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Gauge symmetries emerging from extra dimensions

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We argue that extra dimensions with a properly chosen compactification scheme could be a natural source for emergent gauge symmetries. Actually, some proposed vector field potential terms or polynomial vector field constraints introduced in five-dimensional Abelian and non-Abelian gauge theory are shown to smoothly lead to spontaneous violation of an underlying 5D spacetime symmetry and generate pseudo-Goldstone vector modes as conventional 4D gauge boson candidates. As a special signature, there appear, apart from conventional gauge couplings, some properly suppressed direct multiphoton (multiboson, in general) interactions in emergent QED and Yang-Mills theories whose observation could shed light on their high-dimensional nature. Moreover, in emergent Yang-Mills theories an internal symmetry G also occurs spontaneously broken to its diagonal subgroups once 5D Lorentz violation happens. This breaking originates from the extra vector field components playing a role of some adjoint scalar field multiplet in the 4D spacetime. So, one naturally has the Higgs effect without a specially introduced scalar field multiplet. Remarkably, when being applied to grand unified theories (GUTs) this results in a fact that the emergent GUTs generically appear broken down to the Standard Model just at the 5D Lorentz violation scale M.

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I. INTRODUCTION AND OVERVIEW

Significant progress in understanding the spontaneously broken internal symmetries with accompanying massless scalar Goldstone modes [1] allows one to think that spacetime symmetries, and first of all Lorentz invariance, could also be spontaneously broken so as to generate massless vector and tensor Goldstone modes associated with photons, gravitons and other gauge fields. This has attracted considerable interest over the past fifty years in many different contexts which could be basically classified as the composite models [2–5], constraint-based models [6] and potential-based models [7] (for some later developments see [8–16]). We give below some short formulation of them to make clearer the aims of the present paper.

A. Composite models

Composite models are based on the four-Fermi (or multi-Fermi in general) interaction where the photon and other gauge fields may appear as a fermion-antifermion pair composite state in complete analogy with massless composite scalar fields (identified with pions) in the original Nambu-Jona-Lazinio model [1]. This old idea is better expressed nowadays in terms of effective field theory where the standard QED Lagrangian is readily obtained through the corresponding loop radiative effects due to N fermion species involved [9,10]. One could think, however, that composite models contain too many prerequisites and complications related to the large number of basic fermion species involved, their proper arrangement, nonrenormalizability of the fundamental multi-Fermi Lagrangian, instability under radiative corrections, and so on indefinitely. This approach contains in fact a cumbersome invisible sector which induces the effective emergent theory. A natural question then arises whether one could directly work in the effective vector field theory instead thus having spontaneous Lorentz invariance violation (SLIV) from the outset.

B. Potential-based models

Actually, one could start with a conventional QED-type Lagrangian extended by an arbitrary vector field potential energy terms which explicitly break gauge invariance. For a minimal potential containing bilinear and quartic vector field terms one comes to the Lagrangian

$$L_V = L_{\text{QED}} - \frac{\lambda}{4}(A_\mu A^\mu - n^2M^2)^2, \tag{1}$$

where the mass parameter n^2M^2 stands for the proposed SLIV scale, while n_μ is a properly oriented unit Lorentz vector, $n^2 = n_\mu n^\mu = \pm 1$. This partially gauge invariant model being sometimes referred to as the “bumblebee” model [7] (see also [11] and references therein) means in fact that the vector field A_μ develops a constant background value

$$\langle A_\mu \rangle = n_\mu M \tag{2}$$

and Lorentz symmetry $SO(1,3)$ breaks down at the proposed SLIV scale M to $SO(3)$ or $SO(1,2)$ depending on whether n_μ is timelike $(n^2 = +1)$ or spacelike $(n^2 = -1)$. Expanding the vector field around this vacuum configuration,

$$A_\mu(x) = a_\mu(x) + n_\mu(M + H), \qquad n_\mu a_\mu = 0, \quad (3)$$

one finds that the a_μ field components, which are orthogonal to the Lorentz violating direction n_μ, describe a massless vector Nambu-Goldstone boson, while the H field corresponds to a massive Higgs mode away from the potential minimum. Due to the presence of this mode the model may in principle lead to some physical Lorentz violation in terms of the properly deformed dispersion relations for photon and matter fields involved that appear from the corresponding radiative corrections to their kinetic terms [9]. However, as was argued in [17], a bumblebee-like model appears generally unstable,[1] its Hamiltonian is not bounded from below beyond the constrained phase space determined by the nonlinear condition

$$A_\mu A^\mu = n^2 M^2. \quad (4)$$

With this condition imposed, the massive Higgs mode never appears, the Hamiltonian is positive, and the model is physically equivalent to the nonlinear constraint-based QED, which now we briefly consider.

C. Constraint-based models

This class of models starts directly with the nonlinearly realized Lorentz symmetry for underlying vector field (or vector field multiplet) through the "length-fixing" constraint (4) implemented into conventional gauge invariant theories, both Abelian and non-Abelian ones. This constraint in itself was first studied in the QED framework by Nambu quite a long time ago [6], and in a general context (including loop corrections [19], massive QED framework [20], non-Abelian [21–23] and supersymmetric [24] extensions) in the past decade. The constraint-based models show that, in contrast to the spontaneous violation of internal symmetries, spontaneous Lorentz violation producing vector Goldstone bosons seems not to necessarily imply physical breakdown of Lorentz invariance. Rather, when appearing in a gauge theory framework, this may eventually result in a noncovariant gauge choice in an otherwise gauge invariant and Lorentz invariant theory.

Rather than impose by postulate, the constraint (4) may be implemented into the standard QED Lagrangian L_{QED} through the invariant Lagrange multiplier term

$$L = L_{\mathrm{QED}} - \frac{\lambda}{2}(A_\mu A^\mu - n^2 M^2), \quad n^2 = n_\mu n^\mu = \pm 1, \quad (5)$$

provided that initial values for all fields (and their momenta) involved are chosen so as to restrict the phase space to values with a vanishing multiplier function $\lambda(x)$, $\lambda = 0$. Actually, due to an automatic conservation of the matter current in QED an initial value $\lambda = 0$ will then remain for all time so that the Lagrange multiplier field λ never enters in the physical equations of motions for what follows.[2] It is worth noting that, though the Lagrange multiplier term formally breaks gauge invariance in the Lagrangian (5), this breaking is in fact reduced to the nonlinear gauge choice (4). On the other hand, since gauge invariance is no longer generically assumed, it seems that the vector field constraint (4) might be implemented into the general vector field theory (1) rather than the gauge invariant QED in (5). The point is, however, that both theories are equivalent once the constraint (4) holds. Indeed, due to a simple structure of vector field polynomial in (1), they lead to practically the same equations of motion in both cases.

The constraint (4) is in fact very similar to the constraint appearing in the nonlinear σ-model for pions [25]. It means, in essence, that the vector field A_μ develops some constant background value, $\langle A_\mu(x)\rangle = n_\mu M$, and has a special "Higgsless" expansion around vacuum configuration

$$A_\mu = a_\mu + n_\mu\sqrt{M^2 - n^2 a^2}, \quad n_\mu a_\mu = 0 (a^2 \equiv a_\mu a^\mu) \quad (6)$$

so that Lorentz symmetry formally breaks down, depending on a particular, timelike or spacelike, nature of SLIV mentioned above. The point is, however, that in sharp contrast to the nonlinear σ-model for pions, the nonlinear QED theory ensures that all the physical Lorentz violating effects strictly cancel out among themselves (as was explicitly shown both in the tree [6] and one-loop [19] approximations), due to the starting gauge invariance involved. The noncovariant gauge choice for vector Goldstone bosons shown in (6) appears as the only response of the theory to SLIV.

So to conclude, although it may sound somewhat counterintuitive, one may separate these two aspects: generation of vector Goldstone bosons and physical Lorentz violation. When such a spontaneous violation occurs in the gauge invariant vector field system, this field system generates massless Goldstone modes paying for that just gauge degrees of freedom and leaving the physical ones untouched. As to an observational evidence in favor of emergent theories the only way for SLIV to cause physical Lorentz violation would appear only if gauge invariance in these theories were really broken rather than merely constrained by some gauge condition. Such a violation of gauge invariance could provide the potential-based model

[1] Apart from the instability, the potential-based models were shown [18] to be obstructed from having a consistent ultraviolet completion, whereas the most of viable effective theories possess such a completion.

[2] Interestingly, this solution with the Lagrange multiplier field $\lambda(x)$ being vanished can technically be realized by introducing in the Lagrangian (5) an additional Lagrange multiplier term of the type $\xi\lambda^2$, where $\xi(x)$ is a new multiplier field. One can now easily confirm that a variation of the modified Lagrangian $L + \xi\lambda^2$ with respect to the ξ field leads to the condition $\lambda = 0$, whereas a variation with respect to the basic multiplier field λ preserves the vector field constraint (4).

considered above or some extension of the constraint-based model with high-dimension operators induced by gravity at very small distances [26]. However, in any case, if we are primarily interested in the vector Goldstone generation rather than physical Lorentz violation, it seems more relevant to work in the framework of the constraint-based models rather than in the largely contradictory potential-based ones. We will follow this strategy for the rest of the paper.

