_v = -a_v$ and $\bar{a}_p = a_p$, $\bar{a}_a = a_a$, and also it has been defined the following shorthand notation for the four channels q:

$$\begin{aligned}\sum_q a_q\Gamma_q j_q = &-\alpha g^2 \bar{R}^{\mu\nu}(x-y)\gamma_\mu\sigma_i[(\bar{\psi}(y)\gamma_\nu\sigma_i\psi(x)) + i\gamma_5(\bar{\psi}(y)i\gamma_5\gamma_\nu\sigma_i\psi(x))] \\ &+ 2\alpha g^2 R(x-y)[(\bar{\psi}(y)\psi(x)) + i\gamma_5\sigma_i(\bar{\psi}(y)i\gamma_5\sigma_i\psi(x))].\end{aligned} \tag{13}$$

By turning the (background) quark currents to zero this determinant yields the celebrated Euler Heisenberg effective action for the electromagnetic field [31,32,48,55]. Below, a large quark mass expansion will be performed and the leading quark-quark effective couplings and their dependence on a constant magnetic field will be shown.

III. EXPANSION OF THE DETERMINANT AND EFFECTIVE COUPLINGS

The large quark mass expansion of the determinant will be performed next by neglecting all the quark derivative couplings [56]. A shorthand notation will be used below to improve the reading of the expressions; the gluon kernels will be written shortly: $R \equiv R(x-y)$, $\bar{R}^{\mu\nu} \equiv \bar{R}^{\mu\nu}(x-y)$ and so on. By neglecting terms such as ${\rm Tr}\ln(iS_0^{-1})$ that becomes an irrelevant constant in the generating functional, the dynamical part of expression (12), by considering the anticommutation relations of the Dirac gamma matrices, can then be written as

$$\begin{aligned}S_d \simeq {\rm Tr}\sum_{n=1}^{\infty} d_n\{\tilde{S}_2[&\Delta_A + \xi + \xi_{sb} + \xi_{\rm der} + I_{\rm crossed} + 4(\alpha g^2)^2R^2[j_S(x,y)j_S(y,x) + \gamma_5^2\sigma_i\sigma_j j_P^i(x,y)j_P^j(y,x)] \\ &- (\alpha g^2)^2\bar{R}^{\mu\nu}\bar{R}^{\rho\sigma}\gamma_\mu\gamma_\rho\sigma_i\sigma_j[j_{V\nu}^{\ i}(x,y)j_{V\sigma}^{\ j}(y,x) - \gamma_5^2 j_{A\nu}^{\ i}(x,y)j_{A\sigma}^{\ j}(y,x)]]\}^n,\end{aligned} \tag{14}$$

where the following terms have been defined:

$$\Delta_A = -e^2\hat{Q}^2(A_\mu A^\mu + A^\mu A^\nu\sigma_{\mu\nu}) + \frac{ie}{2}\hat{Q}\sigma_{\mu\nu}F^{\mu\nu}, \tag{15}$$

$$\xi = -e2(\alpha g^2)R\hat{Q}\sigma_i i[A\!\!\!/,\gamma_5]j_P^i(x,y) + e(\alpha g^2)\bar{R}^{\mu\nu}\hat{Q}\sigma_i[[\gamma_\mu, A\!\!\!/]j_{V\nu}^{\ i}(x,y) + \{\gamma_\mu i\gamma_5, A\!\!\!/\}j_{A\nu}^{\ i}(x,y)], \quad (16)$$

$$\xi_{sb} = 4(\alpha g^2)RM^* j_S(x,y) - 2M^*(\alpha g^2)\bar{R}^{\mu\nu}\sigma_i\gamma_\mu j_{V\nu}^{\ i}(x,y), \quad (17)$$

$$\xi_{\mathrm{der}} = -i\alpha g^2\bar{R}^{\mu\nu}(x-y)\sigma_i[[\gamma_\rho,\gamma_\mu]\partial^\rho(j_{V,\nu}^i(y,x)) + i\gamma_5\{\gamma_\rho,\gamma_\mu\}\partial^\rho(j_{A,\nu}^i(y,x))] + 2\alpha g^2 R(x-y)i\sigma_i[\gamma_\rho,\gamma_5]\partial^\rho(j_P^i(y,x)), \quad (18)$$

$$I_{\mathrm{crossed}} = i(\alpha g^2)^2\bar{R}^{\mu\nu}\bar{R}^{\rho\sigma}\{\gamma_\mu\gamma_\rho\gamma_5\sigma_i\sigma_j + \gamma_\rho\gamma_5\gamma_\mu\sigma_j\sigma_i\}j_{V\nu}^{\ i}(x,y)j_{A\sigma}^{\ j}(y,x) + 2(\alpha g^2)R\bar{R}^{\mu\nu}\{\gamma_5\gamma_\mu\gamma_5\sigma_i\sigma_j + \gamma_\mu\gamma_5^2\sigma_j\sigma_i\}j_P^j(x,y)j_{A\nu}^{\ i}(y,x)$$
$$+ 2i(\alpha g^2)R\bar{R}^{\mu\nu}\{\gamma_5\gamma_\mu\sigma_j\sigma_i - \gamma_\mu\gamma_5\sigma_i\sigma_j\}j_P^j(x,y)j_{V\nu}^{\ i}(y,x) + 2(\alpha g^2)R\bar{R}^{\mu\nu}\sigma_i\gamma_\mu j_S(x,y)j_{V\nu}^{\ i}(y,x). \quad (19)$$

The terms Δ_A and ξ contain magnetic-field-dependent terms, and ξ_{sb} presents the symmetry breaking terms since they appear to be proportional to the Lagrangian quark mass m. However if DChSB is considered for the auxiliary scalar field and the corresponding gap equation, this Lagrangian mass is corrected to an effective mass M^*. The other terms above are ξ_{der} with the derivative terms that, with an integration by parts, may produce constant magnetic field contribution when multiplied by ξ, whereas I_{crossed} contains mixing interactions with different quark currents and they produce nonzero terms in the expansion only in higher orders. In this work only the lowest order terms will be investigated, up to the second order in the expansion. The third order of the expansion will have additional factors $\tilde{S}_2$, each of them being $\mathcal{O}(1/M^{*2})$ smaller than the second order ones. In expression (14) the following parameters were defined:

$$d_n = -i\frac{(-1)^{n+1}}{2n}, \quad (20)$$

$$\tilde{S}_2 = 1/(-\partial^2 - M^{*2}). \quad (21)$$

A. First order terms

In the long-wavelength or local limit of the expressions below the effective couplings can be resolved to yield effective coupling constants. In the zero order derivative expansion for the first order expansion the following effective couplings appear:

$$\mathcal{I}_{\mathrm{eff},1} = \Delta M^*\bar{\psi}\psi + g_4[(\bar{\psi}\psi)^2 + (\bar{\psi}\sigma_i i\gamma_5\psi)^2] - g_{4v}[(\bar{\psi}\sigma_i\gamma_\mu\psi)^2 + (\bar{\psi}\sigma_i\gamma_\mu\gamma_5\psi)^2]. \quad (22)$$

