Local and renormalizable framework for the gauge-invariant operator $A^2_{\min}$ in Euclidean Yang-Mills theories in linear covariant gauges

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We address the issue of the renormalizability of the gauge-invariant nonlocal dimension-two operator $A^2_{\min}$, whose minimization is defined along the gauge orbit. Despite its nonlocal character, we show that the operator $A^2_{\min}$ can be cast in local form through the introduction of an auxiliary Stueckelberg field. The localization procedure gives rise to an unconventional kind of Stueckelberg-type action that turns out to be renormalizable to all orders of perturbation theory. In particular, as a consequence of its gauge invariance, the anomalous dimension of the operator $A^2_{\min}$ turns out to be independent from the gauge parameter α entering the gauge-fixing condition, thus being given by the anomalous dimension of the operator A^2 in the Landau gauge.

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I. INTRODUCTION

Dimension-two condensates have been the object of intensive investigations in recent years. These condensates might play an important role in the nonperturbative regime of Euclidean Yang-Mills theories, as pointed out by the considerable amount of results obtained through theoretical and phenomenological studies as well as from lattice simulations [1–34].

For instance, the gluon condensate $\langle A^a_\mu A^a_\mu \rangle$ has been largely investigated in the Landau gauge. As pointed out in [5], this condensate enters the operator product expansion (OPE) of the gluon propagator. Moreover, a combined OPE and lattice analysis has shown that this condensate can account for the $1/Q^2$ corrections that have been reported [18–21,24,26–30,32–34] in the running of the coupling constant and in the gluon correlation functions.

An effective potential for $\langle A^a_\mu A^a_\mu \rangle$ in the Landau gauge has been obtained and evaluated in analytic form at two loops in [7,10,11,15,16], showing that a nonvanishing value of $\langle A^a_\mu A^a_\mu \rangle$ is favored as it lowers the vacuum energy. As a consequence, a dynamical gluon mass is generated. We also recall that, in the Landau gauge, the operator $A^a_\mu A^a_\mu$ is Becchi-Rouet-Stora-Tyutin (*BRST*) invariant on shell, a property that has allowed for an all-orders proof of its multiplicative renormalizability [35]. Its anomalous dimension is not an independent parameter, being expressed as a combination of the gauge β-function and the anomalous dimension of the gauge field A^a_μ [35], namely,

$$\gamma_{A^2}|_{\rm Landau} = \left(\frac{\beta(a)}{a} + \gamma_A^{\rm Landau}(a)\right), \qquad a = \frac{g^2}{16\pi^2}, \quad (1)$$

where $(\beta(a), \gamma_A^{\rm Landau}(a))$ denote, respectively, the β-function and the anomalous dimension of the gauge field A_μ in the Landau gauge. This relation was conjectured and explicitly verified up to three-loop order in [36].

Dimension-two condensates also play an important role within the context of the Gribov-Zwanziger approach to confinement [37–41] as well as for the formation of a dynamical gluon mass within the framework of the Dyson-Schwinger equations in the Landau gauge, as reported in [1,42,43]. These nonperturbative effects give rise to the so-called decoupling solution for the gluon propagator [1,37–39,42,44], i.e., to a propagator that exhibits positivity violation, while attaining a finite nonvanishing value at zero momentum. Until now, this behavior has been in very good agreement with the most recent lattice numerical simulations [45–48]. The generalization of these results to the linear covariant gauges has been worked out recently and can be found in [49–57].

Despite the huge amount of results obtained so far, it seems fair to state that many aspects related to dimension-two operators deserve a better understanding. This is certainly the case for the gauge invariance, a central issue in order to give a precise physical meaning to the corresponding condensates. This is precisely the topic that is studied in the present work. Let us briefly introduce the genuine gauge-invariant dimension-two operator $A^2_{\min}$.

A. Construction and properties of the operator $A^2_{\min}$

The gauge-invariant dimension-two operator $A^2_{\min}$ [1] is constructed by minimizing the functional ${\rm Tr}\int d^4x A^u_\mu A^u_\mu$ along the gauge orbit of A_μ [58–61], namely,

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[1] See Appendix B for more details.

$$A^2_{\min} \equiv \min_{\{u\}} \mathrm{Tr} \int d^4x A^u_\mu A^u_\mu,$$
$$A^u_\mu = u^\dagger A_\mu u + \frac{i}{g} u^\dagger \partial_\mu u. \qquad (2)$$

In particular, the stationary condition of the functional (2) gives rise to a nonlocal transverse field configuration A^h_μ, $\partial_\mu A^h_\mu = 0$, which can be expressed as an infinite series in the gauge field A_μ, i.e.,

$$A^h_\mu = \left(\delta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\partial^2}\right)\phi_\nu, \qquad \partial_\mu A^h_\mu = 0,$$
$$\phi_\nu = A_\nu - ig\left[\frac{1}{\partial^2}\partial A, A_\nu\right] + \frac{ig}{2}\left[\frac{1}{\partial^2}\partial A, \partial_\nu \frac{1}{\partial^2}\partial A\right] + O(A^3). \qquad (3)$$

Remarkably, the configuration A^h_μ turns out to be left invariant by infinitesimal gauge transformations order by order in the gauge coupling g [62] (see also Appendix B) as

$$\delta A^h_\mu = 0,$$
$$\delta A_\mu = -\partial_\mu \omega + ig[A_\mu, \omega]. \qquad (4)$$

Thus, from expression (2) it follows that

$$A^2_{\min} = \mathrm{Tr} \int d^4x A^h_\mu A^h_\mu,$$
$$= \frac{1}{2}\int d^4x \left[A^a_\mu \left(\delta_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\partial^2}\right) A^a_\nu - g f^{abc} \left(\frac{\partial_\nu}{\partial^2}\partial A^a\right)\left(\frac{1}{\partial^2}\partial A^b\right) A^c_\nu\right] + O(A^4). \qquad (5)$$

The gauge-invariant nature of expression (5) can be made manifest by rewriting it in terms of the field strength $F_{\mu\nu}$. In fact, as proven in [58], it turns out that

$$A^2_{\min} = -\frac{1}{2}\mathrm{Tr}\int d^4x \left(F_{\mu\nu}\frac{1}{D^2}F_{\mu\nu} + 2i\frac{1}{D^2}F_{\lambda\mu} \times \left[\frac{1}{D^2}D_\kappa F_{\kappa\lambda}, \frac{1}{D^2}D_\nu F_{\nu\mu}\right] - 2i\frac{1}{D^2}F_{\lambda\mu}\left[\frac{1}{D^2}D_\kappa F_{\kappa\nu}, \frac{1}{D^2}D_\nu F_{\lambda\mu}\right]\right) + O(F^4), \qquad (6)$$

from which the gauge invariance becomes apparent. The operator $(D^2)^{-1}$ in expression (6) denotes the inverse of the Laplacian $D^2 = D_\mu D_\mu$ with D_μ being the covariant derivative [58]. Let us also underline that, in the Landau gauge $\partial_\mu A_\mu = 0$, the operator $(A^h_\mu A^h_\mu)$ reduces to the operator A^2,

$$(A^{h,a}_\mu A^{h,a}_\mu)|_{\mathrm{Landau}} = A^a_\mu A^a_\mu. \qquad (7)$$

B. Aim of the paper and its structure

As already mentioned, the main aim of the present work is to face the issue of the gauge invariance of non-Abelian gauge theories in the presence of dimension-two operators. More precisely, we provide a general and detailed analysis of the gauge-invariant quantity $(A^h_\mu A^h_\mu)$, Eq. (5), within the framework of Euclidean Yang-Mills theories quantized in the class of the linear covariant gauges. We are able to show that, despite its nonlocal character, the operator $(A^h_\mu A^h_\mu)$ can be localized by means of the introduction of an auxiliary Stueckelberg field. Nevertheless, the resulting theory can be seen as a kind of unconventional Stueckelberg model that does not suffer from the known drawbacks, i.e., the nonrenormalizability, of the usual Stueckelberg mass term. Therefore, we end up with a well-defined framework accounting for the existence of a gauge-invariant dimension-two operator.

Relying on an exact BRST invariance, we establish the multiplicative renormalizability of the operator $(A^h_\mu A^h_\mu)$ to all orders of perturbation theory by means of the algebraic renormalization. Moreover, the anomalous dimension of $(A^h_\mu A^h_\mu)$ can be proven to be independent from the gauge parameter α and turns out to be equal to the anomalous dimension of the operator A^2 in the Landau gauge, namely,

$$\gamma_{(A^h)^2} = \gamma_{A^2}|_{\mathrm{Landau}} = \left(\frac{\beta(a)}{a} + \gamma^{\mathrm{Landau}}_A(a)\right), \qquad a = \frac{g^2}{16\pi^2}. \qquad (8)$$

We underline that expression (8) is valid to all orders of perturbation theory, thereby extending the previous one-loop results obtained in [63].

The paper is organized as follows. In Sec. II we present the localization procedure for the operator $(A^h_\mu A^h_\mu)$ within the framework of a BRST-invariant action. In Sec. III we derive the Ward identities and establish the all-order renormalizability of $(A^h_\mu A^h_\mu)$ by means of the algebraic renormalization [64]. In Sec. IV we discuss the anomalous dimensions of $(A^h_\mu A^h_\mu)$ and the composite operator A^h_μ by means of the renormalization group equations (RGEs). Section V contains our conclusion. A few appendixes collect more details about the construction and properties of the operator $(A^h_\mu A^h_\mu)$.

II. A LOCAL FRAMEWORK FOR THE OPERATOR $(A^h_\mu A^h_\mu)$

Our first task will be finding a local framework for the nonlocal operator $(A^h_\mu A^h_\mu)$ of expression (5). For that purpose we start with the standard Faddeev-Popov action of Yang-Mills theory quantized in linear covariant gauges with the inclusion of the mass operator (5) as well as a constraint enforcing the transversality of the field configuration A^h_μ, Eq. (3), i.e., we consider the action

$$S = S_{\rm FP} + \int d^4x \left(\tau^a \partial_\mu A^{h,a}_\mu + \frac{m^2}{2} A^{h,a}_\mu A^{h,a}_\mu \right), \qquad (9)$$

where $S_{\rm FP}$ stands for the Faddeev-Popov action in linear covariant gauges,

$$S_{\rm FP} = \int d^4x \times \left(\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{\alpha}{2} b^a b^a + i b^a \partial_\mu A^a_\mu + \bar{c}^a \partial_\mu D^{ab}_\mu c^b \right), \qquad (10)$$

and where we have introduced the operator $(A^h_\mu A^h_\mu)$ through the mass parameter m^2. Also, the transversality of A^h_μ is enforced by the Lagrange multiplier τ^a.

Since the expression for $(A^h_\mu A^h_\mu)$ given in (5) is an infinite sum of nonlocal terms in the gauge field, the action (9) should be first put in a local form before it can be of any practical use. Following [62,65,66], this goal can be achieved by the introduction of an auxiliary localizing Stueckelberg field ξ^a, whose role is to give, for each gauge field A_μ, its corresponding configuration that minimizes the functional A^2, i.e., A^h_μ. This is most naturally implemented by defining a field h that effectively acts on A_μ as a gauge transformation would act, in order to provide the minimizing configuration A^h, that is,

$$A^h_\mu \equiv A^{h,a}_\mu T^a = h^\dagger A_\mu h + \frac{i}{g} h^\dagger \partial_\mu h, \qquad (11)$$

with

$$h = e^{ig\xi} = e^{ig\xi^a T^a}, \qquad (12)$$

where $\{T^a\}$ are the generators of the gauge group $SU(N)$ and ξ^a is a Stueckelberg field. Therefore, by substituting the expression (11) for A^h in the action (9), we now have a local theory in terms of the field ξ. The price one has to pay to have such a local theory is a nonpolynomial action. Indeed, by expanding (11), one finds an infinite series whose first terms are

$$(A^h)^a_\mu = A^a_\mu - D^{ab}_\mu \xi^b - \frac{g}{2} f^{abc} \xi^b D^{cd}_\mu \xi^d + \mathcal{O}(\xi^3), \qquad (13)$$

where

$$D^{ab}_\mu = \delta^{ab} \partial_\mu - g f^{abc} A^c_\mu \qquad (14)$$

is the covariant derivative in the adjoint representation.

The nonlocal expression (3) for A^h_μ in terms of the gauge field A_μ can be recovered by imposing the transversality condition $\partial_\mu A^h_\mu = 0$, i.e., after taking the divergence of both sides of (13), equating it to 0 and solving for the Stueckelberg field ξ^a [see Eqs. (B21)–(B24) of Appendix B]. This check is not only important for the consistency of the present framework but it also makes it clear that, due to the transversality condition enforced by the Lagrange multiplier τ^a, the Stueckelberg field ξ^a acquires a specific meaning: it is precisely the field that brings a generic gauge configuration A_μ into the gauge-invariant and transverse field configuration A^h_μ that minimizes the functional $A^2_{\rm min}$. As becomes clear in the following, this relevant feature, encoded in the term $\int d^4x \tau^a \partial_\mu A^{h,a}_\mu$, gives rise to deep differences between our construction and the standard Stueckelberg mass term. The latter is known to be a nonrenormalizable theory that has to be treated as an effective field theory [65].

An important feature of A^h_μ, as defined by Eq. (11), is its gauge invariance, that is,

$$A^h_\mu \to A^h_\mu, \qquad (15)$$

as can be seen from the gauge transformations with $SU(N)$ matrix V,

$$A_\mu \to V^\dagger A_\mu V + \frac{i}{g} V^\dagger \partial_\mu V, \qquad h \to V^\dagger h, \qquad h^\dagger \to h^\dagger V. \qquad (16)$$

The local version of the action (9), in terms of the Stueckelberg field ξ^a, is thus given by

$$\begin{aligned} S &= S_{\rm FP} + \int d^4x \left(\tau^a \partial_\mu A^{h,a}_\mu + \frac{m^2}{2} A^{h,a}_\mu A^{h,a}_\mu \right) \\ &= S_{\rm FP} + \int d^4x \left[\tau^a \left(A^a_\mu - D^{ab}_\mu \xi^b - \frac{g}{2} f^{abc} \xi^b D^{cd}_\mu \xi^d \right) \right] \\ &\quad + \frac{m^2}{2} \int d^4x \left(A^a_\mu - D^{ab}_\mu \xi^b - \frac{g}{2} f^{abc} \xi^b D^{cd}_\mu \xi^d \right) \\ &\quad \times \left(A^a_\mu - D^{ae}_\mu \xi^e - \frac{g}{2} f^{aef} \xi^e D^{fg}_\mu \xi^g \right) + \cdots \end{aligned} \qquad (17)$$

Because of the use of the auxiliary Stueckelberg field ξ^a, expression (17) exhibits a nonpolynomial character. At first sight, this feature might seem to jeopardize its renormalizability. Nevertheless, this is not the case, as we prove in the following.

Before entering into the detailed proof of the renormalizability, it is worth addressing the issue of the BRST symmetry as well as taking a look at the propagators of the elementary fields in order to achieve a better understanding of our action as compared to the usual standard massive Stueckelberg theory.

