PHYSICAL REVIEW D **94,** 064021 (2016)

Big-bounce cosmology in the presence of Immirzi field

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(Received 4 July 2016; published 8 September 2016)

The Immirzi parameter is promoted to be a scalar field and the Hamiltonian analysis of the corresponding dynamical system is performed in the presence of gravity. We identified some $SU(2)$ connections, generalizing Ashtekar-Barbero variables, and we rewrite the constraints in terms of them, setting the classical formulation suitable for loop quantization. Then, we consider the reduced system obtained when restricting to a flat isotropic cosmological model. By mimicking loop quantization via an effective semiclassical treatment, we outline how quantum effects are able to tame the initial singularity both in synchronous time and when the Immirzi field is taken as a relational time.

DOI: 10.1103/PhysRevD.94.064021

I. INTRODUCTION

Loop quantum gravity (LQG) [1,2] is probably the most valuable attempt to canonically quantize the gravitational field, essentially in view of its well-known successes: the emergence of a discrete spectrum of areas and volumes, starting from a continuous formulation [3] and the rigorous definition of a kinematical Hilbert space [4], allowed by the properties of cylindrical functionals.

Nonetheless, this proposal is affected by nontrivial shortcomings, like the difficulties in implementing the physical scalar constraint [5], the lack of well-defined classical limit [6] and, overall, the ambiguity of the Immirzi parameter choice [7].

More specifically, different values of such a free parameter of the theory correspond to dealing with nonequivalent representations of the quantum picture [8], since they are not connected by a unitary transformation [9]. Over the years many attempts have been considered to interpret the Immirzi parameter as a topological parameter [10] (supplementing the Holst action [11] by the Nieh-Yan topological term), or to fix it [12] (mainly from black hole entropy calculations, even though later developments ruled out this possibility [13]).

Here, we address the point of view to treat such an Immirzi variable as a real field. In the literature two main classical formulations have been considered, depending on the adopted action: the Holst case, in which one deals with the Holst action with a spacetime dependent Immirzi parameter [14–16], and the Nieh-Yan case, in which the Holst action is supplemented by a term containing the Immirzi field times the Nieh-Yan density [17–20]. Upon solving the equations of motion coming from the variations with respect to spin connections and substituting back in the action the solutions, both of these approaches lead to a theory classically equivalent to Einstein-Hilbert theory with a minimally coupled scalar field if no other matter field is present. However, the latter seems preferable, since in the Holst case some unnatural couplings arise in the presence of spinors. These terms are not present in the Nieh-Yan case, the reason being the theory is fully equivalent to Einstein-Cartan theory. Moreover, a topological interpretation can be given to the Immirzi field as soon as the relaxation to a constant value occurs, by analogy with the θ-angle in QCD. This relaxation, which is also expected to explain the absence of the Immirzi field in low-energy phenomenology, may be induced dynamically by adding a potential term, even though no natural way to derive a suitable potential term is known.

In [21], a more "quantum-oriented" analysis has been performed: starting from the Holst case supplemented by a kinetic and a potential term for the Immirzi field, the Hamiltonian analysis has been provided and some $SU(2)$ connections (analogous to Ashtekar-Barbero-Immirzi variables [22,23]) suitable for loop quantization have been defined. A further merit of this analysis has been the derivation of a natural mechanism which can explain the dynamical relaxation to a constant value starting from a polynomial potential. The limit of this formulation is that the kinetic and potential terms have been added "by hand," while in previous cases they have been inferred by substituting into the original action the solutions of the equations of motion obtained upon variations with respect to spin connections.

Here, we repeat the analysis made in [21] for the Nieh-Yan case; i.e. we define the analogues of the standard

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Ashtekar-Barbero-Immirzi variables starting from the action given in [18], in order to check the final structure of the constraints in Hamiltonian formulation of the theory and to set up the necessary tools for loop quantization. It is worth noting that our theory corresponds to dealing with the Hilbert-Palatini action, since the Immirzi field does not couple with the spin connection and we are not including spinor fields here (in [18] spinor fields are present and the resulting approach is equivalent to Einstein-Cartan theory). In what follows, we refer to the present analysis as Einstein-Cartan representation, having in mind that spinor fields have to be included, dealing with matter sources.

The aim of this restatement of the LQG formulation consists of checking if the Immirzi field can play, in such a scheme, the role of a time variable for the gravitational field evolution, thus also avoiding the requirement of relaxation to a constant value.

