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$X(4140)$, $X(4274)$, $X(4500)$, and $X(4700)$ in the relativized quark model

Qi-Fang Lü*[1] and Yu-Bing Dong[1,2,†]

[1]Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China
[2]Theoretical Physics Center for Science Facilities (TPCSF), CAS, Beijing 100049, China

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We investigate the masses of $cs\bar{c}\,\bar{s}$ tetraquark states in a diquark-antidiquark picture employing the relativized quark model proposed by Godfrey and Isgur. Only the antitriplet diquark states in color space are calculated. The diquark masses are obtained with the relativized potential first, and then the diquark and antidiquark are treated as the usual antiquark and quark, respectively, and the masses of the tetraquark states are obtained by solving the Schrödinger equation with the relativized potential between the diquark and antidiquark. The theoretical uncertainties induced by screening effects are also taken into account. It is found that the resonance of $X(4140)$ can be regarded as the $cs\bar{c}\,\bar{s}$ tetraquark ground states, and the $X(4700)$ can be assigned as the $2S$ excited tetraquark state. When the internal excited diquarks are taken into account, the resonance of $X(4500)$ can be explained as the tetraquark composed of one $2S$ scalar diquark and one scalar antidiquark. In our approach, the $X(4274)$ cannot be explained as a tetraquark state; however, it can be a good candidate of the conventional $\chi_{c1}(3^3P_1)$ state. In addition, other charmoniumlike states $\chi_{c0}(3915)$, $X(4350)$, $X(4630)$, and $X(4660)$, as the $cs\bar{c}\,\bar{s}$ tetraquark states, are also discussed.

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I. INTRODUCTION

Very recently, the LHCb Collaboration performed the first full amplitude analysis of the $B^+ \to J/\psi\phi K^+$ process with pp collision data collected at $\sqrt{s} = 7$ and 8 TeV [1,2]. Besides the confirmation of the two resonances $X(4140)$ and $X(4274)$ in the $J/\psi\phi$ invariant mass, two new structures $X(4500)$ and $X(4700)$ are also observed at the same time. The spin parities are 1^{++} for the $X(4140)$ and $X(4274)$, and 0^{++} for the $X(4500)$ and $X(4700)$. The measured masses and total decay widths are

$$(M;\Gamma)_{X(4140)} = (4146.5 \pm 4.5^{+4.6}_{-2.8}; 83 \pm 21^{+21}_{-14})\ \text{MeV}, \quad (1)$$

$$(M;\Gamma)_{X(4274)} = (4273.3 \pm 8.3^{+17.2}_{-3.6}; 56 \pm 11^{+8}_{-11})\ \text{MeV}, \quad (2)$$

$$(M;\Gamma)_{X(4500)} = (4506 \pm 11^{+12}_{-15}; 92 \pm 21^{+21}_{-20})\ \text{MeV}, \quad (3)$$

$$(M;\Gamma)_{X(4700)} = (4704 \pm 10^{+14}_{-24}; 120 \pm 31^{+42}_{-33})\ \text{MeV}. \quad (4)$$

It should be mentioned that the $X(4140)$ was first reported by the CDF Collaboration in the $J/\psi\phi$ invariant mass distribution of the $B^+ \to J/\psi\phi K^+$ decay in 2009 [3], and then this structure was observed by several collaborations in the next few years [4–10]. In 2011, the CDF Collaboration found the evidence of the $X(4274)$ with approximate significance of 3.1σ [5]. The related peaks of $J/\psi\phi$ mass structures around 4.3 GeV were also reported by LHCb, CMS, D0, and *BABAR* Collaborations [6–9], which may be the same state as the $X(4274)$. It should be noted that the Belle Collaboration measured a narrow $J/\psi\phi$ peak of $X(4350)$ in the double photon collisions [11], which indicates the $X(4350)$ should have the spin parity $J^{PC} = 0^{++}$ or 2^{++} and be a different structure from the $X(4274)$.

