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Semileptonic decays of Λ_b baryons in the relativistic quark model

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Semileptonic Λ_b decays are investigated in the framework of the relativistic quark model based on the quasipotential approach and the quark-diquark picture of baryons. The decay form factors are expressed through the overlap integrals of the initial and final baryon wave functions. All calculations are done without employing nonrelativistic and heavy quark expansions. The momentum transfer dependence of the decay form factors is explicitly determined in the whole accessible kinematical range without any extrapolations or model assumptions. Both the heavy-to-heavy $\Lambda_b \to \Lambda_c \ell \nu_\ell$ and heavy-to-light $\Lambda_b \to p \ell \nu_\ell$ decay branching fractions are calculated. The results agree within error bars with the experimental value of the branching fraction of the $\Lambda_b \to \Lambda_c^+ l^- \bar{\nu}_l$ decay. From the recent LHCb data on the ratio of the branching fractions of the heavy-to-light and heavy-to-heavy semileptonic Λ_b decays the ratio of the Cabibbo-Kobayashi-Maskawa matrix elements $|V_{ub}|/|V_{cb}|$ is obtained. It is consistent with the corresponding ratio determined from the inclusive B meson decays.

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I. INTRODUCTION

In recent years significant experimental progress has been achieved in studying properties of heavy baryons. The masses of all ground states of charmed and bottom baryons have been measured except Ω_b^* [1]. Many decay channels of these baryons were observed and new, more precise data are expected in the near future, since heavy baryons are copiously produced at the LHC. Weak decays of bottom baryons can serve as an additional source for the determination of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements $|V_{cb}|$ and $|V_{ub}|$. Such determination is particularly important since there exists some tension between the values of these matrix elements extracted from exclusive and inclusive bottom meson weak decays [1–3]. Very recently the LHCb Collaboration [4] reported the first measurement of the ratio of the heavy-to-light semileptonic $\Lambda_b \to p l \nu_l$ and heavy-to-heavy semileptonic $\Lambda_b \to \Lambda_c l \nu_l$ decay rates in the constrained kinematical regions, thus providing data for the determination of the ratio of the CKM matrix elements $|V_{ub}/|V_{cb}|$ from baryon decays.

In order to calculate weak decay rates of bottom baryons it is necessary to determine the form factors which parametrize the matrix elements of the weak current between initial and final baryon states. These form factors depend on the momentum transfer from the initial baryon to the final baryon. In the case of semileptonic bottom baryon decays both to heavy and light final baryons the momentum transfer squared q^2 varies in a rather broad kinematical range. Therefore it is very important to explicitly determine the q^2 dependence of decay form factors in the whole kinematical range.

In this paper we study semileptonic Λ_b decays in the framework of the relativistic quark model based on the quasipotential approach and QCD. This model was successfully applied for investigating various meson properties [5,6]. The heavy and strange baryon spectroscopy was studied in the relativistic quark-diquark picture in Refs. [7,8] where masses and wave functions of the ground and excited baryon states were obtained. We also calculated the decay rates of heavy-to-heavy semileptonic baryon transitions [9] using the heavy quark expansion. Both the infinitely heavy quark limit and first-order $1/m_Q$ corrections were considered. It was shown that our model satisfies all model-independent relations following from the heavy quark symmetry [10]. Leading and subleading baryon Isgur-Wise functions were determined. It was found that $1/m_Q$ corrections give larger contributions to heavy baryon decay rates than for heavy meson decay rates. Indeed, for heavy meson decays the heavy quark effective theory (HQET) $\bar{\Lambda}$ parameter is determined by the light quark energy while for heavy baryon decays this parameter is proportional to the light diquark energy which is almost 2 times larger. Here we calculate the weak decay form factors without employing the heavy quark expansion. This allows us to improve previous results and to consider simultaneously heavy-to-heavy and heavy-to-light semileptonic Λ_b decays. It is important to point out that our model provides the explicit q^2 dependence of the weak decay form factors in the whole accessible kinematical range without additional model assumptions and extrapolations. We consistently take into account all relativistic effects including transformations of the baryon wave functions from the rest to the moving reference frame and contributions of the intermediate negative-energy states.

The paper is organized as follows. In the next section we briefly describe the relativistic quark-diquark picture of heavy baryons. Calculation of the weak current matrix elements between baryon states in the quasipotential

approach is discussed in Sec. III. Using this method in Sec. IV we determine the heavy-to-heavy and heavy-to-light weak decay form factors in the whole accessible kinematical range. Semileptonic Λ_b decay rates and other observables are calculated in Sec. V and compared with available experimental data and previous calculations. The determination of the ratio of the CKM matrix elements $|V_{ub}|$ and $|V_{cb}|$ from the recent LHCb data [4] is discussed. We give our conclusions in Sec. VI. Explicit expressions for the decay form factors as overlap integrals of baryon wave functions are listed in the Appendix.

II. RELATIVISTIC QUARK-DIQUARK PICTURE OF BARYONS

We study the semileptonic decays of Λ_Q baryons in the relativistic quark-diquark picture in the framework of the quasipotential approach. The interaction of two quarks in a diquark and the quark-diquark interaction in a baryon are described by the diquark wave function Ψ_d of the bound quark-quark state and by the baryon wave function Ψ_B of the bound quark-diquark state, which satisfy the relativistic quasipotential equation of the Schrödinger type [5]

$$\left(\frac{b^2(M)}{2\mu_R}-\frac{\mathbf{p}^2}{2\mu_R}\right)\Psi_{d,B}(\mathbf{p})=\int\frac{d^3q}{(2\pi)^3}V(\mathbf{p},\mathbf{q};M)\Psi_{d,B}(\mathbf{q}), \tag{1}$$

where the relativistic reduced mass is

$$\mu_R=\frac{M^4-(m_1^2-m_2^2)^2}{4M^3},$$

and the center-of-mass system relative momentum squared on mass shell is

$$b^2(M)=\frac{[M^2-(m_1+m_2)^2][M^2-(m_1-m_2)^2]}{4M^2}.$$

Here M is the bound state diquark or baryon mass, $m_{1,2}$ are the masses of quarks (q_1 and q_2) which form the diquark or of the diquark (d) and quark (q) which form the baryon (B), and $\mathbf{p}$ is their relative momentum.

The kernel $V(\mathbf{p},\mathbf{q};M)$ in Eq. (1) is the quasipotential operator of the quark-quark or quark-diquark interaction which is constructed with the help of the off-mass-shell scattering amplitude, projected onto the positive-energy states. We assume that the effective interaction is the sum of the usual one-gluon exchange term and the mixture of long-range vector and scalar linear confining potentials, where the vector confining potential contains the Pauli term. The resulting quasipotentials are given by the following expressions. The quark-quark (qq) interaction in the diquark is

$$\begin{aligned}V(\mathbf{p},\mathbf{q};M)=\bar u_1(p)\bar u_2(-p)\frac{1}{2}\Big[&\frac{4}{3}\alpha_s D_{\mu\nu}(\mathbf{k})\gamma_1^\mu\gamma_2^\nu\\&+V^V_{\rm conf}(\mathbf{k})\Gamma_1^\mu(\mathbf{k})\Gamma_{2;\mu}(-\mathbf{k})+V^S_{\rm conf}(\mathbf{k})\Big]\\&\times u_1(q)u_2(-q).\end{aligned} \tag{2}$$

The quark-diquark (qd) interaction in the baryon is

$$\begin{aligned}V(\mathbf{p},\mathbf{q};M)=&\frac{\langle d(P)|J_\mu|d(Q)\rangle}{2\sqrt{E_d(p)E_d(q)}}\bar u_q(p)\frac{4}{3}\alpha_s D_{\mu\nu}(\mathbf{k})\gamma^\nu u_q(q)\\&+\psi_d^*(P)\bar u_q(p)J_{d;\mu}\Gamma_q^\mu(\mathbf{k})V^V_{\rm conf}(\mathbf{k})u_q(q)\psi_d(Q)\\&+\psi_d^*(P)\bar u_q(p)V^S_{\rm conf}(\mathbf{k})u_q(q)\psi_d(Q),\end{aligned} \tag{3}$$

where α_s is the QCD coupling constant, $\langle d(P)|J_\mu|d(Q)\rangle$ is the vertex of the diquark-gluon interaction which takes into account the diquark internal structure and $J_{d;\mu}$ is the effective long-range vector vertex of the diquark. The diquark momenta are $P=(E_d(p),-\mathbf{p})$, $Q=(E_d(q),-\mathbf{q})$ with $E_d(p)=\sqrt{\mathbf{p}^2+M_d^2}$. $D_{\mu\nu}$ is the gluon propagator in the Coulomb gauge, $\mathbf{k}=\mathbf{p}-\mathbf{q}$; γ_μ and $u(p)$ are the Dirac matrices and spinors, while $\psi_d(P)$ is the diquark wave function. The factor $1/2$ in the quark-quark interaction accounts for the difference of the color factor compared to the quark-antiquark case.

The effective long-range vector vertex of the quark is defined by

$$\Gamma_\mu(\mathbf{k})=\gamma_\mu+\frac{i\kappa}{2m}\sigma_{\mu\nu}\tilde k^\nu,\qquad \tilde k=(0,\mathbf{k}), \tag{4}$$

where κ is the anomalous chromomagnetic moment of quarks.

In the nonrelativistic limit the vector and scalar confining potentials reduce to

$$\begin{aligned}V^V_{\rm conf}(r)&=(1-\varepsilon)(Ar+B),\\V^S_{\rm conf}(r)&=\varepsilon(Ar+B),\end{aligned} \tag{5}$$

where ε is the mixing coefficient, and the usual Cornell-like potential is reproduced

$$V(r)=-\frac{4}{3}\frac{\alpha_s}{r}+Ar+B. \tag{6}$$

Here we use the QCD coupling constant with freezing

$$\begin{aligned}\alpha_s(\mu^2)&=\frac{4\pi}{\beta_0\ln\frac{\mu^2+M_B^2}{\Lambda^2}},\qquad \beta_0=11-\frac{2}{3}n_f,\\\mu&=\frac{2m_1m_2}{m_1+m_2},\end{aligned} \tag{7}$$

with the background mass $M_B = 2.24\sqrt{A} = 0.95$ GeV and $\Lambda = 413$ MeV [11].

All parameters of the model such as quark masses, parameters of the linear confining potential A and B, the mixing coefficient ε and anomalous chromomagnetic quark moment κ were fixed previously from calculations of meson and baryon properties [5,7]. The constituent quark masses $m_u = m_d = 0.33$ GeV, $m_c = 1.55$ GeV, $m_b = 4.88$ GeV and the parameters of the linear potential $A = 0.18$ GeV^2 and $B = -0.3$ GeV have the usual values of quark models. The value of the mixing coefficient of vector and scalar confining potentials $\varepsilon = -1$ has been determined from the consideration of the heavy quark expansion for the semileptonic heavy meson decays and charmonium radiative decays [5]. The universal Pauli interaction constant $\kappa = -1$ has been fixed from the analysis of the fine splitting of heavy quarkonia ^3P_J- states [5]. Note that the long-range chromomagnetic contribution to the potential, which is proportional to $(1+\kappa)$, vanishes for the chosen value of $\kappa = -1$.

