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Maximally supersymmetric solutions of R^2 supergravity

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There are five maximally supersymmetric backgrounds in four-dimensional off-shell $\mathcal{N}=1$ supergravity, two of which are well known: Minkowski superspace $\mathbb{M}^{4|4}$ and anti–de Sitter superspace $\mathrm{AdS}^{4|4}$. The three remaining supermanifolds support spacetimes of different topology, which are $\mathbb{R}\times S^3$, $\mathrm{AdS}_3\times\mathbb{R}$, and a supersymmetric plane wave isometric to the Nappi-Witten group. As is well known, the Minkowski and anti–de Sitter superspaces are solutions of the Poincaré and anti–de Sitter supergravity theories, respectively. Here we demonstrate that the other three superspaces are solutions of pure R^2 supergravity. We also present a new (probably the simplest) derivation of the maximally supersymmetric backgrounds of off-shell $\mathcal{N}=1$ supergravity.

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I. INTRODUCTION

There exist only five maximally supersymmetric backgrounds in off-shell $\mathcal{N}=1$ supergravity in four dimensions. As purely bosonic backgrounds, the complete list was given by Festuccia and Seiberg [1]. Their results were rederived in [2] using the superspace formalism developed in the mid-1990s [3] (see [4] for a review). As curved $\mathcal{N}=1$ superspaces, all these backgrounds were described in [5]. The algebraic aspects of these backgrounds have recently been studied in [6].

We now list all maximally supersymmetric backgrounds of $\mathcal{N}=1$ supergravity following [5].[1] The simplest and most well known is Minkowski superspace $\mathbb{M}^{4|4}$ [7,8]. It is characterized by the algebra of covariant derivatives

$$\{\mathcal{D}_\alpha,\bar{\mathcal{D}}_{\dot\beta}\}=-2\mathrm{i}\mathcal{D}_{\alpha\dot\beta}, \tag{1.1a}$$

$$\{\mathcal{D}_\alpha,\mathcal{D}_\beta\}=0,\qquad \{\bar{\mathcal{D}}_{\dot\alpha},\bar{\mathcal{D}}_{\dot\beta}\}=0, \tag{1.1b}$$

$$[\mathcal{D}_a,\mathcal{D}_B]=0. \tag{1.1c}$$

The second oldest background is anti–de Sitter (AdS) superspace $\mathrm{AdS}^{4|4}$ [9–11]. It is characterized by the algebra of covariant derivatives

$$\{\mathcal{D}_\alpha,\bar{\mathcal{D}}_{\dot\beta}\}=-2\mathrm{i}\mathcal{D}_{\alpha\dot\beta}, \tag{1.2a}$$

$$\{\mathcal{D}_\alpha,\mathcal{D}_\beta\}=-4\bar{R}M_{\alpha\beta},\qquad \{\bar{\mathcal{D}}_{\dot\alpha},\bar{\mathcal{D}}_{\dot\beta}\}=4R\bar{M}_{\dot\alpha\dot\beta}, \tag{1.2b}$$

$$[\mathcal{D}_a,\mathcal{D}_\beta]=-\frac{\mathrm{i}}{2}\bar{R}(\sigma_a)_{\beta\dot\gamma}\bar{\mathcal{D}}^{\dot\gamma},\qquad [\mathcal{D}_a,\bar{\mathcal{D}}_{\dot\beta}]=\frac{\mathrm{i}}{2}R(\sigma_a)_{\gamma\dot\beta}\mathcal{D}^{\gamma}, \tag{1.2c}$$

$$[\mathcal{D}_a,\mathcal{D}_b]=-|R|^2M_{ab}, \tag{1.2d}$$

with $R=\mathrm{const}$. The Riemann tensor of AdS^4 may be deduced from (1.2d) to be

$$\mathfrak{R}_{abcd}=-|R|^2(\eta_{ac}\eta_{bd}-\eta_{ad}\eta_{bc}). \tag{1.3}$$

The three remaining superspaces are characterized by formally identical anticommutation relations [5]

$$\{\mathcal{D}_\alpha,\mathcal{D}_\beta\}=0,\qquad \{\bar{\mathcal{D}}_{\dot\alpha},\bar{\mathcal{D}}_{\dot\beta}\}=0,\qquad \{\mathcal{D}_\alpha,\bar{\mathcal{D}}_{\dot\beta}\}=-2\mathrm{i}\mathcal{D}_{\alpha\dot\beta}, \tag{1.4a}$$

$$[\mathcal{D}_\alpha,\mathcal{D}_{\beta\dot\beta}]=\mathrm{i}\varepsilon_{\alpha\beta}G^\gamma{}_{\dot\beta}\mathcal{D}_\gamma,\qquad [\bar{\mathcal{D}}_{\dot\alpha},\mathcal{D}_{\beta\dot\beta}]=-\mathrm{i}\varepsilon_{\dot\alpha\dot\beta}G_\beta{}^{\dot\gamma}\bar{\mathcal{D}}_{\dot\gamma}, \tag{1.4b}$$

