PHYSICAL REVIEW D **94,** 064039 (2016)

Gravitational waves from compact sources in a de Sitter background

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(Received 28 October 2015; published 14 September 2016)

The concordance model of cosmology favors a universe with a tiny positive cosmological constant. A tiniest positive constant curvature profoundly alters the asymptotic structure, forcing a relook at a theory of gravitational radiation. Even for compact astrophysical sources, the intuition from Minkowski background is challenged at every step. Nevertheless, at least for candidate sources such as compact binaries, it is possible to quantify the influence of the cosmological constant, as small corrections to the leading order Minkowski background results. Employing suitably chosen Fermi normal coordinates in the static patch of the de Sitter background, we compute the field due to a compact source to first order in Λ. For contrast, we also present the field in the Poincaré patch where the leading correction is of order $\sqrt{\Lambda}$. We introduce a gauge invariant quantity, *deviation scalar*, containing polarization information and compute it in both charts for a comparison.

DOI: 10.1103/PhysRevD.94.064039

I. INTRODUCTION

Asymptotically flat space-times as a model for space-times with compactly supported sources are fashioned after the choice of the Minkowski space-time as the background space-time. This choice constitutes a special case of maximally symmetric background space-times. We could also have a cosmological constant, Λ, in the Einstein equation and take the *de Sitter* ($\Lambda > 0$) or the anti-de Sitter ($\Lambda < 0$) solutions as background. The conformal completion *à la* Penrose immediately reveals the *qualitatively* different structure of the infinity. In particular, *irrespective of the nonzero value* of the cosmological constant, the null infinity—the set consisting of the beginnings and the ends of all inextendible null curves-is *spacelike* for de Sitter and *timelike* for anti-de Sitter [1,2]. This has a drastic effect on the kinds of fluxes that can be used as measures of radiation at infinity in these backgrounds. The asymptotic symmetry groups are different, too [3,4]. It is important to note that *these qualitative* differences are independent of the numerical value (in any suitably chosen units) of the cosmological constant. The quantitative estimates of the deviations from Minkowski background *are* sensitive to the numerical value. This raises the question that, if we choose a background space-time with a nonzero cosmological constant, how does the linearized theory work out? In particular, what are the modifications to the "quadrupole formula(s)?" Can the modifications be obtained as "small" corrections in powers of the cosmological constant?

At this stage, it is worth noting the different facets of the gravitational fields far away from dynamical sources such as astrophysical bodies. The most basic question is: what is the field due to a source at large separations? The very characterization of compact sources presumes a source-free region where vacuum equations, possibly including the cosmological term, hold. Thus, at large separations, we have a natural split of the field into a background and a small deviation caused by the source. The simplest approach is then to linearize the Einstein equation about a background and study its solutions, keeping in mind the inherent nonlinear nature of the theory and hoping for reliable estimates. The linearized equation is a wave equation with a finite propagation speed. Among these linear waves are also the fields due to sources which are computed from the retarded Green function. The Green functions of course depend on the choice of "gauge conditions" on the linear fields, and their explicit form depends on the choice of coordinate chart on the background space-time.

The next level of physical questions relates to physicality of the wave solutions. The general covariance of the theory manifests as a *gauge equivalence* at the linearized level, and this complicates the identification of physical (gauge invariant) attributes of the wave solutions. In the Minkowski background, the linearized Riemann tensor is gauge invariant, and consequently the induced geodesic deviation or tidal distortion is a physical effect of the waves. In the de Sitter background, the linearized Riemann tensor itself is *not* gauge invariant, but thanks to its conformal flatness, a certain *deviation scalar* can be constructed which is gauge invariant. It, too, is related to tidal distortions and contains information about physical attributes of the waves. To the extent that there exists fully gauge fixed solutions with a nonzero tidal distortion, the gravitational waves are "real." All the interferometric detectors measure these distortions in some form or other. Since the waves are capable of doing work, we could ask for a measure of the *energy* carried by the waves.

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The natural strategy for defining a measure of energy through a stress tensor does not work for gravity. There is simply no gauge invariant, tensorial definition of a gravitational stress tensor. There are two approaches taken for a measure of the *flux of gravitational energy*. One is based on an *effective gravitational stress tensor* tailored for the context wherein there are two widely separated scales, $\lambda \ll L$, of spatiotemporal variations of the metric which are used to identify the L-scale component of the metric as a *background metric* and λ-scale component as a small *ripple* [5]. The other approach directly defines the *flux of gravitational radiation* in reference to the *null infinity* using the canonical structure of the space of asymptotically flat/de Sitter solutions of the Einstein equation. This is applicable for all spatially compact sources [6].

A spatially compact source has two natural scales: its physical size R and the scale of its time variation T. For R sufficiently small compared to the distance to the source, d, it is essentially the scale T that is relevant for gravitational radiation, and we may take the corresponding equivalent length scale as $\lambda \sim T$ ($c = 1$ units). On the other hand, the curvature scale of the ambient geometry sufficiently far away from the source provides the scale L. For Minkowski space-time background, $L = \infty$, whereas for nonzero cosmological constant, $L \sim |\Lambda|^{-1/2}$. A sufficiently rapidly varying source is one which has its time scale of variation or equivalent spatial scale $\lambda \ll L$, while a source is distant if $\lambda/d \ll 1$.

Our focus in this work is on sufficiently rapidly varying, distant, spatially compact sources. For current interferometric detectors, the scale $\lambda \sim 10^4$–10^5 meters, the distances d are in the range of kilo to hundreds of mega parsecs ($\sim 10^{19}$–10^{24} meters), while the spatial extents, R, vary over light seconds or less ($\lesssim 10^8$ meters). We would like to note that induced tidal distortions are needed in the direct detection of gravitational waves, regardless of a measure of the energy carried, while for indirect detection based on energy loss due to gravitational radiation, reliable flux measures are crucial. In this work, we focus on the gravitational field and the induced tidal distortion. Computation of flux(es) will be presented in a separate publication. The quadrupole flux based on the canonical approach is already available in Ref. [7].

In obtaining the field due to a compact source, we follow the basic steps which are well known and well understood for Minkowski background: (a) set up the linearized equations; (b) choose a suitable gauge, and obtain a retarded Green function; (c) identify the physical solutions for subsequent computation of geodesic deviation and power radiated; and (d) relate the physical field to appropriate source multipole moments. At each of these steps, we encounter new features compared to the computations in the Minkowski background.

Unlike the Minkowski space-time which admits a natural, global Cartesian chart, de Sitter space-time has several charts appropriate for different situations. The de Sitter space-time defined as the hyperboloid in five-dimensional Minkowski space-time, has a *global chart* of coordinates $(\tau, \chi, \theta, \phi)$, as shown in Fig. 1. There are natural "Poincaré patches" which constitute the causal future (past) of observers and cover "half" of the global chart. For instance, an observer represented by the world line DA has its causal future J^+ spanning the region DBA and is one of the *Poincaré patches*. Since it is appropriate for the cosmological context, we focus on this Poincaré patch. Its boundary denoted by the line AB is the *future null infinity*, $\mathcal{J}^+$. There are two natural coordinate charts for the Poincaré patch, e.g., a *conformal chart* (η, x^i) and a *cosmological chart* (t, x^i). A half of the Poincaré patch admits a timelike Killing vector and is referred to as a *static patch*. This is a natural patch for an isolated body or a black hole with a stationary neighborhood. We present computations in two different charts: suitably defined *Fermi normal coordinates* (FNC) covering the static patch and a conformal chart covering the Poincaré patch; see Fig. 1.

While physical implications should not depend on the choice of charts, their explicit computations do depend on the chosen chart. For convenience as well as for building up intuition, different charts could have different advantages. For instance, the time coordinate of the FNC chart is the Killing parameter of the stationary Killing vector. This reduces the Lie derivative with respect to the Killing vector to a simple coordinate derivative. The metric, too, is obtained as a Taylor series in the curvature, and hence effects due to the cosmological constant can naturally be expected to appear as a power series in Λ. However, this advantage is not available outside the static patch. By contrast, in the conformal chart, the metric is conformal to the Minkowski metric which considerably simplifies the

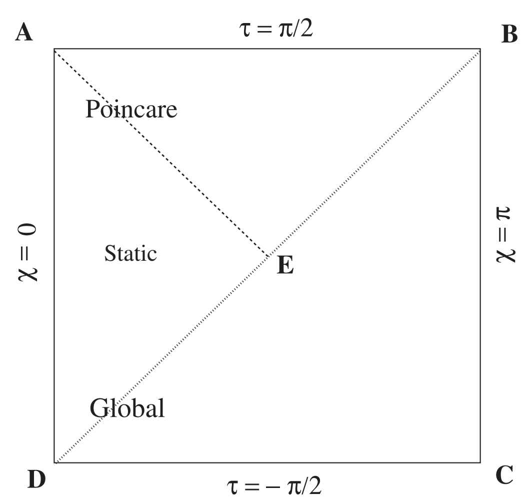

FIG. 1. ABCD denotes the global chart, ABD is a Poincaré patch, while AED is a static patch. The angular coordinates, θ, ϕ are suppressed. The metric in global chart is given by $ds^2 = \frac{3}{\Lambda}\sec^2\tau[-d\tau^2 + d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta d\phi^2)]$.

computations. It is possible to scale out Λ by a suitable choice of variables. However, to get corrections in terms of Λ, one needs to go to the cosmological chart. *A priori*, it is not clear which chart(s) is (are) convenient for what aspect, and we present computations for two choices of charts—the FNC and the conformal chart.

After obtaining the linearized equation, the next step is to choose "a gauge." The natural choice (also used in the Minkowski background) is the *transverse, traceless (TT) gauge*. But there has been another gauge choice [8], which in the conformal chart simplifies the linearized equations as well as subsequent analysis due to its similarity with the Minkowski space-time. This is a gauge which imposes a variant of the transversality condition. We present the solutions in both gauges. The wave propagation has a *tail* term in both gauges. The TT gauge computations are performed in a FNC system and are restricted to order Λ. The tail term is of order Λ^2. In the second gauge, in a large separation regime, the tail integral can be computed explicitly.

Next, to identify the physical fields, one chooses the so-called *synchronous gauge* which sets all fields with at least one temporal index to zero. This steps needs a generalization when the background has a curvature and needs a suitable timelike vector field. Fortunately, such a generalization is available [9] in a neighborhood of a Cauchy surface.

In a curved space-time, the notion of *source multipole moments* needs to be defined appropriately. In the Minkowski background, the coordinates of the global chart are *vectors* under spatial rotations on a constant t hypersurface. In a curved background, the local coordinates have no such property. A suitable definition can be constructed by setting up Fermi normal coordinates. We show that in the FNC chart with TT gauge, the physical fields, and the source moments can be obtained as the Minkowski background results with corrections in powers of $\Lambda \times$ (distant to the source)^2 and present the first correction. The computations are useful and reliable at best up to a distance of about $\Lambda^{-1/2}$ and certainly not up to the null infinity, $\mathcal{J}^+$. The FNC chart is contained within a static patch. For the subset of compact sources we limit ourselves to, this is adequate. Unlike the Minkowski background, the correction terms contain additional types of moments as well as lower order time derivatives of the moments.

The paper is organized as follows. In Sec. II, we recall the linearization procedure together with the associated notion of gauge freedom. We collect the expression for the Ricci tensor up to the quadratic order and give the linearized wave equation for the metric perturbations. We discuss the gauge choices and residual gauge invariance. Section III is divided in two subsections. In the first subsection, we choose the usual transverse, traceless gauge. We present the Hadamard form of the retarded Green function and simplify the expression for field due to a localized source, using the FNC. The leading contribution of the order Λ^0 to the quadrupole field is the same as that in the Minkowski background, and we present the order Λ contributions. Here, appropriate source moments are defined, and the solution in a synchronous gauge is presented. For contrast, in the second subsection, we summarize the computation of the quadrupole field in an alternative gauge [8]. The solution in the synchronous gauge is presented in terms of analogously defined source moments. Here, using the cosmological chart, the corrections appear in powers of $\sqrt{\Lambda}$. In Sec. IV, we present a suitably defined, gauge invariant deviation scalar and compute it for the suitably projected fields. In the final section, Sec. V, we summarize and discuss our results. Some of the technical details are given in the three Appendixes.

II. LINEARIZATION ABOUT DE SITTER BACKGROUND

As noted above, there are several natural patches and charts available in the de Sitter space-time. To introduce perturbations without referring to coordinates,[1] consider a one parameter family of metrics, $g_{\mu\nu}(\epsilon)$, which is differentiable with respect to ϵ at $\epsilon = 0$, and let $\bar{g}_{\mu\nu} \coloneqq g_{\mu\nu}(0)$ be a given solution of the exact Einstein equation. Define *a perturbation* of the exact solution as $h_{\mu\nu} \coloneqq \frac{dg_{\mu\nu}(\epsilon)}{d\epsilon}|_{\epsilon=0}$. As the one parameter families of metrics are varied, we generate the space of perturbations from the corresponding $h_{\mu\nu}$. If every member of the family of metrics solves Einstein equation (with sources and cosmological constant), then the perturbation satisfies a *linear equation* obtained by differentiating the exact equation with respect to ϵ and setting ϵ to zero. Thus, every one parameter family of exact solutions of the Einstein equation gives a solution of the linearized equation. The converse is not always true and is known as the linearization instability problem. In our context, this is not a concern. The general covariance of the Einstein equation implies that every one parameter family of metrics, obtained by diffeomorphisms generated by a vector field on a solution to the Einstein equation, also solves the equation and leads to a corresponding perturbation satisfying the linearized equation. However, these families give the *same physical space-time*. The corresponding perturbations do not give physically distinct, nearby space-times and therefore do not represent *physical perturbations*. These perturbations have the form $h_{\mu\nu} = \mathcal{L}_\xi \bar{g}_{\mu\nu}$ where $\mathcal{L}_\xi$ denotes the Lie derivative. To identify the physical perturbations, we have to "mod out" these perturbations, generated by diffeomorphisms. In other words, physical perturbations are equivalence classes of perturbations:

[1]Sometimes a coordinate system is presumed in which the metric is split into a background plus small perturbations. This obscures the tensorial nature of the perturbation and is avoided as discussed, for example, in Ref. [10].

$$[h_{\mu\nu}] := \{h'_{\mu\nu}/h'_{\mu\nu} = h_{\mu\nu} + \mathcal{L}_\xi \bar{g}_{\mu\nu} \forall \text{ vector fields } \xi\}.$$

More commonly, the expression $h'_{\mu\nu} = h_{\mu\nu} + \mathcal{L}_\xi \bar{g}_{\mu\nu}$ is referred to as a *gauge transformation*, and the equivalence classes are of course the physical perturbations. Thus, by definition of gauge transformations, the linearized equation is gauge invariant. While the perturbations are subjected to these gauge transformations, it should be borne in mind that they are tensors with respect to general coordinate transformations.

While the linearization can be specified in a coordinate-free manner, explicit computations of solutions need coordinates to be introduced. In practice, one begins by writing $g_{\mu\nu}(\epsilon, x) \approx \bar{g}_{\mu\nu}(x) + \epsilon h_{\mu\nu}(x)$ and obtains the linearized equation by substituting this in the full equation and keeping terms to order ϵ. Since we consider perturbations of the source-free de Sitter solution, the matter stress tensor is of order ϵ, while the cosmological constant is of order ϵ^0. Under an infinitesimal diffeomorphism generated by a vector field $\xi^\mu(x)$, $x'^\mu = x^\mu - \epsilon\xi^\mu(x)$, the Lie derivative of the background metric, $\bar{g}_{\mu\nu}(x)$, is given by $\mathcal{L}_\xi \bar{g}_{\mu\nu} = \bar{\nabla}_\mu \xi_\nu + \bar{\nabla}_\nu \xi_\mu$. Here, the $\bar{\nabla}$ denotes the covariant derivative with the Riemann-Christoffel connection of $\bar{g}$ and $\xi_\mu := \bar{g}_{\mu\nu}\xi^\nu$. The gauge transformations thus take the form $h'_{\mu\nu}(x) = h_{\mu\nu}(x) + \bar{\nabla}_\mu \xi_\nu + \bar{\nabla}_\nu \xi_\mu$.

We begin by summarizing the expansions of the connection and Ricci tensor to $o(h^2)$.

In the following, the indices are raised and lowered using the background metric which is taken to be a maximally symmetric one. Background quantities carry an overbar,

$$g^{\mu\nu} = \bar{g}^{\mu\nu} - \epsilon h^{\mu\nu} + \epsilon^2 h^\mu{}_\alpha h^{\alpha\nu} \tag{1}$$

$$\begin{aligned}\Gamma^\lambda{}_{\mu\nu} &= \bar{\Gamma}^\lambda{}_{\mu\nu} + \epsilon\left[\frac{1}{2}\bar{g}^{\lambda\alpha}(\bar{\nabla}_\nu h_{\alpha\mu} + \bar{\nabla}_\mu h_{\alpha\nu} - \bar{\nabla}_\alpha h_{\mu\nu})\right] \\ &\quad - \epsilon^2\left[\frac{1}{2}h^{\lambda\alpha}(\bar{\nabla}_\nu h_{\alpha\mu} + \bar{\nabla}_\mu h_{\alpha\nu} - \bar{\nabla}_\alpha h_{\mu\nu})\right]\end{aligned} \tag{2}$$

$$\begin{aligned}R_{\mu\nu} &= \bar{R}_{\mu\nu} + \epsilon R^{(1)}_{\mu\nu} + \epsilon^2 R^{(2)}_{\mu\nu} \\ R^{(1)}_{\mu\nu} &= -\frac{1}{2}\bar{\Box}h_{\mu\nu} - \frac{1}{2}\bar{\nabla}_\mu\bar{\nabla}_\nu h + \frac{1}{2}(\bar{\nabla}_\mu\bar{\nabla}_\alpha h^\alpha{}_\nu + \bar{\nabla}_\nu\bar{\nabla}_\alpha h^\alpha{}_\mu) \\ &\quad + \frac{1}{2}(\bar{R}_{\mu\alpha}h^\alpha{}_\nu + \bar{R}_{\nu\alpha}h^\alpha{}_\mu) \\ &\quad + \bar{R}_{\mu\alpha\beta\nu}h^{\alpha\beta}, \qquad h := \bar{g}^{\alpha\beta}h_{\alpha\beta};\end{aligned} \tag{3}$$

$$\begin{aligned}R^{(2)}_{\mu\nu} &= \frac{1}{2}h^{\alpha\beta}[\bar{\nabla}_\nu\bar{\nabla}_\mu h_{\alpha\beta} + \bar{\nabla}_\alpha\bar{\nabla}_\beta h_{\mu\nu} - \bar{\nabla}_\alpha\bar{\nabla}_\mu h_{\beta\nu} - \bar{\nabla}_\alpha\bar{\nabla}_\nu h_{\beta\mu}] \\ &\quad - \frac{1}{4}\{2\bar{\nabla}_\alpha h^\alpha{}_\beta - \bar{\nabla}_\beta h\}\{\bar{\nabla}_\mu h_\nu{}^\beta + \bar{\nabla}_\nu h_\mu{}^\beta - \bar{\nabla}^\beta h_{\mu\nu}\} \\ &\quad + \frac{1}{4}(\bar{\nabla}_\mu h^{\alpha\beta} + \bar{\nabla}^\alpha h^\beta{}_\mu - \bar{\nabla}^\beta h^\alpha{}_\mu) \\ &\quad \times (\bar{\nabla}_\nu h_{\alpha\beta} + \bar{\nabla}_\alpha h_{\beta\nu} - \bar{\nabla}_\beta h_{\alpha\nu})\end{aligned} \tag{4}$$

$$\bar{g}^{\mu\nu}R^{(1)}_{\mu\nu} = -\bar{\Box}h + \bar{\nabla}_\mu\bar{\nabla}_\nu h^{\mu\nu}, \tag{5}$$

$$\begin{aligned}G_{\mu\nu} + \Lambda g_{\mu\nu} &= [\bar{G}_{\mu\nu} + \Lambda\bar{g}_{\mu\nu}] + [G^{(1)}_{\mu\nu} + \Lambda h_{\mu\nu}] \\ G^{(1)}_{\mu\nu} + \Lambda h_{\mu\nu} &= -\frac{1}{2}\bar{\Box}h_{\mu\nu} - \frac{1}{2}(\bar{\nabla}_\mu\bar{\nabla}_\nu - \bar{g}_{\mu\nu}\bar{\Box})h \\ &\quad + h_{\mu\nu}\left(\Lambda - \frac{1}{2}\bar{R}\right) + \frac{1}{2}(\bar{\nabla}_\mu\bar{\nabla}_\alpha h^\alpha{}_\nu \\ &\quad + \bar{\nabla}_\nu\bar{\nabla}_\alpha h^\alpha{}_\mu - \bar{g}_{\mu\nu}(\bar{\nabla}_\alpha\bar{\nabla}_\beta h^{\alpha\beta})) + \bar{R}_{\mu\alpha\beta\nu}h^{\alpha\beta} \\ &\quad + \frac{1}{2}(\bar{R}_{\mu\alpha}h^\alpha{}_\nu + \bar{R}_{\nu\alpha}h^\alpha{}_\mu + \bar{g}_{\mu\nu}\bar{R}^{\alpha\beta}h_{\alpha\beta}).\end{aligned} \tag{6}$$

The expressions simplify further for the *maximally symmetric* solution of the background equation, $\bar{G}_{\mu\nu} + \Lambda\bar{g}_{\mu\nu} = 0$. Maximal symmetry implies $\bar{R}_{\mu\alpha\beta\nu} = K(\bar{g}_{\mu\beta}\bar{g}_{\nu\alpha} - \bar{g}_{\mu\nu}\bar{g}_{\alpha\beta})$, while the background equation fixes $K = \Lambda/3$ and the linearized equation becomes[2]

$$\begin{aligned}&-\frac{1}{2}\bar{\Box}h_{\mu\nu} - \frac{1}{2}(\bar{\nabla}_\mu\bar{\nabla}_\nu - \bar{g}_{\mu\nu}\bar{\Box})h + \frac{\Lambda}{3}h_{\mu\nu} + \frac{\Lambda}{6}\bar{g}_{\mu\nu}h \\ &\quad + \frac{1}{2}(\bar{\nabla}_\mu\bar{\nabla}_\alpha h^\alpha{}_\nu + \bar{\nabla}_\nu\bar{\nabla}_\alpha h^\alpha{}_\mu - \bar{g}_{\mu\nu}(\bar{\nabla}_\alpha\bar{\nabla}_\beta h^{\alpha\beta})) = 8\pi T_{\mu\nu}.\end{aligned} \tag{7}$$

It is customary and convenient to use the trace-reversed combination: $\tilde{h}_{\mu\nu} := h_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}h$. Denoting, $B_\mu := \bar{\nabla}_\alpha\tilde{h}^\alpha{}_\mu$, in terms of the tilde variables, the linearized equation takes the form

$$\begin{aligned}&\frac{1}{2}[-\bar{\Box}\tilde{h}_{\mu\nu} + \{\bar{\nabla}_\mu B_\nu + \bar{\nabla}_\nu B_\mu - \bar{g}_{\mu\nu}(\bar{\nabla}^\alpha B_\alpha)\}] \\ &\quad + \frac{\Lambda}{3}[\tilde{h}_{\mu\nu} - \tilde{h}\bar{g}_{\mu\nu}] = 8\pi T_{\mu\nu}.\end{aligned} \tag{8}$$

The divergence of the left-hand side, $\bar{\nabla}^\mu[LHS]_{\mu\nu}$, is identically zero, and thus the source tensor is conserved automatically as it should be. For $\Lambda = 0$, the equation goes over to the flat background equation. Under the gauge transformations, $\tilde{h}_{\mu\nu}$ transforms as

$$\delta\tilde{h}_{\mu\nu}(x) = \bar{\nabla}_\mu\xi_\nu + \bar{\nabla}_\nu\xi_\mu - \bar{g}_{\mu\nu}\bar{\nabla}_\alpha\xi^\alpha,$$

and the linearized equation (8) is explicitly invariant under these as it should be. It is well known that, availing this freedom, it is possible to impose the *transversality condition*, $\bar{\nabla}_\alpha\tilde{h}^\alpha{}_\mu = 0$. The trace can be further gauged away [11] in the absence of sources (or for the traceless stress tensor). The particular choice of arranging $\bar{\nabla}_\alpha\tilde{h}^\alpha{}_\mu = 0 = h$

[2]From now on, the background is taken to be the de Sitter space-time with $\Lambda > 0$, and the units are chosen so that $G = 1 = c$.

is the TT gauge. It simplifies Eq. (8) to (for the traceless stress tensor)

$$-\frac{1}{2}\bar{\Box}\tilde{h}_{\mu\nu}+\frac{\Lambda}{3}\tilde{h}_{\mu\nu}=8\pi T_{\mu\nu}. \tag{9}$$

The transversality condition still allows *residual gauge transformations* generated by vector fields ξ^μ satisfying

$$\delta(\bar{\nabla}_\mu\tilde{h}^\mu{}_\nu)=\bar{\nabla}_\mu(\delta\tilde{h}^\mu{}_\nu)=\bar{\Box}\xi_\nu+\bar{R}_{\alpha\nu}\xi^\alpha=(\bar{\Box}+\Lambda)\xi_\nu=0. \tag{10}$$

If, in addition, the trace (zero or nonzero) is to be preserved, then ξ^μ must further satisfy $\bar{\nabla}_\alpha\xi^\alpha=0$, and this is consistent with the above equation.