Thus, we consider the implementation of the restricted evolutionary theory to the quantization of the isotropic Robertson-Walker Universe, in order to get insight into the nature of the cosmological singularity. As a first step toward such an aim, we mimic loop quantization by considering a semiclassical polymer approach to the considered model. Actually, the obtained Hamiltonian possesses nontrivial features, making its full quantum treatment almost puzzling. The present approach to the isotropic Universe's quantum dynamics is a reliable feasibility test on the implementation of a quantum big-bounce scenario in this revised quantum cosmological framework. The test has been fully successful since we are implementing a well-traced big-bounce picture, having some peculiarities we will discuss in detail below, but strongly resembling the one obtained in loop quantum cosmology (LQC) by Ashtekar *et al.* [24]; see also [25,26].

The issue presented here is really encouraging toward the search for a fully quantum implementation of the model and the proper construction of a semiclassical limit. Finally, we want to stress how the considered Immirzi time is promising in view of a quantum implementation of the so-called Belinski-Khalatnikov-Lifshitz conjecture [27,28], sufficiently near to the singularity, when the spatial gradients are negligible with respect to the system's time evolution. In such a limit, the Immirzi time should be a viable approach and it suggests a new general perspective for investigating the singularity removal in the quantum and semiclassical sectors.

The paper is organized as follows. In Sec. II we perform the Hamiltonian analysis of the formulation with Immirzi field in the Nieh-Yan case, defining the analogues of the Ashtekar-Barbero connections and deriving the expressions of the constraints. In Sec. III we perform the reduction to a flat FRW model and we discuss the classical and the effective semiclassical dynamics, outlining the emergence of a big-bounce scenario. In Sec. IV we comment on the physical content of the model and, in particular, on the interplay between the interpretation of the Immirzi field as a relational time and as an actual physical degree of freedom of the theory. Finally, brief conclusions follow in Sec. V.

II. ASHTEKAR-BARBERO VARIABLES FOR THE IMMIRZI FIELD

The action of LQG in vacuum reads [11] (in units $c = 8\pi G = 1$)

$$S_H = \frac{1}{2}\int_{\mathcal{M}} d^4x\, e e^\mu_I e^\nu_J \left(R^{IJ}{}_{\mu\nu} - \frac{\beta}{2}\epsilon^{IJ}{}_{KL} R^{KL}{}_{\mu\nu} \right), \quad (1)$$

e^μ_I being inverse tetrads of the spacetime manifold and $R^{IJ}{}_{\mu\nu}$ denotes the curvature of the spin connection $\omega^{IJ}{}_\mu$, i.e.

$$R^{IJ}{}_{\mu\nu} = \partial_{[\mu}\omega^{IJ}{}_{\nu]} + \omega^I_{[\mu K}\omega^{JK}{}_{\nu]}, \quad (2)$$

β is the inverse of the Immirzi parameter and it multiplies a term (the Holst term) which does not affect the equations of motion. One can also start from the Einstein-Hilbert action (which in vacuum is equivalent to Einstein-Cartan theory) and show how the Immirzi parameter labels a canonical transformation one can perform on the phase space coordinates. Hence, classically β plays no role and one recovers Einstein-Cartan theory. However, the Holst term (or the corresponding canonical transformation) has a nontrivial effect in phase space, since it allows us to adopt as variables some $SU(2)$ connections, Ashtekar-Barbero variables [22,23], and their conjugate momenta E^a_i, whose explicit expression reads

$$A^i_a = \frac{1}{\beta}K^i_a + \Gamma^i_a \qquad E^a_i = \beta\sqrt{q}e^a_i, \quad (3)$$

e^a_i being inverse triads of the spatial metric q_{ab}, while K^i_a and Γ^i_a are related with the extrinsic and intrinsic curvature of the spatial metric (time and spatial derivatives), respectively. The constraints become

$$G_i \equiv \mathcal{D}_a E^a{}_i = \partial_a E^a_i - \epsilon_{ij}{}^k A^j_a E^a_k = 0 \quad (4)$$

$$\mathcal{V}_a \equiv F^i_{ab} E^b_i \quad (5)$$

$$\mathcal{S} = -\frac{1}{2\sqrt{q}\beta^2}\left\{ F^j{}_{ab} + \left(1 + \frac{1}{\beta^2}\right)\epsilon_{jmn} K^m{}_a K^n{}_b \right\}\epsilon_{jkl} E^a{}_k E^b{}_l, \quad (6)$$

F^i_{ab} being the $SU(2)$ field strength of A^i_a. The constraint G_i coincides with the $SU(2)$ Gauss constraint of a Yang-Mills gauge theory, while $\mathcal{V}_a$ and $\mathcal{S}$ are the vector and scalar constraints, which equal, modulo G_i, the supermomentum and the super-Hamiltonian of the metric formulation, respectively.

