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Pion mean fields and heavy baryons

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We show that the masses of the lowest-lying heavy baryons can be very well described in a pion mean-field approach. We consider a heavy baryon as a system consisting of the $N_c - 1$ light quarks that induce the pion mean field and a heavy quark as a static color source under the influence of this mean field. In this approach we derive a number of *model-independent* relations and calculate the heavy-baryon masses using those of the lowest-lying light baryons as input. The results are in remarkable agreement with the experimental data. In addition, the mass of the Ω_b^* baryon is predicted.

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I. HEAVY BARYONS AND PION MEAN FIELD

In a naive quark model, a heavy baryon consists of a heavy quark and two light quarks. When the mass of the heavy quark $m_Q \to \infty$, the spin of the heavy quark $\boldsymbol{S}_Q$ is conserved, which indicates that the spin of the light-quark degrees of freedom is also conserved: $\boldsymbol{S}_{\mathrm{L}} \equiv \boldsymbol{S} - \boldsymbol{S}_Q$ [1–3]. Because of this heavy-quark spin symmetry, the total spin of the light quarks can be considered as a good quantum number. This suggests that in the first approximation a heavy baryon can be viewed as a bound state of a heavy quark and a diquark. Thus, the flavor $SU(3)_{\mathrm{f}}$ representations of the lowest-lying heavy baryons are $\mathbf{3} \otimes \mathbf{3} = \bar{\mathbf{3}} \oplus \mathbf{6}$, of which the antitriplet has $S_{\mathrm{L}} = 0$ and total $S = 1/2$ and the sextet has $S_{\mathrm{L}} = 1$ with $S = 1/2$ and $S = 3/2$. Since in the limit $m_Q \to \infty$ the heavy quark inside a heavy baryon can be regarded as a static color source, the dynamics of heavy baryons is governed by the light quarks.

It is clear that the complete description of heavy baryons requires more involved treatment of light quarks. In the present paper, we propose to describe the dynamics of the light subsystem in a heavy baryon within a mean-field approach with a *hedgehog* [4] symmetry, motivated by Ref. [5]. Mean-field approximations provide often a simple physical picture, so they have been widely applied in a variety of fields in physics: the Thomas-Fermi approximation in atomic physics, the Ginzburg-Landau theory for superconductivity, the Bethe method in statistical physics, and shell models in nuclear physics, to name a few. In the seminal papers [6], Witten has argued that, to the leading order in $1/N_c$ expansion, the lowest-lying light baryons can be also viewed as bound states of N_c *valence* quarks in a mean field. In the limit of the large number of colors (N_c), the lowest-lying light baryons consist of N_c valence quarks that produce an effective pion mean field, which arises from the vacuum polarization. The N_c valence quarks are influenced by this pion mean field. The chiral-quark soliton model (χQSM) is constructed, based on this picture [7–9]. This mean field and a hedgehog symmetry allow one to derive the effective collective Hamiltonian that includes an explicit breaking of $\mathrm{SU}(3)_{\mathrm{f}}$ symmetry. The Hamiltonian involves the dynamical coefficients, which can be computed explicitly within the χQSM [10] in terms of the relativistic single particle quark states in the soliton background configuration. What will be important in the following is that the quark-soliton configuration has a trivial color structure: it consists of N_c copies of a colorless soliton. This means that in the leading order all dynamical coefficients—so-called *moments of inertia*—of the effective Hamiltonian are proportional to N_c. If the mean field is generated—as in the present case—by $N_c - 1$, rather than by N_c quarks, these coefficients have to be appropriately rescaled.

In the large N_c limit, heavy baryons consist of a heavy quark and $N_c - 1$ light quarks rather than a diquark. In this limit, the $N_c - 1$ valence quarks produce again the pion mean field, and the system can be described as a quark-soliton system. In the case of the light baryons, the $\mathrm{SU}(3)_{\mathrm{f}}$ space of the effective Hamiltonian is subject to a constraint imposed by the N_c valence quarks, $Y' = N_c/3$, that selects the lowest allowed representations: **8** and **10**. In the heavy-baryon case, the constraint is modified $Y' = (N_c - 1)/3$ due to the presence of the $N_c - 1$ valence quarks, and the lowest allowed representations are $\bar{\mathbf{3}}$ and **6**. The model predicts the structure of the symmetry breaking and allows one to compute numerical values of the dynamical coefficients.