III. MATRIX ELEMENT OF THE WEAK CURRENT BETWEEN BARYON STATES

To calculate the heavy Λ_Q baryon decay rate to the heavy or light Λ_q ($q = c$ or u, $\Lambda_u \equiv p$) baryon it is necessary to determine the corresponding matrix element of the weak current between baryon states, which in the quasipotential approach is given by

$$\langle \Lambda_q(p_q)|J_\mu^W|\Lambda_Q(p_Q)\rangle = \int \frac{d^3p d^3q}{(2\pi)^6}\bar{\Psi}_{\Lambda_q \mathbf{p}_q}(\mathbf{p})\Gamma_\mu(\mathbf{p},\mathbf{q})\Psi_{\Lambda_Q \mathbf{p}_Q}(\mathbf{q}), \quad (8)$$

where $\Gamma_\mu(\mathbf{p},\mathbf{q})$ is the two-particle vertex function and $\Psi_{\Lambda \mathbf{p}}$ is the baryon wave function projected onto the positive-energy states of quarks and boosted to the moving reference frame with momentum $\mathbf{p}$. The baryon wave function is the product of the diquark and quark wave functions.

The contributions to Γ come from Figs. 1 and 2. The contribution $\Gamma^{(2)}$ is the consequence of the projection onto the positive-energy states. Note that the form of the relativistic corrections resulting from the vertex function

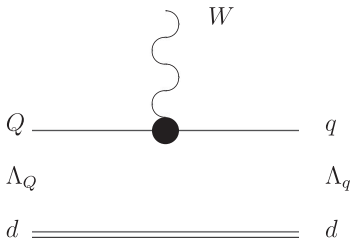

FIG. 1. Lowest-order vertex function $\Gamma^{(1)}$ contributing to the current matrix element (8).

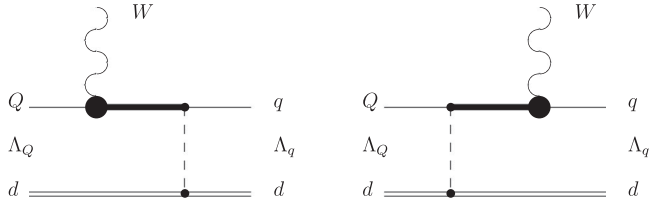

FIG. 2. Vertex function $\Gamma^{(2)}$ taking the quark interaction into account. Dashed lines correspond to the effective potential $\mathcal{V}_{Qd}$ in Eq. (3). Bold lines denote the negative-energy part of the quark propagator.

$\Gamma^{(2)}$ is explicitly dependent on the Lorentz structure of the quark-diquark interaction. For the heavy-to-heavy baryon transitions only $\Gamma^{(1)}$ contributes in the heavy quark limit ($m_Q \to \infty$), while $\Gamma^{(2)}$ gives the subleading-order contributions. The vertex functions are given by

$$\Gamma_\mu^{(1)}(\mathbf{p},\mathbf{q}) = \psi_d^*(p_d)\bar{u}_q(p_q)\gamma_\mu(1-\gamma^5)u_Q(q_Q) \times \psi_d(q_d)(2\pi)^3\delta(\mathbf{p}_d - \mathbf{q}_d), \quad (9)$$

and

$$\Gamma_\mu^{(2)}(\mathbf{p},\mathbf{q}) = \psi_d^*(p_d)\bar{u}_q(p_q)\Big\{\gamma_\mu(1-\gamma^5)\frac{\Lambda_Q^{(-)}(k)}{\epsilon_Q(k)+\epsilon_Q(p_q)} \times \gamma^0\mathcal{V}_{Qd}(\mathbf{p}_d-\mathbf{q}_d) + \mathcal{V}_{Qd}(\mathbf{p}_d-\mathbf{q}_d) \times \frac{\Lambda_q^{(-)}(k')}{\epsilon_q(k')+\epsilon_q(q_Q)}\gamma^0\gamma_\mu(1-\gamma^5)\Big\} \times u_Q(q_Q)\psi_d(q_d), \quad (10)$$

where the superscripts "(1)" and "(2)" correspond to Figs. 1 and 2; $\mathbf{k} = \mathbf{p}_q - \Delta$, $\mathbf{k}' = \mathbf{q}_Q + \Delta$, $\mathbf{\Delta} = \mathbf{P}_q - \mathbf{P}_Q$, $\epsilon(p) = \sqrt{m^2+\mathbf{p}^2}$, and

$$\Lambda^{(-)}(p) = \frac{\epsilon(p) - (m\gamma^0 + \gamma^0(\boldsymbol{\gamma}\mathbf{p}))}{2\epsilon(p)}.$$

The wave functions in the weak current matrix element (8) are not in the rest frame. In the Λ_Q baryon rest frame, the final baryon is moving with the recoil momentum $\mathit{\Delta}$. The wave function of the moving baryon $\Psi_{\Lambda_q \mathit{\Delta}}$ is connected with the wave function in the rest frame $\Psi_{\Lambda_q \mathbf{0}} \equiv \Psi_{\Lambda_q}$ by the transformation [12]

$$\Psi_{\Lambda_q \mathit{\Delta}}(\mathbf{p}) = D_q^{1/2}(R^W_{L_\mathit{\Delta}})D_d^{\mathcal{I}}(R^W_{L_\mathit{\Delta}})\Psi_{B_q \mathbf{0}}(\mathbf{p}), \quad \mathcal{I} = 0, 1, \quad (11)$$

where R^W is the Wigner rotation, L_Δ is the Lorentz boost from the baryon rest frame to a moving one, and the rotation matrix of the quark spin $D^{1/2}(R)$ in the spinor representation is given by

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} D_q^{1/2}(R^W_{L_\Delta}) = S^{-1}(\mathbf{p}_q) S(\mathbf{\Delta}) S(\mathbf{p}), \qquad (12)$$

where

$$S(\mathbf{p}) = \sqrt{\frac{\epsilon(p)+m}{2m}} \left(1 + \frac{\boldsymbol{\alpha}\mathbf{p}}{\epsilon(p)+m}\right)$$

is the usual Lorentz transformation matrix of the four-spinor. The rotation matrix $D^{\mathcal{I}}(R)$ of the diquark with spin $\mathcal{I}$ is equal to $D^0_d(R^W) = 1$ for the scalar diquark and $D^1_d(R^W) = R^W$ for the axial-vector diquark.

IV. FORM FACTORS OF THE Λ_b BARYON DECAYS

The hadronic matrix elements of the vector and axial vector weak currents for the semileptonic decay $\Lambda_Q \to \Lambda_q$ ($Q = b$ and $q = c$ or u) are parametrized in terms of six invariant form factors:

$$\langle \Lambda_q(p',s')|V^\mu|\Lambda_Q(p,s)\rangle = \bar{u}_{\Lambda_q}(p',s')\left[F_1(q^2)\gamma^\mu + F_2(q^2)\frac{p^\mu}{M_{\Lambda_Q}} + F_3(q^2)\frac{p'^\mu}{M_{\Lambda_q}}\right]u_{\Lambda_Q}(p,s),$$
$$\langle \Lambda_q(p',s')|A^\mu|\Lambda_Q(p,s)\rangle = \bar{u}_{\Lambda_q}(p',s')\left[G_1(q^2)\gamma^\mu + G_2(q^2)\frac{p^\mu}{M_{\Lambda_Q}} + G_3(q^2)\frac{p'^\mu}{M_{\Lambda_q}}\right]\gamma_5 u_{\Lambda_Q}(p,s), \qquad (13)$$

where $u_{\Lambda_Q}(p,s)$ and $u_{\Lambda_q}(p',s')$ are Dirac spinors of the initial and final baryon; $q = p' - p$.

Another popular parametrization of these decay matrix elements reads [13,14]

$$\langle \Lambda_q(p',s')|V^\mu|\Lambda_Q(p,s)\rangle = \bar{u}_{\Lambda_q}(p',s')\left[f_1^V(q^2)\gamma^\mu - f_2^V(q^2)i\sigma^{\mu\nu}\frac{q_\nu}{M_{\Lambda_Q}} + f_3^V(q^2)\frac{q^\mu}{M_{\Lambda_Q}}\right]u_{\Lambda_Q}(p,s),$$
$$\langle \Lambda_q(p',s')|A^\mu|\Lambda_Q(p,s)\rangle = \bar{u}_{\Lambda_q}(p',s')\left[f_1^A(q^2)\gamma^\mu - f_2^A(q^2)i\sigma^{\mu\nu}\frac{q_\nu}{M_{\Lambda_Q}} + f_3^A(q^2)\frac{q^\mu}{M_{\Lambda_Q}}\right]\gamma_5 u_{\Lambda_Q}(p,s). \qquad (14)$$

It is easy to find the following relations between these two sets of form factors:

$$f_1^V(q^2) = F_1(q^2) + (M_{\Lambda_Q} + M_{\Lambda_q})\left[\frac{F_2(q^2)}{2M_{\Lambda_Q}} + \frac{F_3(q^2)}{2M_{\Lambda_q}}\right],$$
$$f_2^V(q^2) = -\frac{1}{2}\left[F_2(q^2) + \frac{M_{\Lambda_Q}}{M_{\Lambda_q}}F_3(q^2)\right],$$
$$f_3^V(q^2) = \frac{1}{2}\left[F_2(q^2) - \frac{M_{\Lambda_Q}}{M_{\Lambda_q}}F_3(q^2)\right],$$
$$f_1^A(q^2) = G_1(q^2) - (M_{\Lambda_Q} - M_{\Lambda_q})\left[\frac{G_2(q^2)}{2M_{\Lambda_Q}} + \frac{G_3(q^2)}{2M_{\Lambda_q}}\right],$$
$$f_2^A(q^2) = -\frac{1}{2}\left[G_2(q^2) + \frac{M_{\Lambda_Q}}{M_{\Lambda_q}}G_3(q^2)\right],$$
$$f_3^A(q^2) = \frac{1}{2}\left[G_2(q^2) - \frac{M_{\Lambda_Q}}{M_{\Lambda_q}}G_3(q^2)\right]. \qquad (15)$$

To find the weak decay form factors we need to calculate the matrix element of the weak current between baryon wave functions known from the mass spectra calculations. The general structure of the current matrix element (8) is rather complicated, because it is necessary to integrate both with respect to d^3p and d^3q. The δ function in the expression (9) for the vertex function $\Gamma^{(1)}$ permits us to perform one of these integrations. As a result the contribution of $\Gamma^{(1)}$ to the current matrix element has the usual structure of an overlap integral of baryon wave functions and can be calculated exactly in the whole kinematical range. The situation with the contribution $\Gamma^{(2)}$ is different. Here, instead of a δ function, we have a complicated structure, containing the potential of the quark-diquark interaction in the baryon. Therefore in the general case we cannot get rid of one of the integrations in the contribution of $\Gamma^{(2)}$ to the matrix element (8). Thus it is necessary to use some additional considerations in order to simplify calculations. The main idea is to expand the vertex function $\Gamma^{(2)}$, given by Eq. (10), in such a way that we can get rid of the momentum dependence in the quark energies $\epsilon(p)$. Then it will be possible to use the quasipotential equation (1) in order to perform one of the integrations in the current matrix element (8).

For the heavy-to-heavy $\Lambda_b \to \Lambda_c$ weak transitions, using the fact that both the initial and final baryons contain heavy quarks, one can expand the decay matrix elements in inverse powers of the heavy quark masses. Such an expansion was performed in our model up to subleading order in Ref. [9]. It was found that all heavy quark

symmetry relations are satisfied in our model. However the $1/m_Q$ corrections turn out to be rather large, significantly larger than for the heavy-to-heavy mesons transitions. This is the consequence of the larger value of the expansion parameter $\bar{\Lambda}$. Indeed in the case of baryon decays the parameter $\bar{\Lambda}$ is determined by the light diquark energies [9], while for meson decays it is determined by light quark energies [15]. Therefore consideration of such decays without the heavy quark expansion can significantly improve the precision of predictions. Also such an expansion cannot be applied for the heavy-to-light $\Lambda_b \to p$ weak transitions, since the final baryon contains only light u and d quarks.