These couplings have already been found in Refs. [36,37] and, for these expressions, traces of Dirac and Pauli matrices are taken. The effective coupling constants were defined in the following way:

$$\Delta M^* = -i2(\alpha g^2)\mathrm{Tr}(\tilde{S}_2 R M^*),$$
$$g_4(1,\delta_{ij}) = -i2(\alpha g^2)^2\mathrm{Tr}(\tilde{S}_2 R^2(1,\sigma_i\sigma_j)),$$
$$g_{4v}g^{\nu\sigma}\delta_{ij} = -\frac{i}{2}(\alpha g^2)^2\mathrm{Tr}(\tilde{S}_2\bar{R}^{\mu\nu}\bar{R}^{\rho\sigma}\gamma_\mu\gamma_\rho\sigma_i\sigma_j), \quad (23)$$

where, by performing the trace in Dirac indices, the following kernel can be defined:

$$\bar{R}_\mu^\rho\bar{R}_{\rho\nu} = \bar{R}_{2\mu\nu} = g_{\mu\nu}(R_T + R_L)^2 + 8\frac{k_\mu k_\nu}{k^2}R_T(R_T - R_L). \quad (24)$$

The expression for the effective mass (23) might be ultraviolet divergent or finite depending on the gluon propagator behavior. However the effective couplings constants g_4 and $g_{4,v}$ are finite unless the quark and gluon kernels present an unusual momentum dependence. For gluon propagators written in terms of an effective gluon mass these expressions should also be infrared finite.

B. Second order quark terms up to $\mathcal{O}(\bar{\psi}\Gamma_q\psi)^2$

The second order nonderivative couplings that depend on the magnetic field will be exhibited below. Those terms containing one derivative of quark currents (ξ_{der}) that multiply the vector A^μ either can yield nontrivial contributions to (nonderivative) effective quark couplings if an integration by parts is performed, producing quark couplings to the strength tensor $F^{\mu\nu}$, or it may disappear. The terms that produce nonzero contributions are shown below ($\mathcal{I}_{4q} = I_2 + I_4 + I_{4,\xi} + I_{4,\mathrm{der}} + I_{\mathrm{cross},B}$). The two possible orders of combining structures for each of the term in the expansion will be written as a big anticommutator in most of the terms. Although all the calculations will be performed for the Landau gauge for a constant magnetic field $A_\mu = -B_0(0,0,x,0)$, the electromagnetic field will be carried almost until the last expressions. These terms are the following:

$$
\begin{aligned}
I_2 &= ie^2(\alpha g^2) M^* \mathrm{Tr} \hat{Q}^2 \{\tilde{S}_2(A_\mu A^\mu), \tilde{S}_2 R\} j_S(x,y),\\
I_4 &= -ie^2(\alpha g^2)^2 \mathrm{Tr}\{\tilde{S}_2(A_\mu A^\mu + A^\mu A^\nu \sigma_{\mu\nu}), \tilde{S}_2 R^2\}(\hat{Q}^2 j_S(x,y) j_S(y,x) + \gamma_5^2 \hat{Q}^2 \sigma_i \sigma_j j_P^i(x,y) j_P^j(y,x))\\
&\quad - \frac{i}{4} e^2(\alpha g^2)^2 \mathrm{Tr}\{\tilde{S}_2(A_\mu A^\mu + A^\mu A^\nu \sigma_{\mu\nu}), \tilde{S}_2 \bar{R}^{\rho_1\sigma_1} \bar{R}^{\rho\sigma} \gamma_{\rho_1} \gamma_\rho \hat{Q}^2 \sigma_i \sigma_j\}\\
&\quad \times (j_{V\sigma_1}^{\;i}(x,y) j_{V\sigma}^{\;j}(y,x) - \gamma_5^2 j_{A\sigma_1}^{\;i}(x,y) j_{A\sigma}^{\;j}(y,x))\\
&\quad + \frac{i}{8} ie(\alpha g^2)^2 \mathrm{Tr}\{\tilde{S}_2 F^{\mu_1\nu_1}, \tilde{S}_2 \bar{R}^{\mu\nu} \bar{R}^{\rho\sigma} \sigma_{\mu_1\nu_1} \gamma_\mu \gamma_\rho \hat{Q}^2 \sigma_i \sigma_j\}(j_{V\nu}^{\;i}(x,y) j_{V\sigma}^{\;j}(y,x) - \gamma_5^2 j_{A\nu}^{\;i}(x,y) j_{A\sigma}^{\;j}(y,x)),\\
I_{4,\xi} &= ie^2(\alpha g^2)^2 \mathrm{Tr} \hat{Q}\sigma_i \hat{Q}\sigma_j (\tilde{S}_2 R[\not{A}, \gamma_5] \tilde{S}_2 R[\not{A}, \gamma_5]) j_P^i(x,y) j_P^j(y,x)\\
&\quad + \frac{i}{4} e^2(\alpha g^2)^2 \mathrm{Tr} \hat{Q}\sigma_i \hat{Q}\sigma_j (\tilde{S}_2[\gamma_\mu, \not{A}] \bar{R}^{\mu\nu} \tilde{S}_2[\gamma_\rho, \not{A}] \bar{R}^{\rho\sigma}) j_{V\nu}^{\;i}(x,y) j_{V\sigma}^{\;j}(y,x)\\
&\quad + \frac{i}{4} e^2(\alpha g^2)^2 \mathrm{Tr} \hat{Q}\sigma_i \hat{Q}\sigma_j (\tilde{S}_2\{\gamma_\mu i\gamma_5 \not{A}\} \bar{R}^{\mu\nu} \tilde{S}_2\{\gamma_\rho i\gamma_5, \not{A}\} \bar{R}^{\rho\sigma}) j_{A\nu}^{\;i}(x,y) j_{A\sigma}^{\;j}(y,x)\\
&\quad + \frac{i}{4} e^2(\alpha g^2)^2 \mathrm{Tr} \hat{Q}\sigma_i \hat{Q}\sigma_j (\tilde{S}_2[\gamma_\mu, \not{A}] \bar{R}^{\mu\nu} \tilde{S}_2[\gamma_\rho, \not{A}](i\gamma_5) \bar{R}^{\rho\sigma}) j_{V\nu}^{\;i}(x,y) j_{A\sigma}^{\;j}(y,x)\\
&\quad + i2e(\alpha g^2)^2 \mathrm{Tr} M^*(\tilde{S}_2 \hat{Q}\sigma_i[\gamma_\mu, \not{A}] \bar{R}^{\mu\nu} \tilde{S}_2 R) j_{V\nu}^{\;i}(x,y) j_S(x,y)\\
&\quad + i4(\alpha g^2)^2 \mathrm{Tr}(M^* \tilde{S}_2 R)^2 j_S(x,y) j_S(y,x)\\
&\quad + i(\alpha g^2)^2 \mathrm{Tr}(\sigma_i \sigma_j \gamma_\mu \gamma_\rho M^* \tilde{S}_2 \bar{R}^{\mu\nu} M^* \tilde{S}_2 \bar{R}^{\rho\sigma}) j_{V\nu}^{\;i}(x,y) j_{V\sigma}^{\;j}(y,x),\\
I_{4,\mathrm{der}} &= -i^2 e(\alpha g^2)^2 \mathrm{Tr}\{\tilde{S}_2 R \hat{Q}\sigma_i[\not{A}, \gamma_5], \tilde{S}_2 R \sigma_j[\gamma_5, \gamma_\rho]\} j_P^i(x,y) \partial^\rho(j_P^j(y,x))\\
&\quad + \frac{i^2}{2} e(\alpha g^2)^2 \mathrm{Tr}\{\tilde{S}_2 \bar{R}^{\mu\nu} \hat{Q}\sigma_i([\gamma_\mu, \not{A}]), \tilde{S}_2 \bar{R}^{\mu_2\nu_2}(x-y)\sigma_j[\gamma_\rho, \gamma_{\mu_2}]\} j_{V\nu}^{\;i}(x,y) \partial^\rho(j^j_{V,\nu_2}(y,x))\\
&\quad + \frac{i^2}{2} e(\alpha g^2)^2 \mathrm{Tr}\{\tilde{S}_2 \bar{R}^{\mu\nu} \hat{Q}\sigma_i([\gamma_\mu, \not{A}]), \tilde{S}_2 \bar{R}^{\mu_2\nu_2}(x-y)\sigma_j \gamma_5\{\gamma_\rho, \gamma_{\mu_2}\}\} j_{V\nu}^{\;i}(x,y) \partial^\rho(j^j_{A,\nu_2}(y,x))\\
&\quad + \frac{i^2}{2} e(\alpha g^2)^2 \mathrm{Tr}\{\tilde{S}_2 \bar{R}^{\mu\nu} \hat{Q}\sigma_i\{\gamma_\mu \gamma_5, \not{A}\}, \tilde{S}_2 \bar{R}^{\mu_2\nu_2}(x-y)\sigma_j[\gamma_\rho, \gamma_{\mu_2}]\} j_{A\nu}^{\;i}(x,y) \partial^\rho(j^j_{V,\nu_2}(y,x))\\
&\quad + \frac{i^4}{2} e(\alpha g^2)^2 \mathrm{Tr}\{\tilde{S}_2 \bar{R}^{\mu\nu} \hat{Q}\sigma_i\{\gamma_\mu \gamma_5, \not{A}\}, \tilde{S}_2 \bar{R}^{\mu_2\nu_2}(x-y)\sigma_j \gamma_5\{\gamma_\rho, \gamma_{\mu_2}\}\} j_{A\nu}^{\;i}(x,y) \partial^\rho(j^j_{A,\nu_2}(y,x)),\\
I_{\mathrm{cross},B} &= -\frac{i}{2}(\alpha g^2)^2 e^2 \mathrm{Tr}\left[\tilde{S}_2\left(\hat{Q}^2(A_{\mu_2} A^{\mu_2} + A^{\mu_2} A^{\nu_2} \sigma_{\mu_2\nu_2}) + \frac{ie}{2} \hat{Q} \sigma_{\mu_2\nu_2} F^{\mu_2\nu_2}\right)\right.\\
&\quad \left. \times \tilde{S}_2 \bar{R}^{\mu\nu} \bar{R}^{\rho\sigma}(\gamma_\mu \gamma_\rho \sigma_i \sigma_j + \gamma_\rho \gamma_\mu \sigma_j \sigma_i)\gamma_5\right] j_{V\nu}^{\;i}(x,y) j_{A\sigma}^{\;j}(y,x). \qquad (25)
\end{aligned}
$$

The following traces of isospin and Dirac indices (Tr_F and Tr_D) will be used in the next steps:

$$\mathrm{Tr}_F(\sigma_i \sigma_j) = 2\delta_{ij}, \qquad (26)$$

$$\mathrm{Tr}_F(\hat{Q}\sigma_i \sigma_j) = \frac{1}{3}\delta_{ij} + i\epsilon_{ij3}, \qquad (27)$$

$$\mathrm{Tr}_F(\hat{Q}^2 \sigma_i \sigma_j) = \frac{5}{9}\delta_{ij} + \frac{i}{3}\epsilon_{ij3}, \qquad (28)$$

$$\mathrm{Tr}_D(\gamma^\mu \gamma^\nu) = 4g^{\mu\nu}, \qquad (29)$$

$$\mathrm{Tr}_D(\sigma_{\mu\sigma} \sigma_{\rho\mu_2}) = 4(g_{\mu\mu_2} g_{\sigma\rho} - g_{\mu\rho} g_{\sigma\mu_2}), \qquad (30)$$

$$\mathrm{Tr}_D(\gamma^5 \gamma^\alpha \gamma^\beta \gamma^\delta \gamma^\lambda) = -4i\epsilon^{\alpha\beta\delta\lambda}. \qquad (31)$$

