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Regarding the radion in Randall-Sundrum models with brane curvature

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In Randall-Sundrum models, one typically expects the radion to be the lightest new "gravity" state, as it is dual to a composite pseudo-Goldstone boson associated with conformal symmetry breaking in the IR. Here, we investigate the effects of localized brane curvature on the properties of the radion in Goldberger-Wise stabilized Randall-Sundrum models. We point out that both the radion mass and coupling to brane matter are sensitive to the brane curvature. Radion/Higgs kinetic mixing, via an IR-localized nonminimal coupling to the Higgs, is also investigated, in relation to the ghostlike radion that can occur for $\mathcal{O}(10)$ values of the IR curvature (as required to significantly suppress the first Kaluza-Klein graviton mass). We also discuss a class of IR-localized terms involving the radion. Basic comments regarding the dual four-dimensional theory are offered.

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I. INTRODUCTION

The Randall-Sundrum (RS) model provides a natural means by which to generate hierarchically separated, radiatively stable mass scales [1]. Accordingly, it has received much attention as a candidate solution to the hierarchy problem. The model employs a warped extra dimension, namely a gravitational background, with factorizable geometry, that is sourced by a bulk cosmological constant and nontrivial brane tensions. The use of localized branes in the five-dimensional (5D) spacetime explicitly breaks the 5D diffeomorphism symmetry yet preserves the requisite four-dimensional (4D) symmetry. Consequently, the most-general Lagrangian for the model, consistent with the symmetries, allows localized 4D terms that break the 5D diffeomorphism symmetry.

Included among the set of such terms are the so-called "brane curvature" terms, which can be thought of as localized 4D kinetic terms for the bulk graviton. These terms have received some attention in the literature [2], though generally they are assumed subdominant. Nonetheless, they should appear in the most-general Lagrangian. Recently, the brane curvature terms received attention due to their capacity to suppress the lightest Kaluza-Klein (KK) graviton mass. In particular, it was shown that large brane curvature [i.e., with an $\mathcal{O}(10)$ dimensionless coefficient] can modify the spectrum of KK gravitons such that the lightest KK graviton can have an $\mathcal{O}(10^2)$ GeV mass, while retaining an $\mathcal{O}(\text{TeV})$ lightest KK vector [3–5].

In addition to the KK gravitons, the bulk 5D metric gives rise to a graviscalar fluctuation, known as the radion [6,7]. This field is massless unless the length of the extra dimension is stabilized. After stabilization, it acquires a mass that is sensitive to the backreaction of the stabilizing dynamics. The best-studied method for stabilizing the extra dimension relies on a bulk scalar that develops a nontrivial background value to generate a potential for the radion (as proposed by Goldberger and Wise (GW) [8,9]; also see Ref. [10]).

The common origin of the radion and KK gravitons, as fluctuations of the bulk metric, means both are sensitive to brane curvature terms. Motivated by recent interest in large brane curvature, in this work, we investigate some effects of brane curvature terms on the properties of the radion in a GW-stabilized RS model. We consider the modification to the radion mass and couplings due to the brane curvature and further consider the effects of an IR-localized nonminimal coupling to the Standard Model (SM) Higgs. Our results generalize a number of the corresponding expressions in Ref. [6] to include the effects of brane curvature. We find that, in the GW stabilized model, a nonminimal coupling to the IR Higgs does not allow one to avoid the ghostlike radion that arises for $\mathcal{O}(10)$ values of the IR curvature. However, motivated by models with localized Lorentz-invariance violation, we consider additional IR-localized terms that may help remove the ghostlike radion.

Before proceeding, we note that a number of works recently considered the RS model in relation to the 750 GeV diphoton excess; see, e.g., Ref. [11]. For additional discussion of the spin-2 explanation, see Ref. [12]. In our analysis, we consider the RS model with a UV scale of $M_* \sim \mathcal{O}(M_{\rm Pl})$. However, the results are readily adapted to the little RS model [13] and related warped models [14], for which $M_* \ll M_{\rm Pl}$. For early discussion regarding localized terms in brane models, see Refs. [15,16].

The layout of this paper is as follows. In Sec. II, we describe the setup for our analysis and present some consequences of the brane curvature terms for the case of a massless radion (i.e., a nonstabilized RS model). These results prove useful for subsequent analysis. We explore the radion coupling to brane-localized matter in Sec. III and turn to the more-general case of GW stabilized RS models in Sec. IV. The effects of a nonminimal coupling with an IR-localized SM Higgs are studied in Sec. V, and additional IR-localized terms for the radion are considered in Sec. VI. Comments regarding the interpretation in the dual 4D theory are given in Sec. VII, and we conclude in Sec. VIII.

II. RANDALL-SUNDRUM MODEL WITH BRANE CURVATURE

To study the effects of the brane curvature terms on the metric fluctuations, we employ the interval approach to braneworld gravity [17–19]. This approach enables a transparent treatment of boundary curvature terms, which simply modify the boundary conditions (BCs) for metric fluctuations. However, one must be careful to correctly identify the available gauge freedoms in the presence of such terms (for detailed discussion, see Ref. [20]). Before proceeding, we note that earlier works have considered the effects of brane curvature terms in the RS framework using the orbifold picture [2,21] and for $\mathrm{AdS}_5/\mathrm{AdS}_4$ in the interval approach [18,19]. Let us also note that some content in the following sections has overlap with Ref. [20]. We include it here so the presentation is coherent and (relatively) self-contained and note that (*i*) we present a number of extra results, in relation to IR curvature, that were not given in Ref. [20], due to the focus on UV curvature in that work; (*ii*) in the current presentation, we focus on the case with $M_* \sim M_{\rm Pl}$, relevant for RS models, as opposed to the low UV-scale models of interest in Ref. [20]; (*iii*) we subsequently generalize these results to include a nonminimal coupling with an IR Higgs and additional IR terms for the radion. These calculations reveal the viable parameter space in which the radion is not ghostlike for RS models with brane curvature and an IR-localized Higgs. We find that, although the IR Higgs does modify the viable regions of parameter space, the effect is not large enough to allow $\mathcal{O}(10)$ values of the IR curvature. Consequently, the mass of the lightest KK graviton is not expected to be significantly suppressed relative to the IR scale in models with an IR Higgs.[1] Our results generalize Ref. [6].

The RS model employs a warped extra dimension, labeled by the coordinate $y \in [0, L]$, with a UV (IR) brane of characteristic energy M_* ($e^{-kL}M_*$) located at $y = 0$ ($y = L$). The metric has the form

$$ds^2 = e^{-2ky}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 = G_{MN}dx^M dx^N, \quad (1)$$

where $M, N, \ldots$ ($\mu, \nu, \ldots$) are 5D (4D) Lorentz indices and k denotes the AdS_5 curvature. The corresponding action, including brane-localized curvature terms, is

$$\begin{aligned} S = &\int_{\mathcal{M}} d^5x\sqrt{-G}\{2M_*^3\mathcal{R} - \Lambda\} \\ &+ \sum_i \int d^4x\sqrt{-g_i}\{M_i^2 R_i - V_i/2\} \\ &+ 4M_*^3 \oint_{\partial\mathcal{M}} \sqrt{-g_i}K. \end{aligned} \quad (2)$$

The bulk Ricci scalar $\mathcal{R}$ is constructed with the bulk metric G_{MN}, and M_* is the 5D gravity scale. The brane-localized curvature R_i is constructed with the brane metric g^i_{uv} (the restriction of $G_{\mu\nu}$ to the relevant boundary) and has coefficient M_i on the ith boundary ($i = \mathrm{UV}, \mathrm{IR}$). The last term is the usual Gibbons-Hawking boundary term [22], with K being the extrinsic curvature of the manifold $\mathcal{M}$. This term is included to obtain consistent Einstein equations on the interval [17]. The action includes a bulk cosmological constant Λ and brane tensions V_i, which take their usual RS values, $V_i = -24kM_*^3\theta_i$, with bulk curvature $k = \sqrt{-\Lambda/24M_*^3}$, and we use the notation $\theta_{\rm UV} = -\theta_{\rm IR} = -1$. For future purposes, we define the dimensionless brane curvature coefficients $v_i = M_i^2 k/M_*^3$ and $w_i = V_i/2M_*^3 k$.