It is important to take into account that both $\Lambda_b \to \Lambda_c l\nu_l$ and $\Lambda_b \to p l\nu_l$ decays have a broad kinematical range. The square of the momentum transfer to the lepton pair q^2 varies from 0 to $q^2_{\rm max} \approx 12$ GeV^2 for decays to Λ_c and from 0 to $q^2_{\rm max} \approx 22$ GeV^2 for decays to p. As a result the recoil momentum of the final baryon $|\mathbf{\Delta}|$ is almost always significantly larger than the relative quark momentum in the baryon. Thus one can neglect small relative momentum $|\mathbf{p}|$ with respect to the recoil momentum $|\mathbf{\Delta}|$ in the energies of quarks in energetic final baryons and replace $\epsilon_q(p+\Delta) \equiv \sqrt{m_q^2 + (\mathbf{p}+\mathbf{\Delta})^2}$ by $\epsilon_q(\Delta) \equiv \sqrt{m_q^2 + \mathbf{\Delta}^2}$. It is important to point out that we keep the quark mass in the energies $\epsilon_q(\Delta)$. Thus the resulting expressions are valid both for the heavy-to-light and heavy-to-heavy Λ_b baryon decays. Such replacement is made in the subleading contribution $\Gamma^{(2)}_\mu(\mathbf{p},\mathbf{q})$ only and permits us to perform one of the integrations using the quasipotential equation. As a result, the weak decay matrix elements are expressed through the usual overlap integral of initial and final baryon wave functions. Note that the subleading contributions are proportional to the ratios of baryon binding energies, which are small, to the quark energies and thus turn out to be also small numerically. Therefore we obtain reliable expressions for the form factors in the whole accessible kinematical range. The largest uncertainty, which turns out to be small numerically, occurs for the heavy-to-light transitions in the narrow region near zero recoil of the final light baryon, where the above-discussed replacement is less justified. It is important to emphasize that we consistently take into account all relativistic corrections including boosts of the baryon wave functions from the rest frame to the moving one, given by Eq. (11). The obtained expressions for the form factors are presented in the Appendix (to simplify these expressions we set the long-range anomalous chromomagnetic quark moment $\kappa = -1$).

It is easy to check that for the heavy-to-heavy semileptonic decays one can reproduce the model-independent relations of the HQET [10] by expanding the form factors (A1)–(A22) in inverse powers of the initial and final heavy quark masses. The resulting expressions for the leading and subleading in $1/m_Q$ Isgur-Wise functions coincide with the ones obtained in our previous analysis of heavy baryon decays in the framework of the heavy quark expansion [9]. On the other hand, for the heavy-to-light decays the following HQET relations [16] are also valid:

$$\begin{aligned} F_1(q^2) &= \xi_1^{(0)}(q^2) - \xi_2^{(0)}(q^2), \\ G_1(q^2) &= \xi_1^{(0)}(q^2) + \xi_2^{(0)}(q^2), \\ F_2(q^2) &= G_2(q^2) = 2\xi_2^{(0)}(q^2), \\ F_3(q^2) &= G_3(q^2) = 0. \end{aligned} \tag{16}$$

They arise in the infinitely heavy quark mass limit $m_Q \to \infty$ for the initial heavy Λ_Q baryon only. The other form factor relations found in the additional limits of small and large recoil [17] of the final light Λ_q baryon in the rest frame of the decaying heavy Λ_Q are also satisfied.

For numerical calculations of the form factors we use the quasipotential wave functions of the Λ_b, Λ_c and p baryons obtained in their mass spectra calculations. Note that these calculations were done without the application of nonrelativistic v/c and heavy quark $1/m_Q$ expansions. Therefore the resulting wave functions incorporate nonperturbatively the relativistic quark dynamics in heavy and light baryons. Our results for the masses of these baryons are in good agreement with experimental data [1], which we use in our calculations. We find that the weak decay baryon form factors can be approximated with good accuracy by the following expressions:

$$F(q^2) = \frac{F(0)}{\left(1 - \sigma_1 \frac{q^2}{M^2_{\Lambda_Q}} + \sigma_2 \frac{q^4}{M^4_{\Lambda_Q}} + \sigma_3 \frac{q^6}{M^6_{\Lambda_Q}} + \sigma_4 \frac{q^8}{M^8_{\Lambda_Q}}\right)}. \tag{17}$$

The difference of fitted form factors from the calculated ones does not exceed 0.5%.

The values of $F(0)$, $F(q^2_{\rm max})$ and $\sigma_{1,2,3,4}$ are given in Tables I and II. The evaluation of the theoretical uncertainties of form factor calculations represents an important issue. Of course, the uncertainty of the model itself is not known since it is not directly derived from QCD. We can estimate the errors only within our model. They mostly originate from the uncertainties in the baryon wave

TABLE I. Calculated form factors of the weak $\Lambda_b \to \Lambda_c$ transition.

	$F_1(q^2)$	$F_2(q^2)$	$F_3(q^2)$	$G_1(q^2)$	$G_2(q^2)$	$G_3(q^2)$
$F(0)$	0.719	−0.062	−0.086	0.520	−0.225	0.113
$F(q^2_{\rm max})$	1.62	−0.304	−0.218	1.11	−0.611	0.314
σ_1	1.46	2.28	2.11	1.46	1.56	2.11
σ_2	−4.27	−7.98	−0.99	−3.06	−6.44	−2.49
σ_3	29.1	53.6	17.2	22.3	42.3	25.4
σ_2	−51.1	−87.5	−31.7	−39.9	−73.7	−45.3

TABLE II. Calculated form factors of the weak $\Lambda_b \to p$ transition.

	$F_1(q^2)$	$F_2(q^2)$	$F_3(q^2)$	$G_1(q^2)$	$G_2(q^2)$	$G_3(q^2)$
$F(0)$	0.227	−0.021	−0.013	0.196	−0.076	0.013
$F(q^2_{\max})$	1.50	−0.463	−0.144	0.905	−0.748	0.283
σ_1	0.805	2.80	1.72	0.712	1.06	1.87
σ_2	−3.06	2.14	−2.07	−2.23	−4.49	−2.64
σ_3	4.90	0.60	6.18	2.85	10.2	8.45
σ_2	−1.96	−1.07	−3.37	−0.71	−6.08	−5.24

functions and for the heavy-to-light transitions from the subleading contribution in the low recoil region. For example, to estimate the errors coming from the baryon wave functions we compared the form factors calculated on the basis of the complete relativistic wave functions with the corresponding ones calculated on the basis of the wave functions obtained in the heavy quark limit. As a result we find that the total error of our form factors should be less than 5%.

In Table III the comparison of theoretical predictions for the form factors $f^{V,A}_{1,2,3}(0)$ is given. The calculations in Refs. [13,14] were based on the covariant confined quark model. The authors of Ref. [18] used the light-front quark model and diquark picture, while QCD light-cone sum rules were employed in Ref. [19]. Reasonable agreement between predictions of significantly different approaches for calculating baryon form factors is observed.

We plot the baryon decay form factors in Figs. 3 and 4. The comparison of the $\Lambda_b \to \Lambda_c$ form factor plots in Fig. 3 with our previous calculation within the heavy quark expansion presented in Fig. 5 of Ref. [9] indicate the general consistency, but the values of the form factors $|F_1(q^2_{\max})|$, $|G_{1,2}(q^2_{\max})|$ are somewhat larger and these form factors increase with q^2 more rapidly in the present consideration than in expanded in $1/m_Q$ one. This observation confirms our expectations of the importance of the nonperturbative treatment of the semileptonic heavy-to-heavy baryon form factors.

V. HEAVY-TO-HEAVY AND HEAVY-TO-LIGHT SEMILEPTONIC Λ_b BARYON DECAYS

Now we can use the baryon form factors found in the previous section for the calculation of the Λ_b semileptonic decay rates. For obtaining the corresponding expressions for the decay rates in terms of form factors it is convenient to use the helicity formalism [20].

The helicity amplitudes are expressed in terms of the baryon form factors [20] as

$$
\begin{aligned}
H^{V,A}_{+1/2,0} &= \frac{1}{\sqrt{q^2}}\sqrt{2M_{\Lambda_Q}M_{\Lambda_q}(w\mp 1)}\Big[(M_{\Lambda_Q}\pm M_{\Lambda_q})\mathcal{F}^{V,A}_1(w)\pm M_{\Lambda_q}(w\pm 1)\mathcal{F}^{V,A}_2(w)\pm M_{\Lambda_Q}(w\pm 1)\mathcal{F}^{V,A}_3(w)\Big],\\
H^{V,A}_{+1/2,1} &= -2\sqrt{M_{\Lambda_Q}M_{\Lambda_q}(w\mp 1)}\mathcal{F}^{V,A}_1(w),\\
H^{V,A}_{+1/2,t} &= \frac{1}{\sqrt{q^2}}\sqrt{2M_{\Lambda_Q}M_{\Lambda_q}(w\pm 1)}\Big[(M_{\Lambda_Q}\mp M_{\Lambda_q})\mathcal{F}^{V,A}_1(w)\pm (M_{\Lambda_Q}-M_{\Lambda_q}w)\mathcal{F}^{V,A}_2(w)\pm (M_{\Lambda_Q}w-M_{\Lambda_q})\mathcal{F}^{V,A}_3(w)\Big],
\end{aligned}
\tag{18}
$$

where

$$
w=\frac{M^2_{\Lambda_Q}+M^2_{\Lambda_q}-q^2}{2M_{\Lambda_Q}M_{\Lambda_q}},
$$

the upper (lower) sign corresponds to V (A) and $\mathcal{F}^V_i \equiv F_i$, $\mathcal{F}^A_i \equiv G_i$ $(i=1,2,3)$. $H^{V,A}_{\lambda',\lambda_W}$ are the helicity amplitudes for weak transitions induced by vector (V) and axial-vector (A) currents, where λ' and λ_W are the helicities of the final

TABLE III. Comparison of theoretical predictions for the form factors of weak baryon decays at the maximum recoil point $q^2=0$.