Various theoretical studies on the resonances of $X(4140)$ and $X(4274)$, such as the molecular states [12–29], compact or diquark-antidiquark states [30–40], cusp effects [41,42], dynamically generated resonances [43,44], conventional charmonium [45], and hybrid charmonium states [13,14], have been performed in the literature. Given the $J^{PC} = 1^{++}$ for both states, many molecular and hybrid charmonium interpretations with other quantum numbers can be ruled out [12–14,16–19]. It should be noticed that the cusp effects may explain the structure of the $X(4140)$, but fail to account for the $X(4274)$ [42]. Moreover, the compact tetraquark model, implemented by Stancu, can describe the $X(4140)$ and $X(4274)$ simultaneously [30], while only one $J^{PC} = 1^{++}$ state exists in the color triplet diquark-antidiquark picture in this energy region [32,36,40]. A comprehensive overview about the $X(4140)$ and $X(4274)$ can be found in Ref. [46].

After the new observations of the LHCb Collaboration, several theoretical works have been proposed. By using the QCD sum rule, the $X(4140)$ and $X(4274)$ were interpreted as the S-wave $c\bar{c}s\bar{s}$ tetraquark states with opposite color structures, and the $X(4500)$ and $X(4700)$ as the D-wave $c\bar{c}s\bar{s}$ tetraquark states also with opposite color structures [47]. In Refs. [48,49], the $X(4500)$ is assigned as the first radial excited axial-vector diquark and axial-vector-antidiquark–type tetraquark, the $X(4700)$ was assigned

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[*] lvqifang@ihep.ac.cn
[†] dongyb@ihep.ac.cn

as the tetraquark ground state with a vector-diquark and a vector-antidiquark, and the $X(4140)$ is disfavored as a $c\bar{c}s\bar{s}$ tetraquark state. The possible rescattering effects in the $B^+ \to J/\psi \phi K^+$ are also investigated, which shows that those effects may simulate the structures of the $X(4140)$ and $X(4700)$, but hardly explain the $X(4272)$ and $X(4500)$ [50]. Based on the spin-spin interaction, Maiani *et al.* suggest that the $X(4500)$ and $X(4700)$ are the $2S$ $cs\bar{c}\,\bar{s}$ tetraquark state, the $X(4140)$ is the ground state, and the $X(4274)$ may have quantum number 0^{++} or 2^{++} [51]. Moreover, a detailed calculation is performed by Zhu [52] where the $X(4140)$ and $X(4274)$ may be described simultaneously by adding the up and down quark components. Other studies related to these resonances are also discussed [53–55]. It should be stressed that all those interpretations do not agree with each other, and most of them are obtained with the QCD sum rule and the quark model with only spin-spin interactions. Hence, it is essential to study the four resonance structures, especially the $X(4500)$ and $X(4700)$, in a realistic potential model for a comparison.

In this work, we employ the relativized quark model proposed by Godfrey and Isgur (GI model) to calculate the masses of cs diquark and $cs\bar{c}\,\bar{s}$ tetraquark states. It should be mentioned that this model has been widely used to calculate the masses of the conventional mesons and baryons [56–61]. The obtained wave functions are also employed to estimate their strong decay widths, radiations, decay constants, and leptonic decays [62–74]. It is believed that this model can give a unified description of the low lying mesons and baryons, and therefore, it is suitable to deal with the $cs\bar{c}\,\bar{s}$ tetraquark states, where both heavy-light and heavy-heavy systems are included. To calculate the tetraquark masses, we restrict present calculations in the diquark-antidiquark picture with a color triplet following the route proposed by Ebert, Faustov, and Galkin [75–80]. First of all, the corresponding diquark and antidiquark masses are estimated with this relativized potential. Then, they are treated as the usual pointlike antiquark and quark, respectively. The masses of tetraquark states are, therefore, obtained by solving the Schrödinger-type equation between diquark and antidiquark. This method has been used to discuss the $X(5568)$ observed by D0 Collaboration in our previous work [81–83] and should be appropriate to investigate the abundant charmoniumlike states. Moreover, the color screening effects of the confinement are also considered here. We find that the $X(4140)$ can be assigned as the ground tetraquark state, and the $X(4500)$ and $X(4700)$ are the good candidates of the excited tetraquarks. There is no room left for the $X(4274)$ in the color triplet diquark-antidiquark picture; however, the mass and total decay width of the $X(4274)$ are consistent with the conventional $\chi_{c1}(3^3P_1)$ in the relativized quark model.

This paper is organized as follows. The relativized quark model and the screening potential of confinement are briefly introduced, and the masses of cs diquarks are calculated in Sec. II. In Sec. III, the masses of $cs\bar{c}\,\bar{s}$ tetraquark states and P-wave charmonium are numerically estimated, and discussions are also presented. Finally, we give a short summary of our study in the last section.

