

Pion and η -meson mass splitting at the two-flavor chiral crossoverMarkus Heller^{1,2} and Mario Mitter²¹*Dip. di Fisica “Galileo Galilei”, Via Marzolo 8, I-35131 Padova, Italy*²*Institut für Theoretische Physik, Universität Heidelberg,
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We study the splitting in the screening mass of pions and the η -meson across the chiral crossover. This splitting is determined by the 't Hooft determinant. We use results for the renormalization group scale dependence of the 't Hooft determinant obtained within the functional renormalization group in quenched QCD with two flavors. The scale dependence of the 't Hooft determinant is mapped to its temperature dependence with the help of a Polyakov-quark-meson model. As a result we obtain the temperature dependence of the splitting in the screening mass of pions and the η -meson.

DOI: [10.1103/PhysRevD.94.074002](https://doi.org/10.1103/PhysRevD.94.074002)**I. INTRODUCTION**

The axial $U(1)_A$ -symmetry of quantum chromodynamics is broken by a quantum anomaly. As a consequence the pseudoscalar singlet meson does not appear as a massless mode in the spectrum of QCD in the phase of spontaneously broken chiral symmetry [1,2]. This phenomenon can also be understood in terms of the 't Hooft determinant [3,4]. It is a $U(1)_A$ -symmetry breaking $2N_f$ -quark interaction that gets contributions from fluctuations in the topological charge. Recently, nonperturbative results for the four-quark interaction channels, and in particular for the 't Hooft determinant, have become available from investigations of quenched continuum QCD with two flavors [5], see also [6].

At temperatures of roughly 150–160 MeV, QCD with $2 + 1$ flavors experiences a rapid crossover to a phase that approximately respects chiral symmetry and breaks center symmetry [7–9]. Although the situation is less clear at the finite chemical potential, qualitative changes are expected also at large densities, see e.g. [10–15] for reviews. At temperatures far above the crossover and also at large densities, the effects of the axial anomaly are expected to vanish, since topological charge fluctuations become suppressed [16,17]. Furthermore, it has been argued that the splitting between the mass of the pseudoscalar singlet meson and the pion could become small immediately above the chiral crossover [18]. Experimental signs for such an effective restoration of the $U(1)_A$ -symmetry have been found by [19,20], who report a drop in the in-medium mass of the pseudoscalar singlet meson of at least 200 MeV.

A restoration of the $U(1)_A$ -symmetry would have a qualitative impact on the nature of the phase transition in the two-flavor chiral limit. In the presence of the axial anomaly in terms of the 't Hooft determinant this transition is expected to be of second order in the $O(4)$ universality class [21]. If, however, the $U(1)_A$ -symmetry were restored at the chiral transition, a first order transition or

a second order transition in the universality class of $U(2)_L \times U(2)_R / U(2)_A$ could take place [22–26].

The possibility of an effective restoration of the $U(1)_A$ -symmetry has been addressed in several lattice QCD simulations. A degeneracy in the correlators of the pion and the pseudoscalar singlet meson has been observed in the chirally symmetric phase in a two-flavor simulation with overlap and domain wall fermions [27,28]. On the other hand, [9,29] find effective restoration only at larger temperatures of 196 MeV with $2 + 1$ flavors of domain wall fermions and [30,31], using highly improved staggered fermions, do not see it even at 1.5 times the crossover temperature. The phenomenological implications of the axial anomaly and different scenarios for its fate at the chiral crossover have been investigated with a Dyson-Schwinger approach using models for the quark-gluon interaction [32–35], in the Nambu–Jona-Lasinio (NJL) model [36–39] and with quark-meson [40–42] as well as linear sigma models [43,44].

In this work we use results for the energy-momentum scale dependence of the 't Hooft determinant [5], where the functional renormalization group (RG) in Wetterich's formulation [45] has been used to calculate the effective action of quenched QCD with two flavors. From this functional renormalization group, a coupled set of equations for the 1PI correlation functions, similar to the Dyson-Schwinger equations, can be derived, see e.g. [46–53] for reviews. Additionally, we use the Polyakov-quark-meson (PQM) model [54–56] with two flavors as a qualitative description of the chiral crossover. With the help of this model we derive a mapping of the renormalization group scale dependence of the 't Hooft determinant to its temperature dependence for the investigation of the mass splitting between pseudoscalar singlet η -meson and pion at the chiral crossover.