$$[\mathcal{D}_{\alpha\dot\alpha},\mathcal{D}_{\beta\dot\beta}]=-\mathrm{i}\varepsilon_{\dot\alpha\dot\beta}G_\beta{}^{\dot\gamma}\mathcal{D}_{\alpha\dot\gamma}+\mathrm{i}\varepsilon_{\alpha\beta}G^\gamma{}_{\dot\beta}\mathcal{D}_{\gamma\dot\alpha}, \tag{1.4c}$$

where G_b is covariantly constant,

$$\mathcal{D}_AG_b=0. \tag{1.4d}$$

The difference between these superspaces is encoded in the Lorentzian type of G_a. Since $G^2=G^aG_a$ is constant, the geometry (1.4) describes three different superspaces, $\mathbb{M}^{4|4}_T$, $\mathbb{M}^{4|4}_S$, and $\mathbb{M}^{4|4}_N$, which correspond to the choices $G^2<0$, $G^2>0$, and $G^2=0$, respectively. The Lorentzian

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[1]In all cases, the superspace covariant derivatives $\mathcal{D}_A=(\mathcal{D}_a,\mathcal{D}_\alpha,\bar{\mathcal{D}}^{\dot\alpha})$ have the form $\mathcal{D}_A=E_A{}^M\partial_M+\frac{1}{2}\Omega_A{}^{bc}M_{bc}$, where M_{bc} is the Lorentz generator. In the case of Minkowski superspace, one can choose the Lorentz connection $\Omega_A{}^{bc}$ to vanish, and the inverse vielbein $E_A{}^M$ to have the Akulov-Volkov form [7].

manifolds, which are the bosonic bodies of the superspaces $\mathbb{M}^{4|4}_T$, $\mathbb{M}^{4|4}_S$, and $\mathbb{M}^{4|4}_N$, are $\mathbb{R}\times S^3$, $\mathrm{AdS}_3\times\mathbb{R}$, and a pp-wave spacetime,[2] respectively. The Riemann curvature tensor of these spacetimes is

$$\mathfrak{R}_{abcd} = \frac{1}{4}\{G_c(G_a\eta_{bd} - G_b\eta_{ad}) - G_d(G_a\eta_{bc} - G_b\eta_{ac}) - G^2(\eta_{ac}\eta_{bd} - \eta_{ad}\eta_{bc})\}. \tag{1.5}$$

The superspace $\mathbb{M}^{4|4}_T$ is the universal covering of $\mathcal{M}^{4|4} = \mathsf{SU}(2|1)$. The bosonic body of $\mathcal{M}^{4|4}$ is $\mathsf{U}(2) = (S^1\times S^3)/\mathbb{Z}_2$. The isometry group of $\mathcal{M}^{4|4}$ is $\mathsf{SU}(2|1)\times\mathsf{U}(2)$. One can think of $\mathbb{M}^{4|4}_T$ as a supersymmetric extension of Einstein's static universe. $\mathcal{N}=1$ supersymmetric field theories on $\mathbb{R}\times S^3$ were studied in the mid-1980s by Sen [13]. The superspace $\mathbb{M}^{4|4}_S$ is the universal covering of $\widetilde{\mathcal{M}}^{4|4} = \mathsf{SU}(1,1|1)$. The bosonic body of $\widetilde{\mathcal{M}}^{4|4}$ is $\mathsf{U}(1,1) = (\mathrm{AdS}_3\times S^1)/\mathbb{Z}_2$. The isometry group of $\widetilde{\mathcal{M}}^{4|4}$ is $\mathsf{SU}(1,1|1)\times\mathsf{U}(2)$.

The superspace (1.2) is a maximally supersymmetric solution of anti–de Sitter supergravity described by the action (see, e.g., [3] for a review)

$$S_{\mathrm{SUGRA}} = -\frac{3}{\kappa^2}\int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar\theta E + \left\{\frac{\mu}{\kappa^2}\int \mathrm{d}^4x\mathrm{d}^2\theta\mathcal{E} + \mathrm{c.c.}\right\}, \tag{1.6}$$

where κ is the gravitational coupling constant and μ a cosmological parameter. The integration measures E and $\mathcal{E}$ in (1.6) correspond to the full superspace and its chiral subspace, respectively. The equations of motion corresponding to (1.6) are

$$G_a = 0, \qquad R = \mu; \tag{1.7}$$

see [3] for a pedagogical derivation. Setting $\mu = 0$ in (1.6) gives the action for $\mathcal{N}=1$ Poincaré supergravity [14]. Minkowski superspace (1.1) is a maximally supersymmetric solution of this theory.

In this paper, we are going to show that the superspaces (1.4) are maximally supersymmetric solutions of scale invariant supergravity[3]

$$S = \alpha\int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar\theta E R\bar R + \left\{\beta\int \mathrm{d}^4x\mathrm{d}^2\theta\mathcal{E}R^3 + \mathrm{c.c.}\right\} = \int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar\theta E\{\alpha R\bar R + (\beta R^2 + \bar\beta\bar R^2)\}, \tag{1.8}$$

with α and β a real and a complex dimensionless parameter, respectively. This higher-derivative supergravity model has recently been studied[4] in [15] (see also [16,17].) Along with the supergravity action, both terms in (1.8) have also been discussed in the framework of supersymmetric models for inflation; see [18,19] and references therein.