D. Models with extra spacetime dimensions

Now, after this brief sketch of valuable SLIV models one can see that all of them only suggest a noncovariant description of vector Goldstone bosons where one vector field spacetime component A_μ $(\mu = 0, 1, 2, 3)$ is “Higgs-ified” (3) or constrained (6). It is rather clear that the only way to produce the vector Goldstone bosons in the fully Lorentz covariant way, both in the potential-based and the constraint-based models, would be to enlarge the existing Minkowski spacetime to higher dimensions. Particularly, the spontaneous breakdown of the “five-dimensional Lorentz symmetry” to the ordinary one, $SO(1,4) \to SO(1,3)$, could generate a conventional four-dimensional vector Goldstone vector field A_μ $(\mu = 0, 1, 2, 3)$ that was first argued quite a long ago [8,27,28], though has not been yet worked out in significant detail. Remarkably, the requirement for a fully covariant description of vector Goldstone fields may have, as we will see later, far going consequences for emergent gauge theories. Actually, in contrast to the above-mentioned 4D models with the hidden SLIV, now due to the proposed compactification scheme to physical four dimensions, the starting 5D gauge invariance in these theories appears broken. This does not allow to gauge away from them some possible observational evidence in favor of their emergent nature.

One could try to implement the high-dimensional SLIV program into the brane models [29] with our physical world assumed to be located on a three-dimensional brane embedded in the high-dimensional bulk. However, a serious problem for such theories seems to be how to achieve the localization of emergent gauge fields on the flat brane associated with our world [30]. In this connection, a more attractive possibility seems to be related to a class of extra-dimensional models known as universal extra dimensions (UED) [31–33]. In them the Standard Model fields (or, at least, some essential part of them) are free to propagate through all of the dimensions of space, rather than being confined to our physical spacetime as they typically are in the brane models. Naturally, the UED models look more similar to the original Kaluza-Klein (KK) proposal than somewhat more sophisticated brane model scenarios [29,34]. Phenomenologically, the UED models with the KK parity involved considerably relaxes the constraints from electroweak precision data, allowing for much lower scales of compactification $M_{\mathrm{KK}} = 1/R$ (even up to a few TeV order scale). Another important aspect related to them appears in its ability to provide a natural candidate for the dark matter in the universe. In particular, the lightest KK state can be stable and produced in the early universe with an abundance similar to that of the measured dark matter density. One more attractive feature seems to be that the UEDs, as was mentioned above, could also be a natural source for vector Goldstone bosons associated with photons and other gauge fields, particularly if one proceeds in the five-dimensional UED framework.

E. The present paper

We argue that extra dimensions with a properly chosen compactification scheme could be a natural source for emergent gauge symmetries. We start with a simple QED type theory with the SLIV in five-dimensional (5D) spacetime. This 5D SLIV could appear due to some vector field constraint being a high-dimensional analog of the constraint considered above (4), as is argued in Sec. II. This lead to the spontaneous violation of the 5D Lorentz symmetry at some high scale M that proposedly goes along with a compactification of the 5D spacetime down to physical four dimensions at the comparable scale M_{KK}. This is in fact the symmetrical orbifold compactification S_1/Z_2 under which all spacetime components of the 5D vector field $A_{\bar{\mu}}$ $(\bar{\mu} = 0, 1, 2, 3; 5)$ are taken to be even. The important point is that such a compactification, which breaks the starting 5D gauge invariance to a conventional 4D gauge invariance for the vector field ground modes A_μ^0 $(\mu = 0, 1, 2, 3)$ may significantly contribute into the physical processes involved. As a special signature, there appear, apart from conventional gauge couplings, some properly suppressed direct multiphoton (multiboson, in general) interactions in emergent QED and Yang-Mills theories. This means that they actually possess only a partial gauge invariance whose observation could shed light on their high-dimensional nature. In Sec. III we turn to the Yang-Mills theories where not only spacetime symmetry but also internal symmetry appears spontaneously broken once the 5D SLIV happens. Remarkably, this breaking looks like the breaking that is usually induced by an appropriate adjoint scalar field multiplet incorporated into the vector field theory. Now this breaking originates from extra vector field components. Therefore, one may have a somewhat generic Higgs effect in the 5D SLIV theory which breaks the starting internal symmetry to its diagonal subgroups that we discuss in detail in Sec. IV. The most successful implementation of this phenomena may appear in grand unified theories considered *ab initio* in the five-dimensional spacetime. As a result, these theories have to be naturally broken down to the Standard Model at the 5D Lorentz violation scale M. Finally in Sec. V we conclude.

II. EMERGENT QED STEMMING FROM 5D SPACETIME

For the reader's convenience, we will separate further discussion into the particular steps that are needed for a final formulation of the emergent QED theory in four dimensions.

A. Five-dimensional QED with vector field constraints

We start considering an Abelian $U(1)$ vector field theory in the 5D Minkowski spacetime with an action

$$S = \int \mathrm{L}_{5\mathrm{D}} d^4x dy, \tag{7}$$

where x^μ are conventional 4D coordinates and y describes an extra dimension (which we refer to as the fifth coordinate). The Lagrangian $\mathrm{L}_{5\mathrm{D}}$ is a conventional QED Lagrangian which according to our philosophy also includes some covariant constraint put on five-dimensional vector field $A_{\bar{\mu}}$. This may be implemented, as in the above 4D spacetime case (5), through an appropriate invariant Lagrange multiplier term so that the Lagrangian $\mathrm{L}_{5\mathrm{D}}$ without matter looks as

$$\mathrm{L}_{5\mathrm{D}} = -\frac{1}{4}F_{\bar{\mu}\bar{\nu}}F^{\bar{\mu}\bar{\nu}} - \frac{\lambda}{2}(A_{\bar{\mu}}A^{\bar{\mu}} - \mathrm{n}^2\mathrm{M}_5^2), \quad \mathrm{n}^2 = \mathrm{n}_{\bar{\mu}}\mathrm{n}^{\bar{\mu}} = \pm 1, \tag{8}$$

where $\bar{\mu}, \bar{\nu}$ are 5D indices, while μ, ν are 4D indices ($\bar{\mu}, \bar{\nu} = \mu, \nu; 5 = 0, 1, 2, 3; 5$). The $\lambda(x, y)$ is the Lagrange multiplier function, while the mass parameter M_5 stands for the mass scale where the 5D Lorentz invariance is proposed to appear spontaneously broken along the vacuum direction given now by a properly oriented 5D unit vector $\mathrm{n}_{\bar{\mu}}$ which describes both of the 5D Lorentz violation cases (timelike $\mathrm{n}^2 = 1$ or spacelike $\mathrm{n}^2 = -1$) just by analogy with the known 4D constraints (4) discussed above. To see more detail one has to come to conventional four dimensions. Some lessons which can be retrieved from this tour may appear rather interesting for the 5D SLIV.

Assuming that the extra dimension is compactified as a circle of a radius of R, so that $y \equiv R\theta$, where θ is an angular coordinate $-\pi \leq \theta \leq \pi$, we put the periodicity condition on the starting 5D vector gauge fields taken, $A_{\bar{\mu}}(x, \theta) = A_{\bar{\mu}}(x, \theta + 2\pi)$. This allows for a Fourier expansion as

$$A_{\bar{\mu}}(x, \theta) = \frac{1}{\sqrt{2\pi R}}A_{\bar{\mu}}^0(x) + \sum_{s=1}^{\infty}\frac{1}{\sqrt{\pi R}}[A_{\bar{\mu}}^s(x)\cos(s\theta) + \hat{A}_{\bar{\mu}}^s(x)\sin(s\theta)], \tag{9}$$

where the first term in square brackets describes the modes being even under reflection of the fifth coordinate ($\theta \to -\theta$), while the second one describes the modes which are odd under that reflection. Upon putting $A_{\bar{\mu}}(x, \theta)$ into the action (7) and integration over the extra dimension one gets for kinetic terms of the 4D vector field components

$$\mathrm{L}_{4\mathrm{D,kin}} = \sum_{s=0}\left[-\frac{1}{4}F_{\mu\nu}^s F^{s,\mu\nu} + \frac{1}{2}\left(\partial_\mu A_5^s - \frac{s}{R}\hat{A}_\mu^s\right)^2\right] + (A \leftrightarrow \hat{A}), \tag{10}$$

where taking the fifth-coordinate derivative we have used, as prescribed above, $\partial/\partial y = (1/R)\partial/\partial\theta$. One can see that the terms within round brackets mix even and odd modes. These combinations due to the starting gauge invariance of the 5D theory

$$A_{\bar{\mu}} \to A_{\bar{\mu}} + \partial_{\bar{\mu}}\alpha(x, \theta) \tag{11}$$

provide the mass term arrangement for KK towers. Indeed, with a general parametrization (9) taken for gauge parameter α one has from (11) for even KK modes

$$A_\mu^s \to A_\mu^s + \partial_\mu\alpha^s(x), \qquad A_5^s \to A_5^s - \frac{s}{R}\hat{\alpha}^s(x) \tag{12}$$

and similarly for odd modes. Now, using this gauge freedom to diagonalize the mixed terms in $\mathrm{L}_{5\mathrm{D,kin}}$ by proper fixing the gauges

$$\alpha^s = -(R/s)\hat{A}_5^s, \qquad \hat{\alpha}^s = -(R/s)A_5^s \tag{13}$$

one finally gets

$$\mathrm{L}_{4\mathrm{D,kin}} = \sum_{s=0}\left[-\frac{1}{4}F_{\mu\nu}^s F^{s,\mu\nu} + \frac{1}{2}\left(\frac{s}{R}\right)^2 A_\mu^s A^{s,\mu} + \frac{1}{2}(\partial_\mu A_5^0)^2\right] + (A \leftrightarrow \hat{A}). \tag{14}$$

