

# Lepton flavor violating quarkonium decays

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We argue that lepton flavor violating (LFV) decays  $M \rightarrow \ell_1 \bar{\ell}_2$  of quarkonium states  $M$  with different quantum numbers could be used to put constraints on the Wilson coefficients of effective operators describing LFV interactions at low energy scales. We note that restricted kinematics of the two-body quarkonium decays allows us to select operators with particular quantum numbers, significantly reducing the reliance on the single operator dominance assumption that is prevalent in constraining parameters of the effective LFV Lagrangian. We shall also argue that studies of radiative lepton flavor violating  $M \rightarrow \gamma \ell_1 \bar{\ell}_2$  decays could provide important complementary access to those effective operators.

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## I. INTRODUCTION

Flavor-changing neutral current (FCNC) interactions serve as a powerful probe of physics beyond the standard model (BSM). Since no operators generate FCNCs in the standard model (SM) at tree level, new physics (NP) degrees of freedom can effectively compete with the SM particles running in the loop graphs, making their discovery possible. This is, of course, only true provided the BSM models include flavor-violating interactions.

The observation of charged lepton flavor violating (CLFV) transitions would provide especially clean probes of new physics. This is because in the standard model with massive neutrinos the CLFV transitions are suppressed by the powers of  $m_\nu^2/m_W^2$ , which renders the predictions for their transition rates vanishingly small, e.g.  $\mathcal{B}(\mu \rightarrow e\gamma)_{\text{SM}} \sim 10^{-54}$ . A variety of well-established models on new physics predict significantly larger rates for CLFV transitions [1].

Any new physics scenario which involves lepton flavor violating interactions can be matched to an effective Lagrangian,  $\mathcal{L}_{\text{eff}}$ , whose Wilson coefficients would be determined by the ultraviolet (UV) physics that becomes active at some scale  $\Lambda$ . Below the electroweak symmetry breaking scale, this Lagrangian must be invariant under unbroken  $SU(3)_c \times U(1)_{\text{em}}$  groups. The effective operators would reflect degrees of freedom relevant at the scale at which a given process takes place. If we assume that no new light particles (such as “dark photons” or axions) exist in the low energy spectrum, those operators would be written entirely in terms of the SM degrees of freedom such as leptons:  $\ell_i = \tau, \mu, \text{ and } e$ ; and quarks:  $b, c, s, u, \text{ and } d$ . We shall not consider neutrinos in this paper. We also assume that top quarks have been integrated out.

The effective Lagrangian,  $\mathcal{L}_{\text{eff}}$ , can then be divided into the dipole part,  $\mathcal{L}_D$ ; a part that involves four-fermion interactions,  $\mathcal{L}_{\ell q}$ ; and a gluonic part,  $\mathcal{L}_G$ .

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_D + \mathcal{L}_{\ell q} + \mathcal{L}_G + \dots \quad (1)$$

Here the ellipses denote effective operators that are not relevant for the following analysis. The dipole part in Eq. (1) is usually written as [2]

$$\mathcal{L}_D = -\frac{m_2}{\Lambda^2} [(C_{DR}^{\ell_1 \ell_2} \bar{\ell}_1 \sigma^{\mu\nu} P_L \ell_2 + C_{DL}^{\ell_1 \ell_2} \bar{\ell}_1 \sigma^{\mu\nu} P_R \ell_2) F_{\mu\nu} + \text{H.c.}], \quad (2)$$

where  $P_{R,L} = (1 \pm \gamma_5)/2$  is the right (left) chiral projection operator. The Wilson coefficients would, in general, be different for different leptons  $\ell_i$ .

The four-fermion dimension-six lepton-quark Lagrangian takes the form

$$\begin{aligned} \mathcal{L}_{\ell q} = & -\frac{1}{\Lambda^2} \sum_q [(C_{VR}^{q\ell_1 \ell_2} \bar{\ell}_1 \gamma^\mu P_R \ell_2 + C_{VL}^{q\ell_1 \ell_2} \bar{\ell}_1 \gamma^\mu P_L \ell_2) \bar{q} \gamma_\mu q \\ & + (C_{AR}^{q\ell_1 \ell_2} \bar{\ell}_1 \gamma^\mu P_R \ell_2 + C_{AL}^{q\ell_1 \ell_2} \bar{\ell}_1 \gamma^\mu P_L \ell_2) \bar{q} \gamma_\mu \gamma_5 q \\ & + m_2 m_q G_F (C_{SR}^{q\ell_1 \ell_2} \bar{\ell}_1 P_L \ell_2 + C_{SL}^{q\ell_1 \ell_2} \bar{\ell}_1 P_R \ell_2) \bar{q} q \\ & + m_2 m_q G_F (C_{PR}^{q\ell_1 \ell_2} \bar{\ell}_1 P_L \ell_2 + C_{PL}^{q\ell_1 \ell_2} \bar{\ell}_1 P_R \ell_2) \bar{q} \gamma_5 q \\ & + m_2 m_q G_F (C_{TR}^{q\ell_1 \ell_2} \bar{\ell}_1 \sigma^{\mu\nu} P_L \ell_2 \\ & + C_{TL}^{q\ell_1 \ell_2} \bar{\ell}_1 \sigma^{\mu\nu} P_R \ell_2) \bar{q} \sigma_{\mu\nu} q + \text{H.c.}]. \end{aligned} \quad (3)$$

We note that the tensor operators are often omitted when constraints on the Wilson coefficients in Eq. (3) are derived (see, e.g., [2]). We would like to point out that those are no

less motivated than others in Eq. (3). For example, they would be induced from Fierz rearrangement of operators of the type  $\mathcal{Q} \sim (\bar{q}\ell_2)(\bar{\ell}_1 q)$  that often appear in leptoquark models. Also, as we shall see later, the experimental constraints on those coefficients follow from studying vector meson decays, where the best information on LFV transitions in quarkonia is available.

The dimension seven gluonic operators can be generated either by some high scale physics or by integrating out heavy quark degrees of freedom [2,3],

$$\begin{aligned} \mathcal{L}_G = & -\frac{m_2 G_F}{\Lambda^2} \frac{\beta_L}{4\alpha_s} [(C_{GR}^{\ell_1 \ell_2} \bar{\ell}_1 P_L \ell_2 + C_{GL}^{\ell_1 \ell_2} \bar{\ell}_1 P_R \ell_2) G_{\mu\nu}^a G^{a\mu\nu} \\ & + (C_{GR}^{\ell_1 \ell_2} \bar{\ell}_1 P_L \ell_2 + C_{GL}^{\ell_1 \ell_2} \bar{\ell}_1 P_R \ell_2) G_{\mu\nu}^a \tilde{G}^{a\mu\nu} + \text{H.c.}]. \end{aligned} \quad (4)$$

Here  $\beta_L = -9\alpha_s^2/(2\pi)$  is defined for the number of light active flavors,  $L$ , relevant to the scale of the process, which we take  $\mu \approx 2$  GeV. All Wilson coefficients should also be calculated at the same scale.  $G_F$  is the Fermi constant and  $\tilde{G}^{a\mu\nu} = (1/2)\epsilon^{\mu\nu\alpha\beta} G_{\alpha\beta}^a$  is a dual to the gluon field strength tensor [2].

The experimental constraints on the Wilson coefficients of effective operators in  $\mathcal{L}_{\text{eff}}$  could be obtained from a variety of LFV decays (see, e.g., [1] for a review). Deriving constraints on those Wilson coefficients usually involve an assumption that only one of the effective operators dominates the result. This is not necessarily so in many particular UV completions of the LFV effective field theories (EFTs), so certain cancellations among contributions of various operators are possible. Nevertheless, single operator dominance is a useful theoretical assumption in placing constraints on the parameters of  $\mathcal{L}_{\text{eff}}$ .

In this paper we are going to argue that most of the Wilson coefficients of the effective Lagrangian in Eq. (1) for different  $\ell_i$  could be determined from experimental data on quarkonium decays. In particular, we consider two- and three-body decays of the quarkonia of differing quantum numbers with quarks of various flavors such as  $\Upsilon(nS) \rightarrow \ell_1 \bar{\ell}_2$ ,  $\Upsilon(nS) \rightarrow \gamma \ell_1 \bar{\ell}_2$ , or similar modes. We will note that restricted kinematics of the two-body transitions would allow us to select operators with particular quantum numbers significantly reducing the reliance on the single operator dominance assumption. Finally, we shall argue that studies of radiative lepton flavor violating (RLFV) decays could provide important complementary access to study  $\mathcal{L}_{\text{eff}}$ .