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In the general framework of this mean-field picture, one can extract the moments of inertia from the experimental data on the masses of the lowest-lying light-quark baryons without relying on any model calculation [11,12]. Such an analysis has been performed recently in Ref. [13], and the dynamical parameters have been determined with high accuracy. In the present work, we shall use these values for the description of heavy-baryon masses. Additionally, we shall introduce a spin-spin interaction [14] to remove spin-1/2 and -3/2 degeneracy of the sextet states. The hyperfine coupling—the only parameter undetermined from the light sector—will be fixed from the experimental data.

II. COLLECTIVE HAMILTONIAN

The SU(3) soliton is constructed in terms of a hedgehog [4] ansatz, which couples three first Gell-Mann matrices with a unit space vector $\vec{n}\cdot\vec{\lambda}$. It is an extended object, and therefore its quantization is similar the textbook quantization of a rigid body. This requires one to identify the zero modes, which in the case of a hedgehog correspond to the space rotations and the rotations in the flavor space. Since the soliton lives in the isospin SU(2) subspace of the SU(3) group, the rotation along the hypercharge axis is not dynamical, and the corresponding generalized momentum produces a constraint: the only representations of the SU(3) group that are allowed must contain states with hypercharge Y' (called *right* hypercharge) of which the actual value depends on the number of valence quarks. Moreover, the isospin of the states with hypercharge equal to Y' is equal to the soliton spin. Details can be found in Refs. [10,15]. A general form of the collective rotational Hamiltonian for the light quark takes therefore a form of the quantized symmetric top rotating in the flavor SU(3) space,

$$H^{\mathrm{rot}}_{(p,q)} = M_{\mathrm{sol}} + \frac{1}{2I_1}\sum_{i=1}^{3}\hat{J}_i^2 + \frac{1}{2I_2}\sum_{a=4}^{7}\hat{J}_a^2, \tag{1}$$

where $\hat{J}_i$ are generators of the SU(3) group, the first three components of which correspond to the soliton spin. $I_{1,2}$ stand for the moments of inertia, and M_{sol} is a classical soliton mass. Note that $\hat{J}_8$ corresponding to Y' does not appear in Eq. (1). It is, however, convenient to add and subtract $\hat{J}_8^2$; then, the corresponding eigenvalues of Eq. (1) in the representation $\mathcal{R} = (p,q)$ read

$$\mathcal{E}^{\mathrm{rot}}_{(p,q)} = M_{\mathrm{sol}} + \frac{J(J+1)}{2I_1} + \frac{C_2(p,q) - J(J+1) - 3/4Y'^2}{2I_2}, \tag{2}$$

where C_2 denotes the SU(3) Casimir and J stands for spin. The baryon collective eigenfunctions are expressed in terms of the SU(3) Wigner D functions (see Refs. [10,13] for details). The right hypercharge imposes a constraint on the quantization of the chiral soliton, which for baryons takes the following form: $Y' = N_c/3$. This constraint selects a tower of allowed rotational excitations of the SU(3) hedgehog, which are identical as in the quark model. This has been considered as a success of the collective quantization resulting in a duality between the chiral soliton picture and the constituent quark model. In the case of heavy baryons, as already mentioned, $Y' = (N_c - 1)/3$. Then, the lowest rotational excitations appear to be $(p,q) = (0,1)$ (or $\bar{\mathbf{3}}$) with $S_{\mathrm{L}} = J = 0$ and (2,0) (or **6**) with $S_{\mathrm{L}} = 1$.

III. EXPLICIT SU(3) SYMMETRY BREAKING

The mass splittings in a heavy-baryon multiplet arise from the explicit flavor SU(3) symmetry breaking caused by the strange current quark mass m_s. The collective Hamiltonian of explicit $\mathrm{SU(3)_f}$ symmetry breaking in the light sector [10] reads

$$H_{\mathrm{br}} = \alpha D^{(8)}_{88} + \beta\hat{Y} + \frac{\gamma}{\sqrt{3}}\sum_{i=1}^{3} D^{(8)}_{8i}\hat{J}_i, \tag{3}$$

where α, β, and γ are given in terms of the moments of inertia $I_{1,2}$ and $K_{1,2}$ and the pion-nucleon sigma term $\Sigma_{\pi N} = (m_u + m_d)\langle N|\bar{u}u + \bar{d}d|N\rangle/2 = (m_u + m_d)\sigma$:

$$\alpha = -\frac{2m_s}{3}\sigma - \beta Y', \qquad \beta = -\frac{m_s K_2}{I_2}, \qquad \gamma = \frac{2m_s K_1}{I_1} + 2\beta. \tag{4}$$

In Eq. (4), we have explicitly included Y', which is equal to 1 for the light baryons.