	$f^V_1(0)$	$f^V_2(0)$	$f^V_3(0)$	$f^A_1(0)$	$f^A_2(0)$	$f^A_3(0)$
$\Lambda_b \to \Lambda_c$ this paper	0.526	0.137	0.075	0.505	−0.027	−0.252
[13]	0.549	0.110	−0.023	0.542	0.018	−0.123
[18]	0.5057	0.0994		0.5009	0.0089	
$\Lambda_b \to p$ this paper	0.169	0.050	0.029	0.196	−0.0002	−0.076
[14]	0.080	0.036	−0.005	0.077	−0.001	−0.046
[19]	$0.12^{+0.03}_{-0.04}$	$0.047^{+0.015}_{-0.013}$		$0.14^{+0.03}_{-0.03}$	$-0.016^{+0.007}_{-0.005}$	
[18]	0.1131	0.0356		0.112	0.0097	

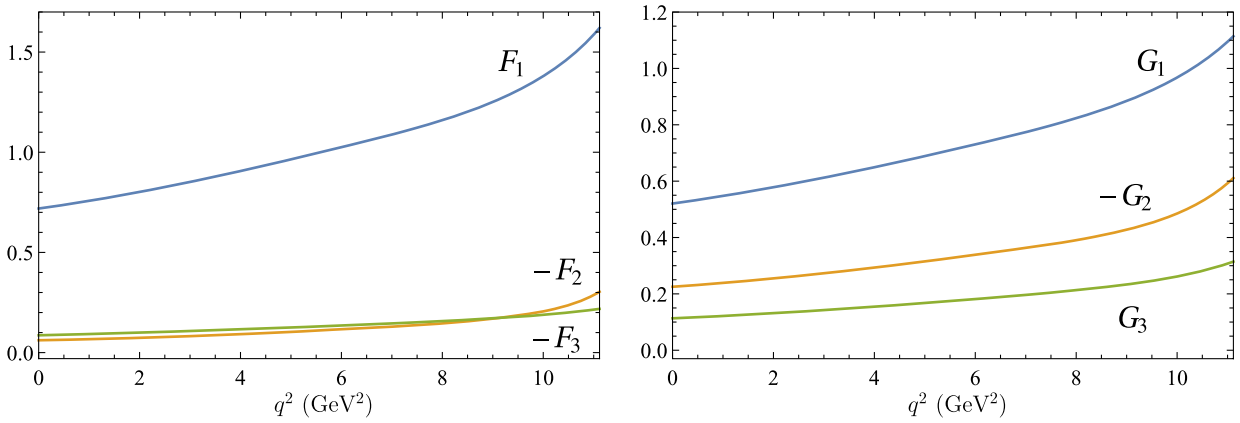

FIG. 3. Form factors of the weak $\Lambda_b \to \Lambda_c$ transition.

baryon and the virtual W boson, respectively. The amplitudes for negative values of the helicities can be obtained using the relation

$$H^{V,A}_{-\lambda',-\lambda_W} = \pm H^{V,A}_{\lambda',\lambda_W}.$$

The total helicity amplitude for the $V-A$ current is then given by

$$H_{\lambda',\lambda_W} = H^{V}_{\lambda',\lambda_W} - H^{A}_{\lambda',\lambda_W}. \tag{19}$$

Following Ref. [13] we write the twofold angular distribution for the decay $\Lambda_Q \to \Lambda_q W^-(\to \ell^-\bar{\nu}_\ell)$

$$\frac{d\Gamma(\Lambda_Q \to \Lambda_q \ell \bar{\nu}_\ell)}{dq^2 d\cos\theta} = \frac{G_F^2}{(2\pi)^3}|V_{qQ}|^2 \frac{\lambda^{1/2}(q^2-m_\ell^2)^2}{48 M^3_{\Lambda_Q} q^2} W(\theta, q^2), \tag{20}$$

with

$$W(\theta,q^2) = \frac{3}{8}\Big\{(1+\cos^2\theta)\mathcal{H}_U(q^2) - 2\cos\theta\mathcal{H}_P(q^2) + 2\sin^2\theta\mathcal{H}_L(q^2) + \frac{m_\ell^2}{q^2}(2\mathcal{H}_s(q^2) + \sin^2\theta\mathcal{H}_U(q^2) + 2\cos^2\theta\mathcal{H}_l(q^2) - 4\cos\theta\mathcal{H}_{SL}(q^2))\Big\}. \tag{21}$$

Here G_F is the Fermi constant, V_{qQ} is the CKM matrix element, $\lambda \equiv \lambda(M^2_{\Lambda_Q}, M^2_{\Lambda_q}, q^2) = M^4_{\Lambda_Q} + M^4_{\Lambda_q} + q^4 - 2(M^2_{\Lambda_Q}M^2_{\Lambda_q} + M^2_{\Lambda_q}q^2 + M^2_{\Lambda_Q}q^2)$, and m_ℓ is the lepton mass ($\ell = e, \mu, \tau$). θ is the angle between the lepton ℓ and W momenta.

The relevant parity-conserving helicity structures are expressed in terms of the total helicity amplitudes (19) by

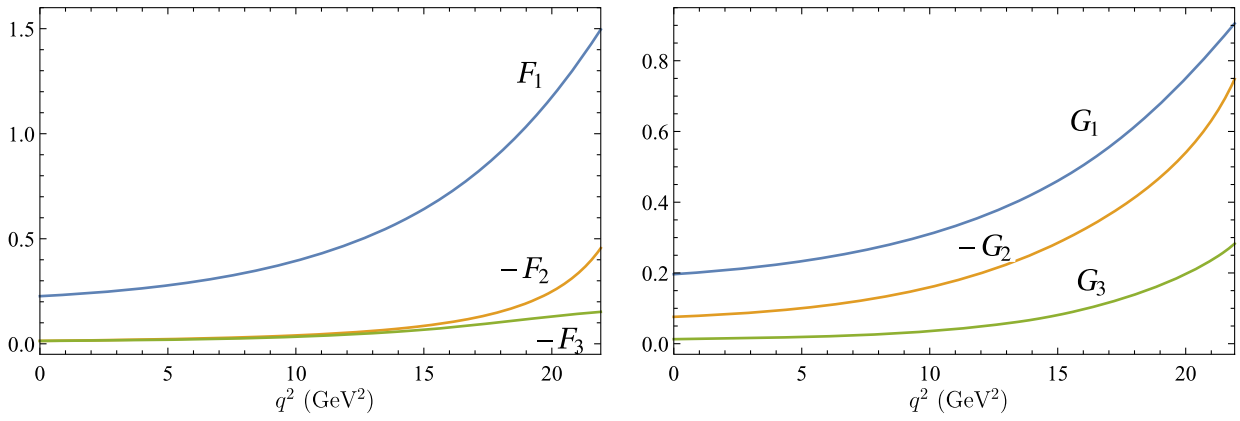

FIG. 4. Form factors of the weak $\Lambda_b \to p$ transition.

$$\mathcal{H}_U(q^2) = |H_{+1/2,+1}|^2 + |H_{-1/2,-1}|^2,$$
$$\mathcal{H}_L(q^2) = |H_{+1/2,0}|^2 + |H_{-1/2,0}|^2,$$
$$\mathcal{H}_S(q^2) = |H_{+1/2,t}|^2 + |H_{-1/2,t}|^2,$$
$$\mathcal{H}_{SL}(q^2) = \mathrm{Re}(H_{+1/2,0}H^\dagger_{+1/2,t} + H_{-1/2,0}H^\dagger_{-1/2,t}), \quad (22)$$

and the parity-violating helicity structures by

$$\mathcal{H}_P(q^2) = |H_{+1/2,+1}|^2 - |H_{-1/2,-1}|^2,$$
$$\mathcal{H}_{L_P}(q^2) = |H_{+1/2,0}|^2 - |H_{-1/2,0}|^2,$$
$$\mathcal{H}_{S_P}(q^2) = |H_{+1/2,t}|^2 - |H_{-1/2,t}|^2. \quad (23)$$

The differential decay rate is obtained by integrating Eq. (20) over $\cos\theta$ [14]

$$\frac{d\Gamma(\Lambda_Q \to \Lambda_q \ell \bar{\nu}_\ell)}{dq^2} = \frac{G_F^2}{(2\pi)^3}|V_{qQ}|^2 \frac{\lambda^{1/2}(q^2 - m_\ell^2)^2}{48 M_{\Lambda_Q}^3 q^2}\mathcal{H}_{\rm tot}(q^2), \quad (24)$$

where

$$\mathcal{H}_{\rm tot}(q^2) = [\mathcal{H}_U(q^2) + \mathcal{H}_L(q^2)]\left(1 + \frac{m_\ell^2}{2q^2}\right) + \frac{3m_\ell^2}{2q^2}\mathcal{H}_S(q^2). \quad (25)$$

Substituting the baryon form factors, calculated in our model in the previous section, into these expressions we obtain corresponding semileptonic differential decay rates which are plotted in Fig. 5.

Many important observables can also be expressed in terms of the helicity amplitudes [14]. The forward-backward asymmetry of the charged lepton is the term linear in $\cos\theta$ in the distribution (21) given by

$$A_{FB}(q^2) = \frac{\frac{d\Gamma}{dq^2}(\text{forward}) - \frac{d\Gamma}{dq^2}(\text{backward})}{\frac{d\Gamma}{dq^2}} = -\frac{3}{4}\frac{\mathcal{H}_P(q^2) + 2\frac{m_\ell^2}{q^2}\mathcal{H}_{SL}(q^2)}{\mathcal{H}_{\rm tot}(q^2)}. \quad (26)$$

The term quadratic in $\cos\theta$ in the distribution (21) is the convexity parameter defined by

$$C_F(q^2) = \frac{1}{\mathcal{H}_{\rm tot}(q^2)}\frac{d^2W(\theta, q^2)}{d(\cos\theta)^2} = \frac{3}{4}\left(1 - \frac{m_\ell^2}{q^2}\right)\frac{\mathcal{H}_U(q^2) - 2\mathcal{H}_L(q^2)}{\mathcal{H}_{\rm tot}(q^2)}. \quad (27)$$

The longitudinal polarization of the final baryon Λ_q reads as

$$P_L(q^2) = \frac{[\mathcal{H}_P(q^2) + \mathcal{H}_{L_P}(q^2)](1 + \frac{m_\ell^2}{2q^2}) + 3\frac{m_\ell^2}{2q^2}\mathcal{H}_{S_P}(q^2)}{\mathcal{H}_{\rm tot}(q^2)}. \quad (28)$$

The plots for these observables are given in Figs. 6–8 for both heavy-to-heavy $\Lambda_b \to \Lambda_c$ and heavy-to-light $\Lambda_b \to p$ semileptonic decays.

Integrating the differential decay rate (24) we get our predictions for the total decay rates and branching ratios which are given in Table IV. We estimate the errors of our calculations of the decay rates and branching fractions divided by the square of the corresponding CKM matrix element $|V_{qQ}|^2$, to be about 10%. For absolute values we use the CKM values $|V_{cb}| = (3.90 \pm 0.15) \times 10^{-2}$, $|V_{ub}| = (4.05 \pm 0.20) \times 10^{-3}$ extracted from our previous analysis of the heavy B and B_s meson decays [6]. In this table we also give our predictions for the average values of the forward-backward asymmetry of the charged lepton $\langle A_{FB}\rangle$, the convexity parameter $\langle C_F\rangle$ and the longitudinal polarization of the final baryon $\langle P_L\rangle$ which are calculated

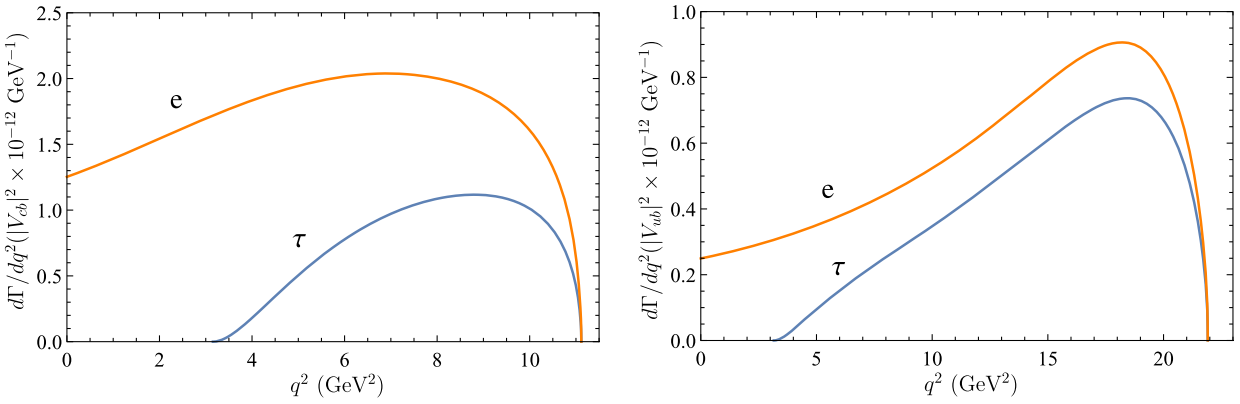

FIG. 5. Predictions for the differential decay rates of the $\Lambda_b \to \Lambda_c \ell \nu_\ell$ (left) and $\Lambda_b \to p \ell \nu_\ell$ (right) semileptonic decays.

