PHYSICAL REVIEW D **94,** 063517 (2016)

Gravitational particle production in oscillating backgrounds and its cosmological implications

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(Received 14 July 2016; published 14 September 2016)

We study the production of light particles due to the oscillation of the Hubble parameter or the scale factor. Any coherently oscillating scalar field, irrespective of its energy fraction in the Universe, imprints such an oscillating feature on them. Not only Einstein gravity but the extended gravity models, such as models with nonminimal (derivative) coupling to gravity and $f(R)$ gravity, lead to oscillation of the scale factor. We present a convenient way to estimate the gravitational particle production rate in these circumstances. Cosmological implications of gravitational particle production, such as dark matter/radiation and moduli problem, are discussed. For example, if the theory is described solely by the standard model plus the Peccei-Quinn sector, the Starobinsky R^2 inflation may lead to an observable amount of axion dark radiation.

DOI: 10.1103/PhysRevD.94.063517

I. INTRODUCTION

A scalar field often plays an important role in cosmology. Inflaton, which drives the accelerated expansion in the early Universe, is an obvious example of such a scalar field [1,2]. The curvaton [3,4], which may offer seeds of the present large-scale structure of the Universe, is another example. A scalar field can also generate the baryon-to-photon ratio in the present Universe [5,6]. These scalar fields typically have to transfer their energy to other components of the Universe, e.g., radiation, during their oscillating regimes. Hence, particle production by a coherently oscillating background is rather common. One of the most prominent examples is (p)reheating after inflation caused by violent oscillation of the inflaton. See Refs. [7–11], for instance.

In most cases studied so far, an explicit coupling between the oscillating scalar field ϕ and another light field χ is introduced in the action. However, the simplest situation is that ϕ and χ interact only through gravity: there are no interactions between them if the cosmic expansion is shut off or they interact only through the evolution of the Universe.[1] It is known that even in such a case, particle production occurs through the change of the cosmic evolution caused by the ϕ field, which is often called "gravitational particle production" [12,13]. Recently, we have pointed out that a (small) oscillation of the Hubble parameter or the scale factor caused by inflaton oscillation generates particles which couple to gravity nonconformally [14]. Such gravitational production takes place even in Einstein gravity, and its effect becomes stronger for some extended gravity theories. Although the production rate is suppressed by the Planck scale, still it can have impacts on cosmology.

In this paper, we extend our previous analysis to cope with more general cases where the oscillation of the scale factor is caused by coherent oscillation of an either dominant or subdominant scalar field. First we consider the system with Einstein gravity and a scalar field coupled minimally with gravity. Then the gravity sectors are extended. As examples, we consider the $f(\phi)R$, $f(R)$, and also $G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ models where ϕ is the scalar field, R is the Ricci scalar, and $G^{\mu\nu}$ is the Einstein tensor, respectively. In these models, the oscillation of the scale factor is more prominent than that in the Einstein gravity, and, hence, gravitational particle production becomes more efficient. We also discuss the cosmological implications of the gravitational particle production such as the dark matter/radiation and the moduli problem in each case.

Before starting the analysis, let us clarify the differences of our study from the existing literature. For example, in Ref. [15], the particle production of the $f(\phi)R$ theory is considered with ϕ being the inflaton. The authors in Refs. [16–18] discussed the particle production of the $f(R)$ theory for the case of $f(R) = R + cR^2$ where the second term is dominant. Therefore, our study has some overlaps with these papers. However, the goal of our paper is to offer a systematic way to estimate the particle production rate. For that purpose, we pay particular attention to an oscillating feature of the scale factor. In fact, we will see that we can treat a wider class of gravity models in a unified way from this viewpoint. It also makes manifest how the background oscillation produces particles coupled with gravity nonconformally. In addition to the

[1]In our terminology, if Planck-suppressed operators involving ϕ and χ are introduced explicitly in the action, they are regarded as explicit couplings, which are not of our interest.

above point, we also discuss the case where the oscillating scalar field is subdominant. A subdominant scalar field can have some cosmological implications, especially in the extended gravity theories, as we will see below.

In this paper, we calculate the particle production rate in the original defining frame [i.e., the Jordan frame for the $f(\phi)R$ and $f(R)$ theories]. This is partly because we cannot go to the Einstein frame in some classes of extended gravity, such as the $G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ theory. In order to estimate the production rate including such a case, we would like to understand what is happening in the original frame in our unified framework. Roughly speaking, we are looking for the effects of Planck-suppressed interactions of the oscillating scalar field on particle production. As expected, it is not so violent compared with the preheating in the usual context [8–11], but still it can play an important role in cosmology, e.g., dark matter/dark radiation production. The effect is prominent in some extended gravity models, and the gravitational coupling itself can be the main source of reheating.

