PHYSICAL REVIEW D **94,** 066012 (2016)

Sound waves in the compactified D0-D4 brane system

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(Received 17 August 2016; published 30 September 2016)

As an extension to our previous work, we study the transport properties of the Witten-Sakai-Sugimoto model in the black D4-brane background with smeared D0 branes (D0-D4/D8 system). Because of the presence of the D0 branes, in the bubble configuration, this model is holographically dual to four-dimensional QCD or Yang-Mills theory with a Chern-Simons term, and the number density of the D0 branes corresponds to the coupling constant (θ angle) of the Chern-Simons term in the dual field theory. In this paper, we accordingly focus on the small number density of the D0 branes to study the sound mode in the black D0-D4 brane system since the coupling of the Chern-Simons term should be quite weak in QCD. Then, we derive its five-dimensional effective theory and analytically compute the speed of sound and the sound wave attenuation in the approach of gauge/gravity duality. Our result shows the speed of sound and the sound wave attenuation are modified by the presence of the D0 branes. Thus, they depend on the θ angle or chiral potential in this holographic description.

DOI: 10.1103/PhysRevD.94.066012

I. INTRODUCTION

Since the gauge/gravity duality or AdS/CFT correspondence was proposed, it has been a valuable tool in analyzing near-equilibrium dynamics of the strongly coupled plasma for a long time [1–5]. Many people believe that the QCD quark-gluon plasma (QGP) has been produced in heavy-ion collision experiments at the Relativistic Heavy-ion Collider in recent years [6–8], and thus the potential application of gauge/gravity or AdS/CFT correspondence for the hydrodynamic description of QGP has become the most important motivation of research in this direction. On the other hand, as one famous top-down prototype of holographic QCD, the Witten-Sakai-Sugimoto model [9–11] introduces a supergravity description based on the geometric background generated by N_c D4 branes compactified on a cycle. Naturally, studying the strongly coupled hydrodynamics of QCD has become one of the most interesting aspects of this holographic model [12–14].

The Witten-Sakai-Sugimoto model is realizing dual [15,16] to a four-dimensional QCD-like theory in the large-N_c limit. Specifically, the D4 branes are compactified on a cycle with appropriate boundary conditions; therefore, the dual field theory is nonconformal and nonsupersymmetric and couples to the Kaluza-Klein field in the adjoint representation. Additionally, there are N_f species of massless flavored quarks introduced by embedding N_f pairs of probe D8/$\overline{\mathrm{D8}}$ branes. In the D4 solitonic solution, the flavor D8/$\overline{\mathrm{D8}}$ branes are connected in the IR region, which holographically corresponds to the broken chiral symmetry in the dual field theory, and the light mesons come from the world volume theory on the connected D8/$\overline{\mathrm{D8}}$ branes in its low-energy effective theory.

Previously, the setup of the original Witten-Sakai-Sugimoto model was parallely implanted into the D4 brane background with smeared D0 branes [17] as an extension (i.e., the D0-D4/D8 brane system). And this system is holographically dual to the QCD or Yang-Mills theory with a topological term (i.e., the Chern-Simons term). It would be more clear if we take into account the action of the D4 branes in the presence of the smeared D0 branes,

$$S_{D_4} = -\mu_4 \mathrm{Tr} \int d^4x dx^4 e^{-\phi} \sqrt{-\det(\mathcal{G}+\mathcal{F})} + \mu_4 \int C_5 + \frac{1}{2}\mu_4 \int C_1 \wedge \mathcal{F} \wedge \mathcal{F}, \tag{1.1}$$

where $\mu_4 = (2\pi)^{-4} l_s^{-5}$, l_s is the size of the string, $\mathcal{G}$ is the induced metric, and $\mathcal{F} = 2\pi\alpha' F$, which is proportional to the gauge field strength on the D4 brane. C_1 and C_5 are the Romand-Romand 1- and 5- forms, respectively, and x^4 represents the periodic direction which is wrapped on the cycle. Obviously, the Yang-Mill action comes from the leading order of the first part in Eq. (1.1) [i.e., the Dirac-Born-Infield action] if it could be expanded by small $\mathcal{F}$. In the bubble background of the Witten-Sakai-Sugimoto model in the D0-D4 background, we have the solution $C_1 \sim \theta dx^4$ [17], and thus the last term in Eq. (1.1) could be integrated as

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$$\int_{S_{x^4}} C_1 \sim \theta, \qquad \int_{S_{x^4}\times\mathbb{R}^4} C_1\wedge\mathcal{F}\wedge\mathcal{F} \sim \theta \int_{\mathbb{R}^4} \mathcal{F}\wedge\mathcal{F}. \quad (1.2)$$

Consequently, the theory on the world volume of the D4 branes holographically corresponds to the QCD or Yang-Mills theory with a topological term as shown in Eq. (1.2). Phenomenologically, this topological term (1.2) may lead to some observable effects such as the chiral magnetic effect (or some other effect in the glueball condensation) [18,19]. Therefore, while the experimental upper bounds on θ are quite small, the θ dependence in QCD or Yang-Mills theory is very interesting. Motivated by these, we have had many previous works on this D0-D4/D8 holographic system such as Refs. [20–22] (also see other people's work about the holographic θ dependence in Refs. [23,24]). Although the dual field theory of this system is less clear[1], in this manuscript, we would like to extend the study to the hydrodynamics in the black D0-D4 system since the θ angle is related to the chiral potential as [25].

The background geometry of the black D0-D4 brane system also satisfies the condition of Refs. [30,31], so the shear viscosity η saturates the universal viscosity bound as in Ref. [32],

$$\frac{\eta}{s} = \frac{1}{4\pi}, \quad (1.3)$$

where s is the entropy density. It shows η/s should not be affected by the presence of the D0 branes (in other words, the θ angle). Hence, we are going to take a next step toward understanding transport phenomena in four-dimensional gauge plasma, i.e., to study the sound waves in this holographic system. And much research of the sound waves by holographic duality could be reviewed, such as Refs. [33,34], also in the original Witten-Sakai-Sugimoto model [12]. Therefore, as a generalization and comparison to the present results in Ref. [12], it would be quite interesting to consider the influence of the θ angle or chiral potential on the sound mode by this holographic system.

In this paper, after this Introduction, we will review the geometry of the black D0-D4 system briefly in Sec. II. Then, we derive the five-dimensionally effective theory of this system in Sec. III. Because of the presence of the D0 branes, it shows there should be a vector field in the effective theory, which is in addition to what has been studied in Ref. [12]. Interestingly, this vector might relate to some other observable effect.[2] Moreover, we find our effective theory is also similar to the resultant theory in Refs. [35,36]. In Sec. IV, we study the fluctuations of the relevant fields in its effective theory and discuss how to simplify the following computations for the sound mode. Then, in Sec. V, the speed of the sound and the sound wave attenuation are accordingly calculated in the hydrodynamic limit. We find they are affected by the presence of the D0 branes, and this may be interpreted as the modification from the θ angle (1.2) in the viewpoint of the dual field theory, or in other words, the speed of the sound and the sound wave attenuation depend on the chiral potential (μ_5) in this holographic description. The final section is the summary and discussion of this paper.