II. RELATIVIZED QUARK MODEL AND MASSES OF *cs* DIQUARKS

The Hamiltonian between the quark and antiquark in the relativized quark model can be expressed as

$$\tilde{H} = H_0 + \tilde{V}(\boldsymbol{p},\boldsymbol{r}), \tag{5}$$

with

$$H_0 = (p^2 + m_1^2)^{1/2} + (p^2 + m_2^2)^{1/2}, \tag{6}$$

$$\tilde{V}(\boldsymbol{p},\boldsymbol{r}) = \tilde{H}_{12}^{\rm conf} + \tilde{H}_{12}^{\rm cont} + \tilde{H}_{12}^{\rm ten} + \tilde{H}_{12}^{\rm so}, \tag{7}$$

where the $\tilde{H}_{12}^{\rm conf}$ includes the spin-independent linear confinement and Coulomb-like interaction, the $\tilde{H}_{12}^{\rm cont}$, $\tilde{H}_{12}^{\rm ten}$, and $\tilde{H}_{12}^{\rm so}$ are the color contact term, the color tensor interaction, and the spin-orbit term, respectively. The $\tilde{H}$ represents that the operator H has taken account of the relativistic effects via the relativized procedure. The explicit forms of those interactions and the details of the relativization procedure can be found in Appendix A of Ref. [56].

Since the GI model is a typical quenched quark model, the coupled channel effects or the screening effects have been ignored. Those effects may influence the excited mass spectrum of mesons, for example, the leptonic decay rates of charmonium [84,85] and the lower mass puzzle of $D_{s0}^*(2317)$ and $D_{s1}(2460)$ [67]. The unquenched properties are reflected by replacing the linear confinement to the screening potential, that is, the $br \to b(1 - e^{-\mu r})/\mu$, where the b denotes the string tension and the μ is the screen parameter [67,68]. The modified formalism with a new screening parameter gives a better description of the charmed and charmed-strange meson spectra. Besides the ground $cs\bar{c}\,\bar{s}$ tetraquark states, the excited ones are also needed to describe the newly observed resonances. Here, we stress that when one deals with the cs diquarks and $cs\bar{c}\,\bar{s}$ tetraquarks, the screening effects should also be considered for completeness.

In present work, only the antitriplet diquark $[\bar{3}_c]_{cs}$ are considered. It should be noticed that the $[6_c]_{cs}$ type diquarks cannot be formed in the GI quark model, since the confinement becomes repulsive in this case. For the quark-quark interaction in the antitriplet diquark system, the relation of $\tilde{V}_{cs}(\boldsymbol{p},\boldsymbol{r}) = \tilde{V}_{c\bar{s}}(\boldsymbol{p},\boldsymbol{r})/2$ is employed. The model parameters employed in our calculations are the same as the ones in the original work [56]. The screening parameter μ varies from 0 to 0.04 GeV, where the $\mu \to 0$ case is equivalent to the linear confinement br, and the

TABLE I. Obtained masses of the cs diquarks. S and A denote scalar and axial-vector diquarks in the ground states, respectively. The notation $n^{2S+1}P_J$ is used to stand for the excited diquarks. The brace and bracket correspond to symmetric and antisymmetric quark contents in flavor, respectively. The units are in MeV.

Quark content	Diquark type	Mass (GI model)	Mass ($\mu = 0.02$ GeV)	Mass ($\mu = 0.04$ GeV)
$[c, s]$	S	2230	2221	2212
$\{c, s\}$	A	2264	2254	2244
$[c, s]$	1^1P_1	2523	2503	2482
$\{c, s\}$	1^3P_0	2518	2496	2475
$[c, s]$	1^3P_1	2529	2508	2486
$[c, s]$	2^1S_0	2624	2593	2563
$\{c, s\}$	2^3S_1	2644	2612	2580
$\{c, s\}$	1^3D_1	2743	2708	2673

$\mu = 0.02$ GeV case can improve the description of the charmed-strange spectrum significantly [67,68].