By resolving the effective coupling constants in the long-wavelength limit, several of the terms above disappear. Besides that, only the momentum derivatives of internal lines will be considered. The nonzero contributions of these expressions can be written as

$$\begin{aligned}\mathcal{L}_{4q} &= \Delta_B M^* \bar{\psi}\psi + g_{4,B}[(\bar{\psi}\psi)^2 + (\bar{\psi}\sigma_i i\gamma_5\psi)^2] + \left(\frac{3g_{4,B}}{5} i\epsilon_{ij3} + g_{ps,B} c_i \delta_{ij}\right)(\bar{\psi}\sigma_i i\gamma_5\psi)(\bar{\psi}\sigma_j i\gamma_5\psi) \\ &\quad + \left[\delta_{ij}\left(g_{4v,B} + g_{4v,B2} c_i + \frac{g_{4v,B-F}}{3} + g_{4v2,B}\right) + i\epsilon_{ij3}\left(\frac{3}{5} g_{4v,B} + g_{4v,B-F} + 3g_{4v2,B}\right)\right] \\ &\quad \times [(\bar{\psi}\sigma_i\gamma_\mu\psi)(\bar{\psi}\sigma_j\gamma^\mu\psi) + (\bar{\psi}\sigma_i\gamma_\mu\gamma_5\psi)(\bar{\psi}\sigma_j\gamma^\mu\gamma_5\psi)] \\ &\quad + g_{s,sb}(\bar{\psi}\psi)^2 + g_{v,sb}(\bar{\psi}\sigma_i\gamma_\mu\psi)^2,\end{aligned} \tag{32}$$

where the following notation was adopted in the terms depending on the coefficients c_i with operators Γ_i:

$$c_i(\bar{\psi}\Gamma_i\psi)^2 = c_1(\bar{\psi}\Gamma_1\psi)^2 + c_2(\bar{\psi}\Gamma_2\psi)^2 + c_3(\bar{\psi}\Gamma_3\psi)^2, \tag{33}$$

the following isospin coefficients being defined:

$$c_1 = -\frac{4}{9}, \qquad c_2 = \frac{4}{9}, \qquad c_3 = \frac{5}{9}, \tag{34}$$

These coefficients are responsible for the pseudoscalar, vector and axial quark-antiquark states (pion, rho, A_1) couplings to the magnetic field. In the first and second lines of expression (32) there are effective couplings dependent on the magnetic field and in the last line those due to the explicit symmetry breaking discussed in Ref. [37]. The couplings $g_{4,B}$ and mainly $g_{ps,B}$ are responsible for extra contributions to the axial current and then they allow for chiral separation effect. Chiral and isospin breaking terms also appear in the vector channel. The effective coupling constants are defined as

$$\Delta_B M^* = -i2e^2(\alpha g^2)\mathrm{Tr}[\hat{Q}^2\tilde{S}_2(A_\mu A^\mu)M^*\tilde{S}_2 R], \tag{35}$$

$$g_{4,B}\left(1; \left(\delta_{ij} + \frac{3}{5} i\epsilon_{ij3}\sigma_3\right)\right) = -i2e^2(\alpha g^2)^2\mathrm{Tr}[\hat{Q}^2\tilde{S}_2 A^\mu A_\mu \tilde{S}_2 R^2](1; \gamma_5^2\sigma_i\sigma_j), \tag{36}$$

$$g_{4v,B}\left(\delta_{ij} + \frac{3}{5} i\epsilon_{ij3}\sigma_3\right) g^{\sigma_1\sigma} = -\frac{i}{2} e^2(\alpha g^2)^2\mathrm{Tr}[\hat{Q}^2\sigma_i\sigma_j\gamma_{\rho_1}\gamma_\rho\tilde{S}_2 A^\mu A_\mu\tilde{S}_2\bar{R}^{\rho_1\sigma_1}\bar{R}^{\rho\sigma}], \tag{37}$$

$$g_{ps,B} c_i\delta_{ij} = -i2e^2(\alpha g^2)^2\mathrm{Tr}[\hat{Q}\sigma_i\hat{Q}\sigma_j(\tilde{S}_2[\not{A},\gamma_5]R)(\tilde{S}_2[\not{A},\gamma_5]R)], \tag{38}$$

$$g_{4v,B2} c_i\delta_{ij} g^{\nu\sigma} = -\frac{i}{4} e^2(\alpha g^2)^2\mathrm{Tr}[\hat{Q}\sigma_i\hat{Q}\sigma_j\tilde{S}_2[\gamma_\mu,\not{A}]\bar{R}^{\mu\nu}\tilde{S}_2[\gamma_\rho,\not{A}]\bar{R}^{\rho\sigma}], \tag{39}$$

$$g_{4v,B-F}(\delta_{ij} + 3i\epsilon_{ij3}\sigma_3)g^{\nu\sigma} = \frac{i}{2} ie(\alpha g^2)^2\mathrm{Tr}[\hat{Q}\sigma_i\sigma_j F^{\mu_1\nu_1}\tilde{S}_2\bar{R}^{\mu\nu}\tilde{S}_2\bar{R}^{\rho\sigma}\sigma_{\mu_1\nu_1}\gamma_\mu\gamma_\rho], \tag{40}$$

$$g_{4v2,B}(\delta_{ij} + 3i\epsilon_{ij3}\sigma_3)g_{\nu\nu_2} = -ie(\alpha g^2)^2\mathrm{Tr}[\hat{Q}\sigma_i\sigma_j(\partial_\sigma A_\rho)[\gamma^\mu,\gamma^\sigma][\gamma^\rho,\gamma^{\mu_2}]\tilde{S}_2\bar{R}_{\mu\nu}\tilde{S}_2\bar{R}_{\mu_2\nu_2}], \tag{41}$$

$$g_{s,sb} = -i4(\alpha g^2)^2\mathrm{Tr}(M^*\tilde{S}_2 R)^2, \tag{42}$$

$$g_{v,sb}\delta_{ij}g^{\nu\sigma} = -i(\alpha g^2)^2\mathrm{Tr}[\sigma_i\sigma_j\gamma_\mu\gamma_\rho M^*\tilde{S}_2\bar{R}^{\mu\nu}\tilde{S}_2 M^*\bar{R}^{\rho\sigma}]. \tag{43}$$