While it is common to choose the TT gauge, it is also possible to make a different choice of gauge [8] in the Poincaré patch of the de Sitter space-time. This will be done in Sec. III B below.

The task now is to obtain the particular solution of the linearized, inhomogeneous equation (8) and extract the *physical solutions*, i.e., solutions satisfying conditions which leave no gauge transformations possible, in the source-free region. Within the perturbative framework, this is obtained at the leading order by using a suitable Green function for the linearized equation on the de Sitter background. The *retarded Green functions* will be determined after some gauge fixing simplifying Eq. (8).

III. RETARDED GREEN FUNCTION

There have been several computations of two point functions for scalar, vector, and tensor fields on de Sitter background [8,11–13]. We will consider two retarded Green functions. In Sec. III A, we impose first the transversality condition and then also the tracelessness condition. We refer to these as the *transverse gauge* and the *TT gauge* respectively. In Sec. III B, following Ref. [8], we choose a gauge which changes the transversality condition by making its right-hand side nonzero. We refer to it as *generalized transverse gauge*. With the tracelessness condition imposed, we refer to it as *generalized TT* gauge. The two computations will provide different views of the physical solutions, in particular the form of the manifestation of the Λ dependence. The computations in the transverse gauge, employing the Hadamard construction [14], follow Ref. [15], while the generalized transverse gauge computations are based on Ref. [8].

A. Transverse and TT gauges

It turns out to be convenient to separate the trace part of the equation and construct the retarded Green function in the TT gauge directly with a source which is traceless.

Imposing the transversality condition, $B_\mu=0$ in Eq. (8) gives

$$\bar{\Box}\tilde{h}_{\mu\nu}-\frac{2\Lambda}{3}[\tilde{h}_{\mu\nu}-\tilde{h}\bar{g}_{\mu\nu}]=-16\pi T_{\mu\nu}, \tag{11}$$

and taking the trace of the above equation gives an equation for the trace, $\tilde{h}$,

$$(\bar{\Box}+2\Lambda)\tilde{h}=-16\pi T, \qquad T\coloneqq\bar{g}_{\mu\nu}T^{\mu\nu}. \tag{12}$$

Subtracting $\frac{1}{4}\bar{g}_{\mu\nu}\times$ Eq. (12) from Eq. (11), we get

$$\bar{\Box}\tilde{h}'_{\mu\nu}-\frac{2\Lambda}{3}\tilde{h}'_{\mu\nu}=-16\pi T'_{\mu\nu}, \qquad \tilde{h}'_{\mu\nu}\coloneqq\tilde{h}_{\mu\nu}-\frac{1}{4}\tilde{h}\bar{g}_{\mu\nu},$$
$$T'_{\mu\nu}\coloneqq T_{\mu\nu}-\frac{1}{4}T\bar{g}_{\mu\nu}. \tag{13}$$

Equation (12) for $\tilde{h}$ is a scalar equation, and its solution is determined by a corresponding Green function with a source which is the trace of the stress tensor. However, we know that in the source-free region, we *can* make a gauge transformation to set the $\tilde{h}$ to zero. Hence, in the region of observational interest, we can gauge away the effect of the trace T. With this understood, we take $\tilde{h}=0$ which gives $\tilde{h}'_{\mu\nu}=\tilde{h}_{\mu\nu}$ and use the traceless $T'_{\mu\nu}$ as the source. For notational simplicity, we drop the prime from the stress tensor. Thus, we focus on the TT gauge equation (9) with a *trace-free stress tensor* as the source.

The equation for the Green function is

$$\bar{\Box}G^{\alpha\beta}{}_{\mu'\nu'}(x,x')-\frac{2\Lambda}{3}G^{\alpha\beta}{}_{\mu'\nu'}(x,x')$$
$$=-4\pi J^{\alpha\beta}{}_{\mu'\nu'}\delta_4(x,x'), \quad \text{where} \tag{14}$$

$$J^{\alpha\beta}{}_{\mu'\nu'}(x,x')\coloneqq\frac{g^\alpha{}_{\mu'}g^\beta{}_{\nu'}+g^\alpha{}_{\nu'}g^\beta{}_{\mu'}}{2}$$
$$-\frac{1}{4}\bar{g}^{\alpha\beta}(x)\bar{g}_{\mu'\nu'}(x'), \quad \text{and} \tag{15}$$

$g^\alpha{}_{\mu'}(x,x')$ denotes the *parallel propagator* along the geodesic connecting x, x'. The tensor $J^{\alpha\beta}{}_{\mu'\nu'}$ is symmetric and traceless in the pairs of indices $\alpha\beta$ and $\mu'\nu'$. The Green's function is obtained using the *Hadamard ansatz*.

The Hadamard ansatz for the retarded Green function for *a general* wave equation is [14]

$$G^{\alpha\beta}{}_{\mu'\nu'}(x,x')=U^{\alpha\beta}{}_{\mu'\nu'}(x,x')\delta_+(\sigma+\epsilon)$$
$$+V^{\alpha\beta}{}_{\mu'\nu'}(x,x')\theta_+(-\sigma-\epsilon), \tag{16}$$

where the space-time points x, x' belong to a *convex normal neighborhood* with x in the chronological future of x'; $\sigma(x,x')$ is the Synge world function which is half the geodesic distance squared between x and x' [15,16]; θ_+, δ_+ are distributions, viewed as functions of x, having support in the chronological future and future light cone of x'

respectively. The small parameter ϵ is introduced to permit differentiation of the distribution and is to be taken to zero in the end. The bitensors U, V are determined by inserting the ansatz in Eq. (14).

Using the relation $\bar{g}^{\alpha\beta}\bar{\nabla}_\alpha\sigma\bar{\nabla}_\beta\sigma = 2\sigma$ and the distributional identities [15],

$$(\sigma+\epsilon)\delta'(\sigma+\epsilon) = -\delta(\sigma+\epsilon),$$
$$(\sigma+\epsilon)\delta''(\sigma+\epsilon) = -2\delta'(\sigma+\epsilon)$$
$$\text{as } \epsilon\to 0: \epsilon\delta'(\sigma+\epsilon)\to 0, \qquad \epsilon\delta''(\sigma+\epsilon)\to 2\pi\delta_4(x,x'), \tag{17}$$

leads to four equations by equating the coefficients of $\theta(-\sigma)$, $\delta(\sigma)$, $\delta'(\sigma)$, and $\delta_4(x,x')$ to zero. The respective equations are

$$\bar{\Box}V^{\alpha\beta}{}_{\mu'\nu'}(x,x') - \frac{2\Lambda}{3}V^{\alpha\beta}{}_{\mu'\nu'}(x,x') = 0, \quad \sigma(x,x')<0; \tag{18}$$

$$2\sigma^\lambda\bar{\nabla}_\lambda V^{\alpha\beta}{}_{\mu'\nu'} + (\bar{\Box}\sigma - 2)V^{\alpha\beta}{}_{\mu'\nu'} = \bar{\Box}U^{\alpha\beta}{}_{\mu'\nu'} - \frac{2\Lambda}{3}U^{\alpha\beta}{}_{\mu'\nu'}, \quad \sigma(x,x')=0; \tag{19}$$

$$(2\sigma^\lambda\bar{\nabla}_\lambda + (\bar{\Box}\sigma - 4))U^{\alpha\beta}{}_{\mu'\nu'} = 0, \quad \sigma(x,x')=0; \tag{20}$$

$$[U^{\alpha\beta}{}_{\mu'\nu'}] = [J^{\alpha\beta}{}_{\mu'\nu'}] = \delta^{(\alpha'}_{\mu'}\delta^{\beta')}_{\nu'} - \frac{1}{4}\bar{g}^{\alpha'\beta'}\bar{g}_{\mu'\nu'}, \quad x = x'. \tag{21}$$

In the above, the quantity enclosed within square brackets denotes its *coincidence limit*—evaluation for the $x = x'$ and super- (sub-)script on σ denotes its covariant derivative.

The last two equations uniquely determine $U^{\alpha\beta}{}_{\mu'\nu'}(x,x')$ on the light cone through x', while the first two equations uniquely determine $V^{\alpha\beta}{}_{\mu'\nu'}(x,x')$ inside and on the light cone through x'. The cosmological constant appears explicitly in these two equations.

1. Determination of $U^{\alpha\beta}{}_{\mu'\nu'}$

Equation (20) is a homogeneous, first order, linear differential equation, and its solution is completely determined by the initial condition provided by Eq. (21). Noting that $\sigma^\lambda\bar{\nabla}_\lambda$ on the parallel propagator and the metric gives zero, we get $\sigma^\lambda\bar{\nabla}_\lambda J^{\alpha\beta}{}_{\mu'\nu'} = 0$.

Hence, the ansatz $U^{\alpha\beta}{}_{\mu'\nu'}(x,x') \coloneqq J^{\alpha\beta}{}_{\mu'\nu'}\tilde{U}(x,x')$ in Eqs. (20) and (21) leads to

$$(2\sigma^\alpha\nabla_\alpha + (\bar{\Box}\sigma - 4))\tilde{U} = 0,$$
$$[\tilde{U}] = 1 \Rightarrow \tilde{U}(x,x') \coloneqq \sqrt{\Delta(x,x')}, \tag{22}$$

where $\Delta(x,x')$ is the (scalarized) Van Vleck determinant or Van Vleck biscalar defined as $\Delta(x,x') \coloneqq -\det(-\sigma_{\alpha\beta'}(x,x'))/\sqrt{g(x)g(x')}$, with g in the denominator denoting the modulus of the determinant of the metric [15]. The biscalar $\tilde{U}$, being de Sitter invariant, depends on x, x' only through the world function $\sigma(x,x')$ which means that value of $\tilde{U}$ along the light cone is the same as its value in the coincidence limit, i.e., $\tilde{U}|_{\sigma=0} = [\tilde{U}] = 1 (= \Delta(x,x')|_{\sigma=0})$ and we need the solution only on the light cone. Thus,

$$U^{\alpha\beta}{}_{\mu'\nu'}(x,x')|_{\sigma=0} \coloneqq J^{\alpha\beta}{}_{\mu'\nu'}|_{\sigma=0}.$$

We *cannot* similarly factor out $J^{\alpha\beta}{}_{\mu'\nu'}$ from $V^{\alpha\beta}{}_{\mu'\nu'}(x,x')$. The reason is that Eq. (19) is an *inhomogeneous* equation and the tensor structure of its right-hand side is not the same as that of $U^{\alpha\beta}{}_{\mu'\nu'}$. Indeed, to order $(\sigma)^2$, we find [15]

$$\left(\bar{\Box} - \frac{2\Lambda}{3}\right)U^{\alpha\beta}{}_{\mu'\nu'} = \left\{-\frac{\Lambda}{6}(4-\bar{\Box}\sigma) - \frac{\Lambda^2\sigma}{9}\right\}J^{\alpha\beta}{}_{\mu'\nu'} \tag{23}$$

$$\begin{aligned} &+ \frac{\Lambda^2}{18}\{\bar{g}^{\alpha\beta}\sigma_{\mu'}\sigma_{\nu'} + \sigma^\alpha\sigma^\beta\bar{g}_{\mu'\nu'} \\ &- \sigma(g^\alpha{}_{\mu'}g^\beta{}_{\nu'} + g^\alpha{}_{\nu'}g^\beta{}_{\mu'}) \\ &+ (g^\alpha{}_{\mu'}\sigma^\beta\sigma_{\nu'} + g^\beta{}_{\mu'}\sigma^\alpha\sigma_{\nu'} + g^\alpha{}_{\nu'}\sigma^\beta\sigma_{\mu'} \\ &+ g^\beta{}_{\nu'}\sigma^\alpha\sigma_{\mu'})\} \\ &\coloneqq \Phi(\sigma)J^{\alpha\beta}{}_{\mu'\nu'} + \frac{\Lambda^2}{18}K^{\alpha\beta}{}_{\mu'\nu'} + o(\sigma^3). \end{aligned} \tag{24}$$

Note that the bitensor $K^{\alpha\beta}{}_{\mu'\nu'}$ is traceless, and just as the bitensor $J^{\alpha\beta}{}_{\mu'\nu'}$, it, too, is annihilated by $\sigma^\lambda\bar{\nabla}_\lambda$.

Noting the coincidence limits, $[\bar{\Box}\sigma] = 4$, $[\sigma] = 0$, $[\sigma^\alpha] = 0$, we see that $[\Phi] = 0 = [K^{\alpha\beta}{}_{\mu'\nu'}]$, and hence the coincidence limit of the left-hand side vanishes.

The coincidence limit of Eq. (19) then implies $[V^{\alpha\beta}{}_{\mu'\nu'}(x,x')] = 0$. However, this does not imply $V^{\alpha\beta}{}_{\mu'\nu'}(x,x')|_{\sigma=0} = 0$. To order σ^2, we can write

$$V^{\alpha\beta}{}_{\mu'\nu'}(x,x') \coloneqq \tilde{V}_1(\sigma)J^{\alpha\beta}{}_{\mu'\nu'} + \tilde{V}_2(\sigma)K^{\alpha\beta}{}_{\mu'\nu'}.$$

This leads to two inhomogeneous differential equations for the biscalars $\tilde{V}_1$, $\tilde{V}_2$. The coincidence limits of these equations, combined with $[\Phi] = 0$, leads to $[\tilde{V}_1] = 0$ and $[\tilde{V}_2] = \frac{\Lambda^2}{108}$. Once again, these values determine these biscalars everywhere on the light cone. Hence, to order σ^2,

$$V^{\alpha\beta}{}_{\mu'\nu'}(x,x')|_{\sigma=0} = \frac{\Lambda^2}{108}K^{\alpha\beta}{}_{\mu'\nu'}|_{\sigma=0} + o(\sigma^3). \tag{25}$$

This shows clearly that the data for characteristic evolution off the light cone are nonzero and hence the tail term is nonzero as well. Equally well, it also shows that the tail term is *at least of order* Λ^2. The Green function is then given by

$$G^{\alpha\beta}{}_{\mu'\nu'}(x,x') = J^{\alpha\beta}{}_{\mu'\nu'}(x,x')\delta_+(\sigma) + V^{\alpha\beta}{}_{\mu'\nu'}\theta_+(-\sigma).$$

We will be computing corrections to order Λ, and hence we do not compute the effect of the tail term in this work. From now on, we restrict to the sharp propagation term only, and only the *trace-free* part of the source stress tensor contributes.

Using the sharp term of the Green function above, the solution to the inhomogeneous equation becomes

$$\tilde{h}^{\alpha\beta}(x) = 4\int_{\text{source}} d^4x' \sqrt{-g(x')}\delta_+(\sigma) J^{\alpha\beta}{}_{\mu'\nu'}(x,x') T^{\mu'\nu'}(x') \tag{26}$$

$$= 4\int_{\text{source}} d^4x' \sqrt{-g(x')}\delta_+(\sigma) g^{\alpha}{}_{\mu'}(x,x') \times g^{\beta}{}_{\nu'}(x,x') T^{\mu'\nu'}(x'). \tag{27}$$

In the second line, we have substituted for $J^{\alpha\beta}{}_{\mu'\nu'}$ and used the fact that the stress tensor is trace free and symmetric.

To proceed further, we employ FNC and *Riemann normal coordinates* (RNC). These coordinate charts are based on the choice of a timelike reference curve γ, a reference point P_0 on it, and an orthonormal tetrad E^α_a at P_0 such that E^α_0 equals the normalized tangent to γ, at P_0. To be definite, let us take the world tube of the spatially compact source to be around the line AD of Fig. 1. The line AD is a timelike geodesic, and we naturally choose the reference curve, γ, to be this line. Denoting the proper time along γ by τ, we choose $P_0 = \gamma(\tau = 0)$, as the reference point. Let E^α_a denote an orthonormal tetrad at P_0 chosen such that E^α_0 is the normalized, geodesic tangent to γ. Fermi transport the tetrad along γ (which is same as parallel transport since γ is a geodesic). Thus, we have an orthonormal tetrad, $e^\alpha{}_a e^\beta{}_b \bar{g}_{\alpha\beta} = \eta_{ab}$, with $e^\alpha{}_0$ equal to the geodesic tangent to γ, all along $\gamma(\tau)$. The corresponding orthonormal cotetrad is denoted as $e^a{}_\alpha$. It follows that, all along $\gamma(\tau)$, $\bar{g}_{\alpha\beta} = \eta_{\alpha\beta}$ and the Christoffel connection is zero. With these choices, the FNC and the RNC are set up as follows (see Fig. 2).

To define the Fermi coordinates of a point P off γ, let β be the unique (spacelike) geodesic from P, *orthogonally* meeting γ at a point $Q = \gamma(\tau_P)$, with a unit affine parameter interval. Its tangent vector, n^α, at Q can be resolved along the triad of spacelike vectors at Q as $n^\alpha \coloneqq \xi^i e^\alpha{}_i$. Its norm gives the proper distance between P and Q, $s^2 \coloneqq n^\alpha n^\beta \eta_{\alpha\beta} = \xi^i\xi^j\delta_{ij}$. The FNC of P are then defined to be (τ_P, ξ^i). Evidently, for points along γ, the spatial coordinates ξ^i are zero. To define the RNC for the same point P as above, construct the unique geodesic starting from P_0 and reaching P in a unit affine parameter interval. This fixes the geodesics tangent vector N^α at P_0. The normal coordinates of P, X^a are then defined through $N^\alpha \coloneqq X^a E^\alpha{}_a$. We will use them in intermediate computations.

Generally, the FNC and the RNC have a domain consisting of points P which have the required unique geodesics from the reference curve/point. By examining the geodesic equation in the global chart, it is easy to see that the RNC's and the FNC's would be valid in the static patch (see also Ref. [17]). In effect, the computations of this subsection are restricted to the static patch.

Our task is to evaluate the terms in the integrand of Eq. (27). The final answer will be expressed in terms of the FNC introduced above.

2. Computation of $\sigma(x,x')$

Let P, P' denote the observation point and a source point respectively. With the base point P_0, we get a geodesic triangle P_0PP' with the $P'P$ geodesic being null and future directed. Let X^a, X'^a denote the RNCs of P and P' respectively. In terms of the RNC set up in this manner, we have to obtain $\sigma(P', P)$. For this, we follow Chap. II of Ref. [16].

The idea is to construct a surface spanning family of geodesics (Fig. 3), interpolating between the geodesics P_0P, P_0P', all originating at P_0 and ending on a point p on the geodesic connecting $P'P_0$. Each of these have their affine parameters, v's, running from 0 to 1. Choose points q' and q on the geodesics P_0P' and P_0P respectively and having the same value of affine parameter, $0 \le v \le 1$. The world function $\sigma(q', q)$ depends *only* on v and gives the desired answer for $v = 1$. When the Riemann tensor is small, i.e., can be treated as order 1 (different from the orders used in the metric expansion), the $\sigma(q', q)$ is expressed as a Taylor expansion, in v, to third order together with the remainder. This gives

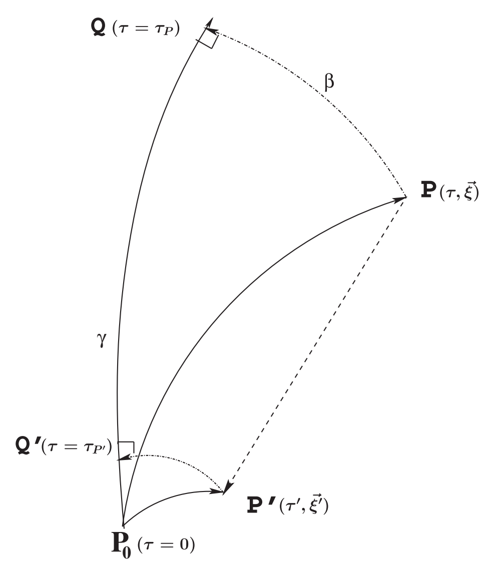

FIG. 2. The definition of Fermi normal coordinates. The dotted line from P to P' is the unique null geodesic for which the parallel propagator is computed in Appendix B. The geodesics P_0P, P_0P' are used in setting up the Riemann normal coordinates. P and P' denote the observation and the source points respectively.