II. REVIEW OF D0-D4 BACKGROUND

We are going to review the black D0-D4 system briefly in this section, and some results have been presented in Refs. [17,20–22,37,38]. In the Einstein frame, the black brane solution of N_c D4 brane with N_0 smeared D0 branes reads [17,38]

$$\begin{aligned} ds^2 &= H_4^{-\frac{3}{8}}[-H_0^{-\frac{7}{8}} f_T(U)(dx^0)^2 + H_0^{\frac{1}{8}}\delta_{ij}dx^i dx^j \\ &\quad + H_0^{\frac{1}{8}}(dS^1)^2] + H_4^{\frac{5}{8}}H_0^{\frac{1}{8}}\left[\frac{dU^2}{f_T(U)} + U^2 d\Omega_4^2\right] \\ e^{-(\Phi-\Phi_0)} &= H_4^{1/4}/H_0^{3/4}, \qquad F_2 = \frac{1}{\sqrt{2!}}\frac{\mathcal{A}}{U^4}\frac{1}{H_0^2}dU\wedge dx^0, \\ F_4 &= \frac{1}{\sqrt{4!}}\mathcal{B}\epsilon_4, \end{aligned} \quad (2.1)$$

where

$$\begin{aligned} &\mathcal{A} = \frac{(2\pi l_s)^7 g_s N_0}{\omega_4 V_4}, \quad \mathcal{B} = \frac{(2\pi l_s)^3 g_s N_c}{\omega_4}, \quad e^{\Phi_0} = g_s, \\ &H_4 = 1 + \frac{U_{Q_4}^3}{U^3}, \quad H_0 = 1 + \frac{U_{Q_0}^3}{U^3}, \quad f_T(U) = 1 - \frac{U_\Lambda^3}{U^3}. \end{aligned} \quad (2.2)$$

We have used g_s, $d\Omega_4$, ϵ_4, and $\omega_4 = 8\pi^2/3$ to represent the string coupling, the line element, the volume form, and the volume of a unit S^4, respectively. U_Λ represents the position of the horizon, and V_4 is the volume of the D4 brane. Notice that x^4 is the periodic direction and the D0-branes have been smeared in the x^i, $i = 1, 2, 3$ and x^4 directions homogeneously. Moreover, for the readers' convenience, the relation between the integration parameters $\mathcal{A}$, $\mathcal{B}$, U_{Q_0}, and U_{Q_4} is given as [17]

[1]While the confined geometry of the original Witten-Sakai-Sugimoto model corresponds to the confinement phase, it is less clear for the deconfined geometry (black brane background) in the dual field theory. It has been discussed in Refs. [26,27] and also in our previous study [28,29]. In this sense, as an implanted version of the original Witten-Sakai-Sugimoto model, the dual field theory of the black D0-D4/D8 system is also less clear.

[2]This additional vector may be related to the chiral vortical separation effect [14]; we would like to do a future study of it in our framework.

$$\mathcal{A} = 3\sqrt{U_{Q_0}^3(U_{Q_0}^3 + U_\Lambda^3)},$$
$$\mathcal{B} = 3\sqrt{U_{Q_4}^3(U_{Q_4}^3 + U_\Lambda^3)}. \tag{2.3}$$

By taking the near-horizon limit, i.e., fixing U/α' and U_Λ/α' where $\alpha' = l_s^2$, we have the relations

$$U_{Q_4}^3 \to \pi\alpha'^{3/2} g_s N_c = \frac{\beta g_{\mathrm{YM}}^2 N_c l_s^2}{4\pi} \equiv R^3,$$
$$H_4(U_\Lambda) \to \frac{R^3}{U_\Lambda^3},$$
$$\beta \to \frac{4\pi}{3} U_\Lambda^{-1/2} R^{3/2} H_0^{1/2}(U_\Lambda), \tag{2.4}$$

where β is the size of the periodic time direction. Consequently, in the near-horizon limit, the black brane solution (2.1) becomes

$$ds^2 = \left(\frac{U}{R}\right)^{\frac{9}{8}} [-H_0^{-\frac{7}{8}} f_T(U)(dx^0)^2 + H_0^{\frac{1}{8}} \delta_{ij} dx^i dx^j + H_0^{\frac{1}{8}} (dS^1)^2] + \left(\frac{R}{U}\right)^{\frac{15}{8}} H_0^{\frac{1}{8}} \left[\frac{dU^2}{f_T(U)} + U^2 d\Omega_4^2\right]$$
$$e^\Phi = g_s \left(\frac{U}{R}\right)^{3/4} H_0^{3/4}. \tag{2.5}$$

Then, the deformed relations in the presence of D0 branes to the variables in QCD are

$$R^3 = \frac{\lambda l_s^2}{2M_{KK}}; \qquad g_s = \frac{\lambda}{2\pi M_{KK} N_c l_s};$$
$$U_\Lambda = \frac{2}{9} M_{KK} \lambda l_s^2 H_0(U_\Lambda), \tag{2.6}$$

where $\lambda = g_{\mathrm{YM}}^2 N_c$ is the ’t Hooft coupling constant and M_{KK} is the mass scale.

III. DIMENSIONAL REDUCTION TO FIVE-DIMENSIONAL THEORY

As we are going to study the hydrodynamics by $\mathrm{AdS}_5/\mathrm{CFT}_4$ duality and the bulk fields in the D0-D4 background are described by ten-dimensional type-IIA supergravity, in this section, let us first employ the standard Kaluza-Klein reduction on $S^1 \times S^4$ as [12,33,36], to derive the five-dimensionally effective theory of this system. In the Einstein frame, the ten-dimensional action of type-IIA supergravity is given as

$$S_{IIA} = \frac{1}{2k_0^2} \int d^{10}x \sqrt{-G} \left[\mathcal{R}^{(10)} - \frac{1}{2} \nabla_M \Phi \nabla^M \Phi - e^{\Phi/2} |F_4|^2 - e^{3\Phi/2} |F_2|^2\right], \tag{3.1}$$

where ϕ is the dilaton and F_4 and F_2 are the Romand-Romand 4- and 2-form field strengths, respectively. We have used G to represent the determinant of ten-dimensional metric G_{MN}, where the indexes (M, N) run from 0 to 9, and the ansatz of the metric

$$ds_{(10)}^2 = G_{MN} dx^M dx^N = e^{-\frac{10}{3}f} g_{ab} dx^a dx^b + e^{2f+8w} (dS^1)^2 + e^{2f-2w} d\Omega_4^2, \tag{3.2}$$

where x^a represents $x^a = \{x^\mu, U\}$, $\mu = 0, 1…3$, would be helpful for the dimensional reduction. Furthermore, we have assumed that the $S^1 \times S^4$ dependence could be trivially reduced, which means the fields f and w do not depend on $S^1 \times S^4$. By using the ansatz (3.2), we could obtain some useful relations, which are

$$\sqrt{-G} = \sqrt{-g} e^{-\frac{10}{3}f} g_4^{1/2},$$
$$\sqrt{-G}|F_4|^2 = \mathcal{B}^2 e^{-8(f-w)} \sqrt{-g} e^{-\frac{10}{3}f} g_4^{1/2},$$
$$\sqrt{-G}|F_2|^2 = \sqrt{-g} F_{ab} F_{cd} g^{ac} g^{bd} e^{\frac{10}{3}f} g_4^{1/2},$$
$$\sqrt{-G} \nabla_M \Phi \nabla^M \Phi = \sqrt{-g} g_4^{1/2} \nabla_a \Phi \nabla^a \Phi, \tag{3.3}$$

where g_4 represents the determinant of the metric on S^4 and g_{ab} is the five-dimensional metric. The relation between the ten-dimensional $\mathcal{R}^{(10)}$ and five-dimensional $\mathcal{R}^{(5)}$ curvature scalar is given as[3]

$$\mathcal{R}^{(10)} = e^{\frac{10}{3}f} \left[\mathcal{R}^{(5)} - 20 g^{ab} \partial_a w \partial_b w - \frac{40}{3} g^{ab} \partial_a f \partial_b f\right] + 12 e^{-2(f-w)}. \tag{3.4}$$