The $SU(2)$ gauge symmetry makes available for quantization some proper techniques of gauge field theories (the use of holonomies and fluxes). Furthermore, on a quantum level, one finds that different values of β label inequivalent quantum sectors (for instance the spectrum of the area operator depends on β [3]); thus the quantum theory is sensible to the Immirzi parameter. This poses the problem of a classically irrelevant parameter which is a quantum ambiguity.

In order to tame this unwanted feature, some approaches have been developed in which the Immirzi parameter is promoted to be a dynamical scalar field [14–16,18–21].

In [18] it has been outlined how if a dynamical Immirzi scalar field $\beta = \beta(x)$ is considered in a formulation in which the Holst term is replaced by the Nieh-Yan topological invariant, the system becomes equivalent to Einstein-Cartan theory with a minimally coupled scalar field, i.e.

$$S = \frac{1}{2}\int_{\mathcal{M}} d^4x\, e\, e^\mu_I e^\nu_J R^{IJ}{}_{\mu\nu} + \frac{3}{4}\int_{\mathcal{M}} d^4x\, e \partial_\mu\beta\partial^\mu\beta. \quad (7)$$

This is the starting point of our analysis. We want to discuss the structure of the phase space in such a theory. Our primary aim is to derive the same kind of $SU(2)$ gauge structure as in a Holst formulation, such that the quantization procedure of LQG can be applied also in the presence of the Immirzi field.

In particular, we can get an $SU(2)$ Gauss constraint also for (7), as soon as the connections and momenta are defined as follows:

$$^{(\beta)}E^a{}_i \equiv \beta(x)\sqrt{q}e^a_i, \quad (8)$$

$$^{(\beta)}A^i{}_a \equiv \Gamma^i{}_a + \frac{1}{2\beta}\epsilon^{ijk}e^b_k e^j_a \partial_b\beta + \frac{1}{\beta}K^i_a. \quad (9)$$

It is worth noting that $\{^{(\beta)}A^i{}_a, {}^{(\beta)}E^a{}_i\}$ still form a couple of canonically conjugate variables, i.e.

$$\{^{(\beta)}E^a_i(x), {}^{(\beta)}A^j_b(y)\} = \delta^a_b\delta^j_i\delta(x,y), \quad (10)$$

and we take them as the coordinates of the gravitational phase space. Other coordinates describe the Immirzi field and we cannot simply take $\{\beta, P\}$, P being the same momentum as that of an ordinary scalar field in metric formulation, since it would have nonvanishing Poisson brackets with $^{(\beta)}A^i{}_a$. For this reason, we defined the scalar field momentum as follows:

$$^{(\beta)}P(x) \equiv P(x) + \frac{1}{\beta}E^a_i K^i_a, \quad (11)$$

and one can explicitly check that the only nonvanishing Poisson brackets are given by (10) and

$$\{^{(\beta)}P(x), \beta(y)\} = \delta(x,y). \quad (12)$$

Eventually the vector constraint takes the form

$$\mathcal{V}_a \equiv {}^{(\beta)}F^i_{ab}{}^{(\beta)}E^b_i + {}^{(\beta)}P\partial_a\beta, \quad (13)$$

and the scalar constraint reads as

$$\begin{aligned}\mathcal{S} \equiv & -\frac{1}{2\beta^2\sqrt{q}}\left\{^{(\beta)}F^j{}_{ab} + \left(1+\frac{1}{\beta^2}\right)\epsilon_{jmn}K^m{}_a K^n{}_b\right\} \\ & \times \epsilon_{jkl}{}^{(\beta)}E^a{}_k{}^{(\beta)}E^b{}_l \\ & + \frac{3}{4\beta^2\sqrt{q}}\left(1+\frac{1}{\beta^2}\right){}^{(\beta)}E^a_k{}^{(\beta)}E^b_k\beta_{,a}\beta_{,b} \\ & - \frac{1}{\beta^3\sqrt{q}}{}^{(\beta)}E^a{}_j{}^{(\beta)}E^b{}_j\nabla_a\beta_{,b} \\ & + \frac{1}{3\sqrt{q}}\left(^{(\beta)}P - \frac{K^j{}_a{}^{(\beta)}E^a{}_j}{\beta^2}\right)^2, \end{aligned} \quad (14)$$

where $^{(\beta)}F^i_{ab}$ is the $SU(2)$ field strength of $^{(\beta)}A^i_a$.

