

Cosmological implications of interacting group field theory models: Cyclic Universe and accelerated expansion

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We study the cosmological implications of interactions between spacetime quanta in the group field theory (GFT) approach to quantum gravity from a phenomenological perspective. Our work represents a first step towards understanding early Universe cosmology by studying the dynamics of the emergent continuum spacetime, as obtained from a fundamentally discrete microscopic theory. In particular, we show how GFT interactions lead to a recollapse of the Universe while preserving the bounce replacing the initial singularity, which has already been shown to occur in the free case. It is remarkable that cyclic cosmologies are thus obtained in this framework without any *a priori* assumption on the geometry of spatial sections of the emergent spacetime. Furthermore, we show how interactions make it possible to have an early epoch of accelerated expansion, which can be made to last for an arbitrarily large number of *e*-folds, without the need to introduce an *ad hoc* potential for the scalar field.

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I. INTRODUCTION

Our current understanding of cosmological data is based on classical models that rely on the validity of Einstein's theory of gravity. However fruitful this approach has been so far, the basic assumptions on which it relies are clearly unjustified from a fundamental point of view, and evidence of their violation is expected to become manifest at the earliest stages of expansion of our Universe. This is in fact where the dynamics of spacetime should more appropriately be given in the framework of a quantum gravity theory and a description of spacetime as a continuum medium should make way for its understanding as fundamentally discrete. Of particular relevance in this sense is the status of the inflationary paradigm. In fact, while its success in providing an explanation for structure growth and solving cosmological puzzles is undeniable, its *ad hoc* underlying assumptions do not find support in a more fundamental theory. More specifically, since the onset of inflation is supposed to take place at Planckian times, the dynamics of the Universe at this stage should find a more suitable formulation so as to take quantum gravitational effects into account. Furthermore, it is conceivable that the quantum dynamics of the gravitational field itself could effectively give rise to dynamical features similar to those of inflationary models, without the need to introduce a new hypothetical field (the inflaton) with an *ad hoc* potential.

The purpose of this article is to bridge the gap between the quantum gravity era and the standard classical cosmological model, following the line of research started in

[1,2]. As in those works, we adopt the group field theory (GFT) approach to quantum gravity [3] which provides a formal and complete definition of spin foam models [4], themselves giving a path integral formulation for loop quantum gravity (LQG) [5,6]. Furthermore, GFT provides a second quantized Fock space reformulation of the kinematical Hilbert space of LQG [7], which also allows to explore quantum geometries encoded by spin network states with many nodes. In the GFT approach spacetime geometry is seen as emerging from the collective behavior of basic building blocks (also called “quanta of geometry”) when taking the macroscopic limit. Such quanta represent the fundamental degrees of freedom of the gravitational field and can be described as simplices equipped with data of group theoretical nature. They can be glued together through their interactions to form simplicial complexes, which in turn correspond to quantum states of spacetime geometry. GFT is completely background independent and general relativity is expected to be recovered dynamically by taking the macroscopic and continuum limit. The particular case of a homogeneous and isotropic universe, which is the one relevant for our applications, can be obtained by studying the dynamics of an isotropic condensate of GFT quanta for a large number of constituents. Exploiting the structure of the kinematical Hilbert space of LQG and, in particular, the discreteness of the spectra of the geometric operators defined thereon [8], the GFT formalism allows for a description of the effective dynamics of the emergent spacetime by means of classical evolution equations for the expectation values of geometric observables. These can be recast in a form that is close to the classical Friedmann equation.

The object of our study is the effect of interactions between the fundamental building blocks, as given in the

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microscopic theory, on the dynamics of the emergent spacetime. It has already been shown in previous works [1,2] that GFT predicts the occurrence of a bounce that replaces the initial spacetime singularity bedeviling classical cosmological models. In this sense, the results of GFT condensate cosmology are reminiscent of those found in loop quantum cosmology (LQC), where the resolution of the initial singularity was shown to be a robust feature [9,10].

The analysis done in Refs. [1,2], however, did not take into account the effect played by GFT interactions. These encode the connectivity of the above mentioned graph structures; in other words, they are responsible for gluing the fundamental building blocks. Here we show, under fairly general assumptions, that the same result holds also in the interacting case. Furthermore, we show that interactions play a substantial role in determining the ultimate fate of the Universe and lead to its recollapse. These two results together imply a cyclic evolution for the Universe. It is remarkable that this prediction has been obtained without making any *a priori* assumptions on the geometry of the spatial slices of the emergent spacetime.

In previous work by some of the authors [2] it has also been shown that the bounce is accompanied by an early era of accelerated expansion. However, here we show that its duration in the free case is subject to very stringent bounds and cannot accommodate a reasonably large number of e -folds. Considering a model with an effective potential which is reminiscent of multicritical models [11], here we show that it is possible to achieve an arbitrarily large number of e -folds by imposing a hierarchy between two interaction coefficients. At the same time, this allows us to select a subclass of models that realize such dynamics, which might give indications to rule out certain models as candidates for the fundamental microscopic theory. It should be stressed that the result follows only from assuming a particular form of the interactions between basic building blocks. A minimally coupled massless scalar field is introduced merely for the purpose of defining a relational clock and by no means must be identified with the inflaton.

The plan of the paper is as follows. In Sec. II we review the effective dynamics of GFT condensate and show how the evolution of macroscopic quantities is extracted from it. We introduce our model and discuss how a sign ambiguity in the coefficient of the kinetic term is fixed by means of phenomenological considerations on the rate of expansion of the Universe at late (relational) times. In Sec. III we show how the higher power interaction term induces a recollapse and we discuss the cyclicity of solutions of the model. Section IV is divided in two parts. In the first part, Sec. IV A, we discuss the case of a noninteracting GFT model and show that it does not support a reasonably large number of e -folds. Then, in Sec. IV B we show how this is made possible by considering suitable interactions terms.

In Sec. V, we discuss how the different terms in the GFT effective potential can be reinterpreted, from the point of view of an effective Friedmann equation, as sources corresponding to effective fluids with particular equations of state. Finally, in Sec. VI, we review the main results of the article and conclude by discussing possible lines for future investigations. In the Appendix we show how to make contact between the relational dynamics employed here and the standard formulation of FLRW (Friedmann-Lemaître-Robertson-Walker) cosmology.