A. BRST invariance

The local action S, Eq. (17), enjoys an exact BRST symmetry,

$$sS = 0, \tag{18}$$

where the nilpotent BRST transformations are given by

$$\begin{aligned} sA^a_\mu &= -D^{ab}_\mu c^b, \\ sc^a &= \frac{g}{2} f^{abc} c^b c^c, \\ s\bar{c}^a &= ib^a, \\ sb^a &= 0, \\ s\tau^a &= 0, \\ s^2 &= 0. \end{aligned} \tag{19}$$

From [67], for the Stueckelberg field we have, with i, j indices associated with a generic representation,

$$sh^{ij} = -igc^a (T^a)^{ik} h^{kj}, \qquad s(A^h)^a_\mu = 0, \tag{20}$$

from which the BRST transformation of the field ξ^a can be evaluated iteratively, yielding

$$s\xi^a = -c^a + \frac{g}{2} f^{abc} c^b \xi^c - \frac{g^2}{12} f^{amr} f^{mpq} c^p \xi^q \xi^r + O(\xi^3). \tag{21}$$

Let us also present a second, equivalent, way of evaluating the BRST transformation of the Stueckelberg field ξ^a. Owing to the dimensionless character of ξ^a, one starts by writing

$$s\xi^a = g^{ab}(\xi) c^b, \tag{22}$$

where $g^{ab}(\xi)$ stands for a generic dimensionless quantity that can be expanded in power series of ξ^a. Imposing now nilpotency of the BRST operator s, i.e.,

$$s^2 \xi^a = s(g^{ab}(\xi) c^b) = 0, \tag{23}$$

one gets the condition

$$\left(\frac{\partial g^{ab}(\xi)}{\partial \xi^m} g^{mp}(\xi) - \frac{\partial g^{ap}(\xi)}{\partial \xi^m} g^{mb}(\xi) \right) = -g^{ac}(\xi)(g f^{cpb}). \tag{24}$$

The above equation can be easily solved order by order by expanding the quantity $g^{ab}(\xi)$ in power series of ξ^a, obtaining

$$g^{ab}(\xi) = -\delta^{ab} + \frac{g}{2} f^{abc} \xi^c - \frac{g^2}{12} f^{acd} f^{cbe} \xi^e \xi^d + O(\xi^3), \tag{25}$$

which gives back precisely expression (21).

Let us end this section by checking out the explicit BRST invariance of A^h_μ. To that purpose, it is better to employ a matrix notation for the fields, i.e.,

$$\begin{aligned} sA_\mu &= -\partial_\mu c + ig[A_\mu, c], \qquad sc = -igcc, \\ sh &= -igch, \qquad sh^\dagger = igh^\dagger c, \end{aligned} \tag{26}$$

with $A_\mu = A^a_\mu T^a$, $c = c^a T^a$, $\xi = \xi^a T^a$. From expression (11) we get

$$\begin{aligned} sA^h_\mu &= igh^\dagger c A_\mu h + h^\dagger(-\partial_\mu c + ig[A_\mu, c])h - igh^\dagger A_\mu ch \\ &\quad - h^\dagger c \partial_\mu h + h^\dagger \partial_\mu (ch) \\ &= igh^\dagger c A_\mu h - h^\dagger (\partial_\mu c) h + igh^\dagger A_\mu ch - igh^\dagger c A_\mu h \\ &\quad - igh^\dagger A_\mu ch - h^\dagger c \partial_\mu h + h^\dagger (\partial_\mu c) h + h^\dagger c \partial_\mu h \\ &= 0. \end{aligned} \tag{27}$$

B. Comparison with the standard Stueckelberg mass term

Let us proceed now by discussing the existing differences between our approach, as expressed by the local action S of Eq. (17), and the usual Stueckelberg mass term. We begin by recalling that the standard Stueckelberg formulation amounts to adding the mass term $\frac{m^2}{2}\int d^4x A^{h,a}_\mu A^{h,a}_\mu$ to the Faddeev-Popov action, yielding thus the following action,

$$S_{\rm Stueck} = S_{\rm FP} + \frac{m^2}{2} \int d^4x A^{h,a}_\mu A^{h,a}_\mu, \tag{28}$$

where $S_{\rm FP}$ is the Faddeev-Popov action of the linear covariant gauges, Eq. (10).

In particular, with respect to expression (17), one notices the absence, in the standard Stueckelberg action (28), of the term $\int d^4x \tau^a \partial_\mu A^{h,a}_\mu$ enforcing the transversality condition $\partial_\mu A^h_\mu = 0$. This means that the Stueckelberg mass term, $\frac{m^2}{2}\int d^4x A^{h,a}_\mu A^{h,a}_\mu$, refers to a generic gauge-invariant field configuration A^h_μ. One sees therefore that, while in the ordinary Stueckelberg action the mass term is related to a generic gauge-invariant configuration A^h_μ, in our case, besides gauge invariance, the configuration A^h_μ is further constrained by the transversality condition $\partial_\mu A^h_\mu = 0$. Therefore, unlike the standard Stueckelberg formulation, our action refers to a very particular and specific mass term, which is the one obtained by mininimizing the operator $A^2_{\rm min}$, as precisely expressed by the presence of the term

$\int d^4x\tau^a\partial_\mu A_\mu^{h,a}$. This is a nontrivial feature of our model, which makes it deeply different from the usual Stueckelberg action (28).

It is instructive to take a look at the propagators of the Stueckelberg field ξ^a which follow from both formulations. In the case of the standard Stueckelberg action, Eq. (28), one obtains

$$\langle\xi^a(p)\xi^b(-p)\rangle_{\rm Stueck} = \delta^{ab}\left(1+\frac{\alpha m^2}{p^2}\right)\frac{1}{m^2p^2}. \qquad (29)$$

This expression captures in a direct and simple way all drawbacks of the standard Stueckelberg formulation, as reviewed in [65]. One notices, in particular, the presence of the mass parameter m^2 in the denominator of (29), a feature that persists even in the Landau gauge, corresponding to $\alpha = 0$, namely,

$$\langle\xi^a(p)\xi^b(-p)\rangle^{\alpha=0}_{\rm Stueck} = \frac{\delta^{ab}}{m^2p^2}. \qquad (30)$$

As one can easily figure out, this property prevents the renormalizability of the standard Stueckelberg formulation [65]. In fact, due to the presence of the parameter m^2 in the denominator of expressions (29) and (30), nonpower-counting renormalizable divergences in the inverse of the mass m^2 show up, invalidating the perturbative loop expansion. As discussed in [65], the theory stemming from the action (28) has to be treated within the realm of an effective nonrenormalizable quantum field theory.

Instead, the inclusion of the term $\int d^4x\tau^a\partial_\mu A_\mu^{h,a}$ leads to a deep modification of the Stueckelberg propagator. In fact, from the quadratic part of the action S, Eq. (17), one gets (see also Appendix C where the complete list of propagators has been given)

$$\langle\xi^a(p)\xi^b(-p)\rangle_S = \alpha\frac{\delta^{ab}}{p^4}. \qquad (31)$$

Expression (31) displays several properties. First of all, unlike the propagator of Eq. (29), one notices the absence of the mass parameter m^2. As far as the UV behavior is concerned, expression (31) does not pose any problem for the validity of the power counting, a property that ensures in fact the all-order renormalizability of the model, as is proven in detail in the next section. Another interesting feature displayed by expression (31) is the decoupling nature of the Stueckelberg field in the Landau gauge, $\alpha = 0$. In fact, from Appendix C, it turns out that

$$\begin{aligned}\langle\xi^a(p)\xi^b(-p)\rangle_S^{\alpha=0} &= \langle A^a_\mu(p)\xi^b(-p)\rangle_S^{\alpha=0} \\ &= \langle A^a_\mu(p)\tau^b(-p)\rangle_S^{\alpha} = 0. \qquad (32)\end{aligned}$$

This is a remarkable property of the Landau gauge, which expresses in terms of Feynman rules the decoupling of the Stueckelberg field ξ^a. It reflects the expected fact that, when $\partial_\mu A_\mu = 0$, the higher order terms of the infinite series (3) become harmless, due to the presence of the divergence $\partial_\mu A_\mu$. Equation (32) reveals in a clear way the deep difference existing between the present formulation and the standard Stueckelberg one for which, even in the Landau gauge, the field ξ^a does not decouple; see Eq. (30). To some extent, property (32) makes almost immediate the perturbative renormalizability of the action S, Eq. (17), in the Landau gauge.

Before ending this section, it is worth spending a few words on the possible implications of the existence of a double pole, at vanishing Euclidean momentum $p^2 = 0$, in the Stueckelberg propagator (31). Even if such a behavior does not pose problems for the UV power counting, it might give rise to unwanted infrared divergences in the explicit loop calculations. For that, a BRST-invariant infrared regularization is presented in the next subsection, relying on a nice property of the BRST transformation of the Stueckelberg field ξ^a. Moreover, we underline the presence, in expression (31), of the gauge parameter α. This is a welcome feature. In fact, owing to the BRST invariance of the theory, it turns out that the correlation functions $\langle O(x)O(y)\rangle$ of BRST-invariant composite operators $O(x)$ are independent from the gauge parameter α; see Ref. [66] for a recent algebraic proof of this statement. This property, combined with the aforementioned BRST-invariant infrared regularization and the decoupling nature of the Stueckelberg field ξ^a in the Landau gauge, ensures that the gauge-invariant correlators $\langle O(x)O(y)\rangle$ are infrared safe. Finally, we restate the Euclidean nature of our construction, i.e., we do not attempt to provide a possible Minkowski interpretation for the action S, Eq. (17). Without entering into details, it suffices to mention that we expect a violation of perturbative unitary in Minkowski space, even if our model displays an exact BRST symmetry. This is precisely corroborated by the presence of a double pole in the propagator of the Stueckelberg field. Multipole fields are known in fact to give problems with perturbative unitarity. A nice example of this is offered by the nonlocal mass operator $F_{\mu\nu}(D^2)^{-1}F_{\mu\nu}$ that has been studied in detail in [68–70]. Similarly to the present case, the nonlocal operator $F_{\mu\nu}(D^2)^{-1}F_{\mu\nu}$ can be cast in local form by introducing a set of suitable auxiliary fields, so that a local formulation can be constructed at the end, enjoying an exact BRST symmetry [68–70]. The resulting action turns out to be renormalizable [68,69]. Nevertheless, it violates perturbative unitarity due to the presence of multipole fields [70]. We point out that the operator $F_{\mu\nu}(D^2)^{-1}F_{\mu\nu}$ is the first term of the infinite series of the gauge-invariant expansion for the operator $A^2_{\rm min}$, as one sees from Eq. (6). We expect thus that the same problems encountered in the analysis of the perturbative unitarity for the operator $F_{\mu\nu}(D^2)^{-1}F_{\mu\nu}$ will show up also in the case of $A^2_{\rm min}$.

Though, as it stands, the Euclidean action S, Eq. (17), turns out to be useful in order to study nonperturbative aspects of confining Euclidean Yang-Mills theories. In particular, expression (17) arises within the context of the BRST-invariant formulation of the Gribov-Zwanziger theory recently achieved in [53,54,66], which takes into account the nonperturbative effects of the Gribov copies. In addition, the action S can be seen as the BRST-invariant extension in linear covariant gauges of the effective model considered by Tissier and Wschebor in the Landau gauge in order to study the positivity violation of the gluon propagator [71,72]. Lastly, as already pointed out in the introduction, action (17) might enable us to investigate the formation of the dimension-two condensate $\langle A^h_\mu A^h_\mu \rangle$ in a BRST-invariant and α-independent way.

C. Infrared BRST-invariant regularization for the Stueckelberg field ξ

As mentioned before, the propagator for the Stueckelberg field in expression (31) could give rise to potential IR divergences when performing explicit loop calculations. Though, as outlined in [66], it turns out to be possible to introduce an IR regularizing mass term for the Stueckelberg field compatible with the BRST invariance. For the benefit of the reader, let us reproduce here the construction of [66]. It relies on a nice property displayed by the BRST transformation of the field ξ^a given in Eqs. (20) and (21), namely,

$$s\left(\frac{\xi^a \xi^a}{2}\right) = -\xi^a c^a, \tag{33}$$

as it follows from Eq. (26), i.e.,

$$s(e^{ig\xi}) = -igce^{ig\xi}. \tag{34}$$

Expanding the exponential in Taylor series, one gets

$$s\left(1 + ig\xi - \frac{g^2}{2}\xi\xi - i\frac{g^3}{3!}\xi\xi\xi + \cdots\right) = -igc\left(1 + ig\xi - \frac{g^2}{2}\xi\xi - i\frac{g^3}{3!}\xi\xi\xi + \cdots\right). \tag{35}$$

Multiplying both sides of Eq. (35) by ξ yields

$$\xi s\left(1 + ig\xi - \frac{g^2}{2}\xi\xi - i\frac{g^3}{3!}\xi\xi\xi + \cdots\right) = -ig\xi c\left(1 + ig\xi - \frac{g^2}{2}\xi\xi - i\frac{g^3}{3!}\xi\xi\xi + \cdots\right). \tag{36}$$

Equating now order by order in g the expression (36) immediately provides Eq. (33).

Because of Eq. (33), we can introduce the following BRST-exact term

$$S_{\mathrm{IRR}} = \int d^4x \frac{1}{2} s(\rho \xi^a \xi^a) = \int d^4x \left(\frac{1}{2} M^4 \xi^a \xi^a + \rho \xi^a c^a\right), \tag{37}$$

where (ρ, M) are constant parameters transforming as

$$s\rho = M^4, \qquad sM^4 = 0. \tag{38}$$

As is apparent, the action $(S + S_{\mathrm{IRR}})$ is BRST invariant, i.e.,

$$s(S + S_{\mathrm{IRR}}) = 0. \tag{39}$$

The parameter ρ has ghost number -1, while M has ghost number 0. From Eq. (37), it turns out that the propagator of the Stueckelberg field ξ^a behaves now like

$$\langle \xi^a(p)\xi^b(-p)\rangle_{S+S_{\mathrm{IRR}}} = \delta^{ab}\frac{\alpha}{p^4 + \alpha M^4}, \tag{40}$$

showing that the mass parameter M introduces an IR regularization in a BRST-invariant way. In Appendix C one finds the whole list of all propagators of the elementary fields evaluated in the presence of the parameters (ρ, M), which have to be set to 0 at the very end of the computation of the correlation functions.