This paper is organised as follows: In Sec. II we briefly discuss the PQM model with two quark flavors and its behavior at the chiral crossover. Section III discusses the

calculation of the 't Hooft determinant [5] and the derivations of its temperature dependence. In Sec. IV we discuss our main result, the mass splitting between η -meson and pion at the chiral crossover and we summarize and conclude in Sec. V.

II. POLYAKOV-QUARK-MESON MODEL

A. Lagrangian

The Euclidean Lagrangian of the two-flavor Polyakov-quark-meson model with axial symmetry breaking term has the form [55]

$$\mathcal{L}_{\text{PQM}} = \bar{q} \left[\not{D} + \frac{h_\pi}{2} (\sigma + i\gamma^5 \vec{\tau} \vec{\pi}) + \frac{h_\eta}{2} (\vec{\tau} \vec{a} + i\gamma^5 \eta) \right] q + \text{tr}(\partial_\mu \Sigma \partial^\mu \Sigma^\dagger) + U(\rho, \xi) + \mathcal{U}(\Phi, \bar{\Phi}), \quad (1)$$

with the meson field $2\Sigma = (\sigma + i\eta) + (\vec{a} + i\vec{\pi})\vec{\tau}$ and Pauli matrices $\vec{\tau}$. The mesonic potential

$$U(\rho, \xi) = m_\rho^2 \rho + m_\xi^2 \xi + g\rho^2 - c\sigma \quad (2)$$

is a function of the chirally symmetric operators $\rho = \text{tr}(\Sigma \Sigma^\dagger)$ and the Kobayashi-Maskawa-'t Hooft determinant $\xi = \det \Sigma + \det \Sigma^\dagger$ [3,4,57]. The latter breaks the $U(1)_A$ -symmetry and leads to a mass splitting between the $(\sigma - \vec{\pi})$ - and $(\eta - \vec{a})$ -mesons. Additionally, an explicit symmetry breaking term c appears in this potential which mimics a nonvanishing current quark mass. Symmetry considerations would allow for another term $\rho_2 = \text{tr}[(\Sigma \Sigma^\dagger)^2]$ in the mesonic potential. For diagonal $\Sigma = \text{diag}(\sigma, \sigma)$, which is the case that is investigated in this work, this invariant becomes however proportional to $\rho^2 \propto \sigma^4$. Furthermore, any contributions to this term from quantum or thermal fluctuations are then added to ρ^2 and g can serve as a counterterm for both operators. For an investigation of the effect of taking the difference between ρ^2 and ρ_2 into account see e.g. [26].

In the PQM model the covariant derivative $\not{D} = \gamma^\mu (\partial_\mu - iA_0 \gamma^0)$ depends only on the nonfluctuating background field A_0 . Therefore also the Polyakov loop, given as the thermal expectation value of the path-ordered and color-traced Wilson loop,

$$\Phi = \frac{1}{N_c} \left\langle \text{tr}_c \mathcal{P} \exp \left[-i \int_0^\beta d\tau A_0(\vec{x}, \tau) \right] \right\rangle_{\beta=1/T}, \quad (3)$$

depends only on A_0 . To provide the gluonic background we use a polynomial ansatz for the effective Polyakov loop potential $\mathcal{U}(\Phi, \bar{\Phi})$ [54,55]

$$\frac{\mathcal{U}(\Phi, \bar{\Phi})}{T^4} = -\frac{b_2(T)}{4} (|\Phi|^2 + |\bar{\Phi}|^2) - \frac{b_3}{6} (\Phi^3 + (\bar{\Phi})^3) + \frac{b_4}{16} (|\Phi|^2 + |\bar{\Phi}|^2)^2 \quad (4)$$

with

$$b_2(T) = a_0 + a_1 \left(\frac{T_0}{T} \right) + a_2 \left(\frac{T_0}{T} \right)^2 + a_3 \left(\frac{T_0}{T} \right)^3 \quad (5)$$

and the same parameters as [56]:

$$\begin{aligned} a_0 &= 6.75 & a_1 &= -1.95 & a_2 &= 2.625 & a_3 &= -7.44 \\ b_3 &= 0.75 & b_4 &= 7.5 \\ T_0 &= 208 \text{ MeV}. \end{aligned} \quad (6)$$