The calculation of the effective 4D Planck mass gives

$$M_{\rm Pl}^2 = \frac{M_*^3}{2k}\{1 + v_{\rm UV} - (1 - v_{\rm IR})e^{-2kL}\}. \quad (3)$$

This includes contributions from both the bulk and brane intrinsic curvatures. Observe that the Planck mass is rather insensitive to the IR curvature, while a constraint of $(1 + v_{\rm UV}) > 0$ is required to ensure positivity of the Planck mass (equivalently, to avoid a ghostlike massless graviton). The different pieces have distinct interpretations in the dual 4D picture, as we discuss in Sec. VII. Variation of the bulk action gives the standard (bulk) equations of motion,

[1] Recall that the first KK graviton mass is only significantly suppressed below the IR scale for large values of the IR curvature [2].

$$\mathcal{R}_{MN} - \frac{1}{2}G_{MN}\mathcal{R} = -\frac{\Lambda}{4M_*^3}G_{MN}. \tag{4}$$

The boundary conditions follow from the variations of the 4D brane action and the Gibbons-Hawking term, combined with surface terms resulting from the variation of the bulk action, giving [18]

$$\left[\frac{v_i}{k}\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right) + \frac{1}{2}g_{\mu\nu}kw_i + \theta_i\sqrt{G^{55}}(g_{\mu\nu,5} - g_{\mu\nu}g_{\alpha\beta,5}g^{\alpha\beta})\right]_{y=y_i} = 0. \tag{5}$$

This equation should be evaluated separately at the boundaries $y = 0, L$. We work in a "straight gauge," defined by $G_{\mu 5} = 0$ [18], without loss of generality. Expanding about the background metric, $G_{MN} = G^0_{MN} + h_{MN}$, with zeroth-order metric $G^0_{\mu\nu} = e^{-2ky}\eta_{\mu\nu}$, and $G^0_{55} = 1$ with $G^0_{\mu 5} = h_{\mu 5} = 0$ in a straight gauge, the boundary conditions give (indices are raised with $g^{\mu\nu} = e^{2ky}\eta^{\mu\nu}$)

$$\left[\frac{v_i}{2k}\{h_{\alpha\mu,\nu}{}^{\alpha} + h_{\alpha\nu,\mu}{}^{\alpha} - h_{\mu\nu,\alpha}{}^{\alpha} - \tilde{h}_{,\mu\nu} - g_{\mu\nu}(h_{\alpha\beta}{}^{\alpha\beta} - \tilde{h}_{,\alpha}{}^{\alpha})\} + \theta_i\{2kh_{\mu\nu} + h_{\mu\nu,5} - g_{\mu\nu}\tilde{h}_{,5} - 3kg_{\mu\nu}h_{55}\}\right]_{y=y_i} = 0. \tag{6}$$

For massive 4D modes, the tensor $h_{\mu\nu}$ can be written as

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu V_\nu + \partial_\nu V_\mu + e^{-2ky}\partial_\mu\partial_\nu\mathcal{S}_1 + G^0_{\mu\nu}\mathcal{S}_2, \tag{7}$$

where $h_{\mu\nu}$ is now transverse traceless with 5 degrees of freedom, $\partial^\alpha h_{\alpha\beta} = \eta^{\alpha\beta}h_{\alpha\beta} = 0$, and V_μ is transverse, $\partial^\alpha V_\alpha = 0$. Also, $\mathcal{S}_1$ and $\mathcal{S}_2$ are scalar degrees of freedom. One can show that the physical massive modes are contained in $h_{\mu\nu}$ [20]. Performing a gauge transformation, with 4D gauge parameter ξ_μ, the transverse component of ξ_μ is used to gauge away V_μ, while the longitudinal part removes one of the scalars. The boundary conditions force the remaining scalar to vanish, absent fine-tuning among the brane curvature terms [20].

Writing the KK expansion for the physical fluctuations as

$$h_{\mu\nu}(x, y) = \kappa_* \sum_n h^{(n)}_{\mu\nu}(x) f_n(y), \tag{8}$$

where κ_* is chosen to give the 4D fields $h^{(n)}_{\mu\nu}$ a canonical mass dimension, the solution in the bulk is

$$f_n(y) = \frac{1}{N_n}\left\{J_2\left(\frac{m_n}{k}e^{ky}\right) + \beta_n Y_2\left(\frac{m_n}{k}e^{ky}\right)\right\}, \tag{9}$$

with m_n the mass of the nth spin-2 KK mode. Applying the boundary conditions gives

$$\beta^i_n = -\frac{J_1(z_i) - (z_i v_i \theta_i/2)J_2(z_i)}{Y_1(z_i) - (z_i v_i \theta_i/2)Y_2(z_i)}, \tag{10}$$

where $z_i = m_n e^{ky_i}/k$. The KK masses follow by enforcing $\beta^{\rm UV}_n = \beta^{\rm IR}_n \equiv \beta_n$. The mass for light IR-localized KK modes has a negligible dependence on the UV brane term—one can essentially take $v_{\rm UV} \approx 0$ without modifying the spectrum. On the other hand, the IR term $v_{\rm IR}$ modifies the KK masses in a nontrivial way [2]. For $v_i \to 0$, the KK masses reduce to the usual RS values [23]. We note that the additional factors of $1/2$ in Eq. (10), relative to Ref. [2], can be removed by rescaling the value of v_i in Eq. (2). This factor reflects the use of an interval rather than an orbifold [much as the brane tensions in Eq. (2) are smaller by a factor of $1/2$, relative to the orbifold picture]. This scaling would introduce a factor of 2 in many equations below, so it is simpler not to rescale. In our notation, the limit of large IR curvature gives a lightest KK graviton with mass approximately given by $m_{G_1} \approx 2e^{-kL}k/\sqrt{v_{\rm IR}/2}$. We note that the IR curvature of $r_L = 10/k$ in Ref. [3] corresponds to $v_{\rm IR} = 20$, while $r_L = 7$ in Ref. [5] corresponds to $v_{\rm IR} \approx 14$, and values of $\gamma_\pi \approx -7.6 < 0$ in Ref. [4] correspond to $v_{\rm IR} > 0$, due to a notational difference. Also, we mention that, in general, the bounds on KK graviton masses are sensitive to the model-building details of a given RS model. In particular, the constraints depend on the size of the coupling between the KK graviton and SM fields and are therefore sensitive to the localization of the SM fields in a given RS model.

The spin-2 spectrum contains the usual UV-localized massless graviton, with profile

$$f_0(y) = e^{-2ky}\sqrt{\frac{2k}{1 - e^{-2kL} + \sum_i v_i e^{-2ky_i}}}. \tag{11}$$

A further massless mode (the scalar radion) is present in the spectrum. This state acquires mass once the length of the extra dimension is stabilized, with the corresponding mass dependent on the backreaction of the stabilizing dynamics [6], as we discuss below for the Goldberger-Wise mechanism. However, it shall prove instructive to first comment on the massless radion, as some results remain useful in the weak backreaction case.

Thus, turning our attention to the graviscalar fluctuations, we note that in a straight gauge one can always use remnant gauge freedom to write the metric fluctuation h_{55} as [18,19,24]

$$h_{55}(x^\mu, y) = F(y)\psi(x^\mu). \tag{12}$$

Here, $F(y)$ is an arbitrary function of y satisfying $\int_0^L dy F(y) \neq 0$. An arbitrary h_{55} can be cast into the form (12) via a general 5D coordinate transformation, $x^M \to x^M + \xi^M$, with $\xi^\mu = 0$ and [18,19,24]

$$\xi^5 = \frac{1}{2}\int^y dy h_{55} - \frac{1}{2}\int^y dy F(y)\psi. \tag{13}$$

The presence of the arbitrary function $F(y)$ is a remnant gauge freedom.

We find it convenient to write the most general form of the metric with background and scalar perturbations as

$$G_{MN} = \begin{pmatrix} a^2[\eta_{\mu\nu} + \nabla_\mu\nabla_\nu P_3 + \eta_{\mu\nu}(2P_2 - aa'P_3{}')] & 0 \\ 0 & 1 + 2P_1 - (a^2 P_3')' \end{pmatrix}. \tag{14}$$

Here, $a(y)$ is the background warp factor, and $P_{1,2,3}$ are spin-zero perturbations that are functions of x^μ and y. This parametrization is motivated by the gauge-invariant forms of Refs. [25,26] and is such that the Einstein equations have a simple structure. For a detailed discussion of the gauge freedoms and the gauge transformations that allow one to write the scalar perturbations in this form, see the Appendix in Ref. [20]. Two of the bulk Einstein equations can be cast as

$$\partial_\mu\partial_\nu(P_1 + 2P_2) = 0 \quad \mu \neq \nu, \tag{15}$$

$$\partial_\mu\left(\frac{a'}{a}P_1 - P_2'\right) = 0 \quad \forall\mu. \tag{16}$$

Taking the integration constants to vanish (the perturbations are localized in x), Eq. (15) relates P_2 and P_1, while Eq. (16) determines the y-dependence of P_1. The remaining bulk Einstein equation reduces to $\Box P_1 = 0$, as expected for a massless 4D field. The perturbation P_3 is completely free in the bulk, reflecting the remnant gauge freedom [20]. This is related to the remnant gauge freedom in the massless sector described in Refs. [18,19], and physical quantities do not depend on (the bulk value of) P_3. Boundary conditions are derived from the two additional boundary Einstein equations:

$$P_3'(y_i) = \frac{-v_i}{a(y_i)[\theta_i k a(y_i) + v_i a'(y_i)]}P_1(y_i). \tag{17}$$

Using the solutions to the above, one can compute the effective 4D action for the physical scalar fluctuation. We perform separation of variables and solve for the profile of P_1, giving

$$P_1 = a^{-2}(y)\psi(x^\mu). \tag{18}$$

This solution is consistent with the boundary conditions (17) for general v_i provided the arbitrary function P_3 has $P_3' \neq 0$ at the boundaries, in accordance with Eq. (17). For the sources in Eq. (2), the solution for the background metric has the standard RS form, $a(y) = e^{-ky}$. Ignoring 4D surface terms (ψ vanishes at $x^\mu \to \infty$) and inserting the solution into the action, one obtains the effective 4D action for scalar perturbations, up to $\mathcal{O}(\psi^2)$, as

$$\mathcal{S}_{\mathcal{O}(\psi^2)} = \int d^4x \left[\frac{3M_*^3}{k}e^{2kL}\left(\frac{1}{1-v_{\rm IR}} - \frac{e^{-2kL}}{1+v_{\rm UV}}\right)\right] \times \left(-\frac{1}{2}\eta^{\mu\nu}\partial_\mu\psi\partial_\nu\psi\right). \tag{19}$$

Note that linear terms in ψ and additional quadratic terms, which appear at intermediate stages of the calculation, cancel out in the final result, providing a check. In particular, higher-order derivative terms present at the quadratic level in individual terms in Eq. (2) cancel out in the full action.