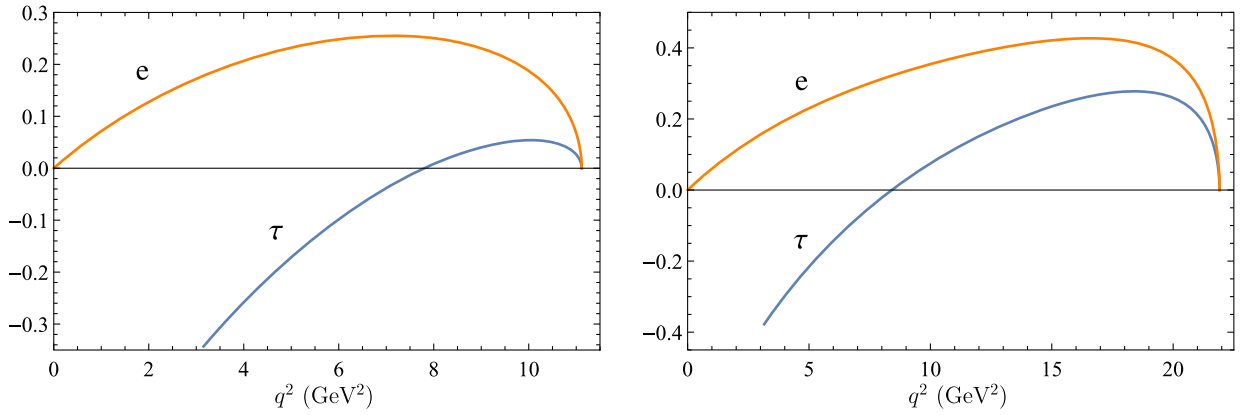

FIG. 6. Predictions for the forward-backward asymmetries $A_{FB}(q^2)$ in the $\Lambda_b \to \Lambda_c \ell^- \nu_\ell$ (left) and $\Lambda_b \to p \ell^- \nu_\ell$ (right) semileptonic decays.

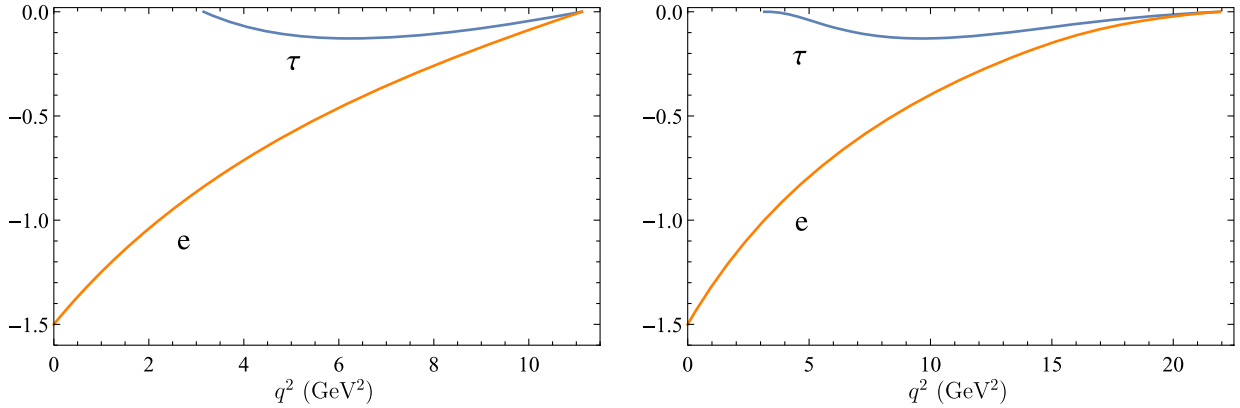

FIG. 7. Predictions for the convexity parameter $C_F(q^2)$ in the $\Lambda_b \to \Lambda_c \ell \nu_\ell$ (left) and $\Lambda_b \to p \ell \nu_\ell$ (right) semileptonic decays.

by separately integrating the numerators and denominators over q^2. Note that these quantities are less sensitive to the uncertainties in the form factor calculations since the errors partially cancel in the ratios of the helicity structures. We find the uncertainties of our predictions for them to be about 3–4%.

We compare our predictions with the results of other theoretical approaches [13,14,16,18,19,21,22] and available

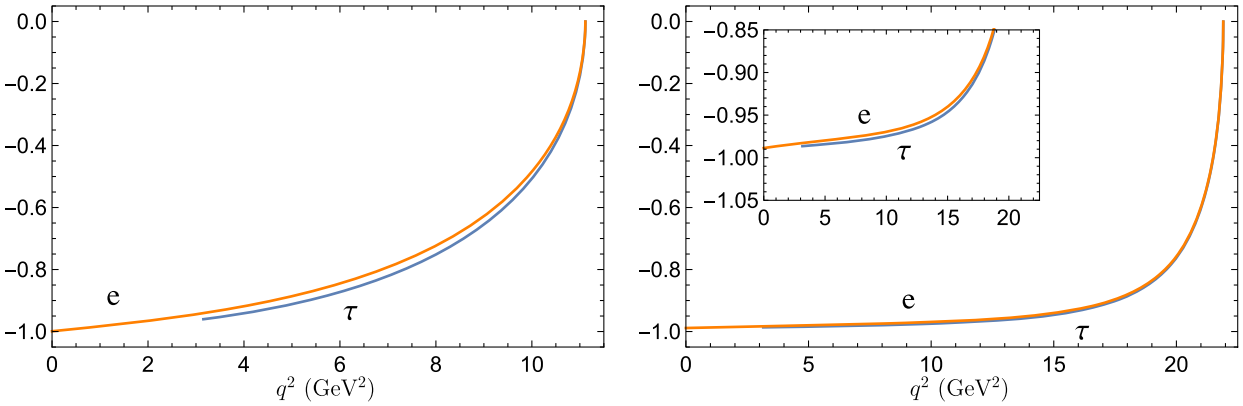

FIG. 8. Predictions for the longitudinal polarization $P_L(q^2)$ of the final baryon in the $\Lambda_b \to \Lambda_c \ell \nu_\ell$ (left) and $\Lambda_b \to p \ell \nu_\ell$ (right) semileptonic decays.

TABLE IV. Predictions for baryon decay rates, branching fractions and asymmetry parameters.

Decay	Γ (ns^{-1})	$\Gamma/\lvert V_{qQ}\rvert^2$ (ps^{-1})	Br (%)	Br$/\lvert V_{qQ}\rvert^2$	$\langle A_{FB}\rangle$	$\langle C_F\rangle$	$\langle P_L\rangle$
$\Lambda_b \to \Lambda_c e\nu_e$	44.2	29.1	6.48	42.6	0.195	−0.57	−0.80
$\Lambda_b \to \Lambda_c \mu\nu_\mu$	44.1	29.0	6.46	42.5	0.189	−0.55	−0.80
$\Lambda_b \to \Lambda_c \tau\nu_\tau$	13.9	9.11	2.03	13.4	−0.021	−0.09	−0.71
$\Lambda_b \to p e\nu_e$	0.306	18.7	0.045	27.4	0.346	−0.32	−0.91
$\Lambda_b \to p \mu\nu_\mu$	0.306	18.7	0.045	27.4	0.344	−0.32	−0.91
$\Lambda_b \to p \tau\nu_\tau$	0.199	12.1	0.029	17.8	−0.185	−0.09	−0.89

TABLE V. Comparison of theoretical predictions for the Λ_b semileptonic decay parameters with available experimental data.

Parameter	this paper	[13,14]	[16]	[21]	[18]	[19]	[22]	Exp. [1]
$\Lambda_b \to \Lambda_c l\nu_l$								
Γ (ns^{-1})	44.2		53.9					
$\Gamma/\lvert V_{cb}\rvert^2$ (ps^{-1})	29.1						$21.5 \pm 0.8 \pm 1.1$	
Br (%)	6.48	6.9		4.83	6.3			$6.2^{+1.4}_{-1.2}$
$\langle A_{FB}\rangle$	0.195	0.18						
$\langle C_F\rangle$	−0.57	−0.63						
$\langle P_L\rangle$	−0.80	−0.82						
$\Lambda_b \to \Lambda_c \tau\nu_\tau$								
Γ (ns^{-1})	13.9		20.9					
$\Gamma/\lvert V_{cb}\rvert^2$ (ps^{-1})	9.11						$7.15 \pm 0.15 \pm 0.27$	
Br (%)	2.03	2.0		1.63				
$\langle A_{FB}\rangle$	−0.021	−0.0385						
$\langle C_F\rangle$	−0.09	−0.10						
$\langle P_L\rangle$	−0.71	−0.72						
$\Lambda_b \to p l\nu_l$								
$\Gamma/\lvert V_{ub}\rvert^2$ (ps^{-1})	18.7	13.3	7.55				$25.7 \pm 2.6 \pm 4.6$	
Br (10^{-4})	4.5	2.9		3.89	2.54	$4.0^{+2.3}_{-2.0}$		
$\langle A_{FB}\rangle$	0.346	0.388						
$\Lambda_b \to p \tau\nu_\tau$								
$\Gamma/\lvert V_{ub}\rvert^2$ (ps^{-1})	12.1	9.6	6.55				$17.7 \pm 1.3 \pm 1.6$	
Br (10^{-4})	2.9	2.1		2.75				
$\langle A_{FB}\rangle$	0.185	0.220						

experimental data [1] in Table V.[1] The most comprehensive results for different decay parameters were previously obtained in the covariant confined quark (CCQ) model [13,14], with which we find the general agreement. The semirelativistic quark model was used in Ref. [16], while the effective Lagrangian approach with form factors calculated on the lattice [22] was employed in Ref. [21]. The authors of Refs. [18,19] made calculations in the light-front quark model and in QCD light-cone sum rules, respectively. The only experimental data are available for the branching ratio of the $\Lambda_b \to \Lambda_c^+ l^- \bar{\nu}_l$ decay ($l = e, \mu$). All theoretical predictions agree well with data within error bars. However, note that lattice calculations [22] give somewhat lower predictions for the branching ratios normalized by the square of the corresponding CKM matrix element for $\Lambda_b \to \Lambda_c$ transitions but give higher results for $\Lambda_b \to p$ transitions than other approaches.

At present the tension between predictions of the Standard Model and experimental data in the B meson sector is observed for the ratio of branching ratios of semileptonic B decays to $D^{(*)}$ mesons involving τ and a muon or electron [1,2]. Therefore it is very important to

[1]We limit our comparison to the recent results only. References to previous predictions and comparison with them can be found, e.g., in Refs. [13,14].

TABLE VI. Predictions for baryon decay rates, branching fractions and asymmetry parameters.