Hence, the only massless vector field is given by the zero mode A_μ^0, while all KK modes acquire a mass by absorbing the scalars A_5^s. This resembles the Higgs mechanism with A_5^s playing the role of the Goldstone bosons associated to the spontaneous 5D spacetime isometry breaking [35]. Remarkably, upon the gauge fixing arrangement (13) made for nonzero KK modes there still remains the $U(1)$ gauge symmetry in the effective 4D theory with a massless gauge field A_μ^0. Apart from that, the massless scalar A_5^0 is also survived. However, this extra degree of freedom appearing at zero level can be projected out from the theory if the starting vector field component $A_5(x, \theta)$ is chosen to be odd under the reflection $\theta \to -\theta$ mentioned above.

B. Looking for a natural compactification

An adequate compactification certainly is a point of our special interest in connection to the 5D SLIV. As is well known, the necessity of producing chiral fermions in four dimensions requires, on general grounds, to consider the orbifold compactification S_1/Z_2 rather than a simple compactification on a circle. This orbifold compactification consists in fact of projecting a circular extra dimension onto a line with two fixed points, $\theta = 0$ and $\theta = \pi$ (or $y = 0$ and $y = \pi R$ for the y coordinate). It removes the unwanted fermionic degrees of freedom, allowing for an existence of chiral fermions [31–33]. Actually, one has to start with two 5D Dirac fermion fields to get independent left-handed and right-handed chiral modes in four dimensions: one field Ψ_1 which has the quantum numbers of the left-handed spinor, and one field Ψ_2 with the quantum numbers of the right-handed spinor.[3] To exclude the additional degrees of freedom one formulates the theory on an orbifold so that Ψ_1 is required to be odd under the $\theta \to -\theta$ orbifold symmetry, while Ψ_2 is even,

$$\Psi_1(x,\theta) = -\gamma^5\Psi_1(x,-\theta), \quad \Psi_2(x,\theta) = \gamma^5\Psi_2(x,-\theta). \tag{15}$$

In these fermion fields initially having a general form (9) only remain the parts

$$\Psi_{1,2}(x,\theta) = \frac{1}{\sqrt{\pi R}}\Psi^0_{L1,R2}(x) + \sum_{s=1}\sqrt{\frac{2}{\pi R}}[\Psi^s_{L1,R2}(x)\cos(s\theta) + \hat{\Psi}^s_{R1,L2}(x)\sin(s\theta)], \tag{16}$$

respectively.[4] As a result, their higher KK modes are four-dimensional vectorlike fermions, while the zero modes are chiral ones being properly determined by the chirality projectors $(1 \mp \gamma^5)/2$. As usual, their gauge couplings are in fact related separately to each of these fermions, whereas in the Yukawa coupling they "work" together.[5]

At this point one must specify how all other fields transform under the proposed orbifold projection. Specifically for vector fields, one usually requires the "asymmetrical" compactification [31–33] according to which the ordinary four components of the 5D vector field $A_{\bar{\mu}}(x,\theta)$ are even under the orbifold transformation, whereas its fifth component is odd. This allows in the gauge invariant theory context to completely remove this component from the theory excluding its zero mode A_5^0 by orbifold projection and gauging away the higher ones A_5^s, as was discussed above. Thus, only massless ground modes A_μ^0, as the Standard Model gauge field candidates, and massive vector KK towers A_μ^s $(s = 1, 2, \ldots)$ are left in the theory. However, as one can readily see, such a procedure differently treating the vector field components explicitly breaks the starting 5D Lorentz invariance that is hardly acceptable if one tries to break it spontaneously. Thus, we propose, in direct contrast to a common practice, the "symmetrical" compactification in which all the 5D vector field components are even under the orbifold transformation

$$A_{\bar{\mu}}(x,-\theta) = A_{\bar{\mu}}(x,\theta), \qquad \bar{\mu} = 0, 1, 2, 3; 5. \tag{17}$$

This, as we see below, may naturally conserve the 5D symmetrical form of all possible nonderivative terms in the starting Lagrangian (8) including the proposed vector field constraint terms that induce the SLIV. Interestingly, such a "partially increased" Lorentz invariance significantly reduces an effective gauge symmetry appearing for vector field components after compactification. Indeed, for the vector field kinetic terms one has now (when all the orbifold-asymmetrical vector field components vanish, $\hat{A}^s_{\bar{\mu}} = 0$)

$$\mathrm{L}_{4\mathrm{D,kin}} = \sum_{s=0}\left[-\frac{1}{4}F^s_{\mu\nu}F^{s,\mu\nu} + \frac{1}{2}(\partial_\mu A_5^s)^2 + \frac{1}{2}\left(\frac{s}{R}A_\mu^s\right)^2\right] \tag{18}$$

and gauge symmetry (12) for KK states, both massive vectors A_μ^s $(s = 1, 2, \ldots)$ and massless scalars A_5^s $(s = 0, 1, 2, \ldots)$, does not work any longer. Only standard gauge invariance for massless ground vector modes A_μ^0 holds

$$A_\mu^0 \to A_\mu^0 + \partial_\mu\alpha^0(x) \tag{19}$$

which looks as if the 5D gauge function α in (11) would not depend on the fifth coordinate and, therefore, only its ground component α^0 was nonzero. These states are completely decoupled from each other. Whereas zero vector field modes being protected by the above gauge invariance are left massless, the massless scalars become eventually massive through all the radiative corrections

[3] Speaking about quantum numbers we have in mind the Standard Model extension of our present QED framework.

[4] The normalization of the fermion field here and all other fields everywhere below is now chosen in accordance with an assumption that the range for the angle variable θ be from 0 to π.

[5] Note that generally after integrating over the fifth coordinate the sum over KK number s of the vector and matter fields in kinetic or interaction terms in the corresponding effective 4D Lagrangian must be zero since this is just conservation of the fifth dimension momentum. This conservation law, being in essence the translational invariance along the extra dimension, appears as an internal symmetry in the 4D KK decomposition, with internal charges, s. Although the introduction of orbifold compactifications breaks the above-mentioned symmetry related to conservation of the fifth dimension momentum, a subgroup of the KK number conservation known as KK parity still remains. In our 5D case compactified on an S_1/Z_2 orbifold, the KK parity is the Z_2 symmetry and can be simply written as $P = (-1)^s$ where s denotes the sth KK mode. Thus, only modes with odd KK number are charged.

involved. Thus, they seem not to produce serious difficulties for the model, as could happen if the extended gauge symmetry (12) providing their masslessness remained. Note also that, though the starting stress-tensor $F_{\bar{\mu}\bar{\nu}}$ does not look invariant under the symmetrical orbifold transformation (17) the final Lagrangian $\mathrm{L}_{4\mathrm{D,kin}}$ appearing upon the compactification really does.[6]

C. Spacetime symmetry breaking phase

Let us now turn to the Lagrange multiplier term in the starting Lagrangian (8) which is proposed to cause spontaneous 5D Lorentz violation. Taking also the multiplier function to be, as all other fields but fermions, symmetrical under orbifold transformation,

$$\lambda(x,\theta) = \frac{1}{\sqrt{\pi R}}\left[\lambda^0(x) + \sqrt{2}\sum_{s=1}\lambda^s(x)\cos(s\theta)\right] \qquad (20)$$

and varying the action with respect to all KK components, λ^0 and λ^s, one has after integration over the angle θ

$$A^0_{\bar{\mu}}A^{0\bar{\mu}} + \sum_{s=1}A^s_{\bar{\mu}}A^{s\bar{\mu}} = \mathrm{n}^2\mathrm{M}^2,$$
$$\sqrt{2}A^0_{\bar{\mu}}A^{s\bar{\mu}} + \sum_{s'=1}A^{s-s'}_{\bar{\mu}}A^{s'\bar{\mu}} = 0 \quad (s = 1,2,\ldots), \qquad (21)$$

respectively.[7] The evident relation was also used between 4D and 5D mass scales, $\mathrm{M}^2 = (\pi R)\mathrm{M}_5^2$. The first constraint in (21) resembles the 4D constraint discussed above (4), while the others are new. Actually we have one constraint for each vector field mode, $A^0_{\bar{\mu}}$ and $A^s_{\bar{\mu}}$ $(s = 1,2,\ldots)$. One can see that, though the symmetrical orbifold compactification (17) taken above for vector field breaks the 5D Lorentz invariance in the Lagrangian (18), it perfectly conserves the 5D invariant form of the constraints (21). They lead in turn to spontaneous violation of 5D Lorentz symmetry and production of massless 4D vector bosons as the corresponding Goldstone modes which, due to the lesser symmetry of the total Lagrangian, are in essence the pseudo-Goldstone modes (see below). In contrast, a conventional asymmetrical orbifold compactification for starting 5D vector field $A_{\bar{\mu}}(x,\theta)$ would explicitly break the Lagrangian 5D form invariance of these constraints[8] and make such an implementation impossible.