The physical radion is defined as $r(x) = \psi(x)N_\psi$, where the normalization constant is

$$N_\psi^2 = \frac{k}{3M_*^3}e^{-2kL}\frac{(1-v_{\rm IR})(1+v_{\rm UV})}{(1+v_{\rm UV}) - (1-v_{\rm IR})e^{-2kL}} = \frac{e^{-2kL}}{6M_{\rm Pl}^2}(1-v_{\rm IR})(1+v_{\rm UV}). \tag{20}$$

We immediately observe from Eq. (19) that the kinetic term is only well behaved for $v_{\rm IR} < 1$, while the UV term suffers no such constraint (one may safely take $v_{\rm UV} \gg 1$, as in Ref. [20]). We assume that a ghostlike radion (a wrong sign kinetic term) signals an instability of the ground state of the theory and that it is desirable to fix this in the traditional way by adding terms to the theory and/or restricting the couplings. Note that the crossover region between parameter space with/without a ghost radion gives a vanishing kinetic term, meaning the theory is strongly coupled; such regions should also be avoided. Regarding parameter space with $(1 + v_{\rm UV}) < 0$, one should use Eq. (19), rather than Eq. (20), to determine whether problems arise, due to the $v_{\rm UV}$-dependence of M_{PL}. Observe that the radion kinetic term is not problematic for $(1 + v_{\rm UV}) < 0$, whereas it is sensitive to the IR term, opposite to the massless graviton (this has a clear interpretation in the dual 4D theory, as discussed below).

It is worth emphasizing a point made above, with regard to the fluctuation P_3. In the limit $v_i \to 0$, one can use the remaining gauge freedom to choose the form of the scalar fluctuations such that the derivative pieces in Eq. (14) vanish, namely $\nabla_\mu \nabla_\nu P_3$ and P_3' [20]. Thus, in the limit of vanishing brane curvature, the standard parametrization of the graviscalar metric fluctuations in RS [6,7,9] is found to be consistent with the boundary conditions in the interval picture. However, for $v_i \neq 0$, one is unable to remove the derivative pieces in Eq. (14) with a gauge choice while *simultaneously* obtaining a solution that is consistent with the boundary conditions [20].

III. RADION COUPLING TO BRANE MATTER

We now turn to the coupling of the radion to brane-localized matter, which depends on the location of the matter. Some expressions presented below generalize results of Ref. [20] for the case with IR curvature. Consider a set of matter fields localized at the boundary $y = y_i$. Expanding the metric in terms of a fluctuation $f_{\mu\nu}$, which only contains the spin-zero parts of the perturbation, $g_{\mu\nu} \to g_{\mu\nu} + f_{\mu\nu}$, integrating over the extra dimension, and scaling the matter fields to bring the kinetic terms to canonical form, the linear fluctuation term is

$$S|_{\mathcal{O}(f)} = -\frac{1}{2} e^{2ky_i} \int d^4x \eta^{\mu\alpha}\eta^{\nu\beta} f_{\mu\nu} T_{\alpha\beta}, \tag{21}$$

where $T_{\mu\nu}$ is written in terms of the flat space metric (and canonical fields). Consider the nonderivative couplings of the graviscalar to $T_{\mu\nu}$,

$$S|_{\mathcal{O}(\psi)} = \frac{e^{2ky_i}}{2}\left[1 - \frac{v_i a'}{(\theta_i ka + v_i a')}\right] \int d^4x \psi T + \ldots, \tag{22}$$

where $T = \eta^{\mu\nu} T_{\mu\nu}$ and we used Eqs. (15)–(18). The coupling of the physical radion r is

$$S|_{\mathcal{O}(r)} = \frac{1}{2}\int d^4x \left(\frac{r}{\Lambda_i}\right) \times T + \ldots, \tag{23}$$

with location-dependent coupling Λ. For matter localized on the brane at $y = y_i$, one has

$$\begin{aligned} \Lambda_i^{-1} &= e^{kL}\left\{\frac{k}{3M_*^3}\frac{1}{(1+v_{\rm UV}) - (1-v_{\rm IR})e^{-2kL}}\right\}^{1/2} \\ &\quad \times \sqrt{\frac{1-\theta_j v_j}{1-\theta_i v_i}} \\ &= \frac{1}{\sqrt{6}}\frac{1}{e^{-kL}M_{\rm Pl}} \times \sqrt{\frac{1-\theta_j v_j}{1-\theta_i v_i}} \quad i \neq j. \end{aligned} \tag{24}$$

One can summarize the brane radion couplings as

$$\Lambda_i = \Lambda_{{\rm RS},i} \times \sqrt{\frac{1-\theta_i v_i}{1-\theta_j v_j}} \quad i \neq j, \tag{25}$$

where $\Lambda_{{\rm RS},i}$ is the usual RS radion coupling for matter on the brane at y_i. Thus, in the limit $v_{\rm UV,IR} \to 0$, one obtains the standard RS results. Note that the IR coupling is

$$\Lambda_{\rm IR}^{-1} = \Lambda_{\rm RS,IR}^{-1} \times \sqrt{\frac{1+v_{\rm UV}}{1-v_{\rm IR}}}, \tag{26}$$

which becomes strongly coupled for $v_{\rm IR} \to 1$. This corresponds to the crossover region between having and avoiding a ghostlike radion, such that the kinetic term vanishes, as mentioned previously.

At first sight, the $v_{\rm IR}$ dependence of these couplings appears unusual. Intuitively, one may expect the IR coupling to diminish for increasing values of $v_{\rm IR}$ and the UV coupling to have limited sensitivity to the size of $v_{\rm IR}$. However, one observes that increasing values of $v_{\rm IR}$ tend to decrease the coupling at the UV brane and increase the coupling at the IR brane. Actually, this behavior is not so surprising. Recall that increasing values of $v_{\rm IR}$ cause the strength of the kinetic term for the unscaled fluctuation $\psi = r/N_\psi$ to increase; see Eq. (19). After scaling ψ, this translates into a suppression of the couplings to the radion r, for increasing $v_{\rm IR}$. For UV-localized matter, this is the only $v_{\rm IR}$ dependence in the coupling, giving the inverse sensitivity of $\Lambda_{\rm UV}^{-1}$ to $v_{\rm IR}$. Note that for $v_{\rm UV} \to 0$, the IR coupling has a simple form, $\Lambda_{\rm IR} = \Lambda_{\rm RS,IR} \times \sqrt{1-v_{\rm IR}}$, while for $v_{\rm IR} \to 1$, one has $\Lambda_{\rm IR}^{-1} \to \infty$, and the theory enters a strong coupling regime. We comment more on this coupling in Sec. VII.

As an additional point, we note that Eq. (24) displays the expected dependence on the UV curvature $v_{\rm UV}$. In the limit $v_{\rm UV} \to \infty$, the UV coupling vanishes, $\Lambda_{\rm UV}^{-1} \to 0$, with the radion expelled from the UV brane. This corresponds to decoupling 4D gravity by sending the 4D Planck scale to infinity. On the other hand, the limit $v_{\rm UV} \to \infty$ has little effect on the IR coupling, which remains as $\Lambda_{\rm IR} \sim e^{-kL} M_*$, namely the characteristic IR scale. This makes intuitive sense, given the interpretation of the radion as a composite dilaton in the dual picture.

The preceding discussion is relevant for brane-localized matter. However, it retains utility for models with bulk fields. The radion couples conformally to matter. In models where SM fermions are treated as zero modes of bulk fermions, they typically remain massless until electroweak symmetry breaking is triggered by an IR-localized Higgs. Consequently, fermion masses arise locally on the IR brane. The mass-induced coupling between the radion and SM fermions therefore occurs locally on the IR brane, with a strength controlled by Eq. (24), giving $\Lambda_{\rm IR}^{-1} \sim e^{kL}\sqrt{k/M_*^3}$. This statement is not sensitive to the localization profile of the zero-mode fermion; information regarding the wave function overlap with the IR brane is encoded in the effective 4D fermion mass. The radion coupling to a fermion f goes like $(m_f/\Lambda_r) \times r\bar{f}f$, being smaller for an electron than a top quark simply because $m_e \ll m_t$, regardless of the origin of this hierarchy (e.g., tiny input Yukawa couplings or suppressed wave function overlap).[2] A similar discussion holds for zero modes of bulk vectors that acquire mass from an IR-localized scalar.

IV. RADION IN GW STABILIZED RS MODELS

In the preceding sections, the radion was massless as no mechanism was employed to stabilize the length of the extra dimension. Here, we briefly discuss the case where the radion acquires mass due to radius stabilization via the Goldberger-Wise mechanism [8]. This approach introduces a bulk scalar Φ, with localized boundary potentials, to generate a potential for the length of the interval. The result is a KK tower of physical scalars that contain an admixture of the KK modes of Φ and the graviscalar. The radion is identified as the lightest mode in this KK tower.