By performing the traces in discrete indices, always by neglecting the quark derivative couplings, and by taking $x = -i\frac{\partial}{\partial q_x}$ the above expressions can be written as

$$\Delta_B M^* = -i\frac{40}{9}(eB_0)^2(\alpha g^2)N_c\mathrm{Tr}'[M^*\tilde{S}_2 x^2\tilde{S}_2 R], \tag{44}$$

$$g_{4,B} = -i\frac{40}{9}(eB_0)^2(\alpha g^2)^2 N_c \mathrm{Tr}'[\tilde{S}_2 x^2 \tilde{S}_2 R^2], \quad (45)$$

$$g_{4v,B} g^{\sigma\sigma_1} = -i\frac{40}{9}(eB_0)^2(\alpha g^2)^2 N_c \mathrm{Tr}'[\tilde{S}_2 x^2 \tilde{S}_2 \bar{R}_2^{\sigma\sigma_1}], \quad (46)$$

$$g_{ps,B} c_j = -ic_j 8(eB_0)^2(\alpha g^2)^2 N_c \mathrm{Tr}'[(\tilde{S}_2 x R)^2], \quad (47)$$

$$g_{4v,B2} g^{\nu\sigma} c_j = -ic_j(eB_0)^2(\alpha g^2)^2 N_c \mathrm{Tr}' \times [\tilde{S}_2 x \bar{R}^{\mu\nu} \tilde{S}_2 x \bar{R}^{\rho\sigma}](4g_{\mu y} g_{y\rho} - 2g_{\mu\rho} g_{yy}), \quad (48)$$

$$g_{4v,B-F} = i\frac{8}{3}(eB_0)(\alpha g^2)^2 N_c \mathrm{Tr}'[\tilde{S}_2 \tilde{S}_2 \bar{R}_2^{xy}], \quad (49)$$

$$g_{4v2,B} = -i16(eB_0)(\alpha g^2)^2 N_c \mathrm{Tr}' \times [T_{xy}[8\tilde{S}_2(R_T - R_L)\tilde{S}_2(R_T - R_L) + \tilde{S}_2(R_T - R_L)\tilde{S}_2(R_T + R_L)]], \quad (50)$$

$$g_{s,sb} = -i32(\alpha g^2)^2 N_c \mathrm{Tr}'(M^* \tilde{S}_2 R)^2, \quad (51)$$

$$g_{v,sb} g^{\nu\sigma} = -i8(\alpha g^2)^2 N_c \mathrm{Tr}'[M^{*2} \tilde{S}_2 \bar{R}^{\nu\mu} \tilde{S}_2 \bar{R}_\mu^\sigma], \quad (52)$$

where $T_{xy} = \frac{k_x k_y}{k^2}$ in expression (50) and Tr' stands for the trace or integral in internal momenta. Due to the structure of $\bar{R}_2^{xy}$ in expression (24) the coupling $g_{4v,B-F}$ is nonzero only for a nonzero transversal component of the gluon propagator; i.e. if $R_T = 0$, it yields $g_{4v,B-F} = 0$.

In Fig. 1, the diagrams corresponding to the one-loop terms presented above are shown. The wavy line with a full dot is a (dressed) nonperturbative gluon propagator, and the short thick line insertions stand for the vector potential whereas the full triangle insertion stands for the magnetic field insertion ($F^{\mu\nu}$). Figure 1(a) shows the contribution to the effective mass due to the magnetic field, whereas Figs. 1(b1)–1(b3) represent the quark-quark effective interactions shown above.

The leading effective mass dependence on the magnetic field, shown in Fig. 1(a), is of the order of $(eB_0)^2/M^{*3}$ instead of the leading correction obtained from the gap equation $\sqrt{eB_0}$ [5]. The leading coupling constants in the expressions above are $g_{4v2,B}$ and $g_{4v,B-F}$ that are linearly proportional to the magnetic field $\partial_\mu A_\nu$, eB_0/M^{*2}. The corresponding diagram is shown in Fig. 1(b3). By extracting $1/M^{*2}$ from $\tilde{S}_2$ in the limit of large quark effective mass, it produces a quantity proportional to the dipole moment coupling itself $eB_0/(2M^*)$. In spite of the absence of a tensor current for the dipolar coupling in the leading effective action, the magnetic field couples directly to the vector or axial currents being a dipolar interaction. All the other couplings—in expressions (35)–(39)—have two insertions $A_\mu A^\mu$ introducing a larger (and suppressing) momentum dependence in internal lines, with corresponding factor $(eB_0)^2/(M^*)^4$. They are smaller in the limit of large quark effective mass.

IV. RATIOS BETWEEN EFFECTIVE COUPLING CONSTANTS

There are few ambiguities in performing numerical estimates of the effective coupling constants found above. The first reason is that the gluon propagator with its infrared behavior is not really well known and results depend strongly on it. Also, one has to choose a way of performing the momenta or energy traces, for example in Euclidean or Minkowski spaces, and this might yield different numerical results. Furthermore, other effects in the gluon sector, such as the B_0 dependence of the quark-gluon coupling or the gluon propagator itself, might be expected to yield B dependence at least of the same order of magnitude as the quark condensate (or quark effective mass) from the gap equation [57]. Due to these reasons numerical estimates will not be presented. Nevertheless, below a few solutions for the gap equation are presented with the intention to justify the approximations done, i.e. to consider the quark effective mass from DChSB as the leading effect of the auxiliary field and the large quark effective mass expansion. With respect to the gap equation, the behavior of the chiral condensate, and therefore of the

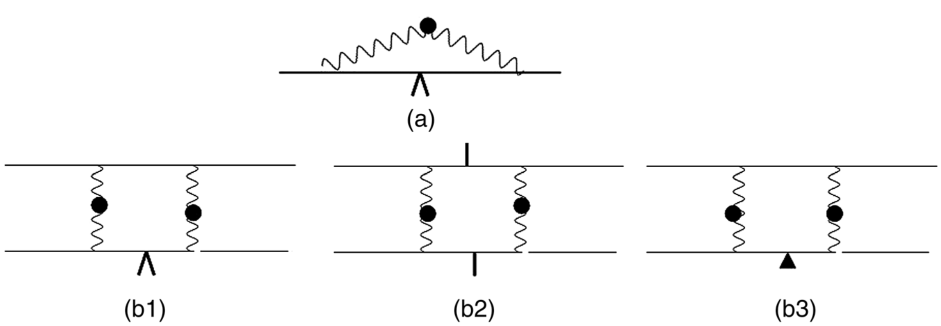

FIG. 1. In these diagrams, the wavy line with a full dot is a (dressed) nonperturbative gluon propagator, and the short bold line insertions for the vector potential, whereas the full triangle is for the magnetic field insertion ($F^{\mu\nu}$). Diagram (a) shows the effective mass due to the magnetic field contribution, whereas diagrams (b1)–(b3) represent all the quark-quark effective interactions shown above.

quark effective mass, under a constant magnetic field has been investigated extensively [2,57,58] and it has been found that the increase of the quark condensate with the (weak) magnetic field is due to the increase of the density of states by accounting the lowest Landau levels with high degeneracy in this regime. Besides that, magnetic catalysis has also been related to the positivity of the scalar QED β function [55].