The organization of this paper is as follows. In Sec. II, we consider the Einstein gravity and show that a (small) oscillating part of the scale factor is induced by the coherently oscillating scalar field even in such a minimal case. We show that this process can be understood as annihilation of the scalar field. In Sec. III, we consider the $f(\phi)R$ theory. In this case, we show that the oscillating part of the scale factor linearly depends on the scalar field in general, and, hence, the scalar field can decay into light particles gravitationally, in contrast to the annihilation process in the previous section. In Sec. IV, we consider the $f(R)$ theory. We find that the situation is rather similar to that of the $f(\phi)R$ theory in this case. In Sec. V, we consider the $G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ theory. In this case, we limit ourselves to the case where the scalar field is subdominant to avoid a gradient instability. Section VI is devoted to the conclusions and discussion.

II. EINSTEIN GRAVITY

In this section, we consider gravitational particle production in Einstein gravity. We will show that the scale factor has an oscillating feature caused by the coherent oscillation of a scalar field ϕ even if it is subdominant. If it dominates the Universe, the amount of produced particles, whose dominant contribution comes from the onset of its oscillation, becomes comparable to that produced by the change of the background geometry [12,14], as expected.

A. Background dynamics

Let us consider the action

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2}M_P^2 R - \frac{1}{2}(\partial\phi)^2 - V(\phi) + \mathcal{L}_M\right), \tag{2.1}$$

where $g = \det(g_{\mu\nu})$ is the determinant of the metric, M_P is the reduced Planck scale, and R is the Ricci scalar. Here and hereafter, we adopt the $(-+++)$ convention for the metric $g_{\mu\nu}$. The scalar field ϕ, which is of our main interest, oscillates coherently and imprints an oscillatory feature in the scale factor. $\mathcal{L}_M$ denotes the Lagrangian for matter other than the scalar ϕ. We assume that $\mathcal{L}_M$ does not depend on ϕ.

The background equation of motion of ϕ is given by

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0, \tag{2.2}$$

where H is the Hubble parameter, and the prime denotes the derivation with respect to ϕ. This is also rewritten as

$$\dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) = 0, \tag{2.3}$$

where $\rho_\phi \equiv \dot{\phi}^2/2 + V$ and $p_\phi \equiv \dot{\phi}^2/2 - V$. The Einstein equation reads

$$3H^2 = \frac{\rho_\phi + \rho_M}{M_P^2}, \tag{2.4}$$

$$3H^2 + 2\dot{H} = -\frac{p_\phi + p_M}{M_P^2}, \tag{2.5}$$

where

$$\rho_M = g_{00}\mathcal{L}_M - 2\frac{\delta\mathcal{L}_M}{\delta g^{00}}, \qquad p_M = \mathcal{L}_M. \tag{2.6}$$

Note that $\delta\mathcal{L}_M/\delta g^{ij} = 0$ for the background part. By using these equations, we obtain

$$\dot{\rho}_M + 3H(\rho_M + p_M) = 0. \tag{2.7}$$

Hereafter, we assume that the matter part satisfies the equation of state $p_M = w\rho_M$. This shows that ρ_M exactly scales as $\rho_M \propto a^{-3(1+w)}$.

The cosmological setup we are considering is as follows. After inflation, inflaton decays and the Universe is dominated by the "matter,"[2]; which is characterized by the energy density ρ_M and the equation of state w. We leave it as a free parameter for a while, although hot thermal plasma with $w = 1/3$ is typically produced by the inflaton decay. The scalar field ϕ begins to oscillate around the time $H = m_\phi$ in a background dominated by ρ_M, with $m_\phi^2 \equiv |(\partial V/\partial\Phi)/\Phi|$ being the effective mass squared of ϕ and Φ being the amplitude of ϕ oscillation. In the following, we consider the deeply oscillating regime $m_\phi \gg H$. We do not necessarily assume that ϕ dominates the Universe in the following discussion. Even if ϕ is subdominant, it induces a

[2]Here and in what follows, matter does not always mean nonrelativistic fluids.

small oscillating feature in the Hubble parameter or the scale factor and leads to particle production, as we will see below.