In Ref. [13], the dynamical parameters α, β, and γ have been determined separately by using the experimental data for the baryon octet masses, the Ω mass, and the mass of the putative pentaquark Θ^+, taking into account isospin symmetry breaking including the electromagnetic interactions [16]. The values of α, β, and γ that have been obtained by χ^2 minimization [13] read as follows:

$$\alpha = -255.03 \pm 5.82\ \mathrm{MeV}, \quad \beta = -140.04 \pm 3.20\ \mathrm{MeV}, \quad \gamma = -101.08 \pm 2.33\ \mathrm{MeV}. \tag{5}$$

When we apply the mean-field approach to heavy baryons, Y' is equal to $Y' = (N_c - 1)/3$. Analogously, as explained previously, the expressions for the moments of inertia and $\Sigma_{\pi N}$ need to be modified by a multiplicative factor of $(N_c - 1)/N_c$. Thus, the m_s mass splittings of the heavy baryons should be calculated in terms of β and γ from Eq. (5), while the value of α should be modified:

$$\alpha \to \bar{\alpha} = \frac{N_c - 1}{N_c}\alpha. \tag{6}$$

The masses of the antitriplet and the sextet baryons (without spin-spin interactions) are then expressed as

$$M_{B,\mathcal{R}}^{Q} = M_{\mathcal{R}}^{Q} + \delta_{\mathcal{R}} Y, \tag{7}$$

where $M_{\mathcal{R}}^{Q} = m_Q + \mathcal{E}_{(p,q)}^{\rm rot}$ is called the center mass of a heavy baryon in representation $\mathcal{R}$. The explicit expressions for $M_{\bar{\mathbf{3}}}^{Q}$ and $M_{\mathbf{6}}^{Q}$ are written respectively as

$$M_{\bar{\mathbf{3}}}^{Q} = m_Q + M_{\rm sol} + \left(\frac{N_c}{N_c-1}\right)\frac{1}{2I_2},$$
$$M_{\mathbf{6}}^{Q} = M_{\bar{\mathbf{3}}}^{Q} + \left(\frac{N_c}{N_c-1}\right)\frac{1}{I_1}, \tag{8}$$

where we have modified the moments of inertia I_1 and I_2 as explained above. The term proportional to the hypercharge Y comes from the explicit $\mathrm{SU}(3)_{\rm f}$ symmetry breaking in Eq. (3). Parameters $\delta_{\bar{\mathbf{3}}}$ and $\delta_{\mathbf{6}}$ are defined as

$$\delta_{\bar{\mathbf{3}}} = \frac{3}{8}\bar{\alpha} + \beta, \qquad \delta_{\mathbf{6}} = \frac{3}{20}\bar{\alpha} + \beta - \frac{3}{10}\gamma. \tag{9}$$

In order to remove the degeneracy between sextet spin-1/2 and −3/2 states, we introduce the spin-spin interaction Hamiltonian expressed as

$$H_{\rm LQ} = \frac{2}{3}\frac{\kappa}{m_Q M_{\rm sol}} \boldsymbol{S}_{\rm L}\cdot \boldsymbol{S}_Q = \frac{2}{3}\frac{\varkappa}{m_Q}\boldsymbol{S}_{\rm L}\cdot\boldsymbol{S}_Q, \tag{10}$$

where κ denotes the flavor-independent hyperfine coupling. The operators $\boldsymbol{S}_{\rm L}$ and $\boldsymbol{S}_Q$ represent the spin operators for the soliton and the heavy quark, respectively. $M_{\rm sol}$ has been incorporated into an unknown coefficient $\varkappa$. The Hamiltonian H_{LQ} does not affect the $\bar{\mathbf{3}}$ states, since in this case $S_{\rm L}=0$. In $\mathbf{6}$ $S_{\rm L}=1$, and it couples to $\boldsymbol{S}_Q$ producing two multiplets $S=1/2$ and $S=3/2$. The respective splittings read as follows,

$$M_{B,\mathbf{6}_{1/2}}^{Q} = M_{B,\mathbf{6}}^{Q} - \frac{2}{3}\frac{\varkappa}{m_Q},$$
$$M_{B,\mathbf{6}_{3/2}}^{Q} = M_{B,\mathbf{6}}^{Q} + \frac{1}{3}\frac{\varkappa}{m_Q}, \tag{11}$$

giving the $3/2-1/2$ splitting

$$M_{B,\mathbf{6}_{3/2}}^{Q} - M_{B,\mathbf{6}_{1/2}}^{Q} = \frac{\varkappa}{m_Q}. \tag{12}$$