With the GW scalar included, the complete action is

$$\mathcal{S} = \int_{\mathcal{M}} d^5x\sqrt{-G}\left\{2M_*^3\mathcal{R} - \frac{1}{2}G^{MN}\partial_M\Phi\partial_N\Phi - V(\Phi)\right\} + 4M_*^3\oint_{\partial\mathcal{M}}\sqrt{-g}K + \sum_i\int d^4x\sqrt{-g_i}\left\{\frac{M_*^3 v_i}{k}R_i - M_*^3 k w_i - \frac{1}{4}t_i g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - \frac{1}{2}\lambda_i(\Phi)\right\}. \quad (27)$$

We include brane kinetic terms for both the gravity (v_i) and scalar (t_i) sectors. $V(\Phi)$ is the bulk potential for the scalar Φ (which subsumes the bulk cosmological constant), and λ_i are brane-localized potentials. The brane tensions kw_i are explicitly separated from the brane potentials, so $\lambda_i(\Phi) = 0$ for the background value of Φ. The general analysis of this system was presented in Ref. [20]. Here, we summarize a few key results, which we subsequently generalize. For a detailed discussion of the methodology, see Ref. [20].

Taking the usual warped metric ansatz,

$$ds^2 = a^2(y)\eta_{\mu\nu}dx^\mu dx^\nu + dy^2, \quad (28)$$

where the warp factor $a(y)$ is to be determined, and allowing the background value of Φ to depend only on y,[3] one can obtain the equations of motion and boundary conditions for the combined gravity-scalar theory. Two of the boundary conditions remain as in Eq. (17), while the other two have the form

$$[t_i\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\Phi) - \sqrt{-g}\lambda_{i,\Phi} - 2\theta_i\sqrt{-G}G^{5N}\partial_N\Phi]_{y=y_i} = 0, \quad (29)$$

when expressed in straight gauge.[4] The effective 4D theory contains the following terms for the KK scalars,

$$\mathcal{S} \supset \mathcal{N}\int d^4x\left(-\frac{1}{2}\eta^{\mu\nu}\partial_\mu\psi\partial_\nu\psi - \frac{1}{2}m^2\psi^2\right), \quad (30)$$

where m^2 is the mass of the KK mode and the normalization factor is

$$\mathcal{N} = 6M_*^3\int_0^L\left(a^2p_1^2 + 24M_*^3\frac{a'^2}{\phi'^2}p_1^2 + 24M_*^3\frac{aa'}{\phi'^2}p_1p_1' + 6M_*^3\frac{a^2}{\phi'^2}p_1'^2\right)dy + 3M_*^3\sum_i\frac{v_i a(y_i)^3p_1(y_i)^2}{ka(y_i)+\theta_i v_i a'(y_i)} + \frac{1}{8}\sum_i t_i\left[12M_*^3\theta_i\frac{2a'(y_i)p_1(y_i)+a(y_i)p_1'(y_i)}{\phi'(y_i)} + \frac{v_i a(y_i)^2\phi'(y_i)p_1(y_i)}{ka(y_i)+\theta_i v_i a'(y_i)}\right]^2. \quad (31)$$

In the above, the form of the background metric is not specified. The point is that the potentials $V(\Phi)$ and $\lambda_i(\Phi)$ cause Φ to obtain a nontrivial background value, which combines with the bulk cosmological constant and the brane tensions to source the metric. To calculate the radion mass, one must specify a particular model by specifying the form for the background scalar. To allow comparison with existing results in the literature, we follow Ref. [6] and consider a perturbed background of the form

$$a(y) = e^{-ky}\left(1 - \frac{l^2}{6}e^{-2uy}\right), \quad (32)$$

$$\phi(y) = 2\sqrt{2}M_*^{3/2}le^{-uy}, \quad (33)$$

valid in the region $y \in [0, L]$. This corresponds to a potential $V(\Phi) = (W_{,\Phi})^2/2 - W^2/6M_*^3$ with $W(\Phi) = 12M_*^3k - u\Phi^2/2$ and the following boundary potentials,

$$\lambda_i(\Phi) = -\theta_i W(\phi_i) - \theta_i W_{,\Phi}(\phi_i)(\Phi - \phi_i) + \gamma_i(\Phi - \phi_i)^2, \quad (34)$$

with constants u, ϕ_i, and γ_i. The length of the extra dimension is now dynamically fixed at $L = u^{-1}\log(\phi_0/\phi_L)$, with the weak backreaction limit defined by $\kappa_*\phi_i/\sqrt{2} \ll 1$. We work to $\mathcal{O}(l^2)$ in the small parameter

[2]Note that for off-shell fermions the radion-fermion coupling contains additional contributions such that the Higgs-like form of the radion coupling to fermions appears in physical amplitudes due to delicate cancellations; see Ref. [27].

[3]That is, we write $\Phi(x^\mu, y) = \phi(y) + P_4(x^\mu, y)$, where $\phi(y)$ is the background value for Φ. See Ref. [20] for more details.

[4]Additional useful forms of the boundary conditions appear in Ref. [20].

$l = \kappa_* \phi_0/\sqrt{2}$, though the expression for ϕ holds to all orders in l.

Writing the metric perturbation as $P_1(x^\mu, y) = p_1(y)\psi(x^m u)$, the solution for $p_1(y)$ is a perturbed form of the massless solution,

$$p_1(y) = \{1 + l^2 f(y)\} \times e^{2ky}. \tag{35}$$

The bulk equation for $f(y)$ is the same as the case without brane curvature terms [6],

$$f'' + 2(k+u)f' = \frac{4}{3}u(u-k)e^{-2uy} - \tilde{m}^2 e^{2ky}, \tag{36}$$

where $m^2 = l^2\tilde{m}^2$. Observe that the radion mass is on the order of the correction to the background—the backreaction must be included to generate a nonzero mass. The solution in the bulk is [6]

$$f'(y) = -\frac{2}{3}u\left(1 - \frac{u}{k}\right)e^{-2uy} - \tilde{m}^2 \frac{1}{4k+2u} e^{2ky} + Ae^{-2(k+u)y}, \tag{37}$$

where A is an integration constant. Working in the limit of stiff brane potentials, $\lambda_{i,\Phi\Phi} \to \infty$, the boundary conditions are [20]

$$\left[f' + \frac{2}{3}ue^{-2uy} + \frac{2}{3}\frac{u^2}{k}e^{-2uy}\frac{\theta_i v_i}{1 - \theta_i v_i}\right]_{y=y_i} = 0, \tag{38}$$

the enforcement of which allows one to determine the mass of the lightest spin-zero state [20]

$$m^2 = \frac{4l^2}{3}\frac{(2k+u)u^2}{k}\left(\frac{1}{1 - v_{\rm IR}} - \frac{e^{-2kL}}{1 + v_{\rm UV}}\right) \times (e^{2(k+u)L} - e^{-2kL})^{-1}. \tag{39}$$

This expression for the radion mass generalizes of the result in Ref. [6] for the case of nonzero brane curvature, $v_i \neq 0$. There are two points worth making. First, one observes that $v_{\rm IR} < 1$ is required for the theory to remain consistent. In particular, values of $0 < v_{\rm IR} < 1$ tend to increase the mass of the radion, relative to the standard RS result. This differs from the case of the KK gravitons, where the increase in $v_{\rm IR}$ corresponds to a reduction in the KK masses (as used recently in relation to the 750 GeV diphoton excess [3–5]). Second, while the mass is sensitive to the effect of the IR curvature, $v_{\rm IR}$, it is rather insensitive to the UV curvature $v_{\rm UV}$.

In cases where it is desirable to have a heavy radion, Eq. (39) might lead one to suppose that we could use the IR curvature to achieve this while avoiding large perturbations to the anti-de Sitter (AdS) background. However, one must be careful, as, although values of $v_{\rm IR}$ close to unity enhance the radion mass, they also approach a strongly coupled regime in the radion interactions (corresponding to the crossover region between having and avoiding a ghostlike radion, where the kinetic term vanishes).

To determine the coupling of the radion to matter in the stabilized extra dimension, one requires the normalization constant $\mathcal{N}$ in Eq. (31). Unsurprisingly, we find

$$\mathcal{N} = \frac{3M_*^3}{k}e^{2kL}\left(\frac{1}{1 - v_{\rm IR}} - \frac{e^{-2kL}}{1 + v_{\rm UV}}\right) + \mathcal{O}(l^2), \tag{40}$$

matching the nonstabilized result in Eq. (20) to leading order. With this expression, one can repeat the calculations of Sec. III to find the radion coupling to matter. To leading order, the results match those in Sec. III, and, in particular, the leading-order radion kinetic term is only well behaved for $v_{\rm IR} < 1$, signaling a ghostlike radion for $v_{\rm IR} > 1$.