Since we consider only the case of vanishing chemical potential, we have $\bar{\Phi} = \Phi$, and the potential $\mathcal{U}(\Phi, \bar{\Phi})$ is therefore a function of Φ alone.

B. Effective potential and meson masses

We calculate the effective potential Ω_{MF} at finite temperature in the extended mean-field approximation [58] which ignores all mesonic fluctuations. The remaining path integral is Gaussian, leading to the determinant of the fermionic kinetic operator. It depends on the Yukawa interactions which we assume to be degenerate $h \equiv h_\pi = h_\eta$. Using $\Phi = \bar{\Phi}$ at $\mu = 0$ the result is

$$\Omega_{MF} = \Omega_{\bar{q}q} + U(\rho, \xi) + \mathcal{U}(\Phi), \quad (7)$$

where

$$\begin{aligned} \Omega_{\bar{q}q} &= -12 \int^\Lambda \frac{d^3 p}{(2\pi)^3} E_q - 8T \int \frac{d^3 p}{(2\pi)^3} \\ &\times \log [1 + 3\Phi e^{-\beta E_q} + 3\bar{\Phi} e^{-2\beta E_q} + e^{-3\beta E_q}]. \end{aligned} \quad (8)$$

Here, $E_q^2 = \vec{p}^2 + \frac{h^2}{2} \rho$ is the quark energy. The potential is regularized with a sharp momentum cutoff Λ and then renormalized by choosing the coefficients of the bare potential Eq. (2) such that the correct value for the pion mass and pion decay constant $\langle \sigma \rangle = f_\pi$ are reproduced at $T = 0$. The numerical results at finite temperature are found to be independent of the sharp momentum cutoff for all values $\Lambda \geq 5 \text{ GeV}$. The temperature dependence of the 't Hooft determinant in this setting is trivial since the quark loop, being a function of ρ only, does not induce any anomalous contributions. Thus, the η -mass would show only a trivial temperature dependence which is proportional to the change in the pion mass in this effective description.

The mesonic screening masses are given by the second derivatives of the effective potential with respect to the corresponding fields

$$m_{\phi_{\text{PQM}}}^2 = \partial_\phi^2 \Omega_{MF}, \quad \phi \in \{\sigma, \vec{\pi}, \eta, \vec{a}\}, \quad (9)$$

evaluated at the field expectation value $\langle \sigma \rangle$. For a potential of the form Eq. (2) this implies

$$\begin{aligned} m_\pi^2 &= m_\rho^2 + m_\xi^2 + g\langle \sigma \rangle^2, \\ m_\sigma^2 &= m_\rho^2 + m_\xi^2 + 3g\langle \sigma \rangle^2, \\ m_\eta^2 &= m_a^2 = m_\rho^2 - m_\xi^2 + g\langle \sigma \rangle^2. \end{aligned} \quad (10)$$

The quark loop respects the full chiral symmetry and therefore it does not contribute to m_ξ^2 . In particular this means that the relation

$$m_\xi^2 = \frac{m_\pi^2 - m_\eta^2}{2} \quad (11)$$

holds for general Ω_{MF} .