III. RENORMALIZABILITY

We are now ready to face the issue of the all-order renormalizability of the action S, Eq. (17). For later convenience, it turns out to be helpful to employ a slightly different parametrization, redefining the gauge parameter α as well as the gauge, Lagrange multiplier, and Stueckelberg fields as

$$A^a_\mu \to \frac{1}{g}A^a_\mu, \quad b^a \to gb^a, \quad \xi^a \to \frac{1}{g}\xi^a, \quad \alpha \to \frac{\alpha}{g^2}. \tag{41}$$

Accordingly, for the field strength and the covariant derivative, we get

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc}A^b_\mu A^c_\nu, \tag{42}$$

$$D^{ab}_\mu = \delta^{ab}\partial_\mu - f^{abc}A^c_\mu, \tag{43}$$

while for the action S

$$S = S_{\mathrm{FP}} + \int d^4x \left(\tau^a \partial_\mu A^{h,a}_\mu + \frac{m^2}{2} A^{h,a}_\mu A^{h,a}_\mu\right), \tag{44}$$

$$S_{\mathrm{FP}} = \int d^4x \left(\frac{1}{4g^2} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{\alpha}{2} b^a b^a + ib^a \partial_\mu A^a_\mu + \bar{c}^a \partial_\mu D^{ab}_\mu c^b\right), \tag{45}$$

where

$$A^h_\mu \equiv A^{h,a}_\mu T^a = h^\dagger A_\mu h + i h^\dagger \partial_\mu h, \qquad h = e^{i\xi^a T^a}. \tag{46}$$

Also, for the BRST transformation, we have

$$\begin{aligned} sA^a_\mu &= -D^{ab}_\mu c^b, \\ sc^a &= \frac{1}{2} f^{abc} c^b c^c, \\ s\bar{c}^a &= i b^a, \\ sb^a &= 0, \\ s\tau^a &= 0, \end{aligned} \tag{47}$$

and

$$\begin{aligned} s\xi^a &= g^{ab}(\xi) c^b, \\ g^{ab}(\xi) &= -\delta^{ab} + \frac{1}{2} f^{abc}\xi^c - \frac{1}{12} f^{acd} f^{cbe} \xi^e \xi^d + O(\xi^3), \end{aligned} \tag{48}$$

with

$$sS = 0. \tag{49}$$

The usefulness of the new parametrization in Eqs. (44) and (45) relies on the property that, acting on the action S with the differential operator $g^2 \frac{\partial}{\partial g^2}$, gives directly the gauge-invariant quantity $\int d^4x F^a_{\mu\nu} F^a_{\mu\nu}$, i.e.,

$$g^2 \frac{\partial S}{\partial g^2} = -\frac{1}{4g^2} \int d^4x F^a_{\mu\nu} F^a_{\mu\nu}, \tag{50}$$

a feature that is helpful in order to write down the parametric form of the most general counterterm allowed by the quantum corrections.

Let us proceed by identifying the Ward identities of the model. To that purpose, following the algebraic renormalization set up [64], we introduce a set of BRST-invariant external sources $(J(x), \mathcal{J}(x)^a_\mu, \Omega^a_\mu(x), L^a(x), K^a(x))$ coupled to the composite operators $(A^h_\mu(x) A^h_\mu(x))$ and $A^h_\mu(x)$ as well as to the nonlinear BRST variation of the fields (A^a_μ, c^a, ξ^a); namely, we consider the classical complete BRST-invariant action Σ defined by

$$\begin{aligned} \Sigma = S_{\rm FP} + \int d^4x \bigg(&\tau^a \partial_\mu A^{h,a}_\mu + \frac{J}{2} A^{h,a}_\mu A^{h,a}_\mu + \mathcal{J}^a_\mu A^{h,a}_\mu \\ &- \Omega^a_\mu D^{ab}_\mu c^b + \frac{1}{2} f^{abc} L^a c^b c^c + K^a g^{ab}(\xi) c^b + \frac{\zeta}{2} J^2 \bigg), \end{aligned} \tag{51}$$

with

$$sJ = s\mathcal{J}^a_\mu = s\Omega^a_\mu = sL^a = sK^a = 0, \tag{52}$$

which ensures the BRST invariance of Σ,

$$s\Sigma = 0. \tag{53}$$

The action S, Eq. (44), can be recovered from Σ, modulo a constant vacuum term $V \frac{\zeta}{2} m^4$, by setting the sources $(J, \mathcal{J}^a_\mu, \Omega^a_\mu, L^a, K^a)$ equal to

$$\begin{aligned} J|_{\rm phys} &= m^2, \\ \mathcal{J}^a_\mu|_{\rm phys} &= \Omega^a_\mu|_{\rm phys} = L^a|_{\rm phys} = K^a|_{\rm phys} = 0, \end{aligned} \tag{54}$$

i.e.,

$$\Sigma|_{\rm phys} = S + V\frac{\zeta}{2} m^4, \tag{55}$$

where V stands for the Euclidean space-time volume. The parameter ζ is a dimensionless free parameter that enables us to take into account possible divergences affecting the vacuum term $J^2(x)$ [7,10,11], allowed by power counting due to the fact that source $J(x)$ has dimension 2. Let us also mention that the vacuum term $\frac{\zeta}{2} J^2$ is required in order to investigate the formation of the dimension-two condensate $\langle A^h_\mu(x) A^h_\mu(x) \rangle$ via evaluation of the corresponding effective potential; see [7,10,11]. In particular, the parameter ζ can be made a function of the coupling constant g in such a way that the generating functional of the correlation functions of the theory obeys a homogeneous renormalization group equation [7,10,11], a result that is employed in Sec. IV in order to determine the anomalous dimensions of the operators $(A^h_\mu A^h_\mu)$ and A^h_μ.

A. Ward identities

The BRST symmetry stated in the previous section can be immediately written as a functional identity. The complete classical action Σ turns out to fulfil the following Ward identities:

(i) The Slavnov-Taylor identity

$$\begin{aligned} \mathcal{S}(\Sigma) \equiv \int d^4x \bigg(&\frac{\delta\Sigma}{\delta\Omega^a_\mu} \frac{\delta\Sigma}{\delta A^a_\mu} + \frac{\delta\Sigma}{\delta L^a} \frac{\delta\Sigma}{\delta c^a} + \frac{\delta\Sigma}{\delta K^a} \frac{\delta\Sigma}{\delta \xi^a} \\ &+ i b^a \frac{\delta\Sigma}{\delta \bar{c}^a} \bigg) = 0. \end{aligned} \tag{56}$$

In view of the algebraic characterization of the counterterm, we introduce the so-called linearized Slavnov-Taylor operator $\mathcal{B}_\Sigma$ [64] defined as

$$\mathcal{B}_\Sigma \equiv \int d^4x \left(\frac{\delta\Sigma}{\delta\Omega^a_\mu}\frac{\delta}{\delta A^a_\mu} + \frac{\delta\Sigma}{\delta A^a_\mu}\frac{\delta}{\delta\Omega^a_\mu} + \frac{\delta\Sigma}{\delta L^a}\frac{\delta}{\delta c^a} + \frac{\delta\Sigma}{\delta c^a}\frac{\delta}{\delta L^a} + \frac{\delta\Sigma}{\delta K^a}\frac{\delta}{\delta\xi^a} + \frac{\delta\Sigma}{\delta\xi^a}\frac{\delta}{\delta K^a} + ib^a\frac{\delta}{\delta\bar{c}^a}\right), \tag{57}$$

which, as the BRST operator s, turns out to be nilpotent,

$$\mathcal{B}_\Sigma\mathcal{B}_\Sigma = 0. \tag{58}$$

(ii) The gauge-fixing condition and the antighost equation [64]

$$\frac{\delta\Sigma}{\delta b^a} = i\partial_\mu A^a_\mu + \alpha b^a, \tag{59}$$

$$\frac{\delta\Sigma}{\delta\bar{c}^a} + \partial_\mu\frac{\delta\Sigma}{\delta\Omega^a_\mu} = 0. \tag{60}$$

In particular, the identity (60) ensures that the antighost field $\bar{c}^a$ and the source Ω^a_μ enter only through the combination

$$\hat{\Omega}^a_\mu = \Omega^a_\mu + \partial_\mu\bar{c}^a. \tag{61}$$

(iii) The τ Ward identity

$$\frac{\delta\Sigma}{\delta\tau^a} - \partial_\mu\frac{\delta\Sigma}{\delta\mathcal{J}^a_\mu} = 0, \tag{62}$$

implying that the field τ^a and the source $\mathcal{J}^a_\mu$ appear only in the combination

$$\hat{\mathcal{J}}^a_\mu = \mathcal{J}^a_\mu - \partial_\mu\tau^a. \tag{63}$$

B. Characterization of the most general counterterm

In order to characterize the most general invariant counterterm that can be freely added to all orders in perturbation theory we follow the setup of the algebraic renormalization [64] and perturb the classical action Σ by adding an integrated local quantity in the fields and sources, Σ^{ct}, with dimension bounded by four and vanishing ghost number. We demand thus that the perturbed action, $(\Sigma + \varepsilon\Sigma^{ct})$, where ε is an expansion parameter, fulfils, to the first order in ε, the same Ward identities obeyed by the classical action Σ, i.e., Eqs. (56), (59), (60), and (62). This requirement gives rise to the set of equations

$$\begin{aligned}
&\mathcal{S}(\Sigma + \varepsilon\Sigma^{ct}) = O(\varepsilon^2),\\
&\frac{\delta}{\delta b^a}(\Sigma + \varepsilon\Sigma^{ct}) = i\partial_\mu A^a_\mu + \alpha b^a + O(\varepsilon^2),\\
&\left(\frac{\delta}{\delta\bar{c}^a} + \partial_\mu\frac{\delta}{\delta\Omega^a_\mu}\right)(\Sigma + \varepsilon\Sigma^{ct}) = O(\varepsilon^2),\\
&\left(\frac{\delta}{\delta\tau^a} - \partial_\mu\frac{\delta}{\delta\mathcal{J}^a_\mu}\right)(\Sigma + \varepsilon\Sigma^{ct}) = O(\varepsilon^2),
\end{aligned} \tag{64}$$

yielding the following constraints on Σ^{ct}:

$$\mathcal{B}_\Sigma\Sigma^{ct} = 0, \tag{65}$$

$$\frac{\delta}{\delta b^a}\Sigma^{ct} = 0, \tag{66}$$

$$\left(\frac{\delta}{\delta\bar{c}^a} + \partial_\mu\frac{\delta}{\delta\Omega^a_\mu}\right)\Sigma^{ct} = 0, \tag{67}$$

$$\left(\frac{\delta}{\delta\tau^a} - \partial_\mu\frac{\delta}{\delta\mathcal{J}^a_\mu}\right)\Sigma^{ct} = 0. \tag{68}$$

From the constraint (66) it follows that Σ^{ct} is independent from the Lagarange multiplier b^a, while Eqs. (67) and (68) ensure that Σ^{ct} depends only on the combinations $\hat{\Omega}^a_\mu = \Omega^a_\mu + \partial_\mu\bar{c}^a$ and $\hat{\mathcal{J}}^a_\mu = \mathcal{J}^a_\mu - \partial_\mu\tau^a$ of Eqs. (61) and (63).

From Eq. (65) one learns that Σ^{ct} belongs to the cohomolgy [64] of the linearized Slavnov-Taylor operator $\mathcal{B}_\Sigma$ in the space of the integrated local quantities in the fields and sources of dimension 4 and ghost number 0. Therefore, we can set

$$\Sigma^{ct} = \Delta + \mathcal{B}_\Sigma\Delta^{(-1)}, \tag{69}$$

where $\Delta^{(-1)}$ denotes a four-dimensional integrated quantity in the fields and sources with ghost number -1. The term $\mathcal{B}_\Sigma\Delta^{(-1)}$ in Eq. (69) corresponds to the trivial solution, i.e., to the exact part of the cohomology of $\mathcal{B}_\Sigma$. Instead, the quantity Δ identifies the nontrivial solution, i.e., the cohomology of $\mathcal{B}_\Sigma$, meaning that $\Delta \neq \mathcal{B}_\Sigma Q$, for some local integrated Q.

From the general results on the cohomology of Yang-Mills theories [64], and with the help of Table I, where the dimension and the ghost number of all fields and sources are displayed, it follows that Δ and $\Delta^{(-1)}$ can be written as

$$\Delta = \int d^4x\left[\frac{c_0}{4g^2}F^a_{\mu\nu}F^a_{\mu\nu} + c_1(\partial_\mu A^{h,a}_\mu)\partial_\nu A^{h,a}_\nu + c_2(\partial_\mu A^{h,a}_\nu)\partial_\mu A^{h,a}_\nu + c_3 f^{abc}A^{h,a}_\mu A^{h,b}_\nu\partial_\mu A^{h,c}_\nu + \lambda^{abcd}A^{h,a}_\mu A^{h,b}_\mu A^{h,c}_\nu A^{h,d}_\nu + \hat{\mathcal{J}}^a_\mu\mathcal{O}^a_\mu(A,\xi) + J\mathcal{O}(A,\xi) + c_4\frac{\zeta}{2}J^2\right], \tag{70}$$

TABLE I. The quantum numbers of fields and sources.

Fields and sources	A	b	c	$\bar{c}$	ξ	τ	Ω	L	K	J	$\mathcal{J}$
Dimension	1	2	0	2	0	2	3	4	4	2	3
Ghost number	0	0	1	−1	0	0	−1	−2	−1	0	0

and

$$\Delta^{(-1)} = \int d^4x[f_1^{ab}(\xi)\hat{\Omega}^a_\mu A^b_\mu + f_2^{ab}(\xi)L^a c^b + f_3^{ab}(\xi)K^a\xi^b], \tag{71}$$

where we have taken into account the gauge-invariant nature of the field A^h_μ, i.e.,

$$\mathcal{B}_\Sigma A^{h,a}_\mu = sA^{h,a}_\mu = 0, \qquad A^{h,a}_\mu \neq \mathcal{B}_\Sigma(\hat{\rho}^a_\mu), \tag{72}$$

for any $\hat{\rho}^a_\mu$. The parameters $(c_0, c_1, c_2, c_3, c_4, \lambda^{abcd})$ in expression (70) are free dimensionless coefficients, while $\mathcal{O}^a_\mu(A,\xi)$ and $\mathcal{O}(A,\xi)$ stand for generic local quantities with dimension 1 and 2 and ghost number 0, respectively, depending only on the fields A^a_μ and ξ^a. Also, the quantities $f_1^{ab}(\xi)$, $f_2^{ab}(\xi)$, and $f_3^{ab}(\xi)$ in expression (71) are arbitrary power series in ξ with ghost number 0, allowed by the dimensionless character of the Stueckelberg field ξ^a. Imposing now the constraint (65), one immediately gets

$$\mathcal{B}_\Sigma \mathcal{O}^a_\mu(A,\xi) = s\mathcal{O}^a_\mu(A,\xi) = 0, \tag{73}$$

$$\mathcal{B}_\Sigma \mathcal{O}(A,\xi) = s\mathcal{O}(A,\xi) = 0, \tag{74}$$

meaning that $\mathcal{O}^a_\mu(A,\xi)$ and $\mathcal{O}(A,\xi)$ have to be BRST invariant. Let us work out in detail the most general solutions of Eqs. (73) and (74), beginning with Eq. (73). Taking into account that the operator $\mathcal{O}^a_\mu(A,\xi)$ has dimension 1, ghost number 0, and carries both color and Lorentz indices, it can be parametrized as

$$\mathcal{O}^a_\mu(A,\xi) = \sigma^{ab}(\xi)A^b_\mu + \omega^{ab}(\xi)\partial_\mu\xi^b, \tag{75}$$

where $(\sigma^{ab}(\xi), \omega^{ab}(\xi))$ are dimensionless quantities in the Stueckelberg field ξ^a. Making use of expression (46), it turns out to be useful to replace A^a_μ by the gauge-invariant field $A^{h,a}_\mu$, upon a redefinition of the quantities $(\sigma^{ab}(\xi), \omega^{ab}(\xi))$, i.e.,

$$\mathcal{O}^a_\mu(A,\xi) = \hat{\sigma}^{ab}(\xi)A^{h,b}_\mu + \hat{\omega}^{ab}(\xi)\partial_\mu\xi^b. \tag{76}$$