Higher-derivative supergravity actions of the type (1.8) have a long history. The α term in (1.8) is generated as a one-loop quantum correction in $\mathcal{N}=1$ supersymmetric field theories coupled to supergravity [20–22]. The component structure of this term was described in [23]. Although the β term in (1.8) breaks the $\mathsf{U}(1)$ R-symmetry, adding such a contribution to the α term is completely natural, keeping in mind that a massless covariantly chiral scalar superfield Φ, $\bar{\mathcal{D}}_{\dot\alpha}\Phi = 0$ is described in supergravity by an action

$$S_{\mathrm{matter}} = \int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar\theta E\left\{\Phi\bar\Phi + \frac{1}{2}\xi(\Phi^2 + \bar\Phi^2)\right\}, \tag{1.9}$$

with ξ a dimensionless parameter. The choice $\xi = 0$ corresponds to the conformal scalar multiplet model which is dual to the improved tensor multiplet [24]. Another natural choice is $\xi = 1$ and corresponds to a nonconformal scalar multiplet which is dual to the free tensor multiplet model [25].

This paper is organized as follows. In Sec. II we briefly discuss the various superspace formulations for $\mathcal{N}=1$ conformal supergravity, and present a new derivation of the maximally supersymmetric backgrounds of off-shell $\mathcal{N}=1$ supergravity. In Sec. III we prove that the curved superspaces described by (1.4) are solutions of the scale invariant supergravity model (1.8). Some concluding comments are given in Sec. IV.

II. A NEW DERIVATION OF THE MAXIMALLY SUPERSYMMETRIC BACKGROUNDS IN OFF-SHELL $\mathcal{N}=1$ SUPERGRAVITY

Every off-shell formulation for $\mathcal{N}=1$ supergravity can be described using the superspace geometry pioneered by Howe [26] 35 years ago and soon after reviewed and further developed in [27]. This curved superspace geometry is based on the structure group $\mathsf{SL}(2,\mathbb{C})\times\mathsf{U}(1)$, and nowadays it is often referred to as $\mathsf{U}(1)$ superspace. The algebra of supergravity covariant derivatives is as follows:

$$\{\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot\alpha}\} = -2\mathrm{i}\mathcal{D}_{\alpha\dot\alpha}, \tag{2.1a}$$

[2] The latter spacetime was shown in [6] to be isometric to the Nappi-Witten group NW_4 [12].

[3] This action is invariant under transformations (2.6) with the parameter σ being real and constant.

[4] Action (1.8) can be rewritten in a manifestly super-Weyl invariant form, as in [15], by introducing a chiral compensator ϕ, $\bar{\mathcal{D}}_{\dot\alpha}\phi = 0$, and replacing R with the super-Weyl invariant chiral scalar $\mathbb{R} = -\frac{1}{4}\phi^{-2}(\bar{\mathcal{D}}^2 - 4R)\bar\phi$ and the full superspace measure E with $E\phi\bar\phi$. Such a superconformal reformulation is sometimes useful, in particular for the component reduction; however it does not offer new insights into the analysis in this paper.

$$\{\mathcal{D}_\alpha, \mathcal{D}_\beta\} = -4\bar{R}M_{\alpha\beta}, \qquad \{\bar{\mathcal{D}}_{\dot\alpha}, \bar{\mathcal{D}}_{\dot\beta}\} = 4R\bar{M}_{\dot\alpha\dot\beta}, \tag{2.1b}$$

$$\begin{aligned}[\mathcal{D}_\alpha, \mathcal{D}_{\beta\dot\beta}] = {}& \mathrm{i}\varepsilon_{\alpha\beta}(\bar{R}\bar{\mathcal{D}}_{\dot\beta} + G^\gamma{}_{\dot\beta}\mathcal{D}_\gamma - (\mathcal{D}^\gamma G^\delta{}_{\dot\beta})M_{\gamma\delta} \\ & + 2\bar{W}_{\dot\beta}{}^{\dot\gamma\dot\delta}\bar{M}_{\dot\gamma\dot\delta}) + \mathrm{i}(\bar{\mathcal{D}}_{\dot\beta}\bar{R})M_{\alpha\beta} \\ & - \frac{\mathrm{i}}{3}\varepsilon_{\alpha\beta}\bar{X}^{\dot\gamma}\bar{M}_{\dot\gamma\dot\beta} + \frac{\mathrm{i}}{2}\varepsilon_{\alpha\beta}\bar{X}_{\dot\beta}\mathbb{J}.\end{aligned} \tag{2.1c}$$

Here the $\mathsf{U}(1)$ generator $\mathbb{J}$ is normalized by

$$[\mathbb{J}, \mathcal{D}_\alpha] = -\mathcal{D}_\alpha, \qquad [\mathbb{J}, \bar{\mathcal{D}}_{\dot\alpha}] = \bar{\mathcal{D}}_{\dot\alpha}. \tag{2.2}$$