That is, $\varkappa$ can be determined by using the center values of the sextet masses. We list the expressions for the heavy-baryon masses in Table I.

IV. MODEL-INDEPENDENT RELATIONS

The mass formulas given in Table I imply relations that do not depend upon actual values of the model parameters—so called *model-independent relations*.[1] An immediate consequence of the mass formulas of Table I is the equal mass splittings separately in the $\bar{\mathbf{3}}$ and $\mathbf{6}$. Note that the mass splittings are independent of the spin and of the heavy-quark mass. These relations are indeed very well satisfied.[2] For the $\bar{\mathbf{3}}$, we have (in MeV)

$$-\delta_{\bar{\mathbf{3}}} = 182.9 \pm 0.3|_{\Xi_c-\Lambda_c} = 173.6 \pm 0.7|_{\Xi_b-\Lambda_b}, \tag{13}$$

which is satisfied with 7% accuracy. In the case of the 6, we have more relations (in MeV):

$$\begin{aligned} -\delta_{\mathbf{6}} &= 123.3 \pm 2.1|_{\Xi_c'-\Sigma_c} = 118.4 \pm 2.7|_{\Omega_c-\Xi_c'} \\ &= 127.8 \pm 0.8|_{\Xi_c^*-\Sigma_c^*} = 120.0 \pm 2.0|_{\Omega_c^*-\Xi_c^*} \\ &= 121.6 \pm 1.3|_{\Xi_b'-\Sigma_b} = 113.0 \pm 1.9|_{\Omega_b-\Xi_b'} \\ &= 121.7 \pm 1.3|_{\Xi_b^*-\Sigma_b^*}. \end{aligned} \tag{14}$$

We see that the equality of splittings is quite accurate (at the 6% level). From the spread of splittings in Eq. (14), we can make the first prediction of the mass of Ω_b^* that is not yet measured, taking as an input the experimental mass of Ξ_b^*:

$$M_{\Omega_b^*} = (6068 - 6083)\ \text{MeV}. \tag{15}$$

Using the mass formulas presented in Table I, we are able to determine the centers of the heavy-baryon masses,

$$M_{\bar{\mathbf{3}}}^{Q} = \frac{M_{\Lambda_Q} + 2M_{\Xi_Q}}{3}, \qquad M_{\mathbf{6}}^{Q} = \frac{M_{\mathbf{6}_{1/2}}^{Q} + 2M_{\mathbf{6}_{3/2}}^{Q}}{3}, \tag{16}$$

where

TABLE I. Expressions for the masses of the heavy baryons.

$\mathcal{R}_J$	B_Q	T	Y	M_{B_Q}
$\bar{\mathbf{3}}_{1/2}$	Λ_Q	0	$\frac{2}{3}$	$\frac{2}{3}\delta_{\bar{\mathbf{3}}} + M_{\bar{\mathbf{3}}}^{Q}$
	Ξ_Q	$\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{1}{3}\delta_{\bar{\mathbf{3}}} + M_{\bar{\mathbf{3}}}^{Q}$
$\mathbf{6}_{1/2}$	Σ_Q	1	$\frac{2}{3}$	$\frac{2}{3}\delta_{\mathbf{6}} - 2\varkappa/3m_Q + M_{\mathbf{6}}^{Q}$
	Ξ_Q'	$\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{1}{3}\delta_{\mathbf{6}} - 2\varkappa/3m_Q + M_{\mathbf{6}}^{Q}$
	Ω_Q	0	$-\frac{4}{3}$	$-\frac{4}{3}\delta_{\mathbf{6}} - 2\varkappa/3m_Q + M_{\mathbf{6}}^{Q}$
$\mathbf{6}_{3/2}$	Σ_Q^*	1	$\frac{2}{3}$	$\frac{2}{3}\delta_{\mathbf{6}} + \varkappa/3m_Q + M_{\mathbf{6}}^{Q}$
	Ξ_Q^*	$\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{1}{3}\delta_{\mathbf{6}} + \varkappa/3m_Q + M_{\mathbf{6}}^{Q}$
	Ω_Q^*	0	$-\frac{4}{3}$	$-\frac{4}{3}\delta_{\mathbf{6}} + \varkappa/3m_Q + M_{\mathbf{6}}^{Q}$

[1]The term "model-independent relations" in the present context was first used in Ref. [11]. It refers to the fact that the operators that appear, e.g., in (3) follow from the hedgehog symmetry, rather than from a specific model.