Applying the same constraints, as they are given in (21), to a possible VEV (vacuum expectation value) of the 5D vector field $A_{\bar{\mu}}(x,\theta)$ expanded in a Fourier cosine series in (9) one could conclude that this VEV may only develop on its ground mode rather than the higher KK ones in order not to be dependent on the extra dimension coordinate. Thus, the starting 5D Lorentz symmetry will break due to the VEV developed solely on the zero modes $\mathbf{A}^0_{\bar{\mu}}$. As to the particular spacetime component $\bar{\mu}$ on which this VEV may develop, we propose that just the spacelike 5D SLIV case $(\mathrm{n}^2 = -1)$ is realized in the present model. Particularly, this symmetry will indeed be spontaneously broken to ordinary Lorentz invariance

$$SO(1,4) \to SO(1,3) \qquad (22)$$

at a scale M

$$\langle A_{\bar{\mu}}\rangle = \mathrm{n}_{\bar{\mu}}\mathrm{M}, \qquad \mathrm{n}^2 = -1 \qquad (23)$$

with the vacuum direction given now by the "unit" vector $\mathrm{n}_{\bar{\mu}}$ with the only nonzero component $\mathrm{n}_{\bar{\mu}} = g_{\bar{\mu}5}$ just along the extra dimension. One can write again, as in the 4D case mentioned above (6), the ground vector field expansion around vacuum configuration stemming from the upper constraint in (21)

$$A^0_{\bar{\mu}} = a_{\bar{\mu}} + \mathrm{n}_{\bar{\mu}}\sqrt{\mathrm{M}^2 + a^2 + (A^s)^2}, \qquad (24)$$

where summation over all repeated indices is taken

$$a^2 \equiv a_{\bar{\mu}}a^{\bar{\mu}}, \qquad (A^s)^2 \equiv \sum_{s=1}A^s_{\bar{\mu}}A^{\bar{\mu}s}, \qquad (25)$$

and also the orthogonality condition for the emergent pseudo-Goldstone modes $a_{\bar{\mu}}$,

$$\mathrm{n}^{\bar{\mu}}a_{\bar{\mu}} = 0, \qquad (26)$$

is supposed. Meanwhile, the effective Higgs field in the model is given by

$$\mathrm{H} = \mathrm{n}^{\bar{\mu}}A^0_{\bar{\mu}} = A^0_5 = \sqrt{\mathrm{M}^2 + a^2 + (A^s)^2}. \qquad (27)$$

Note, as mentioned above, that, while the constraints (21) are formally 5D Lorentz invariant, the vector field kinetic terms in the 4D Lagrangian (18) and also all interaction terms involved possess only ordinary 4D Lorentz

[6]Notably, though the derivative along the extra dimension is not invariant under orbifold reflection ($\partial_5 \to -\partial_5$), it is actually replaced by $\partial_5 \to -is/R$ in the Fourier decomposition of the stress-tensor $F_{\bar{\mu}\bar{\nu}}$ and then is modulo squared in the kinetic terms so that the Lagrangian (18) appears perfectly invariant.

[7]Note that, for convenience and to emphasize the KK mode number conservation[5], we formally included in the sums here and everywhere below the 4D KK modes A^S_μ with possible negative numbers S as well, though for the symmetrical orbifold compactification taken one has $A^S_\mu = A^{|S|}_\mu$ for every value of S.

[8]In this case the first terms in the constraints (21) containing zero modes would have only 4D invariant form, being just $(A^0_\mu)^2$ and $\sqrt{2}A^0_\mu A^s_\mu$, respectively.

invariance once the compactification occurs. This means that all the 4D modes a_μ appeared in the above expansion (24) are in fact pseudo-Goldstone bosons (PGB) related to the accidental symmetry breaking (22) of the constraints (21), rather than true Goldstone vector modes. Remarkably, in contrast to the familiar scalar PGB case [25], these vector PGBs remain strictly massless being protected by gauge invariance (19) surviving after our symmetrical orbifold compactification for massless ground vector mode A^0_μ which coincides with the PGB state a_μ in the expansion (24).

D. Emergent QED: Some immediate consequences

Finally, one can see that the QED theory emerging from the 5D spacetime has a quite simple form, though it contains some extra interaction terms. Indeed, separating ground modes and heavy KK modes in the 4D Lagrangian (18) and putting the expansion (24) one eventually comes to the emergent QED theory in four dimensions (matter terms are omitted)

$$\mathrm{L}_{em}(a,A^s) = -\frac{1}{4}f_{\mu\nu}f^{\mu\nu} + \frac{1}{2}(\partial_\mu \mathrm{H})^2 + \sum_{s=1}\left[-\frac{1}{4}F^s_{\mu\nu}F^{s,\mu\nu} + \frac{1}{2}(\partial_\mu A^s_5)^2 + \frac{1}{2}\left(\frac{s}{R}A^s_\mu\right)^2\right], \tag{28}$$

where we have introduced the stress tensor for PGB modes, $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$, and the "kinetic" term for effective Higgs field H (27). The latter, when properly expanded, gives all possible multiboson couplings

$$\frac{1}{2}(\partial_\mu \mathrm{H})^2 = \frac{1}{2}\frac{(a_\rho \partial_\mu a^\rho + \sum_{s=1}A^s_{\bar\rho}\partial_\mu A^{s,\bar\rho})^2}{\mathrm{M}^2 + a^2 + (A^s)^2} = \frac{1}{2\mathrm{M}^2}\left(a_\rho\partial_\mu a^\rho + \sum\nolimits_{s=1}A^s_{\bar\rho}\partial_\mu A^{s,\bar\rho}\right)^2 \times \left[1 + \sum_{n=1}^{\infty}\left(-\frac{a^2 + (A^s)^2}{\mathrm{M}^2}\right)^n\right] \tag{29}$$

in addition to conventional QED interactions. Thus, starting from the order of $\mathcal{O}(1/\mathrm{M}^2)$ there appear some direct photon-photon scattering couplings and also coupling photons with heavy KK modes in the emergent QED which, therefore, possess only a partial gauge invariance. In contrast to the known 4D Nambu model [6], where the direct photon-photon scattering amplitudes are always canceled by accompanying longitudinal photon exchange terms, in the 5D model they appear alone and consequently are survived. Therefore, their observation could shed light on the emergent nature of QED stemming from 5D spacetime. Interestingly, due to the orbifold symmetry taken for vector fields and fermions (17), (15) the matter fields (both fermions and scalars) when being introduced into the 5D QED do not produce any new "emergent" couplings for their ground modes. So, only vector fields, photons and heavy KK modes acquire some extra direct multiboson interactions (29) when the effective Higgs field H related to the 5D Lorentz violation is properly expanded in the basic Lagrangian (28).

Another crucial prediction of the emergent QED is an existence of the unabsorbed fifth-direction nonzero modes A^s_5 $(s = 1, 2, \ldots)$ being massless at the tree level. Due to KK parity[5] they can be only produced by pairs from an ordinary matter being properly suppressed by the 5D Lorentz violation scale M [as in the photon-photon scattering processes given above in (29)] or by the compactification mass $M_{\mathrm{KK}} \sim 1/R$ (when such a process is caused by the heavy KK mode exchange). On the other hand, any heavy KK state will now rapidly decay into the A^s_5 mode plus ordinary matter that seems to invalidate the dark matter scenario related to extra dimension [33]. However, these A^s_5 modes being no more protected by gauge invariance could in principle acquire large masses through radiative corrections so that the lightest KK state may appear rather stable to provide the measured dark matter density.