Therefore, from condition (73) one gets

$$\frac{\partial\hat{\sigma}^{ab}}{\partial\xi^c}g^{cd}c^dA^{h,b}_\mu + \left(\frac{\partial\hat{\omega}^{ad}}{\partial\xi^b}g^{bc} + \hat{\omega}^{ab}\frac{\partial g^{bc}}{\partial\xi^d}\right)c^c\partial_\mu\xi^d + \hat{\omega}^{ab}g^{bc}\partial_\mu c^c = 0, \tag{77}$$

which immediately gives

$$\frac{\partial\hat{\sigma}^{ab}}{\partial\xi^c} = 0 \Rightarrow \hat{\sigma}^{ab} = b_1\delta^{ab}, \qquad \hat{\omega}^{ab} = 0, \tag{78}$$

where b_1 is a constant. We conclude thus that the most general form for $\mathcal{O}^a_\mu$ is given by

$$\mathcal{O}^a_\mu(A,\xi) = b_1 A^{h,a}_\mu. \tag{79}$$

The same reasoning applies as well to the case of the operator $\mathcal{O}(A,\xi)$ in Eq. (74). Taking into account now that $\mathcal{O}(A,\xi)$ is of dimension 2, we write

$$\mathcal{O}(A,\xi) = \sigma^{ab}(\xi)A^a_\mu A^b_\mu + \omega^a(\xi)\partial_\mu A^a_\mu + \lambda^{ab}(\xi)A^a_\mu\partial_\mu\xi^b + \frac{\rho^{ab}(\xi)}{2}(\partial_\mu\xi^a)\partial_\mu\xi^b + \beta^a(\xi)\partial^2\xi^a, \tag{80}$$

where $(\sigma^{ab}(\xi), \omega^a(\xi), \lambda^{ab}(\xi), \rho^{ab}(\xi), \beta^a(\xi))$ are dimensionless power series in ξ. Again, employing the gauge-invariant variable A^h_μ, we obtain, upon a redefinition of $(\sigma^{ab}, \omega^a, \lambda^{ab}, \rho^{ab}, \beta^a)$,

$$\mathcal{O}(A,\xi) = \hat{\sigma}^{ab}(\xi)A^{h,a}_\mu A^{h,b}_\mu + \hat{\omega}^a(\xi)\partial_\mu A^{h,a}_\mu + \hat{\lambda}^{ab}(\xi)A^{h,a}_\mu\partial_\mu\xi^b + \frac{\hat{\rho}^{ab}(\xi)}{2}(\partial_\mu\xi^a)\partial_\mu\xi^b + \hat{\beta}^a(\xi)\partial^2\xi^a. \tag{81}$$

From Eq. (74) we have

$$0 = \frac{\partial\hat{\sigma}^{ab}}{\partial\xi^d}g^{dc}A^{h,a}_\mu A^{h,b}_\mu c^c + \frac{\partial\hat{\omega}^a}{\partial\xi^c}g^{cb}(\partial_\mu A^{h,a}_\mu)c^b + \left(\frac{\partial\hat{\lambda}^{ab}}{\partial\xi^d}g^{dc} + \hat{\lambda}^{ad}\frac{\partial g^{dc}}{\partial\xi^b}\right)A^{h,a}_\mu(\partial_\mu\xi^b)c^c + \hat{\lambda}^{ac}g^{cb}A^{h,a}_\mu\partial_\mu c^b + \left(\frac{1}{2}\frac{\partial\hat{\rho}^{ab}}{\partial\xi^d}g^{dc} + \hat{\rho}^{ad}\frac{\partial g^{dc}}{\partial\xi^b} + \hat{\beta}^d\frac{\partial^2 g^{dc}}{\partial\xi^a\partial\xi^b}\right)(\partial_\mu\xi^a)(\partial_\mu\xi^b)c^c + \left(\hat{\rho}^{ac}g^{cb} + 2\hat{\beta}^c\frac{\partial g^{cb}}{\partial\xi^a}\right)(\partial_\mu\xi^a)\partial_\mu c^b + \left(\frac{\partial\hat{\beta}^a}{\partial\xi^c}g^{cb} + \hat{\beta}^c\frac{\partial g^{cb}}{\partial\xi^a}\right)(\partial^2\xi^a)c^b + \hat{\beta}^b g^{ba}\partial^2 c^a, \tag{82}$$

from which it follows that

$$\hat{\lambda}^{ab} = \hat{\rho}^{ab} = \hat{\beta}^{a} = 0,$$
$$\frac{\partial \hat{\sigma}^{ab}}{\partial \xi^{d}} = 0 \Rightarrow \hat{\sigma}^{ab} = \frac{b_2}{2}\delta^{ab},$$
$$\frac{\partial \hat{\omega}^{a}}{\partial \xi^{b}} = 0 \Rightarrow \hat{\omega}^{a} = 0 \quad \text{(by color invariance)}, \tag{83}$$

where b_2 is a free coefficient. Finally, for the operator $\mathcal{O}(A,\xi)$, we have

$$\mathcal{O}(A,\xi) = \frac{b_2}{2} A^{h,a}_{\mu} A^{h,a}_{\mu}. \tag{84}$$

Therefore, for the most general counterterm, Eq. (69), we get

$$\Delta = \int d^4x \left[\frac{c_0}{4g^2} F^a_{\mu\nu} F^a_{\mu\nu} + c_1 (\partial_\mu A^{h,a}_{\mu}) \partial_\nu A^{h,a}_{\nu} + c_2 (\partial_\mu A^{h,a}_{\nu}) \partial_\mu A^{h,a}_{\nu} + c_3 f^{abc} A^{h,a}_{\mu} A^{h,b}_{\nu} \partial_\mu A^{h,c}_{\nu} + \lambda^{abcd} A^{h,a}_{\mu} A^{h,b}_{\mu} A^{h,c}_{\nu} A^{h,d}_{\nu} + b_1 \hat{\mathcal{J}}^a_{\mu} A^{h,a}_{\mu} + \frac{b_2}{2} J A^{h,a}_{\mu} A^{h,a}_{\mu} + c_4 \frac{\zeta}{2} J^2 \right], \tag{85}$$

and $\Delta^{(-1)}$ is given by Eq. (71).

It remains now to characterize the coefficients $(c_1, c_2, c_3, \lambda^{abcd})$. To that aim, we rely on an important property of the action S in Eq. (44). When the mass parameter m^2 is set to 0, i.e., $m^2 = 0$, the expression reduces to

$$S_{m^2=0} = S_{\rm FP} + \int d^4x (\tau^a \partial_\mu A^{h,a}_{\mu}), \tag{86}$$

$$S_{\rm FP} = \int d^4x \left(\frac{1}{4g^2} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{\alpha}{2} b^a b^a + i b^a \partial_\mu A^a_\mu + \bar{c}^a \partial_\mu D^{ab}_{\mu} c^b \right), \tag{87}$$

which coincides, modulo the term $\int d^4x \tau^a \partial_\mu A^{h,a}_{\mu}$, with the Faddeev-Popov action $S_{\rm FP}$ of the linear covariant gauges.

Nevertheless, as shown in detail in Appendix A, the additional term $\int d^4x \tau^a \partial_\mu A^{h,a}_{\mu}$ has no consequences on the evaluation of the Green functions of the elementary fields $(A_\mu, b, c, \bar{c})$, meaning that the correlation functions $\langle A_{\mu_1}(x_1)\ldots A_{\mu_n}(x_n)\rangle_{S_{m^2=0}}$ evaluated with the action $S_{m^2=0}$ coincide with those computed with the Faddeev-Popov action $S_{\rm FP}$, namely,

$$\langle A_{\mu_1}(x_1)\ldots A_{\mu_n}(x_n)\rangle_{S_{m^2=0}} = \langle A_{\mu_1}(x_1)\ldots A_{\mu_n}(x_n)\rangle_{S_{\rm FP}}. \tag{88}$$

From this property, it follows that when the external fields $(J, \mathcal{J}, K)$ are set to 0, i.e., $(J, \mathcal{J}, K) \to 0$, the counterterm (85) and (71) has to reduce to that of the Faddeev-Popov action in the presence of the term $\int d^4x \tau^a \partial_\mu A^{h,a}_{\mu}$, namely, to expressions (A14), (A22)of Appendix A and B1 of Appendix B. This requirement gives

$$c_0 = a_0, \qquad c_1 = c_2 = c_3 = 0, \qquad \lambda^{abcd} = 0,$$
$$f_1^{ab} = a_1 \delta^{ab}, \quad f_2^{ab} = a_2 \delta^{ab}, \tag{89}$$

so that for the counterterm Σ^{ct} we obtain

$$\Sigma^{ct} = \int d^4x \left[\frac{a_0}{4g^2} F^a_{\mu\nu} F^a_{\mu\nu} + b_1 \hat{\mathcal{J}}^a_{\mu} A^{h,a}_{\mu} + \frac{b_2}{2} J A^{h,a}_{\mu} A^{h,a}_{\mu} + b_3 \frac{\zeta}{2} J^2 \right] + \mathcal{B}_\Sigma \int d^4x [a_1 \hat{\Omega}^a_\mu A^a_\mu + a_2 L^a c^a + K^a f^a(\xi)], \tag{90}$$

where we have performed the following redefinitions:

$$f^a(\xi) \equiv f_3^{ab}(\xi) \xi^b, \qquad b_3 \equiv c_4. \tag{91}$$

C. Parametric form of the counterterm and renormalization factors

Having determined the most general form of the invariant counterterm, Eq. (90), it remains to check if Σ^{ct} can be reabsorbed in the starting action Σ through a redefinition of parameters, fields, and sources. To that end, let us proceed by casting expression (90) in the so-called parametric form. From the expressions of the linearized Slavnov-Taylor operator $\mathcal{B}_\Sigma$, Eq. (57), we can rewrite the counterterm Σ^{ct} as

$$\Sigma^{ct} = \int d^4x \left(\frac{a_0}{4g^2} F^a_{\mu\nu} F^a_{\mu\nu} + b_1 \mathcal{J}^a_{\mu} A^{h,a}_{\mu} + b_1 \tau^a \partial_\mu A^{h,a}_{\mu} + \frac{b_2}{2} J A^{h,a}_{\mu} A^{h,a}_{\mu} + b_3 \frac{\zeta}{2} J^2 + a_1 A^a_\mu \frac{\delta \Sigma}{\delta A^a_\mu} - i a_1 b^a \partial_\mu A^a_\mu - a_1 \Omega^a_\mu \frac{\delta \Sigma}{\delta \Omega^a_\mu} + a_1 \bar{c}^a \partial_\mu \frac{\delta \Sigma}{\delta \Omega^a_\mu} - a_2 c^a \frac{\delta \Sigma}{\delta c^a} + a_2 L^a \frac{\delta \Sigma}{\delta L^a} + f^a(\xi) \frac{\delta \Sigma}{\delta \xi^a} - K^a \frac{\partial f^a}{\partial \xi^b} \frac{\delta \Sigma}{\delta K^b} \right), \tag{92}$$

where use has been made of the explicit expressions of $\hat{\mathcal{J}}^a_\mu$ and $\hat{\Omega}^a_\mu$ given, respectively, in Eqs. (63) and (61). In order to analyze the different terms of expression (92), we set

$$\Sigma^{ct} = \sum_{n=1}^{7} \Sigma^{ct}_n, \tag{93}$$

with

$$\Sigma_1^{ct} = \int d^4x \frac{a_0}{4g^2} F^a_{\mu\nu} F^a_{\mu\nu},$$
$$\Sigma_2^{ct} = \int d^4x b_1 \mathcal{J}^a_\mu A^{h,a}_\mu,$$
$$\Sigma_3^{ct} = \int d^4x b_1 \tau^a \partial_\mu A^{h,a}_\mu,$$
$$\Sigma_4^{ct} = \int d^4x \left(\frac{b_2}{2} J A^{h,a}_\mu A^{h,a}_\mu + b_3 \frac{\zeta}{2} J^2\right),$$
$$\Sigma_5^{ct} = \int d^4x (-i a_1 b^a \partial_\mu A^a_\mu),$$
$$\Sigma_6^{ct} = \int d^4x a_1 \bar{c}^a \partial_\mu \frac{\delta\Sigma}{\delta\Omega^a_\mu},$$
$$\Sigma_7^{ct} = \int d^4x \left(a_1 A^a_\mu \frac{\delta\Sigma}{\delta A^a_\mu} - a_1 \Omega^a_\mu \frac{\delta\Sigma}{\delta\Omega^a_\mu} + a_2 L^a \frac{\delta\Sigma}{\delta L^a} + f^a(\xi) \frac{\delta\Sigma}{\delta\xi^a} - K^a \frac{\partial f^a}{\partial\xi^b} \frac{\delta\Sigma}{\delta K^b}\right). \tag{94}$$

By noticing that

$$\frac{\partial\Sigma}{\partial g^2} = \frac{\partial}{\partial g^2} \int d^4x \frac{1}{4g^2} F^a_{\mu\nu} F^a_{\mu\nu} = -\frac{1}{g^2} \int d^4x \frac{1}{4g^2} F^a_{\mu\nu} F^a_{\mu\nu}, \tag{95}$$

the term Σ_1^{ct} can be rewritten as

$$\Sigma_1^{ct} = -a_0 g^2 \frac{\partial\Sigma}{\partial g^2}. \tag{96}$$

Taking the variation of the action Σ with respect to $\mathcal{J}^a_\mu$ and τ^a,

$$\frac{\delta\Sigma}{\delta\mathcal{J}^a_\mu} = A^{h,a}_\mu, \qquad \frac{\delta\Sigma}{\delta\tau^a} = \partial_\mu A^{h,a}_\mu, \tag{97}$$

the terms Σ_2^{ct} and Σ_3^{ct} are rewritten as

$$\Sigma_2^{\text{count}} = b_1 \int d^4x \mathcal{J}^a_\mu \frac{\delta\Sigma}{\delta\mathcal{J}^a_\mu},$$
$$\Sigma_3^{\text{count}} = b_1 \int d^4x \tau^a \frac{\delta\Sigma}{\delta\tau^a}. \tag{98}$$

Also, taking the variation of Σ with respect to J, we obtain

$$\frac{\delta\Sigma}{\delta J} = \frac{1}{2} A^{h,a}_\mu A^{h,a}_\mu + \zeta J, \tag{99}$$

from which it follows that Σ_4^{ct} takes the form

$$\Sigma_4^{ct} = \int d^4x \left(b_2 J \frac{\delta\Sigma}{\delta J} + (b_3 - 2b_2) \frac{\zeta}{2} J^2 \right). \tag{100}$$

On the other hand, we also have that

$$\zeta \frac{\partial\Sigma}{\partial\zeta} = \int d^4x \frac{\zeta}{2} J^2. \tag{101}$$