Ratio	this paper	[23]	[21]	[22]	Experiment (LHCb) [4]
R_{Λ_c}	0.313	0.29 ± 0.02	0.3379	$0.3318 \pm 0.0074 \pm 0.0070$	
R_p	0.649		0.7071		
$R_{\Lambda_c p}$	$(0.78 \pm 0.08)\frac{\lvert V_{ub}\rvert^2}{\lvert V_{cb}\rvert^2}$		0.0101	$(1.471 \pm 0.095 \pm 0.109)\frac{\lvert V_{ub}\rvert^2}{\lvert V_{cb}\rvert^2}$	$(1.00 \pm 0.04 \pm 0.08) \times 10^{-2}$

search for the similar decays in the baryon sector. We can define the following ratios of the Λ_b baryon branching fractions:

$$R_{\Lambda_c} = \frac{\mathrm{Br}(\Lambda_b \to \Lambda_c \tau \nu_\tau)}{\mathrm{Br}(\Lambda_b \to \Lambda_c l \nu_l)},$$
$$R_p = \frac{\mathrm{Br}(\Lambda_b \to p \tau \nu_\tau)}{\mathrm{Br}(\Lambda_b \to p l \nu_l)}. \quad (29)$$

Our predictions for these ratios are given in Table VI in comparison with calculations [23] using the QCD sum rule form factors and estimates [21] based on lattice values of weak decay form factors [22]. Results of predictions for R_{Λ_c} are in good agreement, while our R_p value is slightly lower than the Ref. [21] estimate. Note that the lattice determination of form factors is done in the region of small recoils of the final baryon $q^2 \sim q^2_{\mathrm{max}}$ and then their values are extrapolated to the whole kinematical region, which is broad especially for the heavy-to-light $\Lambda_b \to p l \nu_l$ decay. In our model we explicitly determine the form factor q^2 dependence in the whole kinematical range without extrapolations. The possible contributions of new physics to these ratios were analyzed in detail in Refs. [21,23].

Recently the LHCb Collaboration [4] measured the ratio of the heavy-to-heavy and heavy-to-light semileptonic Λ_b decays in the limited interval of q^2

$$R_{\Lambda_c p} = \frac{\int_{15\ \mathrm{GeV}^2}^{q^2_{\mathrm{max}}} \frac{d\Gamma(\Lambda_b \to p \mu \nu_\mu)}{dq^2} dq^2}{\int_{7\ \mathrm{GeV}^2}^{q^2_{\mathrm{max}}} \frac{d\Gamma(\Lambda_b \to \Lambda_c \mu \nu_\mu)}{dq^2} dq^2}. \quad (30)$$

Such a measurement is very important since it allows us for the first time to extract the ratio of CKM matrix elements $|V_{ub}|/|V_{cb}|$ from the Λ_b baryon decays and compare it to the corresponding ratio determined from B and B_s meson decays. Our prediction for the ratio $R_{\Lambda_c p}$ in comparison with the lattice result [22] and experimental value is given in Table VI. From this table we see that our value of the coefficient in front of $|V_{ub}|^2/|V_{cb}|^2$ is significantly lower than the lattice one. This is the result of the above-mentioned deviation of our calculation (and other quark model calculations) from lattice predictions for the values for heavy-to-heavy and heavy-to-light semileptonic Λ_b decays normalized by the square of the CKM matrix element. This deviation even increases in the ratio.

Comparing our result for $R_{\Lambda_c p}$ with experimental data [4] we can extract the ratio of the CKM matrix elements. Using our model value we find

$$\frac{|V_{ub}|}{|V_{cb}|} = 0.113 \pm 0.011|_{\mathrm{theor}} \pm 0.006|_{\mathrm{exp}}, \quad (31)$$

which is in good agreement with the experimental ratio of these matrix elements extracted from inclusive decays [1]

$$\frac{|V_{ub}|_{\mathrm{incl}}}{|V_{cb}|_{\mathrm{incl}}} = 0.105 \pm 0.006, \quad (32)$$

and with the corresponding ratio found in our previous analysis of exclusive semileptonic B and B_s meson decays $[|V_{cb}| = (3.90 \pm 0.15) \times 10^{-2}$, $|V_{ub}| = (4.05 \pm 0.20) \times 10^{-3}]$ [6]

$$\frac{|V_{ub}|}{|V_{cb}|} = 0.104 \pm 0.012. \quad (33)$$

On the other hand, the lattice value for the ratio $R_{\Lambda_c p}$ gives

$$\frac{|V_{ub}|}{|V_{cb}|} = 0.083 \pm 0.004 \pm 0.004, \quad (34)$$

which is in agreement with the corresponding CKM matrix element ratio extracted from the comparison of lattice predictions with data on exclusive B meson decays, but more than 3σ lower than the ratio extracted from inclusive B meson decays [Eq. (32)].

VI. CONCLUSIONS

The semileptonic Λ_b baryon decays were investigated in the framework of the relativistic quark model based on the quasipotential approach and quantum chromodynamics. All parameters of the model had been previously determined from the consideration of meson properties and were kept fixed in the current consideration of the baryon semileptonic decays. The relativistic quark-diquark picture was used for the calculations. The semileptonic decay form factors were obtained both for the heavy-to-heavy and heavy-to-light Λ_b decays. They were expressed as the overlap integrals of the relativistic baryon wave functions which are known from the baryon mass spectra

calculations. All form factors were obtained without employing nonrelativistic or heavy quark expansions. Their momentum transfer dependence was explicitly determined in the whole accessible kinematical range without any extrapolations. All relativistic effects including contributions of the intermediate negative-energy states and relativistic transformations of the wave functions were consistently taken into account.

The helicity formalism was employed for calculating the $\Lambda_b \to \Lambda_c l\nu_l$, $\Lambda_b \to \Lambda_c \tau\nu_\tau$ and $\Lambda_b \to p l\nu_l$, $\Lambda_b \to p\tau\nu_\tau$ decay rates and branching fractions. Different additional observables such as the forward-backward asymmetry A_{FB}, convexity parameter C_F and final baryon polarization P_L were also determined. The obtained results were compared with previous theoretical calculations within significantly different approaches including quark model calculations, QCD light-cone sum rules, lattice simulations [13,14,16,18,19,21,22] and available experimental data [1,4]. Most of our results agree well with the ones obtained within the CCQ model [13,14].

Our value of the branching ratio of semileptonic $\Lambda_b \to \Lambda_c l\nu_l$ decay is in good agreement with the experimental measurement [1]. From the recent LHCb data [4] on the ratio $R_{\Lambda_c p}$ of semileptonic $\Lambda_b \to p\mu\nu_\mu$ to $\Lambda_b \to \Lambda_c \mu\nu_\mu$ decay rates in the constrained momentum transfer q^2 range (30) we find the ratio of the CKM matrix elements $|V_{ub}|/|V_{cb}|$ consistent within error bars with the corresponding ratio determined from inclusive B meson decays [1] and with the one previously obtained from the analysis of the exclusive B and B_s decays in our model [6].

We plan to apply the same approach within our model to the investigation of semileptonic Λ_c decays, rare semileptonic Λ_b decays as well as nonleptonic baryon decays within the factorization approximation.

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APPENDIX: FORM FACTORS OF WEAK $\Lambda_Q \to \Lambda_q$ TRANSITIONS

The final expressions for decay form factors are as follows (the long-range anomalous chromomagnetic quark moment $\kappa = -1$).

1. Vector form factors

$$F_1(q^2) = F_1^{(1)}(q^2) + \varepsilon F_1^{(2)S}(q^2) + (1-\varepsilon) F_1^{(2)V}(q^2), \tag{A1}$$

$$\begin{aligned} F_1^{(1)}(q^2) = \int \frac{d^3p}{(2\pi)^3} \bar{\Psi}_F\left(\mathbf{p} + \frac{2\epsilon_d}{E_F + M_F}\Delta\right) \sqrt{\frac{\epsilon_Q(p)+m_Q}{2\epsilon_Q(p)}} \sqrt{\frac{\epsilon_q(p+\Delta)+m_q}{2\epsilon_q(p+\Delta)}} \\ \times \left\{ 1 + \frac{\epsilon_d}{\epsilon_q(p+\Delta)+m_q}\left[1 + \frac{\epsilon_d}{\epsilon_Q(p)+m_Q}\frac{E_F - M_F}{E_F + M_F}\right] + \frac{\epsilon_d}{\epsilon_Q(p)+m_Q} \right. \\ - \frac{1}{3}\frac{\mathbf{p}^2}{(\epsilon_q(p+\Delta)+m_q)(\epsilon_Q(p)+m_Q)} - \frac{\mathbf{p}\Delta}{E_F+M_F}\left[\frac{1}{\epsilon_q(p+\Delta)+m_q} \right. \\ \left. \left. - \frac{1}{\epsilon_Q(p)+m_Q} + \frac{2M_F}{E_F+M_F}\frac{\epsilon_d}{(\epsilon_q(p+\Delta)+m_q)(\epsilon_Q(p)+m_Q)}\right]\right\} \Psi_I(\mathbf{p}), \end{aligned} \tag{A2}$$

$$\begin{aligned} F_1^{(2)S}(q^2) = -\int \frac{d^3p}{(2\pi)^3} \bar{\Psi}_F\left(\mathbf{p} + \frac{2\epsilon_d}{E_F + M_F}\Delta\right) \sqrt{\frac{\epsilon_Q(p)+m_Q}{2\epsilon_Q(p)}} \sqrt{\frac{\epsilon_q(\Delta)+m_q}{2\epsilon_q(\Delta)}} \\ \times \left\{ \frac{1}{2\epsilon_Q(\Delta)(\epsilon_Q(\Delta)+m_Q)}\left[\epsilon_Q(\Delta) - m_Q + (E_F - M_F)\left(1 - \frac{\epsilon_d}{E_F+M_F}\right) \right. \right. \\ \left. - \frac{\mathbf{p}\Delta}{E_F+M_F}\right][M_I - \epsilon_Q(p) - \epsilon_d(p)] + \frac{1}{2\epsilon_q(\Delta)(\epsilon_q(\Delta)+m_q)}\left[\epsilon_q(\Delta) - m_q \right. \\ + (E_F - M_F)\left(1 - \frac{\epsilon_d}{E_F+M_F}\right) + \frac{\mathbf{p}\Delta}{E_F+M_F}\left]\left[M_F - \epsilon_q\left(\mathbf{p} + \frac{2\epsilon_d}{E_F+M_F}\Delta\right)\right.\right. \\ \left. \left. \left. - \epsilon_d\left(\mathbf{p} + \frac{2\epsilon_d}{E_F+M_F}\Delta\right)\right]\right\} \Psi_I(\mathbf{p}), \end{aligned} \tag{A3}$$

$$F_1^{(2)V}(q^2) = -\int \frac{d^3p}{(2\pi)^3}\bar{\Psi}_F\left(\mathbf{p}+\frac{2\epsilon_d}{E_F+M_F}\Delta\right)\sqrt{\frac{\epsilon_Q(p)+m_Q}{2\epsilon_Q(p)}}\sqrt{\frac{\epsilon_q(\Delta)+m_q}{2\epsilon_q(\Delta)}}$$
$$\times\left\{\frac{1}{2\epsilon_Q(\Delta)(\epsilon_Q(\Delta)+m_Q)}\left(\left[\epsilon_Q(\Delta)-m_Q+(E_F-M_F)\left(1-\frac{\epsilon_d}{E_F+M_F}\right)\right]\right.\right.$$
$$\times\left[1-\frac{\epsilon_d}{m_Q}\frac{E_F-M_F}{E_F+M_F}-\frac{\mathbf{p}\Delta}{E_d(E_F+M_F)}\right]-\left.\frac{\mathbf{p}\Delta}{E_F+M_F}\right)[M_I-\epsilon_Q(p)-\epsilon_d(p)]$$
$$+\frac{1}{2\epsilon_q(\Delta)(\epsilon_q(\Delta)+m_q)}\left(\left[\epsilon_q(\Delta)-m_q+(E_F-M_F)\left(1-\frac{\epsilon_d}{E_F+M_F}\right)\right]\right.$$
$$\times\left[1-\frac{\epsilon_d}{m_q}\frac{E_F-M_F}{E_F+M_F}-\frac{\mathbf{p}\Delta}{E_d(E_F+M_F)}\right]-\left.\frac{\mathbf{p}\Delta}{E_F+M_F}\right)$$
$$\times\left.\left[M_F-\epsilon_q\left(\mathbf{p}+\frac{2\epsilon_d}{E_F+M_F}\Delta\right)-\epsilon_d\left(\mathbf{p}+\frac{2\epsilon_d}{E_F+M_F}\Delta\right)\right]\right\}\Psi_I(\mathbf{p}), \tag{A4}$$