We shall provide calculations of the relevant decay rates and establish constraints, where experimental data are available, on Wilson coefficients of effective operators of the Lagrangian  $\mathcal{L}_{\text{eff}}$  of Eq. (1). In the following sections we assume  $CP$  conservation, which implies that all Wilson coefficients will be treated as real numbers. We shall note that some transitions have not yet been experimentally

studied, so no numerical constraints from those decays are available at the moment. Finally, in studying branching ratios we assume that for a meson,  $M$ , the branching fraction  $\mathcal{B}(M \rightarrow \ell_1 \ell_2) = \mathcal{B}(M \rightarrow \bar{\ell}_1 \ell_2) + \mathcal{B}(M \rightarrow \ell_1 \bar{\ell}_2)$ , unless specified otherwise.

## II. VECTOR QUARKONIUM DECAYS $V \rightarrow \ell_1 \bar{\ell}_2$

There is abundant experimental information on flavor off-diagonal leptonic decays of vector quarkonia, both from the ground and from the excited states (see Table I) [4]. This information can effectively be converted to experimental bounds on Wilson coefficients of vector and tensor operators in Eq. (3), as well as on those of the dipole operators of Eq. (2). Those Wilson coefficients can then be related to model parameters of explicit realizations of UV completions of effective Lagrangian in Eq. (1). The examples of particular new physics models that have been previously suggested to be constrained using vector meson decays  $V \rightarrow \ell_1 \bar{\ell}_2$  include, e.g., [5,6] (for  $Z'$  scenarios), [7–9] (for  $R$ -parity violating supersymmetric models), and [10–12] for other approaches. The most general expression for the  $V \rightarrow \ell_1 \bar{\ell}_2$  decay amplitude can be written as

$$\begin{aligned} A(V \rightarrow \ell_1 \bar{\ell}_2) &= \bar{u}(p_1, s_1) \left[ A_V^{\ell_1 \ell_2} \gamma_\mu + B_V^{\ell_1 \ell_2} \gamma_\mu \gamma_5 + \frac{C_V^{\ell_1 \ell_2}}{m_V} (p_2 - p_1)_\mu \right. \\ &\quad \left. + \frac{iD_V^{\ell_1 \ell_2}}{m_V} (p_2 - p_1)_\mu \gamma_5 \right] v(p_2, s_2) e^\mu(p). \end{aligned} \quad (5)$$

$A_V^{\ell_1 \ell_2}$ ,  $B_V^{\ell_1 \ell_2}$ ,  $C_V^{\ell_1 \ell_2}$ , and  $D_V^{\ell_1 \ell_2}$  are dimensionless constants which depend on the underlying Wilson coefficients of the effective Lagrangian of Eq. (1) as well as on hadronic effects associated with meson-to-vacuum matrix elements or decay constants.

The amplitude of Eq. (5) leads to the branching fraction, which is convenient to represent in terms of the ratio,

$$\begin{aligned} \frac{\mathcal{B}(V \rightarrow \ell_1 \bar{\ell}_2)}{\mathcal{B}(V \rightarrow e^+ e^-)} &= \left( \frac{m_V (1 - y^2)}{4\pi\alpha f_V Q_q} \right)^2 [ |A_V^{\ell_1 \ell_2}|^2 + |B_V^{\ell_1 \ell_2}|^2 \\ &\quad + \frac{1}{2} (1 - 2y^2) (|C_V^{\ell_1 \ell_2}|^2 + |D_V^{\ell_1 \ell_2}|^2) \\ &\quad + y \text{Re}(A_V^{\ell_1 \ell_2} C_V^{\ell_1 \ell_2*} + i B_V^{\ell_1 \ell_2} D_V^{\ell_1 \ell_2*}) ]. \end{aligned} \quad (6)$$

Here  $\alpha$  is the fine structure constant; we neglected the mass of the lighter of the two leptons and set  $y = m_2/m_V$ . The form of the coefficients  $A_V^{\ell_1 \ell_2}$  to  $D_V^{\ell_1 \ell_2}$  depends on the initial state meson. For example, for  $V = \Upsilon(nS)$  ( $b\bar{b}$  states),  $\psi(nS)$  ( $c\bar{c}$  states), or  $\phi$  ( $s\bar{s}$  state), the coefficients are

$$\begin{aligned}
A_V^{\ell_1\ell_2} &= \frac{f_V m_V}{\Lambda^2} [\sqrt{4\pi\alpha} Q_q y^2 (C_{DL}^{\ell_1\ell_2} + C_{DR}^{\ell_1\ell_2}) + \kappa_V (C_{VL}^{q\ell_1\ell_2} + C_{VR}^{q\ell_1\ell_2}) \\
&\quad + 2y^2 \kappa_V \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1\ell_2} + C_{TR}^{q\ell_1\ell_2})], \\
B_V^{\ell_1\ell_2} &= \frac{f_V m_V}{\Lambda^2} [-\sqrt{4\pi\alpha} Q_q y^2 (C_{DL}^{\ell_1\ell_2} - C_{DR}^{\ell_1\ell_2}) - \kappa_V (C_{VL}^{q\ell_1\ell_2} - C_{VR}^{q\ell_1\ell_2}) \\
&\quad - 2y^2 \kappa_V \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1\ell_2} - C_{TR}^{q\ell_1\ell_2})], \\
C_V^{\ell_1\ell_2} &= \frac{f_V m_V}{\Lambda^2} y [\sqrt{4\pi\alpha} Q_q (C_{DL}^{\ell_1\ell_2} + C_{DR}^{\ell_1\ell_2}) + 2\kappa_V \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1\ell_2} + C_{TR}^{q\ell_1\ell_2})], \\
D_V^{\ell_1\ell_2} &= i \frac{f_V m_V}{\Lambda^2} y [-\sqrt{4\pi\alpha} Q_q (C_{DL}^{\ell_1\ell_2} - C_{DR}^{\ell_1\ell_2}) - 2\kappa_V \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1\ell_2} - C_{TR}^{q\ell_1\ell_2})]. \tag{7}
\end{aligned}$$

Here  $Q_q = (2/3, -1/3)$  is the charge of the quark  $q$  and  $\kappa_V = 1/2$  is a constant for pure  $q\bar{q}$  states. It is a good approximation to drop terms proportional to  $y^2$  in Eq. (7) for the heavy quarkonium states. Inspecting the ratio in Eq. (6), one immediately infers that the best constraints could be placed on the four-fermion coefficients,  $C_{VL}^{q\ell_1\ell_2}$  and  $C_{VR}^{q\ell_1\ell_2}$ , as no final state lepton mass suppression exists for those coefficients. Yet, constraints on the dipole coefficients,  $C_{DL}^{\ell_1\ell_2}$  ( $C_{DR}^{\ell_1\ell_2}$ ), are also possible in this case. This would provide NP constraints that are complementary to the ones obtained from the lepton decay experiments, especially for  $\ell = \tau$ , obtained in the radiative  $\tau \rightarrow \mu(e)\gamma$  decays.

The constraints on the Wilson coefficients of tensor operators,  $C_{TL}^{q\ell_1\ell_2}$  ( $C_{TR}^{q\ell_1\ell_2}$ ), in Eq. (7) also depend on the ratio of meson decay constants,

$$\begin{aligned}
\langle 0 | \bar{q} \gamma^\mu q | V(p) \rangle &= f_V m_V \epsilon^\mu(p), \\
\langle 0 | \bar{q} \sigma^{\mu\nu} q | V(p) \rangle &= i f_V^T (\epsilon^\mu p^\nu - p^\mu \epsilon^\nu), \tag{8}
\end{aligned}$$

where  $\epsilon^\mu(p)$  is the  $V$ -meson polarization vector, and  $p$  is its momentum [13].

TABLE I. Available experimental upper bounds on  $\mathcal{B}(V \rightarrow \ell_1 \ell_2)$  and  $\mathcal{B}(\ell_2 \rightarrow \ell_1 \gamma)$  [4,15]. Center dots signify that no experimental constraints are available, and ‘‘FPS’’ means that the transition is forbidden by available phase space. Charge averages of the final states are always assumed.