II. NONLINEAR DYNAMICS OF A GFT CONDENSATE

The dynamics of an isotropic GFT condensate can be described by means of the effective action [1]

$$S = \int d\phi (A |\partial_\phi \sigma|^2 + \mathcal{V}(\sigma)). \quad (1)$$

Here σ is a complex scalar field representing the configuration of the Bose condensate of GFT quanta as a function of relational time ϕ . The form of the effective potential $\mathcal{V}(\sigma)$ can be motivated by means of the microscopic GFT model, and we require it to be bounded from below. There is an ambiguity in the choice of the sign of A , which is not fixed by the microscopic theory and will turn out to be particularly relevant for the cosmological applications of the model.¹ In particular, it can be used to restrict the class of microscopic models by selecting only those that are phenomenologically viable. In fact, as we will show, only models entailing $A < 0$ are sensible from a phenomenological point of view since otherwise one would have faster than exponential expansion.

The dynamics of macroscopic quantities is obtained by computing expectation values on the condensate of the corresponding physical observables in the quantum theory. Introducing some notation is now in order. We define ρ_j as the modulus of the component of the field σ corresponding to the spin- j representation of SU(2), thus introducing its polar form

$$\sigma_j = \rho_j e^{i\theta_j}. \quad (2)$$

The quantity $V_j \sim \ell_p^3 j^{3/2}$ represents an elementary volume determined by the particular SU(2) representation adopted. Since the volume operator is diagonal in the basis of spin representations, one has for its expectation value

$$V(\phi) = \sum_{j \in \mathbb{N}_0/2} V_j \rho_j^2(\phi), \quad (3)$$

as in Refs. [1,13] where it is shown how geometric operators in GFT are constructed from their first quantized

¹This ambiguity has also been discussed earlier in Ref. [12] when exploring the possibility to embed LQC in GFT.

counterpart in LQG [5,8]. The importance of such operators is paramount since they allow us to give a geometric interpretation of the solutions of the theory, e.g., in the condensate phase we consider here. Equation (3) has important applications to homogeneous and isotropic cosmologies. Indeed, it makes it possible to study the evolution of the Universe by means of classical evolution equations, therefore establishing a link with the dynamics of classical FLRW models. The evolution of the volume over (relational) time ϕ is determined from the evolution of all the ρ_j 's. Furthermore, it is reasonable to expect that the condensate field peaks on a particular representation j , in which case the rhs of Eq. (3) would only consist of one term. In this case we will omit the index j in ρ_j and write

$$V(\phi) = V_j \rho^2(\phi). \quad (4)$$

Indeed, such a natural assumption is supported by recent findings in the same setting used here, which explicitly show that GFT condensates form a low-spin phase of many quanta of geometry which are almost entirely characterized by only one spin j (the occupation numbers corresponding to other representations are strongly suppressed), as shown in Ref. [14]. Thus, throughout the article we will work under the assumption that the spin representation j is fixed.

The macroscopic dynamics following from the effective GFT action can be given in terms of effective Friedmann equations, giving the relational evolution of the volume with respect to the scalar field ϕ . Those are obtained by differentiating Eq. (4),

$$\frac{\partial_\phi V}{V} = 2 \frac{\partial_\phi \rho}{\rho}, \quad (5)$$

$$\frac{\partial_\phi^2 V}{V} = 2 \left[\frac{\partial_\phi^2 \rho}{\rho} + \left(\frac{\partial_\phi \rho}{\rho} \right)^2 \right]. \quad (6)$$

Notice that from a macroscopic standpoint Eqs. (5), (6) give the evolution of the emergent spacetime, with ρ playing the role of an auxiliary field (the modulus of σ) whose dynamics is determined by the effective action in Eq. (1).

In this work we consider an effective potential of the following form:

$$\mathcal{V}(\sigma) = B|\sigma(\phi)|^2 + \frac{2}{n}w|\sigma|^n + \frac{2}{n'}w'|\sigma|^{n'}, \quad (7)$$

where we can assume $n' > n$ without loss of generality. The terms in the effective potential can be similarly motivated as in Ref. [1]. The interaction terms appearing in GFT actions are usually defined in such a way that the perturbative expansion of the GFT partition function reproduces that of spin-foam models. Specifically, spin-foam models for 4d quantum gravity are mostly based on interaction terms of power 5, called simplicial. In the case that the GFT field is

endowed by a particular tensorial transformation property, other classes of models can be obtained whose interaction terms, called tensorial, are based on even powers of the modulus of the field. In this light, the particular type of interactions considered here can be understood as mimicking such types of interactions, which is the reason why we will refer to them as pseudosimplicial and pseudotensorial, respectively. In the following we will study their phenomenological consequences, and show how interesting physical effects are determined as a result of the interplay between two interactions of this type. The integer-valued powers n, n' in the interactions will be kept unspecified throughout the article, thus making our analysis retain its full generality. The particular values motivated by the above discussion can be retrieved as particular cases. In the following we will show how different ranges for such powers lead to phenomenologically interesting features of the model, most notably concerning an early era of accelerated expansion in Sec. IV B.

Because $\mathcal{V}(\sigma)$ has to be bounded from below, we require $w' > 0$. The equation of motion of the field σ obtained from Eqs. (1), (7) is

$$-A\partial_\phi^2\sigma + B\sigma + w|\sigma|^{n-2}\sigma + w'|\sigma|^{n'-2}\sigma = 0. \quad (8)$$

Writing the complex field σ in polar form [as in Eq. (2), omitting indices] $\sigma = \rho e^{i\theta}$ one finds (Ref. [1]) that the equation of motion for the angular component leads to the conservation law

$$\partial_\phi Q = 0, \quad \text{with} \quad Q \equiv \rho^2 \partial_\phi \theta, \quad (9)$$

while the radial component satisfies a second-order ordinary differential equation (ODE)

$$\partial_\phi^2 \rho - \frac{Q^2}{\rho^3} - \frac{B}{A}\rho - \frac{w}{A}\rho^{n-1} - \frac{w'}{A}\rho^{n'-1} = 0. \quad (10)$$

The conserved charge Q is proportional to the momentum of the scalar field $\pi_\phi = \hbar Q$ [1]. One immediately observes that for large values of ρ the term $\rho^{n'-1}$ becomes dominant. In order to ensure that Eq. (10) does not lead to drastic departures from standard cosmology at late times [Eq. (5)], the coefficient of such term has to be positive,

$$\mu \equiv -\frac{w'}{A} > 0, \quad (11)$$

which implies, since $w' > 0$, that one must have $A < 0$. In fact, the opposite case $\mu < 0$ would lead to an open cosmology expanding at a faster than exponential rate, which relates to a big rip. Thus, considering $A < 0$, compatibility with the free case (see Refs. [1,2]) demands

$$m^2 \equiv \frac{B}{A} > 0, \quad (12)$$

which in turn implies $B < 0$. The sign of w is *a priori* not constrained, which leaves a considerable freedom in the model. Given the signs of the parameters B and w' , the potential in Eq. (7) can be related to models with spontaneous symmetry breaking in statistical mechanics and quantum field theory. The subleading term in the potential plays an important role in determining an inflationary-like era, as shown below in Sec. IV B.