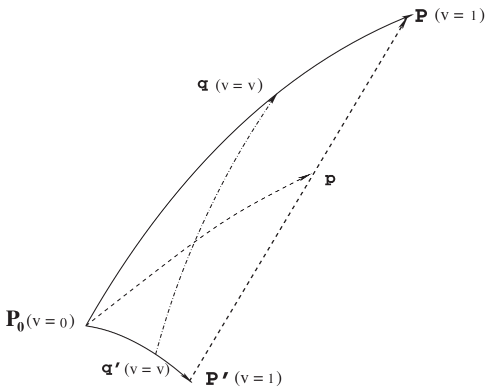

FIG. 3. All lines are the unique geodesics in the Riemann normal neighbourhood. As the point p slides between P' and P, a two-dimensional surface is generated.

$$\sigma(P',P) = \sigma(P_0,P') + \sigma(P_0,P) - \left(g^{\alpha\beta}\frac{\partial\sigma(y,P')}{\partial y^\alpha}\frac{\partial\sigma(y,P)}{\partial y^\beta}\right)\Big|_{P_0} + \frac{1}{6}\int_0^1 dv(1-v)^3\frac{D^4\sigma(q',q)}{Dv^4}. \quad (28)$$

The last term comes from the remainder in the Taylor expansion and contains the modifications due to the nonzero Riemann tensor. This is computed to the first order in the curvature. For maximally symmetric space-time, the computation simplifies. The steps are sketched in Appendix A, and here is the final result expressed in terms of the RNCs of P, P':

$$2\sigma(P,P') = (X-X')\cdot(X-X') - \frac{\Lambda}{9}\{(X\cdot X)(X'\cdot X') - (X\cdot X')^2\} + o(\Lambda^2). \quad (29)$$

Here, the dot product is the Minkowski dot product, $X\cdot Y \coloneqq \eta_{ab}X^aY^b$, etc.

At this stage, we convert the above expression from RNC to FNC. The coordinate transformation between the RNC and the FNC is given by [18]

$$X^0(\tau,\vec{\xi}) = \tau + \tau\frac{R^0{}_{ij0} + R^0{}_{ji0}}{6}\xi^i\xi^j + \cdots = \tau\left(1 - \frac{\Lambda s^2}{9}\right) \quad (30)$$

$$X^i(\tau,\vec{\xi}) = \xi^i + \frac{R^i{}_{0j0}}{6}\xi^j\tau^2 + \frac{R^i{}_{jk0}}{3}\xi^j\xi^k\tau = \xi^i\left(1 - \frac{\Lambda\tau^2}{18}\right) \quad (31)$$

In the second lines, we have used the de Sitter curvature. Substitution in (29) leads to

$$2\sigma(\tau,\vec{\xi},\tau',\vec{\xi}') = \{-(\tau-\tau')^2 + (\vec{\xi}-\vec{\xi}')^2\} + \frac{\Lambda}{9}\{(\tau-\tau')^2(\vec{\xi}^2 + \vec{\xi}'^2 + \vec{\xi}\cdot\vec{\xi}') - (\tau\vec{\xi}' + \tau'\vec{\xi})^2 - (-\tau^2 + \vec{\xi}^2)(-\tau'^2 + \vec{\xi}'^2) + (-\tau\tau' + \vec{\xi}\cdot\vec{\xi}')^2\}, \quad (32)$$

3. Solving the $\delta_+(\sigma)$

We have to solve the $\delta_+(\sigma(P,P'))$ for τ' and eliminate the $d\tau'$ integration. The solution is sought in the form of $\tau' = \tau_0 + \Lambda\tau_1$. The τ_0 is determined by the vanishing of the first braces, and the retarded condition picks out one solution, namely, $\tau_0 = \tau - |\vec{\xi} - \vec{\xi}'|$. The full solution is obtained as

$$\tau'_{\rm ret} \coloneqq \tau_0 + \Lambda\tau_1, \quad \text{where} \quad \tau_0 = \tau - |\vec{\xi}-\vec{\xi}'| \quad (33)$$

$$\tau_1 = -\frac{1}{18}\frac{1}{|\vec{\xi}-\vec{\xi}'|}\{|\vec{\xi}-\vec{\xi}'|^2(\vec{\xi}^2 + \vec{\xi}'^2 + \vec{\xi}\cdot\vec{\xi}') - (\tau(\vec{\xi}+\vec{\xi}') - |\vec{\xi}-\vec{\xi}'|\vec{\xi})^2 - (-\tau^2+\vec{\xi}^2) \times(\vec{\xi}'^2 - (\tau - |\vec{\xi}-\vec{\xi}'|)^2) + (-\tau^2 + \tau|\vec{\xi}-\vec{\xi}'| + \vec{\xi}\cdot\vec{\xi}')^2\}. \quad (34)$$

Now, we introduce the approximation that the source size is much smaller than its distance from observers, i.e., $\vec{\xi}'^2 \ll \vec{\xi}^2 \leftrightarrow s' \ll s$.[3] With this assumption,

$$|\vec{\xi}' - \vec{\xi}| \approx s\sqrt{1 + \frac{s'^2}{s^2} - 2\hat{\xi}\cdot\hat{\xi}'\frac{s'}{s}} \approx s,$$

and keeping only the leading term in powers of s, we get

$$\tau'_{\rm ret} \coloneqq \tau - \left(s + \frac{\Lambda}{18}s^3\right) \eqqcolon \tau - \bar{s}(s) \eqqcolon \tau_{\rm ret}. \quad (35)$$

[3]The spatial coordinates are proportional to the *proper distance* along the corresponding spatial geodesics. This distance is related to but not equal to the "physical distance" equaling the scale factor times the comoving distance. The explicit relation is given in Eq. (C15). Nevertheless, $s' \ll s$ reflects the assumption of the source size being much smaller than the distance to the observer. The cosmological horizon bounding the static chart is at a physical distance of $\sqrt{3/\Lambda}$, and all our s', s are within the static chart.

From this, it follows that

$$\frac{\partial\sigma(\tau,\vec{\xi},\tau',\vec{\xi}')}{\partial\tau'}\bigg|_{\tau'=\tau-s-\frac{\Lambda s^3}{18}} \approx -s\left(1-\frac{\Lambda s^2}{18}\right) \Rightarrow \left|\frac{\partial\sigma}{\partial\tau'}\right|^{-1} \approx \frac{1}{s}\left(1+\frac{\Lambda}{18}s^2\right). \quad (36)$$

Note that the $\tau_{\rm ret}(\tau,s)$ defined above in terms of $\bar{s}$ reflects the non-Minkowskian metric and exactly corresponds to the light cone.

4. Metric and its determinant in FNC

In terms of the FNC, the metric to first order in the curvature is given as [15]

$$g_{00}(\tau,\vec{\xi}) = -1+\frac{\Lambda s^2}{3}, \qquad g_{0i}=0,$$
$$g_{ij} = \delta_{ij} - \frac{\Lambda}{9}(\delta_{ij}s^2 - \xi_i\xi_j). \quad (37)$$

The metric is *static*, and its determinant is given by

$$\sqrt{-g}|_{\rm FNC} \approx 1-\frac{5}{18}\Lambda s^2 = \left(1-\frac{\Lambda s^2}{6}\right)\left(1-\frac{\Lambda s^2}{9}\right). \quad (38)$$

The second factor is the square root of the determinant of the induced metric on a constant τ hypersurface. The metric being static (independent of τ with $g_{0i}=0$) also means that ∂_τ is the stationary Killing vector in the FNC chart.

5. Riemann-Christoffel connection in FNC

We compute this from the metric. Noting that the metric is of the same form as the perturbation about the flat metric, $g_{\mu\nu}=\eta_{\mu\nu}+\delta g_{\mu\nu}$ with $\delta g_{00}=\Lambda s^2/3$, $\delta g_{0i}=0$, $\delta g_{ij} = -\frac{\Lambda}{9}(\delta_{ij}\vec{\xi}^2-\xi_i\xi_j)$, we obtain

$$\Gamma^\mu{}_{\alpha\beta} = \frac{1}{2}(\partial_\alpha\delta g^\mu{}_\beta + \partial_\beta \delta g^\mu{}_\alpha - \partial^\mu\delta g_{\alpha\beta}). \quad (39)$$

Using

$$\delta g^\mu{}_\alpha = -\frac{\Lambda s^2}{3}\delta^\mu_0\delta^0_\alpha - \frac{\Lambda}{9}(\delta^\mu_i\delta^i_\alpha)(\xi^j\xi_j) + \frac{\Lambda}{9}(\delta^\mu_i\xi^i)(\delta^j_\alpha\xi_j),$$

we get

$$\Gamma^\mu{}_{\alpha\beta} = \frac{\Lambda}{18}[-6\{\delta^\mu_0(\delta^0_\alpha\delta^i_\beta\xi_i + \delta^0_\beta\delta^i_\alpha\xi_i) + \delta^0_\alpha\delta^0_\beta\delta^\mu_i\xi^i\}$$
$$-2\{\delta^\mu_i(\delta^i_\alpha\delta^j_\beta\xi_j + \delta^i_\beta\delta^j_\alpha\xi_j) - 2\delta^i_\alpha\delta^i_\beta\delta^\mu_j\xi^j\}]. \quad (40)$$

For future reference, we also give the derivative of the connection,

$$\partial_\gamma\Gamma^\mu{}_{\alpha\beta} = \frac{\Lambda}{18}[-6\{\delta^\mu_0(\delta^0_\alpha\delta^i_\beta\delta_{\gamma i} + \delta^0_\beta\delta^i_\alpha\delta_{\gamma i}) + \delta^0_\alpha\delta^0_\beta\delta^\mu_i\delta^i_\gamma\}$$
$$-2\{\delta^\mu_i(\delta^i_\alpha\delta^j_\beta\delta_{\gamma j} + \delta^i_\beta\delta^j_\alpha\delta_{\gamma j}) - 2\delta^i_\alpha\delta^i_\beta\delta^\mu_j\delta^j_\gamma\}]. \quad (41)$$

The parallel propagator $g^\mu{}_{\alpha'}(P,P')$ is given in Eq. (B4). It involves the coordinate differences $(x'-x)^\beta$ while the coefficients are evaluated at x^α. We need the parallel propagator at the retarded time and in the regime of $s \gg s'$. The coordinate differences are then given as

$$(x'-x)^0 = \tau'-\tau \approx -s, \qquad (x'-x)^i = \xi'^i - \xi^i.$$

Thus, the parallel propagator depends only on $(\tau,\vec{\xi})$.

At this stage, we recall that in the Minkowski background, a simplification is achieved by further imposing the *synchronous gauge* condition, $\tilde{h}_{0\alpha}=0$, which removes the residual gauge freedom of the TT gauge completely, and we are left with only the physical solution: the components $\tilde{h}_{ij}$ satisfying $\partial^i\tilde{h}_{ij}=0=\delta^{ij}\tilde{h}_{ij}$. Is such a simplification available in the de Sitter background?

As a matter of fact, it is a general result [9] that in a globally hyperbolic space-time, given any Cauchy surface, Σ, the normalized, timelike geodesic vector field, η^α, orthogonal to the Cauchy surface allows us to impose the synchronous gauge condition $\tilde{h}_{\alpha\beta}\eta^\beta=0$ in a *normal neighborhood* of Σ. The vector field also provides us with a convenient way to identify the physical components of the solution.

For the static patch we are working in, the hypersurface of constant τ corresponding to the horizontal line through the point E of Fig. 1 is a Cauchy surface, and the required η^α field can be constructed easily to order Λ. For instance, let $\tau=\tau_0$ be the surface Σ_0, with a normalized normal given by $\hat{n}^\alpha \coloneqq (1+\frac{\Lambda}{6}s^2)\delta^\alpha_0$. Then, the vector field η is determined as the solution of an initial value problem:

$$0 = \eta^\beta(\partial_\beta\eta^\alpha + \Gamma^\alpha{}_{\beta\gamma}\eta^\gamma), \qquad \eta^\alpha|_{\Sigma_0} = \hat{n}^\alpha \Rightarrow \quad (42)$$

$$\eta^\alpha = \delta^\alpha{}_0 + \frac{\Lambda}{3}\left(\frac{s^2}{2}\delta^\alpha{}_0 + (\tau-\tau_0)\xi^i\delta^\alpha{}_i\right) + o(\Lambda^2). \quad (43)$$

From this, it follows that in the synchronous gauge,

$$\tilde{h}^{\alpha\beta}\eta_\beta = 0 \Rightarrow \tilde{h}^{00} = \frac{\Lambda}{3}(\tau-\tau_0)\tilde{h}^{0i}\xi_i,$$
$$\tilde{h}^{0i} = \frac{\Lambda}{3}(\tau-\tau_0)\tilde{h}^{ij}\xi_j. \quad (44)$$

Clearly, $\tilde{h}^{00}\sim o(\Lambda^2)$ and can be set to be zero while $\tilde{h}^{0i}$ is completely determined by $\tilde{h}^{ij}$. It will turn out in the next section that *for TT-projected* $\tilde{h}^{ij}$, $\tilde{h}^{0i}=0$. Therefore, we now specialize to the spatial components, $\mu=m$, $\nu=n$.

Keeping only the leading powers in s'/s, the expressions simplify, and we obtain the parallel propagator as

$$g^m{}_{\alpha'}(\tau,\vec{\xi},\tau'_{\rm ret},\vec{\xi}')\approx\delta^m{}_{\alpha'}+\frac{\Lambda s^2}{18}\left[\delta^m{}_{\alpha'}+3\delta^0{}_{\alpha'}\frac{\xi^m}{s}-\delta^j{}_{\alpha'}\frac{\xi_j\xi^m}{s^2}\right]. \tag{45}$$

Note that it is independent of the source point $(\tau',\vec{\xi}')$, thanks to the leading s'/s approximation. It is also independent of τ.

Now, we have assembled all the terms in Eq. (27). The τ' integration exhausts the first factor in the $\sqrt{-g}$, and we get

$$\tilde{h}^{\mu\nu}(\tau,\vec{\xi})=\frac{4}{s}\left(1+\frac{\Lambda s^2}{18}\right)g^\mu{}_{\alpha'}(\vec{\xi})g^\nu{}_{\beta'}(\vec{\xi})\times\int d^3\xi'\sqrt{g_3(\xi')}T^{\alpha'\beta'}(\tau_{\rm ret},\vec{\xi}'). \tag{46}$$

The integral over the source is usually expressed in terms of time derivatives of moments, using the conservation of the stress tensor. To make these integrals well defined, it is convenient and transparent to introduce a suitable orthonormal tetrad and convert the coordinate components to frame components. The frame components are coordinate scalars (although they change under Lorentz transformations), and their integrals are well defined. In the FNC chart, there is a natural choice provided by the $\tau'=$ constant hypersurface passing through the source world tube. At any point on this hypersurface, we have a unique orthonormal triad obtained from the triad on the reference curve by parallel transport along the spatial geodesic. The unit normal, n^α, together with this triad, $e^\alpha{}_m$, $m=1,2,3$, provides the frame, $e^\alpha{}_a$. Explicitly, to order Λ,

$$n^\tau(\tau,\vec{\xi}')=1+\frac{\Lambda s'^2}{6},\quad n^i=0,\qquad e^\tau_m(\tau,\vec{\xi}')=0,$$
$$e^i_m(\tau,\vec{\xi}')=\left(1+\frac{\Lambda s'^2}{18}\right)\delta^i_m-\frac{\Lambda}{18}\xi'^i\xi'_m.$$

In a more compact form (*underlined indices denote frame indices*),

$$e^{\alpha'}_{\underline{a}}\coloneqq\left(1+\frac{\Lambda s'^2}{6}\right)\delta^{\alpha'}{}_\tau\delta^0_{\underline{a}}+\delta^{\alpha'}_i\delta^j_{\underline{a}}\left\{\delta^i_{\underline{j}}\left(1+\frac{\Lambda s'^2}{18}\right)-\frac{\Lambda}{18}\xi'^i\xi'_{\underline{j}}\right\}. \tag{47}$$

It is easy to check that $e^{\alpha'}_{\underline{a}}e^{\beta'}_{\underline{b}}g_{\alpha'\beta'}=\eta_{\underline{a}\,\underline{b}}$. It follows that

$$g^m{}_{\alpha'}(x)e^{\alpha'}_{\underline{a}}(x')\simeq\delta^m_{\underline{a}}\left(1+\frac{\Lambda s^2}{18}\right)+\frac{\Lambda}{6}\delta^0_{\underline{a}}s\xi^m-\frac{\Lambda}{18}\delta^j_{\underline{a}}\xi_{\underline{j}}\xi^m. \tag{48}$$

Defining the frame components of the stress tensor through the relation $T^{\mu\nu}\coloneqq e^\mu{}_{\underline{a}}e^\nu{}_{\underline{b}}\Pi^{\underline{ab}}$ and substituting for $g^m_{\alpha'}e^{\alpha'}_{\underline{a}}(\tau,\vec{\xi},\tau',\vec{\xi}')$, we obtain the final expression for the solution in the synchronous gauge, to leading order in s'/s and to $o(\Lambda)$, as

$$\begin{aligned}\tilde{h}^{mn}(\tau,\vec{\xi})=\frac{4}{s}\left(1+\frac{\Lambda s^2}{18}\right)&\left[\left(1+\frac{\Lambda s^2}{9}\right)\delta^m_{\underline{m}}\delta^n_{\underline{n}}\right.\\&\times\int\sqrt{g_3(\vec{\xi}')}\Pi^{\underline{mn}}(\tau_{\rm ret},\vec{\xi}')\\&+\frac{\Lambda s}{6}\left\{\xi^m\delta^n_{\underline{n}}\int\sqrt{g_3(\vec{\xi}')}\Pi^{\underline{0n}}+\xi^n\delta^m_{\underline{m}}\right.\\&\left.\times\int\sqrt{g_3(\vec{\xi}')}\Pi^{\underline{0m}}\right\}-\frac{\Lambda}{18}\left\{\xi^m\delta^n_{\underline{n}}\xi_{\underline{k}}\right.\\&\left.\left.\times\int\sqrt{g_3(\vec{\xi}')}\Pi^{\underline{kn}}+\xi^n\delta^m_{\underline{m}}\xi_{\underline{k}}\int\sqrt{g_3(\vec{\xi}')}\Pi^{\underline{km}}\right\}\right].\end{aligned} \tag{49}$$

The stress tensor is a function of $(\tau_{\rm ret},\vec{\xi}')$, $\tau_{\rm ret}$ being defined in Eq. (35). The terms in the last three lines will drop out when a suitable TT (transverse, traceless) projection is applied to the above solution to extract its *gauge invariant* content, in Sec. IV. Each of these integrals over the source on a $\tau'=$ constant hypersurface are well defined and give a quantity which is a function of the retarded time and carry only the frame indices. The explicit factors of the mixed-indexed δ's are a constant triad which serve to convert the integrated quantities from frame indices to coordinate indices.

To express the source integrals in terms of moments, we have to consider the conservation equation.

6. Conservation equation

The conservation equation is $\partial_\mu T^{\mu\nu}=-\Gamma^\mu{}_{\mu\lambda}T^{\lambda\nu}-\Gamma^\nu{}_{\mu\lambda}T^{\mu\lambda}$, and we have computed the connection in FNC in Eq. (40). Recalling that the stress tensor is trace free, $T^{\mu\nu}\bar{g}_{\mu\nu}=T^{\mu\nu}(\eta_{\mu\nu}+\delta g_{\mu\nu})=(-T^{00}+T^j{}_j)+T^{\mu\nu}\delta g_{\mu\nu}=0$, we eliminate the spatial trace by using $T^j{}_j=T^{00}-T^{\mu\nu}\delta g_{\mu\nu}$. The second term is order Λ. To within our approximation and *momentarily suppressing the primes on the coordinates*, we find

$$\partial_0T^{00}+\partial_iT^{i0}=\frac{11\Lambda}{9}T^{0i}\xi_i \tag{50}$$

$$\partial_0T^{0i}+\partial_jT^{ji}=\frac{\Lambda}{9}(7T^{ij}\xi_j+T^{00}\xi^i). \tag{51}$$

Taking second derivatives and eliminating T^{0i}, we get

$$\partial^2_{ij}T^{ij} = \partial^2_0 T^{00} + \frac{\Lambda}{9}\{10T^{00} + \xi^j\partial_j T^{00} + 18\xi_i\partial_j T^{ij}\}. \tag{52}$$

Introducing the notation, $\rho \coloneqq \Pi^{\underline{00}}$, $\pi \coloneqq \Pi^{\underline{ij}}\delta_{ij}$, we express the coordinate components of the stress tensor in terms of the frame components as

$$\begin{aligned} T^{00} &= \delta^0{}_{\underline{0}}\delta^0{}_{\underline{0}}\left(1 + \frac{\Lambda s^2}{3}\right)\rho; \\ T^{0i} &= \delta^0{}_{\underline{0}}\left[\left(1 + \frac{2\Lambda s^2}{9}\right)\delta^i{}_{\underline{j}} - \frac{\Lambda}{18}\xi^i\xi_{\underline{j}}\right]\Pi^{\underline{0j}} \\ T^{ij} &= \left[\left(1 + \frac{\Lambda s^2}{9}\right)\delta^i{}_{\underline{k}}\delta^j{}_{\underline{l}} - \frac{\Lambda}{18}(\delta^i{}_{\underline{k}}\xi^j\xi_{\underline{l}} + \delta^j{}_{\underline{l}}\xi^i\xi_{\underline{k}})\right]\Pi^{\underline{kl}}. \end{aligned} \tag{53}$$

In terms of the frame components, the conservation equations take the form (the constant tetrad are suppressed)

$$\partial_\tau\Pi^{\underline{00}} = -\left(1 - \frac{\Lambda s^2}{9}\right)\partial_j\Pi^{\underline{0j}} + \frac{\Lambda}{18}\xi_j\xi^i\partial_i\Pi^{\underline{0j}} + \Lambda\Pi^{\underline{0j}}\xi_j \tag{54}$$

$$\partial_\tau\Pi^{\underline{0i}} = -\left(1 - \frac{\Lambda s^2}{9}\right)\partial_j\Pi^{\underline{ji}} + \frac{\Lambda}{18}\{\xi_j\xi\cdot\partial\Pi^{\underline{ji}} + 15\Pi^{\underline{ij}}\xi_j + 3\rho\xi^i\}. \tag{55}$$

Eliminating $\Pi^{\underline{0i}}$ and using $\pi = \rho$ thanks to the trace-free stress tensor, we get the second order conservation equation as

$$\partial^2_\tau\rho = \left(1 - \frac{2\Lambda s^2}{9}\right)\partial^2_{ij}\Pi^{\underline{ij}} - \frac{\Lambda}{9}[\xi^i\xi_j\partial^2_{ik}\Pi^{\underline{jk}} + 19\xi_i\partial_j\Pi^{\underline{ij}} + 2\xi^j\partial_j\rho + 12\rho]. \tag{56}$$

The usual strategy is to define suitable moments of energy density/pressures and, taking moments of the above equation, express the integral of $\Pi^{\underline{ij}}$ in terms of the moments and its time derivatives. To maintain coordinate invariance, the moment variable (analog of x^i in the Minkowski background) must also be a *coordinate scalar*. Note that in FNC (as in RNC), ξ^i is a contravariant vector. Its frame components naturally provide coordinate scalars. We still have the freedom to multiply these by suitable scalar functions. It is easy to see that the frame components of ξ are the same as the coordinate components at best up to permutations, i.e., $\xi^{\underline{i}} \coloneqq e^{\underline{i}}_j\xi^j = \delta^{\underline{i}}_j\xi^j$. It is also true that $g_{ij}\xi^i\xi^j = \xi^{\underline{i}}\xi^{\underline{j}}\delta_{ij} = s^2$. Hence, suitable functions of s^2 would qualify to be considered as coordinate scalars.