It is worth noting that the vector constraint (13) retains the same form as in the Holst formulation in the presence of a minimally coupled scalar field (see [29,30]). On the contrary, the scalar constraint contains some additional terms. While the theory is classically equivalent to gravity with a minimally coupled scalar field, thus we can perform a change of variables such that the scalar constraint retains the standard form; on a quantum level the choice of variables is crucial and we adopted those suitable for loop quantization. Therefore, while the classical dynamics is equivalent to that of gravity with a minimally coupled scalar field, we expect the quantum dynamics of the Immirzi field to differ significantly. In order to investigate this peculiar dynamics, we consider the symmetry-reduced case of cosmology, in which several simplifications occur. We conclude this section by noting that no contradiction exists between the minimally coupled nature of the original theory and the nonminimal Hamiltonian (14). Indeed, insofar as we remain in the classical sector, the two formulations are equivalent and connected by a canonical transformation. Nonetheless, the nonminimal coupling between the $SU(2)$ connections and the Immirzi field leads, on a quantum level, to a nontrivial dynamics, reliably connected via a nonunitary transformation to the original formulation (6) (indeed, we recall that in LQG different values of the Immirzi parameter, here dynamically emerging, correspond to nonunitary representations).

III. MINISUPERSPACE MODEL

Let us consider the homogeneous and isotropic flat Universe described by the FRW line element

$$ds^2 = -N^2(t)dt^2 + a^2(t)(dx^2 + dy^2 + dz^2), \quad (15)$$

the scale factor $a(t)$ being the only dynamical degree of freedom. One can choose the triads $e^i{}_a = a(t)\delta^i{}_a$, such that the pair of conjugate variables $\{{}^{(\beta)}E, {}^{(\beta)}A\}$ reduces to

$$ {}^{(\beta)}E^a{}_i = p\delta^a{}_i \qquad {}^{(\beta)}A^j{}_b = c\delta^j{}_b, \tag{16} $$

where $\{p, c\}$ are coordinates of the reduced phase space, whose explicit expressions read

$$ |p| = |\beta|a^2 \qquad c = \frac{\dot{a}}{\beta N}, \tag{17} $$

and they form a couple of canonical variables with Poisson brackets given by

$$ \{p, c\} = \frac{1}{3V_0}, \tag{18} $$

V_0 being the fiducial volume of the considered spacetime region. The scalar field phase space coordinates $\{\beta, {}^{(\beta)}P\}$ are restricted to depend on time only, as well.

Since all spatial gradients vanish, the vector constraint (13) holds identically, while the scalar constraint (14) becomes

$$ \mathcal{S} = -3c^2\sqrt{\frac{|p|}{|\beta|}}(\beta^2 - 1) - 2\sqrt{\frac{|\beta|}{|p|}}{}^{(\beta)}Pc + \frac{|\beta|^{3/2}}{3|p|^{3/2}}{}^{(\beta)}P^2. \tag{19} $$

It is worth noting that the kinematical structure coincides with that of LQC [25,26], but the scalar constraint differs significantly. In particular, the second term in (19) is not present in LQC formulation.

The equivalence with the classical dynamics of gravity in the presence of a scalar field can be explicitly demonstrated by computing, through Hamilton equations, the Friedman equation, which can be written as

$$ \left(\frac{\dot{a}}{a}\right)^2 = \frac{{}^{(\beta)}P^2}{3p^3}\frac{|\beta|^5}{(1 + |\beta|)^2} = \frac{\rho}{3}, \tag{20} $$

where ρ obeys the continuity equation $\dot{\rho} = -6\rho\frac{\dot{a}}{a}$, which is equivalent to dealing with a massless scalar field energy density. Moreover, taking β as a clocklike field, one obtains the following dynamics for the scale factor,

$$ a = a(\beta) = a_0 e^{\pm\frac{\beta-\beta_0}{2}}, \tag{21} $$

the initial condition being in $a_0 = a(\beta_0)$, with the classical singularities in $\beta = \pm\infty$, according to the chosen branch.