The connection to the theory of critical phenomena is to be expected from the conjecture that GFT condensates arise through a phase transition from a nongeometric to a geometric phase [15], which could be a possible realization of the geometrogenesis scenario [16]. Despite the lack of a detailed theory near criticality and the fact that the occurrence of the aforementioned phase transition is still a conjecture, there are nonetheless encouraging results coming from the analysis (both perturbative and non-perturbative) of the renormalization group (RG) flow, which shows the existence of IR fixed points in certain models, see Refs. [17,18]. On this ground we will adopt, as a working hypothesis, the formation of a condensate as a result of the phase transition, as in Refs. [1,12,14,19–21].

From Eqs. (11), (12) and defining

$$\lambda \equiv -\frac{w}{A}, \quad (13)$$

we can rewrite Eq. (10) in the form

$$\partial_{\phi}^2 \rho - m^2 \rho - \frac{Q^2}{\rho^3} + \lambda \rho^{n-1} + \mu \rho^{n'-1} = 0, \quad (14)$$

which will be used throughout the rest of this article. The above equation has the form of the equation of motion of a classical point particle with potential (see Fig. 1)

$$U(\rho) = -\frac{1}{2} m^2 \rho^2 + \frac{Q^2}{2\rho^2} + \frac{\lambda}{n} \rho^n + \frac{\mu}{n'} \rho^{n'}. \quad (15)$$

Equation (14) leads to another conserved quantity, E , defined as

$$E = \frac{1}{2} (\partial_{\phi} \rho)^2 + U(\rho), \quad (16)$$

which is referred to as ‘‘GFT energy’’ [1,2]. Its physical meaning, from a fundamental point of view, is yet to be clarified.

III. RECOLLAPSING UNIVERSE

Some properties of the solutions of the model and its consequences for cosmology can already be drawn by means of a qualitative analysis of the solutions of the

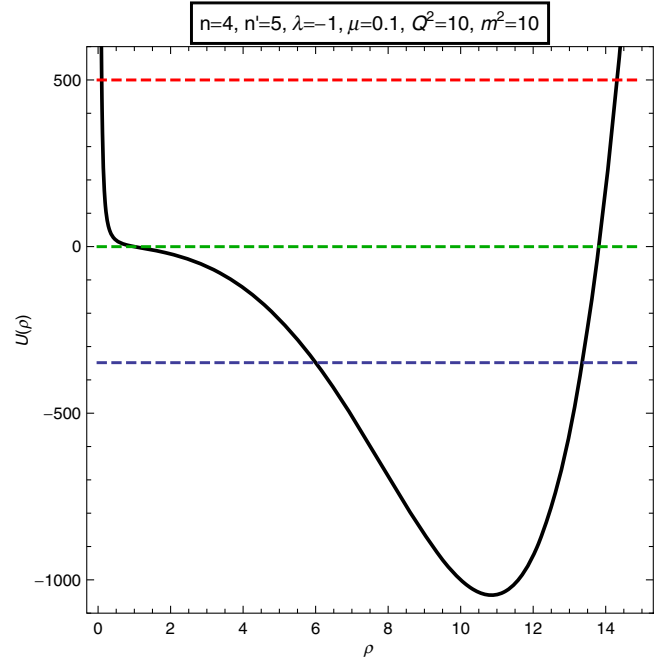


FIG. 1. Plot of the potential $U(\rho)$ [Eq. (15)] for the dynamical system described by Eq. (14) and a particular choice of parameters. The three horizontal curves correspond to different values of the ‘‘GFT energy’’ E , in turn corresponding to different choices of initial conditions for ρ, ρ' . The corresponding orbits in phase space are shown in Fig. 2. Recollapse is generic feature of the model and occurs for any values of the parameters, provided $\mu > 0$ and $Q \neq 0$.

second-order ODE in Eq. (15). In fact, solutions are confined to the positive half-line $\rho > 0$, given the infinite potential barrier at $\rho = 0$ for $Q \neq 0$. Moreover, because $\mu > 0$, the potential in Eq. (15) approaches infinity as ρ takes arbitrarily large values. Therefore, provided that we fix the GFT energy E at a value which is larger than both the absolute minimum and that of (possible) local maxima of the potential $U(\rho)$, the solutions of Eq. (14) turn out to be cyclic motions (see Figs. 1 and 2) describing oscillations around a stable equilibrium point. These, in turn, correspond via Eq. (4) to cyclic solutions for the dynamics of the Universe Eqs. (5), (6) (see Fig. 3).

It is interesting to compare this result with what is known in the case where interactions are disregarded [1,2]. In that case one has that the Universe expands indefinitely and in the limit $\phi \rightarrow \infty$ its dynamics follows the ordinary Friedmann equation for a flat Universe, filled with a massless and minimally coupled scalar field. Therefore, we see that the given interactions in the GFT model induce a recollapse of the Universe, corresponding to the turning point of the motion of ρ , as seen in Fig. 2.