After inserting Eq. (3.4) into Eq. (3.1) and integrating over $S^1 \times S^4$, we obtain the five-dimensional effective action, which takes the form

$$S_{5d} = \frac{\pi \mathcal{V}_4}{k_0^2} \int d^5x \sqrt{-g} \left[\mathcal{R}^{(5)} - \frac{1}{2} g^{ab} \partial_a \Phi \partial_b \Phi - 20 g^{ab} \partial_a w \partial_b w - \frac{40}{3} g^{ab} \partial_a f \partial_b f - \mathcal{P} - e^{\frac{10}{3}f + \frac{3}{2}\Phi} F_{ab} F_{cd} g^{ac} g^{bd}\right], \tag{3.5}$$

where $\mathcal{V}_4$ represents the volume of the 4-sphere and

$$\mathcal{P} = \mathcal{B}^2 e^{\frac{\Phi}{2} - \frac{34}{3}f + 8w} - 12 e^{-\frac{16}{3}f + 2w}. \tag{3.6}$$

The equations of motion for Φ, w, f, and g_{ab} could be obtained from Eq. (3.5) and are as follows:

[3]We will not give the full relation in Eq. (3.4) since there would be some additional total derivatives if imposing the full relation of Eq. (3.4) to the action (3.1). Those terms have thus been dropped. So, only the relevant terms are given in Eq. (3.4).

$$
\begin{aligned}
&g^{ab}\nabla_a\nabla_b f-\frac{3}{80}\frac{\partial\mathcal{P}}{\partial f}-\frac{1}{8}e^{\frac{10}{3}f+\frac{3}{2}\Phi}F_{ab}F_{cd}g^{ac}g^{bd}=0,\\
&g^{ab}\nabla_a\nabla_b w-\frac{1}{40}\frac{\partial\mathcal{P}}{\partial w}=0,\\
&g^{ab}\nabla_a\nabla_b\Phi-\frac{\partial\mathcal{P}}{\partial\Phi}-\frac{3}{2}e^{\frac{10}{3}f+\frac{3}{2}\Phi}F_{ab}F_{cd}g^{ac}g^{bd}=0,\\
&\partial_a[\sqrt{-g}e^{\frac{10}{3}f+\frac{3}{2}\Phi}F^{ab}]=0,\\
&\frac{1}{2}\partial_a\Phi\partial_b\Phi+20\partial_a w\partial_b w+\frac{40}{3}\partial_a f\partial_b f+\frac{1}{3}g_{ab}\mathcal{P}\\
&\quad+\left(2F_{ca}F^c_b-\frac{1}{3}g_{ab}F_{cd}F^{cd}\right)e^{\frac{10}{3}f+\frac{3}{2}\Phi}=\mathcal{R}^{(5d)}_{ab}.
\end{aligned}
\tag{3.7}
$$

We have used F_{ab} to represent the components of the Romand-Romand 2-form F_2. Then, let us consider the five-dimensional ansatz of the metric as

$$
ds^2_{(5)}=-c_1^2dt^2+c_2^2\delta_{ij}dx^idx^j+c_3^2dU^2, \tag{3.8}
$$

which is obtained from the following corresponding ten-dimensional metric (3.2):

$$
\begin{aligned}
ds^2_{(10)}=&\,e^{-\frac{10}{3}f}[-c_1^2dt^2+c_2^2\delta_{ij}dx^idx^j+c_3^2dU^2]\\
&+e^{2f+8w}(dS^1)^2+e^{2(f-w)}d\Omega_4^2.
\end{aligned}
\tag{3.9}
$$

Comparing Eq. (3.9) to the black brane solution of the Witten-Sakai-Sugimoto model in the D0-D4 background (2.1) leads to the relations

$$
\begin{aligned}
f&=\frac{1}{16}\log H_0+\frac{13}{80}\log U,\\
w&=\frac{1}{10}\log U,\\
c_1&=f_T^{1/2}U^{5/6}H_0^{-1/3},\\
c_2&=H_0^{1/6}U^{5/6},\\
c_3&=f_T^{-1/2}H_0^{1/6}U^{-2/3},
\end{aligned}
\tag{3.10}
$$

where we have set $g_s=R=1$ for convenience (as a comparison to Ref. [12]) so that $\mathcal{B}=\sqrt{\frac{9}{2}}$. One can verify the reduced functions in Eq. (3.10) satisfy the five-dimensional effective equations of motion (3.7) consistently. Consequently, we obtain the five-dimensional effective action (3.5) and its solution (3.9), (3.10) of our D0-D4 brane system. However, a difference from the original D4-brane system is that there is an additional vector field C_a in the low-energy effective theory of which the field strength is the Romand-Romand 2-form defined as $F_{ab}=\partial_aC_b-\partial_bC_a$.

IV. FLUCTUATIONS

In this section, let us study the fluctuations of the relevant fields in the black D0-D4 background with the replacements[4]

$$
\begin{aligned}
g_{ab}&\to g_{ab}+h_{ab},\\
f&\to f+\delta f,\\
w&\to w+\delta w,\\
\Phi&\to\Phi+\delta\Phi,\\
C_a&\to C_a+\delta C_a,
\end{aligned}
\tag{4.1}
$$

where $\{h_{ab},\delta f,\delta w,\delta\Phi,\delta C_a\}$ are the fluctuations, while $\{g_{ab},f,w,\Phi,C_a\}$ are the background configurations of the D0-D4 system, i.e., the classical solution of the equations of motion (3.7). For the fluctuations of the metric, we are going to choose the following gauge as [12,33,34,36]:

$$
h_{aU}=0. \tag{4.2}
$$

Furthermore, we have assumed that the fluctuations of the metric depend on $\{t,z,U\}$[5] only; i.e., the system we are considering is $O(2)$ rotationally symmetric in the $x-y$ plane.

In the linearized case, the following sets of the metric are decoupled from each other because of the $O(2)$ symmetry [34]:

$$
\begin{aligned}
&\{h_{12}\},\\
&\{h_{11}-h_{22}\},\\
&\{h_{01},h_{13}\},\\
&\{h_{02},h_{23}\},\\
&\{h_{00},h_{\alpha\alpha}=h_{11}+h_{22},h_{03},h_{33}\}.
\end{aligned}
\tag{4.3}
$$

While the first three sets in Eq. (4.3) are related to the shear modes, the last set corresponds to the sound waves, which is the concern in this manuscript. Besides, there are additional fluctuations as $\{\delta f,\delta w,\delta\Phi,\delta C_a\}$ from the dimension-reductional scalars and vector. As a comparison, let us employ the similar conventions as [12,33,36] by introducing[6]

[4]By the solution for the D0-D4 background (2.1), we have assumed that only one component of C_a is nonzero which is C_t.