$$F_2(q^2) = F_2^{(1)}(q^2)+\varepsilon F_2^{(2)S}(q^2)+(1-\varepsilon)F_2^{(2)V}(q^2), \tag{A5}$$

$$F_2^{(1)}(q^2) = -\int \frac{d^3p}{(2\pi)^3}\bar{\Psi}_F\left(\mathbf{p}+\frac{2\epsilon_d}{E_F+M_F}\Delta\right)\sqrt{\frac{\epsilon_Q(p)+m_Q}{2\epsilon_Q(p)}}\sqrt{\frac{\epsilon_q(p+\Delta)+m_q}{2\epsilon_q(p+\Delta)}}\frac{2M_F}{E_F+M_F}$$
$$\times\left\{\frac{\epsilon_d}{\epsilon_q(p+\Delta)+m_q}\left[1+\frac{\epsilon_d}{\epsilon_Q(p)+m_Q}\frac{E_F-M_F}{E_F+M_F}\right]-\frac{2}{3}\frac{\mathbf{p}^2}{(\epsilon_q(p+\Delta)+m_q)(\epsilon_Q(p)+m_Q)}\right.$$
$$-\frac{\mathbf{p}\Delta}{M_F(\epsilon_q(p+\Delta)+m_q)}\left[1-\frac{1}{2}\frac{\epsilon_d}{\epsilon_Q(p)+m_Q}\frac{E_F-M_F}{E_F+M_F}\right]+\frac{\mathbf{p}\Delta}{\Delta^2}\frac{E_F}{M_F}\frac{E_F+M_F}{\epsilon_Q(p)+m_Q}$$
$$\times\left.\left[1+\frac{\epsilon_d}{\epsilon_q(p+\Delta)+m_q}\frac{E_F-M_F}{E_F+M_F}\right]\right\}\Psi_I(\mathbf{p}), \tag{A6}$$

$$F_2^{(2)S}(q^2) = -\int \frac{d^3p}{(2\pi)^3}\bar{\Psi}_F\left(\mathbf{p}+\frac{2\epsilon_d}{E_F+M_F}\Delta\right)\sqrt{\frac{\epsilon_Q(p)+m_Q}{2\epsilon_Q(p)}}\sqrt{\frac{\epsilon_q(\Delta)+m_q}{2\epsilon_q(\Delta)}}\frac{\mathbf{p}\Delta}{\Delta^2}\frac{E_F}{2\epsilon_Q(\Delta)(\epsilon_Q(\Delta)+m_Q)}$$
$$\times\left[M_I-\epsilon_Q(p)-\epsilon_d(p)+M_F-\epsilon_q\left(\mathbf{p}+\frac{2\epsilon_d}{E_F+M_F}\Delta\right)-\epsilon_d\left(\mathbf{p}+\frac{2\epsilon_d}{E_F+M_F}\Delta\right)\right]\Psi_I(\mathbf{p}), \tag{A7}$$

$$F_2^{(2)V}(q^2) = -\int \frac{d^3p}{(2\pi)^3}\bar{\Psi}_F\left(\mathbf{p}+\frac{2\epsilon_d}{E_F+M_F}\Delta\right)\sqrt{\frac{\epsilon_Q(p)+m_Q}{2\epsilon_Q(p)}}\sqrt{\frac{\epsilon_q(\Delta)+m_q}{2\epsilon_q(\Delta)}}$$
$$\times\left\{\frac{\mathbf{p}\Delta}{\Delta^2}\frac{E_F}{2\epsilon_Q(\Delta)(\epsilon_Q(\Delta)+m_Q)}\left[M_I-\epsilon_Q(p)-\epsilon_d(p)+M_F-\epsilon_q\left(\mathbf{p}+\frac{2\epsilon_d}{E_F+M_F}\Delta\right)-\epsilon_d\left(\mathbf{p}+\frac{2\epsilon_d}{E_F+M_F}\Delta\right)\right]\right.$$
$$+\frac{E_F-M_F}{E_F+M_F}\left(\frac{\epsilon_d}{m_Q}\frac{\epsilon_Q(\Delta)-m_Q+(E_F-M_F)(1-\frac{\epsilon_d}{E_F+M_F})}{\epsilon_Q(\Delta)(\epsilon_Q(\Delta)+m_Q)}[M_I-\epsilon_Q(p)-\epsilon_d(p)]\right.$$
$$+\frac{\epsilon_d}{m_q}\frac{\epsilon_q(\Delta)-m_q+(E_F-M_F)(1-\frac{\epsilon_d}{E_F+M_F})}{\epsilon_q(\Delta)(\epsilon_q(\Delta)+m_q)}\left[M_F-\epsilon_q\left(\mathbf{p}+\frac{2\epsilon_d}{E_F+M_F}\Delta\right)\right.$$
$$\left.\left.\left.-\epsilon_d\left(\mathbf{p}+\frac{2\epsilon_d}{E_F+M_F}\Delta\right)\right]\right\}\Psi_I(\mathbf{p}), \tag{A8}$$

$$F_3(q^2) = F_3^{(1)}(q^2)+\varepsilon F_3^{(2)S}(q^2)+(1-\varepsilon)F_3^{(2)V}(q^2), \tag{A9}$$

$$F_3^{(1)}(q^2) = -\int \frac{d^3p}{(2\pi)^3}\bar{\Psi}_F\left(\mathbf{p}+\frac{2\epsilon_d}{E_F+M_F}\Delta\right)\sqrt{\frac{\epsilon_Q(p)+m_Q}{2\epsilon_Q(p)}}\sqrt{\frac{\epsilon_q(p+\Delta)+m_q}{2\epsilon_q(p+\Delta)}}\frac{2M_F}{E_F+M_F}$$
$$\times\left\{\frac{\epsilon_d}{\epsilon_Q(p)+m_Q}\left[1+\frac{\epsilon_d}{\epsilon_q(p+\Delta)+m_q}\frac{E_F-M_F}{E_F+M_F}+\frac{2\mathbf{p}\Delta}{(E_F+M_F)(\epsilon_q(p+\Delta)+m_q)}\right]\right.$$
$$\left.-\frac{2}{3}\frac{\mathbf{p}^2}{(\epsilon_q(p+\Delta)+m_q)(\epsilon_Q(p)+m_Q)}-\frac{\mathbf{p}\Delta}{\Delta^2}\frac{E_F+M_F}{\epsilon_Q(p)+m_Q}\left[1+\frac{\epsilon_d}{\epsilon_q(p+\Delta)+m_q}\frac{E_F-M_F}{E_F+M_F}\right]\right\}\Psi_I(\mathbf{p}), \quad \text{(A10)}$$

$$F_3^{(2)S}(q^2) = F_3^{(2)V}(q^2)$$
$$= \int \frac{d^3p}{(2\pi)^3}\bar{\Psi}_F\left(\mathbf{p}+\frac{2\epsilon_d}{E_F+M_F}\Delta\right)\sqrt{\frac{\epsilon_Q(p)+m_Q}{2\epsilon_Q(p)}}\sqrt{\frac{\epsilon_q(\Delta)+m_q}{2\epsilon_q(\Delta)}}\frac{\mathbf{p}\Delta}{\Delta^2}\frac{M_F}{2\epsilon_Q(\Delta)(\epsilon_Q(\Delta)+m_Q)}$$
$$\times\left[M_I-\epsilon_Q(p)-\epsilon_d(p)+M_F-\epsilon_q\left(\mathbf{p}+\frac{2\epsilon_d}{E_F+M_F}\Delta\right)-\epsilon_d\left(\mathbf{p}+\frac{2\epsilon_d}{E_F+M_F}\Delta\right)\right]\Psi_I(\mathbf{p}). \quad \text{(A11)}$$

2. Axial-vector form factors

$$G_1(q^2) = G_1^{(1)}(q^2)+\varepsilon G_1^{(2)S}(q^2)+(1-\varepsilon)G_1^{(2)V}(q^2), \quad \text{(A12)}$$

$$G_1^{(1)}(q^2) = \int \frac{d^3p}{(2\pi)^3}\bar{\Psi}_F\left(\mathbf{p}+\frac{2\epsilon_d}{E_F+M_F}\Delta\right)\sqrt{\frac{\epsilon_Q(p)+m_Q}{2\epsilon_Q(p)}}\sqrt{\frac{\epsilon_q(p+\Delta)+m_q}{2\epsilon_q(p+\Delta)}}$$
$$\times\left\{1+\frac{E_F-M_F}{E_F+M_F}\left(\frac{\epsilon_d}{\epsilon_q(p+\Delta)+m_q}\left[1+\frac{\epsilon_d}{\epsilon_Q(p)+m_Q}\right]+\frac{\epsilon_d}{\epsilon_Q(p)+m_Q}\right)\right.$$
$$-\frac{1}{3}\frac{\mathbf{p}^2}{(\epsilon_q(p+\Delta)+m_q)(\epsilon_Q(p)+m_Q)}+\frac{\mathbf{p}\Delta}{E_F+M_F}\left[\frac{1}{\epsilon_q(p+\Delta)+m_q}\right.$$
$$\left.\left.-\frac{1}{\epsilon_Q(p)+m_Q}-\frac{2\epsilon_d}{(\epsilon_q(p+\Delta)+m_q)(\epsilon_Q(p)+m_Q)}\right]\right\}\Psi_I(\mathbf{p}), \quad \text{(A13)}$$

$$G_1^{(2)S}(q^2) = -\int \frac{d^3p}{(2\pi)^3}\bar{\Psi}_F\left(\mathbf{p}+\frac{2\epsilon_d}{E_F+M_F}\Delta\right)\sqrt{\frac{\epsilon_Q(p)+m_Q}{2\epsilon_Q(p)}}\sqrt{\frac{\epsilon_q(\Delta)+m_q}{2\epsilon_q(\Delta)}}$$
$$\times\left\{\frac{1}{2\epsilon_Q(\Delta)(\epsilon_Q(\Delta)+m_Q)}\left[\epsilon_Q(\Delta)-m_Q+(E_F-M_F)\left(1-\frac{\epsilon_d}{E_F+M_F}\right)+\frac{\mathbf{p}\Delta}{E_F+M_F}\right][M_I-\epsilon_Q(p)-\epsilon_d(p)]\right.$$
$$+\frac{1}{2\epsilon_q(\Delta)(\epsilon_q(\Delta)+m_q)}\left[\epsilon_q(\Delta)-m_q+(E_F-M_F)\left(1-\frac{\epsilon_d}{E_F+M_F}\right)+\frac{\mathbf{p}\Delta}{E_F+M_F}\right]$$
$$\left.\times\left[M_F-\epsilon_q\left(\mathbf{p}+\frac{2\epsilon_d}{E_F+M_F}\Delta\right)-\epsilon_d\left(\mathbf{p}+\frac{2\epsilon_d}{E_F+M_F}\Delta\right)\right]\right\}\Psi_I(\mathbf{p}), \quad \text{(A14)}$$