The torsion superfields R, $G_{\alpha\dot\alpha}$, $W_{\alpha\beta\gamma}$, and X_α obey the Bianchi identities:

$$\bar{\mathcal{D}}_{\dot\alpha}R = 0, \qquad \bar{\mathcal{D}}_{\dot\alpha}X_\alpha = 0, \qquad \bar{\mathcal{D}}_{\dot\alpha}W_{\alpha\beta\gamma} = 0, \tag{2.3a}$$

$$X_\alpha = \mathcal{D}_\alpha R - \bar{\mathcal{D}}^{\dot\alpha}G_{\alpha\dot\alpha}, \qquad \mathcal{D}^\alpha X_\alpha = \bar{\mathcal{D}}_{\dot\alpha}\bar{X}^{\dot\alpha}. \tag{2.3b}$$

The reason why the superspace geometry defined by (2.1) is adequate to describe $\mathcal{N} = 1$ conformal supergravity is the fact that the algebra (2.1) does not change under a super-Weyl transformation

$$\mathcal{D}'_\alpha = \mathrm{e}^{\frac{1}{2}L}\left(\mathcal{D}_\alpha + 2(\mathcal{D}^\beta L)M_{\beta\alpha} + \frac{3}{2}(\mathcal{D}_\alpha L)\mathbb{J}\right) \tag{2.4}$$

accompanied by induced transformations of the torsion superfields. The parameter L in (2.4) is a real unconstrained superfield.

Before turning to the derivation of the maximally supersymmetric backgrounds of supergravity, it is worth commenting on other superspace approaches to describe $\mathcal{N} = 1$ conformal supergravity. The $\mathsf{U}(1)$ superspace of [26] is a gauge fixed version of 4D $\mathcal{N} = 1$ conformal superspace [28], in which the entire superconformal algebra $\mathsf{SU}(2,2|1)$ is gauged in superspace [see also [29] for a review of the relationship between the $\mathsf{U}(1)$ and conformal superspaces]. When studying supersymmetric backgrounds of supergravity, it suffices to work with $\mathsf{U}(1)$ superspace, and therefore we do not use conformal superspace in this paper.

The superspace geometry developed by Grimm *et al.* [30] is obtained from (2.1) by setting

$$X_\alpha = 0. \tag{2.5}$$

Under this condition, the $\mathsf{U}(1)$ connection can be gauged away and the structure group reduces to $\mathsf{SL}(2,\mathbb{C})$. Requirement (2.5) can always be achieved by applying a specially chosen super-Weyl transformation (2.4). If such a super-Weyl gauge is chosen, one stays with a residual super-Weyl plus $\mathsf{U}(1)$ gauge freedom given by [31]

$$\mathcal{D}'_\alpha = \mathrm{e}^{\bar\sigma - \sigma/2}(\mathcal{D}_\alpha + (\mathcal{D}^\beta\sigma)M_{\alpha\beta}), \qquad \bar{\mathcal{D}}_{\dot\alpha}\sigma = 0. \tag{2.6}$$

As is well known (see, e.g., [27] for a review), the different off-shell formulations for $\mathcal{N} = 1$ supergravity are obtained by coupling conformal supergravity [described, e.g., using $\mathsf{U}(1)$ superspace] to a compensator. The latter is a chiral scalar in the case of the old minimal formulation [14,32], a real linear superfield for the new minimal formulation [33], and a complex linear superfield for the nonminimal formulation [34,35]. Our analysis of maximally supersymmetric backgrounds of supergravity does not require fixing any specific compensator.

We now recall an important theorem concerning the maximally supersymmetric backgrounds [4,36]. For any (off-shell) supergravity theory in D dimensions, all maximally supersymmetric spacetimes correspond to those supergravity backgrounds which are characterized by the following properties: (i) all Grassmann-odd components of the superspace torsion and curvature tensors vanish, and (ii) all Grassmann-even components of the torsion and curvature tensors are annihilated by the spinor derivatives. In the case of 4D $\mathcal{N} = 1$ supergravity, this theorem means the following:

$$X_\alpha = 0, \tag{2.7a}$$

$$W_{\alpha\beta\gamma} = 0, \tag{2.7b}$$

$$\mathcal{D}_\alpha R = 0 \to \mathcal{D}_A R = 0, \tag{2.7c}$$

$$\mathcal{D}_\alpha G_{\beta\dot\beta} = \bar{\mathcal{D}}_{\dot\alpha}G_{\beta\dot\beta} = 0 \to \mathcal{D}_A G_{\beta\dot\beta} = 0. \tag{2.7d}$$

Equation (2.7d) has an integrability condition that follows from (2.1b). It is

$$0 = \{\bar{\mathcal{D}}_{\dot\alpha}, \bar{\mathcal{D}}_{\dot\beta}\}G_{\gamma\dot\gamma} = 4R\bar{M}_{\dot\alpha\dot\beta}G_{\gamma\dot\gamma} = 2R(\varepsilon_{\dot\gamma\dot\alpha}G_{\gamma\dot\beta} + \varepsilon_{\dot\gamma\dot\beta}G_{\gamma\dot\alpha}), \tag{2.8}$$

and therefore

$$RG_{\alpha\dot\alpha} = 0. \tag{2.9}$$

Equation (2.7a) tells us that all maximally supersymmetric backgrounds are realized in terms of the Grimm-Wess-Zumino superspace geometry [30].