In Eq. (49), we need $\int d^3\xi'\sqrt{g_3(\vec{\xi'})}$. To get this from Eq. (56), we introduce a moment variable $\zeta^i(\vec{\xi})$ and *define* moments of ρ as

$$\begin{aligned} \mathcal{M}^{\underline{i_1 i_2}\cdots\underline{i_n}}(\tau) &\coloneqq \int_{\text{source}} d^3\xi\sqrt{g_3(\vec{\xi})}\zeta^{\underline{i_1}}\cdots\zeta^{\underline{i_n}}\rho(\tau,\vec{\xi}), \\ \zeta^{\underline{i}}(\vec{\xi}) &\coloneqq \left(1 + \frac{\Lambda s^2}{9}\right)\xi^{\underline{i}}, \end{aligned} \tag{57}$$

where the integration is over the support of the source on the constant—τ hypersurface.

Multiplying Eq. (56) by $\sqrt{g_3(\vec{\xi'})}\zeta^{\underline{i_1}}\ldots\zeta^{\underline{i_n}}$ and integrating over the source, we get

$$\begin{aligned} \ddot{\mathcal{M}}^{\underline{i_1}\cdots\underline{i_n}} &= \int d^3\xi\,\Pi^{\underline{ij}}\partial^2_{ij}\left(\left(1 + \frac{(n-3)\Lambda s^2}{9}\right)\xi^{\underline{i_1}\cdots\underline{i_n}}\right) \\ &\quad - \frac{\Lambda}{9}\left[\int d^3\xi\,\Pi_{\underline{j}}{}^{\underline{k}}\partial^2_{ik}(\xi^{\underline{i}}\xi^{\underline{j}}\xi^{\underline{i_1}\cdots\underline{i_n}}) \right. \\ &\quad + 19\int d^3\xi\,\Pi^{\underline{ij}}\partial_j(\xi_{\underline{i}}\xi^{\underline{i_1}\cdots\underline{i_n}}) \\ &\quad \left. + 2\int d^3\xi\,\rho\partial_j(\xi^{\underline{j}}\xi^{\underline{i_1}\cdots\underline{i_n}}) + 12\int d^3\xi\,\rho\xi^{\underline{i_1}\cdots\underline{i_n}}\right]. \end{aligned} \tag{58}$$

There are no factors of $(1 - \Lambda s^2/9)$ in the terms enclosed by the square brackets since there is already an explicit prefactor of Λ.

The first few moments satisfy,

$$\partial^2_\tau\mathcal{M} = \frac{\Lambda}{3}\mathcal{M}, \quad (\text{‘Mass conservation’}) \tag{59}$$

$$\partial^2_\tau\mathcal{M}^{\underline{i}} = \frac{2\Lambda}{3}\mathcal{M}^{\underline{i}} + \frac{2\Lambda}{3}\int d^3\xi\Pi^{\underline{ij}}\xi_{\underline{j}}, \quad (\text{‘Momentum conservation’}) \tag{60}$$

$$\partial^2_\tau\mathcal{M}^{\underline{ij}} = 2\int_{\text{source}} d^3\xi\sqrt{g_3(\vec{x})}\Pi^{\underline{ij}} + \Lambda\mathcal{M}^{\underline{ij}} + \Lambda\int d^3\xi\xi_{\underline{k}}(\Pi^{\underline{ki}}\xi^{\underline{j}} + \Pi^{\underline{kj}}\xi^{\underline{i}}). \tag{61}$$

There are additional types of integrals over $\Pi^{\underline{0n}}$ and $\xi\cdot\Pi\cdot\xi$ in Eq. (49). But these come with an explicit factor of Λ which simplifies the calculation. These can be expressed in terms of different moments using both the second order conservation equation (56) and the first order one (54). In particular, taking the fourth moment and tracing over a pair gives [to order $(\Lambda)^0$],

$$\int d^3\xi \xi_{\underline{k}}(\Pi^{\underline{km}}\xi^{\underline{n}} + \Pi^{\underline{kn}}\xi^{\underline{m}}) = \frac{1}{4}[\delta_{\underline{rs}}\partial_\tau^2 \mathcal{M}^{\underline{mnrs}} - 2\mathcal{M}^{\underline{mn}} - 2\mathcal{N}^{\underline{mn}}], \quad \text{where}$$

$$\mathcal{N}^{\underline{mn}} := \int d^3\xi \sqrt{g_3(\vec{\xi})}\,\Pi^{\underline{mn}} s^2. \quad (62)$$

Likewise, taking the first moment of Eq. (54), we get

$$\int d^3\xi \sqrt{g_3(\vec{\xi})}\Pi^{\underline{0n}} = \partial_\tau \mathcal{M}^{\underline{n}}. \quad (63)$$

Collecting all these, we write the solution in the form

$$\begin{aligned}\tilde{h}^{mn}(\tau,\vec{\xi}) = \delta^m{}_{\underline{m}}\delta^n{}_{\underline{n}}\Bigg[&\left(\frac{2}{s}\partial_\tau^2\mathcal{M}^{\underline{mn}}\right) \\ &-\frac{\Lambda}{3s}\left(\xi_{\underline{k}}\frac{\xi^{\underline{m}}\partial_\tau^2\mathcal{M}^{\underline{kn}} + \xi^{\underline{n}}\partial_\tau^2\mathcal{M}^{\underline{km}}}{3} - s^2\partial_\tau^2\mathcal{M}^{\underline{mn}}\right) \\ &+\frac{\Lambda}{s}\left(-\mathcal{M}^{\underline{mn}} + \mathcal{N}^{\underline{mn}} - \frac{1}{2}\delta_{\underline{rs}}\partial_\tau^2\mathcal{M}^{\underline{mnrs}}\right) \\ &+\frac{2\Lambda}{3}(\xi^{\underline{m}}\partial_\tau\mathcal{M}^{\underline{n}} + \xi^{\underline{n}}\partial_\tau\mathcal{M}^{\underline{m}})\Bigg]. \quad (64)\end{aligned}$$

The moments on the right-hand side are all evaluated at the retarded τ, and we have displayed the constant triad. The constant triad plays no role here, but a similar one in the next subsection is important.

There are several noteworthy points:

(1) The leading term has exactly the same form as for the usual flat space background. The correction terms involve the first, the second, and the fourth moments as well as a new type of moment $\mathcal{N}^{\underline{mn}}$. We will see in the next section that the term involving ξ^i will drop out in a TT projection.

(2) There are terms which have no time derivative of any of the moments and hence can have constant (in time) field. This is a new feature not seen in the Minkowski background. *A priori*, such a term is permitted even in the Minkowski background. For instance, if $\partial_\tau T^{\alpha\beta} = 0$, i.e., the source is static, then Eq. (46) or Eq. (49) implies that $\partial_\tau \tilde{h}^{mn} = 0$, and hence the solution can have a τ-independent piece. However, in this case ($\Lambda = 0$), the conservation equation (61) equation relates the field to the double τ derivative of the quadrupole moment which vanishes for a static source. It reflects the physical expectation that a static source does not radiate. Does this expectation change in a curved background?

In a general curved background, "staticity" could be defined in a coordinate invariant manner only if there is a timelike Killing vector, say, T. A source would then be called static if the Lie derivative of the stress tensor vanishes, $\mathcal{L}_T T^{\alpha\beta} = 0$. In the de Sitter background, in the static patch we are working in, the stationary Killing vector is precisely ∂_τ. Hence, from the definition of moments (57), (47), it follows that for a static source, $\partial_\tau T^{\alpha\beta} = 0$, all its moments would be τ independent. However, the conservation equations (59) for the zeroth moment[4] contradict this, unless $\mathcal{M}$ itself vanishes. Hence, we cannot even have strictly static (test) sources in a curved background. Thus, in the specific case of the de Sitter background, the nonderivative terms in Eq. (61) do not indicate the possibility of the time-independent field $\tilde{h}^{mn}$.

For a very slowly varying source—so that we can neglect the derivative terms—we can have a leftover, slowly varying field, falling off as $\sim\Lambda/s$. Such a field has a very long wavelength and is not "radiative" in the static patch. To isolate radiative fields, one should probe the vicinity of the null infinity which is beyond the extent of the static patch. For typical rapidly changing sources ($\lambda \ll s$) the τ-derivative terms dominate over these terms, and in the context of present focus, we *drop them hereafter*.

The remaining terms that survive the TT projection all have a second order τ derivative. Similar features also arise in the cosmological chart in the next subsection.

(3) The mass conservation equation can be immediately integrated and have exponentially growing and decaying components. The scale of this time variation is $\sim(\Lambda)^{-1/2}$ which is extremely slow, about the age of the Universe. These equations do not depend on the Green function at all and are just consequences of the matter conservation equation for small curvature. We are working in a static patch of the space-time, so the time variation is not driven by the time dependence of the background geometry. It is the background *curvature* that is responsible for the changes in the matter distribution and hence its moments. In effect, this confirms that test matter cannot remain static in a *curved* background even if the background is static. In a flat background, there is no work done on the test matter, and hence the sources' mass and linear momenta are conserved (the zeroth and the first moment are time independent).

For contrast, in the next subsection, we recall the computation in the generalized transverse gauge [8]. This subsection has the tail contribution explicitly available, and the correction terms are in powers of $\sqrt{\Lambda}$.

[4]The nonzero curvature always does "work" on the test matter, and the "mass of the matter" alone is not conserved.

B. Generalized transverse gauge in Poincaré patch

The computation takes advantage of the conformally flat form of the metric in the conformal chart and makes a choice of a generalized transverse gauge to simplify the linearized equation. We summarize them for convenience and present the radiative solution.

In the conformal chart (see Fig. 4), the coordinates and the metric take the form

$$z^0 = H^{-1}\sinh(Ht) + Hr^2\frac{e^{Ht}}{2},$$
$$z^1 = H^{-1}\cosh(Ht) - Hr^2\frac{e^{Ht}}{2}, \qquad (\therefore\ z^0 + z^1 > 0),$$
$$z^i = e^{Ht}x^i, \qquad i = 2,3,4, \qquad r^2 \coloneqq \sum_i (x^i)^2, \qquad t,\ x^i \in \mathbb{R}; \tag{65}$$

$$ds^2 = -dt^2 + e^{2Ht}\sum_{i=2}^{4}(dx^i)^2. \quad \text{The substitution,}$$
$$\eta \coloneqq -H^{-1}e^{-Ht} \Rightarrow, \tag{66}$$

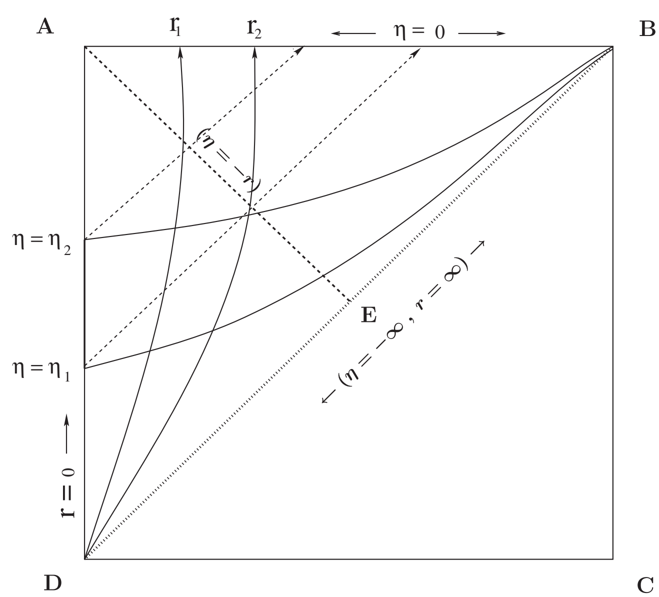

FIG. 4. The full square is the Penrose diagram of de Sitter space-time with generic point representing a 2-sphere. The Poincaré patch labeled ABD is covered by the conformal chart (η, r, θ, ϕ). The line BD does not belong to the chart. The line AB is the *future null infinity*, $\mathcal{J}^+$, and the line AE is the *cosmological horizon*. Two constant η spacelike hypersurfaces are shown with $\eta_2 > \eta_1$. The two constant r timelike hypersurfaces have $r_2 > r_1$. The two dotted lines at 45 deg denote the paths of gravitational waves emitted at $\eta = \eta_1, \eta_2$ on the world line at $r = 0$, through the source. During the interval (η_1, η_2), the source is "active," i.e., varying rapidly enough to be in the detectable range of frequencies. The region AED is a static patch.

$$ds^2 = \frac{1}{H^2\eta^2}\left[-d\eta^2 + \sum_i (dx^i)^2\right], \qquad \eta \in (-\infty, 0),$$
$$H \coloneqq \sqrt{\frac{\Lambda}{3}}. \tag{67}$$

The conformally flat form leads to a great deal of simplification. The $\mathcal{J}^+$ is approached as $\eta \to 0_-$, while the $\eta \to -\infty$ corresponds to the Friedmann-Lemaître-Robertson-Walker singularity.

In this chart, the de Sitter d'Alembertian can be conveniently expressed in terms of the Minkowski d'Alembertian, leading to

$$\begin{aligned} 0 = \Omega^{-2}\Bigg[&\Box\tilde{h}_{\mu\nu} + \frac{2}{\eta}\{(\delta^0_\mu\partial^\sigma\tilde{h}_{\sigma\nu} + \delta^0_\nu\partial^\sigma\tilde{h}_{\sigma\mu}) \\ &+ (-\partial_0\tilde{h}_{\mu\nu} + \partial_\mu\tilde{h}_{0\nu} + \partial_\nu\tilde{h}_{0\mu})\} \\ &+ \frac{2}{\eta^2}\{\delta^0_\mu\delta^0_\nu\tilde{h}_{\alpha\beta}\eta^{\alpha\beta} + \eta_{\mu\nu}\tilde{h}_{00} + 2(\delta^0_\mu\tilde{h}_{0\nu} + \delta^0_\nu\tilde{h}_{0\mu})\}\Bigg] \\ &- \left(\frac{2\Lambda}{3}\right)[\tilde{h}_{\mu\nu} - \eta_{\mu\nu}\tilde{h}_{\alpha\beta}\eta^{\alpha\beta}] \\ &- \left\{(\partial_\mu B_\nu + \partial_\nu B_\mu - \eta_{\mu\nu}\partial^\alpha B_\alpha) + \frac{2}{\eta}(\delta^0_\mu B_\nu + \delta^0_\nu B_\mu)\right\}, \end{aligned}$$
$$\Omega^2 \coloneqq \frac{1}{H^2\eta^2} = \frac{3}{\Lambda\eta^2}. \tag{68}$$

The left-hand side will be $-16\pi T_{\mu\nu}$ in the presence of matter.

While the transverse gauge will eliminate the B_μ terms, it still keeps the linearized equation in a form that mixes different components of $\tilde{h}_{\mu\nu}$. A different choice of B_μ achieves decoupling of these components. Taking B_μ of the form $f(\eta)\tilde{h}_{0\mu}$ shows that for the choice $f(\eta) \coloneqq \frac{2\Lambda}{3}\eta$, the equation (with source included) simplifies to [8]

$$\begin{aligned} -16\pi T_{\mu\nu}\Omega^2 = \Box\tilde{h}_{\mu\nu} - \frac{2}{\eta}\partial_0\tilde{h}_{\mu\nu} - \frac{2}{\eta^2}\{&\delta^0_\mu\delta^0_\nu\tilde{h}^\alpha_\alpha - \tilde{h}_{\mu\nu} \\ &+ \delta^0_\mu\tilde{h}_{0\nu} + \delta^0_\nu\tilde{h}_{0\mu}\}, \quad \text{with} \end{aligned} \tag{69}$$

$$0 = \partial^\alpha\tilde{h}_{\alpha\mu} + \frac{1}{\eta}\delta^0_\mu\tilde{h}^\alpha_\alpha, \qquad \tilde{h}^\alpha_\alpha \coloneqq \tilde{h}_{\alpha\beta}\eta^{\alpha\beta}$$
$$\text{(gauge fixing condition).} \tag{70}$$

From now on in this subsection, the tensor indices are raised/lowered with the Minkowski metric.

It turns out to be convenient to work with new variables, $\chi_{\mu\nu} \coloneqq \Omega^{-2}\tilde{h}_{\mu\nu}$. All factors of Ω^2 *and* Λ drop out of the equations, and $\chi_{\mu\nu}$ satisfies [8]

$$-16\pi G T_{\mu\nu} = \Box\chi_{\mu\nu} + \frac{2}{\eta}\partial_0\chi_{\mu\nu} - \frac{2}{\eta^2}(\delta^0_\mu\delta^0_\nu\chi_\alpha{}^\alpha + \delta^0_\mu\chi_{0\nu} + \delta^0_\nu\chi_{0\mu}), \tag{71}$$

$$0 = \partial^\alpha\chi_{\alpha\mu} + \frac{1}{\eta}(2\chi_{0\mu} + \delta^0_\mu\chi_\alpha{}^\alpha) \quad \text{(gauge condition)}. \tag{72}$$

Under the gauge transformations generated by a vector field ξ^μ, the $\chi_{\mu\nu}$ transform as

$$\delta\chi_{\mu\nu} = (\partial_\mu\underline{\xi}_\nu + \partial_\nu\underline{\xi}_\mu - \eta_{\mu\nu}\partial^\alpha\underline{\xi}_\alpha) - \frac{2}{\eta}\eta_{\mu\nu}\underline{\xi}_0,$$
$$\underline{\xi}_\mu \coloneqq \Omega^{-2}\xi_\mu = \eta_{\mu\nu}\xi^\nu. \tag{73}$$

The gauge condition (72) is preserved by the transformation generated by a vector field ξ^μ satisfying

$$\Box\underline{\xi}_\mu + \frac{2}{\eta}\partial_0\underline{\xi}_\mu - \frac{2}{\eta^2}\delta^0_\mu\underline{\xi}_0 = 0, \tag{74}$$

and Eq. (71) is invariant under the gauge transformations generated by these restricted vector fields.

It is further shown in Ref. [8] that the residual invariance is exhausted by setting $\chi_{0i} = 0 = \hat{\chi}(\coloneqq \chi_{00} + \chi_i{}^i)$. The gauge condition (72) then implies $\partial^0\chi_{00} = 0$, and by choosing it to be zero at some initial $\eta =$ constant hypersurface, we can take $\chi_{00} = 0$ as well. Thus, the physical solutions satisfy conditions $\partial^i\chi_{ij} = 0 = \chi^i{}_i$, and it suffices to focus on Eq. (71) for $\mu, \nu = i, j$.

To obtain the inhomogeneous solution, we return to the equations satisfied by the χ_{00}, χ_{0i}, and χ_{ij}, which are decoupled, and we are interested only in the χ_{ij} equation:

$$\Box\chi_{ij} + \frac{2}{\eta}\partial_0\chi_{ij} = -16\pi T_{ij}, \qquad \partial_i\chi^i{}_j = 0 = \chi_i{}^i.$$

The corresponding, retarded Green function is defined by

$$\left(\Box + \frac{2}{\eta}\partial_0\right)G_R(\eta, x; \eta', x') = -\frac{\Lambda}{3}\eta^2\delta^4(x - x') \tag{75}$$

and is given by [8]

$$G_R(\eta, x; \eta' x') = \frac{\Lambda}{3}\eta\eta'\frac{1}{4\pi}\frac{\delta(\eta - \eta' - |x - x'|)}{|x - x'|} + \frac{\Lambda}{3}\frac{1}{4\pi}\theta(\eta - \eta' - |x - x'|). \tag{76}$$

The particular solution is given by

$$\chi_{ij}(\eta, x) = 16\pi\int_{\text{source}}\frac{d\eta' d^3x'}{\frac{\Lambda}{3}\eta'^2}G_R(\eta, x; \eta' x')T_{ij}(\eta', x') \tag{77}$$

$$= 4\int d\eta' d^3x' \frac{\eta}{\eta'}\frac{\delta(\eta - \eta' - |x - x'|)}{|x - x'|}T_{ij}(\eta', x') + 4\int d\eta' d^3x' \frac{1}{\eta'^2}\theta(\eta - \eta' - |x - x'|)T_{ij}(\eta', x') \tag{78}$$

$$= 4\int d^3x' \frac{\eta}{|x - x'|(\eta - |x - x'|)}T_{ij}(\eta', x')|_{\eta' = \eta - |x - x'|} + 4\int d^3x' \int_{-\infty}^{\eta - |x - x'|} d\eta' \frac{T_{ij}(\eta', x')}{\eta'^2}. \tag{79}$$

The spatial integration is over the matter source confined to a compact region and is finite. The second term in Eqs. (78) and (79) is the tail term.

It is possible to put the solution in the same form as in the case of flat background, in terms of suitable Fourier transforms with respect to η [8]. However, we work with the $(\eta, \vec{x})$-space.