Conventionally, the diquarks are classified into two groups: the ground states locating in the $1S$ wave and the ones with internal excitations. For the diquarks lying in the ground states, the spin parities are $J^P = 0^+$ and $J^P = 1^+$, named the scalar diquark and the axial-vector one, respectively. For the diquarks with internal excitations, we only consider the $1P$, $2S$, and $1D$ waves in this work and restrict the total angular momentum $J = 0$ or 1. With the spectrum notation, the excited diquarks can be, respectively, denoted as 1^1P_1, 1^3P_0, 1^3P_1, 2^1S_0, 2^3S_1, and 1^3D_1. Here, we use the Gaussian expansion method to solve the Hamiltonian (5) with the $\tilde{V}_{cs}(\boldsymbol{p}, \boldsymbol{r})$ potential [86]. The obtained masses of the cs diquarks are presented in Table I. It can be seen that the diquark masses decrease with the parameter μ becoming larger. While the μ varies from 0 to 0.02 GeV, the diquark masses change about 10 MeV for ground states, and 20–35 MeV for the excited states. Since the $\mu = 0.02$ GeV case can give a better description of the $c\bar{s}$ meson spectra [67], we prefer to adopt the diquark masses at this value to calculate the tetraquark states and present the $\mu \to 0$ and 0.04 GeV cases as the theoretical uncertainties.

III. MASSES OF $cs\bar{c}\,\bar{s}$ TETRAQUARK STATES AND P-WAVE CHARMONIUM

A. Tetraquarks composed of S and A diquarks

In this work, a diquark is treated as a pointlike antitriple state or we assume the distance between diquark and antidiquark is large enough [79,87–93]. Therefore, we restrict our calculations of low lying tetraquark states within the $N \equiv 2(n + n_d + n_{\bar{d}}) + L + L_d + L_{\bar{d}} \leq 2$ shell, where the $n(L)$, $n_d(L_d)$, and $n_{\bar{d}}(L_{\bar{d}})$ are the radial (orbital) quantum numbers of the relative motion, the diquark, and the antidiquark, respectively.

First of all, we calculate the tetraquark ground states composed of the S and A type diquarks, where the $l_d = l_{\bar{d}} = 0$. Their mass spectra are listed in Table II. The theoretical errors induced by the screen effects are about 20–30 MeV. It should be noted that the relativized quark model also has other uncertainties [56]. Combining these uncertainties, we do not expect the total theoretical errors to be smaller than 50 MeV in the present work. It can be seen that the lowest state is the $J^P = 0^+$ $A\bar{A}$ type diquark-antidiquark configuration rather than the $S\bar{S}$ type. This order of mass spectrum is different from the results of Refs. [76,77], in which the $J^P = 0^+$ $S\bar{S}$ type is the lowest one. We know that there exist fine splitting for the $A\bar{A}$ states via the spin-spin interaction. The coefficients are −2, −1, and 2 for the 0^+, 1^+, and 2^+ $A\bar{A}$ states, respectively, if the spin-spin interaction is treated perturbatively. Although the mass of the A type diquark is higher than that of the S type, the larger fine splitting caused by the spin-spin interaction can suppress the $J^P = 0^+$ $A\bar{A}$ state to be the lowest one. The lowest mass of the 0^{++} state is 3962 MeV, which is consistent with the $\chi_{c0}(3915)$. Hence, it is possible to assign the $\chi_{c0}(3915)$ as the lightest $cs\bar{c}\,\bar{s}$ state, which has been proposed in Ref. [40].

The other obvious feature of our results is that the mass gap between the $1S$ 0^{++} doublet is larger, while the gap between the $2S$ states is extremely small and the theoretical errors overlap with each other. In the charmed mesons and charmonium sector, the splitting between the $2S$ states is much smaller than the $1S$ doublet [94] as well as the predicted charmed-strange meson spectrum [56,67]. Since the spin-spin splitting is smaller for the $2S$ states, the lowest $2S$ $A\bar{A}$ with coefficient −2 becomes higher and close to the $2S$ $S\bar{S}$ state. Their predicted masses are 4703 and 4733 MeV, which are in a good agreement with the $X(4700)$. The obtained two $2S$ 0^{++} states are too close to distinguish by the masses, and more information about $X(4700)$ is needed. In Ref. [51], the $X(4500)$ and $X(4700)$ are attributed to the $2S$ doublet. However, the 200 MeV mass gap prohibits the $X(4500)$ and $X(4700)$ as the same doublet, and only the $X(4700)$ is favored as the $2S$ states in the present work.