It is well known in the classical theory that such a recollapse follows as a simple consequence of the closed topology of three-space. In the GFC framework instead, the topology of space(time) is not fixed at the outset, but should

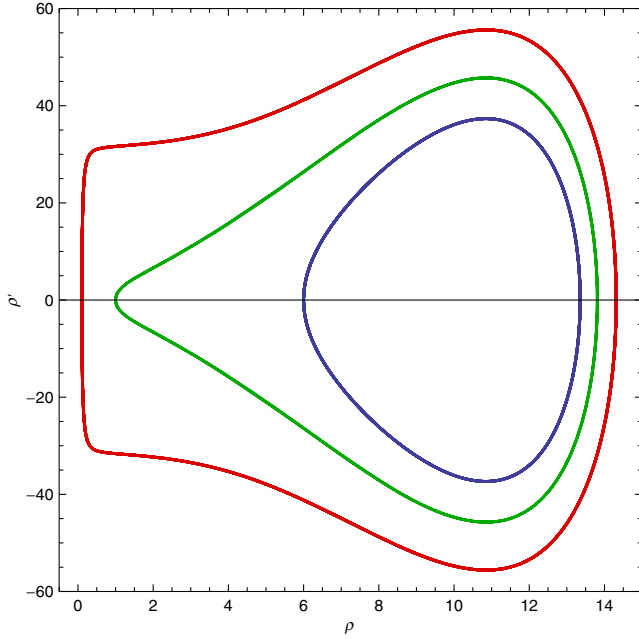


FIG. 2. Phase portrait of the dynamical system given by Eq. (14). Orbits have energy given by the corresponding color lines as Fig. 1. Orbits are periodic and describe oscillations around the stable equilibrium point (center fixed point) given by the absolute minimum of the potential $U(\rho)$. This is a general feature of the model which does not depend on the particular choice of parameters, provided Eq. (11) is satisfied.

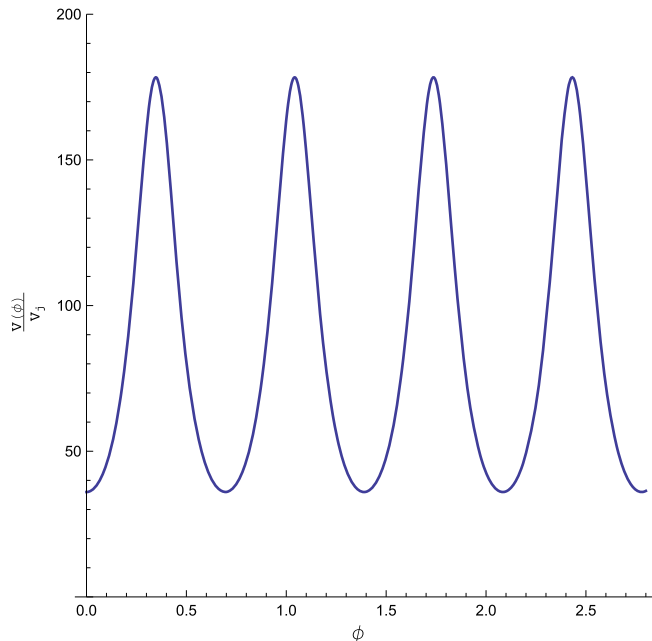


FIG. 3. Plot of the volume of the Universe as a function of relational time ϕ (in arbitrary units), corresponding to the blue orbit in Fig. 2. As a generic feature of the interacting model the Universe undergoes a cyclic evolution and its volume has a positive minimum, corresponding to a bounce.

rather be reconstructed from the behavior of the system in the macroscopic limit. In other words, the simple condensate ansatz used here does not provide any information about the topology of spatial sections of the emergent spacetime which, as it is well known, play an important role in the dynamics of classical cosmological models. Any topological information must therefore come from additional input. A possible strategy one could follow is to work with generalized condensates encoding such information [13]. Here, instead, we propose that the closedness of the reconstructed space need not be encoded in the condensate ansatz as an input, but is rather determined by the dynamics as a consequence of the GFT interactions. Hence, allowing only interactions that are compatible with reproducing a given spatial topology, one may recover the classical correspondence between closed spatial topology and having a finitely expanding Universe.

IV. GEOMETRIC INFLATION

Cosmology obtained from GFT displays a number of interesting features concerning the initial stage of the evolution of the Universe, which mark a drastic departure from the standard FLRW cosmologies. In particular, the initial big-bang singularity is replaced by a regular bounce (see Refs. [1,2]), followed by an era of accelerated expansion [2]. Similar results were also obtained in the early LQC literature, see, e.g., Refs. [9,22]. However, it is not obvious *a priori* that they must hold for GFT as well. Nonetheless, it is remarkable that these two different approaches yield qualitatively similar results for the dynamics of the Universe near the classical singularity, even though this fact by itself does not necessarily point at a deeper connection between the two.

In the model considered in this paper, our results have a purely quantum geometric origin and do not rely on the assumption of a specific potential for the minimally coupled scalar field,² which is taken to be massless and introduced for the sole purpose of having a relational clock. This is quite unlike inflation, which instead heavily relies on the choice of the potential and initial conditions for the inflaton in order to predict an era of accelerated expansion with the desired properties.

In this section we investigate under which conditions on the interaction potential of the GFT model it is possible to obtain an epoch of accelerated expansion that could last long enough, so as to account for the minimum number of e -folds required by standard arguments. The number of e -folds is given by

²This is also the case in LQC, see Ref. [22]. However, the number of e -folds computed in that framework turns out to be too small in order to supplant inflation [23].

$$N = \frac{1}{3} \log \left(\frac{V_{\text{end}}}{V_{\text{bounce}}} \right), \quad (17)$$

where V_{bounce} is the volume of the Universe at the bounce and V_{end} is its value at the end of the era of accelerated expansion. A necessary condition for it to be called an inflationary era is that the number of e -folds must be large enough, namely, $N \gtrsim 60$.

Using Eq. (4) we rewrite Eq. (17) as

$$N = \frac{2}{3} \log \left(\frac{\rho_{\text{end}}}{\rho_{\text{bounce}}} \right), \quad (18)$$

with an obvious understanding of the notation. This formula is particularly useful since it allows us to derive the number of e -folds only by looking at the dynamics of ρ .