III. EMERGENT YANG-MILLS THEORY

We now consider Yang-Mills theory in the five-dimensional Minkowski spacetime with the vector field Lagrangian

$$\mathcal{L}_{5\mathrm{D}} = -\frac{1}{4}\mathrm{Tr}|\mathbf{F}_{\bar\mu\bar\nu}|^2 + \lambda[\mathrm{Tr}(\mathbf{A}_{\bar\mu}\mathbf{A}^{\bar\mu}) - \mathbf{n}^2\mathrm{M}_5^2],$$
$$\mathbf{F}_{\bar\mu\bar\nu} = \partial_{\bar\mu}\mathbf{A}_{\bar\nu} - \partial_{\bar\nu}\mathbf{A}_{\bar\mu} + ig_5[\mathbf{A}_{\bar\mu}, \mathbf{A}_{\bar\nu}] \quad (\bar\mu, \bar\nu = \mu, \nu; 5). \tag{30}$$

This non-Abelian internal symmetry case is supposed to be given by a general local group G with generators t^i ($[t^i, t^j] = if^{ijk}t^k$ and $\mathrm{Tr}(t^i t^j) = \delta^{ij}$ where f^{ijk} are structure constants and $i, j, k = 0, 1, \ldots, N-1$). The corresponding 5D vector fields which transform according to its adjoint representation are given in the proper matrix form $\mathbf{A}_{\bar\mu} = \mathbf{A}^i_{\bar\mu}t^i$, while the possible matter fields (fermions, for definiteness) could be presented in the fundamental representation column ψ^r $(r = 0, 1, \ldots, d-1)$ of G. According to our philosophy, the starting theory, as in the above Abelian case, also contains some covariant constraint put on 5D vector field $\mathbf{A}_{\bar\mu}$ that causes at the scale M_5 a spontaneous violation of the 5D Lorentz invariance involved. This is arranged through the Lagrange multiplier term in the Lagrangian (30) with the multiplier function $\lambda(x, y)$ depending in general on all five coordinates. The vacuum direction is given now by a properly oriented unit rectangular matrix $\mathbf{n}^i_{\bar\mu}$ which describes in general both of the 5D Lorentz violation cases (timelike or spacelike)

$$\mathbf{n}_{\bar{\mu}} = \mathbf{n}^i_{\bar{\mu}} t^i, \qquad \mathbf{n}^2 = \mathbf{n}_{\bar{\mu}} \mathbf{n}^{\bar{\mu}} = \pm 1. \tag{31}$$

Decomposing all fields in the Lagrangian (30) in a Fourier cosine series along the fifth coordinate one has

$$\mathbf{A}_{\bar{\mu}}(x,\theta) = \frac{1}{\sqrt{\pi R}}\left[\mathbf{A}^0_{\bar{\mu}}(x) + \sqrt{2}\sum_{s=1}^{\infty}\mathbf{A}^s_{\bar{\mu}}(x)\cos(s\theta)\right],$$
$$\lambda(x,\theta) = \frac{1}{\sqrt{\pi R}}\left[\lambda^0(x) + \sqrt{2}\sum_{s=1}^{\infty}\lambda^s(x)\cos(s\theta)\right], \tag{32}$$

where it was again proposed that they all five vector field components $\mathbf{A}_{\bar{\mu}}$, as well as the multiplier function λ, are even under orbifold transformation

$$\mathbf{A}_{\bar{\mu}}(x,-\theta) = \mathbf{A}_{\bar{\mu}}(x,\theta), \qquad \lambda(x,-\theta) = \lambda(x,\theta). \tag{33}$$

Then integrating the action over the angle θ and varying the resulting 4D Lagrangian $\mathcal{L}_{4\mathrm{D}}$ with respect to the zero and higher KK modes, λ^0 and λ^s of the multiplier function $\lambda(x,\theta)$ one obtains all possible constraints put on the properly normalized vector field 4D modes[7]

$$\mathrm{Tr}(\mathbf{A}^0_{\bar{\mu}}\mathbf{A}^{0\bar{\mu}}) + \sum_{s=1}\mathrm{Tr}(\mathbf{A}^s_{\bar{\mu}}\mathbf{A}^{s\bar{\mu}}) = \mathbf{n}^2\mathrm{M}^2,$$
$$\sqrt{2}\mathrm{Tr}(\mathbf{A}^0_{\bar{\mu}}\mathbf{A}^{s\bar{\mu}}) + \sum_{s'=1}\mathrm{Tr}(\mathbf{A}^{s-s'}_{\bar{\mu}}\mathbf{A}^{s'\bar{\mu}}) = 0 \quad (s = 1,2,\ldots), \tag{34}$$

where the evident relation was also used between 4D and 5D mass scales, $\mathrm{M}^2 = (\pi R)\mathrm{M}_5^2$. Eventually, we have one constraint for each vector field mode, $\mathbf{A}^0_{\bar{\mu}}$ and $\mathbf{A}^s_{\bar{\mu}}$ $(s = 1,2,\ldots)$, while the final 4D Lagrangian (with the Lagrange multiplier term omitted) may be written as

$$\mathcal{L}_{4\mathrm{D}} = -\frac{1}{4}\sum_{s=0}\mathrm{Tr}(|\bar{\mathbf{F}}^s_{\mu\nu}|^2 - 2|\bar{\mathbf{F}}^s_{\mu 5}|^2)$$
$$+ \sum_{s=1}\mathcal{O}[(\mathbf{A}^0)^2(\mathbf{A}^s)^2, (\mathbf{A}^0\mathbf{A}^s)(\mathbf{A}^s)^2, (\mathbf{A}^s)^2(\mathbf{A}^s)^2], \tag{35}$$

where we truncated the vector field covariant derivatives ignoring in the stress tensors $\bar{\mathbf{F}}^s_{\mu\nu}$ and $\bar{\mathbf{F}}^s_{\mu 5}$ the commutator terms for nonzero KK modes (appearing in the Lagrangian in the indicated orders) that are unessential for the further analysis. One can see that, though the symmetrical orbifold compactification (33) taken above for vector field multiplet $\mathbf{A}_{\bar{\mu}}$ breaks the 5D Lorentz invariance in the Lagrangian (35), it perfectly conserves the 5D invariant form of the constraints (34). They lead in turn to spontaneous violation of 5D Lorentz symmetry and production of massless 4D vector bosons as the corresponding pseudo-Goldstone modes related to the total symmetry breaking.

Let us consider this 5D SLIV phenomenon in more detail. Applying the same constraints (34) to possible VEV of the 5D vector field multiplet $\mathbf{A}_{\bar{\mu}}(x,\theta)$ expanded in a Fourier series in (32) one could conclude that, as in the above Abelian case, this VEV may only develop on its ground mode rather than the higher KK ones in order not to be dependent on the extra dimension coordinate. Thus, the starting 5D Lorentz symmetry breaks due to the VEV developed solely on the zero modes $\mathbf{A}^0_{\bar{\mu}}$. As to the particular spacetime component $\bar{\mu}$ on which this VEV may develop, we propose in what follows the spacelike 5D SLIV $(\mathbf{n}^2 = -1)$ in the theory, thus taking the case $\mathbf{n}^2 = -1$ in (31). However, there is one special point in the non-Abelian theory framework (with an internal symmetry group G introduced) that has been studied before in a conventional 4D spacetime [21–23]. Namely, although we only propose the $SO(1,4)\times G$ invariance of the Lagrangian (35), the vector field constraint (34) (or, equally, some possible polynomial potential terms which could be included into the starting Lagrangian $\mathcal{L}_{5\mathrm{D}}$) possesses in fact much higher accidental global symmetry $SO(N,4N)$ determined by the dimensionality N of the G group adjoint representation to which the vector field multiplet $\mathbf{A}_{\bar{\mu}}$ belongs. This symmetry is indeed spontaneously broken

$$SO(N,4N) \to SO(N,4N-1) \tag{36}$$

at a scale M

$$\langle \mathbf{A}^i_{\bar{\mu}}\rangle = \mathbf{n}^i_{\bar{\mu}}\mathrm{M} \tag{37}$$

with the vacuum direction given by the matrix $\mathbf{n}^i_{\mu}$ describing now the 5D spacelike SLIV case, $\mathbf{n}^2 = -1$. Without loss of generality, this matrix can be written in the factorized "two-vector" form $\mathbf{n}^i_{\bar{\mu}} = n_{\bar{\mu}}\mathbf{I}^i$ where $n_{\bar{\mu}}$ is the unit Lorentz vector which is oriented in 5D spacetime so as to be parallel to the vacuum matrix $\mathbf{n}^i_{\bar{\mu}}$, while $\mathbf{I}^i$ is the unit vector in the internal space $(\mathbf{I}^i\mathbf{I}^i = 1)$. This matrix $\mathbf{n}^i_{\mu}$ has in fact only one nonzero element subject to the appropriate $SO(N,4N)$ rotation. This is, specifically,

$$\mathbf{n}^i_{\bar{\mu}} = n_{\bar{\mu}}\mathbf{I}^i = g_{\bar{\mu}5}\delta^{ii_0}, \tag{38}$$

provided that the vacuum expectation value (37) is developed along the i_0 direction in the internal space and along the $\bar{\mu} = 5$ direction, respectively, in the 5D Minkowski spacetime.