From Table II, it can be seen that the $X(4140)$ is a good candidate of the lowest 1^{++} state. The predicted mass of the lowest 1^{++} state is 4195 MeV, which is the same value as that obtained in compact tetraquark picture by Stancu [30]. In the compact tetraquark scenario, there also exists a higher 1^{++} state, which may be assigned as the $X(4274)$ [30]. By considering the $[6_c]_{cs}$ type diquarks, the diquark and antidiquark picture can also give the good description of the $X(4274)$ [47]. However, as emphasized in Sec. II, the $[6_c]_{cs}$ type diquarks cannot be formed due to the repulsive confinement in the GI quark model. Only one $1S$ 1^{++} state exists in the $[\bar{3}_c]_{cs} \otimes [3_c]_{\bar{c}\,\bar{s}}$ diquark-antidiquark picture, and no room is left for the

TABLE II. Masses of $cs\bar{c}\,\bar{s}$ tetraquark states composed of the S and A diquarks and antidiquarks in $1S$, $1P$, $2S$, and $1D$ waves. In the $A\bar{S}$ case, the linear combinations together with $S\bar{A}$ are understood to form the eigenstates of charge conjugation [40]. The units are in MeV.

J^{PC}	Diquark	Antidiquark	$n+1$	S	L	Mass (GI model)	Mass ($\mu = 0.02$ GeV)	Mass ($\mu = 0.04$ GeV)	Exotic candidate
$\vert 0^{++}\rangle$	S	$\bar{S}$	1	0	0	4185	4164	4143	
$\vert 0^{++}\rangle$	A	$\bar{A}$	1	0	0	3984	3962	3940	$\chi_{c0}(3915)$
$\vert 1^{++}\rangle$	A	$\bar{S}$	1	1	0	4217	4195	4173	$X(4140)$
$\vert 1^{+-}\rangle$	A	$\bar{S}$	1	1	0	4217	4195	4173	
$\vert 1^{+-}\rangle$	A	$\bar{A}$	1	1	0	4139	4117	4095	
$\vert 2^{++}\rangle$	A	$\bar{A}$	1	2	0	4325	4302	4278	
$\vert 0^{--}\rangle$	A	$\bar{S}$	1	1	1	4599	4572	4545	
$\vert 0^{-+}\rangle$	A	$\bar{S}$	1	1	1	4599	4572	4545	
$\vert 0^{-+}\rangle$	A	$\bar{A}$	1	1	1	4595	4567	4540	
$\vert 1^{--}\rangle$	A	$\bar{S}$	1	1	1	4633	4605	4578	$X(4630)$
$\vert 1^{--}\rangle$	S	$\bar{S}$	1	0	1	4632	4604	4577	$X(4630)$
$\vert 1^{-+}\rangle$	A	$\bar{S}$	1	1	1	4633	4605	4578	
$\vert 1^{-+}\rangle$	A	$\bar{A}$	1	1	1	4680	4651	4622	
$\vert 1^{--}\rangle$	A	$\bar{A}$	1	0	1	4679	4651	4622	$X(4660)$
$\vert 1^{--}\rangle$	A	$\bar{A}$	1	2	1	4599	4571	4543	
$\vert 2^{-+}\rangle$	A	$\bar{S}$	1	1	1	4691	4662	4633	
$\vert 2^{-+}\rangle$	A	$\bar{A}$	1	1	1	4706	4677	4648	
$\vert 2^{--}\rangle$	A	$\bar{S}$	1	1	1	4691	4662	4633	
$\vert 2^{--}\rangle$	A	$\bar{A}$	1	2	1	4702	4673	4643	
$\vert 3^{--}\rangle$	A	$\bar{A}$	1	2	1	4735	4705	4675	
$\vert 0^{++}\rangle$	S	$\bar{S}$	2	0	0	4767	4733	4700	$X(4700)$
$\vert 0^{++}\rangle$	A	$\bar{A}$	2	0	0	4736	4703	4671	$X(4700)$
$\vert 1^{++}\rangle$	A	$\bar{S}$	2	1	0	4798	4764	4730	
$\vert 1^{+-}\rangle$	A	$\bar{S}$	2	1	0	4798	4764	4730	
$\vert 1^{+-}\rangle$	A	$\bar{A}$	2	1	0	4784	4750	4716	
$\vert 2^{++}\rangle$	A	$\bar{A}$	2	2	0	4870	4833	4797	
$\vert 0^{++}\rangle$	A	$\bar{A}$	1	2	2	4949	4912	4876	
$\vert 1^{++}\rangle$	A	$\bar{S}$	1	1	2	4931	4896	4860	
$\vert 1^{++}\rangle$	A	$\bar{A}$	1	2	2	4962	4925	4881	
$\vert 1^{+-}\rangle$	A	$\bar{S}$	1	1	2	4931	4896	4860	
$\vert 1^{+-}\rangle$	A	$\bar{A}$	1	1	2	4969	4932	4896	
$\vert 2^{++}\rangle$	S	$\bar{S}$	1	2	2	4930	4895	4859	
$\vert 2^{++}\rangle$	A	$\bar{S}$	1	1	2	4952	4916	4879	
$\vert 2^{++}\rangle$	A	$\bar{A}$	1	0	2	4989	4952	4915	
$\vert 2^{++}\rangle$	A	$\bar{A}$	1	2	2	4982	4945	4907	
$\vert 2^{+-}\rangle$	A	$\bar{S}$	1	1	2	4952	4916	4879	
$\vert 2^{+-}\rangle$	A	$\bar{A}$	1	1	2	4992	4954	4917	
$\vert 3^{++}\rangle$	A	$\bar{S}$	1	1	2	4980	4943	4906	
$\vert 3^{++}\rangle$	A	$\bar{A}$	1	2	2	5001	4963	4926	
$\vert 3^{+-}\rangle$	A	$\bar{S}$	1	1	2	4980	4943	4906	
$\vert 3^{+-}\rangle$	A	$\bar{A}$	1	1	2	4999	4961	4924	
$\vert 4^{++}\rangle$	A	$\bar{A}$	1	2	2	5009	4972	4934	