$\ell_1 \ell_2$	$\mu\tau$	$e\tau$	$e\mu$
$\mathcal{B}(\Upsilon(1S) \rightarrow \ell_1 \ell_2)$	$6.0 \times 10^{-6}$	...	...
$\mathcal{B}(\Upsilon(2S) \rightarrow \ell_1 \ell_2)$	$3.3 \times 10^{-6}$	$3.2 \times 10^{-6}$	...
$\mathcal{B}(\Upsilon(3S) \rightarrow \ell_1 \ell_2)$	$3.1 \times 10^{-6}$	$4.2 \times 10^{-6}$	...
$\mathcal{B}(J/\psi \rightarrow \ell_1 \ell_2)$	$2.0 \times 10^{-6}$	$8.3 \times 10^{-6}$	$1.6 \times 10^{-7}$
$\mathcal{B}(\phi \rightarrow \ell_1 \ell_2)$	FPS	FPS	$4.1 \times 10^{-6}$
$\mathcal{B}(\ell_2 \rightarrow \ell_1 \gamma)$	$4.4 \times 10^{-8}$	$3.3 \times 10^{-8}$	$5.7 \times 10^{-13}$

While the decay constants,  $f_V$ , are known, both experimentally from leptonic decays and theoretically from lattice or QCD sum rule calculations, for a variety of states  $V$ , the tensor (transverse) decay constant,  $f_V^T$ , has only recently been calculated for the charmonium  $J/\psi$  state with the result  $f_{J/\psi}^T(2 \text{ GeV}) = (410 \pm 10) \text{ MeV}$  [13]. In the absence of the estimate for  $f_V^T$ , we follow the suggestion made in Ref. [14] and assume that  $f_V^T = f_V$ . This seems to be the case for the  $J/\psi$  state [13] to better than 10%. We present numerical values of the decay constants in Table II. Note that the ratio of Eq. (6) is largely independent of the values of the decay constants.

Choosing other initial states would make it possible to constrain other combinations of the Wilson coefficients in Eq. (1). This is important for the NP models where several LFV operators would contribute, especially in the case where no operator gives *a priori* dominant contribution. For example, choosing  $V$  to be the  $\rho$  meson with  $\rho \sim (u\bar{u} - d\bar{d})/\sqrt{2}$  gives

$$\begin{aligned}
A_\rho^{e\mu} &= \frac{f_\rho m_\rho}{\Lambda^2} y^2 \sqrt{2\pi\alpha} (Q_u - Q_d) (C_{DL}^{\ell_1\ell_2} + C_{DR}^{\ell_1\ell_2}), \\
B_\rho^{e\mu} &= -\frac{f_\rho m_\rho}{\Lambda^2} y^2 \sqrt{2\pi\alpha} (Q_u - Q_d) (C_{DL}^{\ell_1\ell_2} - C_{DR}^{\ell_1\ell_2}), \\
C_\rho^{e\mu} &= \frac{f_\rho m_\rho}{\Lambda^2} y \sqrt{2\pi\alpha} (Q_u - Q_d) (C_{DL}^{\ell_1\ell_2} + C_{DR}^{\ell_1\ell_2}), \\
D_\rho^{e\mu} &= -i \frac{f_\rho m_\rho}{\Lambda^2} y \sqrt{2\pi\alpha} (Q_u - Q_d) (C_{DL}^{\ell_1\ell_2} - C_{DR}^{\ell_1\ell_2}). \tag{9}
\end{aligned}$$

Here we imposed isospin symmetry on the NP operators and their coefficients, which resulted in the cancellation of the four-fermion operator contribution. The restricted kinematics of the decay implies that only  $\mu e$  operators can be constrained. The corresponding results for  $V = \omega \sim (u\bar{u} + d\bar{d})/\sqrt{2}$  decay can be obtained from Eq. (7) by substituting  $Q_q \rightarrow (Q_u + Q_d)/\sqrt{2}$  and using  $\kappa_\omega = 1/\sqrt{2}$ . Again, the restricted kinematics of the decay implies that

TABLE II. Vector meson decay constants used in the calculation of branching ratios  $\mathcal{B}(V \rightarrow \ell_1 \bar{\ell}_2)$ . The transverse decay constants are set  $f_V^T = f_V$  except for  $J/\psi$ , which has  $f_{J/\psi}^T = (410 \pm 10)$  MeV [10,13,16–19].

State	$\Upsilon(1S)$	$\Upsilon(2S)$	$\Upsilon(3S)$	$J/\psi$	$\psi(2S)$	$\phi$	$\rho(\omega)$
$f_V$ [MeV]	$649 \pm 31$	$481 \pm 39$	$539 \pm 84$	$418 \pm 9$	$294 \pm 5$	$241 \pm 18$	$209.4 \pm 1.5$

TABLE III. Constraints on the Wilson coefficients of four-fermion operators. Center dots signify that no experimental data are available to produce a constraint; “FPS” means that the transition is forbidden by phase space. Note that no experimental data are available for higher excitations of  $\psi$ .

Wilson coefficient [GeV <sup>-2</sup> ]	Leptons	Initial state (quark)				
	$\ell_1 \ell_2$	$\Upsilon(1S)(b)$	$\Upsilon(2S)(b)$	$\Upsilon(3S)(b)$	$J/\psi(c)$	$\phi(s)$
$ C_{VL}^{q\ell_1\ell_2}/\Lambda^2 $	$\mu\tau$	$5.6 \times 10^{-6}$	$4.1 \times 10^{-6}$	$3.5 \times 10^{-6}$	$5.5 \times 10^{-5}$	FPS
	$e\tau$	...	$4.1 \times 10^{-6}$	$4.1 \times 10^{-6}$	$1.1 \times 10^{-4}$	FPS
	$e\mu$	...	...	...	$1.0 \times 10^{-5}$	$2 \times 10^{-3}$
$ C_{VR}^{q\ell_1\ell_2}/\Lambda^2 $	$\mu\tau$	$5.6 \times 10^{-6}$	$4.1 \times 10^{-6}$	$3.5 \times 10^{-6}$	$5.5 \times 10^{-5}$	FPS
	$e\tau$	...	$4.1 \times 10^{-6}$	$4.1 \times 10^{-6}$	$1.1 \times 10^{-4}$	FPS
	$e\mu$	...	...	...	$1.0 \times 10^{-5}$	$2 \times 10^{-3}$
$ C_{TL}^{q\ell_1\ell_2}/\Lambda^2 $	$\mu\tau$	$4.4 \times 10^{-2}$	$3.2 \times 10^{-2}$	$2.8 \times 10^{-2}$	1.2	FPS
	$e\tau$	...	$3.3 \times 10^{-2}$	$3.2 \times 10^{-2}$	2.4	FPS
	$e\mu$	...	...	...	4.8	$1 \times 10^4$
$ C_{TR}^{q\ell_1\ell_2}/\Lambda^2 $	$\mu\tau$	$4.4 \times 10^{-2}$	$3.2 \times 10^{-2}$	$2.8 \times 10^{-2}$	1.2	FPS
	$e\tau$	...	$3.3 \times 10^{-2}$	$3.2 \times 10^{-2}$	2.4	FPS
	$e\mu$	...	...	...	4.8	$1 \times 10^4$

TABLE IV. Constraints on the dipole Wilson coefficients from the  $1^{--}$  quarkonium decays and radiative lepton transitions  $\ell_2 \rightarrow \ell_1 \gamma$ . Center dots signify that no experimental data are available to produce a constraint; “FPS” means that the transition is forbidden by phase space.

Dipole Wilson coefficient [GeV <sup>-2</sup> ]	Leptons	Initial state					$\ell_2 \rightarrow \ell_1 \gamma$
	$\ell_1 \ell_2$	$\Upsilon(1S)(b)$	$\Upsilon(2S)(b)$	$\Upsilon(3S)(b)$	$J/\psi(c)$	$\phi(s)$	
$ C_{DL}^{\ell_1\ell_2}/\Lambda^2 $	$\mu\tau$	$2.0 \times 10^{-4}$	$1.6 \times 10^{-4}$	$1.4 \times 10^{-4}$	$2.5 \times 10^{-4}$	FPS	$2.6 \times 10^{-10}$
	$e\tau$	...	$1.6 \times 10^{-4}$	$1.6 \times 10^{-4}$	$5.3 \times 10^{-4}$	FPS	$2.7 \times 10^{-10}$
	$e\mu$	...	...	...	$1.1 \times 10^{-3}$	0.2	$3.1 \times 10^{-7}$
$ C_{DR}^{q\ell_1\ell_2}/\Lambda^2 $	$\mu\tau$	$2.0 \times 10^{-4}$	$1.6 \times 10^{-4}$	$1.4 \times 10^{-4}$	$2.5 \times 10^{-4}$	FPS	$2.6 \times 10^{-10}$
	$e\tau$	...	$1.6 \times 10^{-4}$	$1.6 \times 10^{-4}$	$5.3 \times 10^{-4}$	FPS	$2.7 \times 10^{-10}$
	$e\mu$	...	...	...	$1.1 \times 10^{-3}$	0.2	$3.1 \times 10^{-7}$

only  $\mu e$  operators interacting with up and down quarks can be constrained. Since we imposed isospin symmetry, it is convenient to use  $m_q = (m_u + m_d)/2$ .