C. Crossover

Once the model parameters have been fixed to yield physical values for the observables, the Polyakov-quark-meson model gives a qualitative description for the QCD crossover at finite temperature. To demonstrate this, we show the order parameters in Fig. 1. The normalized chiral condensate shows a rapid change to a very small value around temperatures of 170 to 180 MeV, which is the usual value found in these types of models for the temperature of the chiral crossover. The Polyakov loop, Φ , increases at the same temperatures, indicating a crossover to a deconfined phase of broken center symmetry. When comparing to results from lattice QCD [8,9], the crossover temperature in this model is too large by about 20 MeV. To some extent this can be remedied by improving the parametrization of the Polyakov loop potential and adding a strange quark [42,59], but it is still an indication for the qualitative nature of the description provided by the PQM model. To improve

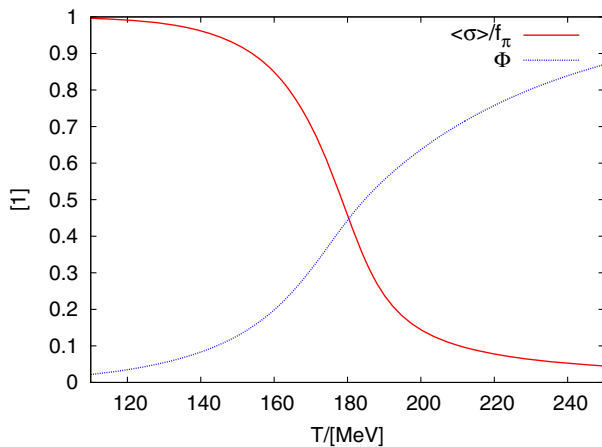


FIG. 1. Order parameters for the chiral, $\langle \sigma \rangle / f_\pi$, and deconfinement transition, Φ , as functions of the temperature T .

its applicability, this model can be embedded in a full QCD calculation via the dynamical hadronization technique [49,60,61]. Such an approach has been used e.g. quantitatively in quenched QCD in [5] and qualitatively also for unquenched QCD in [62].

III. 't HOOFT DETERMINANT

We use results for the 't Hooft determinant from a calculation within quenched QCD with two quark flavors [5]. This approach uses only the strong coupling strength and bare quark mass as input at perturbative momentum scales. From this input, the effective action $\Gamma[\phi]$ is calculated in a vertex expansion by solving the Wetterich equation [45]

$$\partial_k \Gamma_k[\phi] = \frac{1}{2} \text{Tr} \left[\frac{1}{\Gamma_k^{(2)} + R_k} \partial_k R_k \right]. \quad (12)$$

By the integration of infinitesimal momentum shells, the resulting trajectory for the effective average action, $\Gamma_k[\phi]$, connects the renormalized perturbative action $S[\phi] = \lim_{k \rightarrow \Lambda} \Gamma_k[\phi]$, defined in terms of strong coupling strength and bare quark mass at some large momentum Λ , with the full quantum effective action $\Gamma[\phi] = \lim_{k \rightarrow 0} \Gamma_k[\phi]$. The momentum-shell integration is controlled by the regulator function R_k which acts as a momentum dependent mass term.

As the momentum shell integration approaches non-perturbative momenta below $\mathcal{O}(1)$ GeV, four-quark interactions are created via two-gluon exchange diagrams, e.g. [53]. If these four-quark interactions are approximated by a momentum-independent coupling constant, chiral symmetry breaking is signalled by a singularity in this coupling. This divergence is a consequence of the emergence of the pion pole in the spontaneously broken phase. To avoid this divergence, one has to include momentum dependencies in the four-quark interaction. One possibility to do so with manageable effort is to use the dynamical hadronization technique [49,60,61], see also [5,62,63] for recent applications. In each momentum shell integration step, this technique rewrites the four-quark interactions in terms of meson exchange. This naturally includes the correct momentum dependence close to the pion pole, thus avoiding any singularities.