It is unsurprising that the normalization factor $\mathcal{N}$ is dominated by the $\mathcal{O}(l^0)$ terms in the weak-backreaction limit. The $\mathcal{O}(l^2)$ corrections from the first two lines of Eq. 3 should clearly be subdominant for a weak backreaction. However, one may ask if the brane-localized kinetic terms for the GW scalar could be chosen sufficiently large to overcome the $\mathcal{O}(l^2)$ suppression, potentially modifying the sign of the radion kinetic term in the presence of large IR curvature. It is interesting to evaluate these t_i-dependent terms explicitly,

$$\mathcal{N} = \sum_i \frac{t_i l^2 M_*^3}{4}\left[\frac{3}{u}Ae^{-(k+u)y_i} - \frac{3(m^2/l^2)e^{(3k+u)y_i}}{u(4k+2u)} + \frac{2ue^{(k-u)y_i}}{k(1 - \theta_i v_i)}\right]^2 + \ldots, \tag{41}$$

where the dots denote everything bar the $\mathcal{O}(l^2)$ t_i-dependent terms. Here, m^2 is the radion mass in Eq. (39), and A is the constant in Eq. (37). Using the explicit expressions for m^2 and A to evaluate the $\mathcal{O}(l^2)$ t_i-dependent terms, one finds that the terms shown in Eq. (41) vanish identically. Thus, to $\mathcal{O}(l^2)$, the normalization factor $\mathcal{N}$ is independent of the brane kinetic terms t_i. One concludes that it does not appear possible to employ large values of t_i to avoid the ghostlike radion that occurs for $v_{\rm IR} \sim \mathcal{O}(10)$.

V. RADION-HIGGS KINETIC MIXING

In the above, we considered a GW stabilized RS model with brane curvature. The scalar spectrum consisted of a KK tower of massive scalars, the lightest of which is the radion. For model building purposes, one would subsequently add the SM fields to the warped space. In particular, one adds the SM Higgs boson, which should be localized at (or toward) the IR brane in order to solve the hierarchy problem. In general, the Higgs will mix with the KK scalars, the most important consequence of which is the mixing between the Higgs and the lightest mode, namely the radion.[5] In addition to the evident phenomenological implications, the radion-Higgs mixing has a further consequence. We observed previously that the radion mass in the GW stabilized setup is only well behaved for relatively moderate values of the IR curvature, $v_{\rm IR} < 1$. This observation is important with regard to efforts to suppress the lightest KK graviton mass by employing values of $v_{\rm IR} \sim \mathcal{O}(10)$, as such models could suffer from instabilities. It is also important for models seeking to generate a heavier radion, as the IR curvature term can be used to increase the radion mass. These conclusions, however, are drawn prior to the inclusion of the Higgs-radion mixing. In this section, we discuss the modifications to these observations due to Higgs-radion mixing. The results in this section generalize a number of results in Ref. [6] to include nonzero brane curvature.

We are interested in the case where the SM is added to the warped space. For present purposes, we assume an IR-localized SM Higgs,

$$S \supset -\int d^4x\sqrt{g_{\rm IR}}\{(D^\mu H)^\dagger(D_\mu H) + V(H)\} + \xi\int d^4x\sqrt{g_{\rm IR}}|H|^2 R_{\rm IR}, \tag{42}$$

where we include an IR-localized nonminimal coupling. There are two ways one can proceed to analyze the system of Sec. IV with the SM Higgs added. The most-general analysis involves deriving the full equations of motion and boundary conditions for the gravity $+\,\Phi + H$ system and deriving the new KK spectrum for the scalar sector (comprised of the radion, Φ, and H). Alternatively, one can treat the SM Higgs as a small perturbation on the previously derived background solution and derive the leading-order mixing effects between the Higgs and the KK scalars. Here, we make a simple observation which allows an intermediate approach.

Expanding the Higgs around its vacuum expectation value (VEV), one has

$$S \supset \frac{\xi}{2}\int d^4x\sqrt{g_{\rm IR}}e^{2kL}(h^2 + 2vh + v^2)R_{\rm IR}, \tag{43}$$

where the Higgs is rescaled to the canonical kinetic form, $H \to e^{kL}H$, with $v \ll M_{\rm Pl}$ being the warped-down SM VEV, $v \simeq 246$ GeV. Observe that the nonminimal IR coupling gives two different physical effects. The $\mathcal{O}(v)$ term induces Higgs-radion kinetic mixing, requiring one to diagonalize the scalar kinetic terms, as discussed below. On the other hand, the $\mathcal{O}(v^2)$ term does not induce kinetic mixing but instead gives a new contribution to the *total* IR brane curvature. To treat this term, we can rewrite the IR curvature as

$$S \supset \frac{M_*^3}{k}\int d^4x\sqrt{-g_{\rm IR}}\{v_{\rm IR} + \xi v_H\}R_{\rm IR}, \tag{44}$$

with the dimensionless coefficient $v_H = (kv^2)/(2e^{-2kL}M_*^3)$ parametrizing the Higgs contribution to the effective IR-localized curvature. This makes it clear that the $\mathcal{O}(v^2)$ term in the nonminimal coupling can be incorporated in our earlier analysis by the replacement $v_{\rm IR} \to v_{\rm IR} + \xi v_H$ in the action (27). The results obtained via this approach reduce to those obtained by the alternative method of treating this term as a perturbation (see the Appendix). Let us also emphasize that the KK graviton masses are sensitive to the total IR curvature and are thus sensitive to the value of v_H for $\xi \neq 0$.

With the above observation, one easily includes the effects of the Higgs-induced IR curvature into the full equations of motion and boundary conditions, following the analysis of Ref. [20] (as outlined in the preceding section). To quote a few key results, the expression for the Planck mass becomes

$$M_{\rm Pl}^2 = \frac{M_*^3}{2k}\{1 + v_{\rm UV} - (1 - v_{\rm IR} - \xi v_H)e^{-2kL}\}, \tag{45}$$

along with a related change to the massless graviton profile. One of the IR boundary conditions changes to

$$P_3'(y_{\rm IR}) = \frac{-(v_{\rm IR} + \xi v_H)}{a(y_{\rm IR})[ka(y_{\rm IR}) + (v_{\rm IR} + \xi v_H)a'(y_{\rm IR})]}P_1(y_{\rm IR}), \tag{46}$$

and the leading-order expression for the radion normalization factor becomes

$$\mathcal{N} = \frac{3M_*^3}{k}e^{2kL}\left(\frac{1}{1 - (v_{\rm IR} + \xi v_H)} - \frac{e^{-2kL}}{1 + v_{\rm UV}}\right) + \mathcal{O}(l^2). \tag{47}$$

The stiff brane-potential limit expression for the boundary conditions, Eq. (38), also changes, and the new $\mathcal{O}(l^2)$ expression for the radion mass is

$$m^2 = \frac{4l^2}{3}\frac{(2k+u)u^2}{k}\left(\frac{1}{1 - v_{\rm IR} - \xi v_H} - \frac{e^{-2kL}}{1 + v_{\rm UV}}\right) \times (e^{2(k+u)L} - e^{-2kL})^{-1}. \tag{48}$$

With the $\mathcal{O}(v^2)$ term incorporated into the full equations of motion and boundary conditions, we now treat the $\mathcal{O}(v)$ term as a perturbation on the new background solution.[6] The calculation makes use of the following result for the linear terms in the radion fluctuation,

[5] Note that the KK scalars can still influence radion production and decay mechanisms, with the size of the effect dependent on the size of the mass splitting between the radion mass and the KK scalars.

[6] Note that we are not performing an expansion in the parameter v here; references to $O(v)$ and $\mathcal{O}(v^2)$ terms in the nonminimal coupling are made purely for labeling purposes.

$$\sqrt{g_i}R_i = \frac{3ka^3(y_i)p_1(y_i)\Box\psi}{ka(y_i)+\theta_i v_i a'(y_i)} + \ldots, \tag{49}$$

where one should use the total brane curvature for the IR brane, $v_i \to v_{\rm IR} + \xi v_H$. Using this result to extract the kinetic mixing gives

$$S \supset \frac{3\xi}{(1 - v_{\rm IR} - \xi v_H)} \int d^4x e^{2kL} v h \Box\psi. \tag{50}$$

We perform a partial rescaling of the radion kinetic term,

$$\psi = r \times \sqrt{\frac{(1+v_{\rm UV})}{6M_{\rm Pl}^2 e^{-2kL}}}, \tag{51}$$

such that the kinetic mixing term is

$$S \supset 3\xi \frac{v}{\Lambda_{\rm RS,IR}} \frac{\sqrt{1+v_{\rm UV}}}{(1 - v_{\rm IR} - \xi v_H)} \int d^4x h \Box r \equiv \frac{A}{B} \int d^4x h \Box r, \tag{52}$$

where we define $B \equiv (1 - v_{\rm IR} - \xi v_H)$. This partial scaling allows our results to be readily compared with Ref. [6].