For $|\vec{x}| \gg |\vec{x}'|$, we can approximate $|\vec{x} - \vec{x}'| \approx r \coloneqq |\vec{x}|$. This allows us separate out the $\vec{x}'$ dependence from the $\eta - |x - x'|$. In the first term, this leads to the spatial integral over $T_{ij}(\eta - r, x')$, while in the second term, we can interchange the order of integration again leading to the same spatial integral. The spatial integral of T_{ij} can be simplified using moments. This is done through the matter conservation equation using the conformally flat form of the metric,

$$\partial^\mu T_{\mu 0} + \frac{1}{\eta}(T_{00} + T^i_i) = 0, \qquad \partial^\mu T_{\mu i} + \frac{2}{\eta}T_{0i} = 0. \tag{80}$$

Taking derivatives of these equations to eliminate T_{0i}, we get

$$\partial^i\partial^j T_{ij} = \partial^2_\eta T_{00} - \frac{1}{\eta}\partial_\eta(T_{00} + T_i{}^i) + \frac{3}{\eta^2}(T_{00} + T_i{}^i) - \frac{2}{\eta}\partial_\eta T_{00}. \tag{81}$$

As in the previous section, we introduce a tetrad to define the frame components of the stress tensor. The conformal form of the metric suggests a natural choice, ($\sqrt{\Lambda/3} \eqqcolon H$),

$$f^\alpha_{\underline{0}} \coloneqq -H\eta(1, \vec{0}), \quad f^\alpha_{\underline{m}} \coloneqq -H\eta\delta^\alpha_{\underline{m}} \quad \Leftrightarrow \quad f^\alpha_{\underline{a}} \coloneqq -H\eta\delta^\alpha_{\underline{a}}, \tag{82}$$

The corresponding components of the stress tensor are given by

$$\rho \coloneqq P_{\underline{00}} = T_{\alpha\beta} f^\alpha{}_{\underline{0}} f^\beta{}_{\underline{0}} = H^2\eta^2 T_{00}\delta^0{}_{\underline{0}}\delta^0{}_{\underline{0}},$$
$$P_{\underline{ij}} \coloneqq T_{\alpha\beta} f^\alpha{}_{\underline{i}} f^\beta{}_{\underline{j}} = H^2\eta^2 T_{ij}\delta^i{}_{\underline{i}}\delta^j{}_{\underline{j}}; \tag{83}$$

$$P_{\underline{0i}} \coloneqq T_{\alpha\beta} f^\alpha{}_{\underline{0}} f^\beta{}_{\underline{i}}, \qquad \pi \coloneqq P_{\underline{ij}}\delta^{\underline{ij}}. \tag{84}$$

In terms of these, the conservation equations take the form (suppressing the constant tetrad)

$$0 = \partial_\eta P^{\underline{00}} + \partial_i P^{\underline{i0}} - \frac{1}{\eta}(3P^{\underline{00}} + \pi) \tag{85}$$

$$0 = \partial_\eta P^{\underline{0i}} + \partial_j P^{\underline{ji}} - \frac{1}{\eta} 4P^{\underline{0i}}. \tag{86}$$

It is convenient to go over to the cosmological chart $(t, \vec{x})$ and convert the ∂_η to ∂_t using the definitions: $\eta \coloneqq -H^{-1}e^{-Ht}$. This leads to $\partial_\eta = e^{Ht}\partial_t \coloneqq a(t)\partial_t$,

$$0 = \partial_t\rho + \frac{1}{a}\partial_i P^{\underline{0i}} + H(3\rho + \pi); \tag{87}$$

$$0 = \partial_t P^{\underline{0i}} + \frac{1}{a}\partial_j P^{\underline{ij}} + 4HP^{\underline{0i}}; \tag{88}$$

$$0 = \partial_t^2\rho - \frac{1}{a^2}\partial^2_{ij}P^{\underline{ij}} + 8H\partial_t\rho + H\partial_t\pi + 5H^2(3\rho + \pi). \tag{89}$$

As before, we define the moments of the two rotational scalars, ρ, π, by integrating over the source distribution at $\eta =$ constant hypersurface. The determinant of the induced metric on these hypersurfaces is $a^3(\eta)$. The tetrad components of the moment variable are given by $\bar{x}^{\underline{i}} \coloneqq f^{\underline{i}}_\alpha x^\alpha = -(\eta H)^{-1}\delta^{\underline{i}}{}_j x^j = a(t)x^{\underline{i}}$. The moments are defined by

$$Q^{\underline{i_1}\cdots\underline{i_n}}(t) \coloneqq \int_{\mathrm{Source}(t)} d^3x a^3(t)\rho(t,\vec{x})\bar{x}^{\underline{i_1}}\cdots\bar{x}^{\underline{i_n}}, \tag{90}$$

$$\bar{Q}^{\underline{i_1}\cdots\underline{i_n}}(t) \coloneqq \int_{\mathrm{Source}(t)} d^3x a^3(t)\pi(t,\vec{x})\bar{x}^{\underline{i_1}}\cdots\bar{x}^{\underline{i_n}}. \tag{91}$$

Taking the second moment of Eq. (81) and lowering the frame indices, we get

$$\int d^3x a^3(t)P_{\underline{ij}}(t,x) = \frac{1}{2}[\partial_t^2 Q_{\underline{ij}} - 2H\partial_t Q_{\underline{ij}} + H\partial_t\bar{Q}_{\underline{ij}}]. \tag{92}$$

Let us write the solution, Eq. (79), in terms of the cosmological chart, incorporating the approximation $|\vec{x}'| \ll |\vec{x}|$,

$$\chi_{ij}(\eta,x) = 4\frac{\eta}{r(\eta-r)}\int d^3x' T_{ij}(\eta',x')\Big|_{\eta'=\eta-r} + 4\int d^3x'\int_{-\infty}^{\eta-r} d\eta'\,\frac{T_{ij}(\eta',x')}{\eta'^2}. \tag{93}$$

Define the *retarded time*, t_{ret}, through $(\eta - r) \coloneqq -H^{-1}e^{-Ht_{\mathrm{ret}}}$, and set $\bar{a} \coloneqq a(t_{\mathrm{ret}})$. Then, we have $\eta = -(aH)^{-1}$, $(\eta - r) = -(\bar{a}H)^{-1}$. Using these,

$$\frac{\eta}{\eta-r}T_{ij}(\eta-r,x') = a(t)^{-1}\bar{a}^3 P_{\underline{ij}}(t_{\mathrm{ret}},x'),$$
$$d\eta'\frac{1}{\eta'^2}T_{ij}(\eta',x') = H^2 dt' a^3(t')P_{\underline{ij}}(t',x'). \tag{94}$$

All terms involve only the $\int a^3 P_{ij}$ which is obtained above. With these, the solution takes the form

$$\chi_{\underline{ij}}(t,r) \approx \frac{2}{ra(t)}[\partial_{t'}^2 Q_{\underline{ij}} - 2H\partial_{t'}Q_{\underline{ij}} + H\partial_{t'}\bar{Q}_{\underline{ij}}]\Big|_{t_{\mathrm{ret}}} + 2H^2\{\partial_{t'}Q_{\underline{ij}} - 2HQ_{\underline{ij}} + H\bar{Q}_{\underline{ij}}\}\Big|_{t_{\mathrm{ret}}} - 2H^2\{\partial_{t'}Q_{\underline{ij}} - 2HQ_{\underline{ij}} + H\bar{Q}_{\underline{ij}}\}\Big|_{-\infty}. \tag{95}$$

We have restored the constant triad and used the definition $\chi_{\underline{ij}} \coloneqq \delta_{\underline{i}}{}^i\delta_{\underline{j}}{}^j\chi_{ij}$. The first term in Eq. (95) is the contribution of the sharp term, and the remaining terms are from the tail. The tail contribution has separated into a term which depends on the retarded time just as the sharp term does, and the contribution from the history is given by the limiting value in the last line.

This expression is valid *as a leading term for* $|\vec{x}| \gg |\vec{x}'|$. (For the Hulse-Taylor system, the physical size is about 3 light seconds, and it is about 20,000 light years away, giving $|x'|/|x| \sim 10^{-12}$.) We work with this expression in the following and suppress the $\approx$ sign.

We write $a^{-1} = \bar{a}^{-1}(\frac{\bar{a}}{a}) = \bar{a}^{-1}(1 - Hr\bar{a}) = \bar{a}^{-1} - rH$ in the first term to make manifest the dependence on retarded time t_{ret}. The solution is then expressed as

$$\chi_{\underline{ij}}(t,r) \approx \frac{2}{r\bar{a}}\{\partial_t^2 Q_{\underline{ij}} - 2H\partial_t Q_{\underline{ij}} + H\partial_t\bar{Q}_{\underline{ij}}\} - 2H\{\partial_t^2 Q_{\underline{ij}} - 3H\partial_t Q_{\underline{ij}} + H\partial_t\bar{Q}_{\underline{ij}} + 2H^2 Q_{\underline{ij}} - H^2\bar{Q}_{\underline{ij}}\} - 2H^2\{\partial_{t'}Q_{\underline{ij}} - 2HQ_{\underline{ij}} + H\bar{Q}_{\underline{ij}}\}\Big|_{-\infty}. \tag{96}$$

A few remarks are in order:

(1) In the conformal chart, there is no explicit dependence on the cosmological constant, and it is not a suitable chart for exploring the subtle limit of vanishing cosmological constant [6,7]. Hence, we changed to the cosmological chart and exhibited the solution with explicit powers of H. Although the solution in Eq. (79) showed the presence of a tail

term as an integral over the history of the source, in the final expression, the field depends only on the properties of the source at the retarded time t_{ret} which was defined through $(\eta - r)$ except for the limiting value in the last line.

(2) Unlike the FNC chart, here the *tail contribution* has moments without a time derivative which naively indicates that for "static" sources, there could be a nonzero field. A coordinate invariant way of specifying staticity of a source is to refer to the Killing parameter of a stationary Killing vector in its vicinity, e.g., $T\cdot\partial \coloneqq -H(\eta\partial_\eta + x^i\partial_i) = \partial_t - Hx^i\partial_i$. (This also equals the ∂_τ of the FNC.) A static source satisfies $\mathcal{L}_T T_{\mu\nu} = T\cdot\partial T_{\mu\nu} - 2HT_{\mu\nu} = 0$. Explicitly, $\mathcal{L}_T f^{\alpha}{}_{\underline{a}} = 0$, and hence for a static source, $\mathcal{L}_T P_{\underline{ab}} = 0$. Furthermore, the Lie derivative of the moment variable $x^{\underline{i}} = ax^i$ also vanishes as does that of the volume element. Hence, $\mathcal{L}_T Q_{\underline{ij}} = 0$. Since the moments are coordinate scalars and independent of spatial coordinates, their Lie derivative is just ∂_t. Hence, for static sources, $\partial_t Q_{\underline{ij}} = 0 = \partial_t \bar{Q}_{\underline{ij}}$ (indeed *all* moments will be independent of t). For constant moments, there is a cancellation between the terms in the second and the third line of Eq. (95), and the field vanishes. The boundary term at $t = -\infty$ is essential for this cancellation.

However, the conservation equations for the zeroth and the first moments are

$$\partial_t Q + H\bar{Q} = 0,$$
$$\partial_t^2 Q_{\underline{i}} + H\partial_t\bar{Q}_{\underline{i}} - H^2(Q_{\underline{i}} - \bar{Q}_{\underline{i}}) = 0. \tag{97}$$

The equation for the zeroth moment can be derived directly from (87). These again show that in a curved background, test matter cannot remain static.

For very slowly varying moments, the sharp contribution is negligible, while the tail has a contribution, not falling off as r^{-1}. In FNC, the slowly varying contribution is in the sharp term but could not be thought of as "radiation." The absence of such a contribution in the sharp term in Eq. (95) suggests that the slowly varying sharp term of FNC [Eq. (64)] would not survive as radiation at $\mathcal{J}^+$, though of course this cannot be analyzed within the FNC chart. The surviving tail contribution has been thought of as inducing a linear memory effect in Ref. [19].

The contribution from the $t = -\infty$ boundary is in any case a constant and does not play any role in any physical observables which typically involve time derivatives. With this understood, we now suppress these boundary contributions.

(3) To link with Ref. [7], the final step involves replacement of ∂_t by the Lie derivative with respect to the stationary Killing vector. Using $\mathcal{L}_T Q_{\underline{ij}}(t_{\text{ret}}) = (\partial_t - Hx^i\partial_i)(t_{\text{ret}})\partial_{t_{\text{ret}}} Q_{\underline{ij}}(t_{\text{ret}}) = \partial_{t_{\text{ret}}} Q_{\underline{ij}}(t_{\text{ret}})$ and $\mathcal{L}_T \delta_i{}^{\underline{i}} = T\cdot\partial\delta_i{}^{\underline{i}} - H\delta_i{}^{\underline{i}}$, we get

$$\begin{aligned}\mathcal{L}_T Q_{ij} &= \mathcal{L}_T(\delta_i{}^{\underline{i}}\delta_j{}^{\underline{j}} Q_{\underline{ij}})\\ &= (\mathcal{L}_T\delta_i{}^{\underline{i}}\delta_j{}^{\underline{j}})Q_{\underline{ij}} + \delta_i{}^{\underline{i}}\delta_j{}^{\underline{j}}\partial_{t_{\text{ret}}} Q_{\underline{ij}}\\ &= \partial_{t_{\text{ret}}} Q_{ij} - 2HQ_{ij}.\end{aligned} \tag{98}$$

This is where the constant triad plays a role, unlike in the FNC chart where $\mathcal{L}_T = \partial_\tau$ on all tensors. With these translations, our solution in Eq. (96) takes the forms

$$\begin{aligned}\chi_{ij}(t,r) = {}&\frac{2}{r\bar{a}}\delta_i{}^{\underline{i}}\delta_j{}^{\underline{j}}[\partial_t^2 Q_{\underline{ij}} - 2H\partial_t Q_{\underline{ij}} + H\partial_t\bar{Q}_{\underline{ij}}](t)\\ &-2H\delta_i{}^{\underline{i}}\delta_j{}^{\underline{j}}[\partial_t^2 Q_{\underline{ij}} - 3H\partial_t Q_{\underline{ij}} + 2H^2 Q_{\underline{ij}}\\ &+H\partial_t\bar{Q}_{\underline{ij}} - H^2\bar{Q}_{\underline{ij}}](t),\end{aligned} \tag{99}$$

$$\begin{aligned}= {}&\frac{2}{r\bar{a}}[\mathcal{L}_T^2 Q_{ij} + 2H\mathcal{L}_T Q_{ij} + H\mathcal{L}_T\bar{Q}_{ij} + 2H^2\bar{Q}_{ij}]\\ &-2H[\mathcal{L}_T^2 Q_{ij} + H\mathcal{L}_T Q_{ij} + H\mathcal{L}_T\bar{Q}_{ij} + H^2\bar{Q}_{ij}].\end{aligned} \tag{100}$$

The terms on the right hand side are evaluated at $t = t_{\text{ret}}$. Both terms have the same derivatives of moments appearing in them and, on combining, lead to a coefficient of the form $((r\bar{a})^{-1} - H)$. Thus, in each order in H, the effect of the tail is to reduce the amplitude. Equation (100) matches with the solution given by Ashtekar *et al.* [7], and the $\Lambda \to 0$ limit of the solution goes over to the Minkowski background solution.

In the next section, we define the gauge invariant deviation scalar to compare the computations done in the two charts.

IV. TIDAL DISTORTIONS

The two solutions presented above were obtained in two different gauges. With a further choice of synchronous gauge, we could restrict the solutions to the spatial components alone. While these conditions fix the gauge completely, these spatial components still have to satisfy certain "spatial transversality and trace-free" conditions. The solutions obtained above do not satisfy these conditions and hence do not represent solutions of the original linearized Einstein equation. Their dependence on the retarded time and the "radial" coordinate, however, offers an easy way to construct solutions which *do* satisfy these spatial-TT conditions [18]. In flat background, this is achieved by the *algebraic* TT projector (defined below), and the method extends to the de Sitter background as well.

For χ_{ij}, the spatial-TT conditions have the form $\partial^j\chi_{ji} = 0 = \delta^{ij}\chi_{ij}$, which has exactly the same form as in the case of the Minkowski background. To deduce their form for the $\tilde{h}^{ij}$, consider $\tilde{h}^{\mu\nu}$ satisfying the TT gauge condition and the synchronous gauge condition: $\bar{\nabla}_\mu\tilde{h}^{\mu\nu} = 0 = \tilde{h}^{\mu\nu}\bar{g}_{\mu\nu}, \tilde{h}^{\alpha 0} = \frac{\Lambda}{3}(\tau-\tau_0)\tilde{h}^{\alpha i}\xi^j\delta_{ij}$. These imply $\tilde{h}^{\mu\nu} = h^{\mu\nu}$,

$$h^{00} = o(\Lambda^2), \qquad h^{0i} = \frac{\Lambda}{3}(\tau-\tau_0)h^{ij}\xi^k\delta_{jk};$$

$$h^{ij}\delta_{ij} = -\frac{\Lambda}{9}h^{ij}\xi_i\xi_j. \tag{101}$$

Furthermore,

$$\bar{\nabla}_\mu h^{\mu\nu} = \partial_\mu h^{\mu\nu} + \bar{\Gamma}^\mu{}_{\mu\lambda}h^{\lambda\nu} + \bar{\Gamma}^\nu{}_{\mu\lambda}h^{\mu\lambda} = 0 \quad \Rightarrow$$
$$\partial_\mu h^{\mu\nu} = \frac{5\Lambda}{9}h^{i\nu}\xi_i + \frac{2\Lambda}{3}\delta_0^\nu h^{0i}\xi_i + \frac{2\Lambda}{9}\delta_i^\nu(h^{ij} - h^{kl}\delta_{kl}\delta^{ij})\xi_j \quad \Rightarrow \tag{102}$$

$$\partial_j(h^{ji}\xi_i) = -\frac{\Lambda}{3}(\tau-\tau_0)\partial_\tau(h^{ij}\xi_i\xi_j) + \frac{5\Lambda}{9}\xi_i\xi_j h^{ji} \quad (\nu = 0) \tag{103}$$

$$\partial_j h^{ji} = -\frac{\Lambda}{3}(\tau-\tau_0)\partial_\tau(h^{ij}\xi_j) + \frac{4\Lambda}{9}\xi_j h^{ji} \quad (\nu = i). \tag{104}$$

Multiplying (104) by ξ_i, subtracting from (103), and using (101) implies that $h^{ij}\xi_i\xi_j = 0 = h^{ij}\delta_{ij}$. The $\xi_i\times$ (103) then implies that $\partial_j(h^{ij}\xi_i) = 0$, satisfying Eq. (103) identically. *Provided* $h^{ij}\xi_j = 0$, the spatial transversality condition $\partial_j h^{ij} = 0$ will be satisfied. The TT projector defined below will ensure $h^{ij}\xi_j = 0$ to the *leading order in* s^{-1}. Hence, the spatial transversality will also hold for the projected h^{ij} to the leading order in s^{-1}. The projector being local (algebraic) in space-time while the spatial TT conditions are nonlocal (differential), the projector ensures the condition only for large s. Elsewhere, the condition must be satisfied by adding solutions of the homogeneous wave equation. However, we need the explicit forms of the solution only in the large s regions for which the projector suffices.

As in the case of the Minkowski background, corresponding to each spatial, unit vector $\hat{n}$, define the projectors,

$$P^i_j(\hat{n}) := \delta^i{}_j - \hat{n}^i\hat{n}_j,$$
$$\Lambda^{ij}{}_{kl} := \frac{1}{2}(P^i{}_kP^j{}_l + P^i{}_lP^j{}_k - P^{ij}P_{kl}). \tag{105}$$

Contraction with $\hat{n}$ gives zero, and the trace of Λ projector in either pair of indices vanishes. From any X^{kl}, the Λ projector gives $X^{ij}_{TT} := \Lambda^{ij}{}_{kl}X^{kl}$ which is trace free and is transverse to the unit vector $\hat{n}$. For the FNC fields, we choose $\hat{n}^i := \vec{\xi}^i/s$, and for the conformal chart fields, we choose $\hat{n}^i = -H\eta\vec{x}^i/r$. When $\tilde{h}^{ij}_{TT}$ is substituted in Eq. (104), the condition reduces to the spatial transversality, $\partial_j\tilde{h}^{ij}_{TT} = 0$. The χ^{TT}_{ij} also satisfies the same condition: $\partial^j\chi^{TT}_{ij} = 0$.

Since $\hat{n}$ is a radial unit vector, It follows that $\partial_j\Lambda^{ij}{}_{kl} = \frac{1}{2r}(P^i{}_k\hat{n}_l + P^i{}_l\hat{n}_k)$ which is down by a power of r (or s for FNC). Therefore, to the leading order in r^{-1}, $\partial_j\tilde{h}^{ij}_{TT} = \Lambda^{ij}{}_{kl}\partial_j\tilde{h}^{kl}$.

Noting that the retarded solutions have a form $\sim f^{ij}(\tau-s)/s$, we get

$$\partial_j\left[\frac{f^{ij}(\tau-s)}{s}\right] = -\frac{1}{s^2}\xi_j(\partial_\tau f^{ij} + s^{-1}f^{ij})$$
$$\approx -\hat{\xi}_j\partial_\tau\left[\frac{f^{ij}(\tau-s)}{s}\right] + o\left(\frac{1}{s^2}\right),$$
$$\hat{\xi}_j := s^{-1}\xi_j.$$

It follows immediately that to the leading order in s^{-1} (or r^{-1}), $\partial_j\tilde{h}^{ij}_{TT} \approx -\partial_\tau(\hat{\xi}_j\tilde{h}^{ij}_{TT}) = 0$ (and likewise $\partial^j\chi^{TT}_{ij} = 0$). Note that, although to begin with the spatial TT conditions in FNC look different from those of the conformal chart, they have the same form after the corresponding Λ projections. Thus, for the Λ-projected h^{ij}, too, $h^{\alpha 0} = 0$, $\forall\ \alpha$. There is no plane wave assumption or spatial Fourier transform needed for this projection. Of course, the Λ projector only ensures that the gauge conditions are satisfied to the leading order in $r^{-1}(s^{-1})$. These Λ-projected fields represent physical perturbations, and gauge invariant observables of interest can be computed using these.

From now on, the solutions will be in the synchronous gauge and with TT projection implicit: $\tilde{h}^{\tau\beta} = 0, \tilde{h}^{ij} \leftrightarrow \tilde{h}^{ij}_{TT}$ and $\chi_{\eta\alpha} = 0, \chi_{ij} \leftrightarrow \chi^{TT}_{ij}$. In particular, $\tilde{h}^{ij} = h^{ij}$.