Contrasting Eq. (6) with the experimental data from Ref. [4] we can constrain the Wilson coefficients of the Lagrangian Eq. (1). Assuming single operator dominance, the results can be found in Table III. The Wilson coefficients of dipole operators can be found in Table IV.

It is important to note that some of the bounds presented in Tables III and IV are rather weak and might not even look physically meaningful, especially the ones coming from  $\phi$  decays. In fact, assuming Wilson coefficients  $C \sim 1$  seems to imply that the new physics scale  $\Lambda/\sqrt{C}$  only extends to

several MeVs, clearly breaking the EFT paradigm that assumes local operators up to scales of several TeVs. A correct interpretation of those entries in Tables III and IV is that existing data simply do not allow one to place strong

TABLE V. Available experimental limits on  $\mathcal{B}(P \rightarrow \ell_1 \ell_2)$  [4]. Note that no constraints for the heavy quark pseudoscalar states such as  $\eta_{b(c)}$  are available. Only phase space allowed transitions are shown.

$\ell_1 \ell_2$	$e\mu$
$\mathcal{B}(\eta \rightarrow \ell_1 \ell_2)$	$6 \times 10^{-6}$
$\mathcal{B}(\eta' \rightarrow \ell_1 \ell_2)$	$4.7 \times 10^{-4}$
$\mathcal{B}(\pi^0 \rightarrow \ell_1 \ell_2)$	$3.6 \times 10^{-10}$

constraints on the combination Wilson coefficients. This is rather common in EFT analyses of new physics phenomena; see, e.g., [20].

As one can see from Eq. (7), there is a practical limitation on the two-body vector meson decays. Only a subset of the Wilson coefficients is selected by the quantum numbers of the initial state and can be probed. This fact can be turned into virtue if experimental information on LFV decays of quarkonium states with other quantum numbers is available.

### III. PSEUDOSCALAR QUARKONIUM DECAYS $P \rightarrow \ell_1 \bar{\ell}_2$

Constraints on other Wilson coefficients of the effective Lagrangian in Eq. (1) could be obtained by considering decays of pseudoscalar mesons with quantum numbers  $0^{-+}$ , which include states like  $\eta_{b(c)}$ ,  $\eta$ ,  $\eta'$ , and their excitations (see Table V). These decays would be sensitive to axial and pseudoscalar operators, providing information about  $C_{PL}^{q\ell_1\ell_2}$  ( $C_{PR}^{q\ell_1\ell_2}$ ) and/or  $C_{AL}^{q\ell_1\ell_2}$  ( $C_{AR}^{q\ell_1\ell_2}$ ) in Eq. (3) as well as to gluonic operators of Eq. (4). The  $\eta_{b(c)}$  states could be abundantly produced at the LHCb experiment directly in gluon-gluon fusion interactions [21]. In case of the  $\eta_c$  and its excitations, another production mechanism would include nonleptonic  $B$  decays, as the corresponding branching ratios for nonleptonic  $B$  decays into  $\eta_c$  and kaons are reasonably large, of order of per mille [4].

Similar to the decays of vector mesons considered in Sec. II, one can write the most general expression for the  $P \rightarrow \ell_1 \bar{\ell}_2$  decay amplitude as

$$A(P \rightarrow \ell_1 \bar{\ell}_2) = \bar{u}(p_1, s_1)[E_P^{\ell_1\ell_2} + iF_P^{\ell_1\ell_2}\gamma_5]v(p_2, s_2) \quad (10)$$

with  $E_P^{\ell_1\ell_2}$  and  $F_P^{\ell_1\ell_2}$  being dimensionless constants which depend on the Wilson coefficients of operators in Eq. (1) and various decay constants.

The amplitude of Eq. (10) leads to the branching ratio for flavor off-diagonal leptonic decays of pseudoscalar mesons,

$$\mathcal{B}(P \rightarrow \ell_1 \bar{\ell}_2) = \frac{m_P}{8\pi\Gamma_P}(1-y^2)^2[|E_P^{\ell_1\ell_2}|^2 + |F_P^{\ell_1\ell_2}|^2]. \quad (11)$$

Here  $\Gamma_P$  is the total width of the pseudoscalar state. We have once again neglected the mass of the lighter lepton and set  $y = m_2/m_P$ . Calculating  $E_P^{\ell_1\ell_2}$  and  $F_P^{\ell_1\ell_2}$  for  $P = \eta_b$  ( $b\bar{b}$  state) and  $\eta_c$  ( $c\bar{c}$  state), the coefficients are

$$\begin{aligned} E_P^{\ell_1\ell_2} &= y \frac{m_P}{4\Lambda^2} [-if_P [2(C_{AL}^{q\ell_1\ell_2} + C_{AR}^{q\ell_1\ell_2}) \\ &\quad - m_P^2 G_F (C_{PL}^{q\ell_1\ell_2} + C_{PR}^{q\ell_1\ell_2})] + 9G_F a_P (C_{GL}^{\ell_1\ell_2} + C_{GR}^{\ell_1\ell_2})], \\ F_P^{\ell_1\ell_2} &= -y \frac{m_P}{4\Lambda^2} [f_P [2(C_{AL}^{q\ell_1\ell_2} - C_{AR}^{q\ell_1\ell_2}) \\ &\quad - m_P^2 G_F (C_{PL}^{q\ell_1\ell_2} - C_{PR}^{q\ell_1\ell_2})] + 9iG_F a_P (C_{GL}^{\ell_1\ell_2} - C_{GR}^{\ell_1\ell_2})]. \end{aligned} \quad (12)$$

The hadronic matrix elements in Eq. (12) are defined as [3]

$$\begin{aligned} \langle 0 | \bar{q} \gamma^\mu \gamma_5 q | P(p) \rangle &= -if_P p^\mu, \\ \langle 0 | \frac{\alpha_s}{4\pi} G^{a\mu\nu} \tilde{G}_{\mu\nu}^a | P(p) \rangle &= a_P. \end{aligned} \quad (13)$$

Here  $p$  is the momentum of the meson. For heavy quarks  $q = c, b$  one expects the matrix elements of gluonic operators in Eq. (13) to be quite small. This can be visualized by noting that in the heavy quark limit  $\eta_{b(c)}$  is a small state of size  $(m_{b(c)}v)^{-1}$ , and it has a small overlap with soft gluons, whose Compton wavelength is of the order of  $\Lambda_{\text{QCD}}^{-1}$ , as  $m_{b(c)}v \gg \Lambda_{\text{QCD}}$ . Here  $v$  is the velocity of heavy quarks. Thus, for the remainder of this paper, we shall set  $a_{\eta_{b(c)}} = 0$ . The constraints on the Wilson coefficients of gluonic operators could be obtained either from studying lepton flavor violating  $\eta'$  decays (for  $\mu e$  currents) or from the corresponding tau decays. We use  $a_\eta = -0.022 \pm 0.002 \text{ GeV}^3$  and  $a_{\eta'} = -0.057 \pm 0.002 \text{ GeV}^3$  [22]. The numerical values of the other pseudoscalar decay constants used in the calculations can be found in Table VI. For the light quark states, such as  $\eta$  and  $\eta'$ , the corresponding expressions are a bit more involved,

$$\begin{aligned} E_P^{\ell_1\ell_2} &= y \frac{m_P}{4\Lambda^2} [-if_P^{u/d} \kappa_1^P [2(C_{AL}^{u/d\ell_1\ell_2} + C_{AR}^{u/d\ell_1\ell_2}) \\ &\quad - G_F m_P^2 (C_{PL}^{u/d\ell_1\ell_2} + C_{PR}^{u/d\ell_1\ell_2})] \\ &\quad - if_P^s \kappa_2^P [2(C_{AL}^{s\ell_1\ell_2} + C_{AR}^{s\ell_1\ell_2}) - G_F m_P^2 (C_{PL}^{s\ell_1\ell_2} + C_{PR}^{s\ell_1\ell_2})] + 9G_F a_P (C_{GL}^{\ell_1\ell_2} + C_{GR}^{\ell_1\ell_2})], \\ F_P^{\ell_1\ell_2} &= y \frac{m_P}{4\Lambda^2} [-f_P^{u/d} \kappa_1^P [2(C_{AL}^{u/d\ell_1\ell_2} - C_{AR}^{u/d\ell_1\ell_2}) \\ &\quad - G_F m_P^2 (C_{PL}^{u/d\ell_1\ell_2} - C_{PR}^{u/d\ell_1\ell_2})] \\ &\quad - f_P^s \kappa_2^P [2(C_{AL}^{s\ell_1\ell_2} - C_{AR}^{s\ell_1\ell_2}) - G_F m_P^2 (C_{PL}^{s\ell_1\ell_2} - C_{PR}^{s\ell_1\ell_2})] - 9iG_F a_P (C_{GL}^{\ell_1\ell_2} - C_{GR}^{\ell_1\ell_2})], \end{aligned} \quad (14)$$

TABLE VI. Pseudoscalar meson decay constants used in the calculation of branching ratios  $\mathcal{B}(P \rightarrow \ell_1 \bar{\ell}_2)$  [4,13,22,23].