A. Renormalization group scale dependence of the 't Hooft determinant

To be able to identify the resonant four-quark interaction channel a fierz-complete basis with ten basis elements has been used in [5]. The momentum dependence of the resonant pion channel has been taken into account via the dynamical hadronization technique. The choice of only hadronizing this single channel uses the knowledge that the $U(1)_A$ -anomaly will break the symmetry between pions and η -meson. A more thorough investigation of the

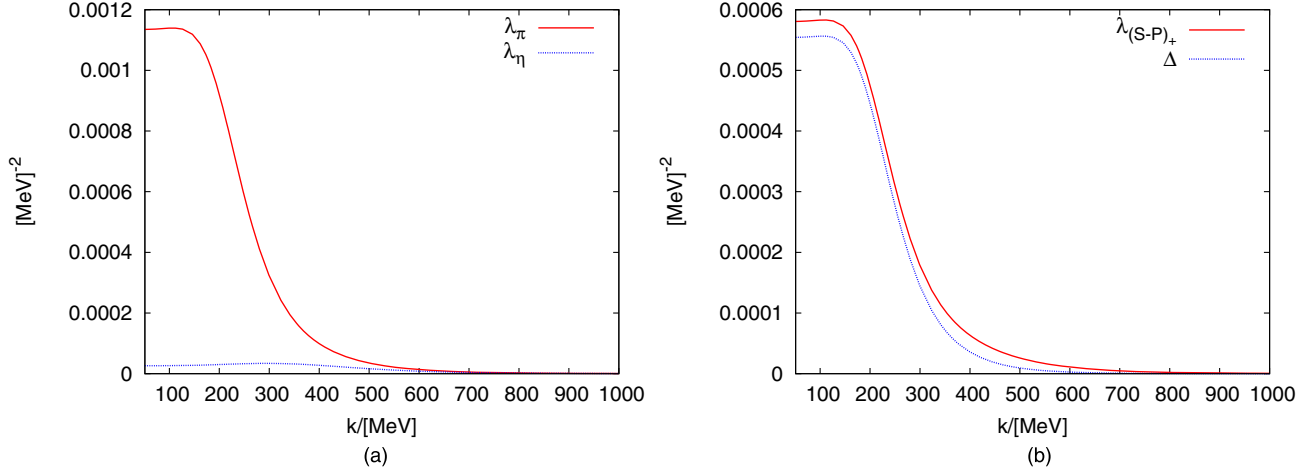


FIG. 2. Four-quark interactions as functions of renormalization group scale k [5].

implications of this choice and the $U(1)_A$ -anomaly will be presented elsewhere.

Here we are especially interested in the four-quark channels corresponding to the exchange of $\sigma - \pi$ -mesons (λ_π) and $\eta - \bar{a}$ -mesons (λ_η)

$$\lambda_\pi ((\bar{q}q)^2 - (\bar{q}\gamma^5\bar{\tau}q)^2), \quad (13)$$

$$\lambda_\eta (-(\bar{q}\gamma^5q)^2 + (\bar{q}\bar{\tau}q)^2). \quad (14)$$

These two channels can also be parametrized as

$$\begin{aligned} & \frac{\lambda_{(S-P)_+}}{4} ((\bar{q}q)^2 - (\bar{q}\gamma^5\bar{\tau}q)^2 - (\bar{q}\gamma^5q)^2 + (\bar{q}\bar{\tau}q)^2) \\ & + \frac{\Delta}{4} ((\bar{q}q)^2 - (\bar{q}\gamma^5\bar{\tau}q)^2 + (\bar{q}\gamma^5q)^2 - (\bar{q}\bar{\tau}q)^2), \end{aligned} \quad (15)$$

where the first term with coupling $\lambda_{(S-P)_+}$ has the full chiral symmetry and the second term, proportional to Δ , corresponds to the 't Hooft determinant in the case of two quark flavors. Consequently, the difference of the pion and η -meson four-quark couplings is directly proportional to the strength of the 't Hooft determinant

$$\Delta = \frac{\lambda_\pi - \lambda_\eta}{2}. \quad (16)$$

The corresponding results for these two channels are shown in Fig. 2. These results have been obtained in the truncation of [5] and have been rescaled such that a unit residue at the pion pole is guaranteed. Additionally the wave function renormalizations of pions and η -meson have been assumed to be degenerate. We clearly see that the four-quark interactions reach a considerable strength only at nonperturbative momenta. In this regime, the 't Hooft

determinant is almost as strong as the symmetric four-quark channel. As a consequence, the pion channel becomes dominant, whereas the η -meson is comparably heavy. Although hardly visible in Fig. 2, the η -pion splitting is already present at scales above the chiral symmetry breaking scale. One possible source for this splitting is the previously mentioned asymmetric choice of only hadronizing the pion- σ -meson channel.