As an illustration, we consider the deviation induced in the nearby geodesics, as tracked by a freely falling observer. Thus, we consider a congruence of timelike geodesics of the *background space-time* and consider the tidal effects of a transient gravitational wave.

We begin with the observation that for all space-times satisfying $R_{\mu\nu} = \Lambda g_{\mu\nu}$ (which include the de Sitter background as well as its linearized perturbations in source-free regions) and for vectors u, Z, Z' satisfying $u\cdot Z = u\cdot Z' = Z\cdot Z' = 0$, the definition of the Weyl tensor implies

$$C_{\alpha\beta\mu\nu} - R_{\alpha\beta\mu\nu} = -\frac{\Lambda}{3}(g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\nu}g_{\beta\mu}) \Rightarrow \tag{106}$$

$$R_{\alpha\beta\mu\nu}Z'^\alpha u^\beta Z^\mu u^\nu = C_{\alpha\beta\mu\nu}Z'^\alpha u^\beta Z^\mu u^\nu + \frac{\Lambda}{3}\{(u\cdot u)(Z'\cdot Z) - (u\cdot Z')(u\cdot Z)\} \tag{107}$$

$$\therefore \quad D(u, Z, Z') := -R_{\alpha\beta\mu\nu} Z'^{\alpha} u^{\beta} Z^{\mu} u^{\nu} = -C_{\alpha\beta\mu\nu} Z'^{\alpha} u^{\beta} Z^{\mu} u^{\nu}. \tag{108}$$

The last equation shows the gauge invariance of $D(u, Z', Z)$. This is because the gauge transform of the Weyl tensor for the background is zero and the gauge transform of the $Z'uZu$ factor (it depends on the perturbation through the normalizations) does not contribute since the Weyl tensor of the de Sitter background itself is zero. Notice that $D(u, Z, Z')$ is symmetric in $Z \leftrightarrow Z'$ and is the component of acceleration of one deviation vector Z, along another orthogonal deviation vector.

A suitably chosen congruence of timelike geodesics, $u^{\alpha}\partial_{\alpha}$, provides a required pair of orthogonal deviation vectors for the gauge invariant observable $D(u, Z', Z)$ which we now refer to as *deviation scalar*. Since deviation vectors are always defined with respect to a geodesic congruence, we leave the argument u implicit and restore it in the final expressions. The deviation scalar is related to Weyl scalars as noted in Ref. [20]. We compute this for the Λ-projected solutions given in (64) and (96). Note that the dot products in the above equations involve the perturbed metric, $(\bar{g} + h)_{\mu\nu}$. For the explicit choices that we will make below, we denote the observer and the deviation vectors in the form $u = \bar{u} + \delta u, Z = \bar{Z} + \delta Z, Z' = \bar{Z}' + \delta Z'$ with the "barred" quantities normalized using the background metric while the "delta" quantities are treated as of the same order as the perturbed field. Thus,

$$\bar{u} \cdot \bar{\nabla} \bar{u}^{\alpha} = 0, \qquad \bar{u} \cdot \partial \bar{Z}^{\alpha} = \bar{Z} \cdot \partial \bar{u}^{\alpha}, \qquad \bar{u} \cdot \partial \bar{Z}'^{\alpha} = \bar{Z}' \cdot \partial \bar{u}^{\alpha};$$
$$\bar{u} \cdot \bar{u} = -1, \qquad \bar{u} \cdot \bar{Z} = \bar{u} \cdot \bar{Z}' = \bar{Z}' \cdot \bar{Z} = 0. \tag{109}$$

The delta quantities have to satisfy conditions so that the full quantities satisfy the requisite orthogonality relations with respect to the perturbed metric.

The deviation scalar is then given by

$$\begin{aligned} -D(Z', Z) &:= (\bar{g}_{\alpha\beta} + h_{\alpha\beta})(\bar{R}^{\alpha}{}_{\lambda\mu\nu} + R^{(1)\alpha}{}_{\lambda\mu\nu}) \\ &\quad \times (\bar{Z}'^{\beta} \bar{u}^{\lambda} \bar{Z}^{\mu} \bar{u}^{\nu} + \delta(Z'^{\beta} u^{\lambda} Z^{\mu} u^{\nu})) \\ &= \bar{g}_{\alpha\beta} \bar{R}^{\alpha}{}_{\lambda\mu\nu} \bar{Z}'^{\beta} \bar{u}^{\lambda} \bar{Z}^{\mu} \bar{u}^{\nu} + \bar{g}_{\alpha\beta} R^{(1)\alpha}{}_{\lambda\mu\nu} \bar{Z}'^{\beta} \bar{u}^{\lambda} \bar{Z}^{\mu} \bar{u}^{\nu} \\ &\quad + \bar{g}_{\alpha\beta} \bar{R}^{\alpha}{}_{\lambda\mu\nu} \delta(Z'^{\beta} u^{\lambda} Z^{\mu} u^{\nu}) \\ &\quad + h_{\alpha\beta} \bar{R}^{\alpha}{}_{\lambda\mu\nu} \bar{Z}'^{\beta} \bar{u}^{\lambda} \bar{Z}^{\mu} \bar{u}^{\nu} \end{aligned} \tag{110}$$

$$\begin{aligned} \therefore D(Z', Z) &= -R^{(1)\alpha}{}_{\lambda\mu\nu} \bar{Z}'_{\alpha} \bar{u}^{\lambda} \bar{Z}^{\mu} \bar{u}^{\nu} + \frac{\Lambda}{3} \bar{g}_{\alpha\beta} (\bar{Z}'^{\beta} \delta Z^{\alpha} + \delta Z'^{\beta} \bar{Z}^{\alpha}) \\ &\quad + \frac{\Lambda}{3} h_{\alpha\beta} \bar{Z}'^{\alpha} \bar{Z}^{\beta} \\ &= -R^{(1)\alpha}{}_{\lambda\mu\nu} \bar{Z}'_{\alpha} \bar{u}^{\lambda} \bar{Z}^{\mu} \bar{u}^{\nu} + \frac{\Lambda}{3} \delta(g_{\alpha\beta} Z'^{\alpha} Z^{\beta}). \end{aligned} \tag{111}$$

Here, $R^{(1)}$ refers to the Riemann tensor linear in $h_{\mu\nu}$. In Eq. (110), the first term vanishes thanks to the properties of the barred quantities, while in the third term, only one factor has a delta quantity. The only contributions that survive in the third and the fourth terms are the ones with $\bar{u}^2 = -1$. These terms combine (note the full metric in the last term), and Eq. (111) reflects this. Next,

$$R^{(1)\alpha}{}_{\lambda\mu\nu} = \bar{\nabla}_{\mu} \Gamma^{(1)\alpha}{}_{\nu\lambda} - \bar{\nabla}_{\nu} \Gamma^{(1)\alpha}{}_{\mu\lambda} \tag{112}$$

$$\Gamma^{(1)\alpha}{}_{\nu\lambda} = \frac{1}{2} \bar{g}^{\alpha\beta} (\bar{\nabla}_{\lambda} h_{\beta\nu} + \bar{\nabla}_{\nu} h_{\beta\lambda} - \bar{\nabla}_{\beta} h_{\nu\lambda}); \tag{113}$$

$$\begin{aligned} \therefore \; D(u, Z', Z) &= -\frac{1}{2} [\bar{Z}'^{\alpha} \bar{u}^{\lambda} \bar{Z}^{\mu} \bar{u}^{\nu} (\bar{\nabla}_{\mu} \bar{\nabla}_{\lambda} h_{\alpha\nu} - \bar{\nabla}_{\mu} \bar{\nabla}_{\alpha} h_{\nu\lambda} \\ &\quad - \bar{\nabla}_{\nu} \bar{\nabla}_{\lambda} h_{\alpha\mu} + \bar{\nabla}_{\nu} \bar{\nabla}_{\alpha} h_{\mu\lambda} \\ &\quad + [\bar{\nabla}_{\mu}, \bar{\nabla}_{\nu}] h_{\alpha\lambda})] + \frac{\Lambda}{3} \delta(g_{\alpha\beta} Z'^{\alpha} Z^{\beta}). \end{aligned} \tag{114}$$

Evaluating the commutator in the last term within the square brackets, we write it as $\frac{\Lambda}{3} h_{\alpha\beta} \bar{Z}'^{\alpha} \bar{Z}^{\beta}$. To proceed further, we need to make the choice of the congruence, the deviation vectors, and the delta quantities. This is done in the respective charts.

A natural class of timelike geodesics of the background geometry is suggested in the conformal chart. From Eq. (C5), we know that the curves $x^i = x_0^i$ are timelike geodesics. The corresponding, normalized velocity is given by $\bar{u}^{\alpha}(\eta, x^i) := -H\eta(1, \vec{0})$. The same family of geodesics is given in FNC as $\bar{u}^{\alpha} = (1 + \Lambda s^2/3, \sqrt{\Lambda/3}\vec{\xi})$. From now on, we will use $\Lambda/3 =: H^2$ for ease of comparison.

A. FNC chart of the static patch

From the explicit choice of the freely falling observer, we get the following consequences:

$$\bar{u} \cdot \bar{Z} = 0 \Rightarrow \bar{Z}^0 = H\vec{\xi} \cdot \vec{\bar{Z}},$$
$$\bar{u} \cdot \bar{Z}' = 0 \Rightarrow \bar{Z}'^0 = H\vec{\xi} \cdot \vec{\bar{Z}}'; \tag{115}$$

$$\vec{\xi} \cdot \vec{\bar{Z}} = 0 = \vec{\xi} \cdot \vec{\bar{Z}}' \Rightarrow \bar{Z}^0 = 0 = \bar{Z}'^0; \tag{116}$$

$$\bar{Z} \cdot \bar{Z}' = 0 \Rightarrow \vec{\bar{Z}}' \cdot \vec{\bar{Z}} = 0; \tag{117}$$

$$\bar{u} \cdot \partial \bar{Z}^{\alpha} = \bar{Z} \cdot \partial \bar{u}^{\alpha} \Rightarrow \bar{u} \cdot \partial \bar{Z}^i = H\bar{Z}^i,$$
$$\bar{u} \cdot \partial \bar{Z}'^{\alpha} = \bar{Z}' \cdot \partial \bar{u}^{\alpha} \Rightarrow \bar{u} \cdot \partial \bar{Z}'^i = H\bar{Z}'^i; \tag{118}$$

$$\partial_{\tau} \bar{Z}^i = H(\bar{Z}^i - \vec{\xi} \cdot \vec{\partial} \bar{Z}^i),$$
$$\partial_{\tau} \bar{Z}'^i = H(\bar{Z}'^i - \vec{\xi} \cdot \vec{\partial} \bar{Z}'^i). \tag{119}$$

In the second equation above, we have made the further choice; namely, the spatial parts of $\bar{Z}, \bar{Z}'$ are orthogonal to the radial direction $\vec{\xi}$ as well.

The idea is to bring the deviation vectors across the derivatives. Using the properties above and $\tilde{h}_{ij}\bar{u}^i = 0$ which holds thanks to the TT projection, Eq. (114) gives

$$D(Z', Z) - \frac{\Lambda}{3}\delta(g_{\alpha\beta}Z'^{\alpha}Z^{\beta}) = \left[\frac{1}{2}(\bar{u}\cdot\bar{\nabla})^2 - H(\bar{u}\cdot\bar{\nabla})\right](\tilde{h}^{TT}_{ij}\bar{Z}'^i\bar{Z}^j). \quad (120)$$

The second term on the left-hand side of the above equation vanishes.

To see this, we collect the equations satisfied by the δ quantities,

$$g_{\alpha\beta}u^{\alpha}u^{\beta} = -1 \Rightarrow \bar{u}_{\alpha}\delta u^{\alpha} = 0 \quad (121)$$

$$\begin{aligned} g_{\alpha\beta}u^{\alpha}Z^{\beta} = 0 &\Rightarrow \bar{u}_{\alpha}\delta Z^{\alpha} + \bar{Z}_i\delta u^i = 0 \\ g_{\alpha\beta}u^{\alpha}Z'^{\beta} = 0 &\Rightarrow \bar{u}_{\alpha}\delta Z'^{\alpha} + \bar{Z}'_i\delta u^i = 0 \\ u\cdot\nabla u^{\alpha} = 0 &\Rightarrow \bar{u}\cdot\bar{\nabla}\delta u^{\alpha} = -\delta u\cdot\bar{\nabla}\bar{u}^{\alpha} \end{aligned} \quad (122)$$

$$\begin{aligned} u\cdot\partial Z^{\alpha} - Z\cdot\partial u^{\alpha} = 0 &\Rightarrow \bar{u}\cdot\bar{\nabla}\delta Z^{\alpha} \\ &= \bar{Z}^i\bar{\nabla}_i\delta u^{\alpha} + \delta Z\cdot\bar{\nabla}\bar{u}^{\alpha} - \delta u\cdot\bar{\nabla}\bar{Z}^{\alpha} \\ u\cdot\partial Z'^{\alpha} - Z'\cdot\partial u^{\alpha} = 0 &\Rightarrow \bar{u}\cdot\bar{\nabla}\delta Z'^{\alpha} \\ &= \bar{Z}'^i\bar{\nabla}_i\delta u^{\alpha} + \delta Z'\cdot\bar{\nabla}\bar{u}^{\alpha} - \delta u\cdot\bar{\nabla}\bar{Z}'^{\alpha} \\ g_{\alpha\beta}Z^{\alpha}Z'^{\beta} = 0 &\Rightarrow \bar{Z}_{\alpha}\delta Z'^{\alpha} + \bar{Z}'_i\delta Z^i \\ &\quad + \tilde{h}^{TT}_{ij}\bar{Z}'^i\bar{Z}^j = 0. \end{aligned} \quad (123)$$

Equations (121) serve to give the zeroth components of the δ-vectors in terms of their spatial components. Equations (122) are evolution equations along the geodesic for the δ-vectors and preserve the previous three equations. The last equation (123) is needed for the gauge invariance of the deviation scalar. The spatial components of δ-vectors are still free. Demanding that Eq. (123) is preserved along the observer geodesic leads to

$$(\bar{Z}'_i\bar{Z}^j + \bar{Z}_i\bar{Z}'^j)\bar{\nabla}_j\delta u^i = -(\bar{u}\cdot\bar{\nabla} - 2H)(\tilde{h}^{TT}_{ij}\bar{Z}'^i\bar{Z}^j). \quad (124)$$

Here, we have used the evolution equations for $\delta Z, \delta Z'$, Eq. (123), and $\bar{\nabla}_j\bar{u}^i = H\delta^i_j + o(H^3)$. This equation together with the evolution equation for δu^i can be taken to restrict δu^i, and we are still left free with the δZ^i, $\delta Z'^i$ subject only to Eq. (123). This equation precisely sets the second term on the left-hand side of Eq. (120) to zero.

Thus, we obtain the deviation scalar as a simple expression,

$$\begin{aligned} D(u, Z', Z) &= \left[\frac{1}{2}(\bar{u}\cdot\bar{\nabla})^2 - H(\bar{u}\cdot\bar{\nabla})\right]Q, \\ Q &:= (\tilde{h}^{TT}_{ij}\bar{Z}'^i\bar{Z}^j) \quad \text{with}, \\ \bar{u}\cdot\bar{\nabla}Q &= \bar{u}\cdot\partial Q = ((1 + H^2s^2)\partial_{\tau} + H\xi^i\partial_i)Q. \end{aligned} \quad (125)$$

For subsequent comparison, it is more convenient to take the deviation vectors across the derivatives, using $\bar{u}\cdot\bar{\nabla}\bar{Z}^i = H\bar{Z}^i$, etc. The deviation scalar is then given by

$$D(u, Z', Z) = \bar{Z}'^i\bar{Z}^j\left[\frac{1}{2}(\bar{u}\cdot\bar{\nabla})^2 + H(\bar{u}\cdot\bar{\nabla})\right]\tilde{h}^{TT}_{ij} \quad \text{with} \quad (126)$$

$$\bar{u}\cdot\bar{\nabla}\tilde{h}^{TT}_{ij} = \bar{u}\cdot\partial\tilde{h}^{TT}_{ij} + o(H^3). \quad (127)$$

Substituting the solution (64) gives

$$\begin{aligned} D(u, Z', Z) = \frac{1}{s}\Bigg[&\left(1 - 2Hs + \frac{7}{2}H^2s^2\right)\partial^4_{\tau}\mathcal{M}^{TT}_{ij} \\ &- H^2s\partial^3_{\tau}\mathcal{M}^{TT}_{ij} - H^2\partial^2_{\tau}\mathcal{M}^{TT}_{ij} \\ &- \frac{3H^2}{4}\partial^4_{\tau}\mathcal{M}^{TT}_{ijkl}\delta^{kl}\Bigg]\bar{Z}'^i\bar{Z}^j. \end{aligned} \quad (128)$$

The τ derivatives are evaluated at the retarded time $(\tau - \bar{s}(s))$ defined in Eq. (35).

B. Conformal chart of the Poincaré patch

For the solution in the generalized transverse gauge, the full metric has the form $g_{\mu\nu} = \Omega^2(\eta_{\mu\nu} + \chi_{\mu\nu})$, $\Omega^2 = 3\Lambda^{-1}\eta^{-2} = H^{-2}\eta^{-2}$. We can then use the Weyl transformation property of the Riemann tensor and obtain the full curvature in terms of the curvature of $(\eta + \chi)$ metric plus extra terms depending on *derivatives* of $\ln(\Omega)$. From these derivatives, Λ drops out, and the full curvature (and hence the relative acceleration) is completely independent of Λ. Explicitly,

$$\begin{aligned} R_{\alpha\lambda\mu\nu}[\Omega^2(\eta + \chi)] = \Omega^2\Bigg[&R_{\alpha\lambda\mu\nu}[\eta + \chi] + \frac{1}{\eta^2}\{\hat{g}_{\alpha\mu}\hat{g}_{\nu\lambda} - \hat{g}_{\alpha\nu}\hat{g}_{\mu\lambda}\} \\ &+ \frac{1}{\eta}\{\hat{g}_{\alpha\nu}\hat{\Gamma}^0{}_{\mu\lambda} - \hat{g}_{\alpha\mu}\hat{\Gamma}^0{}_{\nu\lambda} + \hat{g}_{\mu\lambda}\hat{g}_{\alpha\beta}\hat{\Gamma}^{\beta}{}_{0\nu} \\ &- \hat{g}_{\nu\lambda}\hat{g}_{\alpha\beta}\hat{\Gamma}^{\beta}{}_{0\mu}\}\Bigg] \quad \text{where,} \\ \hat{g}_{\mu\nu} &= \eta_{\mu\nu} + \chi_{\mu\nu}, \\ \hat{\Gamma}^{\alpha}_{\mu\nu}|_{o(\chi)} &= \frac{1}{2}(\partial_{\nu}\chi^{\alpha}_{\mu} + \partial_{\mu}\chi^{\alpha}_{\nu} - \partial^{\alpha}\chi_{\mu\nu}). \end{aligned} \quad (129)$$

The definition of the deviation scalar and its invariance remains the same. We also choose the same geodesic congruence in the background space-time so that $u^{\alpha} = -H\eta\delta^{\alpha}_0$. As before, we choose two mutually orthogonal deviation vectors, Z, Z' and write $D(Z', Z) = -R_{\alpha\beta\mu\nu} Z'^{\alpha} u^{\beta} Z^{\mu} u^{\nu}$. Using the Weyl transformation given above, we write

$$\hat{D}(\hat{Z}', \hat{Z}) = \left[R_{\alpha\lambda\mu\nu}[\hat{g}] + \frac{1}{\eta^2}\{\hat{g}_{\alpha\mu}\hat{g}_{\nu\lambda} - \hat{g}_{\alpha\nu}\hat{g}_{\mu\lambda}\} + \frac{1}{\eta}\{\hat{g}_{\alpha\nu}\hat{\Gamma}^0_{\mu\lambda} - \hat{g}_{\alpha\mu}\hat{\Gamma}^0_{\nu\lambda} + \hat{g}_{\mu\lambda}\hat{g}_{\alpha\beta}\hat{\Gamma}^{\beta}_{0\nu} - \hat{g}_{\nu\lambda}\hat{g}_{\alpha\beta}\hat{\Gamma}^{\beta}_{0\mu}\}\right] \hat{Z}'^{\alpha}\hat{u}^{\lambda}\hat{Z}^{\mu}\hat{u}^{\nu}, \quad (130)$$

where we have defined new *scaled variables* as $u^{\alpha} \coloneqq |\Omega|^{-1}\hat{u}^{\alpha}$, $Z^{\alpha} \coloneqq |\Omega|^{-1}\hat{Z}^{\alpha}$, $Z'^{\alpha} \coloneqq |\Omega|^{-1}\hat{Z}'^{\alpha}$, and $D(Z', Z) \coloneqq \Omega^{-2}\hat{D}(\hat{Z}', \hat{Z})$. This removes all the explicit factors of Ω^2, and we get an expression for the scaled deviation scalar, defined by perturbations about *Minkowski* background, with explicit additional terms.