Since there is no notion of proper time, a sensible definition of acceleration can only be given in relational terms. In particular, we seek a definition that agrees with the standard one given in ordinary cosmology. As in Ref. [2] we can therefore define the acceleration as

$$\alpha(\rho) \equiv \frac{\partial_\phi^2 V}{V} - \frac{5}{3} \left(\frac{\partial_\phi V}{V} \right)^2. \quad (19)$$

Hence, from Eqs. (5), (6) one gets the following expression for the acceleration α as a function of ρ for a generic potential:

$$\alpha(\rho) = -\frac{2}{\rho^2} \left\{ \partial_\phi U(\rho) \rho + \frac{14}{3} [E - U(\rho)] \right\}. \quad (20)$$

Using Eq. (15) one finally has for our model

$$\begin{aligned} \alpha(\rho) = & -\frac{2}{\rho^2} \left[\frac{14}{3} E + \left(1 - \frac{14}{3n'} \right) \mu \rho^{n'} + \frac{4m^2 \rho^2}{3} \right. \\ & \left. + \left(1 - \frac{14}{3n} \right) \lambda \rho^n - \frac{10Q^2}{3\rho^2} \right]. \end{aligned} \quad (21)$$

Therefore, the sign of the acceleration is opposite to that of the polynomial

$$s(\rho) = P(\rho) + \left(3 - \frac{14}{n} \right) \lambda \rho^{n+2} + \left(3 - \frac{14}{n'} \right) \mu \rho^{n'+2}, \quad (22)$$

where we defined

$$P(\rho) = 4m^2 \rho^4 + 14E\rho^2 - 10Q^2. \quad (23)$$

In the following we will study in detail the properties of the era of accelerated expansion. The free case will be discussed in Sec. IV A, whereas the role of interactions in allowing for an inflationary-like era will be the subject of Sec. IV B.

A. The noninteracting case

In this case the acceleration is given by

$$\alpha(\rho) = -\frac{2}{3\rho^4} P(\rho). \quad (24)$$

The bounce occurs when ρ reaches its minimum value, *i.e.* when $U(\rho) = E$, leading to

$$\rho_{\text{bounce}}^2 = \frac{1}{m^2} (\sqrt{E^2 + m^2 Q^2} - E). \quad (25)$$

A straightforward calculation shows that $\alpha(\rho_{\text{bounce}}) > 0$ as expected. The era of accelerated expansion ends when $P(\rho)$ vanishes, which happens at a point $\rho_\star > \rho_{\text{bounce}}$, which is given by

$$\rho_\star = \frac{1}{4m^2} (\sqrt{49E^2 + 40m^2 Q^2} - 7E). \quad (26)$$

We can then use Eqs. (18), (25), and (26) to determine the energy E as a function of the number of e -folds N . Reality of E thus leads to the following bounds on N

$$\frac{1}{3} \log \left(\frac{10}{7} \right) \leq N \leq \frac{1}{3} \log \left(\frac{7}{4} \right), \quad (27)$$

that is,

$$0.119 \lesssim N \lesssim 0.186. \quad (28)$$

Such tight bounds, holding for all values of the parameters m^2 and Q^2 , rule out the free case as a candidate to replace the standard inflationary scenario in cosmology.

B. The interacting case: The multicritical model

In this subsection we investigate the consequences of interactions for the evolution of the Universe. In particular, we show how the interplay between the two interaction terms in the effective potential [Eq. (7)] makes it possible to have an early epoch of accelerated expansion, which lasts as long as in inflationary models. Before studying their effect, we want to discuss how the occurrence of such interaction terms could be motivated from the GFT perspective. In principle, one could have infinitely many interaction terms given by some power of the GFT field. However, only a finite number of them will be of relevance at a specific scale, as dictated by the behavior of the fundamental theory under the RG flow.

In a continuum and large-scale limit, new terms in the action could be generated whereas others might become irrelevant. In this sense, one might speculate that, *e.g.*, in addition to the five-valent simplicial interaction term the effective potential includes another term which becomes relevant on a larger scale. Ultimately, rigorous RG

arguments will of course have the decisive word regarding the possibility of obtaining such terms from the fundamental theory. Nevertheless, by studying the phenomenological features of such potentials and extracting physical consequences from the corresponding cosmological solutions, we aim at clarifying the map between the fundamental microscopic and effective macroscopic dynamics of the theory. At the same time, our results might help to shed some light onto the subtle issue of the physical meaning of such interaction terms.

Hereafter we assume the hierarchy $\mu \ll |\lambda|$, since otherwise an inflationary era cannot be easily accommodated. This means that the higher-order term in the interaction potential $\mathcal{V}(\sigma)$ becomes relevant only for very large values of the condensate field σ , hence, of the number of quanta representing the basic building blocks of quantum spacetime. Consequently, the dynamics in the immediate vicinity of the bounce is governed by the parameters of the free theory and the subleading interaction term.

To begin with, let us start by fixing the value of the GFT energy. We require the Universe to have a Planckian volume at the bounce. Since the volume is given by Eq. (4), this is done by imposing $\rho_{\text{bounce}} = 1$. Such a condition also fixes the value of the GFT energy to

$$E = U(\rho_{\text{bounce}} = 1). \quad (29)$$

In fact, we demand that ρ_{bounce} is the minimal value of ρ which is compatible with the GFT energy E available to the system. Hence, we also have the condition $\partial_\rho U(\rho_{\text{bounce}} = 1) \leq 0$. Notice that this is trivially satisfied in the free case. In the interacting case (holding the hierarchy $\mu \ll |\lambda|$) one can therefore use it to obtain a bound on λ ,

$$\lambda \leq m^2 + Q^2. \quad (30)$$

For our purposes and in order to carry over our analysis in full generality, it is convenient to introduce the definitions

$$\alpha \equiv \left(3 - \frac{14}{n}\right)\lambda, \quad (31)$$

$$\beta \equiv \left(3 - \frac{14}{n'}\right)\mu. \quad (32)$$

The acceleration Eq. (21) can thus be written as

$$\mathbf{a}(\rho) = -\frac{2}{\rho^4} [P(\rho) + \alpha\rho^{n+2} + \beta\rho^{n'+2}]. \quad (33)$$

As pointed out before, $\mathbf{a} > 0$ has to hold at the bounce. The first thing to be observed is that $\alpha < 0$ is a necessary condition in order to have enough e -folds. In fact, if this

were not the case, the bracket in Eq. (33) would have a zero at a point $\rho_{\text{end}} < \rho_\star$ [cf. Eq. (26)], thus leading to a number of e -folds that is even smaller than the corresponding one in the free case. Furthermore, it is possible to constrain the value of μ in a way that leads both to the aforementioned hierarchy and to the right value for N , which we consider as fixed at the outset. In order to do so, we solve Eq. (18) with respect to ρ_{end} , having fixed the bounce at $\rho_{\text{bounce}} = 1$,

$$\rho_{\text{end}} = \rho_{\text{bounce}} e^{\frac{3}{2}N}. \quad (34)$$

The end of inflation occurs when the polynomial in the bracket in Eq. (33) has a zero. Since $\rho_{\text{end}} \gg 1$, it is legitimate to determine this zero by taking into account only the two highest powers in the polynomial, with respect to which all of the other terms are negligible. We therefore have

$$\alpha\rho_{\text{end}}^{n+2} + \beta\rho_{\text{end}}^{n'+2} \approx 0, \quad (35)$$

which, using Eq. (34), leads to

$$\beta = -\alpha e^{-\frac{3}{2}N(n'-n)}. \quad (36)$$

The last equation is consistent with the hierarchy $\mu \ll |\lambda|$ and actually fixes the value of μ once λ , n , n' and N are assigned. Furthermore one has $\beta > 0$ which, together with Eqs. (11), (32), implies $n' > \frac{14}{3}$. Importantly, this means that $n' = 5$ is the lowest possible integer compatible with an inflationary-like era. This particular value is also interesting in another respect because in GFT typically only specific combinatorially nonlocal interactions that are minimally of such a power allow for an interpretation in terms of simplicial quantum gravity [3,24].