Henceforth, we will extract an oscillating part of the scale factor, which is important for the gravitational particle production. In particular, we will express it in terms of ϕ explicitly. To do so, we divide quantities into the oscillation-averaged part, which only evolves due to the Hubble expansion and the rapidly oscillating part with frequency of order $\sim m_\phi$:

$$H = \langle H\rangle + \delta H, \tag{2.8}$$

$$a = \langle a\rangle + \delta a, \tag{2.9}$$

$$\rho_\phi = \langle \rho_\phi\rangle + \delta\rho_\phi, \tag{2.10}$$

$$\rho_M = \langle \rho_M\rangle + \delta\rho_M. \tag{2.11}$$

Here the bracket $\langle\ldots\rangle$ denotes the oscillation average, and quantities with δ denote the oscillating part. We treat the oscillating parts as perturbations and keep only terms up to first order in them. This treatment is justified in the deeply oscillating regime.

We first note that ρ_M exactly scales as $\rho_M \propto a^{-3(1+w)}$ and, also, $\dot\rho_\phi \sim \mathcal{O}(H\rho_\phi)$. Thus, we can use the Virial theorem for ϕ in the limit $m_\phi \gg H$ and take the oscillation average to obtain

$$\dot{\langle\rho_\phi\rangle} + \frac{6n}{n+2}\langle H\rangle\langle\rho_\phi\rangle = 0, \tag{2.12}$$

where we have assumed $V \sim \phi^n$ dominates in the potential.[3] This implies that ρ_ϕ scales as $\langle\rho_\phi\rangle \propto a^{-6n/(n+2)}$. In order to extract the oscillating part, the following equation is useful:

$$\dot H = -\sum_{i=\phi,M}\frac{\rho_i + p_i}{2M_P^2}. \tag{2.13}$$

From this expression, we obtain the oscillating part of the Hubble parameter as

$$\delta\dot H \simeq -\frac{1}{2M_P^2}(\dot\phi^2 - \langle\dot\phi^2\rangle + (1+w)\delta\rho_M). \tag{2.14}$$

Let us make an order-of-magnitude estimation to understand its approximated behavior. We have $\delta\rho_M/\rho_M \sim \delta a/a \sim \mathcal{O}(\delta H/m_\phi)$ since the relation $\rho_M \propto a^{-3(1+w)}$ holds exactly. Therefore, we obtain $\delta\rho_M/M_P^2 \lesssim \mathcal{O}(H^2\delta H/m_\phi) \ll m_\phi\delta H$. Thus, the last term on the rhs of Eq. (2.14) can be neglected, and, hence, we find

$$\delta\dot H \simeq -\frac{1}{2M_P^2}(\dot\phi^2 - \langle\dot\phi^2\rangle). \tag{2.15}$$

It can be expressed as

$$\delta\dot H + \frac{6n}{n+2}\langle H\rangle\delta H \simeq -\frac{1}{n+2}\frac{1}{M_P^2}\left(\frac{d}{dt} + 3H\right)(\phi\dot\phi), \tag{2.16}$$

where we have used the oscillating part of Eq. (2.4). A similar equation was derived in Ref. [14] in the case where ϕ dominates the Universe. In contrast, here, we have not necessarily assumed ϕ domination. In Eq. (2.16), only the relevant terms are the first terms of the lhs and rhs. This is because δH and $\phi\dot\phi$ are oscillating functions with frequency $\sim m_\phi$, and, hence, the second terms of the lhs and rhs are suppressed by $\mathcal{O}(H/m_\phi)$. Thus, we arrive at

$$\delta H \simeq -\frac{1}{n+2}\frac{\phi\dot\phi}{M_P^2}. \tag{2.17}$$

By integrating this, we obtain

$$\frac{a(t)}{\langle a(t)\rangle} \simeq 1 - \frac{1}{2(n+2)}\frac{\phi^2 - \langle\phi^2\rangle}{M_P^2}. \tag{2.18}$$

This equation explicitly relates the oscillating part of the scale factor to the (subdominant) oscillating scalar field. Note that ϕ^2 appears regardless of the exponent n of the potential.