One can readily see that, in response to this breaking (36) the $5N-1$ massless modes according to a number of broken generators are therefore produced. Actually, due to the symmetry reduction in the post-compactification Lagrangian (35) all these Goldstone modes $\mathbf{a}^i_{\bar{\mu}}$ are in fact pseudo-Goldstone bosons related to breaking of the accidental $SO(N,4N)$ symmetry of the SLIV constraints (34). They are excited along the directions being orthogonal to the vacuum determined by the above unit vector $\mathbf{n}^i_{\bar{\mu}}$

$$\mathbf{n}_{\bar{\mu}}^i \mathbf{a}_{\bar{\mu}}^i = \mathbf{0} \quad (i = i_0, 1, \ldots, N-1). \tag{39}$$

These PGBs include N four-component vector modes $\mathbf{a}_\mu^i$ which complete the adjoint vector field multiplet of the internal symmetry group G. Again as in the Abelian case, these vector PGBs, in sharp contrast to the familiar scalar PGB case [25], remain strictly massless being protected by the non-Abelian gauge invariance in the final Lagrangian (35) where an actual symmetry $SO(1,3) \otimes G$ still remains.[9] Apart from them, there are the $N-1$ massless scalar modes $\phi^i \equiv \mathbf{a}_5^i$ $(i = 1, \ldots, N-1)$. In contrast to vector bosons, they are not protected by any gauge symmetry and consequently will get masses through the radiative corrections.

One can write again, as in the above Abelian case (24), the ground vector field expansion around vacuum configuration stemming from the upper constraint in (34)

$$\mathbf{A}_{\bar{\mu}}^{0i} = \mathbf{a}_{\bar{\mu}}^i + \mathbf{n}_{\bar{\mu}}^i \sqrt{\mathrm{M}^2 - \mathbf{n}^2\mathbf{a}^2 - \mathbf{n}^2(\mathbf{A}^s)^2},$$
$$(\mathbf{a}^2 \equiv \mathbf{a}_{\bar{\mu}}^i \mathbf{a}^{\bar{\mu}i}, (\mathbf{A}^s)^2 \equiv \mathbf{A}_{\bar{\mu}}^{si} \mathbf{A}^{\bar{\mu}si}), \tag{40}$$

where summation over all repeated indices is taken, and also the orthogonality condition (39) for the emergent pseudo-Goldstone $\mathbf{a}_{\bar{\mu}}^i$ is supposed. Meanwhile, the effective Higgs term in the expansion (40)

$$\mathcal{H} = \mathbf{A}_{\bar{\mu}}^{0i} \mathbf{n}_{\bar{\mu}}^i = \sqrt{\mathrm{M}^2 - \mathbf{n}^2\mathbf{a}^2 - \mathbf{n}^2(\mathbf{A}^s)^2}$$
$$= \mathrm{M} + \mathcal{O}(\mathbf{a}^2/\mathrm{M}, (\mathbf{A}^s)^2/\mathrm{M}) \tag{41}$$

induces masses for some set of vector fields inside of the multiplet $\mathbf{a}_\mu^i$. Putting the expansion (40) into the Lagrangian (35) one eventually comes to the emergent 4D Yang-Mills theory stemming from the 5D spacetime with all possible vector field couplings involved. They are given for the ground modes by the truncated stress tensors presented in the Lagrangian (35)

$$\bar{\mathbf{F}}_{\mu\nu}^0 = \partial_\mu \mathbf{a}_\nu - \partial_\nu \mathbf{a}_\mu + ig[\mathbf{a}_\mu, \mathbf{a}_\nu] \tag{42}$$

and

$$\bar{\mathbf{F}}_{\mu 5}^0 = \partial_\mu(\phi + \mathbf{l}\mathcal{H}) + ig([\mathbf{a}_\mu, \phi] + \mathcal{H}[\mathbf{a}_\mu, \mathbf{l}]) \tag{43}$$

while for the higher modes $(s = 1, 2, \ldots)$ by the truncated tensors

$$\frac{\bar{\mathbf{F}}_{\mu\nu}^s}{\sqrt{2}} = \partial_\mu \mathbf{A}_\nu^s - \partial_\nu \mathbf{A}_\mu^s + ig([\mathbf{A}_\mu^s, \mathbf{a}_\nu] + [\mathbf{a}_\mu, \mathbf{A}_\nu^s]) \tag{44}$$

and

$$\frac{\bar{\mathbf{F}}_{\mu 5}^s}{\sqrt{2}} = \partial_\mu \mathbf{A}_5^s + ig([\mathbf{A}_\mu^s, \phi] + \mathcal{H}[\mathbf{A}_\mu^s, \mathbf{l}]) - i\frac{s}{R}\mathbf{A}_\mu^s, \tag{45}$$

where an effective 4D gauge coupling constant $g = g_5/\sqrt{\pi R}$ has been introduced. Such a form of these tensors readily follows upon an integration of the corresponding action over the extra dimension where also the normalization for ground and higher modes is properly taken into account.

Note that the starting theory (30) without the Lagrange multiplier term is invariant under the 5D non-Abelian gauge transformations of the vector field multiplet

$$\mathbf{A}_{\bar{\mu}}' = \mathbf{A}_{\bar{\mu}} + i[\alpha, \mathbf{A}_{\bar{\mu}}] + \partial_{\bar{\mu}}\alpha, \tag{46}$$

where the gauge parameter $\alpha = \alpha^i t^i$ is also proposed to have a symmetrical cosine expansion as the vector field components $\mathbf{A}_{\bar{\mu}}$. After compactification for a particular s component $(s = 0, 1, \ldots)$ it turns to

$$\delta \mathbf{A}_\mu^s = i\sum_{s'=0}[\alpha^{s'}, \mathbf{A}_\mu^{s-s'}] + \partial_\mu \alpha^s,$$
$$\delta \mathbf{A}_5^s = i\sum_{s'=0}[\alpha^{s'}, \mathbf{A}_5^{s-s'}] \tag{47}$$

showing that in the "rotation" part of each KK mode (ground state or higher KK mode) contributes all other states as well. Remarkably, for the symmetrical orbifold compactification taken the fifth-direction modes $\mathbf{A}_5^{s,i}$ are only rotated under the internal symmetry group transformations thus behaving themselves as the matter fields rather than the gauge field components.[10] This means that they cannot be gauged away from the theory. Therefore, as in the Abelian case, one has eventually, apart from the massless ground modes $\mathbf{A}_{\bar{\mu}}^{0,i} = \mathbf{a}_{\bar{\mu}}^i = (\mathbf{a}_\mu^i, \phi^i)$, the massive vector KK modes $\mathbf{A}_\mu^{s,i}$ and the massless "scalars" $\mathbf{A}_5^{s,i}$ $(s = 1, 2, \ldots)$. Moreover, the latter modes $\mathbf{A}_\mu^{s,i}$ and $\mathbf{A}_5^{s,i}$ break in essence the starting gauge invariance (47) down to a conventional gauge invariance related solely to the ground vector field modes in the 4D Lagrangian (35)

$$\delta \mathbf{A}_\mu^s = i[\alpha^0, \mathbf{A}_\mu^s] + \partial_\mu \alpha^s \delta^{s0}, \qquad \delta \mathbf{A}_5^s = i[\alpha^0, \mathbf{A}_5^s] \tag{48}$$

while all other modes are only "rotated" by the internal group symmetry generators. Actually, again as in the above Abelian case, this looks as if the 5D gauge function α in (46) would not depend on the fifth coordinate and, therefore, only its ground component α^0 is nonzero. This restricted gauge invariance makes it possible to uncover some direct observational effects related to the 5D SLIV, in

[9] Actually, the internal symmetry group G eventually appears spontaneously broken to its diagonal subgroups (see below).

[10] This will allow us later (Sec. IV) to treat its ground mode multiplet $\mathbf{A}_5^{0,i}$ $(i = 0, 1, \ldots, N-1)$ as an independent adjoint Higgs field multiplet in the theory considered.

contrast to the completely hidden spontaneous Lorentz violation appearing in a conventional 4D spacetime [21–23].

These effects are essentially determined by the stress tensors $\bar{\mathbf{F}}^0_{\mu 5}$ and $\bar{\mathbf{F}}^s_{\mu 5}$, (43) and (45), related to the fifth direction. Again, as in the Abelian case, apart from conventional gauge couplings presented in (42), one has direct multiboson (multiphoton in particular) couplings following from the kinetic term of the effective scalar $\mathcal{H}$ in the Lagrangian (35)

$$\frac{1}{2}(\partial_\mu \mathcal{H})^2 = \frac{1}{2}\frac{(\mathbf{a}^i_\rho \partial_\mu \mathbf{a}^{\rho i} + \sum_{s=1}\mathbf{A}^s_{\bar{\rho} i}\partial_\mu \mathbf{A}^{s,\bar{\rho} i})^2}{\mathrm{M}^2 + \mathbf{a}^2 + (\mathbf{A}^s)^2}. \qquad (49)$$

As a matter of fact, there appear an infinite number of the properly suppressed direct vector boson-boson scattering couplings and also couplings of these ground mode bosons with heavy KK towers in the Lagrangian. Thus, the emergent Yang-Mills theory, just as the emergent QED considered above, actually possesses only a partial gauge invariance whose observation could be of primary interest. Likewise, an existence of the unabsorbed fifth-direction nonzero modes $\mathbf{A}^{s,i}_5$ $(s = 1, 2, \ldots)$ being massless at the tree level appears as a somewhat unavoidable prediction of the model. Again, due to KK parity they will be only produced by pairs from an ordinary matter being properly suppressed by the 5D Lorentz violation scale M, as is shown in (49) for their possible production in boson-boson scattering and other processes. Also, any heavy KK state will now rapidly decay into the $\mathbf{A}^{s,i}_5$ modes plus ordinary matter. However, all these massless fifth-direction states in the model, the ground modes $\mathbf{A}^{0,i}_5 = \mathbf{a}^i_5$ and towers $\mathbf{A}^{s,i}_5$, being no more protected by the restricted gauge invariance (48) could in principle acquire quite large masses (see some details in Sec. IV), thus escaping the direct observation.