State	$\eta_b$	$\eta_c$	$\eta, u(d)$	$\eta, s$	$\eta', u(d)$	$\eta', s$	$\pi$
$f_p^q$ [MeV]	$667 \pm 6$	$387 \pm 7$	$108 \pm 3$	$-111 \pm 6$	$89 \pm 3$	$136 \pm 6$	$130.41 \pm 0.20$

TABLE VII. Constraints on the Wilson coefficients from pseudoscalar meson decays. Center dots signify that no experimental data are available to produce a constraint; “FPS” means that the transition is forbidden by phase space.

Wilson coefficient	Leptons		Initial state				
	$\ell_1 \ell_2$	$\eta_b$	$\eta_c$	$\eta(u/d)$	$\eta(s)$	$\eta'(u/d)$	$\eta'(s)$
$ C_{AL}^{q\ell_1\ell_2}/\Lambda^2 $	$\mu\tau$	...	...	FPS	FPS	FPS	FPS
	$e\tau$	...	...	FPS	FPS	FPS	FPS
	$e\mu$	...	...	$3 \times 10^{-3}$	$2 \times 10^{-3}$	$2.1 \times 10^{-1}$	$1.9 \times 10^{-1}$
$ C_{AR}^{q\ell_1\ell_2}/\Lambda^2 $	$\mu\tau$	...	...	FPS	FPS	FPS	FPS
	$e\tau$	...	...	FPS	FPS	FPS	FPS
	$e\mu$	...	...	$3 \times 10^{-3}$	$2 \times 10^{-3}$	$2.1 \times 10^{-1}$	$1.9 \times 10^{-1}$
$ C_{PL}^{q\ell_1\ell_2}/\Lambda^2 $	$\mu\tau$	...	...	FPS	FPS	FPS	FPS
	$e\tau$	...	...	FPS	FPS	FPS	FPS
	$e\mu$	...	...	$2 \times 10^3$	$1 \times 10^3$	$3.9 \times 10^4$	$3.6 \times 10^4$
$ C_{PR}^{q\ell_1\ell_2}/\Lambda^2 $	$\mu\tau$	...	...	FPS	FPS	FPS	FPS
	$e\tau$	...	...	FPS	FPS	FPS	FPS
	$e\mu$	...	...	$2 \times 10^3$	$1 \times 10^3$	$3.9 \times 10^4$	$3.6 \times 10^4$

TABLE VIII. Constraints on the pseudoscalar gluonic Wilson coefficients. Center dots signify that no experimental data are available to produce a constraint. No data for other lepton species are available.

Gluonic Wilson coefficient [GeV $^{-2}$ ]	Leptons		Initial state			
	$\ell_1 \ell_2$	$\eta_b$	$\eta_c$	$\eta$	$\eta'$	
$ C_{GL}^{\ell_1\ell_2}/\Lambda^2 $	$e\mu$	...	...	$2 \times 10^2$	$5.0 \times 10^3$	
$ C_{GR}^{\ell_1\ell_2}/\Lambda^2 $	$e\mu$	...	...	$2 \times 10^2$	$5.0 \times 10^3$	

where  $\kappa_1^{\eta'} = 1/\sqrt{3}$ ,  $\kappa_2^{\eta'} = -\sqrt{2/3}$ ,  $\kappa_1^{\eta} = \sqrt{2/3}$ , and  $\kappa_2^{\eta} = 1/\sqrt{3}$ . It is important to note that, if observed, a simultaneous fit to several light quark meson decays could independently constrain Wilson coefficients of effective operators in Eq. (1), as follows from Eq. (14).

The resulting constraints on the Wilson coefficients could be found in Tables VII and VIII. Note that no

TABLE IX. Decay constants of Eq. (13) for the scalar quarkonium decays, derived from the quark model calculation of [24]. We follow [24] and do not assign uncertainty to the quark model estimates of the decay constants. Masses and measured widths are from [4], and unmeasured widths (in brackets) are calculated as in [24,26].

State	$\chi_{c0}(1P)$	$\chi_{b0}(1P)$	$\chi_{b0}(2P)$
$m_S$ [MeV]	$3414.75 \pm 0.31$	$9859.44 \pm 0.52$	$10232.5 \pm 0.6$
$\Gamma_S$ [MeV]	$10.5 \pm 0.6$	(1.35)	(0.247 $\pm$ 0.097)
$f_S$ [MeV]	$\approx 887$	$\approx 423$	$\approx 421$

experimental constraints on the  $b$  and  $c$  currents are available, as the corresponding transitions  $\eta_{b(c)} \rightarrow \ell_1 \bar{\ell}_2$  have not yet been experimentally studied. Also, constraints on the Wilson coefficients of gluonic operators in Table VIII are significantly weaker than those available from tau decays [3]. Finally, just as in Sec. II, large entries in Tables VII and VIII do not imply a breakdown of the EFT description of LFV decays, but signify that existing data do not allow us to place strong constraints on the combination of relevant Wilson coefficients.

#### IV. SCALAR QUARKONIUM DECAYS $S \rightarrow \ell_1 \bar{\ell}_2$

Scalar quarkonium decays would ideally allow one to probe the Wilson coefficients of the scalar quark density operators in Eq. (3). The corresponding  $p$ -wave states  $\chi_{q0}$ , where  $q = b, c$  could effectively be produced directly either in gluon-gluon fusion at the LHC or in the radiative decays of  $\Upsilon(2S)$ ,  $\Upsilon(3S)$ , or corresponding  $\psi$  states. It is

important to note that the corresponding branching ratios for, say,  $\psi(2S) \rightarrow \gamma\chi_{c0}$  are rather large, of the order of 10%. Finally, they could also be produced in  $B$  decays at flavor factories.

Since Wilson coefficients of other operators could be better probed in the processes discussed in Secs. II and III, in this section we shall concentrate on the contributions of operators that could not be probed in the decays of vector or pseudoscalar quarkonium states.

The most general expression for the  $S \rightarrow \ell_1\bar{\ell}_2$  decay amplitude looks exactly like Eq. (10), with obvious modifications for the scalar decay,

$$A(S \rightarrow \ell_1\bar{\ell}_2) = \bar{u}(p_1, s_1)[E_S^{\ell_1\ell_2} + iF_S^{\ell_1\ell_2}\gamma_5]v(p_2, s_2). \quad (15)$$

$E_S^{\ell_1\ell_2}$  and  $F_S^{\ell_1\ell_2}$  are dimensionless constants. The branching ratio, which follows from Eq. (15), is

$$\mathcal{B}(S \rightarrow \ell_1\bar{\ell}_2) = \frac{m_S}{8\pi\Gamma_S}(1-y^2)^2[|E_S^{\ell_1\ell_2}|^2 + |F_S^{\ell_1\ell_2}|^2]. \quad (16)$$

Here  $\Gamma_S$  is the total width of the scalar state and  $y = m_2/m_S$ . The coefficients  $E_S^{\ell_1\ell_2}$  and  $F_S^{\ell_1\ell_2}$  are

$$\begin{aligned} E_S^{\ell_1\ell_2} &= y \frac{m_S G_F}{4\Lambda^2} [2if_S m_S m_q (C_{SL}^{q_1 l_2} + C_{SR}^{q_1 l_2}) \\ &\quad + 9a_S (C_{GL}^{q_1 l_2} + C_{GR}^{q_1 l_2})], \\ F_S^{\ell_1\ell_2} &= y \frac{m_S G_F}{4\Lambda^2} [2f_S m_S m_q (C_{SL}^{q_1 l_2} - C_{SR}^{q_1 l_2}) \\ &\quad - 9ia_S (C_{GL}^{q_1 l_2} - C_{GR}^{q_1 l_2})]. \end{aligned} \quad (17)$$

The hadronic matrix elements in Eq. (17) are defined as

$$\begin{aligned} \langle 0|\bar{q}q|S(p)\rangle &= -im_S f_S, \\ \langle 0|\frac{\alpha_s}{4\pi}G^{a\mu\nu}G_{\mu\nu}^a|S(p)\rangle &= a_S. \end{aligned} \quad (18)$$

Note that we introduced an extra minus sign and a factor of  $m_S$  compared to [24] for the scalar quark density to have uniform units for all matrix elements of quark currents. For the same reasons as in the pseudoscalar case, one expects that the gluonic matrix elements in Eq. (17) for the heavy quark states  $\chi_{c0}$  or  $\chi_{b0}$  are small, so we set  $a_S = 0$  from now on. This means that the Wilson coefficients of the gluonic operators are better probed in LFV tau decays, where the low energy theorems [3] or experimental data [2] could be used to constrain relevant gluonic matrix elements.