B. Particle production rate

In the previous subsection, we obtained

$$a(t) \simeq \langle a(t)\rangle\left(1 - \frac{1}{2(n+2)}\frac{\phi^2 - \langle\phi^2\rangle}{M_P^2}\right). \tag{2.19}$$

Now let us estimate the particle production rate due to the oscillating part of the scale factor a. Intuitively, in the present case, such a particle production may be understood as the pair annihilation of ϕ, since the oscillating part of the scale factor depends quadratically on ϕ. Thus, we call it "gravitational annihilation" [14].[4] On the other hand, we

[3] If ϕ oscillates around the finite vacuum expectation value ϕ_0, ϕ should be interpreted as its deviation from the potential minimum.

[4] In Ref. [15], the gravitational annihilation due to $f(\phi)R$ coupling was discussed. There it was claimed that this effect does not exist in the Einstein gravity limit. This is not true, however, as shown here and also in Ref. [14]: the gravitational annihilation takes place even in Einstein gravity. Recently, Refs. [19,20] considered dark matter production by the gravitational annihilation of particles in a thermal bath.

will see that the oscillating part of the scale factor depends linearly on the scalar field for extended gravity theories such as the $f(\phi)R$ and $f(R)$ models. In contrast to the present case, we can view it as the decay of the scalar field, and, hence, we will use the word "gravitational decay" in such cases.

Below we consider particle production of a minimally coupled scalar and graviton. The production of fermions and vector bosons is suppressed by their masses and couplings because they are classically Weyl invariant and do not feel the oscillation of the scale factor in the massless limit.

1. Scalar

First we consider a scalar field χ which minimally couples with gravity

$$S = \int d^4x\sqrt{-g}\left(-\frac{1}{2}(\partial\chi)^2 - \frac{1}{2}m_\chi^2\chi^2\right), \tag{2.20}$$

with $m_\chi \ll m_\phi$. In the standard model (SM), only the Higgs boson would be a minimally coupled scalar. Here we do not limit ourselves to the case where χ is the Higgs boson but consider general scalar fields. By using the master formula (A26) derived in Appendix A, the number density of χ particles produced during one Hubble time after ϕ begins to oscillate is given by.[5]

$$n_\chi(t) \simeq \frac{C}{32\pi H}\left(\frac{1}{n+2}\right)^2\left(\frac{m_\phi^2\Phi^2}{M_P^2}\right)^2, \tag{2.21}$$

where Φ denotes the oscillation amplitude of ϕ. This result may be translated into the effective annihilation process of ϕ particles with a rate of

$$\Gamma_{\phi\phi\to\chi\chi} \equiv n_\phi\langle\sigma v\rangle_{\phi\phi\to\chi\chi} \simeq \frac{C}{16\pi}\left(\frac{1}{n+2}\right)^2\frac{\Phi^2}{M_P^2}\frac{m_\phi^3}{M_P^2}. \tag{2.22}$$

Taking into account the Hubble expansion, one can easily see that the largest contribution comes from the very beginning of the ϕ oscillation at $H = m_\phi$ unless w is unlikely large and/or n is so small. Note that even if χ obtains a Hubble-induced mass term, this production mechanism becomes effective soon after the ϕ oscillation.

2. Graviton

Next we apply our formalism to the graviton production. The graviton action is given by

$$S = \int d\tau d^3x a^2(t)\frac{M_P^2}{8}\left[\left(\frac{\partial h_{ij}}{\partial\tau}\right)^2 - (\partial_k h_{ij})^2\right], \tag{2.23}$$

where τ is the conformal time, and h_{ij} is the metric perturbation satisfying the transverse and traceless conditions $h_{ii} = \partial_i h_{ij} = 0$. The indices i, j, and k run the space coordinates. Hence, the production rate is similar to the minimal scalar, except for the factor 2 corresponding to the two polarization states of the graviton:

$$n_h(t) \simeq \frac{C}{16\pi H}\left(\frac{1}{n+2}\right)^2\left(\frac{m_\phi^2\Phi^2}{M_P^2}\right)^2. \tag{2.24}$$