[5]The coordinate x^μ could be identified as $\{t,x,y,z\}$.

[6]It would not be confused with Eqs. (1.1) and (1.2) if we use the same $\mathcal{F}$ to represent the fluctuation of the function f here.

$$
\begin{aligned}
&h_{00}=e^{-i\omega t+iqx_3}c_1^2H_{tt}, \qquad h_{03}=e^{-i\omega t+iqx_3}c_2^2H_{tz}, \qquad h_{\alpha\alpha}=e^{-i\omega t+iqx_3}c_2^2H_{\alpha\alpha}, \qquad h_{33}=e^{-i\omega t+iqx_3}c_2^2H_{zz},\\
&\delta f=e^{-i\omega t+iqx_3}\mathcal{F}, \qquad \delta w=e^{-i\omega t+iqx_3}\mathcal{W}, \qquad \delta\Phi=e^{-i\omega t+iqx_3}\varphi, \qquad \delta C_a=e^{-i\omega t+iqx_3}\mathcal{C}_a,
\end{aligned}
\tag{4.4}
$$

where the definitions of the functions c_1 and c_2 have been given in Eq. (3.10) and $\{H_{tt},H_{tz},H_{\alpha\alpha},H_{zz},\mathcal{F},\mathcal{W},\varphi,\mathcal{C}_a\}$ are the functions which depend on the radial coordinate U only. By inserting Eq. (4.1) and (4.4) into Eq. (3.7) and expanding all the equations of motion at a linearized level, we obtain five ordinary differential equations as in the Appendix, and the relevant equations are collected once we evaluate Eqs. (A1)–(A5) by Eqs. (3.9), (3.10), and (4.4),

$$
\begin{aligned}
0={}&H_{tt}''+\left[\ln\frac{c_1^2c_2^3}{c_3}\right]'H_{tt}'-[\ln c_1]'H_{ii}'-\frac{c_3^2}{c_1^2}\left(q^2\frac{c_1^2}{c_2^2}H_{tt}+\omega^2H_{ii}+2\omega qH_{tz}\right)\\
&-\frac{2}{3}c_3^2\left(\frac{\partial\mathcal{P}}{\partial f}\mathcal{F}+\frac{\partial\mathcal{P}}{\partial w}\mathcal{W}+\frac{\partial\mathcal{P}}{\partial\Phi}\varphi\right)+\left(\frac{6c_1'c_2'}{c_1c_2}-\frac{2c_1'c_3'}{c_1c_3}+\frac{2c_1''}{c_1}+\frac{2}{3}c_3^2\mathcal{P}\right)H_{tt}\\
&+\frac{4F_{tU}}{9c_1^2}e^{\frac{10}{3}f+\frac{3}{2}\Phi}(12\hat{F}_{tU}+20F_{tU}\mathcal{F}+9F_{tU}\varphi),
\end{aligned}
\tag{4.5}
$$

$$
\begin{aligned}
0={}&H_{tz}''+\left[\ln\frac{c_2^5}{c_1c_3}\right]'H_{tz}'+q\omega\frac{c_3^2}{c_2^2}H_{\alpha\alpha}+\left(\frac{2c_1'c_2'}{c_1c_2}+\frac{4c_2'^2}{c_2^2}-\frac{2c_2'c_3'}{c_2c_3}+\frac{2c_2''}{c_2^2}+\frac{2}{3}c_3^2\mathcal{P}\right)H_{tz}\\
&\times\frac{4F_{tU}}{3c_1^2}e^{\frac{10}{3}f+\frac{3}{2}\Phi}\left(3\hat{F}_{3U}\frac{c_1^2}{c_2^2}+F_{tU}H_{tz}\right),
\end{aligned}
\tag{4.6}
$$

$$
\begin{aligned}
0={}&H_{aa}''+\left[\ln\frac{c_1c_2^5}{c_3}\right]'H_{aa}'+\frac{c_3^2}{c_1^2}\left(\omega^2-q^2\frac{c_1^2}{c_2^2}\right)H_{aa}+(H_{zz}'-H_{tt}')[\ln c_2^2]'\\
&+\left(\frac{4c_1c_3c_2'^2+2c_2c_3c_1'c_2'-2c_2c_1c_2'c_3'}{c_1c_2^2c_3}+\frac{2c_2''}{c_2}+\frac{2}{3}c_3^2\mathcal{P}\right)H_{aa}+\frac{4}{3}c_3^2\left(\frac{\partial\mathcal{P}}{\partial f}\mathcal{F}+\frac{\partial\mathcal{P}}{\partial w}\mathcal{W}+\frac{\partial\mathcal{P}}{\partial\Phi}\varphi\right)\\
&+\frac{4F_{tU}}{9c_1^2}e^{\frac{10}{3}f+\frac{3}{2}\Phi}(20F_{tU}\mathcal{F}+12\hat{F}_{tU}+6F_{tU}H_{tt}+3F_{tU}H_{aa}+9F_{tU}\varphi),
\end{aligned}
\tag{4.7}
$$

$$
\begin{aligned}
0={}&H_{zz}''+\left[\ln\frac{c_1c_2^4}{c_3}\right]'H_{zz}'+(H_{aa}'-H_{tt}')[\ln c_2]'+\frac{c_3^2}{c_1^2}\left[\omega^2H_{zz}+2\omega qH_{tz}+q^2\frac{c_1^2}{c_2^2}(H_{tt}-H_{aa})\right]\\
&+H_{zz}\left(\frac{c_1'c_2'}{c_1c_2}+\frac{c_2'^2}{c_2^2}-\frac{c_2'c_3'}{c_2c_3}+\frac{2c_2''}{c_2}+\frac{2}{3}c_3^2\mathcal{P}\right)+\frac{2}{3}c_3^2\left(\frac{\partial\mathcal{P}}{\partial f}\mathcal{F}+\frac{\partial\mathcal{P}}{\partial w}\mathcal{W}+\frac{\partial\mathcal{P}}{\partial\Phi}\varphi\right)\\
&+\frac{2F_{tU}}{9c_1^2}e^{\frac{10}{3}f+\frac{3}{2}\Phi}(12\hat{F}_{tU}+20F_{tU}\mathcal{F}+6F_{tU}H_{tt}+6F_{tU}H_{zz}+9F_{tU}\varphi),
\end{aligned}
\tag{4.8}
$$

$$
\begin{aligned}
0={}&\mathcal{F}''+\left[\ln\frac{c_1c_2^3}{c_3}\right]'\mathcal{F}'+\frac{1}{2}f'(H_{ii}'-H_{tt}')+\frac{c_3^2}{c_1^2}\left(\omega^2-q^2\frac{c_1^2}{c_2^2}\right)\mathcal{F}-\frac{3}{80}c_3^2\left(\frac{\partial^2\mathcal{P}}{\partial f^2}\mathcal{F}+\frac{\partial^2\mathcal{P}}{\partial f\partial w}\mathcal{W}+\frac{\partial^2\mathcal{P}}{\partial f\partial\Phi}\varphi\right)\\
&+\frac{F_{tU}}{24c_1^2}e^{\frac{10}{3}f+\frac{3}{2}\Phi}(12\hat{F}_{tU}+20F_{tU}\mathcal{F}+6F_{tU}H_{tt}+9F_{tU}\varphi),
\end{aligned}
\tag{4.9}
$$