The mixed kinetic Lagrangian contains the terms

$$\mathcal{L} \supset \frac{1}{2}(r,h)\begin{pmatrix} B^{-1} & 0 \\ 2AB^{-1} & 1 \end{pmatrix}\begin{pmatrix} \Box r \\ \Box h \end{pmatrix}, \tag{53}$$

which are diagonalized by the following $GL(2)$ transformation,

$$\begin{pmatrix} r \\ h \end{pmatrix} = \begin{pmatrix} \mathcal{Z}^{-1} & 0 \\ -A(B\mathcal{Z})^{-1} & 1 \end{pmatrix}\begin{pmatrix} r' \\ h' \end{pmatrix}, \tag{54}$$

where

$$\mathcal{Z}^2 \equiv B^{-1} - (A/B)^2 = \frac{1 - v_{\rm IR} - \xi v_H - 9\xi^2\gamma^2(1+v_{\rm UV})}{(1 - v_{\rm IR} - \xi v_H)^2}. \tag{55}$$

Here, we adopt the notation of Ref. [6]:

$$\gamma = \frac{v}{\Lambda_{\rm RS,IR}} = \frac{v}{\sqrt{6}e^{-kL}M_{\rm Pl}}. \tag{56}$$

The quantity $\mathcal{Z}^2$ in Eq. (55) corresponds to the coefficient of the radion kinetic term after the kinetic mixing is diagonalized. It should be strictly positive to ensure the kinetic term is positive definite and avoid a ghostlike radion. Equation (55) generalizes the result in Ref. [6] for the case with localized brane curvature. We can consider various limits of this expression. The limit $\gamma^2 \ll 1$ gives

$$\mathcal{Z}^2 = (1 - v_{\rm IR})^{-2}\left\{1 - v_{\rm IR} + \frac{3\xi\gamma^2}{2}[1 - 6\xi(1+v_{\rm UV})]\right\} + \ldots. \tag{57}$$

Taking the further limit of vanishing brane curvature, $v_{\rm IR,UV} \to 0$, gives

$$\mathcal{Z}^2 = 1 + \frac{3}{2}\xi\gamma^2(1-6\xi), \tag{58}$$

which matches the expression in Ref. [6].[7]

Returning the curvature terms, $v_{\rm IR,UV} \neq 0$, and demanding that the radion is not ghostlike, Eq. (57) shows that one should restrict ξ to the range

$$\xi_- \le \xi \le \xi_+, \tag{59}$$

where

$$\xi_\pm = \frac{1}{12(1+v_{\rm UV})}\left\{1 \pm \left[1 + \frac{16(1+v_{\rm UV})(1-v_{\rm IR})}{\gamma^2}\right]^{1/2}\right\}. \tag{60}$$

This expression also generalizes Ref. [6]. Note that it appears difficult to select values of ξ consistent with $v_{\rm IR} \sim \mathcal{O}(10)$. The only hope arises for values of $0 < (1+v_{\rm UV}) \ll 1$, specifically, with $(1+v_{\rm UV}) = \epsilon \times \gamma^2/[16(v_{\rm IR}-1)] < 1$ for small ϵ.[8] However, this solution is misleading; it gives $\xi_\pm \propto (v_{\rm IR}-1)\gamma^{-2}$, such that the original expansion in $\gamma \ll 1$ cannot be trusted for ξ in the range $\xi_- < \xi < \xi_+$, given that γ is multiplied by a factor of ξ or ξ^2 in Eq. (57). This failure to find values of ξ that avoid a ghostlike radion for $v_{\rm IR} \sim \mathcal{O}(10)$ is best understood via Eq. (55), which gives the more-general constraint for avoiding a ghostlike radion in the presence of brane curvature and a nonminimal coupling to an IR Higgs, namely

$$v_{\rm IR} + \xi v_H + 9\xi^2\gamma^2(1+v_{\rm UV}) < 1. \tag{61}$$

One notes immediately that no solution with $\mathcal{O}(10)$ IR curvature appears possible, given that the last term on the left-hand side is positive definite. Naively, one may expect that a cancellation could be arranged between the terms $v_{\rm IR}$ and ξv_H, such that $v_{\rm IR} \sim \mathcal{O}(10)$ is allowed. However, the brane curvature relevant for modifying the KK graviton mass is the total IR curvature, so a suppressed KK graviton mass requires $v_{\rm IR} + \xi v_H \sim \mathcal{O}(10)$ in the presence of the nonminimal coupling. Thus, no such cancellation is available. We conclude that it appears difficult to reconcile the

[7] After correcting for a notational difference.
[8] Recall that $(1+v_{\rm UV})$ must be strictly positive, to avoid a ghostlike massless 4D graviton.

constraint in Eq. (61) with $\mathcal{O}(10)$ values of the total IR brane curvature. Small values of the IR curvature, consistent with the above constraints, remain viable.

In the basis with Higgs-radion kinetic mixing, the scalar mass terms are diagonal: $\mathcal{L} \supset -m^2 r^2/2 - m_h^2 h^2/2$. However, diagonalizing the kinetic terms induces mass mixing. Defining the physical mass eigenstates as

$$\begin{pmatrix} r_m \\ h_m \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} r' \\ h' \end{pmatrix}, \quad (62)$$

the mass eigenvalues are

$$m_\pm^2 = \frac{1}{[2(B\mathcal{Z})^2]^{-1}} \{m^2 B^2 + m_h^2 B\mathcal{Z}(A^2 + B\mathcal{Z}) \pm \Delta_m\},$$

where

$$\Delta_m = [(m^2 B^2 + m_h^2 B\mathcal{Z}(A^2 - B\mathcal{Z}))^2 - 4m_h^4 A^2 (B\mathcal{Z})^2]^{1/2}, \quad (63)$$

and, as previously, m^2 is the radion mass prior to mixing, $B = (1 - v_{\rm IR} - \xi v_H)$, $A = 3\xi\gamma\sqrt{1 + v_{\rm UV}}$, and $\mathcal{Z}$ is defined by Eq. (55). The identification of the physical Higgs and radion with $m_\pm$ depends on the mass ordering; the physical radion has mass m_+ (m_-) if it is heavier (lighter) than the physical Higgs. The mixing angle is

$$\tan 2\theta = \frac{2AB\mathcal{Z}m_h^2}{m^2 B^2 + m_h^2 B\mathcal{Z}(A^2 - B\mathcal{Z})}. \quad (64)$$

Writing the above results for the mixing angle and eigenmasses in terms of the explicit expressions for A, B, and $\mathcal{Z}$ produces cumbersome expressions that are not particularly enlightening. One can consider various limits of the results, however. As an example, for $\gamma \ll 1$, the mixing angle reduces to

$$\tan 2\theta = \frac{6m_h^2 \xi\gamma\sqrt{1 + v_{\rm UV}}}{m^2(1 - v_{\rm IR}) - m_h^2}, \quad (65)$$

which is the generalization of the result in Ref. [6] for $v_{\rm IR,UV} \neq 0$.

Our results show that the inclusion of Higgs-radion mixing via an IR-localized nonminimal coupling does not provide a means for avoiding a ghostlike radion in a GW-stabilized RS model with large IR curvature. Of course, the radion mass is dependent on the backreaction of the stabilizing dynamics, and one may wonder if the ghostlike radion can be avoided in the case of a strong backreaction, perhaps with different stabilizing dynamics. We have nothing insightful to say regarding this possibility, though we note that the strong backreaction would also affect the mass of the KK gravitons. Other possibilities include taking the Higgs "off the wall" [21] and into the bulk or considering warped models with a different mechanism of stabilization. In Ref. [28], the effects of bulk SM fields on the radion couplings were studied; however, it would be interesting to study this scenario with additional brane-localized curvature terms. Leaving these points aside, we now turn our attention to some alternative IR terms.

VI. ADDITIONAL IR TERMS FOR THE RADION

In a certain sense, the use of branes in the RS model means the brane-localized action needs only satisfy the 4D diffeomorphism symmetry. This allows a number of additional terms that, in general, should be included in the most-general Lagrangian. This fact was already invoked to motivate the study of brane curvature terms and the nonminimal coupling to the IR Higgs. Motivated by the work of Ref. [29], in this section, we comment on a class of brane terms involving the radion.

Reference [29] considered explicit brane-localized mass terms for the spin-2 metric fluctuations $h_{\mu\nu}$.[9] Such terms explicitly break the 5D general coordinate invariance and essentially force one to choose a gauge. Given recent interest in RS models with large IR curvature, and the inherent problem of the ghostlike radion, here we comment on localized terms for the scalar metric perturbation h_{55}, which preserve the local 4D symmetry but break the 5D general coordinate invariance. Such terms could arise in the presence of a bulk gauge field which obtains a VEV along its fifth direction, as studied in Ref. [31] in the context of Lorentz-invariance violating warped models. Reference [31] considered terms of the form $u^A u^B \mathcal{R}_{AB}$, denoting the VEV of the vector field as $u^A = (0,0,0,0,1)$, and showed that including such couplings in the bulk leads to a mass for the graviton. Thus, such couplings should be prohibited in the bulk. However, allowing the couplings locally on the branes preserves the massless graviton and generates brane-localized interactions of the form $\lambda\mathcal{R}_{55}$, which break 5D diffeomorphism invariance while retaining the 4D symmetry. With the 5D symmetry broken on the branes, one expects the brane Lagrangians to contain the most-general set of operators consistent with local 4D diffeomorphism invariance. Some of these terms can be of interest with regard to the ghostlike radion, as we now discuss.