Relation (2.9) (actually its θ-independent part) was given in [1] without derivation. Let us also show that (2.9) is a simple consequence of the general analysis given in Sec. 6.4 of [3]. Consider a background superspace $(\mathcal{M}^{4|4}, \mathcal{D})$. A supervector field $\xi = \xi^B E_B = \xi^b E_b + \xi^\beta E_\beta + \bar\xi_{\dot\beta}\bar{E}^{\dot\beta}$ on $(\mathcal{M}^{4|4}, \mathcal{D})$ is called Killing if

$$\delta_{\mathcal{K}}\mathcal{D}_A = [\mathcal{K}, \mathcal{D}_A] = 0,$$
$$\mathcal{K} := \xi^B(z)\mathcal{D}_B + \frac{1}{2}K^{bc}(z)M_{bc} + \mathrm{i}\tau(z)\mathbb{J}, \qquad (2.10)$$

for some Lorentz (K^{bc}) and R-symmetry (τ) parameters. All parameters ξ^β, K^{bc}, τ are determined in terms of ξ^b.

Let $\xi = \xi^A E_A$ be a conformal Killing supervector field of $(\mathcal{M}^{4|4}, \mathcal{D})$. As demonstrated in Sec. 6.4 of [3], its explicit form is

$$\xi^A = (\xi^a, \xi^\alpha, \bar{\xi}_{\dot\alpha}) = \left(\xi^a, -\frac{\mathrm{i}}{8}\bar{\mathcal{D}}_{\dot\beta}\xi^{\dot\beta\alpha}, -\frac{\mathrm{i}}{8}\mathcal{D}^\beta\xi_{\beta\dot\alpha}\right), \qquad (2.11)$$

where the vector component $\xi_{\alpha\dot\alpha}$ is real and obeys the equation [3]

$$\mathcal{D}_{(\alpha}\xi_{\beta)\dot\beta} = 0, \qquad (2.12)$$

which implies

$$(\mathcal{D}^2 + 2\bar{R})\xi_{\alpha\dot\alpha} = 0. \qquad (2.13)$$

In accordance with (2.7d), $G_{\alpha\dot\alpha}$ is covariantly constant, and hence it is a solution of (2.12). Then (2.13) reduces to (2.9).

III. MAXIMALLY SUPERSYMMETRIC SOLUTIONS OF PURE R^2 SUPERGRAVITY

We now prove that the curved superspaces described by (1.4) are solutions[5] of the scale invariant supergravity model (1.8). For this we will use the background-field method for $\mathcal{N} = 1$ supergravity as developed by Grisaru and Siegel [38] and further elaborated in [3].

We denote infinitesimal increments of the supergravity prepotentials by H^a and σ, where H^a is real unconstrained and σ is covariantly chiral, $\bar{\mathcal{D}}_{\dot\alpha}\sigma = 0$. The variations of various supergravity functionals under such an infinitesimal change in the prepotentials was computed in Sec. 5.6 of [3] (see also [21]). The results we need here are

$$\begin{aligned}\delta\int &\mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar\theta E R\bar{R}\\ &= -\frac{1}{4}\int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar\theta E\{\sigma\mathcal{D}^2R + \bar\sigma\bar{\mathcal{D}}^2\bar{R}\}\\ &\quad+\frac{1}{2}\int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar\theta E H^{\alpha\dot\alpha}\bigg\{2R\bar{R}G_{\alpha\dot\alpha}\\ &\quad-\frac{1}{6}(\mathcal{D}^2R + \bar{\mathcal{D}}^2\bar{R}) + \frac{\mathrm{i}}{6}\mathcal{D}_{\alpha\dot\alpha}(\bar{\mathcal{D}}^2\bar{R} - \mathcal{D}^2R)\\ &\quad+\frac{2}{3}R\overleftrightarrow{D}_{\alpha\dot\alpha}\bar{R} + \frac{1}{3}(\mathcal{D}_\alpha R)\bar{\mathcal{D}}_{\dot\alpha}\bar{R}\bigg\}, \qquad (3.1a)\end{aligned}$$

$$\begin{aligned}\delta\int &\mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar\theta E R^2\\ &= 3\int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar\theta E(\sigma - \bar\sigma)R^2\\ &\quad+\int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar\theta E H^{\alpha\dot\alpha}\{G_{\alpha\dot\alpha} - \mathrm{i}\mathcal{D}_{\alpha\dot\alpha}\}R^2. \qquad (3.1b)\end{aligned}$$

It is seen that both variations (3.1a) and (3.1b) vanish for the backgrounds (1.4). If the parameter β in (1.8) is nonzero, $\beta \neq 0$, the anti–de Sitter superspace (1.2) is not a solution of the equations of motion for (1.8).