$$
0=\mathcal{W}''+\left[\ln\frac{c_1c_2^3}{c_3}\right]'\mathcal{W}'+\frac{1}{2}w'(H_{ii}'-H_{tt}')+\frac{c_3^2}{c_1^2}\left(\omega^2-q^2\frac{c_1^2}{c_2^2}\right)\mathcal{W}-\frac{1}{40}c_3^2\left(\frac{\partial^2\mathcal{P}}{\partial w\partial f}\mathcal{F}+\frac{\partial^2\mathcal{P}}{\partial w^2}\mathcal{W}+\frac{\partial^2\mathcal{P}}{\partial w\partial\Phi}\varphi\right),
\tag{4.10}
$$

$$
\begin{aligned}
0={}&\varphi''+\left[\ln\frac{c_1c_2^3}{c_3}\right]'\varphi'+\frac{1}{2}\Phi'(H_{ii}'-H_{tt}')+\frac{c_3^2}{c_1^2}\left(\omega^2-q^2\frac{c_1^2}{c_2^2}\right)\varphi-c_3^2\left(\frac{\partial^2\mathcal{P}}{\partial\Phi\partial f}\mathcal{F}+\frac{\partial^2\mathcal{P}}{\partial\Phi\partial w}\mathcal{W}+\frac{\partial^2\mathcal{P}}{\partial\Phi^2}\varphi\right)\\
&+\frac{F_{tU}}{2c_1^2}e^{\frac{10}{3}f+\frac{3}{2}\Phi}(12\hat{F}_{tU}+20F_{tU}\mathcal{F}+6F_{tU}H_{tt}+9F_{tU}\varphi),
\end{aligned}
\tag{4.11}
$$

where $\hat{F}_{ab} = \mathcal{A} e^{i\omega t - iqx_3}(\partial_a \delta C_b - \partial_b \delta C_a)$.[7] Besides, there are three additional first-order constraints which come from association with the (partially) fixed diffeomorphism invariance,[8]

$$0 = \omega\left(H'_{ii} + \left[\ln\frac{c_2}{c_1}\right]' H_{ii}\right) + q\left(H'_{tz} + 2\left[\ln\frac{c_2}{c_1}\right]' H_{tz}\right) + \omega\left(\frac{80}{3} f'\mathcal{F} + 40w'\mathcal{W} + \Phi'\varphi\right), \tag{4.12}$$

$$0 = q\left(H'_{tt} - \left[\ln\frac{c_2}{c_1}\right]' H_{tt}\right) + \frac{c_2^2}{c_1^2}\omega H'_{tz} - qH'_{aa} + 4i\hat{F}_{tz}\frac{F_{tU}}{c_1^2} e^{\frac{10}{3}f + \frac{3}{2}\Phi} - q\left(\frac{80}{3} f'\mathcal{F} + 40w'\mathcal{W} + \Phi'\varphi\right), \tag{4.13}$$

$$\begin{aligned} 0 = {} & [\ln c_1 c_2^2]' H'_{ii} - [\ln c_2^3]' H'_{tt} + \frac{c_3^2}{c_1^2}\left[\omega^2 H_{ii} + 2\omega q H_{tz} + q^2 \frac{c_1^2}{c_2^2}(H_{tt} - H_{aa})\right] \\ & + c_3^2\left(\frac{\partial \mathcal{P}}{\partial f}\mathcal{F} + \frac{\partial \mathcal{P}}{\partial w}\mathcal{W} + \frac{\partial \mathcal{P}}{\partial \Phi}\varphi\right) - \left(\frac{80}{3} f'\mathcal{F}' + 40w'\mathcal{W}' + \Phi'\varphi'\right) + \frac{2}{3} c_3^2 \mathcal{P}(H_{ii} - H_{tt}) \\ & + \left(\frac{c'_1 c'_2}{c_1 c_2} + \frac{2c'^2_2}{c_2^2} - \frac{c'_2 c'_3}{c_2 c_3} + \frac{c''_2}{c_2}\right) H_{ii} + \left(\frac{c'_1 c'_3}{c_1 c_3} - \frac{3c'_1 c'_2}{c_1 c_2} - \frac{c''_1}{c_1}\right) H_{tt} \\ & + \frac{F_{tU}}{3c_1^2} e^{\frac{10}{3}f + \frac{3}{2}\Phi}(12\hat{F}_{tU} + 20F_{tU}\mathcal{F} + 10F_{tU}H_{tt} + 2F_{tU}H_{aa} + 2F_{tU}H_{zz} + 9F_{tU}\varphi). \end{aligned} \tag{4.14}$$

Notice that we do not give the relations of the fluctuations from the equation of motion for the vector field C_a, since this vector part corresponds to the diffusive or transverse channel [34], which are less relevant to the sound modes [34]. Therefore, we will not attempt to discuss more about the vector part, and thus we can simply set $\delta C_a = 0$ if studying the sound mode only. Nevertheless, the surviving equations of motion from Eqs. (4.5)–(4.14) are still complicated. On the other hand, the parameter $\mathcal{A}$ related to the coupling constant of the topological term in the dual field theory as [as shown in (1.2)] could be very small, thus it simplifies the calculation greatly if considering the leading order in small $\mathcal{A}$ expansion of all the equations in Eqs. (4.5)–(4.14). Then, if we introduce the gauge-invariant variables as [12,33,34,36]

$$\begin{aligned} & Z_H = 4\frac{q}{\omega} H_{tz} + 2H_{zz} - H_{aa}\left(1 - \frac{q^2}{\omega^2}\frac{c'_1 c_1}{c'_2 c_2}\right) + 2\frac{q^2}{\omega^2}\frac{c_1^2}{c_2^2} H_{tt}, \qquad Z_f = \mathcal{F} - \frac{f'}{[\ln c_2^4]'} H_{aa}, \\ & Z_w = \mathcal{W} - \frac{w'}{[\ln c_2^4]'} H_{aa}, \qquad Z_\Phi = \varphi - \frac{\Phi'}{[\ln c_2^4]'} H_{aa} \end{aligned} \tag{4.15}$$

with a new coordinate

$$x = \frac{c_1}{c_2}, \tag{4.16}$$

then we find the decoupled equations of motion for Z's by imposing Eqs. (4.5)–(4.14) in the small $\mathcal{A}$ expansion as

$$\begin{aligned} 0 = {} & \frac{d^2 Z_H}{dx^2} + \left[\frac{3q^2(2x^2 - 1) + 5\omega^2}{x(5\omega^2 - q^2(3 + 2x^2))} + \mathcal{A}^2 h_1(x)\right]\frac{dZ_H}{dx} \\ & + \left[\frac{4}{9}\frac{(-\omega^2 + q^2 x^2)(q^2(3 + 2x^2) - 5\omega^2) - 18q^2 U_\Lambda x^2 (1 - x^2)^{5/3}}{(5\omega^2 - q^2(3 + 2x^2))(1 - x^2)^{5/3} x^2 U_\Lambda} + \mathcal{A}^2 h_2(x)\right] Z_H \\ & + \left[\frac{4}{15}\frac{q^2(-3q^2 + 5\omega^2)}{\omega^2(5\omega^2 - q^2(3 + 2x^2))} + \mathcal{A}^2 g_1(x)\right]\kappa + \mathcal{O}(\mathcal{A}^4), \\ 0 = {} & \frac{d^2\kappa}{dx^2} + \left[\frac{1}{x} + \mathcal{A}^2 g_2(x)\right]\frac{d\kappa}{dx} + \left[\frac{4(\omega^2 - q^2 x^2)}{9U_\Lambda x^2 (1 - x^2)^{5/3}} + \mathcal{A}^2 g_3(x)\right]\kappa + \mathcal{O}(\mathcal{A}^4), \end{aligned} \tag{4.17}$$