Finally, we note that no constraints on the Wilson coefficients of the scalar currents in  $\mathcal{L}_{\text{eff}}$  are available, as the corresponding transitions  $\chi_{b(c)0} \rightarrow \ell_1\bar{\ell}_2$  have not yet been experimentally studied.

## V. THREE BODY VECTOR QUARKONIUM DECAYS $V \rightarrow \gamma\ell_1\bar{\ell}_2$

Addition of a photon to the final state certainly reduces the number of the events available for studies of LFV decays, especially since no compensating mechanisms seem to be present (cf. [25]). However, it also makes it possible for other operators in  $\mathcal{L}_{\text{eff}}$  that were not considered in two-body decays of vector quarkonium to contribute. This makes the analysis of RLFV decays a worthwhile exercise, especially for the decays of the vector quarkonium states.

### A. Resonant transitions

The resonant two-body radiative transitions of vector states  $V \rightarrow \gamma(M \rightarrow \ell_1\bar{\ell}_2)$  could be used to study two-body decays considered above, provided the corresponding branching ratios for the radiative decays are large enough. Since vector states are abundantly produced in  $e^+e^-$  annihilation, these decays could provide a powerful tool to study LFV transitions at flavor factories.

If the soft photon can be tagged in an effective manner at  $B$  factories and  $(p_{\ell_1} + p_{\ell_2})^2 \approx m_M^2$ , the combined branching ratio factorizes<sup>1</sup> and can be written as

$$\mathcal{B}(V \rightarrow \gamma\ell_1\bar{\ell}_2) = \mathcal{B}(V \rightarrow \gamma M)\mathcal{B}(M \rightarrow \ell_1\bar{\ell}_2), \quad (19)$$

where the scalar decays ( $M = \chi_{q0}$ )  $\mathcal{B}(\chi_{q0} \rightarrow \ell_1\bar{\ell}_2)$  have been studied in Sec. IV, while the corresponding pseudoscalar transitions ( $M = \eta_q$ )  $\mathcal{B}(\eta_q \rightarrow \ell_1\bar{\ell}_2)$  have been studied in Sec. III.

The resonant RLFV decays are quite useful for studies of scalar heavy meson decays, as the corresponding branching ratios are large, of order of a few percent [4]. In charm,

$$\begin{aligned} \mathcal{B}(\psi(2S) \rightarrow \gamma\chi_{c0}(1P)) &= 9.99 \pm 0.27\%, \\ \mathcal{B}(\psi(3770) \rightarrow \gamma\chi_{c0}(1P)) &= 0.73 \pm 0.09\%. \end{aligned}$$

The corresponding radiative transitions in the beauty sector are also rather large,

$$\begin{aligned} \mathcal{B}(\Upsilon(2S) \rightarrow \gamma\chi_{b0}(1P)) &= 3.8 \pm 0.4\%, \\ \mathcal{B}(\Upsilon(3S) \rightarrow \gamma\chi_{b0}(1P)) &= 0.27 \pm 0.04\%, \\ \mathcal{B}(\Upsilon(3S) \rightarrow \gamma\chi_{b0}(2P)) &= 5.9 \pm 0.6\%. \end{aligned} \quad (20)$$

<sup>1</sup>This equation implicitly assumes that the state  $M$  is narrow, which is an excellent approximation for the scalar heavy quarkonium states considered in this paper (see Table IX). A complete Dalitz plot analysis would be required for wider states.

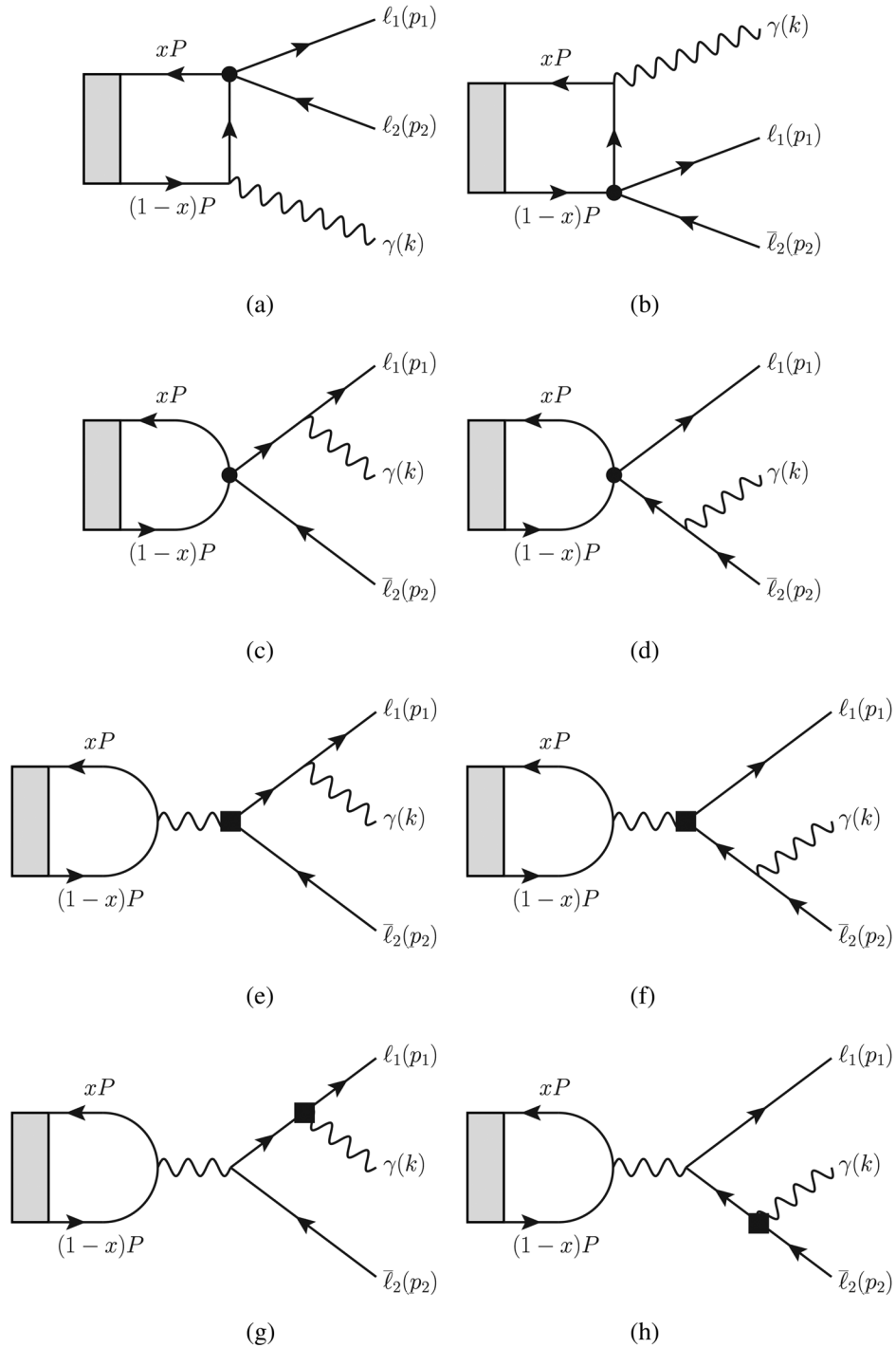


FIG. 1. Feynman diagrams for  $\mathcal{A}(V \rightarrow \gamma \ell_1 \bar{\ell}_2)$ . The black circles represent the four fermion LFV vertex, the black boxes represent the dipole LFV vertex, and the grey boxes represent the quarkonium bound state.

A rough estimate [24] shows that with the integrated luminosity of  $\mathcal{L} = 250 \text{ fb}^{-1}$  the number of produced  $\chi_b$  states could reach tens of millions. Thus, studies of LFV transitions of  $\chi_b$  states could result in a solid bound on the Wilson coefficients of the scalar operators in  $\mathcal{L}_{\text{eff}}$ .

Similar radiative transitions to the pseudoscalar states are generally smaller. However, since the pseudoscalar  $0^{-+}$  states are lighter than the  $1^{-}$  ones, the radiative

transition rates could still reach a percent level in charm,

$$\mathcal{B}(J/\psi \rightarrow \gamma \eta_c) = 1.7 \pm 0.4\%,$$

$$\mathcal{B}(\psi(2S) \rightarrow \gamma \eta_c) = 0.34 \pm 0.05\%.$$

The corresponding branching ratios in the  $b$  sector are in a sub permille level and cannot effectively be used to study LFV decays of the  $\eta_b$  states.