Thus,

$$\Sigma_4^{ct} = b_2 \int d^4x J \frac{\delta\Sigma}{\delta J} + (b_3 - 2b_2)\zeta \frac{\partial\Sigma}{\partial\zeta}. \tag{102}$$

Now, considering the gauge-fixing equation (59), we can rewrite Σ_5^{ct} as

$$\Sigma_5^{ct} = \int d^4x \left(-a_1 b^a \frac{\delta\Sigma}{\delta b^a} + a_1 \alpha b^a b^a \right). \tag{103}$$

Furthermore, from

$$2\alpha \frac{\partial\Sigma}{\partial\alpha} = \int d^4x \alpha b^a b^a, \tag{104}$$

one gets

$$\Sigma_5^{ct} = -a_1 \int d^4x b^a \frac{\delta\Sigma}{\delta b^a} + 2a_1\alpha \frac{\partial\Sigma}{\partial\alpha}. \tag{105}$$

The term Σ_6^{ct} can be immediately rewritten using the antighost equation (60) as

$$\Sigma_6^{ct} = -a_1 \int d^4x \bar{c}^a \frac{\delta\Sigma}{\delta\bar{c}^a}. \tag{106}$$

Putting together all expressions, for the parametric form of the counterterm we obtain

$$\begin{aligned}\Sigma^{ct} = {} & -a_0 g^2 \frac{\partial\Sigma}{\partial g^2} + (b_3 - 2b_2)\zeta \frac{\partial\Sigma}{\partial\zeta} + 2a_1\alpha \frac{\partial\Sigma}{\partial\alpha} \\ & + \int d^4x \Big(a_1 A^a_\mu \frac{\delta\Sigma}{\delta A^a_\mu} - a_1 b^a \frac{\delta\Sigma}{\delta b^a} \\ & - a_1 \bar{c}^a \frac{\delta\Sigma}{\delta\bar{c}^a} - a_2 c^a \frac{\delta\Sigma}{\delta c^a} + b_1 \tau^a \frac{\delta\Sigma}{\delta\tau^a} + f^a(\xi) \frac{\delta\Sigma}{\delta\xi^a} \\ & - a_1 \Omega^a_\mu \frac{\delta\Sigma}{\delta\Omega^a_\mu} + a_2 L^a \frac{\delta\Sigma}{\delta L^a} \\ & + b_1 \mathcal{J}^a_\mu \frac{\delta\Sigma}{\delta\mathcal{J}^a_\mu} + b_2 J \frac{\delta\Sigma}{\delta J} - K^b \frac{\partial f^b}{\partial\xi^a} \frac{\delta\Sigma}{\delta K^a} \Big),\end{aligned} \tag{107}$$

which can be finally written as

$$\Sigma^{ct} = \mathcal{R}\Sigma, \tag{108}$$

with $\mathcal{R}$ being the differential operator,

$$\mathcal{R} = -a_0 g^2 \frac{\partial}{\partial g^2} + (b_3 - 2b_2)\zeta \frac{\partial}{\partial \zeta} + 2a_1 \alpha \frac{\partial}{\partial \alpha} + \int d^4x \Big(a_1 A^a_\mu \frac{\delta}{\delta A^a_\mu} - a_1 b^a \frac{\delta}{\delta b^a} - a_1 \bar{c}^a \frac{\delta}{\delta \bar{c}^a} - a_2 c^a \frac{\delta}{\delta c^a} + b_1 \tau^a \frac{\delta}{\delta \tau^a} + f^a(\xi) \frac{\delta}{\delta \xi^a} - a_1 \Omega^a_\mu \frac{\delta}{\delta \Omega^a_\mu} + a_2 L^a \frac{\delta}{\delta L^a} + b_1 \mathcal{J}^a_\mu \frac{\delta}{\delta \mathcal{J}^a_\mu} + b_2 J \frac{\delta}{\delta J} - K^b \frac{\partial f^b}{\partial \xi^a} \frac{\delta}{\delta K^a} \Big). \tag{109}$$

The usefulness of expression (108) relies on the fact that it immediately provides the redefinition of the fields, parameter, and sources needed to show that the counterterm Σ^{ct} can be in fact reabsorbed into the starting action, namely,

$$\Sigma(\Phi) + \varepsilon \Sigma^{ct}(\Phi) = \Sigma(\Phi_0) + O(\varepsilon^2), \tag{110}$$

where ε is an expansion parameter, Φ is a shorthand notation for the fields, parameters, and sources, while Φ_0 stands for the corresponding redefinitions. From Eq. (108) it is apparent that the redefined fields, parameters, and sources are given by

$$\Phi_0 = (1 + \varepsilon \mathcal{R})\Phi. \tag{111}$$

In fact, using (111), it is almost immediate to prove that

$$\Sigma[\Phi_0] = \Sigma[\Phi + \varepsilon \mathcal{R}\Phi] = \Sigma[\Phi] + \varepsilon \mathcal{R}(\Sigma) + O(\varepsilon^2), \tag{112}$$

showing that the counterterm Σ^{ct} can be reabsorbed into the starting action Σ.

By direct inspection of Eq. (112), for the renormalization factors one finds

$$\begin{aligned} &A_0 = Z_A^{1/2} A, \quad b_0 = Z_b^{1/2} b, \quad c_0 = Z_c^{1/2} c, \\ &\bar{c}_0 = Z_{\bar{c}}^{1/2} \bar{c}, \quad \xi^a_0 = Z^{ab}_\xi(\xi)\xi^b, \quad \tau_0 = Z_\tau^{1/2}\tau, \\ &\Omega_0 = Z_\Omega \Omega, \quad L_0 = Z_L L, \quad K^a_0 = Z^{ab}_K(\xi) K^b, \\ &J_0 = Z_J J, \quad \mathcal{J}_0 = Z_{\mathcal{J}} \mathcal{J}, \\ &g_0 = Z_g g, \quad \alpha_0 = Z_\alpha \alpha, \quad \zeta_0 = Z_\zeta \zeta, \end{aligned} \tag{113}$$

where

$$Z_g = 1 + \varepsilon a_0, \tag{114}$$

$$Z_A^{1/2} = 1 + \varepsilon a_1, \tag{115}$$

$$Z_c^{1/2} = 1 - \varepsilon a_2, \tag{116}$$

$$Z_{\mathcal{J}} = 1 + \varepsilon b_1, \tag{117}$$

$$Z_J = 1 + \varepsilon b_2, \tag{118}$$

$$Z_\zeta = 1 + \varepsilon(b_3 - 2b_2), \tag{119}$$

$$Z^{ab}_\xi = \delta^{ab} + \varepsilon f^{ab}_3, \tag{120}$$

$$Z^{ab}_K = \delta^{ab} - \varepsilon \frac{\partial f^b}{\partial \xi^a} = \delta^{ab} - \varepsilon \left(\frac{\partial f^{bc}_3}{\partial \xi^a} \xi^c + f^{ba}_3 \right), \tag{121}$$

and

$$\begin{aligned} &Z_\alpha = Z_A, \quad Z_b^{1/2} = Z_{\bar{c}}^{1/2} = Z_\Omega = Z_A^{-1/2}, \\ &Z_\tau^{1/2} = Z_{\mathcal{J}}, \quad Z_L = Z_c^{-1/2}. \end{aligned} \tag{122}$$

Observe that in Eqs. (120) and (121) we have used the definition $f^a(\xi) = f^{ab}_3(\xi)\xi^b$ introduced in Eq. (91). We also underline that, according to (120) and (121), the renormalization factors of the Stueckelberg field ξ^a and of the corresponding source K^a are nonlinear, i.e., they are power series in ξ^a. This is an expected feature, due to the dimensionless character of the Stueckelberg field, a feature common to other renormalizable models displaying massless fields as, for example, $N = 1$ super Yang-Mills theory in superspace; see [73].

IV. THE ANOMALOUS DIMENSIONS OF $(A^h_\mu A^h_\mu)$ AND A^h_μ

Let us address now the issue of the anomalous dimensions of the operators $(A^{h,a}_\mu A^{h,a}_\mu)$ and $A^{h,a}_\mu$. As a consequence of their gauge invariance, their anomalous dimensions turn out to be independent from the gauge parameter α, a result that can be established at the algebraic level through the use of the extended BRST technique [64]. (See also the recent proof given in [66].) In particular, due to its α-independence, the anomalous dimension of $(A^{h,a}_\mu A^{h,a}_\mu)$ is the same as that computed in the Landau gauge, i.e., for $\alpha = 0$. Moreover, taking into account that, in the Landau gauge, the operator $(A^{h,a}_\mu A^{h,a}_\mu)$ reduces to $(A^a_\mu A^a_\mu)$, we expect that the anomalous dimension $\gamma_{(A^h)^2}$ of $(A^{h,a}_\mu A^{h,a}_\mu)$ should be equal to the anomalous dimension $\gamma_{A^2}|_{\rm Landau}$ of the operator $(A^a_\mu A^a_\mu)$ in the Landau gauge, namely,

$$\gamma_{(A^h)^2} = \gamma_{A^2}|_{\rm Landau} = -\left(\frac{\beta(a)}{a} + \gamma_A^{\rm Landau}(a) \right), \quad a = \frac{g^2}{16\pi^2}, \tag{123}$$

where $(\beta(a), \gamma_A^{\rm Landau}(a))$ denote, respectively, the β-function and the anomalous dimension of the gauge field A_μ in the Landau gauge.[2]

[2]For an all-order algebraic proof of the relationship

$$\gamma_{A^2}|_{\rm Landau} = -\left(\frac{\beta(a)}{a} + \gamma_A^{\rm Landau}(a) \right), \qquad a = \frac{g^2}{16\pi^2},$$

see [35].

A similar property is expected in the case of the operator A^h_μ, namely,

$$\gamma_{A^h} = \gamma_{A^h}|_{\alpha=0} = \gamma_A^{\rm Landau}(a), \tag{124}$$

i.e., the anomalous dimension of A^h_μ should equal that of the gauge field A^a_μ in the Landau gauge. Therefore, both $\gamma_{(A^h)^2}$ and γ_{A^h} are not independent parameters of the theory.

Let us give a formal proof of Eqs. (123) and (124) by making use of the RGE that, owing to the renormalizability and to the BRST invariance of the theory, reads

$$\mu\frac{\partial\Gamma}{\partial\mu} + \beta_{g^2}\frac{\partial\Gamma}{\partial g^2} - \gamma_A\mathcal{N}_A\Gamma - \gamma_c\mathcal{N}_c\Gamma - \gamma_{(A^h)^2}\int d^4x J\frac{\delta\Gamma}{\delta J} - \gamma_{A^h}\int d^4x\left(\mathcal{J}^a_\mu\frac{\delta\Gamma}{\delta\mathcal{J}^a_\mu} + \tau^a\frac{\delta\Gamma}{\delta\tau^a}\right) - \int d^4x\left(\gamma^{ab}_\xi(\xi)\xi^b\frac{\delta\Gamma}{\delta\xi^a} + \gamma^{ab}_K(\xi)K^b\frac{\delta\Gamma}{\delta K^a}\right) = 0, \tag{125}$$

where

$$\mathcal{N}_A = \int d^4x\left(A^a_\mu\frac{\delta}{\delta A^a_\mu} - b^a\frac{\delta}{\delta b^a} - \bar{c}^a\frac{\delta}{\delta\bar{c}^a} - \Omega^a_\mu\frac{\delta}{\delta\Omega^a_\mu}\right) + 2\alpha\frac{\partial}{\partial\alpha},$$
$$\mathcal{N}_c = \int d^4x\left(c^a\frac{\delta}{\delta c^a} - L^a\frac{\delta}{\delta L^a}\right),$$
$$\gamma^{ab}_\xi = (Z^{-1}_\xi)^{ac}\mu\frac{\partial}{\partial\mu}Z^{cb}_\xi,$$
$$\gamma^{ab}_K = (Z^{-1}_K)^{ac}\mu\frac{\partial}{\partial\mu}Z^{cb}_K. \tag{126}$$

Let us act now on the RGE with the test operator

$$\frac{\delta^2}{\delta\mathcal{J}^a_\mu(x)\delta\mathcal{J}^b_\nu(y)}, \tag{127}$$

and set all fields and sources equal to 0. A simple algebraic calculation gives

$$0 = \mu\frac{\partial}{\partial\mu}\langle A^{h,a}_\mu(x)A^{h,b}_\nu(y)\rangle + \beta_{g^2}\frac{\partial}{\partial g^2}\langle A^{h,a}_\mu(x)A^{h,b}_\nu(y)\rangle - 2\gamma_A\alpha\frac{\partial}{\partial\alpha}\langle A^{h,a}_\mu(x)A^{h,b}_\nu(y)\rangle - 2\gamma_{A^h}\langle A^{h,a}_\mu(x)A^{h,b}_\nu(y)\rangle. \tag{128}$$

Moreover, due to the α-independence of the gauge-invariant correlation function $\langle A^{h,a}_\mu(x)A^{h,b}_\nu(y)\rangle$, it follows that

$$\frac{\partial}{\partial\alpha}\langle A^{h,a}_\mu(x)A^{h,b}_\nu(y)\rangle = 0. \tag{129}$$

Thus,

$$\mu\frac{\partial}{\partial\mu}\langle A^{h,a}_\mu(x)A^{h,b}_\nu(y)\rangle + \beta_{g^2}\frac{\partial}{\partial g^2}\langle A^{h,a}_\mu(x)A^{h,b}_\nu(y)\rangle - 2\gamma_{A^h}\langle A^{h,a}_\mu(x)A^{h,b}_\nu(y)\rangle = 0. \tag{130}$$

In addition, from (129) we can make direct use of the Landau gauge, namely,

$$\langle A^{h,a}_\mu(x)A^{h,b}_\nu(y)\rangle = \langle A^{h,a}_\mu(x)A^{h,b}_\nu(y)\rangle_{\alpha=0} = \langle A^{h,a}_\mu(x)A^{h,b}_\nu(y)\rangle_{\rm Landau}. \tag{131}$$

Therefore,

$$\langle A^{h,a}_\mu(x)A^{h,b}_\nu(y)\rangle = \frac{\int[D\phi]A^{h,a}_\mu(x)A^{h,b}_\nu(y)e^{-S_{(\alpha,m^2=0)}}}{\int[D\phi]e^{-S_{(\alpha,m^2=0)}}}, \tag{132}$$

with $[D\phi] \equiv DADbDcD\bar{c}D\xi D\tau$ and

$$S_{(\alpha,m^2=0)} = \int d^4x\left(\frac{1}{4g^2}F^a_{\mu\nu}F^a_{\mu\nu} + ib^a\partial_\mu A^a_\mu + \bar{c}^a\partial_\mu D^{ab}_\mu c^b + \tau^a\partial_\mu A^{h,a}_\mu\right). \tag{133}$$

Integrating out the fields $(\tau, b, c, \bar{c})$, we get

$$\langle A^{h,a}_\mu(x)A^{h,b}_\nu(y)\rangle = \frac{\int DAD\xi\delta(\partial_\mu A^h_\mu)\delta(\partial_\mu A_\mu)\det(-\partial\cdot D)A^{h,a}_\mu(x)A^{h,b}_\nu(y)e^{-S_{\rm YM}}}{\int DAD\xi\delta(\partial_\mu A^h_\mu)\delta(\partial_\mu A_\mu)\det(-\partial\cdot D)e^{-S_{\rm YM}}}. \tag{134}$$