$X(4274)$ [40,51]. Maiani *et al.* also discuss the possibility that the $X(4274)$ may have 0^{++} or 2^{++} quantum numbers [51]. If so, the $X(4274)$ seems to be the candidate of the $1S$ 2^{++} state with 4302 MeV in the present work. In addition, the obtained masses of the $1P$ tetraquark states are above 4.5 GeV. The four 1^{--} states are all around 4.6 GeV, which may correspond to the $X(4630)$ and $X(4660)$. The $1D$ tetraquarks are above 4.9 MeV, which are much higher than the energy region with which we are concerned.

To sum up, by considering the tetraquarks composed of S and A diquarks, we assign the $X(4140)$ as the $1S$ 1^{++} $cs\bar{c}\,\bar{s}$ tetraquark state and the $X(4700)$ as the $2S$ 0^{++} $cs\bar{c}\,\bar{s}$ tetraquark state within the $S\bar{S}$ or $A\bar{A}$ type. There is no room left for the $X(4274)$ and $X(4500)$ in our present interpretation.

B. Tetraquarks composed of internal excited diquarks

Besides the S and A type diquarks, the internal excited diquarks have also been considered to account for the newly observed structures [47,48]. We adopt the diquark masses listed in Table I to calculate the 0^{++} and 1^{++} tetraquark states. The results are presented in Table III.

For the 0^{++} tetraquark states, there are several combinations around 4.5 GeV, such as one $2S$ diquark and one $\bar{S}$ type antidiquark, and one P-wave excited diquark and one P-wave antidiquark. In Ref. [47], the $X(4500)$ is explained as one D-wave diquark and one S-wave antidiquark. In our present calculation, the central value of one D-wave diquark and one S-wave antidiquark case is 4410 MeV, which seems to be smaller than the experimental data. We, therefore, prefer to interpret the $X(4500)$ as the tetraquark state composed of one 2^1S_0 diquark and one $\bar{S}$ type antidiquark with the predicted mass 4516 MeV, although the one P-wave diquark and the one P-wave antidiquark-type tetraquark state cannot be simply excluded. It is also possible to explain the $X(4700)$ as the $^3P_0$ diquark-antidiquark state, while the ground tetraquark state with vector-diquark vector-antidiquark is favored in Ref. [48].

The lowest state is the tetraquark composed of one 2^3S_1 diquark and one A type antidiquark with mass 4315 MeV. As emphasized in Sec. I, a narrow $J/\psi\phi$ peak $X(4350)$ in the double photon collisions has been observed [11], which indicates the $X(4350)$ should have the spin parity $J^{PC} = 0^{++}$ or 2^{++}. We believe that the $X(4350)$ is a good candidate of the lowest 0^{++} tetraquark with internal excited diquarks. In this scheme, the $\chi_{c0}(3915)$ is the ground $A\bar{A}$ tetraquark state, the $X(4700)$ is the $2S$ $A\bar{A}$ or $S\bar{S}$ state, the

TABLE III. Masses of $cs\bar{c}\,\bar{s}$ tetraquark states composed of internal excited diquarks. When the diquark and antidiquark are in different types, the linear combinations are understood to form the eigenstates of charge conjugation [40]. The units are in MeV.