We perform a first analysis on the quantum dynamics, by mimicking the quantization procedure adopted in LQC. In particular, we discuss the classical implications of the replacement

$$ c \longrightarrow \frac{\sin\mu c}{\mu}, \tag{22} $$

in the scalar constraint (19), where the polymer parameter μ is fixed according to

$$ \mu^2 = \frac{\Delta}{|p|^\alpha} \qquad \Delta \equiv 2\sqrt{3}\pi l_P^2, \tag{23} $$

Δ being the minimum area gap eigenvalue in LQG, while α is a quantum ambiguity. In particular, in what follows we will consider the most relevant cases in LQC, namely $\alpha = 0$ and $\alpha = 1$ corresponding to the so-called μ_0 and $\bar{\mu}$ schemes, respectively.

The motivation for this analysis comes from LQC, where the replacement (22) on a classical level is able to capture the main semiclassical nontrivial effect, i.e. the emergence of the bounce replacing the initial singularity [31,32].

Hence, we consider the classical dynamics generated by the modified scalar constraint

$$ \mathcal{S}_{sc} = -3\frac{\sin^2\mu c}{\mu^2}\sqrt{\frac{|p|}{|\beta|}}(|\beta|^2 - 1) - 2\frac{\sin\mu c}{\mu^2}\sqrt{\frac{|\beta|}{|p|}}{}^{(\beta)}P + \frac{|\beta|^{3/2}}{3|p|^{3/2}}{}^{(\beta)}P^2, \tag{24} $$

from which the following Friedman equation can be inferred,

$$ \left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho}{3}\left(\frac{1 + |\beta|}{|\beta|}\right)^2\left(\sqrt{1 - \frac{\rho}{\rho_C}} - \frac{|\beta|}{1 + |\beta|}\right)^2, \tag{25} $$

$\rho_C \equiv \frac{3|\beta|^3}{\mu^2|p|}$ being the critical density at which the bounce occurs in LQC, and the equations for $\dot{\beta}$ and $\dot{\rho}$ are given respectively by

$$ \dot{\beta} = \frac{2{}^{(\beta)}P}{3p^{3/2}}\frac{|\beta|^{5/2}}{1 + |\beta|} \qquad \dot{\rho} = -6\rho\frac{\dot{a}}{a}. \tag{26} $$

It is worth noting that the bounce is predicted also from (25) (see Fig. 1), but at a smaller energy density ${}^{(\beta)}\rho_C$ with respect to LQC:

$$ {}^{(\beta)}\rho_C = \rho_C\frac{|\beta|(|\beta| + 2)}{(1 + |\beta|)^2} < \rho_C. \tag{27} $$

Furthermore the continuity equation for ρ still holds, with the ratio $\frac{\dot{a}}{a}$ now described by (25).

If we take β as a clocklike field, the scale factor behaves as follows:

$$ a(\beta) = \left[\frac{A}{4(a_0\beta)^{2+\alpha}}e^{-\frac{2+\alpha}{2}(\beta-\beta_0)} + a_0^{2+\alpha}e^{\frac{2+\alpha}{2}(\beta-\beta_0)}\right]^{1/(2+\alpha)}, \tag{28} $$

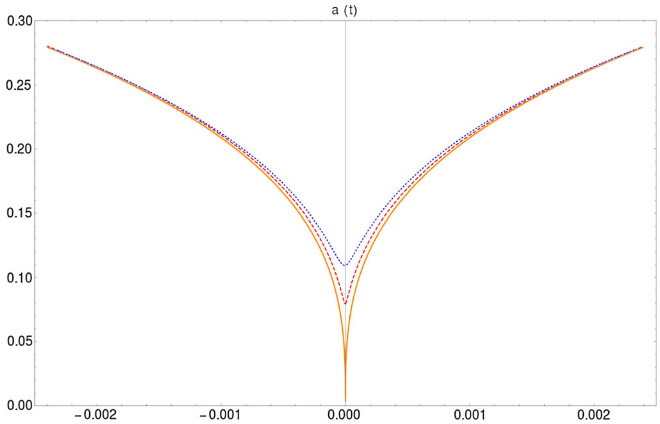

FIG. 1. Physical scale factor as function of t. The solid lines are the two classical branches, reaching the singularity $(\mathrm{a}=0)$ for $t=0$. The dotted and dashed lines are the solutions of the polymer cosmological Hamiltonian for $\alpha=0$, 1, respectively. Positive and negative branches are plotted and the singularity is smoothly removed, matching both solutions.