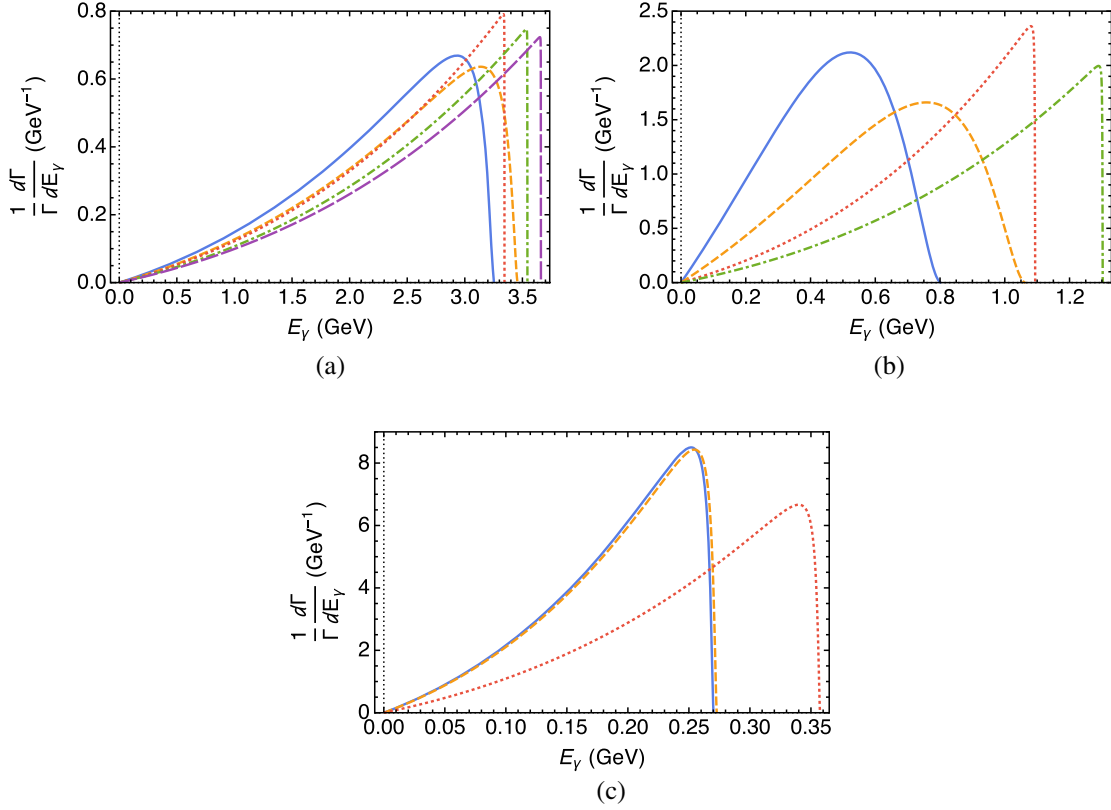


FIG. 2. Differential decay rates as functions of photon energy  $E_\gamma$  for axial operators. Plotted decay rates are for (a)  $\Upsilon(1S) \rightarrow \gamma\mu\tau$  or  $\gamma\eta\tau$  (solid blue curve),  $\Upsilon(2S) \rightarrow \gamma\mu\tau$  or  $\gamma\eta\tau$  (short-dashed gold curve),  $\Upsilon(3S) \rightarrow \gamma\mu\tau$  or  $\gamma\eta\tau$  (dotted red curve),  $\Upsilon(1S) \rightarrow \gamma e\mu$  (dot-dashed green curve),  $\Upsilon(2S) \rightarrow \gamma e\mu$  and  $\Upsilon(3S) \rightarrow \gamma e\mu$  (long-dashed purple curve); (b)  $J/\psi \rightarrow \gamma\mu\tau$  or  $\gamma\eta\tau$  (solid blue curve),  $\psi(2S) \rightarrow \gamma\mu\tau$  or  $\gamma\eta\tau$  (short-dashed gold curve),  $J/\psi \rightarrow \gamma e\mu$  (dotted red curve),  $\psi(2S) \rightarrow \gamma e\mu$  (dot-dashed green curve); (c)  $\rho \rightarrow \gamma e\mu$  (solid blue curve),  $\omega \rightarrow \gamma e\mu$  (short-dashed gold curve),  $\phi \rightarrow \gamma e\mu$  (dotted red curve).

## B. Nonresonant transitions

Nonresonant three-body radiative decays of vector states  $V \rightarrow \gamma\ell_1\bar{\ell}_2$  could be used to constrain the scalar operators, which are not accessible in the two-body decays of vector or pseudoscalar states. Since the final state now includes the photon, it is no longer possible to express all of the hadronic effects in terms of the decay constants. The constraints would then depend on a set of  $V \rightarrow \gamma$  form factors that are not well known. We shall discuss those in a future publication [27].

Here we would provide information about  $C_{SL}^{q\ell_1\ell_2}(C_{SR}^{q\ell_1\ell_2})$ , but at the expense of introducing model dependence. We shall calculate the transition  $V \rightarrow \gamma\ell_1\bar{\ell}_2$  choosing a particular model to describe the effective quark-antiquark distribution function [25].

In principle, besides the Wilson coefficients of the scalar operators, nonresonant RLFV decays could be used to obtain information about vector, axial, pseudoscalar, and tensor operators and thus  $C_{VL}^{q\ell_1\ell_2}(C_{VR}^{q\ell_1\ell_2})$ ,  $C_{AL}^{q\ell_1\ell_2}(C_{AR}^{q\ell_1\ell_2})$ ,  $C_{PL}^{q\ell_1\ell_2}(C_{PR}^{q\ell_1\ell_2})$ , and  $C_{TL}^{q\ell_1\ell_2}(C_{TR}^{q\ell_1\ell_2})$ . However, because these operators can be constrained using much simpler two-body decays of vector and pseudoscalar states

(see Secs. II and III) without significant model dependence, and with better statistics, we shall focus here mainly on the scalar operators, leaving the other constraints to future work [27]. In principle, a calculation of the amplitude  $\mathcal{A}(V \rightarrow \gamma\ell_1\bar{\ell}_2)$  involves evaluation of the eight diagrams shown in Fig. 1. Since the initial state is a  $1^{--}$  vector meson, the contributions of the axial, scalar, and pseudoscalar are contained in diagrams 1(a) and 1(b). The diagrams 1(c) and 1(d) contain the vector and tensor operator contributions, and 1(e) and 1(f) are generated by the dipole operator contributions. By the same arguments as above, we shall also ignore those in this paper.

A calculation of  $\mathcal{A}(V \rightarrow \gamma\ell_1\bar{\ell}_2)$  presented in this paper involves a model to describe the quark-antiquark wave function of the quarkonium state [25]. We choose to follow [25,28–30] and write it as

$$\Psi_V = \frac{I_c}{\sqrt{6}} \Phi_V(x)(m_V\gamma^\alpha + ip^\beta\sigma^{\alpha\beta})\epsilon^\alpha(p). \quad (21)$$

Here  $I_c$  is the identity matrix in color space,  $x$  is the quarkonium momentum fraction carried by one of the constituent quarks, and  $p$  is the momentum of the vector