All these predictions may be equally expected from both Abelian and non-Abelian theory cases. However, there is one point being particularly specific to the emergent Yang-Mills theory. This is the generic Higgs mechanism appearing in a non-Abelian theory which leads to an automatic internal symmetry reduction when the 5D symmetry spacetime symmetry spontaneously breaks down to a conventional Lorentz invariance.

IV. INTERNAL SYMMETRY REDUCTION IN FOUR DIMENSIONS

We have seen above that if the starting 5D theory possesses some non-Abelian internal symmetry G this symmetry, simultaneously with the underlying spacetime symmetry, occurs spontaneously broken. As a result, some pseudo-Goldstone vector bosons emerging during symmetry breaking process (36) may acquire masses. This breaking itself appears similar to the breaking which is usually induced by an introduction into the theory of an appropriate adjoint scalar field multiplet. Now such a multiplet originates from extra vector field components. Particularly, in the 5D theory the role of such a scalar field multiplet plays the multiplet composed from the fifth component $(\mathbf{A}^0_5)^i$ of the zero-mode vector field $(\mathbf{A}^0_\mu)^i$, whose VEV is given by Eqs. (37) and (38) depending on the direction $i = i_0$ in the internal space along which G symmetry appears broken. They through their covariant derivatives give masses to the 4D ground vector field modes $\mathbf{a}^i_\mu$. Therefore, one may have the Higgs effect in the 5D SLIV theory without a specially introduced Higgs field.

Let us consider it in more detail. Rewriting the starting field expansion (40) for particular components we receive

$$(\mathbf{A}^0_\mu)^i = \mathbf{a}^i_\mu,$$
$$(\mathbf{A}^0_5)^i = \phi^i + \mathbf{l}^i\mathcal{H}, \qquad \mathbf{l}^i = \delta^{ii_0}, \qquad \mathbf{l} = \mathbf{l}^i t^i = t^{i_0}, \quad (50)$$

where for the emergent pseudo-Goldstone modes $\mathbf{a}^i_\mu$ and ϕ^i work the orthogonality conditions along an internal symmetry breaking direction

$$\mathbf{a}^i_\mu \mathbf{l}^i = \phi^i \mathbf{l}^i = 0. \qquad (51)$$

One can readily see that the covariant derivative (43) and (45) in the Lagrangian $\mathcal{L}_{4\mathrm{D}}$ (35) generates some vector field masses stemming from the first constant term in decomposition of the effective Higgs field $\mathcal{H}$ in (41). Note that these mass terms being proportional to the 5D Lorentz breaking mass scale M cannot be gauged away since the restricted 4D gauge invariance (48) is only left in the Lagrangian $\mathcal{L}_{4\mathrm{D}}$ after compactification. Meanwhile, this gauge invariance is spontaneously broken by itself, as follows from the covariant derivative term (43). Using a proper unitary gauge one can decouple the extra ϕ scalar multiplet from the four-dimensional vector fields $\mathbf{a}^i_\mu$. As a result, they are getting mass terms of the type

$$\mathcal{L}_m(\mathbf{a}_\mu) = \frac{1}{2}g^2\mathrm{M}^2\mathrm{Tr}[\mathbf{a}_\mu, \mathbf{l}]^2 = \frac{1}{2}g^2\mathrm{M}^2\mathbf{a}^i_\mu\mathbf{a}^j_\mu\mathrm{Tr}\{[t^{i_0}, t^i]\cdot[t^{i_0}, t^j]\}$$
$$= \frac{1}{2}g^2\mathrm{M}^2\mathbf{a}^i_\mu(f^{i_0}f^{i_0})_{ij}\mathbf{a}^j_\mu, \qquad (52)$$

where the structure constants $f^{i_0 ik}$ in the above commutators are written in the matrix form $f^{i_0}_{ik}$ and matrix product $(f^{i_0}f^{i_0})_{ij}$ always appears diagonal for any internal symmetry breaking direction i_0. It can easily be seen that these masses crucially depend on this direction so that all the ground vector field modes related to the corresponding broken generators of the internal symmetry group G receive the masses of the order of the 5D Lorentz scale M. The masses vanish when there is a vanishing commutator $[t^{i_0}, t^i] = 0$ in (52). This means that massless vector bosons only occur when the index i belongs to appropriate diagonal subgroups of the symmetry group G.

Remarkably, the spontaneous breaking of internal symmetry also modifies the masses of KK towers involved, as follows from the covariant derivative (45). Indeed, properly writing out the commutators one comes to

$$\bar{\mathbf{F}}^{s,i}_{\mu 5} = \partial_\mu \mathbf{A}^{s,i}_5 - g f^{ijk} \mathbf{A}^{s,j}_\mu (\phi^k + \mathrm{M}\mathbf{l}^k) - i\frac{s}{R}\mathbf{A}^{s,i}_\mu, \quad \mathbf{l}^k = \delta^{ki_0}. \tag{53}$$

Unfortunately, in contrast to the previous case, we have no conventional gauge invariance to separate scalar and vector modes. Instead, one can redefine the scalar fields in such a way to separate them in the momentum space[11]

$$\mathbf{A}^{s,i}_5 \to \mathbf{A}^{s,i}_5 + ig\mathrm{M}\frac{k^\mu f^{i_0 ij}\mathbf{A}^{s,j}_\mu}{k^2}. \tag{54}$$

After their substitution into covariant derivative one has diagonalized massless scalars $\mathbf{A}^{s,i}_5$ and massive vector towers $\mathbf{A}^{s,i}_\mu$ with modified kinetic terms determined by the scale M

$$|\bar{\mathbf{F}}^{s}_{\mu 5}|^2 = (k_\mu \mathbf{A}^{s,i}_5)^2 + (g\mathrm{M})^2\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right)\mathbf{A}^{s,i}_\mu (f^{i_0} f^{i_0})_{ij}\mathbf{A}^{s,j}_\mu. \tag{55}$$

So, collecting both types of mass terms for towers in (53) one has

$$\mathcal{L}_m(\mathbf{A}^s_\mu) = \sum_{s=1} \mathbf{A}^{s,i}_\mu \left[\frac{1}{2}\left(\frac{s}{R}\right)^2 \delta_{ij} + g^2\mathrm{M}^2 (f^{i_0} f^{i_0})_{ij}\right]\mathbf{A}^{s,j}_\mu. \tag{56}$$

We see that masses of towers for each number s are significantly influenced by an internal symmetry breaking along the direction determined by the generator $\mathbf{l} = \mathbf{l}^i t^i = t^{i_0}$. Particularly, all towers related to the corresponding broken generators of the group G will receive the large extra masses of the order of the 5D Lorentz scale M.

The most successful implementation of this phenomena may appear in grand unified theories considered *ab initio* in the five-dimensional spacetime. Once the 5D SLIV is applied, along with the compactification to the physical world, the adjoint "scalar field" multiplet composed from the extra vector field components, $(\mathbf{A}^0_5)^i$, will break the grand unified theory (GUT) down to the Standard Model so that all "nondiagonal" vector bosons [say, X- and Y-bosons in the $SU(5)$ theory] get large mass terms being of the order of the scale M. Thus, the scale M of the 5D SLIV can be identified with the grand unification scale M_{GUT} when emergent GUTs are considered. This is in sharp contrast to the Abelian internal symmetry case, where the 5D SLIV scale M is arbitrary and could even be of the order of a few TeV.

The point is, however, that due to the high symmetry of the constraints (34) one has in reality a vacuum degeneracy when applying them to the internal symmetry breaking in the GUTs. Indeed, the first constraint in (34) written for the proposed spacelike SLIV ($\mathbf{n}^2 = -1$) as

$$\mathrm{Tr}(\mathbf{A}^0_{\bar\mu}\mathbf{A}^{0\bar\mu}) = \mathrm{M}^2[1 + \mathcal{O}((\mathbf{A}^s)^2/\mathrm{M}^2)], \tag{57}$$

where we also ignored all the higher KK modes, explicitly demonstrates such a degeneracy in the internal space. Meanwhile, due to violation of the starting gauge invariance (46) in the post-compactification stage, the radiative corrections will induce in general all possible potential terms in the Lagrangian $\mathcal{L}_{4D}$ (35)

$$\mathcal{U}(\mathbf{A}) = \frac{m^2_{\mathbf{A}}}{2}\mathrm{Tr}(\mathbf{A}^0_{\bar\mu}\mathbf{A}^{0\bar\mu}) + \frac{\lambda_{\mathbf{A}}}{4}[\mathrm{Tr}(\mathbf{A}^0_{\bar\mu}\mathbf{A}^{0\bar\mu})]^2 + \frac{\lambda'_{\mathbf{A}}}{4}\mathrm{Tr}(\mathbf{A}^0_{\bar\mu}\mathbf{A}^{0\bar\mu}\mathbf{A}^0_{\bar\nu}\mathbf{A}^{0\bar\nu}), \tag{58}$$

where again the nonzero KK modes were omitted and some optional vector field mass parameter ($m^2_{\mathbf{A}}$) and coupling constants ($\lambda_{\mathbf{A}}$, $\lambda'_{\mathbf{A}}$) introduced (higher order terms are ignored). Now, one can readily see that the first two terms in the potential $\mathcal{U}$ only add some constants to the Lagrangian because of the constraint (57), whereas the third one in fact makes the lifting vacuum degeneracy in a theory considered.