(we remind that we are just considering the two cases $\alpha=1,0$), where we fixed initial conditions $a(\beta_0)=a_0$ for positive β_0, while the constant A determines the magnitude of the quantum corrections [see the comparison with the $+$ sign classical solution in (21)] and it reads explicitly

$$A=\frac{\Delta\beta_0^2}{9(\beta_0+1)^2}{}^{(\beta)}P^2(\beta_0). \tag{29}$$

$a(\beta)$ is plotted in Fig. 2, in which also the solution for negative β_0 values is drawn, and it outlines how the solution splits into two separate branches for positive and negative values of β. Each branch has its own bounce and they both diverge in $\beta\to 0$.

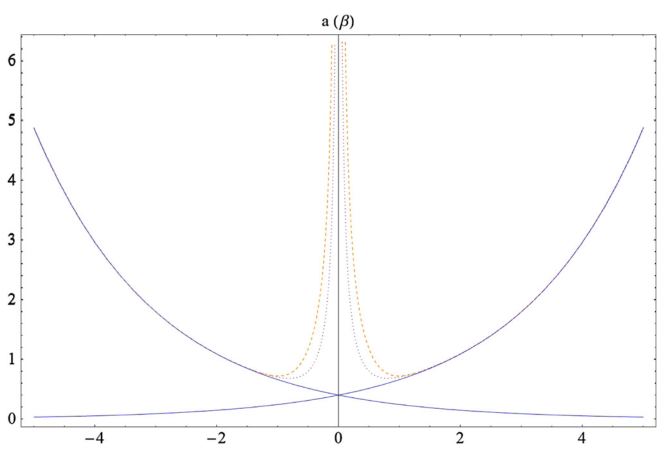

FIG. 2. Physical scale factor as function of β: the solid lines are the two classical branches, reaching the singularity $(\mathrm{a}=0)$ for $\beta=\pm\infty$. The dotted and dashed lines are the solutions of the polymer cosmological Hamiltonian for $\alpha=0$, 1, respectively. In both cases a divergence in $\beta=0$ arises, while singularity is removed in each solution.

IV. PHYSICAL OUTLOOKS ON THE MODEL

In view of comparing the present approach with the standard LQC singularity removal [24], we observe that both formulations rely on a scalar relational time variable: here we describe the cosmological evolution in terms of the Immirzi field, while LQC refers to a matter clock constructed by a minimally coupled scalar field. We stress how the semiclassical predictions of these two approaches mainly overlap in characterizing a big-bounce dynamics, apart from slightly different values of the associated critical energy density. However, the nonminimally coupled character of the Immirzi field, in the $SU(2)$ variable representation, is reflected by a modified scalar constraint with respect to LQC (19). The symmetry reduction associated to the Universe's homogeneity seems to weaken the difference in the corresponding outputs of the minimal and nonminimally coupled theories. But, we can infer that in a full quantization procedure, based on $SU(2)$ variables of LQG, the present formulation leads to nontrivial implications in the implementation of the scalar constraint (14), requiring attention for the possible improvements or shortcomings with respect to the standard formulation.

We also stress that, in the present cosmological framework the Immirzi parameter, being a relational time, needs no longer to be fixed to a specific value. It must be thought of as an evolutionary degree of freedom for the restated LQC, similarly to a matter scalar degree of freedom. Again, such a point of view could appear too ambitious for the full theory, retaining peculiar inhomogeneous couplings between the Immirzi field and the $SU(2)$ variables.

In a general framework, appropriate to describe local quantum space properties, we are naturally led to suggest, as in the Introduction, a relaxation of the Immirzi field to a fixed value. The real mechanism able to explain such a relaxation must probably be searched for in the coupling of the proposed theory with matter, especially when a potential term for the scalar field can be suitably generated. Clearly such a potential term, privileging a value of the Immirzi field in correspondence to a "vacuum configuration," can also come from extended approaches to general relativity, relying on extensions of the Hilbert-Palatini (and then of the Einstein-Cartan) Lagrangian for the spacetime geometry.