Our considerations so far leave open the following two possibilities:

- (i) $\lambda < 0$ and $n \geq 5$ ($n' > n$), which in the case of $n = 5$ could correspond to the just-mentioned simplicial interaction term and the higher-order n' term, could possibly be generated in the continuum and large-scale limit of the theory and becomes dominant for very large ρ . For even n' it mimics so-called tensorial interactions.
- (ii) $\lambda > 0$ and $2 < n < 5$ ($n' \geq 5$), which for $n' = 5$ could allow a connection to simplicial quantum gravity and would remain dominant for large ρ over the n term, which in the case $n = 4$ is reminiscent of an interaction of tensorial type.

However, this is not yet enough in order to guarantee an inflationlike era. In fact we have to make sure that there is no intermediate stage of deceleration occurring between the bounce at $\rho_b = 1$ and ρ_{end} , i.e., that $\mathbf{a}(\rho)$ stays positive in the interval between these two points. In other words, we want to make sure that ρ_{end} is the only zero of the acceleration lying to the right of ρ_b . In fact, $\mathbf{a}(\rho)$ starts

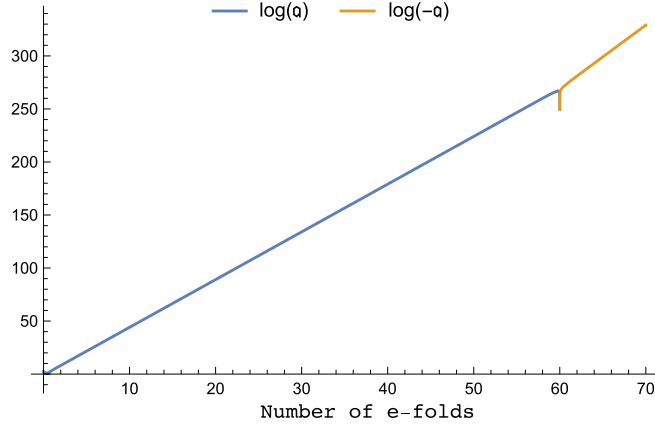


FIG. 4. Inflationary era supported by GFT interactions in the multicritical model. The blue (orange) curve represents the graph of the logarithm of the acceleration (minus the acceleration) as a function of the number of e -folds, in the case $\lambda < 0$. The plot refers to the particular choice of parameters $n = 5$, $n' = 6$, $m = 1$, $Q = 1$, and $\lambda = -3$. The value of μ is determined from Eq. (36) by requiring the number of e -folds to be $N = 60$. There is a logarithmic singularity at $N \approx 60$, marking the end of the accelerated expansion.

positive at the bounce and has a minimum when $P(\rho)$ becomes of the same order of magnitude of the term containing the power ρ^{n+2} [see Eq. (33)]. Thus we see that we have to require that the local minimum of $\alpha(\rho)$ [i.e., the maximum of the polynomial in brackets in Eq. (33)] is positive (resp. negative). As ρ increases further, the acceleration increases again until it reaches a maximum when the contribution coming from the term containing $\rho^{n'+2}$ becomes of the same order of magnitude of the other terms. Thereafter the acceleration turns into a decreasing function all the way until $\rho \rightarrow +\infty$ and, therefore, has a unique zero. Positivity of the local minimum of $\alpha(\rho)$ translates into a further constraint on parameters space. By direct inspection, it is possible to see that the latter case listed above does not satisfy such condition for any value of the parameters of the model. Therefore we conclude that λ must be negative if the acceleration is to keep the same sign throughout the inflationary era. The evolution of the acceleration as a function of relational time ϕ is shown in Figs. 4–6 for some specific choices of the parameters. It is worthwhile stressing that the behavior of the model in the case $\lambda < 0$ is nevertheless generic and therefore does not rely on the specific choice of parameters. Furthermore, by adjusting the value of N and the other parameters in Eq. (36), it is possible to achieve any desirable value of e -folds during inflation.

Our comments in this section apply to the multicritical model with the effective potential Eq. (7) but do not hold in a model with only one interaction term. In fact in that case it is not possible to prevent the occurrence of an intermediate era of deceleration between ρ_b and ρ_{end} , the latter giving the

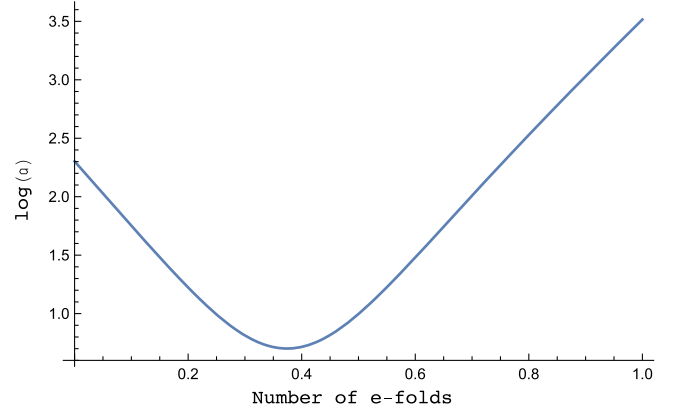


FIG. 5. The behavior of the acceleration close to the bounce.