B. Temperature dependence of the 't Hooft determinant

As discussed already, the quark-meson model can be derived from QCD within the functional renormalization group approach via the dynamical hadronization technique. Within this approach, mesons are introduced as auxiliary fields. The momentum dependence of the dynamically created four-quark interactions is then rewritten in terms of the exchange of meson ϕ , i.e.

$$\Gamma_{(\bar{q}q)^2}^{(4)} \rightarrow \Gamma_{(\bar{q}q)\phi}^{(3)} (\Gamma_{\phi\phi}^{(2)})^{-1} \Gamma_{(\bar{q}q)\phi}^{(3)}. \quad (17)$$

The couplings of the four-quark channels corresponding to pion and η -meson exchange, λ_π and λ_η , are therefore related to the corresponding Yukawa interactions and meson masses via the relations

$$\lambda_\pi = \frac{h_\pi^2}{2m_\pi^2} \quad \lambda_\eta = \frac{h_\eta^2}{2m_\eta^2}, \quad (18)$$

which hold at each renormalization group scale k .

For calculating the temperature dependence of the η -meson mass we use the temperature independent approximation $h \equiv h_\pi = h_\eta$. For given h , the temperature dependence of the meson masses, $m_\phi(T)$, is then directly proportional to temperature dependence of the corresponding four-quark interaction strength, $\lambda_\phi(T)$.

Consequently, we are left with calculating the temperature dependence of $\lambda_\eta(T)$. To do so, we assume that the temperature dependence of a given coupling strength can be mapped onto its renormalization group scale dependence, since both quantities are energy-momentum scales. A similar approximation has been used successfully for obtaining the strong running coupling as a function of temperature and magnetic field in [64]. To calculate the precise relation between the temperature and RG-scale dependence we use the temperature dependence of the pion mass, $m_{\pi,\text{PQM}}(T)$, as obtained within the PQM model. The pion mass gives us the corresponding four-quark interaction strength $\lambda_{\pi,\text{PQM}}(T)$ via (18) with $h_\pi = h$. Finally, we use the corresponding coupling $\lambda_{\pi,\text{QCD}}(k)$, obtained in the approach presented in [5], to relate the RG scale k with the temperature T by demanding $\lambda_{\pi,\text{PQM}}(T) \equiv \lambda_{\pi,\text{QCD}}(k)$. This condition maps each temperature T to a corresponding RG scale $k(T)$ and we obtain finally

$$m_\eta^2(T) = \frac{h^2}{2} \left(\frac{h^2}{2m_{\pi,\text{PQM}}^2(T)} - 2\Delta_{\text{QCD}}(k(T)) \right)^{-1}, \quad (19)$$

from the RG running of the 't Hooft determinant

$$\Delta_{\text{QCD}}(k) := \frac{(\lambda_{\pi,\text{QCD}}(k) - \lambda_{\eta,\text{QCD}}(k))}{2}. \quad (20)$$

IV. η - AND π -MESON MASS SPLITTING AT CHIRAL CROSSOVER

We show the temperature dependence of the mesonic curvature masses, defined by the second derivative of the effective potential, in Fig. 3. At small temperatures our value for the η -mass is $m_\eta = 880$ MeV. This agrees within the given errors with lattice QCD which gives approximately 819(127) MeV for the corresponding pole mass [65]. One uncertainty in our calculation stems from using the curvature mass instead of the pole mass, which is far away from Euclidean momenta. Furthermore, we used the assumption that the wave function renormalization as well as the Yukawa coupling of the η -meson is degenerate with the one of the pion.