[7] By this definition, there would be an i factor in $\hat{F}_{tz}$ once we calculate the derivative with respect to z or x_3. In this sense, Eq. (4.13) is a real equation although there is an i factor.

[8] In fact, there are more additional equations from the vanished components of the linearized Ricci tensor. We have checked that those equations determine the vanished components of δF_{ab}. As a result, the nonzero and relevant components of δF_{ab} mixed to the sound mode are only δF_{tz}, δF_{zU}, and δF_{tU}. So, the nonzero components of δC_a could be δC_t only if C_U and δC_U are gauged by C_U, $\delta C_U = 0$.

where

$$\kappa = 48Z_w + 9Z_\phi + 52Z_f,$$
$$h_1(x) = \frac{2(q^4(54-33x^2+84x^4+20x^6)-10q^2(9+16x^4)\omega^2+125x^2\omega^4)}{45xU_\Lambda^6(q^2(3+2x^2)-5\omega^2)^2},$$
$$h_2(x) = \frac{4}{1215x^2(-1+x^2)^2U_\Lambda^7(q^2(3+2x^2)-5\omega^2)} \times [5q^6x^2(1-x^2)^{1/3}(-27+36x^4+16x^6)+125(3-4x^2)(1-x^2)^{1/3}\omega^6 + 15q^2\omega^2(27(-1+x^2)^3(-3+2x^2)U_\Lambda+5(1-x^2)^{1/3}(-6-x^2+12x^4)\omega^2) - 3q^4(27(-1+x^2)^3(-9+4x^2)U_\Lambda+5(1-x^2)^{1/3}(-9-30x^2+32x^4+32x^6)\omega^2)]. \quad (4.18)$$

We will not give the explicit formula of the functions $g_{1,2,3}(x)$ used in Eq. (4.17) since they are too messy and lengthy. Moreover, in the next section, it will be clear that the functions $g_{1,2,3}(x)$ are actually less useful to the calculations for the sound mode because the sound mode is relevant to Z_H only.

V. HYDRODYNAMIC LIMIT IN THE SMALL $\mathcal{A}$ EXPANSION

In this section, let us study the physical fluctuation equations (4.17) in the hydrodynamic limit, i.e., $\omega \to 0, q \to 0$, but with $\frac{\omega}{q}$ fixed as a constant. As in many discussions, only the leading and next-to-leading solution (in the small q expansion) of Eq. (4.17) is needed. On the hand, since the sound mode is relevant to Z_H instead of κ, so, similar to the discussions and calculations in Refs. [12,33,34,36], we can simply choose $\kappa = 0$ as the solution for Eq. (4.17) consistently,[9] and for Z_H, we find that at the horizon $x \to 0_+$, $Z_H \to x^{\pm\frac{i\omega}{2\pi T}}x^{-\mathcal{A}}$.[10] By imposing the incoming boundary condition on all physical modes, we assume that

$$Z_H = x^{-\frac{i\omega}{2\pi T}}x^{-\mathcal{A}}z_H, \quad (5.1)$$

where z_H must be regular at the horizon. Additionally, since we are interested in the hydrodynamic pole dispersion relation in the stress-energy correlation, it would be convenient to parametrize the $\boldsymbol{\omega}$ and $\mathfrak{q}$ as

$$\boldsymbol{\omega} = v_s\mathfrak{q} - i\mathfrak{q}^2\Gamma, \quad (5.2)$$

where

$$\boldsymbol{\omega} = \frac{\omega}{2\pi T}, \qquad \mathfrak{q} = \frac{q}{2\pi T} \quad (5.3)$$

and v_s and Γ are the speed of sound and the sound wave attenuation, respectively, which would be determined from the pole dispersion relation. Without the loss of generality, we can choose the boundary condition for z_H as [12]

$$z_H|_{x\to 0_+} = 1, \qquad z_H|_{x\to 1_-} = 0. \quad (5.4)$$

By expanding z_H with small $\mathfrak{q}$, we assume

$$z_H = z_{H,0} + i\mathfrak{q}z_{H,1}. \quad (5.5)$$

Inserting Eqs. (5.1)–(5.4) into Eq. (4.17) in the small $\mathfrak{q}$ and $\mathcal{A}$ expansion with $\kappa = 0$, we obtain the following equations for $z_{H,0}$ and $z_{H,1}$ as

$$0 = z''_{H,0} - \frac{6x^2+5v_s^2-3}{x(2x^2-5v_s^2+3)}z'_{0,H} + \frac{8}{2x^2-5v_s^2+3}z_{0,H} + \mathcal{A}\left[\left(\frac{1}{x^2}+\frac{6x^2+5v_s^2-3}{x^2(2x^2-5v_s^2+3)}\right)z_{0,H} - \frac{2}{x}z'_{0,H}\right] \quad (5.6)$$

for leading order in $\mathcal{O}(\mathfrak{q}^0)$ and

$$0 = z''_{H,1} + \frac{3-5v_s^2-6x^2}{x(3-5v_s^2+2x^2)}z'_{H,1} + \frac{8}{3-5v_s^2+2x^2}z_{H,1} + \frac{2v_s(40x^2\Gamma+20x^2v_s^2-25v_s^4+30v_s^2-4x^4-12x^2-9)}{x(2x^2-5v_s^2+3)^2}z'_{H,0} - \frac{8v_s(-2x^2+5v_s^2-3+10\Gamma)}{(2x^2-5v_s^2+3)^2}z_{H,0} - \mathcal{A}\left[\frac{2z'_{H,1}}{x} - \frac{8}{3-5v_s^2+2x^2}z_{H,1} + \frac{2v_s(40x^2\Gamma+20x^2v_s^2-25v_s^4+30v_s^2-4x^4-12x^2-9)}{x^2(2x^2-5v_s^2+3)^2}z_{H,0}\right]. \quad (5.7)$$

[9]It has been discussed that $\kappa = 0$ could be a solution for the $\mathcal{A} = 0$ case [12], so it is also consistent with this solution for κ in the small $\mathcal{A}$ case.