Reference [29] employed a gauge with $h_{\mu 5} = h_{55} = 0$. After fixing the gauge, a boundary mass was added for the metric perturbation, with the corresponding $\mathcal{O}(h^2)$ brane Lagrangian having the form $\sim\{h_{\mu\nu}h^{\mu\nu} - (h^\mu_\mu)^2\}$. In this work, we employ a straight gauge with $h_{\mu 5} = 0$, while $h_{55} \neq 0$. By analogy with Ref. [29], we consider $\mathcal{O}(h^2)$ brane-localized terms for the radion that break the 5D Lorentz symmetry locally on the brane. In particular, the

[9] Note that Ref. [30] considered an explicit bulk mass for the graviton in RS models.

most-general Lagrangian, consistent with localized Lorentz-invariance violation on the IR brane, contains the following additional terms IR terms, which are consistent with 4D Lorentz invariance:

$$\delta\mathcal{S} = -3\xi_\partial \frac{M_*^3}{k}\int d^4x\sqrt{-g_{\rm IR}}g^{\mu\nu}\partial_\mu h_{55}\partial_\nu h_{55} - \xi_m k\int d^4x\sqrt{-g_{\rm IR}}h_{55}^2\Phi^2. \tag{66}$$

Treating these terms as perturbations on the background, the first term is an IR-localized kinetic term for h_{55}, which gives a new contribution to the kinetic Lagrangian,[10]

$$\delta\mathcal{S} \supset -\frac{1}{2}\frac{8\xi_\partial}{(1-v_{\rm IR})^2}\left(\frac{3M_*^3}{k}e^{2kL}\right)\int d^4x(\eta^{\mu\nu}\partial_\mu\psi\partial_\nu\psi), \tag{67}$$

while the second term gives a new contribution to the radion mass,

$$\delta\mathcal{S} \supset -\frac{1}{2}\frac{4e^{4kL}\phi^2}{(1-v_{\rm IR})^2}\int d^4x\psi^2. \tag{68}$$

Let us focus on the kinetic term first, taking the limit $\xi_m \ll 1$. Combining the new kinetic term with the preexisting kinetic terms gives

$$\mathcal{S}+\delta\mathcal{S} \supset \int d^4x\left(-\mathcal{N}'\frac{1}{2}\eta^{\mu\nu}\partial_\mu\psi\partial_\nu\psi - \mathcal{N}\frac{1}{2}m^2\psi^2\right), \tag{69}$$

where the normalization factor is now

$$\begin{aligned}\mathcal{N}' &= \frac{3M_*^3}{k}e^{2kL}\left(\frac{1}{1-v_{\rm IR}}+\frac{8\xi_\partial}{(1-v_{\rm IR})^2}-\frac{e^{-2kL}}{1+v_{\rm UV}}\right)+\mathcal{O}(l^2)\\ &= \mathcal{N}+\frac{3M_*^3}{k}e^{2kL}\frac{8\xi_\partial}{(1-v_{\rm IR})^2}+\mathcal{O}(l^2).\end{aligned} \tag{70}$$

Interestingly, the new contribution to the kinetic term can apparently cure the problem of a ghostlike radion for large IR curvature, provided one has

$$(v_{\rm IR}-1) < 8\xi_\partial. \tag{71}$$

Thus, for values of $v_{\rm IR}\approx 15$, which can achieve an $\mathcal{O}(10^2)$ GeV KK graviton, one obtains the constraint $\xi_\partial > 14/8 = 1.75$. For our parametrization of the IR kinetic term in Eq. (66), it appears possible to avoid a ghostlike radion with $\xi_\partial = \mathcal{O}(1)$. Note that the radion mass is now $m_r^2 = (\mathcal{N}/\mathcal{N}')m^2$, which is nontachyonic for the parameter space that avoids a ghost radion; the product $\mathcal{N}\times m^2$ is positive for $v_{\rm IR} > 1$ (both $\mathcal{N}$ and m^2 are negative for $v_{\rm IR} > 1$). Consequently, provided ξ_∂ satisfies Eq. (71), one has $\mathcal{N}' > 0$ to ensure the radion kinetic term is positive definite and $m_r^2 > 0$ is automatically positive. Thus, the IR kinetic term for the radion in Eq. (66), which is consistent with the 4D symmetries of the theory, may help avoid the ghostlike radion that occurs for large values of $v_{\rm IR}$.

With this observation, we can reconsider the radion coupling to IR matter to include the effects of the IR-localized kinetic term. We find that the IR coupling is modified to take the form

$$\Lambda_{\rm IR}^{-1} \simeq \Lambda_{\rm RS,IR}^{-1}\times\sqrt{\frac{1+v_{\rm UV}}{v_{\rm IR}-1}}\left[\frac{8\xi_\partial}{(v_{\rm IR}-1)}-1\right]^{-1/2}, \tag{72}$$

where we write the result for the case of $v_{\rm IR} > 1$, assuming ξ_∂ is chosen to ensure positivity of the radion kinetic term. The key point here is that avoiding the ghostlike radion has produced a brane coupling that is also well behaved for $v_{\rm IR} > 1$.

Turning now to the IR-localized mass term, in the limit where the new kinetic piece is negligible, $\xi_\partial \ll 1$, the quadratic action for the radion is

$$\mathcal{S} \supset \mathcal{N}\int d^4x\left(-\frac{1}{2}\eta^{\mu\nu}\partial_\mu\psi\partial_\nu\psi - \frac{1}{2}(m^2+\delta m^2)\psi^2\right), \tag{73}$$

where $\mathcal{N}$ is the prior normalization factor and the new mass correction from the localized IR action is

$$\delta m^2 \simeq \frac{64\xi_m}{3(1-v_{\rm IR})}l^2k^2e^{-2(k+u)L}. \tag{74}$$

The mass m^2 was found earlier in Eq. (39). Observe that the mass correction has the same parametric dependence on the warp factor, the backreaction, and the IR curvature as m^2, namely $\delta m^2 \propto (1-v_{\rm IR})^{-1}l^2e^{-2(k+u)L}$.

At the end of Sec. IV, we saw that the IR brane curvature term could not be used to significantly enhance the radion mass without making the radion interactions strongly coupled. In this regard, it is interesting to note the effects of the brane mass term for h_{55}. Working in the limit of small backreaction, $u \ll k$, and comparing Eq. (74) to Eq. (39), we see that $\delta m^2/m^2 \sim 8k^2\xi_m/u^2$, seemingly allowing one to increase the mass. Of course, if the boundary mass becomes too large, one should incorporate it into the full BCs.

We emphasize that our comments in this section, regarding additional IR terms for h_{55}, are motivated by scenarios with brane-localized 5D Lorentz-invariance violation. The operators we studied form part of the most-general set of brane terms that break 5D diffeomorphism

[10]Here, $v_{\rm IR}$ can include the contribution from the IR Higgs, if desired.

invariance, while retaining the 4D symmetry, and are expected to appear in models with localized Lorentz-invariance violation. Our main point is to mention that such operators may offer hope of avoiding a ghostlike radion. It would be interesting to undertake more-detailed studies of warped models with brane-localized Lorentz-invariance violation to explore these matters further.

VII. COMMENTS ON AdS/CFT

According to the AdS/CFT correspondence [32], RS models are thought to be dual to strongly coupled 4D theories that are (approximately) conformal for energies $M_* > E > e^{-kL}M_*$ [33]. Conformal symmetry is broken explicitly in the UV by a cutoff (dual to the UV brane) and spontaneously in the IR (dual to the IR brane). UV- (IR-) localized fields in the 5D picture are dual to fundamental (composite) fields in the 4D theory. More precisely, the UV value of a given bulk field in the 5D picture is dual to a fundamental field that is external to the strongly coupled 4D sector (see, e.g., Ref. [34]). Here, we make a few basic comments regarding RS models with brane curvature.[11]

Recall that the effective 4D Planck mass is

$$M_{\rm Pl}^2 = \frac{M_*^3 v_{\rm UV}}{2k} + \frac{M_*^3}{2k}\{1-(1-v_{\rm IR})e^{-2kL}\}, \qquad (75)$$

including contributions from both the bulk and brane curvatures. The different pieces have distinct interpretations in the dual 4D picture. The UV brane contribution, $M_{\rm UV}^2 = v_{\rm UV}M_*^3/k$, results from a UV-localized curvature term. As such, it corresponds to a kinetic term for the fundamental spin-2 particle associated with the UV restriction of the bulk 5D graviton [36]. The true massless graviton does not correspond exactly to this fundamental spin-2 field but instead contains a small admixture of the massive spin-2 composite states. This admixture is tiny, however, as is evident by the high degree of UV localization for the massless zero-mode in the RS picture—that is, the UV value of the bulk graviton field is overwhelmingly dominated by the value of the zero mode (i.e., massless graviton).

The origin of this "fundamental" contribution to the Planck mass is separate from the dual conformal field theory (CFT) dynamics. For $v_{\rm IR} \to 0$, however, the remaining pieces in Eq. (75) encode a dynamically generated contribution to the Planck scale, induced by CFT loops; i.e., in the limit $M_{\rm UV}^2 \to 0$, the Planck mass (equivalently, massless graviton kinetic term) is fully induced by CFT loops. Taking the further limit $L \to \infty$, the RS expression for the Planck scale is $M_{\rm Pl}^2 \sim M_*^3/k$, which should correspond to the induced Planck mass from a CFT with UV cutoff k. The latter has the form $M_{\rm Pl}^2 \sim ck^2$, with c being uniquely determined by the corresponding central charge of the CFT. The holographic calculation of c via 5D supergravity gives $(M_*/k)^3$ [37], so $M_{\rm Pl}^2 \sim M_*^3/k$ is in agreement with the RS result.[12] For finite L, the dual CFT has a further source of conformal symmetry breaking in the IR, at the scale $M_{\rm IR} = e^{-kL}k$. Now, the CFT-induced Planck mass is modified due to CFT symmetry breaking scale in the IR, giving $M_{\rm Pl}^2 \sim c(k^2 - M_{\rm IR}^2)$, in agreement with the limit $v_{\rm UV,IR} \to 0$ of Eq. (3).