Employing the result given in Appendix B, see Eqs. (B22) and (B23), the equation $\partial_\mu A^h_\mu = 0$ can be solved iteratively for ξ^a yielding

$$\xi = \frac{1}{\partial^2}\partial_\mu A_\mu + i\frac{g}{\partial^2}\left[\partial A, \frac{\partial A}{\partial^2}\right] + i\frac{g}{\partial^2}\left[A_\mu, \partial_\mu\frac{\partial A}{\partial^2}\right] + \frac{i}{2}\frac{g}{\partial^2}\left[\frac{\partial A}{\partial^2}, \partial A\right] + O(A^3), \tag{135}$$

so that we can integrate over ξ^a, obtaining

$$\langle A^{h,a}_\mu(x)A^{h,b}_\nu(y)\rangle = \frac{\int DA\delta(\partial_\mu A_\mu)\det(-\partial\cdot D)A^{h,a}_\mu(x)A^{h,b}_\nu(y)e^{-S_{\rm YM}}}{\int DA\delta(\partial_\mu A_\mu)\det(-\partial\cdot D)e^{-S_{\rm YM}}}, \tag{136}$$

where A^h_μ is now given by, see Eq. (B24) of Appendix B,

$$A^h_\mu = A_\mu - \frac{1}{\partial^2}\partial_\mu\partial A - ig\frac{\partial_\mu}{\partial^2}\left[A_\nu, \partial_\nu\frac{\partial A}{\partial^2}\right] - i\frac{g}{2}\frac{\partial_\mu}{\partial^2}\left[\partial A, \frac{1}{\partial^2}\partial A\right] + ig\left[A_\mu, \frac{1}{\partial^2}\partial A\right] + i\frac{g}{2}\left[\frac{1}{\partial^2}\partial A, \frac{\partial_\mu}{\partial^2}\partial A\right] + O(A^3). \tag{137}$$

However, due to the presence in Eq. (136) of the delta function $\delta(\partial_\mu A_\mu)$, all terms containing a divergence ∂A vanish, namely,

$$\begin{aligned}\langle A^{h,a}_\mu(x)A^{h,b}_\nu(y)\rangle &= \frac{\int DA\delta(\partial_\mu A_\mu)\det(-\partial\cdot D)A^{h,a}_\mu(x)A^{h,b}_\nu(y)e^{-S_{\rm YM}}}{\int DA\delta(\partial_\mu A_\mu)\det(-\partial\cdot D)e^{-S_{\rm YM}}} \\ &= \frac{\int DA\delta(\partial_\mu A_\mu)\det(-\partial\cdot D)A^{a}_\mu(x)A^{b}_\nu(y)e^{-S_{\rm YM}}}{\int DA\delta(\partial_\mu A_\mu)\det(-\partial\cdot D)e^{-S_{\rm YM}}} \\ &= \langle A^a_\mu(x)A^b_\nu(y)\rangle_{\rm Landau}.\end{aligned} \tag{138}$$

Thus, the RGE for the correlation function $\langle A^{h,a}_\mu(x)A^{h,b}_\nu(y)\rangle$ becomes

$$\mu\frac{\partial}{\partial\mu}\langle A^a_\mu(x)A^b_\nu(y)\rangle_{\rm Landau} + \beta_{g^2}\frac{\partial}{\partial g^2}\langle A^a_\mu(x)A^b_\nu(y)\rangle_{\rm Landau} - 2\gamma_{A^h}\langle A^a_\mu(x)A^b_\nu(y)\rangle_{\rm Landau} = 0, \tag{139}$$

which proves Eq. (124). Of course, the same reasoning can be applied to Eq. (123).

V. CONCLUSIONS

In this work we have provided a study of the gauge-invariant nonlocal operator $A^2_{\rm min}$,

$$A^2_{\rm min} = {\rm Tr}\int d^4x A^h_\mu A^h_\mu, \tag{140}$$

with A^h_μ being the transverse configuration, $\partial_\mu A^h_\mu = 0$, given in expression (3).

Despite the highly nonlocal character, we have shown that a fully local setup for both operators $(A^h_\mu A^h_\mu)$ and A^h_μ can be constructed, giving rise to a local and BRST-invariant action S, Eq. (17). The main tool in order to achieve such a local formulation has been the introduction of an auxiliary Stueckelberg field ξ^a, Eqs. (11) and (12).

As pointed out in Sec. II, the transversality condition, $\partial_\mu A^h_\mu = 0$, plays an important role, giving rise to deep differences between our formulation and the conventional Stueckelberg one, which is known to be nonrenormalizable. Unlike the conventional Stueckelberg formulation, the novel action S, Eq. (17), has been proven to be renormalizable to all orders, as shown in detail in Sec. III. Furthermore, owing to the gauge invariance of $(A^h_\mu A^h_\mu)$ and A^h_μ, the corresponding anomalous dimensions, $(\gamma_{(A^h)^2}, \gamma_{A^h})$, turn out to be independent from the gauge parameter α entering the gauge-fixing condition, being given by

$$\begin{aligned}\gamma_{(A^h)^2} &= \gamma_{A^2}|_{\rm Landau} = -\left(\frac{\beta(a)}{a} + \gamma^{\rm Landau}_A(a)\right), \quad a = \frac{g^2}{16\pi^2}, \\ \gamma_{A^h} &= \gamma_{A^h}|_{\alpha=0} = \gamma^{\rm Landau}_A(a),\end{aligned} \tag{141}$$

where $(\beta(a), \gamma^{\rm Landau}_A(a))$ denote, respectively, the β-function and the anomalous dimension of the gauge field A_μ in the Landau gauge. We see therefore that $(\gamma_{(A^h)^2}, \gamma_{A^h})$ are not independent parameters of the theory.

The present results can open the road to several future investigations. For instance, the possibility of having at our disposal a local and renormalizable framework might enable us to investigate the formation, through the computation of the effective potential [7,10,11,15,16], of the gauge-invariant dimension-two condensate $\langle A^h_\mu A^h_\mu\rangle$. This result might yield a better understanding, within a manifestly BRST-invariant set up, of the relevance of the condensate $\langle A^h_\mu A^h_\mu\rangle$ for the formation of the dynamical gluon mass [7,10,11,15,16] as well the analysis of the $\frac{1}{Q^2}$ corrections in the gluon correlation functions within the OPE expansion, as reported in [18–21,24,26–30,32–34] in the case of the Landau gauge.

Another topic worth mentioning is the study of the BRST-invariant and α-independent correlation function

$$\langle A^h_\mu(x) A^h_\mu(y)\rangle, \tag{142}$$

within the local present setup. Because of its α-independence, expression (142) can be seen as the natural generalization, in the case of the covariant linear gauges, of the two-point function $\langle A_\mu(x)A_\mu(y)\rangle_{\rm Landau}$ studied in the renormalizable massive Yang-Mills model in the Landau gauge considered in [71,72]. As such, expression (142) might provide information about the occurrence of positivity violation, already observed in the Landau gauge [71,72]. This could, in principle, shed some light on the important question regarding the physical significance of positivity violation and its relation to phenomenological properties, since this issue might now be studied in a systematic gauge-parameter invariant manner, instead of relying solely on gauge-dependent quantities, like the gluon propagator. In this sense, expression (142) might be regarded as a powerful and practical tool to detect the positivity violation, in linear covariant gauges, of the two-point gluon correlation function within a BRST-invariant formulation. Moreover, it would be interesting to find out whether $\langle A^h_\mu(x)A^h_\mu(y)\rangle$ develops complex-conjugated poles—as seems to occur in Gribov-type fits to lattice data for the gluon propagator—or real ones, with a negative residue being the cause of the positivity violation in the latter scenario. Even though the framework presented here represents a well-defined analytical setup for the study of correlation functions of the operator A^h, the determination of the nature of the poles requires a fully nonperturbative analysis that is currently accessible only through lattice input, so numerical studies of expression (142) are very welcome.

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APPENDIX A: A REVIEW ON THE RENORMALIZATION OF THE YANG-MILLS ACTION IN LINEAR COVARIANT GAUGES

When the mass parameter m^2 is set to 0, the action S in Eq. (45) reduces to

$$S_{m^2=0} = S_{\rm FP} + \int d^4x (\tau^a \partial_\mu A^{h,a}_\mu), \tag{A1}$$

where $S_{\rm FP}$ is

$$S_{\rm FP} = \int d^4x \left(\frac{1}{4g^2} F^a_{\mu\nu}F^a_{\mu\nu} + \frac{\alpha}{2} b^a b^a + i b^a \partial_\mu A^a_\mu + \bar{c}^a \partial_\mu D^{ab}_\mu c^b\right), \tag{A2}$$

i.e., $S_{m^2=0}$ coincides, modulo the term $\int d^4x \tau^a \partial_\mu A^{h,a}_\mu$, with the usual Faddeev-Popov action of the linear covariant gauge. Evidently, the action $S_{m^2=0}$ is left invariant by the BRST transformations given in Eqs. (47) and (48),

$$s S_{m^2=0} = 0. \tag{A3}$$

Nevertheless, when $m^2 = 0$, the additional term $\int d^4x \tau^a \partial_\mu A^{h,a}_\mu$ has no consequences on the evaluation of the Green functions of the elementary fields $(A_\mu, b, c, \bar{c})$. More precisely, it turns out that the correlation functions $\langle A_{\mu_1}(x_1)\ldots A_{\mu_n}(x_n)\rangle_{S_{m^2=0}}$ evaluated with the action $S_{m^2=0}$ coincide with those computed with the Faddeev-Popov action $S_{\rm FP}$, namely,

$$\langle A_{\mu_1}(x_1)\ldots A_{\mu_n}(x_n)\rangle_{S_{m^2=0}} = \langle A_{\mu_1}(x_1)\ldots A_{\mu_n}(x_n)\rangle_{S_{\rm FP}}. \tag{A4}$$

The statement (A4) can be checked by means of the functional integral. Let us consider expression

$$\langle A_{\mu_1}(x_1)\ldots A_{\mu_n}(x_n)\rangle_{S_{m^2=0}} = \frac{\int [D\phi] A_{\mu_1}(x_1)\ldots A_{\mu_n}(x_n) e^{-S_{m^2=0}}}{\int [D\phi] e^{-S_{m^2=0}}}, \tag{A5}$$

where $[D\phi]$ stands for integration over all fields, i.e., $[D\phi] = DADbDcD\bar{c}D\xi D\tau$. Integrating over the field τ, one gets

$$\langle A_{\mu_1}(x_1)\ldots A_{\mu_n}(x_n)\rangle_{S_{m^2=0}} = \frac{\int DADbDcD\bar{c}D\xi \delta(\partial_\mu A^h_\mu) A_{\mu_1}(x_1)\ldots A_{\mu_n}(x_n) e^{-S_{\rm FP}}}{\int DADbDcD\bar{c}D\xi \delta(\partial_\mu A^h_\mu) e^{-S_{\rm FP}}}. \tag{A6}$$

Making use of the result given in Appendix (B), see Eqs. (B22) and (B23), the equation $\partial_\mu A^h_\mu = 0$ can be solved iteratively for ξ^a yielding

$$\xi = \frac{1}{\partial^2}\partial_\mu A_\mu + i\frac{g}{\partial^2}\left[\partial A, \frac{\partial A}{\partial^2}\right] + i\frac{g}{\partial^2}\left[A_\mu, \partial_\mu \frac{\partial A}{\partial^2}\right] + \frac{i}{2}\frac{g}{\partial^2}\left[\frac{\partial A}{\partial^2}, \partial A\right] + O(A^3), \tag{A7}$$

so that expression (A6) can be written as

$$\langle A_{\mu_1}(x_1)\ldots A_{\mu_n}(x_n)\rangle_{S_{m^2=0}} = \frac{\int DADbDcD\bar{c}D\xi\delta(\xi - [\text{power series in } A])A_{\mu_1}(x_1)\ldots A_{\mu_n}(x_n)e^{-S_{\rm FP}}}{\int DADbDcD\bar{c}D\xi\delta(\xi - [\text{power series in } A])e^{-S_{\rm FP}}}. \tag{A8}$$

Observing now that the Faddeev-Popov action $S_{\rm FP}$, Eq. (A1), does not contain any dependence from the Stueckelberg field, it follows that the integration over the variable ξ in Eq. (A6) is straightforward, giving

$$\langle A_{\mu_1}(x_1)\ldots A_{\mu_n}(x_n)\rangle_{S_{m^2=0}} = \frac{\int DADbDcD\bar{c}A_{\mu_1}(x_1)\ldots A_{\mu_n}(x_n)e^{-S_{\rm FP}}}{\int DADbDcD\bar{c}e^{-S_{\rm FP}}}, \tag{A9}$$

which proves the statement (A4). The same reasoning applies to other Green's functions containing the elementary fields $(b, c, \bar{c})$. In summary, all Green's functions of the elementary fields $(A_\mu, b, c, \bar{c})$ evaluated with the action (A1) are exactly the same as those computed with the Faddeev-Popov action (A2).

In particular, from this result it follows that the action (A1) is renormalizable, the most general counterterm being given, modulo terms in the variable τ, by the usual counterterm of the linear covariant gauges.

Let us give a closer look at the possible local BRST-invariant counterterm $S^{ct}_{m^2=0}$ affecting the action $S_{m^2=0}$ at the quantum level. $S^{ct}_{m^2=0}$ is a local integrated quantity in the fields bounded by dimension 4. Moreover, it is useful to notice that, besides the BRST invariance, Eq. (A3), the action $S_{m^2=0}$ is constrained by the additional Ward identity

$$\int d^4x \frac{\delta S_{m^2=0}}{\delta \tau^a} = 0, \tag{A10}$$

which implies that the variable τ can enter only through a space-time derivative, i.e., $\partial_\mu \tau^a$. Therefore, owing to the previous considerations, and taking into account that the field τ has dimension 2, for the counterterm $S^{ct}_{m^2=0}$ we write

$$S^{ct}_{m^2=0} = S^{ct}_{\rm FP} - \int d^4x (\partial_\mu \tau^a)\mathcal{O}^a_\mu(A, \xi), \tag{A11}$$

where $S^{ct}_{\rm FP}$ is the usual local BRST-invariant counterterm of the Faddeev-Popov action in linear covariant gauges and where $\mathcal{O}^a_\mu(A, \xi)$ is a local quantity of dimension 1. From BRST invariance, we immediately get

$$s\mathcal{O}^a_\mu(A, \xi) = 0, \tag{A12}$$

whose general solution, see Eqs. (73)–(79), is

$$\mathcal{O}^a_\mu(A, \xi) = b_1 A^{h,a}_\mu, \tag{A13}$$

with b_1 being an arbitrary coefficient. Thus, for the most general counterterm corresponding to $S|_{m^2=0}$ we have

$$S^{ct}_{m^2=0} = S^{ct}_{\rm FP} - b_1 \int d^4x (\partial_\mu \tau^a) A^{h,a}_\mu. \tag{A14}$$

Let us end this subsection by providing the expression of the Faddeev-Popov counterterm $S^{ct}_{\rm FP}$, as derived form the algebraic renormalization procedure [64].