We conclude by observing that, in principle, the two different interpretations of the Immirzi field, i.e. the cosmological one (dealing with an evolutionary degree of freedom) and the more general case of a local quantization of the space geometry, do not conflict with each other. In fact, they are associated to different physical contexts or, to say it better, to different stages of the Universe's evolution. In other words, we can argue that near the big bounce, the Immirzi field was a real dynamical degree of freedom, later relaxing to a vacuum configuration, as a result of the Universe cooling during expansion.

V. CONCLUSIONS

We performed the Hamiltonian analysis of the model presented in [18], in which the Immirzi parameter is promoted to be a scalar field. We outlined how it is possible to recover some $SU(2)$ connections, playing the role of Ashtekar-Barbero variables. The corresponding momenta describe the triads of the spatial metric times the squared Immirzi field and emerge as basic variables in loop quantization. Inspired by this achievement we investigate the cosmological implications of the model. We identified the analogues of reduced phase space variables of LQC and we mimic loop quantization by an effective treatment, based on replacing the reduced connection variable with its polymerlike version. We considered two choices of the polymer parameter, corresponding to the so-called μ_0 and $\bar{\mu}$ schemes in LQC. The results of this analysis show how the singularity is replaced by a bounce, occurring at lower energy density with respect to the critical energy density in LQC. We also investigated the possibility of taking β as a relational time. The equation for the scale factor as a function of β can be analytically solved and the initial singularity is still replaced by a bounce, but a subtlety arises. The scale factor diverges in $\beta = 0$, such that one gets two disconnected branches for positive and negative values of the Immirzi field. Further investigations are needed in order to test whether such a divergence is tamed in a full quantum treatment or is a proper feature of the model.

We conclude by coming back to a subtle question, concerning the double interpretation of the Immirzi field, as it emerges from the present analysis: on the cosmological level it behaves as a relational time, while in the full LQC framework it is a dynamical degree of freedom, to be frozen somehow to a specific constant value. The possibility of dealing with the Immirzi field as a relational time, for the early Universe's evolution, is clearly a consequence of the simplifications associated to the symmetry restrictions of the considered minisuperspace model: the $SU(2)$ algebra of the LQC variables reduces to a $U(1)$-like formulation of the discrete cosmological space. In the general case, the Immirzi field is still a dynamical degree of freedom, but its interpretation as a physical clock can encounter some shortcomings, for instance due to the presence of its momentum gradients in the nonminimal Hamiltonian (14). However, we think, as already pointed out in Sec. II, that the two possible interpretations correspond simply to different stages of the Universe's dynamics.

When the Universe emerges from the big bang/bounce, the space is globally homogeneous and the analysis of Sec. III naturally applies: the Immirzi field is a dynamical degree of freedom, playing the role of a relational time, up to a classical limit is reached. After this phase, the Universe expands and cools, so that we suggest that the Immirzi field starts to feel the influence of a potential term (due to additional degrees of freedom, like the one offered by extended gravitational theories). The resulting self-interacting field is frozen to a vacuum value by a phase transition of the Universe, due to decreasing temperature (i.e. a symmetry breaking takes place across the Universe). Then the Immirzi field reduces to the standard Immirzi parameter of LQC and its value can affect the calculation concerning local spacetime realization on the quantum level; for instance its role in the determination of black hole entropy can be evaluated [13,33,34]. The scenario proposed above has supporting points, corresponding, on one hand, to the possibility of neglecting, near enough to the singularity, the self-interacting scalar field potential (for a study concerning its derivation in $f(R)$ extended gravity, see [35]), and, on the other hand, the so-called Belinsky-Khalatnikov-Lifshitz conjecture [28,36–38], which guesses the possibility of reducing the primordial cosmological Wheeler superspace to the product of minisuperspaces, one in each space point (for a quantum discussion of the implications of such a conjecture, see [39] and [40]). In other words, the proposed minisuperspace scenario, based on a Klein-Fock Immirzi field, could possess a much greater degree of generality (especially if extended to a Bianchi I cosmology).

Clearly, our conjecture requires, to be viable, the individuation of a convincing model for the potential term generation, also able to account for a specific value assumed by the Immirzi field on the vacuum configuration.

ACKNOWLEDGMENTS

The work of F. C. was supported by funds provided by the National Science Center under the agreement DEC12 2011/02/A/ST2/00294.

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