Diquark	Antidiquark	$n+1$	S	L	Mass (GI model)	Mass ($\mu = 0.02$ GeV)	Mass ($\mu = 0.04$ GeV)	Exotic candidate
$\vert \mathbf{0}^{++}\rangle$								
2^1S_0	$\bar{S}$	1	0	0	4558	4516	4475	$X(4500)$
2^3S_1	$\bar{A}$	1	0	0	4358	4315	4271	$X(4350)$
1^3D_1	$\bar{A}$	1	0	0	4456	4410	4363	
1^1P_1	$\overline{1^1P_1}$	1	0	0	4490	4448	4405	
1^1P_1	$\overline{1^3P_1}$	1	0	0	4496	4453	4409	
1^3P_0	$\overline{1^3P_0}$	1	0	0	4727	4682	4639	
1^3P_1	$\overline{1^3P_1}$	1	0	0	4501	4458	4413	
1^1P_1	$\bar{S}$	1	1	1	4875	4837	4799	
1^1P_1	$\bar{A}$	1	1	1	4842	4805	4767	
1^3P_0	$\bar{A}$	1	1	1	4870	4831	4793	
1^3P_1	$\bar{S}$	1	1	1	4880	4842	4803	
1^3P_1	$\bar{A}$	1	1	1	4848	4810	4771	
$\vert \mathbf{1}^{++}\rangle$								
2^1S_0	$\bar{A}$	1	1	0	4589	4547	4505	
2^3S_1	$\bar{S}$	1	1	0	4577	4534	4492	
2^3S_1	$\bar{A}$	1	1	0	4505	4461	4418	
1^3D_1	$\bar{S}$	1	1	0	4671	4626	4580	
1^3D_1	$\bar{A}$	1	1	0	4600	4554	4508	
1^1P_1	$\overline{1^3P_0}$	1	1	0	4732	4689	4646	
1^1P_1	$\overline{1^3P_1}$	1	1	0	4640	4598	4554	
1^3P_0	$\overline{1^3P_1}$	1	1	0	4738	4693	4649	
1^1P_1	$\bar{S}$	1	1	1	4892	4855	4816	
1^1P_1	$\bar{A}$	1	0	1	4923	4885	4847	
1^1P_1	$\bar{A}$	1	1	1	4924	4886	4847	
1^1P_1	$\bar{A}$	1	2	1	4845	4808	4770	
1^3P_0	$\bar{S}$	1	0	1	4902	4863	4824	
1^3P_0	$\bar{A}$	1	1	1	4904	4864	4825	
1^3P_1	$\bar{S}$	1	1	1	4898	4859	4820	
1^3P_1	$\bar{A}$	1	0	1	4929	4890	4850	
1^3P_1	$\bar{A}$	1	1	1	4929	4890	4850	
1^3P_1	$\bar{A}$	1	2	1	4850	4813	4774	

$X(4350)$ is the $2^3S_1\bar{A}$ tetraquark state, and the $X(4500)$ is the $2^1S_0\bar{S}$ tetraquark state. The absent 0^{++} one without orbital excitation is the ground $S\bar{S}$ tetraquark state. The $X(4140)$ is also the ground state 1^{++} $A\bar{S}$ tetraquark state without orbital excitation. The predicted 1^{++} tetraquark states with internal excited diquarks are much higher than the $X(4274)$, which disfavors the $X(4274)$ as the 1^{++} tetraquark state.

Besides the observed $cs\bar{c}\,\bar{s}$ tetraquark candidates, there exist many other predicted ones in this scheme. To search for these states, their production mechanisms and decay behaviors should also be investigated simultaneously. The tetraquark states may be produced via the complex mechanism and be rich in some specific processes [92]. The total decay widths are also the essential properties, since the much broader structures can hardly be detected. Further studies on these tetraquark states are needed both theoretically and experimentally.