In accordance with (2.7b), all maximally supersymmetric backgrounds of $\mathcal{N} = 1$ supergravity are conformally flat.[6] Therefore all of them are solutions of the equations of motion for $\mathcal{N} = 1$ conformal supergravity described by the chiral action [40,41]

$$I_{\mathrm{CSG}} = \int \mathrm{d}^4x\mathrm{d}^2\theta\mathcal{E}W^{\alpha\beta\gamma}W_{\alpha\beta\gamma} + \mathrm{c.c.}$$

The scale invariant supergravity action (1.8) corresponds to the old minimal formulation for $\mathcal{N} = 1$ supergravity. Within the new minimal formulation for $\mathcal{N} = 1$ supergravity, the construction of pure R^2 supergravity has new features [15,42]. In particular, the gauge field, which is associated with the $\mathsf{U}(1)$ factor of the structure group $\mathsf{SL}(2,\mathbb{C})\times\mathsf{U}(1)$, propagates. The specific feature of this supergravity formulation is that its conformal compensator $\mathbb{G}$ is a real linear superfield, $(\bar{\mathcal{D}}^2 - 4R)\mathbb{G} = (\mathcal{D}^2 - 4\bar{R})\mathbb{G} = 0$, with the super-Weyl transformation law $\mathbb{G}' = \mathrm{e}^{2L}\mathbb{G}$ under (2.4). In $\mathsf{U}(1)$ superspace, any dynamical system is described by a super-Weyl invariant action involving, in general, the compensator. Pure R^2 supergravity is described by the super-Weyl invariant action

$$\begin{aligned}S &= \gamma\int \mathrm{d}^4x\mathrm{d}^2\theta\mathcal{E}\mathbb{X}^\alpha\mathbb{X}_\alpha + \mathrm{c.c.},\\ \mathbb{X}_\alpha &= X_\alpha + \frac{3}{4}(\bar{\mathcal{D}}^2 - 4R)\mathcal{D}_\alpha\ln\mathbb{G}, \qquad (3.2)\end{aligned}$$

for some coupling constant γ. Here $\mathbb{X}_\alpha$ transforms as a primary superfield[7] under the super-Weyl transformations, $\mathbb{X}'_\alpha = \mathrm{e}^{\frac{3}{2}L}\mathbb{X}_\alpha$. The super-Weyl invariance allows us to choose one of the two gauge conditions: either (i) $X_\alpha = 0$, or (ii) $\mathbb{G} = 1$. Choosing the latter (which implies $R = \bar{R} = 0$) reduces the chiral integrand in (3.2) to $X^\alpha X_\alpha$. It is then clear that setting $X_\alpha = 0$ solves the supergravity equations. As was explained in Sec. II, the $\mathsf{U}(1)$ connection is flat, Eq. (2.7a), for all maximally supersymmetric backgrounds

[5] For other solutions of R^2 supergravity, see, e.g., [37].

[6] This is not true for some maximally supersymmetric backgrounds of $\mathcal{N} = 2$ supergravity [39].

[7] This follows from the super-Weyl transformation of X_α given, e.g., in [27].

of $\mathcal{N}=1$ supergravity. We conclude that the backgrounds (1.4) are maximally supersymmetric solutions of pure R^2 supergravity within the new minimal formulation.

IV. CONCLUDING COMMENTS

It is instructive to compare the maximally supersymmetric backgrounds (1.2) and (1.4) with their counterparts for three-dimensional $\mathcal{N}=2$ supergravity.

In three dimensions, the maximally supersymmetric backgrounds of off-shell $\mathcal{N}=2$ supergravity were classified in [43], and also reviewed and elaborated in [4]. The three-dimensional analogue of (1.2) is the (1,1) AdS superspace [44]. The three-dimensional analogues of the backgrounds (1.4) are given by the following algebra of covariant derivatives $\mathcal{D}_A=(\mathcal{D}_a,\mathcal{D}_\alpha,\bar{\mathcal{D}}^\alpha)$:

$$\{\mathcal{D}_\alpha,\mathcal{D}_\beta\}=0,\qquad \{\bar{\mathcal{D}}_\alpha,\bar{\mathcal{D}}_\beta\}=0, \tag{4.1a}$$

$$\{\mathcal{D}_\alpha,\bar{\mathcal{D}}_\beta\}=-2\mathrm{i}(\gamma^c)_{\alpha\beta}(\mathcal{D}_c-2\mathcal{S}M_c-\mathrm{i}\mathcal{C}_c\mathbb{J}) + 4\varepsilon_{\alpha\beta}(\mathcal{C}^cM_c-\mathrm{i}\mathcal{S}\mathbb{J}), \tag{4.1b}$$

$$[\mathcal{D}_a,\mathcal{D}_\beta]=\mathrm{i}\varepsilon_{abc}(\gamma^b)_\beta{}^\gamma\mathcal{C}^c\mathcal{D}_\gamma+(\gamma_a)_\beta{}^\gamma\mathcal{S}\mathcal{D}_\gamma, \tag{4.1c}$$

$$[\mathcal{D}_a,\bar{\mathcal{D}}_\beta]=-\mathrm{i}\varepsilon_{abc}(\gamma^b)_\beta{}^\gamma\mathcal{C}^c\bar{\mathcal{D}}_\gamma+(\gamma_a)_\beta{}^\gamma\mathcal{S}\bar{\mathcal{D}}_\gamma, \tag{4.1d}$$

$$[\mathcal{D}_a,\mathcal{D}_b]=4\varepsilon_{abc}(\mathcal{C}^c\mathcal{C}_d+\delta^c{}_d\mathcal{S}^2)M^d. \tag{4.1e}$$