Let us turn again to the emergent $SU(5)$ GUT case. For the constraint (57) and radiative corrections ignored there appears a twofold vacuum degeneracy in the theory: one vacuum with $SU(3) \times SU(2) \times U(1)$ symmetry (corresponding to the Standard Model) and another one with symmetry $SU(4) \times U(1)$. Interestingly, this resembles the vacuum degeneracy problem in supersymmetric GUTs [37]. However, while there is no way to split this vacuum degeneracy in the pure SUSY context, the situation is radically changed in the emergent GUTs due to the radiative corrections involved. Actually, one can readily confirm that for the positive coupling constants $\lambda'_{\mathbf{A}}$ in the potential (58) the $SU(3) \times SU(2) \times U(1)$ vacuum is definitely dominated in the emergent $SU(5)$ theory. Remarkably, the alternative $SU(4) \times U(1)$ vacuum may only exist for the negative constants $\lambda'_{\mathbf{A}}$, thus in an unstable theory case, that is principally unacceptable. Although we do not calculate here the above radiatively induced potential (58), it seems natural to propose that it

[11]Note that, though now one may not put a unitary gauge to get rid of the massless scalar fields $\mathbf{A}^{s,i}_5$, these fields [particularly, the Goldstone ones for i values determined by nonzero matrix elements $f^{i_0 ij}$ in (54)] correspond in fact to the unphysical particles in the sense that they could not appear as incoming or outgoing lines in Feynman graphs. In a somewhat similar context of the Standard Model formulated in the axial gauge this was first argued in [36].

may not destabilize the emergent $SU(5)$ theory, so that the coupling constant $\lambda'_{\mathbf{A}}$ always appears positive. Thus, as a result of the degeneracy lifting, just the Standard Model vacuum is generically chosen once the 5D SLIV occurs.

Due to radiative corrections, the X- and Y-bosons of $SU(5)$, apart from the masses presented above in (52), receive extra mass contributions (being proportional to $\lambda'_{\mathbf{A}}\mathrm{M}^2$) from the last term in the potential (58). Likewise, all the diagonal fifth-direction ground modes $\mathbf{A}_5^{0,i} = \mathbf{a}_5^i$ with the $SU(3) \times SU(2)$ assignment $(8,1)+(1,3)+(1,1)$ receive the same order masses $\mathcal{O}(\lambda'_{\mathbf{A}}\mathrm{M}^2)$, while the nondiagonal Goldstone ones with the assignment $(3,2)+(\bar{3},2)$ appear massless though nonobservable, as was mentioned before[11]. Analogously, an inclusion into the radiatively induced potential (58) the higher KK mode terms will produce masses for still being massless fifth-direction $\mathbf{A}_5^{s,i}$ towers as well. So, eventually all the starting 5D vector field modes, apart from the gauge bosons of $SU(3) \times SU(2) \times U(1)$, acquire masses in the 4D emergent $SU(5)$ GUT being automatically broken to the Standard Model.

V. CONCLUSION

We have argued that the spontaneously broken extra dimensional spacetime symmetry could be a natural source for emergent vector bosons associated with photons and other gauge fields. Indeed, the only way to produce such bosons in a fully Lorentz covariant way would be to enlarge the existing Minkowski spacetime to higher dimensions. As a matter of fact, all four-dimension models only suggest a noncovariant description of vector Goldstone bosons where one of vector field spacetime component A_μ $(\mu = 0, 1, 2, 3)$ is inevitably Higgs-ified. Moreover, the spontaneous breakdown of Lorentz symmetry itself may appear hidden from observation when considered in a gauge invariant theory framework.

The essential point is that an extra dimensional spacetime is eventually reduced to a conventional four dimensions due to some compactification pattern proposed. However, while the kinetic terms of the vector (and other) fields will only possess a standard Lorentz symmetry after compactification, their potential terms [or, equally, the polynomial vector field constraints like (21) and (34)] may still have the higher symmetrical form if the compactification pattern is properly chosen. This consequently induces the high-dimensional SLIV due to which massless pseudo-Goldstone states are generated as gauge boson candidates. So, an adequate choice of a compactification mechanism is a crucial point when considering extra dimensions as a possible source for a generation of emergent gauge theories. However, while a simple compactification on a circle conserves the starting spacetime symmetry for vector field constraints like (21) and (34), the orbifold compactification S_1/Z_2 introduced to have chiral fermions in four dimensions may in general explicitly break this symmetry down to a conventional 4D Lorentz invariance.

Actually, for a conventional asymmetrical orbifold compactification when ordinary four components of $A_{\bar{\mu}}$ are taken to be even under the orbifold transformation, whereas its fifth component is odd, the 5D Lorentz symmetry is turned out to be explicitly broken, though the 5D gauge symmetry (12) still remains. Eventually, one has the theory without extra vector field components, A_5^0 and A_5^s $(s = 1, 2, \ldots)$ since the ground mode A_5^0 vanishes, while higher A_5^s modes appear absorbed by the 4D massive KK towers A_μ^s. Without extra vector field components, the nonlinear constraints (21) and (34) will cause the VEV on one of the ordinary components of the 4D vector field ground mode A_μ^0 and $\mathbf{A}_\mu^0$ $(\mu = 0, 1, 2, 3)$, respectively. Thus, we come even in the 5D spacetime to the SLIV picture appearing in the four-dimensional Nambu model and its generalizations (which was intensively discussed above in Sec. I). Due to the starting 5D gauge symmetry which really remains after compactification, the SLIV inducing constraints (21) and (34) will be simply converted into the gauge fixing conditions so that such models have no observational consequences unless this symmetry is explicitly broken by some external sources.

In this connection, the 5D SLIV model developed above is entirely based on the symmetrical orbifold compactification S_1/Z_2 under which all spacetime components of the 5D vector field $A_{\bar{\mu}}(x, \theta)$ are taken to be even. Interestingly, such a "partially increased" Lorentz invariance happens to significantly reduce an effective gauge symmetry appearing for vector field components after compactification. The starting gauge symmetry (12) for KK states, both massive vectors A_μ^s $(s = 1, 2, \ldots)$ and massless scalars A_5^s $(s = 0, 1, 2, \ldots)$, does not work any longer. Only standard gauge invariance (19) for massless ground vector modes A_μ^0 holds. This allows to uncover a number of possible observational evidences in favor of emergent QED and Yang-Mills theories which cannot be gauged away as in 4D SLIV theories. They include, apart from conventional gauge couplings, the properly suppressed direct multiphoton (multiboson, in general) interactions. This means that emergent gauge theories actually possess only a partial gauge invariance whose observation could shed light on their high-dimensional nature. Another crucial prediction is an existence of the unabsorbed fifth-direction nonzero modes A_5^s $(s = 1, 2, \ldots)$ being massless at the tree level. Due to KK parity they can be only produced by pairs from an ordinary matter being properly suppressed by the 5D Lorentz violation scale M or by the compactification mass $M_{\mathrm{KK}} \sim 1/R$. On the other hand, any heavy KK state will now rapidly decay into the A_5^s mode plus ordinary matter that seems to invalidate the dark matter scenario related to extra dimension [33]. However, these A_5^s modes being no more protected by gauge invariance could in principle acquire large masses through radiative corrections so that the lightest KK state may appear rather stable to provide the measured dark matter density.

All the above, while largely spoken relative to the emergent QED, is equally applicable to both Abelian and non-Abelian cases. However, there is one point being particularly specific to Yang-Mills theory. In this case, due to 5D SLIV, together with the spacetime symmetry breaking, the non-Abelian internal symmetry group G also occurs spontaneously broken. As a result, all non-diagonal emergent vector bosons appearing during the symmetry breaking process (36) may acquire masses. This breaking originates from the extra vector field components playing a role of some adjoint scalar field multiplet in the 4D spacetime. Therefore, one may have the generic Higgs effect in the 5D SLIV theory which breaks the starting internal symmetry G to its diagonal subgroups. When being applied to grand unified theories this results in a fact that the emergent GUTs automatically appear broken down to the Standard Model just at the 5D Lorentz violation scale M. So, a spontaneous breakdown of a high-dimensional spacetime symmetry to a conventional Lorentz invariance may determine an internal symmetry pattern at low energies, and also control an admissible proton decay rate and, consequently, an acceptable matter-antimatter asymmetry in the early universe. We may return to this interesting scenario elsewhere.

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