[2]All model-independent relations are checked using the data from [17] quoted in Tables II and III. For isospin multiplets, an average mass is used.

$$M^Q_{\mathbf{6}_{1/2}} = \frac{3M_{\Sigma_Q} + 2M_{\Xi'_Q} + M_{\Omega_Q}}{6},$$
$$M^Q_{\mathbf{6}_{3/2}} = \frac{3M_{\Sigma^*_Q} + 2M_{\Xi^*_Q} + M_{\Omega^*_Q}}{6}, \tag{17}$$

with $M^Q_{\mathbf{6}_{1/2,3/2}}$ given in Eq. (11). Equation (17) cannot be used in the b sector, because we do not know $M_{\Omega^*_b}$. Fortunately, we can determine the centers of the multiplets without invoking Ω_Q masses. Defining

$$S(\Sigma_Q) = \frac{M_{\Sigma_Q} + 2M_{\Sigma^*_Q}}{3} = M^Q_{\mathbf{6}} + \frac{2}{3}\delta_{\mathbf{6}},$$
$$S(\Xi_Q) = \frac{M_{\Xi'_Q} + 2M_{\Xi^*_Q}}{3} = M^Q_{\mathbf{6}} - \frac{1}{3}\delta_{\mathbf{6}}, \tag{18}$$

we have

$$M^Q_{\mathbf{6}} = \frac{S(\Sigma_Q) + 2S(\Xi_Q)}{3}. \tag{19}$$

For the sextet in the c sector, Eqs. (16) and (19) can be regarded as a model-independent relation,

$$M^c_{\mathbf{6}} = 2579.6 \pm 0.4|_{\mathrm{Eq.}(16)} = 2580.8 \pm 0.5|_{\mathrm{Eq.}(19)}, \tag{20}$$

in MeV. Relation (20) is fulfilled with unprecedented accuracy. For the **3** and for the **6** in the b sector, we have

$$M^c_{\bar{\mathbf{3}}} = (24087.4 \pm 0.2)|_{\mathrm{Eq.}(16)}\ \mathrm{MeV},$$
$$M^b_{\bar{\mathbf{3}}} = (5735.2 \pm 0.4)|_{\mathrm{Eq.}(16)}\ \mathrm{MeV},$$
$$M^b_{\mathbf{6}} = (5908.0 \pm 0.3)|_{\mathrm{Eq.}(19)}\ \mathrm{MeV}. \tag{21}$$

Apart from equal splittings in **6** (14), mass formulas of Table I admit a sum rule:

$$M_{\Omega^*_Q} = 2M_{\Xi'_Q} + M_{\Sigma^*_Q} - 2M_{\Sigma_Q}. \tag{22}$$

Equation (22) yields (2764.5 ± 3.1) MeV for $M_{\Omega^*_c}$, which is 1.4 MeV below the experiment, and predicts

$$M_{\Omega^*_b} = (6076.8 \pm 2.25)\ \mathrm{MeV}, \tag{23}$$

which falls in the range of Eq. (15).

Equation (8) provides yet another model-independent relation, allowing one to determine the moment of inertia I_1 either from the c or from the b sector,

$$\frac{1}{I_1} = \frac{2}{3}(M^Q_{\mathbf{6}} - M^Q_{\bar{\mathbf{3}}}) = 114.7|_c = 115.2|_b, \tag{24}$$

in MeV.[3] The reason for the equality of splittings between the multiplet centers can be traced back to the fact that it comes only from the energy of the rotational excitations, which are flavor blind in the present approach. Moreover, the effects of the $\mathrm{SU}(3)_{\mathrm{f}}$ symmetry breaking are simply the same both for the charm and bottom baryons, since they are solely due to the presence of the light quarks inside a heavy baryon. The relation of Eq. (24) is indeed very accurate, but it undershoots by 30% the value of $1/I_1$ extracted from the light sector equal to 160 MeV.