Here M_c denotes the Lorentz generator (defined in [43]) and the $\mathsf{U}(1)$ generator $\mathbb{J}$ is defined similarly to (2.2). The scalar $\mathcal{S}$ and vector $\mathcal{G}_b$ components of the torsion tensor are constrained by

$$\mathcal{D}_A\mathcal{S}=0,\qquad \mathcal{D}_\alpha\mathcal{C}_b=0\Rightarrow\mathcal{D}_a\mathcal{C}_b=2\varepsilon_{abc}\mathcal{C}^c\mathcal{S}, \tag{4.2}$$

and hence $\mathcal{C}^b\mathcal{C}_b=\text{const}$. We point out that the solution with $\mathcal{C}_a=0$ corresponds to the (2,0) AdS superspace [44]. However, here we are interested in the case $\mathcal{C}_b\neq0$. When both $\mathcal{S}$ and $\mathcal{C}_b$ are nonvanishing, the above curved superspace is a maximally supersymmetric solution of topologically massive type II supergravity [4]. In the case $\mathcal{S}=0$ and $\mathcal{C}_b\neq0$, the above superspace is a solution of three-dimensional R^2 supergravity [45].

One of the most interesting properties of the maximally supersymmetric backgrounds (1.4) is that they allow for the Maxwell-Goldstone multiplet models which describe partial $\mathcal{N}=2\to\mathcal{N}=1$ supersymmetry breaking [5] and reduce to the Bagger-Galperin model [46] in the flat limit, $G_a\to0$.

The $\mathcal{N}=2$ analogue of the scale invariant supergravity (1.8) was given in [47]. It is of interest to see which rigid $\mathcal{N}=2$ maximally supersymmetric backgrounds [39] are solutions of this theory.

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[1] G. Festuccia and N. Seiberg, Rigid supersymmetric theories in curved superspace, J. High Energy Phys. 06 (2011) 114.
[2] S. M. Kuzenko, Symmetries of curved superspace, J. High Energy Phys. 03 (2013) 024.
[3] I. L. Buchbinder and S. M. Kuzenko, *Ideas and Methods of Supersymmetry and Supergravity or a Walk Through Superspace* (IOP, Bristol, 1995); Revised Edition: 1998.
[4] S. M. Kuzenko, Supersymmetric spacetimes from curved superspace, *Proc. Sci.*, CORFU2014 (2015) 140 [arXiv:1504.08114].
[5] S. M. Kuzenko and G. Tartaglino-Mazzucchelli, Nilpotent chiral superfield in $N=2$ supergravity and partial rigid supersymmetry breaking, J. High Energy Phys. 03 (2016) 092.
[6] P. de Medeiros, J. Figueroa-O'Farrill, and A. Santi, Killing superalgebras for Lorentzian four-manifolds, J. High Energy Phys. 06 (2016) 106.
[7] V. P. Akulov and D. V. Volkov, Goldstone fields with spin 1/2, Teor. Mat. Fiz. **18**, 39 (1974) [Theor. Math. Phys. **18**, 28 (1974)].
[8] A. Salam and J. A. Strathdee, Super-gauge transformations, Nucl. Phys. **B76**, 477 (1974).
[9] B. W. Keck, An alternative class of supersymmetries, J. Phys. A **8**, 1819 (1975).
[10] B. Zumino, Nonlinear realization of supersymmetry in de Sitter space, Nucl. Phys. **B127**, 189 (1977).
[11] E. A. Ivanov and A. S. Sorin, Superfield formulation of OSp(1,4) supersymmetry, J. Phys. A **13**, 1159 (1980).
[12] C. R. Nappi and E. Witten, A WZW Model Based on a Nonsemisimple Group, Phys. Rev. Lett. **71**, 3751 (1993).
[13] D. Sen, Supersymmetry in the space-time $R\times S^3$, Nucl. Phys. **B284**, 201 (1987).
[14] J. Wess and B. Zumino, Superfield Lagrangian for supergravity, Phys. Lett. **74B**, 51 (1978).