At temperatures close to the crossover we see the usual behavior of the pion and σ -meson masses, which become degenerate above the transition. The mass of the η -meson shows a drop at the chiral transition. This is in accordance with experimental results for the in-medium mass [19,20]. A similar drop in the η -meson mass is found in the (2 + 1)-flavor version of the quark-meson model without temperature dependence in the 't Hooft determinant coupling [40,41]. Since the mesonic 't Hooft determinant is of order Σ^{N_f} in the meson field, the drop in the (2 + 1)-flavor η -meson mass can be attributed solely to the melting of the light condensate. The presented two-flavor case, on the other hand, requires a genuine temperature dependence in

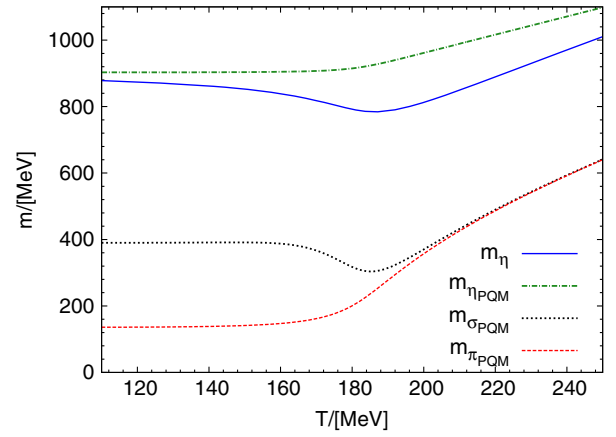


FIG. 3. Meson screening masses as functions of the temperature T .

the strength of the 't Hooft determinant coupling to reproduce a similar drop. This is seen by comparing to the temperature dependence of the η -meson mass obtained from the PQM model alone. In this approximation the mass is monotonous in the temperature, since m_ξ^2 is temperature independent, whereas m_ρ^2 grows with the temperature. Therefore, the drop in the mass of the η -meson is clearly a QCD effect. On the other hand, it has recently been found in a study with 2 + 1 quark flavors that mesonic fluctuations might also have a visible and counteracting effect on the strength of the 't Hooft determinant [66].

Above the chiral crossover, we still see a large splitting between the masses of the pion and the η -meson, which is in contrast to some lattice simulations reporting a fast reduction of the mass splitting above the chiral crossover [27,28]. Several assumptions within our calculation can have an influence on the mass splitting above the chiral crossover. First, our procedure of mapping the renormalization group scale dependence on the temperature introduces uncertainties precisely at the chiral crossover. Second, the restoration of chiral symmetry usually happens at too large temperatures and too slowly in the quark-meson model. Consequently, the temperature points above the crossover should actually be compressed in Fig. 3, which would entail a faster effective restoration of $U(1)_A$. Finally, the asymmetric hadronization procedure used in [5] could be responsible for a too large mass splitting in the chirally symmetric phase. In future investigations, the η -channel should therefore be dynamically hadronized as well.

V. SUMMARY AND CONCLUSION

We have investigated the mass splitting between pions and η -meson across the chiral crossover. To this end, we have used results for the renormalization group scale dependence of the 't Hooft determinant from two-flavor quenched QCD. From these results the temperature dependence for the 't Hooft determinant has been

approximated by matching the temperature dependence of the pion mass in a mean-field approximation of the Polyakov-quark-meson model to its renormalization group scale dependence from the result in quenched QCD [5].

We find a drop in the mass of the η -meson at the chiral crossover which is compatible with experimental results for the in-medium η -meson mass [19,20]. In the case of two flavors, this drop is the consequence of a genuine temperature dependence in the strength of the 't Hooft determinant coupling. A large splitting between pion and η -meson mass is found at temperatures above the chiral transition. This might be caused by the slow restoration of chiral symmetry at large temperatures within the used model together with the procedure used for mapping the renormalization group scale to the temperature. Additionally, the asymmetric choice in the dynamical hadronization procedure of [5] and the fact that we used curvature masses instead of pole masses might play a role.

For future investigations it would be interesting to improve the calculation [5] with respect to axial anomaly effects and dynamically hadronize the η -meson as well. It would be interesting to include mesonic fluctuations in the PQM model and see whether the found temperature dependence in the 't Hooft determinant coupling strength is sufficiently strong to change the order of the chiral transition in the chiral limit. Furthermore, a continuation of the current approach to Minkowski space similar to [67] would be desirable.

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