Another set of model-independent relations is not directly related to the specifics of the soliton model but provides a test of our assumption concerning the spin interactions of Eq. (12),

$$\frac{\varkappa}{m_c} = 64.5 \pm 0.8|_{\Sigma_c} = 69.1 \pm 2.1|_{\Xi_c} = 70.7 \pm 2.6|_{\Omega_c}$$
$$\frac{\varkappa}{m_b} = 20.2 \pm 1.9|_{\Sigma_b} = 20.3 \pm 0.1|_{\Xi_b} \tag{25}$$

(in MeV). Equation (25) provides yet another prediction for the Ω^*_b mass,

$$M_{\Omega^*_b} = M_{\Omega_b} + \frac{\varkappa}{m_b} = (6068.3 \pm 2.1)\ \mathrm{MeV}, \tag{26}$$

in good agreement with Eq. (23). From the ratios of the spin splittings (25), we can determine the ratio of the heavy-quark masses:

$$\frac{m_c}{m_b} = 0.29 - 0.31. \tag{27}$$

The experimental values of the $\overline{\mathrm{MS}}$ heavy-quark masses lead to $m_c/m_b = 0.305$ inserted, where both masses m_Q are evaluated at the renormalization point $\mu = m_Q$ [17]. Of course, heavy-quark masses in the effective models, like the one considered in this paper, may differ from the QCD masses. It is therefore encouraging that we get the mass ratio close to the ratio of the QCD masses.

V. MASSES OF HEAVY BARYONS

Having determined $\varkappa/m_Q$, using the numerical values of $\bar{\alpha}$, β, and γ from Eqs. (5) and (6), we can predict the masses of the lowest-lying ($\bar{\mathbf{3}}$ and **6**) heavy baryons. As we have already mentioned, the determination of I_1 from the heavy-quark sector and from the light sector differ by 30%. Therefore, in the following, we shall use the center masses $M^Q_{\bar{\mathbf{3}}}$ and $M^Q_{\mathbf{6}}$ as given by Eqs. (20) and (21).

As a first check, let us compare the values of δ parameters determined from the light sector through Eqs. (9),

$$\delta_{\bar{\mathbf{3}}} = -203.8 \pm 3.5\ \mathrm{MeV},$$
$$\delta_{\mathbf{6}} = -135.2 \pm 3.3\ \mathrm{MeV}, \tag{28}$$

with the values following from the heavy sector given in Eqs. (13) and (14). We see that the light sector values (28)

[3] A similar relation is found in Ref. [18] with a different factor and in a different context.

TABLE II. The results of the masses of the charmed baryons in comparison with the experimental data.

$\mathcal{R}_J^Q$	B_c	Mass	Experiment [17]	Deviation ξ_c
$\bar{\mathbf{3}}_{1/2}^c$	Λ_c	2272.5 ± 2.3	2286.5 ± 0.1	-0.006
	Ξ_c	2476.3 ± 1.2	2469.4 ± 0.3	0.003
$\mathbf{6}_{1/2}^c$	Σ_c	2445.3 ± 2.5	2453.5 ± 0.1	-0.003
	Ξ_c'	2580.5 ± 1.6	2576.8 ± 2.1	0.001
	Ω_c	2715.7 ± 4.5	2695.2 ± 1.7	0.008
$\mathbf{6}_{3/2}^c$	Σ_c^*	2513.4 ± 2.3	2518.1 ± 0.8	-0.002
	Ξ_c^*	2648.6 ± 1.3	2645.9 ± 0.4	0.001
	Ω_c^*	2783.8 ± 4.5	2765.9 ± 2.0	0.006

underestimate the heavy-quark determination (13) and (14) by approximately 13%. Interestingly, the ratio $\delta_{\bar{3}}/\delta_{\mathbf{6}} = 1.5$ is almost exactly equal to the ratio of the average splittings as given in Eqs. (13) and (14). The accuracy of the predictions given in Eq. (28) deserves a comment. Equal splittings in $\bar{\mathbf{3}}$ or $\mathbf{6}$ are analogous to the Gell-Mann–Okubo mass formulas for the light-baryon decuplet and follow solely from the $\mathrm{SU}(3)_{\mathrm{f}}$ group properties. However, the relation between the splittings in $\bar{\mathbf{3}}$ and $\mathbf{6}$ is a complicated dynamical question. The fact that chiral dynamics with an input from the light-baryon sector alone reproduces $\delta_{\bar{3}}$ and $\delta_{\mathbf{6}}$ with good accuracy is therefore by far not trivial. The fact that the ratio $\delta_{\bar{3}}/\delta_{\mathbf{6}}$ is reproduced even better suggests a multiplicative modification of these parameters by a common factor, which may be due e.g. to a slight change of m_s in the heavy-baryon environment.