[15] S. Ferrara, A. Kehagias, and M. Porrati, $\mathcal{R}^2$ supergravity, J. High Energy Phys. 08 (2015) 001.
[16] C. Kounnas, D. Lüst, and N. Toumbas, R^2 inflation from scale invariant supergravity and anomaly free superstrings with fluxes, Fortschr. Phys. **63**, 12 (2015).
[17] S. Ferrara, M. Porrati, and A. Sagnotti, Scale invariant Volkov-Akulov supergravity, Phys. Lett. B **749**, 589 (2015).
[18] S. V. Ketov and A. A. Starobinsky, Embedding $(R+R^2)$-inflation into supergravity, Phys. Rev. D **83**, 063512 (2011).
[19] S. Ferrara, R. Kallosh, and A. Van Proeyen, On the supersymmetric completion of $R+R^2$ gravity and cosmology, J. High Energy Phys. 11 (2013) 134.
[20] I. L. Buchbinder and S. M. Kuzenko, Matter superfields in external supergravity: Green functions, effective action and superconformal anomalies, Nucl. Phys. **B274**, 653 (1986).
[21] I. L. Buchbinder and S. M. Kuzenko, Quantization of the classically equivalent theories in the superspace of simple supergravity and quantum equivalence, Nucl. Phys. **B308**, 162 (1988).
[22] I. L. Buchbinder and S. M. Kuzenko, Nonlocal action for supertrace anomalies in superspace of $N=1$ supergravity, Phys. Lett. B **202**, 233 (1988).
[23] S. Theisen, Fourth-order supergravity, Nucl. Phys. **B263**, 687 (1986); **B269**, 744(E) (1986).
[24] B. de Wit and M. Roček, Improved tensor multiplets, Phys. Lett. B **109**, 439 (1982).
[25] W. Siegel, Gauge spinor superfield as a scalar multiplet, Phys. Lett. **85B**, 333 (1979).
[26] P. S. Howe, A superspace approach to extended conformal supergravity, Phys. Lett. **100B**, 389 (1981); Supergravity in superspace, Nucl. Phys. **B199**, 309 (1982).
[27] S. J. Gates, Jr., M. T. Grisaru, M. Roček, and W. Siegel, *Superspace, or One Thousand and One Lessons in Supersymmetry* (Benjamin/Cummings, Reading, MA, 1983).
[28] D. Butter, $N=1$ conformal superspace in four dimensions, Ann. Phys. (Amsterdam) **325**, 1026 (2010).
[29] D. Butter and S. M. Kuzenko, A dual formulation of supergravity-matter theories, Nucl. Phys. **B854**, 1 (2012).
[30] R. Grimm, J. Wess, and B. Zumino, Consistency checks on the superspace formulation of supergravity, Phys. Lett. **73B**, 415 (1978); A complete solution of the Bianchi identities in superspace, Nucl. Phys. **B152**, 255 (1979).
[31] P. S. Howe and R. W. Tucker, Scale invariance in superspace, Phys. Lett. **80B**, 138 (1978).
[32] K. S. Stelle and P. C. West, Minimal auxiliary fields for supergravity, Phys. Lett. **74B**, 330 (1978); S. Ferrara and P. van Nieuwenhuizen, The auxiliary fields of supergravity, Phys. Lett. **74B**, 333 (1978).
[33] M. F. Sohnius and P. C. West, An alternative minimal off-shell version of $N=1$ supergravity, Phys. Lett. **105B**, 353 (1981).
[34] P. Breitenlohner, A geometric interpretation of local supersymmetry, Phys. Lett. **67B**, 49 (1977); Some invariant Lagrangians for local supersymmetry, Nucl. Phys. **B124**, 500 (1977).
[35] W. Siegel and S. J. Gates, Jr., Superfield supergravity, Nucl. Phys. **B147**, 77 (1979).
[36] S. M. Kuzenko, J. Novak, and G. Tartaglino-Mazzucchelli, Symmetries of curved superspace in five dimensions, J. High Energy Phys. 10 (2014) 175.
[37] A. Cisterna, M. Hassane, and J. Oliva, Exact solutions in $\mathcal{R}^2$ SUGRA, Phys. Rev. D **92**, 104012 (2015).
[38] M. T. Grisaru and W. Siegel, Supergraphity (I). Background field formalism, Nucl. Phys. **B187**, 149 (1981); Supergraphity (II). Manifestly covariant rules and higher loop finiteness, Nucl. Phys. **B201**, 292 (1982).
[39] D. Butter, G. Inverso, and I. Lodato, Rigid 4D $\mathcal{N}=2$ supersymmetric backgrounds and actions, J. High Energy Phys. 09 (2015) 088.
[40] W. Siegel, Solution to constraints in Wess-Zumino supergravity formalism, Nucl. Phys. **B142**, 301 (1978).
[41] B. Zumino, Supergravity and superspace, in *Recent Developments in Gravitation—Cargèse 1978*, edited by M. Lévy and S. Deser (Plenum Press, New York, 1979), pp. 405–459.
[42] S. Cecotti, S. Ferrara, M. Porrati, and S. Sabharwal, New minimal higher derivative supergravity coupled to matter, Nucl. Phys. **B306**, 160 (1988).
[43] S. M. Kuzenko, U. Lindström, M. Roček, I. Sachs, and G. Tartaglino-Mazzucchelli, Three-dimensional $N=2$ supergravity theories: From superspace to components, Phys. Rev. D **89**, 085028 (2014).
[44] S. M. Kuzenko and G. Tartaglino-Mazzucchelli, Three-dimensional $N=2$ (AdS) supergravity and associated supercurrents, J. High Energy Phys. 12 (2011) 052.
[45] S. M. Kuzenko, J. Novak, and G. Tartaglino-Mazzucchelli, Higher derivative couplings and massive supergravity in three dimensions, J. High Energy Phys. 09 (2015) 081.
[46] J. Bagger and A. Galperin, A new Goldstone multiplet for partially broken supersymmetry, Phys. Rev. D **55**, 1091 (1997).
[47] S. M. Kuzenko and J. Novak, On curvature squared terms in $N=2$ supergravity, Phys. Rev. D **92**, 085033 (2015).