$$
\begin{aligned}
G_1^{(2)V}(q^2) = &-\int \frac{d^3p}{(2\pi)^3}\bar{\Psi}_F\left(\mathbf{p}+\frac{2\epsilon_d}{E_F+M_F}\Delta\right)\sqrt{\frac{\epsilon_Q(p)+m_Q}{2\epsilon_Q(p)}}\sqrt{\frac{\epsilon_q(\Delta)+m_q}{2\epsilon_q(\Delta)}}\\
&\times\left\{\frac{1}{2\epsilon_Q(\Delta)(\epsilon_Q(\Delta)+m_Q)}\left(\left[\epsilon_Q(\Delta)-m_Q+(E_F-M_F)\left(1-\frac{\epsilon_d}{E_F+M_F}\right)\right]\right.\right.\\
&\times\left[1+\frac{\epsilon_d}{m_Q}\frac{E_F-M_F}{E_F+M_F}-\frac{\mathbf{p}\Delta}{E_d(E_F+M_F)}\right]+\left.\frac{\mathbf{p}\Delta}{E_F+M_F}\right)[M_I-\epsilon_Q(p)-\epsilon_d(p)]\\
&+\frac{1}{2\epsilon_q(\Delta)(\epsilon_q(\Delta)+m_q)}\left(\left[\epsilon_q(\Delta)-m_q+(E_F-M_F)\left(1-\frac{\epsilon_d}{E_F+M_F}\right)\right]\right.\\
&\times\left[1-\frac{\epsilon_d}{m_q}\frac{E_F-M_F}{E_F+M_F}-\frac{\mathbf{p}\Delta}{E_d(E_F+M_F)}\right]-\left.\frac{\mathbf{p}\Delta}{E_F+M_F}\right)\\
&\times\left.\left[M_F-\epsilon_q\left(\mathbf{p}+\frac{2\epsilon_d}{E_F+M_F}\Delta\right)-\epsilon_d\left(\mathbf{p}+\frac{2\epsilon_d}{E_F+M_F}\Delta\right)\right]\right\}\Psi_I(\mathbf{p}),
\end{aligned}
\tag{A15}
$$

$$
G_2(q^2) = G_2^{(1)}(q^2)+\varepsilon G_2^{(2)S}(q^2)+(1-\varepsilon)G_2^{(2)V}(q^2),
\tag{A16}
$$

$$
\begin{aligned}
G_2^{(1)}(q^2) = &-\int \frac{d^3p}{(2\pi)^3}\bar{\Psi}_F\left(\mathbf{p}+\frac{2\epsilon_d}{E_F+M_F}\Delta\right)\sqrt{\frac{\epsilon_Q(p)+m_Q}{2\epsilon_Q(p)}}\sqrt{\frac{\epsilon_q(p+\Delta)+m_q}{2\epsilon_q(p+\Delta)}}\frac{2M_F}{E_F+M_F}\\
&\times\left\{\frac{\epsilon_d}{\epsilon_q(p+\Delta)+m_q}\left[1+\frac{\epsilon_d}{\epsilon_Q(p)+m_Q}\right]\right.\\
&-\frac{\mathbf{p}\Delta}{M_F(\epsilon_q(p+\Delta)+m_q)}\left[1-\frac{\epsilon_d}{\epsilon_Q(p)+m_Q}\frac{M_F}{E_F+M_F}\right]\\
&-\left.\frac{\mathbf{p}\Delta}{\Delta^2}\frac{E_F}{M_F}\frac{E_F+M_F}{\epsilon_Q(p)+m_Q}\left[1+\frac{\epsilon_d}{\epsilon_q(p+\Delta)+m_q}\right]\right\}\Psi_I(\mathbf{p}),
\end{aligned}
\tag{A17}
$$

$$
\begin{aligned}
G_2^{(2)S}(q^2) = &-\int \frac{d^3p}{(2\pi)^3}\bar{\Psi}_F\left(\mathbf{p}+\frac{2\epsilon_d}{E_F+M_F}\Delta\right)\sqrt{\frac{\epsilon_Q(p)+m_Q}{2\epsilon_Q(p)}}\sqrt{\frac{\epsilon_q(\Delta)+m_q}{2\epsilon_q(\Delta)}}\\
&\times\left\{\frac{\mathbf{p}\Delta}{E_F+M_F}\left(\frac{1}{\epsilon_Q(\Delta)(\epsilon_Q(\Delta)+m_Q)}[M_I-\epsilon_Q(p)-\epsilon_d(p)]\right.\right.\\
&+\left.\left.\frac{1}{\epsilon_q(\Delta)(\epsilon_q(\Delta)+m_q)}\left[M_F-\epsilon_q\left(\mathbf{p}+\frac{2\epsilon_d}{E_F+M_F}\Delta\right)-\epsilon_d\left(\mathbf{p}+\frac{2\epsilon_d}{E_F+M_F}\Delta\right)\right]\right)\right.\\
&-\frac{\mathbf{p}\Delta}{\Delta^2}\frac{E_F}{2\epsilon_Q(\Delta)(\epsilon_Q(\Delta)+m_Q)}\left[M_I-\epsilon_Q(p)-\epsilon_d(p)\right.\\
&+\left.M_F-\epsilon_q\left(\mathbf{p}+\frac{2\epsilon_d}{E_F+M_F}\Delta\right)-\epsilon_d\left(\mathbf{p}+\frac{2\epsilon_d}{E_F+M_F}\Delta\right)\right]\Psi_I(\mathbf{p}),
\end{aligned}
\tag{A18}
$$

$$G_2^{(2)V}(q^2) = -\int \frac{d^3p}{(2\pi)^3}\bar{\Psi}_F\left(\mathbf{p}+\frac{2\epsilon_d}{E_F+M_F}\Delta\right)\sqrt{\frac{\epsilon_Q(p)+m_Q}{2\epsilon_Q(p)}}\sqrt{\frac{\epsilon_q(\Delta)+m_q}{2\epsilon_q(\Delta)}}$$
$$\times\left\{\frac{1}{\epsilon_Q(\Delta)(\epsilon_Q(\Delta)+m_Q)}\left(\left[\epsilon_Q(\Delta)-m_Q+(E_F-M_F)\left(1-\frac{\epsilon_d}{E_F+M_F}\right)\right]\right.\right.$$
$$\times\left[\frac{\mathbf{p}\Delta}{E_d(E_F+M_F)}-\frac{\epsilon_d}{m_Q}\frac{E_F-M_F}{E_F+M_F}\right]-\left.\frac{\mathbf{p}\Delta}{E_F+M_F}\right)[M_I-\epsilon_Q(p)-\epsilon_d(p)]$$
$$+\frac{1}{\epsilon_q(\Delta)(\epsilon_q(\Delta)+m_q)}\left(\left[\epsilon_q(\Delta)-m_q-(E_F-M_F)\left(1-\frac{\epsilon_d}{E_F+M_F}\right)\right]\right.$$
$$\times\left[-\frac{\mathbf{p}\Delta}{E_d(E_F+M_F)}+\frac{\epsilon_d}{m_q}\frac{E_F-M_F}{E_F+M_F}\right]+\left.\frac{\mathbf{p}\Delta}{E_F+M_F}\right)\left[M_F-\epsilon_q\left(\mathbf{p}+\frac{2\epsilon_d}{E_F+M_F}\Delta\right)\right.$$
$$\left.-\epsilon_d\left(\mathbf{p}+\frac{2\epsilon_d}{E_F+M_F}\Delta\right)\right]-\frac{\mathbf{p}\Delta}{\Delta^2}\frac{E_F}{2\epsilon_Q(\Delta)(\epsilon_Q(\Delta)+m_Q)}\left[M_I-\epsilon_Q(p)-\epsilon_d(p)\right.$$
$$\left.\left.+M_F-\epsilon_q\left(\mathbf{p}+\frac{2\epsilon_d}{E_F+M_F}\Delta\right)-\epsilon_d\left(\mathbf{p}+\frac{2\epsilon_d}{E_F+M_F}\Delta\right)\right]\right\}\Psi_I(\mathbf{p}), \tag{A19}$$

$$G_3(q^2) = G_3^{(1)}(q^2)+\varepsilon G_3^{(2)S}(q^2)+(1-\varepsilon)G_3^{(2)V}(q^2), \tag{A20}$$

$$G_3^{(1)}(q^2) = \int \frac{d^3p}{(2\pi)^3}\bar{\Psi}_F\left(\mathbf{p}+\frac{2\epsilon_d}{E_F+M_F}\Delta\right)\sqrt{\frac{\epsilon_Q(p)+m_Q}{2\epsilon_Q(p)}}\sqrt{\frac{\epsilon_q(p+\Delta)+m_q}{2\epsilon_q(p+\Delta)}}\frac{2M_F}{E_F+M_F}$$
$$\times\left\{\frac{\epsilon_d}{\epsilon_Q(p)+m_Q}\left[1+\frac{\epsilon_d}{\epsilon_q(p+\Delta)+m_q}-\frac{\mathbf{p}\Delta}{(E_F+M_F)(\epsilon_q(p+\Delta)+m_q)}\right]\right.$$
$$\left.-\frac{\mathbf{p}\Delta}{\Delta^2}\frac{E_F+M_F}{\epsilon_Q(p)+m_Q}\left[1+\frac{\epsilon_d}{\epsilon_q(p+\Delta)+m_q}\right]\right\}\Psi_I(\mathbf{p}), \tag{A21}$$

$$G_3^{(2)S}(q^2) = G_3^{(2)V}(q^2) = -\int \frac{d^3p}{(2\pi)^3}\bar{\Psi}_F\left(\mathbf{p}+\frac{2\epsilon_d}{E_F+M_F}\Delta\right)\sqrt{\frac{\epsilon_Q(p)+m_Q}{2\epsilon_Q(p)}}\sqrt{\frac{\epsilon_q(\Delta)+m_q}{2\epsilon_q(\Delta)}}$$
$$\times\frac{\mathbf{p}\Delta}{\Delta^2}\frac{M_F}{2\epsilon_Q(\Delta)(\epsilon_Q(\Delta)+m_Q)}\left[M_I-\epsilon_Q(p)-\epsilon_d(p)+M_F\right.$$
$$\left.-\epsilon_q\left(\mathbf{p}+\frac{2\epsilon_d}{E_F+M_F}\Delta\right)-\epsilon_d\left(\mathbf{p}+\frac{2\epsilon_d}{E_F+M_F}\Delta\right)\right]\Psi_I(\mathbf{p}), \tag{A22}$$

where

$$|\mathbf{\Delta}| = \sqrt{\frac{(M_I^2+M_F^2-q^2)^2}{4M_I^2}-M_F^2},$$

superscripts (1) and (2) correspond to vertex functions $\Gamma^{(1)}$ and $\Gamma^{(2)}$, S and V correspond to the scalar and vector confining potentials, ϵ_d is the diquark energy,

$$E_F = \sqrt{M_F^2+\mathbf{\Delta}^2}, \qquad \epsilon_q(\Delta) = \sqrt{m_q^2+\mathbf{\Delta}^2}, \qquad \epsilon_q(p+\lambda\Delta) = \sqrt{m_q^2+(\mathbf{p}+\lambda\Delta)^2} \quad (q=b,c,u,d),$$

subscripts I and F denote the initial Λ_Q and final Λ_q baryons, and the subscript q corresponds to a c or u quark for the final Λ_c or p, respectively.

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