For notational simplicity, we will suppress the hats in the following and restore them in the final equation. The background quantities, denoted by overbars refer to the Minkowski metric, and the corresponding δ quantities are treated as of the same order as the perturbation χ^{TT}_{ij}. In particular, $\bar{u}^{\alpha} = \delta^{\alpha}{}_0$, $\bar{u} \cdot \partial \bar{u}^{\alpha} = 0$, $\bar{u} \cdot \partial \bar{Z}^{\alpha} = \bar{Z} \cdot \partial \bar{u}^{\alpha}$ and similarly for $\bar{Z}'^{\alpha}$. Proceeding exactly as before, we deduce

$$\bar{u}^i = \bar{Z}^0 = \bar{Z}'^0 = \bar{Z}^i \bar{Z}'^j \delta_{ij} = 0, \qquad \partial_{\eta}\bar{Z}^i = \partial_{\eta}\bar{Z}'^i = 0; \quad (131)$$

$$\delta u^0 = 0 = \delta Z^0 - \bar{Z}_i \delta u^i = \delta Z'^0 - \bar{Z}'_i \delta u^i, \qquad \bar{Z}'_i \delta Z^i + \bar{Z}_i \delta Z'^i + \chi_{ij}\bar{Z}'^i \bar{Z}^j = 0; \quad (132)$$

$$\partial_{\eta}\delta Z^{\alpha} = \bar{Z}^i \partial_i \delta u^{\alpha} - \delta u^i \partial_i \bar{Z}^i, \qquad \partial_{\eta}\delta Z'^{\alpha} = \bar{Z}'^i \partial_i \delta u^{\alpha} - \delta u^i \partial_i \bar{Z}'^i; \quad (133)$$

$$(\bar{Z}'_i \bar{Z}^j + \bar{Z}_i \bar{Z}'^j)\partial_j \delta u^i = -\partial_{\eta}(\chi_{ij}\bar{Z}'^i \bar{Z}^j), \qquad \partial_{\eta}\delta u^i = 0. \quad (134)$$

As before, demanding preservation of the last of the normalization conditions in (132) under η evolution gives conditions on δu^i given in Eq. (134). These are used in simplifying Eq. (130). The η^{-2} term of this equation vanishes as before, while the η^{-1} coefficient gives only one contribution. In the first term, $R(\hat{g})$ gets replaced by $R^{(1)}$ which is linear in χ^{TT}_{ij}. This leads to (restoring the hats)

$$\hat{D}(\hat{Z}', \hat{Z}) = \frac{1}{2}\left(\partial^2_{\eta}\chi^{TT}_{ij} - \frac{1}{\eta}\partial_{\eta}\chi^{TT}_{ij}\right)\bar{\hat{Z}}'^i \bar{\hat{Z}}^j. \quad (135)$$

Noting that χ_{ij} is a function of η only through $\eta_{\rm ret} = \eta - r$, we can replace ∂_{η} by $\partial_{\eta_{\rm ret}} \eqqcolon \partial_{\bar{\eta}}$. Going to the cosmological chart via the definitions $\eta = -H^{-1}e^{-Ht} \eqqcolon -H^{-1}a^{-1}$ and $\bar{\eta} \coloneqq \eta - r \coloneqq -H^{-1}\bar{a}^{-1}$ which defines the retarded time $\bar{t}$ through $\bar{a} = a(\bar{t})$, we replace $\bar{\partial}_{\eta} = \bar{a}\partial_{\bar{t}}$. This leads to

$$\hat{D}(\hat{Z}', \hat{Z}) = \frac{\bar{\hat{Z}}'^i \bar{\hat{Z}}^j}{2}\bar{a}^2\left(\partial^2_{\bar{t}} + H\left(1 + \frac{a}{\bar{a}}\right)\partial_{\bar{t}}\right)\chi^{TT}_{ij} \quad \text{with } \chi_{ij} \text{ from Eq. (99).}$$

To express the deviation scalar in terms of the Killing time τ, we observe that on scalars, $\mathcal{L}_T f = T \cdot \partial f$, while on tensorial functions of the retarded time,

$$\mathcal{L}_T Q_{ij}(\bar{t}) = ((\partial_t - Hx^i\partial_i)(\bar{t}))(\partial_{\bar{t}} Q_{ij}(\bar{t})) - 2HQ_{ij}(\bar{t}), \quad \text{with } (\partial_t - Hx^i\partial_i)(\bar{t}) = 1.$$

After a straightforward computation, we get

$$\hat{D}(u, \hat{Z}', \hat{Z}) = (\bar{\hat{Z}}'^i \bar{\hat{Z}}^j)\left(\frac{\bar{a}^2}{ra}\right)[\mathcal{L}^4_T Q_{ij} + 6H\mathcal{L}^3_T Q_{ij} + 11H^2\mathcal{L}^2_T Q_{ij} + 6H^3\mathcal{L}_T Q_{ij}$$
$$+ H\mathcal{L}^3_T \bar{Q}_{ij} + 6H^2\mathcal{L}^2_T \bar{Q}_{ij} + 11H^3\mathcal{L}_T \bar{Q}_{ij} + 6H^4\bar{Q}_{ij}]. \quad (136)$$

To compare the deviation scalars computed above, we need to ensure that we use the "same" deviation vectors. Since the same observer is used, the deviation vectors are defined the same way with the only exception of their normalization. So let us use[5] *normalized deviation vectors* $Z^i \coloneqq \gamma\hat{Z}^i$ where $\hat{Z}^i\hat{Z}^j\delta_{ij} = 1$. Then, $Z^2 = 1$ determines γ. In the FNC,

[5]We now suppress the overbars on the deviation vectors to avoid cluttering.

$\gamma = (1 + H^2 s^2/6)$, whereas in the conformal chart, $\gamma = |\Omega|^{-1}$. Thus, in the conformal chart, the hatted deviation vectors are already normalized. In the FNC, we need to replace the deviation vectors by $(1 + H^2 s^2/6) \times \hat{Z}$, and in the conformal chart, we write $\hat{D}(\hat{Z}', \hat{Z}) = \Omega^2 D(Z', Z)$. In the conformal chart, we retain terms up to order H^2 only, and since the FNC calculation uses a traceless stress tensor, we take $\bar{Q}$ moments to equal the Q moments. The two expressions are given below (recall that in FNC, $\mathcal{L}_T$ on all tensors reduces to ∂_τ):

$$D^{\rm FNC}(u,Z',Z) = \frac{1}{s}\left(1+\frac{H^2 s^2}{3}\right)\left[\left(1-2Hs+\frac{7}{2}H^2 s^2\right)\partial_\tau^4 \mathcal{M}_{ij}^{TT} - H^2 s \partial_\tau^3 \mathcal{M}_{ij}^{TT} - H^2 \partial_\tau^2 \mathcal{M}_{ij}^{TT} - \frac{3H^2}{4}\partial_\tau^4 \mathcal{M}_{ijkl}^{TT}\delta^{kl}\right]\hat{Z}'^i \hat{Z}^j$$
$$= \frac{1}{s}\left(1-2Hs+\frac{23}{6}H^2 s^2\right)\left[\partial_\tau^4 \mathcal{M}_{ij}^{TT} - H^2 s \partial_\tau^3 \mathcal{M}_{ij}^{TT} - H^2 \partial_\tau^2 \mathcal{M}_{ij}^{TT} - \frac{3H^2}{4}\partial_\tau^4 \mathcal{M}_{ijkl}^{TT}\delta^{kl}\right]\hat{Z}'^i \hat{Z}^j \qquad (137)$$

$$D^{\rm Conf}(u,Z',Z)|_{o(H^2)} = \left(\frac{\bar{a}^2}{a^2}\frac{1}{ra}\right)\Bigg|_{o(H^2)} [\mathcal{L}_T^4 Q_{ij} + 7H\mathcal{L}_T^3 Q_{ij} + 17H^2 \mathcal{L}_T^2 Q_{ij} + 17H^3 \mathcal{L}_T Q_{ij} + 6H^4 Q_{ij}]\hat{Z}'^i \hat{Z}^j$$
$$= \frac{1}{s}\left(1-2Hs+\frac{19}{6}H^2 s^2\right)[\mathcal{L}_T^4 Q_{ij}^{TT} + 7H\mathcal{L}_T^3 Q_{ij}^{TT} + 17H^2 \mathcal{L}_T^2 Q_{ij}^{TT}]\hat{Z}'^i \hat{Z}^j. \qquad (138)$$

Equations (128) and (136) give the deviation scalars in the two charts. The comparable expressions are given in (137) and (138). These are obtained for the specific choice of the congruence of the de Sitter background: $\bar{u}^\alpha(\eta, x^i) \coloneqq -H\eta(1, \vec{0})$.

We have obtained two different looking expressions for the same, gauge invariant deviation scalar. The difference can be attributed to the definition of moments. They have been defined on two different spatial hypersurfaces—the $\tau =$ constant in FNC and the $\eta =$ constant in the conformal chart. In the conformal chart solution, there is no truncation of powers of H (in the leading r approximation), and it includes the contribution of both the sharp and the tail terms. By contrast, the FNC chart computation is obtained as an expansion in H only up to the quadratic order. Furthermore, it includes only the contribution of the sharp term. While it is possible to relate the frame components of the stress tensor in the two charts, the relation among the moments is nontrivial and is not obtained here.

We have defined a gauge invariant quantity and illustrated how to compute it. It depends on a timelike geodesic congruence and two mutually orthogonal deviation vectors. At the linearized level, it also depends on the $\hat{n}$ direction used in the TT projection. What information about the wave does it contain? To see this, consider the simpler case of Minkowski background, and choose the congruence so that $\bar{u}^\alpha = (1, \vec{0})$. It follows that at the linearized level, the quantity

$$A_{\alpha\beta}(\eta_{\mu\nu} + h_{\mu\nu}) \coloneqq -R_{\alpha\mu\beta\nu}(\eta_{\mu\nu} + h_{\mu\nu})u^\mu u^\nu$$
$$\approx -R^{(1)}_{\alpha\mu\beta\nu}(h)\bar{u}^\mu \bar{u}^\nu = -R^{(1)}_{\alpha 0\beta 0}(h) \qquad (139)$$

is symmetric in $\alpha \leftrightarrow \beta$ and spatial, i.e., $A_{00} = 0 = A_{0i}$. When the transient wave $h_{\mu\nu}$ is in synchronous gauge and TT projected, the matrix $A_{ij}(h^{TT}_{kl})$ is also transverse. This is because the Λ projector can be taken across the derivatives up to terms down by powers of r. Explicitly,

$$A_{ij}(h^{TT}) \approx \frac{1}{2}\Lambda_{ij}{}^{kl}(\hat{n})\partial_0^2 h_{kl}, \qquad A_{ij}\delta^{ij} = 0. \qquad (140)$$

Since the deviation vectors, too, are taken to be transverse, in effect the deviation scalar reduces to $D(u, Z', Z) \approx \hat{\bar{Z}}'^a A_{ab}(h)\hat{\bar{Z}}^b$ where a, b take two values and the real, symmetric matrix A_{ab} is traceless. With respect to an arbitrarily chosen basis $\{\hat{e}_1, \hat{e}_2\}$ in the plane transverse to the wave direction, $\hat{n}$, we can define the $+$ and the $\times$ polarizations by setting the matrix $A \coloneqq h_+\sigma_3 + h_\times\sigma_1$. If $\hat{\bar{Z}}$ makes an angle ϕ with $\hat{e}_1$, then the unit deviation vectors are given by $\hat{\bar{Z}} = (\cos(\phi), \sin(\phi))$, $\hat{\bar{Z}}' = (-\sin(\phi), \cos(\phi))$. It follows that

$$D(u, Z', Z) = -h_+ \sin(2\phi) + h_\times \cos(2\phi). \qquad (141)$$

Thus, for a pair of bases $(\hat{e}_1, \hat{e}_2)$ and $(\hat{\bar{Z}}, \hat{\bar{Z}}')$, determination of the deviation scalar gives one relation between the amplitudes of the two polarizations. A similar determination at another detector location gives a second relation, thereby providing amplitudes of individual polarizations.

A natural choice for $\hat{e}_1, \hat{e}_2$ would be the unit vectors provided by the Right Ascension/Declination coordinate system used by astronomers, at the $\hat{n}$ direction. The basis of unit deviation vectors could be constructed in many ways, for instance, by using the wave direction $\hat{n}$ and one of the arms of the interferometer which form a plane. Its unit normal may be taken as $\hat{\bar{Z}}$, and then $\hat{n} \times \hat{\bar{Z}}$ can be taken as $\hat{\bar{Z}}'$. To avoid the exceptional case where the wave is incident along the chosen arm of the interferometer, one could repeat the procedure with the other arm. The construction gives ϕ at the detector location. Suffice it to say that measurement of the deviation scalar for appropriate deviation vectors at two or more detectors would constitute a measurement of the amplitudes of individual polarizations of a gravitational wave.

To be useful in observations, the deviation scalar must be computed for congruence related to a specific interferometer (Earth-based ones are not in freefall, and the space-based ones would be) and related to the waveform. These details are beyond the scope of the present work.

V. SUMMARY AND DISCUSSION

Let us begin by recalling the main motivation for this work. The concordance model of cosmology favors dark energy modelled conveniently in terms of a positive cosmological constant which is about 10^{-29} gm/cc or about $10^{-52}\,\mathrm{m}^{-2}$ in the geometrized units with $G = 1 = c$. In the vicinity of any astrophysical sources, this density is extremely small, and only over vast distances of matter-free regions, we may expect its effects to be felt. Over distances of typical, detectable compact sources of gravitational waves—about megaparsecs—its effect may be estimated to be of order $\sqrt{\Lambda}r \sim 10^{-4}$. (This is comparable to the fourth order Post-Newtonian corrections for a $v/c \sim$ 0.2–0.3 and is relevant for direct detection of gravitational waves.) On the other hand, the asymptotic structure of $\mathcal{J}^+$—the final destination for all massless radiation—is qualitatively different for *arbitrarily small* values of Λ and has a significant impact on asymptotic symmetry groups and the fluxes associated with them. Does this affect *indirect detection* of gravitational waves, e.g., in the orbital decays of binary pulsars? To the extent that the Hulse-Taylor pulsar observations have already vindicated the quadrupole formula computed in Minkowski background, one does not expect the radically different nature of $\mathcal{J}^+$ to play any significant role in such indirect detections. A physically relevant question then is how the effects of positive Λ are to be estimated quantitatively. Our main motivation has been to address this question.

In the Introduction, we noted the different features and issues that arise: multiple charts, gauges, identification of physical perturbations, source multipole moments, and energy measures. We considered two different charts (FNC and conformal) and two different gauge choices (TT and generalized TT) and defined the corresponding synchronous gauges to identify the physical components, and these were expressed in terms of the appropriately defined source moments.

A strategy to determine of the waveform of a transient gravitational waves, e.g., using an interferometer, always selects a frequency window of sensitivity and corresponding class of sources. For the class of sources we have assumed (rapidly varying and distant), it seems sufficient to confine attention to a region maximally up to the cosmological horizon. The physical distance (e.g., luminosity distance) from the source to the cosmological horizon, e.g., $\eta = -r$ in the conformal chart, is $\sqrt{3/\Lambda\eta^2}r = \sqrt{3/\Lambda}$. This contains typical, currently detectable sources and thus should suffice for estimation. We obtained the corresponding fields, to order Λ, using Fermi normal coordinates based near the compact source, and this is given in Eq. (64). For a subsequent comparison, we also computed the field in the conformal/cosmological charts. It is given in Eq. (96).

By contrast, an indirect detection via observation of orbital decays of binary systems is premised on the energy lost due to gravitational radiation. This is typically the inspiral phase of the binary system and has much lower frequencies (about 10^{-5} Hz for Hulse-Taylor). This is beyond the capabilities of Earth-based interferometers, and one has to appeal to the energy carried away by gravitational waves. The energy flux calculations are ideally done at infinity. For these, done in the conformal/cosmological chart, we refer the reader to Refs. [6,7]. As mentioned in the Introduction, there are two distinct prescriptions, and it would be useful to compare them. Flux computations and comparisons will be dealt with in a separate publication.

In the Minkowski background analysis, tail terms appear at higher orders of perturbations, and these are understood to be due to scattering off the curvature generated at the lower orders. In the de Sitter background, curvature effects are felt by the perturbations at the linear order itself. This is manifested in both the gauges. In the generalized transverse gauge, the tail term is explicitly available and plays a crucial role at the null infinity [7]. In the TT gauge, however, the tail term itself is order Λ^2 and within the FNC patch does not seem likely to give significant contribution by cumulative effects. However, it remains to compute this explicitly.

As a byproduct of expressing the retarded solution in terms of the source moments, we also saw (not surprisingly) that the "mass" (zeroth moment) and the "momentum" (first moment) are not conserved, thanks to the curvature of the de Sitter background. More generally, it also implied that static (test) sources cannot exist in a curved background. This is just a consequence of the conservation equation in a curved background, quite independent of any gravitational waves.

In the Minkowski background, geodesic deviation acceleration, to the linearized order, is gauge invariant and is used to infer the waveform. In a general curved background, its gauge invariance is lost. However, for a conformally flat background, the component of a deviation vector along another, orthogonal deviation vector defines a gauge invariant function, $D(u, Z, Z')$, which we termed as a deviation scalar. In the simpler context of a flat background, we saw that its measurement at two or more detectors would give the amplitudes of individual polarizations. Its determination could provide useful information on the polarization of gravitational waves even for a nonzero cosmological constant.

We computed the deviation scalar, for the solutions given in the two charts. This is a new result. The expressions obtained (137), (138) are different. The comparison is expected to be possible when both charts overlap and only up to order $\Lambda \sim H^2$. In FNC, we have computed only the sharp term. However, it is not clear if the "sharp" contribution can be identified in a chart-independent manner. So in the conformal chart, we took the full field and restricted its contribution to order Λ. While we compute the same observable, a chart dependence or, more precisely, a dependence on the spatial hypersurface enters through the definition of source moments. There is also a choice of moment variable involved ($\zeta^{\underline{i}}$ in FNC). Thus, the solutions are given in terms of source moments which are defined on *different spatial hypersurfaces*. As such, they cannot be compared immediately. An explicit model system for which the two different moments are computed should help clarify some of these aspects and show the equality of the deviation scalar computed in two ways. This needs to be checked.

Lastly, we comment on the lessons from these computations. Even a smallest cosmological constant (positive or negative) immediately brings up the more than one "natural" choices of charts in a given patch. Quite apart from the qualitatively distinct structure of the respective $\mathcal{J}^+$, even the local (near source) analysis reveals different issues to be faced. The FNC is very natural to the local analysis and goes through the same way for anti-de Sitter as well. It naturally gives the answer as corrections to the corresponding Minkowski answer, in *powers of* Λ. This is also seen the Bondi-Sachs chart [20]. From the intuition from Minkowski background analysis, neighborhood of infinity is the natural place for characterizing *radiation* in a gauge invariant manner. Then, the conformal chart (for de Sitter) is a natural choice. And here the corrections to the Minkowski answer are obtained in powers of $\sqrt{\Lambda}$. This difference in the powers of Λ was seen in the solutions obtained in Eqs. (64) and (96). However, it is meaningless to compare the gauge fixed fields. For this purpose, the gauge invariant deviation scalar was computed and compared. The manifest dependence of the corrections on Λ does distinguish a local (neighborhood of source) form from the one in the asymptotic region.

To conclude, linearization about the de Sitter background provides a simplified arena for an extension of the computational steps from a flat background to a curved background. The weak gravitational waves can be computed as corrections in powers of the cosmological constant. There is a gauge invariant observable that could provide information about the amplitudes of the two polarizations. More precise computations at least for a model source are needed for a quantitative estimate of corrections to the waveforms. If the Λ corrections could be identified from the signal, it could provide an independent measurement of the cosmological constant.

ACKNOWLEDGMENTS

We would like to thank Abhay Ashtekar for helpful discussions during his visit to Chennai and useful suggestions. We also acknowledge discussions with Alok Laddha and K. G. Arun. We would like to thank Yi-Zen Chu for correspondence and for pointing out an erroneous boundary term in Eq. (79) which has been corrected.

APPENDIX A: TRIANGLE LAW FOR WORLD FUNCTION

We sketch the steps that go in the computation of the world function between the observation event and a source event, $\sigma(P', P)$, given in Eq. (29). In reference to Fig. 3, we want to compute $\phi \coloneqq \frac{1}{3}\int_0^1 dv(1-v)^3 \frac{D^4\sigma(q',q)}{Dv^4}$ [16].

Let u denote the parameter along the geodesics connecting q', q as they vary along the geodesics P_0P' and P_0P. These geodesics are all parametrized such that they begin at $q'(u_1, v)$ and end at $q(u_2, v)$. In general, $\sigma(q', q)$ is a function of u_1, u_2, v. But since u_1, u_2 are the same for all such pairs, we have $\sigma(q', q) = \sigma(v)$. Therefore,

$$\frac{D\sigma(v)}{Dv} = \frac{dx'^{\alpha}}{dv}\frac{\partial\sigma}{\partial x'^{\alpha}} + \frac{dx^{\alpha}}{dv}\frac{\partial\sigma}{\partial x^{\alpha}} \coloneqq \sigma_{\alpha'}V^{\alpha'} + \sigma_{\alpha}V^{\alpha}. \quad \text{(A1)}$$

The V's denote the tangent vectors at the respective end points, while the prime on the component labels indicates which end point is implied. The suffix on the σ denotes the covariant derivative at the corresponding point. Since σ is a (bi)scalar, its covariant derivative equals the partial derivative.

The second and higher derivatives of σ with respect to v are computed similarly, noting that $\frac{DV^{\alpha}}{Dv} = \frac{DV^{\alpha'}}{Dv} = 0$ since $P_0 \to P'$, $P_0 \to P$ are both geodesics and v is the affine parameter along them. We also note the property of the world function [16], $\sigma_{\alpha'\beta} = \sigma_{\beta\alpha'}$. This leads to

$$\frac{D^2\sigma}{Dv^2} = \sigma_{\alpha'\beta'}V^{\alpha'}V^{\beta'} + \sigma_{\alpha\beta}V^{\alpha}V^{\beta} + 2\sigma_{\alpha'\beta}V^{\alpha'}V^{\beta} \quad \text{(A2)}$$

$$\begin{aligned}\frac{D^4\sigma}{Dv^4} &= \sigma_{\alpha'\beta'\mu'\nu'}V^{\alpha'}V^{\beta'}V^{\mu'}V^{\nu'} + 4\sigma_{\alpha'\beta'\mu'\nu}V^{\alpha'}V^{\beta'}V^{\mu'}V^{\nu} \\ &\quad + 6\sigma_{\alpha'\beta'\mu\nu}V^{\alpha'}V^{\beta'}V^{\mu}V^{\nu} + 4\sigma_{\alpha'\beta\mu\nu}V^{\alpha'}V^{\beta}V^{\mu}V^{\nu} \\ &\quad + \sigma_{\alpha\beta\mu\nu}V^{\alpha}V^{\beta}V^{\mu}V^{\nu}. \end{aligned} \quad \text{(A3)}$$

We have not written the third derivative as we do not need it.