C. *P*-wave charmonium

According to the Particle Data Group [94], the four $1P$ charmonium states and $\chi_{c2}(2P)$ have been well established. The assignment of the $\chi_{c0}(3915)$ is in debate. The $\chi_{c1}(2^3P_1)$ state is related with the nature of the mysterious $X(3872)$, which can hardly be solved in the near future. The mass spectrum and strong decay behavior of the $3P$ states are also investigated in the Refs. [63,64]. Here, we present the mass spectrum of P-wave charmonium up to 5 GeV in Fig. 1.

The predicted mass of the $\chi_{c1}(3^3P_1)$ state is 4261 MeV, in good agreement with the $X(4274)$. With the quark pair creation model, the total decay width of the $\chi_{c1}(3^3P_1)$ is 39 MeV [63], which is consistent with the experimental width $56 \pm 11^{+8}_{-11}$ MeV. In the relativized quark model, it can be seen that for the 1^{++} states around 4.2 GeV, one tetraquark state corresponds to the $X(4140)$, and the charmonium stands for the $X(4274)$. More arguments can be found in Ref. [50], where the $X(4274)$ is suggested as the $\chi_{c1}(3^3P_1)$ state.

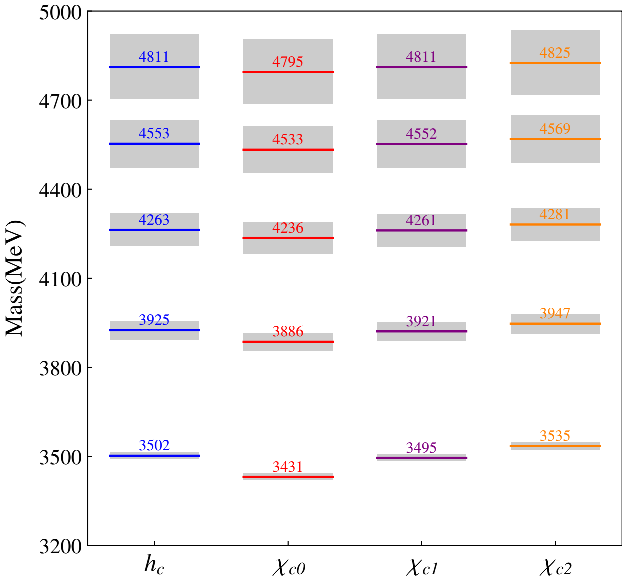

FIG. 1. Mass of the P-wave charmonium up to 5 GeV.

We notice that the $X(4500)$ and $X(4700)$ also lie in the energy regions of the $\chi_{c0}(4^3P_0)$ and $\chi_{c0}(5^3P_0)$. The screening effects are small for the low lying charmonium, while the uncertainties become much larger for the $4P$ and $5P$ states. These uncertainties of mass spectra prohibit us from giving a reliable conclusion on the $\chi_{c0}(4^3P_0)$ and $\chi_{c0}(5^3P_0)$ states. Other information, such as the strong and radiative decay behaviors of these higher charmonium, are needed. Moreover, under the factorization ansatz, the 1^{++} charmonium can be produced more easily than the 0^{++} charmonium in the $B^+ \to K^+ + X$ process [50], which indicates the 1^{++} $X(4274)$ is suitable for the conventional charmonium, but the 0^{++} $X(4500)$ and $X(4700)$ are not.

IV. SUMMARY

In this work, we investigate the masses of $cs\bar{c}\,\bar{s}$ tetraquark states in the diquark-antidiquark picture using the relativized quark model proposed by Godfrey and Isgur. The diquark and antidiquark masses are obtained with the relativized potential first, and then the diquark and antidiquark are treated as the usual pointlike antiquark and quark, respectively. Here, only the antitriplet diquark $[\bar{3}_c]_{cs}$ is considered. The masses of tetraquark states are obtained by solving the Schrödinger-type equation between diquark and antidiquark. The color screening effects are also added in the present work.

It is found that the resonance $X(4140)$ can be regarded as the $A\bar{S}$ type tetraquark state, and the $X(4700)$ can be assigned as the $2S$ $A\bar{A}$ or $S\bar{S}$ state. When the internal excited diquarks are considered, the $X(4500)$ can be explained as the tetraquark composed of one $2S$ scalar diquark and one scalar antidiquark. The $X(4274)$ as the tetraquark state is not favored; however, it could be a good candidate of the conventional $\chi_{c1}(3^3P_1)$ state. Other charmoniumlike states $\chi_{c0}(3915)$, $X(4350)$, $X(4630)$, and $X(4660)$ are also discussed in this work. We expect our assignments can be tested by future experiments.

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