1. Renormalizability of the Faddeev-Popov action in linear covariant gauges

Following [64], in order to determine the most general invariant counterterm $S^{ct}_{\rm FP}$ affecting the Faddeev-Popov action in linear covariant gauges, Eq. (A2), we start from the complete classical action

$$\Sigma_0 = S_{\rm FP} + \int d^4x \left(-\Omega^a_\mu D^{ab}_\mu c^b + \frac{1}{2} f^{abc} L^a c^b c^c \right), \tag{A15}$$

where we have introduced the external sources (Ω^a_μ, L^a) coupled to the nonlinear BRST variations of the fields (A^a_μ, c^a); see Eqs. (47) and (48).

The action Σ_0 obeys the following set of Ward identities [64]:

$$\int d^4x \left(\frac{\delta \Sigma_0}{\delta \Omega^a_\mu} \frac{\delta \Sigma_0}{\delta A^a_\mu} + \frac{\delta \Sigma_0}{\delta L^a} \frac{\delta \Sigma_0}{\delta c^a} + ib^a \frac{\delta \Sigma_0}{\delta \bar{c}^a} \right) = 0, \tag{A16}$$

$$\frac{\delta \Sigma_0}{\delta b^a} = i\partial_\mu A^a_\mu + \alpha b^a, \tag{A17}$$

$$\frac{\delta \Sigma_0}{\delta \bar{c}^a} + \partial_\mu \frac{\delta \Sigma_0}{\delta \Omega^a_\mu} = 0, \tag{A18}$$

from which it turns out [64] that the most general local invariant counterterm Σ^{ct}_0 contains three free parameters (a_0, a_1, a_2), being given by the expression

$$\Sigma^{ct}_0 = a_0 \int d^4x \frac{1}{4g^2} F^a_{\mu\nu} F^a_{\mu\nu} + \mathcal{B}_{\Sigma_0} \int d^4x (a_1(\Omega^a_\mu + \partial_\mu \bar{c}^a) A^a_\mu + a_2 L^a c^a), \tag{A19}$$

where $\mathcal{B}_{\Sigma_0}$ is the nilpotent linearized Slavnov-Taylor operator,

$$\mathcal{B}_{\Sigma_0} = \int d^4x \left(\frac{\delta \Sigma_0}{\delta \Omega^a_\mu} \frac{\delta}{\delta A^a_\mu} + \frac{\delta \Sigma_0}{\delta A^a_\mu} \frac{\delta}{\delta \Omega^a_\mu} + \frac{\delta \Sigma_0}{\delta L^a} \frac{\delta}{\delta c^a} + \frac{\delta \Sigma_0}{\delta c^a} \frac{\delta}{\delta L^a} + ib^a \frac{\delta}{\delta \bar{c}^a} \right), \tag{A20}$$

$$\mathcal{B}_{\Sigma_0}\mathcal{B}_{\Sigma_0} = 0. \tag{A21}$$

Expression (A19) can be conveniently written in parametric form [64] as

$$\Sigma_0^{ct} = -a_0 g^2 \frac{\partial \Sigma_0}{\partial g^2} + 2\alpha a_1 \frac{\partial \Sigma_0}{\partial \alpha} + \int d^4x \left(a_1 A_\mu^a \frac{\delta \Sigma_0}{\delta A_\mu^a} - a_1 b^a \frac{\delta \Sigma_0}{\delta b^a} - a_1 \bar{c}^a \frac{\delta \Sigma_0}{\delta \bar{c}^a} - a_1 \Omega_\mu^a \frac{\delta \Sigma_0}{\delta \Omega_\mu^a} - a_2 c^a \frac{\delta \Sigma_0}{\delta c^a} + a_2 L^a \frac{\delta \Sigma_0}{\delta L^a} \right), \tag{A22}$$

which is suitable for establishing the renormalizablity of the starting action Σ_0, i.e., to check that Σ_0^{ct} can be reabsorbed in Σ_0 through a redefinition of the fields, parameters, and sources, according to

$$\Sigma_0[A, b, c, \bar{c}, \Omega, L, g^2, \alpha] + \varepsilon \Sigma_0^{ct} = \Sigma_0[A_0, b_0, c_0, \bar{c}_0, \Omega_0, L_0, g_0^2, \alpha_0] + O(\varepsilon^2), \tag{A23}$$

with ε standing for an expansion parameter and where the label 0 denotes the redefined parameters, fields, and sources. By direct inspection of Eq. (A23), it follows that the counterterm Σ_0^{ct} can be reabsorbed through the following redefinitions:

$$g_0^2 = Z_{g^2} g^2, \qquad A_0 = Z_A^{1/2} A, \qquad c_0 = Z_c^{1/2} c, \tag{A24}$$

with

$$Z_{g^2} = 1 - \varepsilon a_0, \quad Z_A^{1/2} = 1 + \varepsilon a_1, \quad Z_c^{1/2} = 1 - \varepsilon a_2, \tag{A25}$$

and

$$\alpha_0 = Z_A \alpha, \quad b_0 = Z_A^{-1/2} b, \quad \bar{c}_0 = Z_A^{-1/2} \bar{c}, \quad \Omega_0 = Z_A^{-1/2} \Omega, \quad L_0 = Z_c^{-1/2} L, \tag{A26}$$

exhibiting the multiplicative all-orders renormalizability of the Faddeev-Popov action in linear covariant gauges.

Finally, setting the external sources (Ω_μ^a, L^a) to 0, for the counterterm $S_{\rm FP}^{ct}$, Eq. (A14), one gets

$$S_{\rm FP}^{ct} = \Sigma_0^{ct}|_{\Omega=L=0} = -a_0 g^2 \frac{\partial S_{\rm FP}}{\partial g^2} + 2\alpha a_1 \frac{\partial S_{\rm FP}}{\partial \alpha} + \int d^4x \left(a_1 A_\mu^a \frac{\delta S_{\rm FP}}{\delta A_\mu^a} - a_1 b^a \frac{\delta S_{\rm FP}}{\delta b^a} - a_1 \bar{c}^a \frac{\delta S_{\rm FP}}{\delta \bar{c}^a} - a_2 c^a \frac{\delta S_{\rm FP}}{\delta c^a} \right). \tag{A27}$$

APPENDIX B: PROPERTIES OF THE FUNCTIONAL $f_A[u]$

In this appendix we recall some useful properties of the functional $f_A[u]$,

$$f_A[u] \equiv \mathrm{Tr} \int d^4x A_\mu^u A_\mu^u = \mathrm{Tr} \int d^4x \left(u^\dagger A_\mu u + \frac{i}{g} u^\dagger \partial_\mu u \right) \times \left(u^\dagger A_\mu u + \frac{i}{g} u^\dagger \partial_\mu u \right). \tag{B1}$$

For a given gauge field configuration A_μ, $f_A[u]$ is a functional defined on the gauge orbit of A_μ. Let $\mathcal{A}$ be the space of connections A_μ^a with finite Hilbert norm $\|A\|$, i.e.,

$$\|A\|^2 = \mathrm{Tr} \int d^4x A_\mu A_\mu = \frac{1}{2} \int d^4x A_\mu^a A_\mu^a < +\infty, \tag{B2}$$

and let $\mathcal{U}$ be the space of local gauge transformations u such that the Hilbert norm $\|u^\dagger \partial u\|$ is finite too, namely,

$$\|u^\dagger \partial u\|^2 = \mathrm{Tr} \int d^4x (u^\dagger \partial_\mu u)(u^\dagger \partial_\mu u) < +\infty. \tag{B3}$$

The following proposition holds [58–61]:

Proposition: The functional $f_A[u]$ achieves its absolute minimum on the gauge orbit of A_μ.This proposition means that there exists a $h \in \mathcal{U}$ such that

$$\delta f_A[h] = 0, \tag{B4}$$

$$\delta^2 f_A[h] \geq 0, \tag{B5}$$

$$f_A[h] \leq f_A[u], \quad \forall\ u \in \mathcal{U}. \tag{B6}$$

The operator $A^2_{\rm min}$ is thus given by

$$A^2_{\rm min} = \min_{\{u\}} \mathrm{Tr} \int d^4x A_\mu^u A_\mu^u = f_A[h]. \tag{B7}$$

Let us take a look at the two conditions (B4) and (B5). To evaluate $\delta f_A[h]$ and $\delta^2 f_A[h]$ we set[3]

$$v = h e^{ig\omega} = h e^{ig\omega^a T^a}, \tag{B8}$$

$$[T^a, T^b] = i f^{abc} T^c, \qquad \mathrm{Tr}(T^a T^b) = \frac{1}{2} \delta^{ab}, \tag{B9}$$

where ω is an infinitesimal Hermitian matrix and we compute the linear and quadratic terms of the expansion of the functional $f_A[v]$ in power series of ω. Let us first obtain an expression for A_μ^v,

[3]The case of the gauge group $SU(N)$ is considered here.

$$\begin{aligned} A^v_\mu &= v^\dagger A_\mu v + \frac{i}{g} v^\dagger \partial_\mu v \\ &= e^{-ig\omega} h^\dagger A_\mu h e^{ig\omega} + \frac{i}{g} e^{-ig\omega} (h^\dagger \partial_\mu h) e^{ig\omega} + \frac{i}{g} e^{-ig\omega} \partial_\mu e^{ig\omega} \\ &= e^{-ig\omega} A^h_\mu e^{ig\omega} + \frac{i}{g} e^{-ig\omega} \partial_\mu e^{ig\omega}. \end{aligned} \tag{B10}$$

Expanding up to the order ω^2, we get

$$\begin{aligned} A^v_\mu &= \left(1 - ig\omega - g^2\frac{\omega^2}{2}\right) A^h_\mu \left(1 + ig\omega - g^2\frac{\omega^2}{2}\right) + \frac{i}{g}\left(1 - ig\omega - g^2\frac{\omega^2}{2}\right)\partial_\mu\left(1 + ig\omega - g^2\frac{\omega^2}{2}\right) \\ &= \left(1 - ig\omega - g^2\frac{\omega^2}{2}\right)\left(A^h_\mu + igA^h_\mu\omega - g^2 A^h_\mu\frac{\omega^2}{2}\right) + \frac{i}{g}\left(1 - ig\omega - g^2\frac{\omega^2}{2}\right)\left(ig\partial_\mu\omega - \frac{g^2}{2}(\partial_\mu\omega)\omega - \frac{g^2}{2}\omega(\partial_\mu\omega)\right) \\ &= A^h_\mu + igA^h_\mu\omega - \frac{g^2}{2}A^h_\mu\omega^2 - ig\omega A^h_\mu + g^2\omega A^h_\mu\omega - \frac{g^2}{2}\omega^2 A^h_\mu + \frac{i}{g}\left(ig\partial_\mu\omega - \frac{g^2}{2}(\partial_\mu\omega)\omega - \frac{g^2}{2}\omega\partial_\mu\omega + g^2\omega\partial_\mu\omega\right) + O(\omega^3), \end{aligned} \tag{B11}$$

from which it follows that

$$A^v_\mu = A^h_\mu + ig[A^h_\mu, \omega] + \frac{g^2}{2}[[\omega, A^h_\mu], \omega] - \partial_\mu\omega + i\frac{g}{2}[\omega, \partial_\mu\omega] + O(\omega^3). \tag{B12}$$

We now evaluate

$$\begin{aligned} f_A[v] &= \mathrm{Tr}\int d^4x A^u_\mu A^u_\mu \\ &= \mathrm{Tr}\int d^4x\bigg[\left(A^h_\mu + ig[A^h_\mu, \omega] + \frac{g^2}{2}[[\omega, A^h_\mu], \omega] - \partial_\mu\omega + i\frac{g}{2}[\omega, \partial_\mu\omega] + O(\omega^3)\right) \\ &\quad \times \left(A^h_\mu + ig[A^h_\mu, \omega] + \frac{g^2}{2}[[\omega, A^h_\mu], \omega] - \partial_\mu\omega + i\frac{g}{2}[\omega, \partial_\mu\omega] + O(\omega^3)\right)\bigg] \\ &= \mathrm{Tr}\int d^4x\bigg\{A^h_\mu A^h_\mu + igA^h_\mu[A^h_\mu, \omega] + g^2 A^h_\mu\omega A^h_\mu\omega - \frac{g^2}{2}A^h_\mu A^h_\mu\omega^2 - \frac{g^2}{2}A^h_\mu\omega^2 A^h_\mu - A^h_\mu\partial_\mu\omega \\ &\quad + i\frac{g}{2}A^h_\mu[\omega, \partial_\mu\omega] + ig[A^h_\mu, \omega]A^h_\mu - g^2[A^h_\mu, \omega][A^h_\mu, \omega] - ig[A^h_\mu, \omega]\partial_\mu\omega + g^2\omega A^h_\mu\omega A^h_\mu \\ &\quad - \frac{g^2}{2}A^h_\mu\omega^2 A^h_\mu - \frac{g^2}{2}\omega^2 A^h_\mu A^h_\mu - \partial_\mu\omega A^h_\mu - ig\partial_\mu\omega[A^h_\mu, \omega] + \partial_\mu\omega\partial_\mu\omega + i\frac{g}{2}[\omega, \partial_\mu\omega]A^h_\mu\bigg\} + O(\omega^3) \\ &= f_A[h] - \mathrm{Tr}\int d^4x\{A^h_\mu, \partial_\mu\omega\} + \mathrm{Tr}\int d^4x\bigg(g^2 A^h_\mu\omega A^h_\mu\omega - \frac{g^2}{2}A^h_\mu A^h_\mu\omega^2 - \frac{g^2}{2}A^h_\mu\omega^2 A^h_\mu \\ &\quad - g^2[A^h_\mu, \omega][A^h_\mu, \omega] + g^2\omega A^h_\mu\omega A^h_\mu - \frac{g^2}{2}A^h_\mu\omega^2 A^h_\mu - \frac{g^2}{2}\omega^2 A^h_\mu A^h_\mu\bigg) + \mathrm{Tr}\int d^4x(\partial_\mu\omega\partial_\mu\omega \\ &\quad + i\frac{g}{2}[\omega, \partial_\mu\omega]A^h_\mu - ig\partial_\mu\omega[A^h_\mu, \omega] - ig[A^h_\mu, \omega]\partial_\mu\omega + i\frac{g}{2}A^h_\mu[\omega, \partial_\mu\omega]) + O(\omega^3) \\ &= f_A[h] + 2\int d^4x tr(\omega\partial_\mu A^h_\mu) + \int d^4x tr\{2g^2\omega A^h_\mu\omega A^h_\mu - 2g^2 A^h_\mu A^h_\mu\omega^2 - g^2(A^h_\mu\omega - \omega A^h_\mu)(A^h_\mu\omega - \omega A^h_\mu)\} \\ &\quad + \int d^4x tr\bigg(\partial_\mu\omega\partial_\mu\omega + i\frac{g}{2}\omega\partial_\mu\omega A^h_\mu - i\frac{g}{2}\partial_\mu\omega\omega A^h_\mu \\ &\quad - ig\partial_\mu\omega A^h_\mu\omega + ig\partial_\mu\omega\omega A^h_\mu - igA^h_\mu\omega\partial_\mu\omega + ig\omega A^h_\mu\partial_\mu\omega + i\frac{g}{2}A^h_\mu\omega\partial_\mu\omega - i\frac{g}{2}A^h_\mu\partial_\mu\omega\omega\bigg) + O(\omega^3) \\ &= f_A[h] + 2\mathrm{Tr}\int d^4x(\omega\partial_\mu A^h_\mu) + \mathrm{Tr}\int d^4x(\partial_\mu\omega\partial_\mu\omega + ig\omega\partial_\mu\omega A^h_\mu - ig\partial_\mu\omega\omega A^h_\mu - 2ig\partial_\mu\omega A^h_\mu\omega + 2ig\partial_\mu\omega\omega A^h_\mu) \\ &\quad + O(\omega^3). \end{aligned} \tag{B13}$$