The desired world function is, using Taylor expansion with a remainder, about P_0 $(v = 0)$,

$$\sigma(P', P) = \sigma(\bar{v}) = \sigma(0) + \bar{v}\frac{D\sigma}{Dv}\bigg|_0 + \frac{1}{2}\bar{v}^2\frac{D^2\sigma}{Dv^2}\bigg|_0 + \frac{1}{6}\bar{v}^3\frac{D^3\sigma}{Dv^3}\bigg|_0 + \frac{1}{6}\int_0^1 dv(1-v)^3\frac{D^4\sigma(q', q)}{Dv^4}. \quad \text{(A4)}$$

It is known that $\sigma(0) = 0 = \frac{D\sigma}{Dv}(0) = \frac{D^3\sigma}{Dv^3}(0)$. The coincidence limits of the second derivatives of σ are given by $[\sigma_{\alpha'\beta'}] = [\sigma_{\alpha\beta}] = g_{\alpha\beta}$ and $[\sigma_{\alpha'\beta}] = [\sigma_{\alpha\beta'}] = -g_{\alpha'\beta} = -g_{\alpha\beta'}$ and $\bar{v}V^{\alpha'} = -g^{\alpha'\beta'}\sigma_{\beta'}$ and $\bar{v}V^{\alpha} = +g^{\alpha\beta}\sigma_{\beta}$ [16]. This leads to

$$\begin{aligned}\bar{v}^2\frac{D\sigma}{Dv^2}\bigg|_0 &= g_{\alpha\beta}(\bar{v}V^{\alpha})(\bar{v}V^{\beta}) + g_{\alpha'\beta'}(\bar{v}V^{\alpha'})(\bar{v}V^{\beta'}) \\ &\quad - 2g_{\alpha'\beta}(\bar{v}V^{\alpha'})(\bar{v}V^{\beta}) \\ &= g^{\alpha\beta}\sigma_{\alpha}\sigma_{\beta} + g^{\alpha'\beta'}\sigma_{\alpha'}\sigma_{\beta'} + 2g^{\alpha'\beta}\sigma_{\alpha'}\sigma_{\beta} \\ &= 2\sigma(P_0, P) + 2\sigma(P_0, P') \\ &\quad - 2\sigma_{\alpha}(P_0, P')\sigma^{\alpha}(P_0, P). \end{aligned} \quad \text{(A5)}$$

In the last line, we have used $2\sigma = g^{\alpha\beta}\sigma_{\alpha}\sigma_{\beta}$. Substituting in Eq. (A4), we get

$$\begin{aligned}\sigma(P', P) &= \sigma(P_0, P') + \sigma(P_0, P) \\ &\quad - \left(g^{\alpha\beta}\frac{\partial\sigma(y, P')}{\partial y^{\alpha}}\frac{\partial\sigma(y, P)}{\partial y^{\beta}}\right)\bigg|_{P_0} \\ &\quad + \frac{1}{6}\int_0^1 dv(1-v)^3\frac{D^4\sigma(q', q)}{Dv^4}. \end{aligned} \quad \text{(A6)}$$

To compare with the triangle law, we denote $\overrightarrow{PQ}^2 \coloneqq 2\sigma(P, Q)$. Then, the above equation can be written as

$$\overrightarrow{P'P}^2 = \overrightarrow{P_0P'}^2 + \overrightarrow{P_0P}^2 - 2\overrightarrow{P_0P'}\cdot\overrightarrow{P_0P} + \phi. \quad \text{(A7)}$$

To evaluate ϕ, we need to evaluate the fourth order covariant derivatives of the world function. These are obtained in terms of the parallel propagator and integrals of curvature. To state the result, we introduce the notation

$$\text{Parallel propagator}: X_{\parallel}^{\alpha}(p) \coloneqq g^{\alpha}{}_{\beta'}(p', p)X_{\parallel}^{\beta'}(p') \quad \text{where} \quad V^{\gamma}\nabla_{\gamma}X_{\parallel}^{\alpha} = 0, \quad \text{(A8)}$$

$$\text{Symmetrized Riemann}: S_{\alpha\beta\mu\nu} \coloneqq -\frac{1}{3}(R_{\alpha\mu\beta\nu} + R_{\alpha\nu\beta\mu}). \quad \text{(A9)}$$

The parallel propagator $g^{\alpha}{}_{\beta'}(p', p)$ is a bitensor, and its indices are raised/lowered by the metric at the respective points.

It is convenient to introduce a tetrad basis, $E^{\alpha}{}_{a}, E^{a}{}_{\alpha}$, at p' and define it at p by parallel transporting it along the geodesic from p' to p. The parallel propagator is then given by $g^{\alpha}{}_{\beta'}(p', p) = E^{\alpha}{}_{a}(p)E^{a}{}_{\beta'}(p')$. Denoting the components with respect to these parallelly transported tetrad by Latin indices, the second order covariant derivatives of the world function are given by (Eq. (97) of Ref. [16])

$$\sigma_{a'b'}(q', q) = g_{a'b'}(q') + \frac{3}{2}\frac{1}{u_2 - u_1}\int_{u_1}^{u_2} du(u_2 - u)^2 \times S_{abcd}(u)U^cU^d(u) \quad \text{(A10)}$$

$$\sigma_{a'b}(q', q) = g_{a'b}(q') + \frac{3}{2}\frac{1}{u_2 - u_1}\int_{u_1}^{u_2} du(u_2 - u)(u - u_1) \times S_{abcd}(u)U^cU^d(u) \quad \text{(A11)}$$

$$\sigma_{ab}(q', q) = g_{ab}(q') + \frac{3}{2}\frac{1}{u_2 - u_1} \times \int_{u_1}^{u_2} du(u - u_1)^2 S_{abcd}(u)U^cU^d(u). \quad \text{(A12)}$$

Note that the tetrad components of the parallel propagator are just η_{ab} while the tetrad components of the geodesic tangent vectors U^a are constant along the geodesics and may be taken out of the integration. These expression have corrections at the second order in curvature.

The fourth covariant derivatives have a similar form but now involve covariant derivatives of the symmetrized Riemann tensor. In our context of maximally symmetric background, all these covariant derivatives of the Riemann tensor vanish, and the expressions simplify drastically. In particular, the third covariant derivatives are all absent as they involve the covariant derivatives of the Riemann tensor and the index distribution also gets restricted thanks to the symmetries of the Riemann tensor. This leads to (Eq. (117) of Ref. [16])

$$\sigma_{a'b'c'd'}(q', q) = \frac{3}{(u_2 - u_1)^3}\int_{u_1}^{u_2} du(u_2 - u)^2 S_{abcd}(u), \quad \text{(A13)}$$

$$\sigma_{a'b'c'd}(q', q) = -\frac{3}{(u_2 - u_1)^3}\int_{u_1}^{u_2} du(u_2 - u)^2 S_{abcd}(u), \quad \text{(A14)}$$

$$\sigma_{a'b'cd}(q', q) = \frac{3}{(u_2 - u_1)^3}\int_{u_1}^{u_2} du(u_2 - u)^2 S_{abcd}(u). \quad \text{(A15)}$$

These again have a correction at the second order in curvature. Note that the tetrad components refer to the tetrad derived from an arbitrary choice at q', by parallel transport along the geodesic $q' \to q$.

In Sec. III, we choose a tetrad at the base point of the RNC, P_0, and set it up elsewhere by parallel transporting along the geodesics emanating from P_0. This gives the tetrad $E^{\alpha'}{}_{a'}$ at q'. However, the tetrad at q, $E^{\alpha}{}_{a}$ is not equal to $\tilde{E}^{\alpha}{}_{a}$—the one obtained from $E^{\alpha'}{}_{a'}$ by parallel transport along the $q' \to q$ geodesic. They are related through the holonomy group element along the closed curve $q \to P_0 \to q' \to q$: $\tilde{E}^{a}{}_{\alpha} = H^{\beta}{}_{\alpha}\tilde{E}^{a}{}_{\beta}$. Because of the smallness of the curvature, $H^{\beta}{}_{\alpha}$ differs from the identity element by a term of order Λ. In short, the error committed in replacing the tetrad components of curvature relative to the $q' \to q$ parallelly transported tetrad by those derived from tetrad at P_0 will be of second order in the curvature, i.e., order Λ^2.

With this understood, we regard all the tetrad components in the fourth covariant derivatives to be relative to the tetrad derived from P_0. Equation (136) of Ref. [16] then gives

$$\phi = \phi_0 = \frac{3}{(u_2 - u_1)^3}\int_0^1 dw(1-w)^3 \int_{u_1}^{u_2} du \times \{(u_2 - u)^2 + (u - u_1)^2\} \times \{S_{a'b'cd}\bar{v}^4 V^{a'} V^{b'} V^c V^d\}(u, w). \tag{A16}$$

The tetrad components of the symmetrized Riemann tensor simplify further thanks to the maximal symmetry.

$$S_{abcd}(u,w) = -\frac{1}{3}(R_{\alpha\mu\beta\nu} + R_{\alpha\nu\beta\mu})E^{\alpha}{}_{a}E^{\beta}{}_{b}E^{\mu}{}_{c}E^{\nu}{}_{d}(u,w). \tag{A17}$$

$$= -\frac{\Lambda}{9}(g_{\alpha\beta}g_{\mu\nu} - g_{\alpha\nu}g_{\mu\beta} + g_{\alpha\beta}g_{\nu\mu} - g_{\alpha\mu}g_{\nu\beta})E^{\alpha}{}_{a}E^{\beta}{}_{b}E^{\mu}{}_{c}E^{\nu}{}_{d}(u,w) \tag{A18}$$

$$= -\frac{\Lambda}{9}[2(E_a\cdot E_b)(E_c\cdot E_d) - (E_a\cdot E_d)(E_b\cdot E_c) - (E_a\cdot E_c)(E_b\cdot E_d)]$$

$$= -\frac{\Lambda}{9}[2\eta_{ab}\eta_{cd} - \eta_{ad}\eta_{bc} - \eta_{ac}\eta_{bd}] \because \text{(orthonormality of the tetrad)}. \tag{A19}$$

Consequently, the symmetrized Riemann tensor comes out of the integrals. The vectors $V^a, V^{a'}$ are independent of u because they come from the expansion of $\sigma(v)$ and are independent of v since they are geodesic tangents and refer to the parallelly transported tetrad. The terms enclosed in the second pair of braces come out of the integration, and we get

$$\phi = \frac{3}{(u_2 - u_1)^3}\left[\int_0^1 dw(1-w)^3 \int_{u_1}^{u_2} du\{(u_2-u)^2 + (u-u_1)^2\}\right]\{S_{a'b'cd}\bar{v}^4 V^{a'}V^{b'}V^cV^d\} \tag{A20}$$

$$= \left[\frac{1}{2}\right]\{S_{a'b'cd}X^{a'}X^{b'}X^cX^d\}, \qquad \bar{v}V^* =: X^* (= \text{corresponding RNC}) \tag{A21}$$

$$= -\frac{\Lambda}{9}(X^2X'^2 - (X\cdot X')^2). \tag{A22}$$

Notice that the reference to the choice of the tetrad $E^{\alpha}{}_{a}$ has disappeared.

APPENDIX B: CALCULATION OF THE PARALLEL PROPAGATOR

In the main text, we needed the parallel propagator $g^{\mu}{}_{\alpha'}(x,x')$ along the null geodesic from the observation point P to a source point P'. To this end, introduce an arbitrary tetrad $e^{\mu}_{a}(P)$ and its inverse cotetrad $e^{a}_{\alpha}(P)$ which is parallel transported along the null geodesic. These will drop out at the end. The parallel propagator is then given by

$$g^{\mu}{}_{\alpha}(x,x') = e^{\mu}{}_{a}(x)e^{a}{}_{\alpha}(x').$$

The geodesic satisfies the equation

$$\frac{d^2x^\mu}{d\lambda} + \Gamma^{\mu}{}_{\alpha\beta}(x(\lambda))\frac{dx^\alpha}{d\lambda}\frac{dx^\beta}{d\lambda} = 0; \qquad x^\mu(0) = x^\mu(P) := \hat{x}^\mu, \quad \dot{x}^\mu(0) = \hat{t}^\mu.$$

The parallel transported cotetrad satisfies the equation

$$\frac{de^a_\alpha}{d\lambda} - \Gamma^{\gamma}_{\alpha\beta}\frac{dx^\beta}{d\lambda}e^a_\gamma = 0; \qquad e^a{}_\alpha(0) = e^a{}_\alpha(P) := \hat{e}^a{}_\alpha.$$

These are solved by Taylor expanding in the affine parameter λ and determining the coefficients. Denoting the evaluations at $\lambda = 0$ by hatted quantities, we write

$$e^a{}_\alpha(\lambda) = \hat{e}^a{}_\alpha + \lambda \dot{e}^a{}_\alpha(0) + \frac{\lambda^2}{2}\ddot{e}^a{}_\alpha(0)\cdots \tag{B1}$$

$$x^\mu(\lambda) = \hat{x}^\mu + \lambda\hat{t}^\mu + \frac{\lambda^2}{2}(-\hat{\Gamma}^\mu{}_{\alpha\beta}\hat{t}^\alpha\hat{t}^\beta) + \frac{\lambda^3}{6}(-\partial_\gamma\hat{\Gamma}^\mu{}_{\alpha\beta}\hat{t}^\alpha\hat{t}^\beta\hat{t}^\gamma) + \cdots \tag{B2}$$

In the last equation, we have used the geodesic equation. By differentiating the geodesic equation, the higher order terms in $x^\mu(\lambda)$ are determined. We note that the connection is order Λ and linear in coordinates. So more than the first derivative of the connection is not needed. In the Taylor expansion of x^μ, we have shown only the terms to order Λ. Substituting these expansions in the parallel transport equation determines the solution as

$$e^a{}_\alpha(\lambda) = \hat{e}^a{}_\mu\left[\delta^\mu_\alpha + (\lambda\hat{t}^\beta)\hat{\Gamma}^\mu{}_{\alpha\beta} + \frac{1}{2}(\lambda\hat{t}^\gamma)(\lambda\hat{t}^\beta)\partial_\gamma\hat{\Gamma}^\mu{}_{\alpha\beta}\right]. \tag{B3}$$

From the Taylor expansions of x^μ and $e^a{}_\alpha$, we eliminate $\lambda\hat{t}$ and obtain the parallel tetrad in terms of the coordinates. To the linear order in Λ, this simply replaces $\lambda\hat{t}^\beta$ by $(x' - x)^\beta$. The parallel propagator is then given by

$$g^\mu{}_{\alpha'}(P, P') = \hat{\delta}^\mu{}_{\alpha'} + \hat{\Gamma}^\mu{}_{\alpha'\beta'}(x' - x)^{\beta'} + \frac{1}{2}\partial_{\gamma'}\hat{\Gamma}^\mu{}_{\alpha'\beta'}(x' - x)^{\gamma'}(x' - x)^{\beta'} + o(\Lambda^2). \tag{B4}$$

We have used primed indices for notational consistency for bitensors. The hatted quantities are the coincidence limits.

Notice that the arbitrary tetrad introduced at the beginning has disappeared. We have not used any specific property of the Fermi or Riemann normal coordinates, except for the order Λ. In the main text, we have used the Fermi normal coordinates, and the connection together with its derivative are given in Eqs. (40) and (41).

APPENDIX C: FNC $\leftrightarrow$ CONFORMAL CHART TRANSFORMATIONS

We used two different charts in presenting the quadrupole field, the FNC restricted to the static patch and the conformal coordinates covering the Poincaré patch which overlaps with the static patch. To relate these two sets of coordinates, (τ, ξ^i) and (η, x^i), consider the geodesic equation in the conformal coordinates. In conformal coordinates,

$$ds^2 = \frac{\alpha^2}{\eta^2}\left[-d\eta^2 + \sum_i (dx^i)^2\right], \qquad \alpha^2 = \frac{3}{\Lambda}; \tag{C1}$$

$$\Gamma^0{}_{00} = -\frac{1}{\eta}, \qquad \Gamma^0{}_{0j} = 0, \qquad \Gamma^0{}_{ij} = -\frac{\delta_{ij}}{\eta}, \tag{C2}$$

$$\Gamma^i{}_{00} = 0, \qquad \Gamma^i{}_{0j} = -\frac{1}{\eta}\delta^i{}_j, \qquad \Gamma^k{}_{ij} = 0. \tag{C3}$$

The geodesic equation splits as

$$0 = \frac{d^2\eta}{d\lambda^2} - \frac{1}{\eta}\left(\frac{d\eta}{d\lambda}\right)^2 - \frac{\delta_{ij}}{\eta}\frac{dx^i}{d\lambda}\frac{dx^j}{d\lambda}, \tag{C4}$$

$$0 = \frac{d^2x^i}{d\lambda^2} - \frac{2}{\eta}\frac{d\eta}{d\lambda}\frac{dx^i}{d\lambda}; \qquad \Rightarrow \frac{d\vec{x}}{d\lambda} = \eta^2\vec{C},$$

$$\therefore\ \vec{x}(\lambda) = \vec{C}\int_0^\lambda d\lambda'\eta^2(\lambda') + \vec{x}_0,$$

$$\text{where } \vec{C} \text{ is a constant vector, and} \tag{C5}$$

$$0 = \frac{d^2\eta}{d\lambda^2} - \frac{1}{\eta}\left(\frac{d\eta}{d\lambda}\right)^2 - \vec{C}^2\eta^3. \tag{C6}$$

The choice $\vec{x}_0 = \vec{0}$ corresponds to radial geodesics.

To define FNC, we have to choose one timelike geodesic of which the proper time provides the time coordinate, τ. We choose this to be the line AD in Fig. 1. This corresponds to the choice $\vec{x}_0 = 0$ and $\vec{C} = 0$. The η equation can be immediately integrated to give the *reference geodesic* as

$$\eta_*(\tau) = -\sqrt{\frac{3}{\Lambda}}e^{-\tau\sqrt{\Lambda/3}}, \qquad \vec{x}_*(\tau) = \vec{0}. \tag{C7}$$

For future convenience, we have chosen an integration constant to be $-\sqrt{3/\Lambda}$, while the integration constant in the exponent is determined by the proper time condition (norm $= -1$) which makes τ to be one of the FNC.

To determine ξ^i coordinates, we consider spatial geodesics, emanating orthogonally from the reference geodesic. Clearly, we consider a radial geodesic, $\vec{x}_0 = \vec{0}$, and defining $\vec{x}(\sigma) \coloneqq \hat{C}r(\sigma)$ where $\hat{C} \coloneqq \vec{C}/|\vec{C}|$. The geodesic is determined by solving the equation for $\eta(\sigma)$ with initial conditions reflecting the orthogonality, $-d_\tau\eta_* d_\sigma\eta + d_\tau r_* d_\sigma r = 0$,

$$d^2_\sigma\eta - \frac{(d_\sigma\eta)^2}{\eta} - \vec{C}^2\eta^3 = 0, \qquad \eta(0) = \eta_*(\tau),$$
$$d_\sigma\eta(0) = 0, \qquad r(0) = 0,$$
$$d_\sigma r(0) = \gamma. \tag{C8}$$

Let P be the point with conformal coordinates (η_P, r_P) and FNC (τ, s). Taking the norm of the initial tangent vector to be s^2, the pairs of coordinates are related as

$$\eta_P \coloneqq \eta(\sigma = 1), \qquad r_P \coloneqq r(\sigma = 1), \qquad s^2 = \frac{3}{\Lambda\eta^2(0)}\gamma^2.$$

Using the first integral of the r-equation, we get

$$d_\sigma r(0) = |\vec{C}|\eta^2(0) = \gamma = \sqrt{\frac{\Lambda}{3}}|\eta(0)|s$$
$$= se^{-\tau\sqrt{\Lambda/3}} \Rightarrow |\vec{C}| = s\frac{\Lambda}{3}e^{\tau\sqrt{\Lambda/3}}. \qquad \text{(C9)}$$

To obtain (η_P, r_P), we need to solve the η-equation.

For this, we first take out a scale ζ by defining $\eta(\sigma) \coloneqq \zeta y(\sigma)$ which gives $y'' - y'^2/y - |\vec{C}|^2\zeta^2 y^3 = 0$, and choosing $\zeta = \eta_*(\tau)$, we get $|\vec{C}|^2\zeta^2 = \Lambda s^2/3 \eqqcolon \epsilon$. The desired coordinates are then given by

$$r_P \coloneqq r(\sigma = 1) = se^{-\tau\sqrt{\Lambda/3}}\int_0^1 d\sigma' y^2(\sigma') \qquad \text{(C10)}$$

$$\eta_P \coloneqq \eta(\sigma = 1) = -\sqrt{\frac{3}{\Lambda}}e^{-\tau\sqrt{\Lambda/3}}y(\sigma = 1) \quad \text{with,} \qquad \text{(C11)}$$

$$0 = y'' - \frac{y'^2}{y} - \epsilon y^3, \qquad y(0) = 1, \qquad y'(0) = 0,$$
$$\epsilon \coloneqq \frac{\Lambda}{3}s^2. \qquad \text{(C12)}$$

To order ϵ, the solution for $y(\sigma) \coloneqq y_0(\sigma) + \epsilon y_1(\sigma)$ is obtained as $y(\sigma) = 1 + \epsilon\sigma^2/2$ which leads to the coordinate transformation,

$$r(\tau, s) = se^{-\tau\sqrt{\Lambda/3}}\left(1 + \frac{\Lambda s^2}{9}\right),$$
$$\eta(\tau, s) = -\sqrt{\frac{3}{\Lambda}}e^{-\tau\sqrt{\Lambda/3}}\left(1 + \frac{\Lambda s^2}{6}\right). \qquad \text{(C13)}$$

For inverting the transformation, it is more convenient to use the combinations $a(\eta) \coloneqq -\sqrt{3/\Lambda}\eta^{-1}$, $A(\tau) \coloneqq e^{\tau\sqrt{\Lambda/3}}$ so that

$$r(A, s) = \frac{s}{A}\left(1 + \frac{\Lambda}{9}s^2\right),$$
$$a(A, s) = A\left(1 - \frac{\Lambda}{6}s^2\right) \qquad \text{(C14)}$$

$$s(a, r) = (ra)\left(1 + \frac{\Lambda}{18}(ra)^2\right),$$
$$A(a, r) = a\left(1 + \frac{\Lambda}{6}(ra)^2\right). \qquad \text{(C15)}$$

Note that ra is physical distance such as the commonly used luminosity distance in cosmology, while s is also a physical distance but along a spatial geodesic. Equation (C15) gives the relation between them.

From these relations, it is easy to verify that the stationary Killing vector field

$$-\sqrt{3/\Lambda}T \coloneqq \eta\partial_\eta + x^i\partial_i = \eta\partial_\eta + r\partial_r = \partial_\tau. \qquad \text{(C16)}$$

For completeness, we list the transformations between the conformal chart and the FNC chart in the static patch, up to order H^2,

$$\eta(\tau, \xi^i) \coloneqq -\frac{e^{-H\tau}}{H}\left(1 + \frac{H^2 s^2}{2}\right),$$
$$x^i(\tau, \xi^i) \coloneqq \xi^i e^{-H\tau}\left(1 + \frac{H^2 s^2}{3}\right) \qquad \text{(C17)}$$

$$e^{-H\tau}(\eta, x^i) \coloneqq -\eta H\left(1 - \frac{r^2}{2\eta^2}\right),$$
$$\xi^i(\eta, x^i) \coloneqq -\frac{x^i}{\eta H}\left(1 + \frac{r^2}{6\eta^2}\right). \qquad \text{(C18)}$$

With these, it can be checked that the two metrics go into each other.

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