In the following, we shall use $M_{\mathbf{6}}^c = 2580.8$ MeV and Eq. (21) for the multiplet centers, δ parameters from Eq. (28), and the average values for the hyperfine splittings: $\varkappa/m_c = (68.1 \pm 1.1)$ MeV and $\varkappa/m_b = (20.3 \pm 1.0)$ MeV. In order to quantify the quality of the predictions, we introduce deviation $\xi_Q = (M_{\mathrm{th}}^{B_Q} - M_{\mathrm{exp}}^{B_Q})/M_{\mathrm{exp}}^{B_Q}$, where $M_{\mathrm{th}}^{B_Q}$ represents the prediction of the present work, whereas $M_{\mathrm{exp}}^{B_Q}$ stands for the experimental value. The results presented in Tables II and III are in remarkable agreement with the experimental data within 0.7% or less. Note that the uncertainties in Tables II and III include those from the multiplet centers, $\varkappa/m_Q$, α, β, and γ.

TABLE III. The results of the masses of the bottom baryons in comparison with the experimental data.

$\mathcal{R}_J^Q$	B_b	Mass	Experiment [17]	Deviation ξ_b
$\bar{\mathbf{3}}_{1/2}^b$	Λ_b	5599.3 ± 2.4	5619.5 ± 0.2	-0.004
	Ξ_b	5803.1 ± 1.2	5793.1 ± 0.7	0.002
$\mathbf{6}_{1/2}^b$	Σ_b	5804.3 ± 2.4	5813.4 ± 1.3	-0.002
	Ξ_b'	5939.5 ± 1.5	5935.0 ± 0.05	0.001
	Ω_b	6074.7 ± 4.5	6048.0 ± 1.9	0.004
$\mathbf{6}_{3/2}^b$	Σ_b^*	5824.6 ± 2.3	5833.6 ± 1.3	-0.002
	Ξ_b^*	5959.8 ± 1.2	5955.3 ± 0.1	0.001
	Ω_b^*	6095.0 ± 4.4	–	–

In the last row of Table III, the mass of the Ω_b^* is predicted,

$$M_{\Omega_b^*} = (6095.0 \pm 4.4 \pm 24)\ \mathrm{MeV}, \tag{29}$$

where ± 24 MeV corresponds to the overall accuracy of of the model which we assume to be, as for the other bottom states, within the 0.4% range. This result lies slightly above the other predictions obtained in Eqs. (15), (23), and (26).

VI. SUMMARY AND OUTLOOK

In the present paper, we have applied a pion mean-field approach with hedgehog symmetry to the description of the heavy-baryon masses, which essentially assumes that the heavy quark (c or b) is surrounded by a pion mean field or a light-quark soliton produced from the $N_c - 1$ valence quarks. This assumption leads to a number of model-independent predictions: (i) the soliton quantization forcing heavy baryons to have the $\mathrm{SU}(3)_{\mathrm{f}}$ structure $\bar{\mathbf{3}}$ with spin $1/2$ and the $\mathbf{6}$ with spin $1/2$ and $3/2$ with approximate degeneracy of the sextets, (ii) equal mass splittings within the multiplets given in Eqs. (13) and (14) that do not depend on the heavy-quark mass, (iii) equal splittings between $\bar{\mathbf{3}}$ and 6 for the c and b sectors (24), and (iv) the sum rule that allows one to calculate the mass of Ω_Q^* in Eq. (22).

We have completed the model by adding the hyperfine interaction that is inversely proportional to the heavy-quark mass in Eq. (10). This assumption proved to very accurate as shown in Eq. (27), and the pertinent coefficient has been determined from the mass splittings in Eq. (25).