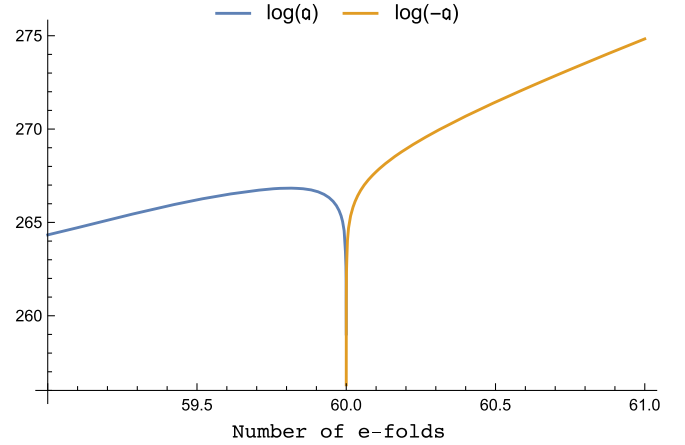


FIG. 6. The behavior of the acceleration at the end of inflation.

scale at which the higher-order interaction term becomes relevant.

One last remark is in order: inflation was shown to be a feature of multicritical GFT models but only at the price of a fine-tuning in the value of the parameter μ [see Eq. (36)].

V. INTERACTIONS AND THE FINAL FATE OF THE UNIVERSE

It is possible to recast the dynamical equations for the volume of the Universe in a form that bears a closer resemblance to the standard Friedmann equation, as shown in Ref. [2]. In fact, the Hubble expansion rate can be expressed as (see the Appendix for more details)

$$H = \frac{1}{3} \frac{\partial_\phi V}{V^2} \pi_\phi. \quad (37)$$

From Eq. (5) and the proportionality between the momentum of the scalar field and Q , we have

$$H^2 = \frac{4}{9} \frac{\hbar^2 Q^2}{V^2} \left(\frac{\partial_\phi \rho}{\rho} \right)^2. \quad (38)$$

The term in bracket can thus be interpreted as a dynamical effective gravitational constant as in Ref. [2]. Alternatively, using Eqs. (4), (16), and (15), the last equation [Eq. (38)] becomes (considering the case with only one interaction term, namely, $\lambda = 0$)

$$H^2 = \frac{8\hbar^2 Q^2}{9} \left[\frac{\varepsilon_m}{V^2} + \frac{\varepsilon_E}{V^3} + \frac{\varepsilon_Q}{V^4} + \frac{\varepsilon_\mu}{V^{3-n'/2}} \right], \quad (39)$$

where we defined

$$\varepsilon_E = V_j E, \quad (40)$$

$$\varepsilon_m = \frac{m^2}{2}, \quad (41)$$

$$\varepsilon_Q = -\frac{Q^2}{2} V_j^2, \quad (42)$$

$$\varepsilon_\mu = -\frac{\mu}{n'} V_j^{1-n'/2}. \quad (43)$$

The exponents of the denominators in Eq. (39) can be related to the w coefficients in the equation of state $p = w\varepsilon$ of some effective fluids, with energy density ε and pressure p . Each term scales with the volume as $\propto V^{-(w+1)}$.

It is worth pointing out that Eq. (39) makes clearer the correspondence with the framework of ekpyrotic models,³ where one has the gravitational field coupled to matter fields with $w > 1$. Such models have been advocated as a possible alternative to inflation, see Ref. [25]. We observe that at early times (i.e., small volumes) the occurrence of the bounce is determined by the negative sign of ε_Q , which is also the term corresponding to the highest w . However, while this is sufficient to prevent the classical singularity, it is not enough to guarantee that the minimum number of e -folds is reached at the end of the accelerated expansion, as shown in Sec. IV A. In fact, the role of interactions is crucial in that respect, as our analysis in Sec. IV B has shown.

In the rest of this section we focus instead on the consequences of having interactions in the GFT model for the evolution of the Universe at late times. As we have already seen in Sec. III, a positive μ entails a recollapsing Universe. This should also be clear from Eq. (39). In particular, we notice that the corresponding term in the equation is an increasing function of the volume for $n' > 6$. This is quite an unusual feature for a cosmological model, where all energy components (with the exception of the cosmological constant) are diluted by the expansion of the Universe. For $n' = 6$ one finds instead a cosmological constant term. It is also possible to have the interactions reproduce the classical curvature term $\propto \frac{\kappa}{V^{2/3}}$ by choosing

$n' = \frac{14}{3}$; this is, however, not allowed if one restricts to integer powers in the interactions [3].

Our analysis shows that only $\lambda < 0$ leaves room for an era of accelerated expansion analogous to that of inflationary models. In order for this to be possible, one must also have $n' > n \geq 5$. Moreover, if one rules out phantom energy (i.e., $w < -1$), there is only one case which is allowed, namely, $n = 5$, $n' = 6$. Then, during inflation, the Universe can be described as dominated by a fluid with equation of state $w = -\frac{1}{2}$. After the end of inflation its energy content also receives a contribution from a negative cosmological constant, which eventually leads to a recollapse. It is remarkable that this particular case selects an interaction term that is in principle compatible with the simplicial interactions that have been extensively considered in the GFT approach. However, it must be pointed out that the realization of the geometric inflation picture also imposes strong restrictions on the type of interactions one can consider, as well as on their relative strength.

VI. SUMMARY AND OUTLOOK

We investigated the phenomenological consequences of interacting GFT models for the dynamics of the early Universe, which in this framework is seen as emerging from the collective behavior of “quanta of geometry.” The dynamical equations for the GFT condensate are classical and are obtained from an effective action, whose form can be motivated from the microscopic theory. The dynamics of the condensate in turn determines a Friedmann-like dynamics for the geometric observables corresponding to classical dynamical variables.