[10]As $\mathcal{A} > 0$, it means $x^{-\mathcal{A}}$ is also singular if $x \to 0$. This behavior at the horizon is a bit different from the original D4-brane system; however, it should be consistent in the small $\mathcal{A}$ limit.

for next-to-leading order in $\mathcal{O}(\mathbf{q}^1)$. Then, the solution for Eq. (5.6) can be obtained as

$$z_{H,0} = \frac{(3-3\mathcal{A}-5v_s^2+5\mathcal{A}v_s^2-2x^2-2\mathcal{A}x^2)C_1}{1-5\mathcal{A}-5v_s^2+5\mathcal{A}v_s^2} + \frac{(3-3\mathcal{A}-5v_s^2+5\mathcal{A}v_s^2-2x^2-2\mathcal{A}x^2)}{2(1+\mathcal{A})^2(1-5\mathcal{A}-5v_s^2+5\mathcal{A}v_s^2)} \times x^{\mathcal{A}}\left[\frac{1}{\mathcal{A}} - \frac{4(-3+5v_s^2)}{(-1+\mathcal{A})(-3+5v_s^2+2x^2+\mathcal{A}(3-5v_s^2+2x^2))}\right]C_2, \tag{5.8}$$

where $C_{1,2}$ are two integration constants. We impose the boundary condition (5.4) for $z_{H,0}$,

$$z_{H,0}|_{x\to 1^-} = 0. \tag{5.9}$$

Besides, in order to compare our solution (5.8) with Ref. [12], we further require

$$z_{H,0}|_{x\to 0^+} = 1. \tag{5.10}$$

Thus, the relation between the integration constants $C_{1,2}$ is obtained as

$$C_2 = -\frac{2\mathcal{A}(-1+\mathcal{A}^2)^2C_1^2}{4\mathcal{A}+C_1-2\mathcal{A}C_1+\mathcal{A}^2C_1}. \tag{5.11}$$

The solution (5.8) should be definitely able to return to [12] in the small $\mathcal{A}$ limit, and in this sense, we have the following extra relations:

$$C_1 = \frac{1-5v_s^2}{3-5v_s^2}, \qquad C_2 \sim \mathcal{O}(\mathcal{A}^4). \tag{5.12}$$

Accordingly, it yields

$$v_s \simeq \frac{1}{\sqrt{5}} - \frac{2\mathcal{A}}{\sqrt{5}} + \mathcal{O}(\mathcal{A}^2). \tag{5.13}$$

So, the speed of the sound wave is shifted shown as Eq. (5.13). Then, the equation of motion for $z_{H,1}$ could be obtain from Eq. (5.7) by inserting the solution of (5.8), (5.13), (5.11), and (5.12). However, the resultant equation from Eq. (5.7) is too complicated to solve analytically. Hence, we expand $z_{H,1}$ in the small $\mathcal{A}$ case as

$$z_{H,1}(x) = \mathcal{X}(x) + \mathcal{A}\mathcal{Y}(x) + \mathcal{O}(\mathcal{A}^2). \tag{5.14}$$

By the expansion (5.14), we obtain the decoupled equation for $\mathcal{X}(x)$ in leading order $\mathcal{O}(\mathcal{A}^0)$ as

$$0 = \mathcal{X}'' + \frac{(1-3x^2)}{x(1+x^2)}\mathcal{X}' + \frac{4}{1+x^2}\mathcal{X} + \frac{8-20\Gamma}{\sqrt{5}(1+x^2)}, \tag{5.15}$$

and the equation for $\mathcal{Y}(x)$ in the next-to-leading order $\mathcal{O}(\mathcal{A}^1)$ is

$$0 = \mathcal{Y}'' + \frac{-2(-1+x^2)^2\mathcal{X}' + (1-2x^2-3x^4)\mathcal{Y}'}{x(1+x^2)^2} + \frac{4(-1+x^2)^2\mathcal{X} + 4(1+x^2)\mathcal{Y}}{(1+x^2)^2} + \frac{4-4x^6+x^4(-22+125\Gamma)+x^2(-46+205\Gamma)}{2\sqrt{5}x^2(1+x^2)^2}. \tag{5.16}$$

The solution for Eqs. (5.15) and (5.16) could be found as

$$\begin{aligned}\mathcal{X}(x) &= \frac{1}{\sqrt{5}}(5\Gamma-2) + D_1(-1+x^2) + D_2(-2-\ln x + x^2\ln x)\\ \mathcal{Y}(x) &= (-1+x^2)E_1 + E_2(-2-\ln x+x^2\ln x) + \frac{1}{40}(2\sqrt{5}-165\sqrt{5}\Gamma\\ &\quad -160D_2 - 160D_2\ln x - 32\sqrt{5}\ln x - 8\sqrt{5}\ln^2 x + 8\sqrt{5}x^2\ln^2 x - 40D_2\ln^2 x + 40x^2D_2\ln^2 x),\end{aligned} \tag{5.17}$$

where $D_{1,2}$ and $E_{1,2}$ are the integration constants. Since we have required that $z_{H,1}$ must be regular at the horizon, it yields

$$E_2 = \frac{1}{\sqrt{5}\mathcal{A}}, \qquad D_2 = -\frac{1}{\sqrt{5}}. \tag{5.18}$$

Then, imposing the boundary condition (5.4) to (5.14) with Eq. (5.17), we obtain

$$D_1 = -\mathcal{A}E_1,$$
$$\Gamma \simeq \frac{2}{5} + \frac{4}{5}\mathcal{A} + \mathcal{O}(\mathcal{A}^2), \tag{5.19}$$

which shows how the sound wave attenuation is also shifted by $\mathcal{A}$.

VI. SUMMARY AND DISCUSSION

In this paper, first we studied the five-dimensional effective theory of our black D0-D4 system by the dimensional reduction (Kaluza-Klein reduction). It contains an additional vector field to the five-dimensional effective theory of the original D4-brane system as [12]. Then, employing a technique similar to that in Refs. [12,33], the physical fluctuations in the five-dimensional effective theory of our D0-D4 system was also studied. Although the dual field theory of the black D0-D4 system is not completely clear (which is also the case for the original D4-brane system), we calculated the speed of sound and the sound wave attenuation in the hydrodynamic limit by using our five-dimensional effective theory. Particularly, we focused on the small $\mathcal{A}$ expansion in our calculations since the parameter $\mathcal{A}$ is related to the θ angle (the number density of D0 branes), which should be very small in QCD.

While our solution of the gauge-invariant variable (5.8), (5.14) is quite different, it could return to [12] if expanded by small $\mathcal{A}$. Accordingly, it is allowed to compare our result with Ref. [12]. Equations (5.13) and (5.19) show the speed of sound and the sound wave attenuation are all shifted by the presence of the D0 branes, and they return to the results in [12] consistently if setting $\mathcal{A} = 0$ (i.e., no D0 branes). Because of Eq. (1.2), Eqs. (5.13) and (5.19) could be interpreted as the modification from the θ angle or the chiral potential to the speed of sound and the sound wave attenuation. In hydrodynamics, the speed of sound depends on the mass of fermions and bosons and also the temperature [33]. But our holographic result suggests an additional θ dependence or chiral potential dependence if the topological term of QCD or Yang-Mills theory is considered, and as the leading-order modification from the topological term (the θ angle), it shows that the speed of sound decreases while the sound wave attenuation increases.[11]

Besides, we need to keep in mind the simplification and the approximation used in our calculations. First, we do not consider the fluctuations from the vector since this part is less relevant to the sound mode [13,34], so we simply turn off this part. However, this may lead to some observable effects such as the chiral vortical separation effect in hydrodynamics. Thus, a future study about it would be interesting and natural. Second, the solution for κ (the combination of the gauge invariant variable with Z_w, Z_f, and Z_Φ) has been chosen as $\kappa = 0$. This solution for κ is a rough choice, while it is consistent with its equation of motion (4.17) (and also consistent with its boundary condition in Ref. [12]). Therefore, a further improvement to take into account the solution of κ is also needed, although it might not change the results about the sound modes qualitatively.