To solve the gap equation (10) a longitudinal (confining) effective gluon propagator was chosen of the form of $g^2 R^{\mu\nu}_{ab}(k) = K_F/(k^2 + M_g^2)^2 g^{\mu\nu}\delta_{ab}$ for $K_F = 8\pi^3 M^2/9$ [43,59]. This effective propagator incorporates the large strength of the running coupling constant and, to some extent, some of the relatively important issues of the ultraviolet and infrared behavior of the gluon confining propagator [43]. With this gluon propagator the gap equation, as well as all the expressions for the effective quark masses and effective coupling constants, is finite, i.e. free of ultraviolet and infrared divergences. For example consider $M_g = M \simeq 378$ MeV, that is of the order of the values discussed in Ref. [43] in spite of being relatively smaller than the usual theoretical and lattice findings [41,47]. For a current quark mass $m = 10$ MeV, the gap equation (10) is nonzero only for the scalar auxiliary field s as defined, and it yields, for $B_0 = 0$, $s_0 \simeq 210$ MeV for which $M^* \simeq 220$ MeV. For a weak magnetic field $z = \frac{eB_0}{M^{*2}} = 0.1$ the gap equation yields $M^*(z = 0.1) = 227$ MeV. By considering $M_G = 511$ MeV, which is closer to the values obtained in lattice QCD, it yields $M^*(z = 0) = 300$ MeV and for weak magnetic field $M^*(z = 0.1) = 309$ MeV.

The effective coupling constants presented above can exhibit simple relations in the limit of large effective masses. For some of these effective coupling constants, this is achieved in specific limits of the gluon kernels. In the limit of very large quark effective mass, i.e. for $\tilde{S}_2 \to 1/M^{*2}$, some of these ratios are independent of the chosen component for the gluon propagator [$R_T(x - y)$ or $R_L(x - y)$], i.e.

$$\frac{\Delta M_B^*}{\Delta M^*} \sim \frac{5}{9}\frac{(eB_0)^2}{M^{*4}}, \qquad \frac{g_{4,B}}{g_4} \sim \frac{5}{9}\frac{(eB_0)^2}{M^{*4}},$$
$$\frac{g_{ps,B}}{g_{4,B}} \sim \frac{9}{5}, \qquad \frac{g_{4v,B2}}{g_{4v,B}} \sim \frac{27}{40}. \tag{53}$$

For other effective coupling constants, still in the limit of very large effective quark mass M^*, it is possible to obtain simple relations by considering particular relative contributions of the longitudinal and transversal components of the gluon propagator. In the following it will be considered that any of the two components present an effective gluon mass. The ratios will be computed in the limit of large masses $R_{T/L} \sim 1/M_G^n$ for $n = 2, 4$, by keeping $M_G > M^*$.

If it is assumed $R_L = 0$, then the following ratios are obtained:

$$\left(\frac{g_{4,B}}{g_{4v,B}}\right)^T \sim \frac{3}{4}, \qquad \left(\frac{g_{4,B-F}}{g_{4,B}}\right)^T \sim \frac{3}{20}\frac{M^{*2}}{eB_0},$$
$$\left(\frac{g_{4,B}}{g_{4v2,B}}\right)^T \sim \frac{80}{9}\frac{eB_0}{M^{*2}}, \tag{54}$$

whereas for $R_T = 0$ it yields

$$\left(\frac{g_{4,B}}{g_{4v,B}}\right)^L \sim \frac{1}{4}, \qquad \left(\frac{g_{4,B-F}}{g_{4,B}}\right)^L \sim 0,$$
$$\left(\frac{g_{4,B}}{g_{4v2,B}}\right)^L \sim \frac{80}{63}\frac{eB_0}{M^{*2}}. \tag{55}$$

All the coupling constants of the order of B_0 or B_0^2 are smaller than the NJL coupling, from expression (23), since a large effective quark mass expansion has been done: i.e. $\frac{(eB_0)}{M^{*2}} < 1$ or $\frac{(eB_0)}{M^{*2}} \ll 1$. However, by increasing the magnetic field strength this expansion still may be reliable up to some limit by computing higher orders terms (nth order expansion). This produces further quark-quark effective interactions dependent on B_0^{n-j} where $j = 0, 1, 2\ldots n$. Consequently the complete account of the Landau orbits that could be done for the quark kernel [2,58] emerges as a series in powers of the magnetic field in agreement with [60].

V. SUMMARY AND CONCLUSIONS

By departing from a (dressed) one-gluon exchange mechanism for the quark-quark interaction, different leading quark-quark effective interactions due to polarization were derived in the presence of a weak magnetic field, i.e. $eB_0 \ll M^{*2}$. The relevant assumption for the GCM is that the gluon propagator is dressed by nonperturbative effects due to the non-Abelian character of gluon interactions. The one-loop BFM method was applied with a correction due to the auxiliary field method. However only the leading effect of the auxiliary fields was considered, that is the correction to the quark effective mass. The one-loop quark effective action in the presence of the background field was expanded for large quark effective mass and weak magnetic field up to the second order in quark bilinears and to leading order in the magnetic field. The (leading) first and second order B_0-dependent terms provided corrections to the background quark mass and effective interactions such as the usual NJL and vector NJL ones, besides new chiral and isospin symmetry breaking terms. They correspond to the different couplings of the magnetic field to pseudoscalar, vector and axial isospin triplets states. The set of B_0-dependent interactions from expressions (32) is given by

$$\mathcal{L}_{4q} = \Delta_B M^* \bar{\psi}\psi + g_{4,B}[(\bar{\psi}\psi)^2 + (\bar{\psi}\sigma_i i\gamma_5\psi)^2] + \left(\frac{3g_{4,B}}{5} i\epsilon_{ij3} + g_{ps,B}c_i\delta_{ij}\right)(\bar{\psi}\sigma_i i\gamma_5\psi)(\bar{\psi}\sigma_j i\gamma_5\psi)$$
$$+ \left[\delta_{ij}\left(g_{4v,B} + g_{4v,B2}c_i + \frac{g_{4v,B-F}}{3} + g_{4v2,B}\right) + i\epsilon_{ij3}\left(\frac{3}{5}g_{4v,B} + g_{4v,B-F} + 3g_{4v2,B}\right)\right]$$
$$\times [(\bar{\psi}\sigma_i\gamma_\mu\psi)(\bar{\psi}\sigma_j\gamma^\mu\psi) + (\bar{\psi}\sigma_i\gamma_\mu\gamma_5\psi)(\bar{\psi}\sigma_j\gamma^\mu\gamma_5\psi)]$$
$$+ g_{s,sb}(\bar{\psi}\psi)^2 + g_{v,sb}(\bar{\psi}\sigma_i\gamma_\mu\psi)^2. \quad (56)$$