In this article we considered an effective potential including two interaction terms besides the quadratic one, the latter being already present in the free theory. An ambiguity in the kinetic term, represented by the factor A , is fixed by requiring the expansion of the Universe to not be faster than exponential at large volumes. A general prediction of the model is the occurrence of a recollapse when the higher-order interaction term becomes codominant. Results that have already been obtained in the free theory (Refs. [1,2]) survive in the interacting case, in particular for what concerns the occurrence of a bounce and an early epoch of accelerated expansion. The former result, together with the recollapse induced by interactions, leads to cyclic cosmologies. A more detailed analysis of the latter, instead, leads to the conclusion that, in the free case, the era of accelerated expansion does not last for a number of e -folds which is at least as large as in inflationary models. This is instead made possible when suitable interaction terms are taken into account, as in the multi-critical model considered here. Indeed, we showed that one can attain an arbitrary number of e -folds as the Universe accelerates after the bounce. Furthermore, having an inflationary-like expansion imposes a restriction on the

³We are grateful to Martin Bojowald for this observation.

class of viable models. In fact, this is possible only for $\lambda < 0$ and $5 \leq n < n'$, and when one has the hierarchy $\mu \ll |\lambda|$. Reasonable phenomenological arguments lead us to select only the case $n = 5$, $n' = 6$ as physical. The two powers can be related, respectively, to simplicial interactions, commonly considered in the GFT framework, and to a negative cosmological constant. While the result is encouraging as a first step towards a quantum geometric description of the inflationary era, a few remarks are in order. In fact, it must be pointed out that it comes at the price of a fine-tuning in the coupling constant of the higher-order interaction term. From this point of view, it shares one of the major difficulties of ordinary inflationary models. Furthermore, an inflationary-like era does not seem to be a generic property of GFT models, but in fact requires interactions of a suitable form.

Future work must be devoted to incorporating other degrees of freedom in the description of the effective dynamics of the emergent spacetime. In fact, their phenomenological signatures, in particular for what concerns the seeds for the growth of structures, are crucial in order to be able to give a definite answer to the problem of finding a valid alternative to the inflationary paradigm, which might come from quantum geometry.

Cyclic cosmologies were obtained under fairly general assumptions on the effective potential, namely, its boundedness from below. Furthermore, no input was given about the geometry of the spatial sections of the reconstructed spacetime. Given the classical correspondence existing between a finitely expanding Universe and closed spatial topology, it is intriguing whether this purely dynamical result bears any implications on the geometry of the emergent spacetime. In particular it would be worth investigating the existence of such a generalized correspondence by extending our analysis and studying, e.g., the implications of the peculiar combinatorial structure of GFT interactions for the geometry of the emergent spacetime and its dynamics. In fact, this further step would be required in order to properly take into account the effects of anisotropies.

In this article we considered a model that can in principle exhibit multicritical behavior, which is reminiscent of analogous models discussed in statistical field theory to model systems with a more complex phase structure [11]. Multicriticality has not been considered in the GFT literature so far. Nevertheless, it would be worth studying the possible implications of such models from a fundamental GFT point of view and relate them to the geometrogenesis scenario.

It is our hope that the interplay between determining phenomenological constraints, such as those obtained in this work, and a fundamental approach involving, e.g., RG arguments, might help to single out the correct microscopic theory (or even a family of such theories). Further work must come from both directions in a common effort to

develop an appropriate framework for studying early Universe cosmology, which correctly takes into account the quantum dynamics of all the relevant degrees of freedom of the gravitational field.

We would like to draw the attention of the reader to another important aspect concerning the interpretation of the results obtained in this work, with respect to the physical significance and the role played by the operator counting the number of nodes in the spin network, whose expectation value is given by $\mathcal{N} = \sum_j \rho_j^2$. Looking back at Eq. (3), one sees that in the case in which the system lies just in one representation with fixed j (as supported by the findings in Ref. [14]), this quantity is proportional to the total volume of the universe. It is therefore clear that only its relative variations are observable, and turn out to be proportional to the Hubble expansion rate.

As a closing remark, we would like to stress that the GFT condensate cosmology approach is not the only one that has been suggested in the literature to extract the cosmological sector of LQG from a covariant formulation of its dynamics. In fact, spin foam cosmology (SFC) [26] makes use of the spin foam expansion [4] and is thus an expansion in terms of the number of degrees of freedom. Initially, SFC was studied using the simplest cellular decomposition of the three-sphere, given by the so-called dipole graph [27], and was later on extended to more general regular graphs [28]. Central to this approach is the assumption that the relevant physics is encoded within a fixed number of quanta of geometry, whereas in GFT condensate cosmology there is no *a priori* restriction on the number of quanta which—in principle—can be large. As suggested in [19,21], a GFT condensate could also be constituted by means of dipoles or even more complicated building blocks; it would be important to repeat our analysis for those and compare the results to the ones obtained from SFC.

APPENDIX

Here we review how the effective equations that give the relational evolution of the volume of the Universe, Eqs. (5), (6), can be recast in a form that is closer to that of ordinary FLRW models. In fact, in the framework considered in this article, spacetime is an emergent concept. Hence, there is no natural notion of proper time and evolution of physical observables is more appropriately defined in terms of a matter clock, here represented by a massless scalar which is minimally coupled to the gravitational field. It is nonetheless worth showing the relation that exists between the two different descriptions in the classical theory, where they are both well defined; by considering the classical relation between the velocity of the scalar field ϕ and its canonically conjugate momentum π_ϕ ,

$$\pi_\phi = V \dot{\phi}, \quad (\text{A1})$$

together with the relation between the proper volume and the scale factor

$$V \propto a^3, \quad (\text{A2})$$

one can express the Hubble expansion rate as

$$H = \frac{\dot{a}}{a} = \frac{1}{3} \frac{\dot{V}}{V} = \frac{1}{3} \pi_\phi \frac{\partial_\phi V}{V^2}. \quad (\text{A3})$$

The last equation is the same as Eq. (37), which leads to the modified Friedmann equation of Eq. (38). In a similar fashion, it is also possible to get the Raychaudhuri equation for the acceleration. In fact, taking two derivatives of Eq. (A2), one is lead to

$$\frac{\ddot{a}}{a} = \frac{1}{3} \left[\frac{\ddot{V}}{V} - \frac{2}{3} \left(\frac{\dot{V}}{V} \right)^2 \right]. \quad (\text{A4})$$

Furthermore, using Eq. (A1), one finds

$$\dot{V} = \partial_\phi V \dot{\phi} = \partial_\phi V \frac{\pi_\phi}{V} \quad (\text{A5})$$

and

$$\ddot{V} = \left(\frac{\pi_\phi}{V} \right)^2 \left[\partial_\phi^2 V - \frac{(\partial_\phi V)^2}{V} \right]. \quad (\text{A6})$$

From Eqs. (A3), (A4), and (A6), one has

$$\frac{\ddot{a}}{a} = \frac{1}{3} \left(\frac{\pi_\phi}{V} \right)^2 \left[\frac{\partial_\phi^2 V}{V} - \frac{5}{3} \left(\frac{\partial_\phi V}{V} \right)^2 \right]. \quad (\text{A7})$$

The last equation justifies the definition of the acceleration given in Eq. (19).

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