Next, we have used the three parameters extracted from the light-baryon sector in Eq. (5) to calculate heavy-baryon masses. The soliton has been assumed to be exactly the same as in the case of the light baryons with one exception: all moments of intertia have been rescaled by a factor of $(N_c - 1)/N_c$, because there are $N_c - 1$ rather than N_c valence quarks in a heavy baryon. With this modification, the predictions for the heavy-baryon masses turned out to be within 0.5% range when compared with the experiment data. Unfortunately, the splitting of the multiplet centers of mass equal to $3/2I_1$ turned out to be 30% off the light-baryon prediction. This result suggests that, apart from the $(N_c - 1)/N_c$ factor, moments of inertia can be further modified in the presence of the heavy quark. These modifications to the large extent cancel in the ratios that enter Eq. (4) except for the $\Sigma_{\pi N}$ term in the definition of α. We have checked, however, that varying $\Sigma_{\pi N}$ by $\pm 30\%$ changes $\delta_{\bar{3}}$ by ± 8.6 MeV and $\delta_{\mathbf{6}}$ by ± 3.5 MeV. Such a change will not affect the quality of the predictions for the heavy-baryon masses presented in this paper.

Finally, we have presented four different predictions of the Ω_b^* mass: (i) from the spread of the m_s splittings in $\mathbf{6}$ (15), (ii) from the Ω_Q^* sum rule (23), (iii) from the hyperfine splittings (26), and (iv) finally in Eq. (29). All these estimates are consistent and point to the value of $M_{\Omega_b^*}$

within the range of Eq. (15). Only the prediction of Eq. (29) based on the parameters of the light-baryons sector was obtained as slightly higher than the other ones. We anticipate that the mass of Ω_b^* will soon be measured at the LHC.

The present work has clear physical implications. The mean fields of the pion play indeed a crucial role in explaining not only the masses of the lowest-lying baryons in the light-quark sector but also those of the heavy baryons. The feedback of the heavy quark on the light sector may be of order of 30%, but it largely cancels in the heavy-baryon splittings. This aspect of the pion mean field deserves further study.

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[1] N. Isgur and M. B. Wise, Phys. Lett. B **232**, 113 (1989).
[2] N. Isgur and M. B. Wise, Phys. Rev. Lett. **66**, 1130 (1991).
[3] H. Georgi, Phys. Lett. B **240**, 447 (1990).
[4] T. H. R. Skyrme, Proc. R. Soc. A **260**, 127 (1961).
[5] D. Diakonov, arXiv:1003.2157.
[6] E. Witten, Nucl. Phys. **B160**, 57 (1979); **B223**, 422 (1983); **B223**, 433 (1983).
[7] D. Diakonov, V. Y. Petrov, and P. V. Pobylitsa, Nucl. Phys. **B306**, 809 (1988).
[8] C. V. Christov, A. Blotz, H.-Ch. Kim, P. Pobylitsa, T. Watabe, T. Meissner, E. Ruiz Arriola, and K. Goeke, Prog. Part. Nucl. Phys. **37**, 91 (1996).
[9] D. Diakonov, arXiv:hep-ph/9802298.
[10] A. Blotz, D. Diakonov, K. Goeke, N. W. Park, V. Petrov, and P. V. Pobylitsa, Nucl. Phys. **A555**, 765 (1993).
[11] G. S. Adkins and C. R. Nappi, Nucl. Phys. **B249**, 507 (1985).
[12] D. Diakonov, V. Petrov, and M. V. Polyakov, Z. Phys. A **359**, 305 (1997).
[13] G. S. Yang and H.-Ch. Kim, Prog. Theor. Phys. **128**, 397 (2012).
[14] Ya. B. Zeldovich and A. D. Sakharov, Yad. Fiz. **4**, 395 (1966); Sov. J. Nucl. Phys. **4**, 283 (1967); Acta Phys. Hung. **22**, 153 (1967).
[15] E. Guadagnini, Nucl. Phys. **B236**, 35 (1984); P. O. Mazur, M. A. Nowak, and M. Praszalowicz, Phys. Lett. **147B**, 137 (1984); S. Jain and S. R. Wadia, Nucl. Phys. **B258**, 713 (1985).
[16] G. S. Yang, H.-Ch. Kim, and M. V. Polyakov, Phys. Lett. B **695**, 214 (2011).
[17] K. A. Olive *et al.* (Particle Data Group Collaboration), Chin. Phys. C **38**, 090001 (2014).
[18] A. Momen, J. Schechter, and A. Subbaraman, Phys. Rev. D **49**, 5970 (1994).