The mass correction $\Delta_B M$ is positive as expected from the usual magnetic catalysis analysis from the NJL-type gap equation. However this effective mass contribution for a weak field is of the order of $(eB_0)^2/M^{*3}$ whereas the leading contribution from the gap equation for a weak B field is of the order of $\sqrt{eB_0}$. The gap equation for the auxiliary field was found to depend on the magnetic field as usually investigated for NJL or GCM-type models. Almost all the effective coupling constants are of the order of $(eB_0)^2/M^{*4}$ except two of them, $g_{4v2,B}$ and $g_{4v,B-F}$, are $\mathcal{O}(eB_0/M^{*2})$, corresponding therefore to dipolar couplings in spite of the absence of the tensor current. These two effective couplings are the leading ones being that $g_{4v,B-F}$ is nonzero only if the gluon propagator has a transversal component. There are overall corrections to the NJL and vector-NJL coupling constants, respectively, given by $g_{4,B}$ and $g_{4v,B}$, $g_{4v,B-F}$ and $g_{4v2,B}$. The effective coupling constant $g_{4,B}$ enhances the strength of the quark scalar interaction. This might be seen as an increase of the strength of quark interactions that produce dynamical chiral symmetry breaking. Although this may suggest that DChSB can be obtained for zero NJL coupling constant ($g_4 \to 0$) when $g_{4,B} \neq 0$, this might be misleading in the sense that in the present development both effective couplings have the same physical origin, namely the quark polarization with a quark-gluon coupling g^2. The physical content of magnetic catalysis would be clearer in this sense by considering a different mechanism for one of the two effective interactions (g_4 or $g_{4,B}$). The effective coupling $g_{4v2,B}$ is also positive and therefore it might contribute to the vector condensation in the vacuum [11]. However some new couplings appear signaling the emergence of pseudoscalar and vector or axial multiplets with a different interaction with the magnetic field, i.e. different electric charge (+, − and 0). These couplings therefore break chiral and isospin symmetries. In particular the effective couplings $g_{4,B}$ and $g_{ps,B}$ yield pion interactions with the magnetic field. The vector couplings to the magnetic field are $g_{4v,B}$, $g_{4v,B2}$, $g_{4v,B-F}$ and $g_{4v2,B}$ providing the different couplings in the vector and axial channels therefore related to the ρ and A_1 triplets. The two couplings due to the explicit symmetry breaking, $g_{s,sb}$ and $g_{v,sb}$, have already been investigated in Ref. [37]. The analytical ratios exhibited in Sec. IV are very specific to the limit in which the large quark effective mass is smaller than an effective gluon mass that is expected to be present in a nonperturbative gluon propagator [47]. Other limits could be considered and will be presented elsewhere. The main sources of possible improvements are the simplified momentum dependence of the internal lines and the inclusion of auxiliary fields which however will produce numerically smaller contributions. Higher order interactions of the expansion of the quark determinant considered in this manuscript yield corrections for stronger magnetic fields with increasing powers of eB_0 for the effective coupling constants. Alternatively, the whole summation over the Landau levels for internal quark lines (quark kernel) can be considered for arbitrary values of the magnetic field. Pion and quark B_0-dependent effective interactions will be investigated elsewhere.

ACKNOWLEDGMENTS

The author thanks I. Shovkovy for a short conversation and acknowledges partial financial support from CNPq.

[1] J. O. Andersen, W. R. Naylor, and A. Tranberg, Rev. Mod. Phys. 88, 025001 (2016).
[2] V. A. Miransky and I. A. Shovkovy, Phys. Rep. 576, 1 (2015).
[3] K. Tuchin, Adv. High Energy Phys. 2013, 490495 (2013).
[4] V. A. Miransky and I. A. Shovkovy, Phys. Rev. D 66, 045006 (2002).
[5] V. P. Gusynin, V. A. Miransky, and I. A. Shovkovy, Nucl. Phys. B462, 249 (1996).
[6] G. S. Bali, F. Bruckmann, G. Endrodi, Z. Fodor, S. D. Katz, and A. Schafer, Phys. Rev. D 86, 071502 (2012).