Turning on the IR term, $v_{\rm IR} \neq 0$, the additional term in Eq. (3) encodes a change to the CFT-induced Planck mass due to some modification of the IR dynamics. While it is difficult to make precise statements about the strongly coupled sector in the dual 4D theory, it seems clear that the IR-localized brane curvature is dual to some modification of the kinetic terms for the spin-2 composite states. Given that the massless graviton is largely comprised of the fundamental spin-2 field, one may not expect that modifying the composite spin-2 kinetic terms would affect the kinetic term for the massless graviton. However, the massless graviton contains a small admixture to the composite spin-2 states, and a modification to the kinetic terms for the spin-2 composites should induce a highly suppressed modification of the kinetic term for the massless graviton—i.e., it should generate a mixing-suppressed contribution to the Planck mass. This naive expectation is borne out by Eq. (75), where the suppressing factor e^{-2kL} encodes the tiny mixing between the fundamental graviton and the spin-2 composites. Indeed, explicit calculations, in the so-called holographic basis, show that the mixing between the fundamental spin-2 state and the lightest composite spin-2 state is $\sin^2\theta_g \sim e^{-2kL}$ [34], in agreement with the above.[13]

Based on an inspection of the 4D Planck mass in Eq. (75), one may naively interpret the effect of the IR term as corresponding to a change in the effective IR scale of the broken CFT. It is instructive to consider this point. The standard expression for the Planck mass in RS models, without brane curvature terms, can be written as

$$M_{\rm Pl}^2 = \left(\frac{M_*}{2k}\right)^3 \times (k^2 - M_{\rm IR}^2). \qquad (76)$$

If one shifts the IR brane to a new location, $L \to L + \delta L$, the IR scale shifts accordingly to $M'_{\rm IR} = e^{-k(L+\delta L)}k$, modifying the expression for the Planck scale,

[11] To the extent that the following discussion contains useful content, it is, in part, attributable to Ref. [35]. Any errors, however, are the responsibility of the authors.

[12] In the language of a dual large-$\mathcal{N}$ gauge theory, the induced Planck scale is $\sim k^2\mathcal{N}^2$, where $\mathcal{N}^2 \sim (M_*/k)^3$ relates to the number of colors in the dual CFT.

[13] Note that a massless mode from a bulk vector with IR kinetic term does not have this severe suppression of the IR-term dependence, as the fundamental/composite mixing is much larger in the spin-1 sector.

$$M_{\rm Pl}^2 = \left(\frac{M_*}{2k}\right)^3 \times (k^2 - (M'_{\rm IR})^2). \qquad (77)$$

Comparing this expression to Eq. (75), it appears that the same effect can be obtained by including an IR brane curvature term with coefficient $v_{\rm IR}$, while keeping the brane fixed at $y = L$. Specifically, for $v_{\rm IR} < 1$, we define

$$M_{\rm IR}^{\rm eff} = \sqrt{1 - v_{\rm IR}} e^{-kL} k = \sqrt{1 - v_{\rm IR}} M_{\rm IR}, \qquad (78)$$

such that the 4D Planck mass Eq. (3) is written as

$$M_{\rm Pl}^2 = \left(\frac{M_*}{2k}\right)^3 \times (k^2 - (M_{\rm IR}^{\rm eff})^2), \qquad (79)$$

where we take $v_{\rm UV} = 0$ to focus on the effect of the IR term. Comparing with the standard RS result (76), it appears that the effect of the IR curvature term is to modify the effective IR scale. In particular, values in the range $0 < v_{\rm IR} < 1$ tend to decrease the effective IR scale in a way that appears similar to the increase in length $L \to L + \delta L$ with $\delta L = (\frac{-1}{2k}) \times \log(1 - v_{\rm IR})$.

If this were correct, one could immediately deduce some additional consequences of the IR curvature. In RS models, the radion couples conformally to IR-localized fields as $(r/\Lambda_{\rm RS})T$, where T is the trace of the stress-energy tensor and $\Lambda_{\rm RS}$ is a dimensionful coupling on the order of the IR scale, $\Lambda_{\rm RS} \sim M_{\rm IR}$. With this information, one can guess the effect of the IR curvature term on the coupling of the radion to IR-localized fields:

$$\Lambda_{\rm RS} \sim M_{\rm IR} \to \Lambda \sim M_{\rm IR}^{\rm eff} = \sqrt{1 - v_{\rm IR}} M_{\rm IR}. \qquad (80)$$

Thus, values of $0 < v_{\rm IR} < 1$, which tend to decrease $M_{\rm IR}^{\rm eff}$, would tend to *increase* the coupling of the radion to IR fields, as this goes like $\Lambda^{-1} \sim (M_{\rm IR}^{\rm eff})^{-1}$. Conversely, values of $v_{\rm IR} < 0$ tend to decrease the strength with which the radion couples. In Sec. III, we explicitly calculated the radion coupling to IR matter in the presence of IR curvature and obtained a result in agreement with this naive guess.[14] It is interesting that the above interpretation of the IR term allows one to foreshadow our conclusions so easily. Similarly, the interpretation of the IR curvature term as modifying the effective IR scale in the gravity sector suggests that the KK graviton masses should decrease for $0 < v_{\rm IR} < 1$, consistent with explicit calculations [2].

While the above line of reasoning may have utility, one should refrain from taking the interpretation of a modification to the IR confinement scale too seriously. This is evidenced by the failure of the IR curvature term to modify the KK masses for other bulk fields; i.e., the KK decomposition of a bulk vector gives a spectrum that is insensitive to the presence of an IR curvature term, implying that the spin-1 composite spectrum is not sensitive to this modification. Thus, the interpretation in terms of a change to the IR scale appears to be a mere coincidence—the IR curvature represents a change to the kinetic terms for the composite states, which affects the massless graviton kinetic term via mixing, in a way that *mimics* the effect of a modification to the IR/confinement scale.

Regarding the radion, it is interesting to note that the IR curvature affects the graviton and radion kinetic terms in different ways. The radion is highly IR localized and is dual to a dilaton that is overwhelmingly composite. This situation is opposite to that of the graviton. Thus, the IR curvature, which encodes a modification to the kinetic terms for the spin-2 and dilaton sectors, should induce an unsuppressed change to the dilaton kinetic term. This behavior is seen already in Eq. (19). The radion kinetic term is highly sensitive to the IR curvature, whereas it is relatively insensitive to the UV curvature, opposite to the massless graviton. These different sensitivities of the radion and graviton to the IR and UV curvature are consistent with the dual picture.

VIII. CONCLUSION

The most general Lagrangian for RS models includes brane-localized curvature terms on both the UV and IR branes. These terms can modify the spectrum of KK gravitons, as studied recently in relation to models with an $\mathcal{O}(10^2)$ GeV KK graviton [3–5]. The brane curvature also has consequences for the properties of the radion. In this work, we investigated some of these properties for a general RS model, both with and without GW stabilization. We showed that the brane curvature can modify the radion mass and couplings. Furthermore, demanding a nonghostlike radion gives a restriction on the allowed parameter space for the curvature terms. We investigated the effects of a nonminimal IR coupling with the SM Higgs to determine the parameter space consistent with a nonghostlike radion. Our results generalize a number of expressions in Ref. [6] to the case with nonzero brane curvature. Unfortunately, the resulting modifications did not remove the ghost-radion encountered for $\mathcal{O}(10)$ values of the IR curvature. Motivated by models with brane-localized Lorentz-invariance violation, we also considered additional IR terms for the radion, showing that such terms offered some hope for avoiding the ghost radion. Our results suggest it could be interesting to further study such models.

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[14]Note that the effective coupling for the nonminimal term $hvR_{\rm IR}$ also has the expected form based on the above reasoning, once the radion kinetic term is brought to canonical form.

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APPENDIX: WEAK BOUNDARY CURVATURE LIMIT

In our analysis, we included the Higgs-induced IR curvature in the full equations of motion and boundary conditions, arriving at an action, to quadratic order in the radion, with the form

$$\mathcal{S} \supset \mathcal{N} \int d^4x \left(-\frac{1}{2}\eta^{\mu\nu}\partial_\mu\psi\partial_\nu\psi - \frac{1}{2}m^2\psi^2 \right), \tag{A1}$$

with normalization factor

$$\mathcal{N} = \frac{3M_*^3}{k} e^{2kL} \left(\frac{1}{1-(v_{\rm IR}+\xi v_H)} - \frac{e^{-2kL}}{1+v_{\rm UV}} \right) + \mathcal{O}(l^2). \tag{A2}$$

In the limit $v_H \ll 1$, an expansion to $\mathcal{O}(v_H)$ gives

$$\mathcal{N} = \frac{3M_*^3}{k} e^{2kL} \left(\frac{1}{1-v_{\rm IR}} - \frac{e^{-2kL}}{1+v_{\rm UV}} \right) + \frac{3M_*^3}{k} e^{2kL} \frac{\xi v_H}{(1-v_{\rm IR})} + \dots \tag{A3}$$

The $\mathcal{O}(v_H)$ piece of the radion kinetic term agrees with that obtained by treating the nonminimal coupling term $\sim\xi v^2 R_{\rm IR}$ as a perturbation on the background obtained without the IR Higgs.

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