ACKNOWLEDGMENTS

We would like to thank Chao Wu and Yi Yang for valuable comments and discussions. We thank the Tsinghua University, Beijing, for its kind hospitality during the program "String 2016," in which this work was finalized. Wenhe Cai is supported partially by the National Natural Science Foundation of China No. 11305168.

APPENDIX: THE EQUATIONS OF MOTION FOR THE RELEVANT FIELDS

In this Appendix, we collect the equations of motion for the fluctuations in the five-dimensional effective theory. From Eq. (3.7), at the linearized level, the equations for the fluctuations are

$$\begin{aligned} &g^{ab}[\partial_a\partial_b\delta f - (\Gamma^c{}_{ab}\partial_c\delta f + \Gamma^{(1)c}_{ab}\partial_c f)] - h^{ab}\nabla_a\nabla_b f - \frac{3}{80}\left(\frac{\partial^2\mathcal{P}}{\partial f\partial\Phi}\delta\Phi + \frac{\partial^2\mathcal{P}}{\partial f^2}\delta f + \frac{\partial^2\mathcal{P}}{\partial f\partial w}\delta w\right) \\ &\quad - \frac{1}{8}e^{\frac{10}{3}f+\frac{3}{2}\Phi}\left(2F^{ab}\delta F_{ab} - h^{ac}g^{bd}F_{ab}F_{cd} - F_{ab}F_{cd}g^{ac}h^{bd} + \frac{10}{3}F_{ab}F^{ab}\delta f + \frac{3}{2}F_{ab}F^{ab}\delta\Phi\right) = 0, \end{aligned} \tag{A1}$$

$$g^{ab}[\partial_a\partial_b\delta w - (\Gamma^c{}_{ab}\partial_c\delta w + \Gamma^{(1)c}_{ab}\partial_c w)] - h^{ab}\nabla_a\nabla_b w - \frac{1}{40}\left(\frac{\partial^2\mathcal{P}}{\partial w\partial\Phi}\delta\Phi + \frac{\partial^2\mathcal{P}}{\partial w^2}\delta w + \frac{\partial^2\mathcal{P}}{\partial w\partial f}\delta f\right) = 0, \tag{A2}$$

$$\begin{aligned} &g^{ab}[\partial_a\partial_b\delta\Phi - (\Gamma^c{}_{ab}\partial_c\delta\Phi + \Gamma^{(1)c}_{ab}\partial_c\Phi)] - h^{ab}\nabla_a\nabla_b\Phi - \left(\frac{\partial^2\mathcal{P}}{\partial\Phi^2}\delta\Phi + \frac{\partial^2\mathcal{P}}{\partial\Phi\partial f}\delta f + \frac{\partial^2\mathcal{P}}{\partial\Phi\partial w}\delta w\right) \\ &\quad - \frac{3}{2}e^{\frac{10}{3}f+\frac{3}{2}\Phi}\left(2F^{ab}\delta F_{ab} - h^{ac}g^{bd}F_{ab}F_{cd} - F_{ab}F_{cd}g^{ac}h^{bd} + \frac{10}{3}F_{ab}F^{ab}\delta f + \frac{3}{2}F_{ab}F^{ab}\delta\Phi\right) = 0, \end{aligned} \tag{A3}$$

[11] We noted a recent work [39], which studies the same holographic system, after submitting the first version of this manuscript to the arXiv, and our result is qualitatively similar to Ref. [39].

$$\partial_a\left[\sqrt{-g}e^{\frac{10}{3}f+\frac{3}{2}\Phi}\left(\frac{10}{3}F^{ab}\delta f+\frac{3}{2}F^{ab}\delta\Phi+g^{ac}g^{bd}\delta F_{cd}-F_{cd}h^{ca}g^{bd}-F_{cd}g^{ca}h^{bd}\right)\right]=0, \tag{A4}$$

$$\begin{aligned}&\frac{40}{3}(\partial_a f\partial_b\delta f+\partial_a\delta f\partial_b f)+20(\partial_a w\partial_b\delta w+\partial_a\delta w\partial_b w)+\frac{1}{2}(\partial_a\Phi\partial_b\delta\Phi+\partial_a\delta\Phi\partial_b\Phi)\\&+\frac{1}{3}g_{ab}\left(\frac{\partial\mathcal{P}}{\partial f}\delta f+\frac{\partial\mathcal{P}}{\partial w}\delta w+\frac{\partial\mathcal{P}}{\partial\Phi}\delta\Phi\right)+\frac{1}{3}h_{ab}\mathcal{P}+\left(\frac{10}{3}\delta f+\frac{3}{2}\delta\Phi\right)\left(2F_{ca}F^c{}_b-\frac{1}{3}g_{ab}F_{cd}F^{cd}\right)e^{\frac{10}{3}f+\frac{3}{2}\Phi}\\&+\left(2\delta F_{ca}F^c{}_b+2F^c{}_a\delta F_{cb}-2F_{ca}F_{db}h^{cd}-\frac{2}{3}g_{ab}F^{cd}\delta F_{cd}+\frac{2}{3}g_{ab}F^d{}_cF_{de}h^{ce}-\frac{1}{3}h_{ab}F_{cd}F^{cd}\right)e^{\frac{10}{3}f+\frac{3}{2}\Phi}=\mathcal{R}^{(1)}_{ab},\end{aligned} \tag{A5}$$

where $\Gamma^c{}_{ab}$ and $\Gamma^{(1)c}_{ab}$ are defined as

$$\begin{aligned}h^{ab}&=g^{ac}g^{bd}h_{cd},\\ \Gamma^c{}_{ab}&=\frac{1}{2}g^{dc}(\partial_b g_{da}+\partial_a g_{db}-\partial_d g_{ab}),\\ \Gamma^{(1)c}_{ab}&=\frac{1}{2}[g^{cd}(\partial_a h_{db}+\partial_b h_{ad}-\partial_d h_{ab})-h^{cd}(\partial_a g_{db}+\partial_b g_{ad}-\partial_d g_{ab})]\end{aligned} \tag{A6}$$

and $\mathcal{R}^{(1)}_{ab}$ is defined as

$$\mathcal{R}^{(1)}_{ab}=\partial_a\Gamma^{(1)c}_{cb}-\partial_b\Gamma^{(1)c}_{ca}+\Gamma^{(1)c}_{ad}\Gamma^d{}_{cb}+\Gamma^c{}_{ad}\Gamma^{(1)d}_{cb}-\Gamma^{(1)c}_{bd}\Gamma^d{}_{ca}-\Gamma^c{}_{bd}\Gamma^{(1)d}_{ca}. \tag{A7}$$

Equations (4.5)–(4.14) could be obtained from Eqs. (A1)–(A5).

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