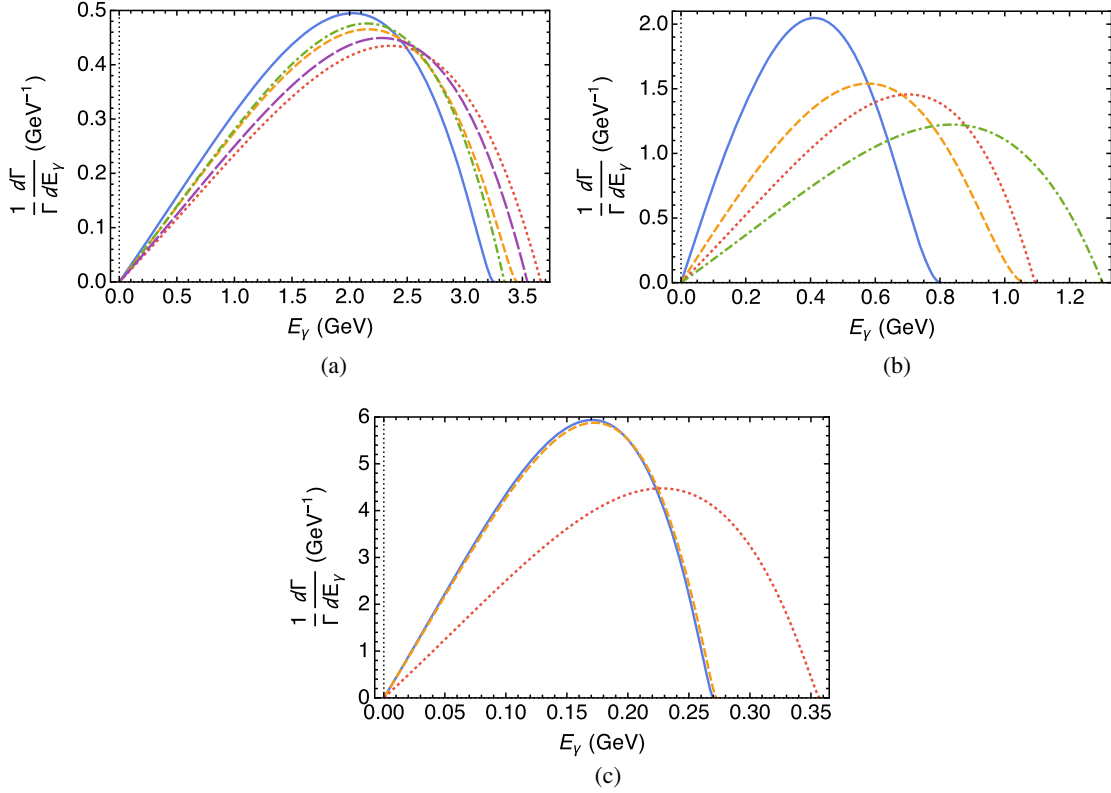


FIG. 3. Differential decay rates as functions of photon energy  $E_\gamma$  for scalar/pseudoscalar operators. Plotted decay rates are for (a)  $\Upsilon(1S) \rightarrow \gamma\mu\tau$  or  $\gamma\eta\tau$  (solid blue curve),  $\Upsilon(2S) \rightarrow \gamma\mu\tau$  or  $\gamma\eta\tau$  (short-dashed gold curve),  $\Upsilon(3S) \rightarrow \gamma\mu\tau$ ,  $\gamma\eta\tau$ , or  $\gamma\eta\mu$  (dotted red curve),  $\Upsilon(1S) \rightarrow \gamma\eta\mu$  (dot-dashed green curve),  $\Upsilon(2S) \rightarrow \gamma\eta\mu$  (long-dashed purple curve); (b)  $J/\psi \rightarrow \gamma\mu\tau$  or  $\gamma\eta\tau$  (solid blue curve),  $\psi(2S) \rightarrow \gamma\mu\tau$  or  $\gamma\eta\tau$  (short-dashed gold curve),  $J/\psi \rightarrow \gamma\eta\mu$  (dotted red curve),  $\psi(2S) \rightarrow \gamma\eta\mu$  (dot-dashed green curve); (c)  $\rho \rightarrow \gamma\eta\mu$  (solid blue curve),  $\omega \rightarrow \gamma\eta\mu$  (short-dashed gold curve),  $\phi \rightarrow \gamma\eta\mu$  (dotted red curve).

meson. The distribution amplitude,  $\Phi_V(x)$ , in Eq. (21) is defined as

$$\Phi_V(x) = \frac{f_V}{2\sqrt{6}} \delta(x - 1/2), \quad (22)$$

where  $f_V$  is a decay constant defined in Eq. (8). We chose the simplest wave function which makes the approximation that each constituent quark carries half the meson's momentum, which is a good approximation for the heavy quark states such as  $\Upsilon(nS)$  or  $J/\psi$ . The nonlocal matrix

element that is relevant for the radiative transition is then expressed in terms of an integral over momentum fraction,

$$\langle 0 | \bar{q} \Gamma^\mu q | V \rangle = \int_0^1 \text{Tr}[\Gamma^\mu \Psi_V] dx. \quad (23)$$

We can now calculate the total and differential decay rates. Assuming single operator dominance, the axial, scalar, and pseudoscalar operators lead to the following differential decay rates:

$$\begin{aligned} \frac{d\Gamma_{V \rightarrow \gamma \ell_1 \bar{\ell}_2}^A}{dm_{12}^2} &= \frac{1}{9} \frac{\alpha Q_q^2 f_V^2}{(4\pi)^2 \Lambda^4} (C_{AL}^2 + C_{AR}^2) \frac{(m_V^2 - m_{12}^2)(2m_V^2 y^2 + m_{12}^2)(m_V^2 y^2 - m_{12}^2)^2}{m_V m_{12}^6}, \\ \frac{d\Gamma_{V \rightarrow \gamma \ell_1 \bar{\ell}_2}^S}{dm_{12}^2} &= \frac{1}{24} \frac{\alpha Q_q^2 f_V^2 G_F^2 m_V}{(4\pi)^2 \Lambda^4} (C_{SL}^2 + C_{SR}^2) \frac{y^2 (m_V^2 - m_{12}^2)(m_V^2 y^2 - m_{12}^2)^2}{m_{12}^2}, \\ \frac{d\Gamma_{V \rightarrow \gamma \ell_1 \bar{\ell}_2}^P}{dm_{12}^2} &= \frac{1}{24} \frac{\alpha Q_q^2 f_V^2 G_F^2 m_V}{(4\pi)^2 \Lambda^4} (C_{PL}^2 + C_{PR}^2) \frac{y^2 (m_V^2 - m_{12}^2)(m_V^2 y^2 - m_{12}^2)^2}{m_{12}^2}. \end{aligned} \quad (24)$$

Here  $y$  and  $Q_q$  are defined to be the same as in Sec. II, and we follow the usual definition of the Mandelstam variable  $m_{12}^2 = (p_1 + p_2)^2$  [4], where momentum  $p_1$  and  $p_2$  correspond to  $\ell_1$  and  $\ell_2$ . Note that in writing Eqs. (24) and (25) we

suppressed some of the indices of the Wilson coefficients (i.e.,  $C_{SL}^{q\ell_1\ell_2} \rightarrow C_{SL}$ ) for brevity. The total decay rates for the RLFV transitions can be found by integrating Eq. (24) over  $m_{12}^2$ , which gives

$$\begin{aligned}\Gamma_A(V \rightarrow \gamma\ell_1\bar{\ell}_2) &= \frac{1}{18} \frac{\alpha Q_q^2}{(4\pi)^2} \frac{f_V^2 m_V^3}{\Lambda^4} (C_{AL}^2 + C_{AR}^2) f(y^2), \\ \Gamma_S(V \rightarrow \gamma\ell_1\bar{\ell}_2) &= \frac{1}{144} \frac{\alpha Q_q^2}{(4\pi)^2} \frac{f_V^2 G_F^2 m_V^7}{\Lambda^4} (C_{SL}^2 + C_{SR}^2) y^2 f(y^2), \\ \Gamma_P(V \rightarrow \gamma\ell_1\bar{\ell}_2) &= \frac{1}{144} \frac{\alpha Q_q^2}{(4\pi)^2} \frac{f_V^2 G_F^2 m_V^7}{\Lambda^4} (C_{PL}^2 + C_{PR}^2) y^2 f(y^2),\end{aligned}\quad (25)$$

where  $f(y^2) = 1 - 6y^2 - 12y^4 \log(y) + 3y^4 + 2y^6$ . We can use Eq. (25) to normalize differential decay distributions, so that they are independent of the unknown Wilson coefficients and plot the normalized decay distributions under the assumption of a single operator dominance. We show differential photon spectra in  $V \rightarrow \gamma\ell_1\bar{\ell}_2$  decay in Fig. 2 for the axial operators, and in Fig. 3 for the scalar or pseudoscalar ones.

Since no experimental constraints are available for the RLFV decays of vector quarkonia, we cannot yet place any constraints on the Wilson coefficients from those transitions.

## VI. CONCLUSIONS

Lepton flavor violating transitions provide a powerful engine for new physics searches. Any new physics model that incorporates flavor and involves flavor-violating interactions at high energy scales can be cast in terms of the effective Lagrangian of Eq. (1) at low energies. We argued that Wilson coefficients of this Lagrangian could effectively be probed by studying decays of quarkonium states with different spin-parity quantum numbers, providing

complementary constraints to those obtained from tau and mu decays [1,31].

The proposed framework allows us to select two-body quarkonium decays in such a way that only operators with particular quantum numbers are probed, significantly reducing the reliance on the single operator dominance assumption that is prevalent in constraining the parameters of the effective LFV Lagrangian. We also argued that studies of RLFV decays could provide important complementary access to those effective operators.

With new data coming from the LHC experiments and the Belle II experiment, we strongly encourage our colleagues to provide experimental constraints on both the LFV and the RLFV transitions discussed in this paper.

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