Thus,

$$\begin{aligned} f_A[v] &= f_A[h] + 2\mathrm{Tr}\int d^4x(\omega\partial_\mu A^h_\mu) \\ &\quad + \mathrm{Tr}\int d^4x(\partial_\mu\omega\partial_\mu\omega + ig\omega\partial_\mu\omega A^h_\mu - ig\partial_\mu\omega\omega A^h_\mu \\ &\quad - ig(\partial_\mu\omega)A^h_\mu\omega + ig(\partial_\mu\omega)\omega A^h_\mu) + O(\omega^3) \\ &= f_A[h] + 2\mathrm{Tr}\int d^4x(\omega\partial_\mu A^h_\mu) \\ &\quad + \mathrm{Tr}\int d^4x\{\partial_\mu\omega(\partial_\mu\omega - ig[A^h_\mu,\omega])\} + O(\omega^3). \end{aligned} \tag{B14}$$

Finally

$$\begin{aligned} f_A[v] &= f_A[h] + 2\mathrm{Tr}\int d^4x(\omega\partial_\mu A^h_\mu) \\ &\quad - \mathrm{Tr}\int d^4x\omega\partial_\mu D_\mu(A^h)\omega + O(\omega^3), \end{aligned} \tag{B15}$$

so that

$$\begin{aligned} &\delta f_A[h] = 0 \Rightarrow \partial_\mu A^h_\mu = 0, \\ &\delta^2 f_A[h] > 0 \Rightarrow -\partial_\mu D_\mu(A^h) > 0. \end{aligned} \tag{B16}$$

We see therefore that the set of field configurations fulfilling conditions (B16), i.e., defining relative minima of the functional $f_A[u]$, belongs to the so-called Gribov region Ω, which is defined as

$$\Omega = \{A_\mu | \partial_\mu A_\mu = 0 \quad \text{and} \quad -\partial_\mu D_\mu(A) > 0\}. \tag{B17}$$

Let us proceed now by showing that the transversality condition, $\partial_\mu A^h_\mu = 0$, can be solved for $h = h(A)$ as a power series in A_μ. We start from

$$A^h_\mu = h^\dagger A_\mu h + \frac{i}{g}h^\dagger\partial_\mu h, \tag{B18}$$

with

$$h = e^{ig\phi} = e^{ig\phi^a T^a}. \tag{B19}$$

Let us expand h in powers of ϕ,

$$h = 1 + ig\phi - \frac{g^2}{2}\phi^2 + O(\phi^3). \tag{B20}$$

From Eq. (B18) we have

$$\begin{aligned} A^h_\mu &= A_\mu + ig[A_\mu,\phi] + g^2\phi A_\mu\phi - \frac{g^2}{2}A_\mu\phi^2 \\ &\quad - \frac{g^2}{2}\phi^2 A_\mu - \partial_\mu\phi + i\frac{g}{2}[\phi,\partial_\mu] + O(\phi^3). \end{aligned} \tag{B21}$$

Thus, condition $\partial_\mu A^h_\mu = 0$ gives

$$\begin{aligned} \partial^2\phi &= \partial_\mu A + ig[\partial_\mu A_\mu,\phi] + ig[A_\mu,\partial_\mu\phi] + g^2\partial_\mu\phi A_\mu\phi \\ &\quad + g^2\phi\partial_\mu A_\mu\phi + g^2\phi A_\mu\partial_\mu\phi - \frac{g^2}{2}\partial_\mu A_\mu\phi^2 \\ &\quad - \frac{g^2}{2}A_\mu\partial_\mu\phi\phi - \frac{g^2}{2}A_\mu\phi\partial_\mu\phi - \frac{g^2}{2}\partial_\mu\phi\phi A_\mu \\ &\quad - \frac{g^2}{2}\phi\partial_\mu\phi A_\mu - \frac{g^2}{2}\phi^2\partial_\mu A_\mu + i\frac{g}{2}[\phi,\partial^2\phi] + O(\phi^3). \end{aligned} \tag{B22}$$

This equation can be solved iteratively for ϕ as a power series in A_μ, namely,

$$\begin{aligned} \phi &= \frac{1}{\partial^2}\partial_\mu A_\mu + i\frac{g}{\partial^2}\left[\partial A, \frac{\partial A}{\partial^2}\right] + i\frac{g}{\partial^2}\left[A_\mu, \partial_\mu\frac{\partial A}{\partial^2}\right] \\ &\quad + \frac{i}{2}\frac{g}{\partial^2}\left[\frac{\partial A}{\partial^2}, \partial A\right] + O(A^3), \end{aligned} \tag{B23}$$

so that

$$\begin{aligned} A^h_\mu &= A_\mu - \frac{1}{\partial^2}\partial_\mu\partial A - ig\frac{\partial_\mu}{\partial^2}\left[A_\nu, \partial_\nu\frac{\partial A}{\partial^2}\right] \\ &\quad - i\frac{g}{2}\frac{\partial_\mu}{\partial^2}\left[\partial A, \frac{1}{\partial^2}\partial A\right] + ig\left[A_\mu, \frac{1}{\partial^2}\partial A\right] \\ &\quad + i\frac{g}{2}\left[\frac{1}{\partial^2}\partial A, \frac{\partial_\mu}{\partial^2}\partial A\right] + O(A^3). \end{aligned} \tag{B24}$$

Expression (B24) can be written in a more useful way, given in Eq. (3). In fact

$$\begin{aligned} A^h_\mu &= \left(\delta_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\partial^2}\right)\left(A_\nu - ig\left[\frac{1}{\partial^2}\partial A, A_\nu\right] + \frac{ig}{2}\left[\frac{1}{\partial^2}\partial A, \partial_\nu\frac{1}{\partial^2}\partial A\right]\right) + O(A^3) \\ &= A_\mu - ig\left[\frac{1}{\partial^2}\partial A, A_\mu\right] + \frac{ig}{2}\left[\frac{1}{\partial^2}\partial A, \partial_\mu\frac{1}{\partial^2}\partial A\right] - \frac{\partial_\mu}{\partial^2}\partial A + ig\frac{\partial_\mu}{\partial^2}\partial_\nu\left[\frac{1}{\partial^2}\partial A, A_\nu\right] - i\frac{g}{2}\frac{\partial_\mu}{\partial^2}\partial_\nu\left[\frac{\partial A}{\partial^2}, \frac{\partial_\nu}{\partial^2}\partial A\right] + O(A^3) \\ &= A_\mu - \frac{\partial_\mu}{\partial^2}\partial A + ig\left[A_\mu, \frac{1}{\partial^2}\partial A\right] + \frac{ig}{2}\left[\frac{1}{\partial^2}\partial A, \partial_\mu\frac{1}{\partial^2}\partial A\right] + ig\frac{\partial_\mu}{\partial^2}\left[\frac{\partial_\nu}{\partial^2}\partial A, A_\nu\right] + i\frac{g}{2}\frac{\partial_\mu}{\partial^2}\left[\frac{\partial A}{\partial^2}, \partial A\right] + O(A^3), \end{aligned} \tag{B25}$$

which is precisely expression (B24). The transverse field given in Eq. (3) enjoys the property of being gauge invariant order by order in the coupling constant g. Let us work out the transformation properties of ϕ_ν under a gauge transformation,

$$\delta A_\mu = -\partial_\mu \omega + ig[A_\mu, \omega]. \tag{B26}$$

We have, up to the order $O(g^2)$,

$$\begin{aligned}\delta\phi_\nu &= -\partial_\nu\omega + ig\left[\frac{1}{\partial^2}\partial A, \partial_\nu\omega\right] - i\frac{g}{2}\left[\omega, \partial_\nu\frac{1}{\partial^2}\partial A\right] \\ &\quad - i\frac{g}{2}\left[\frac{\partial A}{\partial^2}, \partial_\nu\omega\right] + O(g^2) \\ &= -\partial_\nu\omega + i\frac{g}{2}\left[\frac{1}{\partial^2}\partial A, \partial_\nu\omega\right] + i\frac{g}{2}\left[\partial_\nu\frac{1}{\partial^2}\partial A, \omega\right] \\ &\quad + O(g^2). \end{aligned} \tag{B27}$$

Therefore,

$$\delta\phi_\nu = -\partial_\nu\left(\omega - i\frac{g}{2}\left[\frac{\partial A}{\partial^2}, \omega\right]\right) + O(g^2), \tag{B28}$$

from which the gauge invariance of A^h_μ is established.

Finally, let us work out the expression of $A^2_{\rm min}$ as a power series in A_μ.

$$\begin{aligned} A^2_{\rm min} &= {\rm Tr}\int d^4x A^h_\mu A^h_\mu \\ &= {\rm Tr}\int d^4x\left[\phi_\mu\left(\delta_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\partial^2}\right)\phi_\nu\right] \\ &= {\rm Tr}\int d^4x\left[\left(A_\mu - ig\left[\frac{1}{\partial^2}\partial A, A_\mu\right]\right.\right. \\ &\quad \left. + \frac{ig}{2}\left[\frac{1}{\partial^2}\partial A, \partial_\mu\frac{1}{\partial^2}\partial A\right]\right) \\ &\quad \times\left(\delta_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\partial^2}\right)\left(A_\nu - ig\left[\frac{1}{\partial^2}\partial A, A_\nu\right]\right. \\ &\quad \left.\left. + \frac{ig}{2}\left[\frac{1}{\partial^2}\partial A, \partial_\nu\frac{1}{\partial^2}\partial A\right]\right)\right] \\ &= \frac{1}{2}\int d^4x\left[A^a_\mu\left(\delta_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\partial^2}\right)A^a_\nu\right. \\ &\quad \left. - 2gf^{abc}\frac{\partial_\nu\partial A^a}{\partial^2}\frac{\partial A^b}{\partial^2}A^c_\nu - gf^{abc}A^a_\nu\frac{\partial A^b}{\partial^2}\frac{\partial_\nu\partial A^c}{\partial^2}\right] \\ &\quad + O(A^4), \end{aligned} \tag{B29}$$

leading to the result quoted in Eq. (5).

We conclude this appendix by noting that, due to gauge invariance, $A^2_{\rm min}$ can be rewritten in a manifestly invariant way in terms of $F_{\mu\nu}$ and the covariant derivative D_μ [58]; see Eq. (6).

APPENDIX C: PROPAGATORS OF THE ELEMENTARY FIELDS

In order to evaluate the tree-level two-point functions of the theory, we start from the local action

$$S = S_{\rm FP} + \int d^4x\left(\tau^a\partial_\mu A^{h,a}_\mu + \frac{m^2}{2}A^{h,a}_\mu A^{h,a}_\mu\right) + S_{\rm IRR}, \tag{C1}$$

where $S_{\rm FP}$ is the Faddeev-Popov term of the linear covariant gauges, Eq. (10), and $S_{\rm IRR}$ stands for the BRST-invariant infrared regularizing mass term for the Stueckelberg field, namely,

$$S_{\rm IRR} = \int d^4x\frac{1}{2}s(\rho\xi^a\xi^a) = \int d^4x\left(\frac{1}{2}M^4\xi^a\xi^a + \rho\xi^a c^a\right). \tag{C2}$$

From the quadratic part of expression (C1), one finds the following set of tree-level propagators in momentum space:

$$\langle A^a_\mu(p)A^b_\nu(-p)\rangle = \frac{1}{p^2+m^2}\delta^{ab}\mathcal{P}_{\mu\nu} + \frac{\alpha}{p^2}\frac{p_\mu p_\nu}{p^2}, \tag{C3}$$

$$\langle A^a_\mu(p)b^b(-p)\rangle = -\frac{p^2}{p^4+\alpha M^4}\delta^{ab}p_\mu, \tag{C4}$$

$$\langle A^a_\mu(p)\xi^b(-p)\rangle = i\frac{\alpha\delta^{ab}}{p^4+\alpha M^4}p_\mu, \tag{C5}$$

$$\langle A^a_\mu(p)\tau^b(-p)\rangle = -i\frac{\alpha M^4}{p^2(p^4+\alpha M^4)}p_\mu\delta^{ab}, \tag{C6}$$

$$\langle b^a(p)b^b(-p)\rangle = \frac{M^4}{p^4+\alpha M^4}\delta^{ab}, \tag{C7}$$

$$\langle b^a(p)\xi^b(-p)\rangle = i\frac{p^2\delta^{ab}}{p^4+\alpha M^4}, \tag{C8}$$

$$\langle b^a(p)\tau^b(-p)\rangle = -i\frac{M^4}{p^2}\delta^{ab}, \tag{C9}$$

$$\langle\bar{c}^a(p)A^b_\mu(-p)\rangle = -i\frac{\rho\alpha}{p^2(p^4+\alpha M^4)}\delta^{ab}p_\mu, \tag{C10}$$

$$\langle\bar{c}^a(p)b^b(-p)\rangle = i\frac{\rho}{p^4+\alpha M^4}\delta^{ab}, \tag{C11}$$

$$\langle\bar{c}^a(p)\tau^b(-p)\rangle = \frac{\rho}{p^4+\alpha M^4}\delta^{ab}, \tag{C12}$$

$$\langle\bar{c}^a(p)\xi^b(-p)\rangle = \frac{\rho\alpha}{p^2(p^4+\alpha M^4)}\delta^{ab}, \tag{C13}$$

$$\langle \xi^a(p)\xi^b(-p)\rangle = \frac{\alpha\delta^{ab}}{p^4+\alpha M^4}, \tag{C14}$$

$$\langle \xi^a(p)\tau^b(-p)\rangle = \frac{p^2}{p^4+\alpha M^4}\delta^{ab}, \tag{C15}$$

$$\langle \tau^a(p)\tau^b(-p)\rangle = -\left(\frac{m^2(p^4-\alpha M^4)+M^4p^2}{p^2(p^4+\alpha M^4)}\right)\delta^{ab}, \tag{C16}$$

$$\langle \bar{c}^a(p)c^b(-p)\rangle = \frac{1}{p^2}\delta^{ab}, \tag{C17}$$

with $\mathcal{P}_{\mu\nu} = (\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2})$ being the transverse projector. All other propagators that have not been listed above are vanishing. Let us also recall that the parameters M and ρ that regularize the propagation of the Stueckelberg field in the infrared have to be set to 0 at the end of any actual calculation.

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