[7] N. Mueller, J. A. Bonnet, and C. S. Fischer, Phys. Rev. D **89**, 094023 (2014).
[8] K. Fukushima and Y. Hidaka, Phys. Rev. Lett. **110**, 031601 (2013).
[9] F. Bruckmann, G. Endrodi, and T. G. Kovacs, J. High Energy Phys. 04 (2013) 112.
[10] B. Chatterjee, H. Mishra, and A. Mishra, Phys. Rev. D **91**, 034031 (2015).
[11] M. N. Chernodub, Phys. Rev. Lett. **106**, 142003 (2011).
[12] K. Fukushima, D. E. Kharzeev, and H. J. Warringa, Phys. Rev. D **78**, 074033 (2008).
[13] D. E. Kharzeev, Prog. Part. Nucl. Phys. **75**, 133 (2014); Annu. Rev. Nucl. Part. Sci. **65**, 193 (2015).
[14] E. V. Gorbar, V. A. Miransky, and I. A. Shovkovy, Phys. Lett. B **695**, 354 (2011); For QED see E. V. Gorbar, V. A. Miransky, I. A. Shovkovy, and X. Wang, Phys. Rev. D **88**, 025043 (2013); Lifang Xia, E. V. Gorbar, V. A. Miransky, and I. A. Shovkovy, Phys. Rev. D **90**, 085011 (2014).
[15] J. Chao, P. Chu, and M. Huang, Phys. Rev. D **88**, 054009 (2013); L. Yu, H. Liu, and M. Huang, Phys. Rev. D **90**, 074009 (2014).
[16] For example in P.-C. Chu, X. Wang, L. W. Chen, and M. Huang, Phys. Rev. D **91**, 023003 (2015).
[17] U. Vogl and W. Weise, Prog. Part. Nucl. Phys. **27**, 195 (1991).
[18] S. P. Klevansky, Rev. Mod. Phys. **64**, 649 (1992).
[19] E. J. Ferrer, V. de la Incera, and A. Sanchez, Phys. Rev. Lett. **107**, 041602 (2011); E. J. Ferrer, V. de la Incera, and X. J. Wen, Phys. Rev. D **91**, 054006 (2015); E. J. Ferrer, V. de la Incera, I. Portillo, and M. Quiroz, Phys. Rev. D **89**, 085034 (2014).
[20] R. L. S. Farias, K. P. Gomes, G. Krein, and M. B. Pinto, Phys. Rev. C **90**, 025203 (2014).
[21] M. Ferreira, P. Costa, O. Lourenco, T. Frederico, and C. Providencia, Phys. Rev. D **89**, 116011 (2014).
[22] A. Ayala, C. A. Dominguez, L. A. Hernández, M. Loewe, and R. Zamora, Phys. Lett. B **759**, 99 (2016).
[23] C.-F. Li, L. Yang, X. J. Wen, and G. X. Peng, Phys. Rev. D **93**, 054005 (2016).
[24] M. A. Andreichikov, V. D. Orlovsky, and Y. A. Simonov, Phys. Rev. Lett. **110**, 162002 (2013).
[25] E. J. Ferrer, V. de la Incera, and X. J. Wen, Phys. Rev. D **91**, 054006 (2015).
[26] A. Rabhi and C. Providencia, Phys. Rev. C **83**, 055801 (2011).
[27] A. Ayala, M. Loewe, and R. Zamora, Phys. Rev. D **91**, 016002 (2015).
[28] H. Abuki, R. Gatto, and M. Ruggieri, Phys. Rev. D **80**, 074019 (2009).
[29] P. C. Chu, X. Wang, L. W. Chen, and M. Huang, Phys. Rev. D **91**, 023003 (2015).
[30] P. Costa, M. Ferreira, D. P. Menezes, J. Moreira, and C. Providencia, Phys. Rev. D **92**, 036012 (2015); D. P. Menezes, M. P. Pinto, L. B. Castro, P. Costa, and C. Providencia, Phys. Rev. C **89**, 055207 (2014).
[31] W. Heisenberg and H. Euler, Z. Phys. **98**, 714 (1936) [translated into English: arXiv:physics/0605038.
[32] J. Schwinger, Phys. Rev. **82**, 664 (1951).
[33] Y. Nambu, Prog. Theor. Phys. **5**, 82 (1950).
[34] S. Ozaki, Phys. Rev. D **89**, 054022 (2014); S. Ozaki, T. Arai, K. Hattori, and K. Itakura, Phys. Rev. D **92**, 016002 (2015); B. V. Galilo and S. N. Nedelko, Phys. Rev. D **84**, 094017 (2011); S. Ozaki, Phys. Rev. D **89**, 054022 (2014).
[35] A. Paulo, Jr. and F. L. Braghin, Phys. Rev. D **90**, 014049 (2014).
[36] F. L. Braghin, Eur. Phys. J. A **52**, 134 (2016).
[37] F. L. Braghin, Phys. Lett. B **761**, 424 (2016).
[38] D. Ebert, H. Reinhardt, and M. K. Volkov, Prog. Part. Nucl. Phys. **33**, 1 (1994).
[39] C. D. Roberts, R. T. Cahill, and J. Praschifka, Ann. Phys. (N.Y.) **188**, 20 (1988).
[40] B. Van den Bossche, arXiv:nucl-th/9807010.
[41] D. Binosi, L. Chang, J. Papavassiliou, and C. D. Roberts, Phys. Lett. B **742**, 183 (2015), and references therein.
[42] K.-I. Kondo, Phys. Rev. D **57**, 7467 (1998).
[43] J. M. Cornwall, Phys. Rev. D **83**, 076001 (2011).
[44] P. Maris and C. D. Roberts, Int. J. Mod. Phys. E **12**, 297 (2003); P. Tandy, Prog. Part. Nucl. Phys. **39**, 117 (1997).
[45] K. Higashijima, Phys. Rev. D **29**, 1228 (1984); V. A. Miransky, Sov. J. Nucl. Phys. **38**, 280 (1983).
[46] K.-I. Aoki, M. Bando, T. Kugo, M. G. Mitchard, and H. Nakatani, Prog. Theor. Phys. **84**, 683 (1990).
[47] A. C. Aguilar, D. Binosi, and J. Papavassiliou, Phys. Rev. D **84**, 085026 (2011); **88**, 074010 (2013).
[48] C. Itzikson and J.-B. Zuber, *Quantum Field Theory* (McGraw-Hill, New York, 1985).
[49] A. A. Andrianov, D. Espriu, and R. Tarrach, Nucl. Phys. **B533**, 429 (1998).
[50] O. Oliveira and P. Bicudo, J. Phys. G **38**, 045003 (2011).
[51] L. F. Abbott, Acta Phys. Pol. B **13**, 33 (1982).
[52] S. Weinberg, *The Quantum Theory of Fields* (Cambridge University Press, Cambridge, England, 1996), Vol. II.
[53] H. Kleinert, in *Erice Summer Institute 1976, Understanding the Fundamental Constituents of Matter*, edited by A. Zichichi (Plenum Press, New York, 1978).
[54] F. L. Braghin, Eur. Phys. J. Plus **130**, 195 (2015).
[55] G. S. Bali, F. Bruckmann, G. Endrődi, Z. Fodor, S. D. Katz, S. Krieg, A. Schäfer, and K. K. Szabó, J. High Energy Phys. 02 (2012) 044; G. Endrodi, J. High Energy Phys. 04 (2013) 023.
[56] U. Mosel, *Path Integrals in Field Theory: An Introduction* (Springer-Verlag, Berlin, 2004).
[57] G. S. Bali, F. Bruckmann, G. Endrődi, F. Gruber, and A. Schäfer, J. High Energy Phys. 04 (2013) 130.
[58] P. Watson and H. Reinhardt, Phys. Rev. D **89**, 045008 (2014).
[59] J. Greensite, *An Introduction to the Confinement Problem* (Springer, Heildelberg, 2011).
[60] T.-K. Chyi, C.-W. Hwang, W. F. Kao, G.-L. Lin, K.-W. Ng, and J.-J. Tseng, Phys. Rev. D **62**, 105014 (2000).