C. Cosmological implications

The gravitational annihilation of a subdominant scalar field ϕ yields the abundance given in (2.21), but the gravitational annihilation of the inflaton also gives a significant contribution [12,14]. The ratio of χ abundance produced by a subdominant scalar field ϕ to that produced by the inflaton is estimated as

$$\frac{n_\chi^{(\phi)}}{n_\chi^{(\mathrm{inf})}} \simeq \epsilon\frac{m_\phi}{H_{\mathrm{inf}}}\left(\frac{\phi_i}{M_P}\right)^4, \tag{2.25}$$

where

$$\epsilon = \min\left[1, \sqrt{m_\phi/\Gamma_{\mathrm{inf}}}\right], \tag{2.26}$$

with H_{inf} and Γ_{inf} being the Hubble scale at the end of inflation and the inflaton decay rate, respectively. Here we have assumed that the inflaton oscillation behaves as nonrelativistic matter, and the inflaton decays into radiation. Also, ϕ is assumed to be subdominant at the onset of its oscillation. The dominant contribution comes from the gravitational annihilation of the inflaton since $m_\phi < H_{\mathrm{inf}}$ in order for ϕ to begin coherent oscillation after inflation. In this case, cosmological implications were studied in [14], and we briefly discuss them here. The energy-density-to-entropy ratio of χ with a sizable mass term is estimated to be

$$\frac{\rho_\chi^{(\mathrm{inf})}}{s} \simeq \frac{1}{\Delta}\frac{9C}{512\pi}\frac{m_\chi T_{\mathrm{R}} H_{\mathrm{inf}}}{M_P^2} \simeq 1\times10^{-9}\ \mathrm{GeV}\frac{C}{\Delta}\left(\frac{m_\chi}{10^6\ \mathrm{GeV}}\right) \times\left(\frac{T_{\mathrm{R}}}{10^{10}\ \mathrm{GeV}}\right)\left(\frac{H_{\mathrm{inf}}}{10^{14}\ \mathrm{GeV}}\right), \tag{2.27}$$

where s is the entropy density, T_{R} is the reheating temperature, and Δ denotes the dilution factor due to the late decay of ϕ, which is given by[6]

[5]If ϕ dominates the Universe, we have $n_\chi(t) \sim (\mathrm{const}) \times H^3$, as found in [12,14].

[6]Here we have introduced some interactions that induce a complete decay of ϕ, in order for the ϕ oscillation not to dominate the Universe. For simplicity, we assume that this interaction does not involve χ.

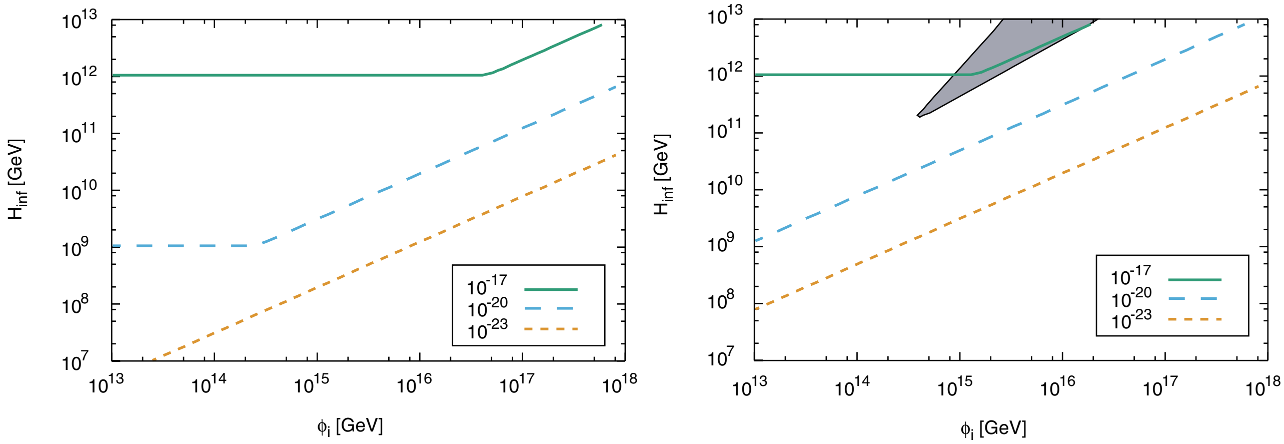

FIG. 1. Contour plot of Y_χ produced by the inflaton for $m_\phi = H_{\rm inf}/10$ (left) and $m_\phi = H_{\rm inf}/1000$ (right) on the plane of $(\phi_i, H_{\rm inf})$. We fix the decay rate of ϕ as $\Gamma_\phi = m_\phi^3/128\pi M_P^2$ and the reheating temperature as $T_{\rm R} = 10^{10}$ GeV. The shaded region is excluded due to too large curvature perturbation if ϕ remains light during inflation.

$$\Delta = \max\left[1, \sqrt{H_{\rm dom}/\Gamma_\phi}\right]. \tag{2.28}$$

Here, $H_{\rm dom}$ is the Hubble parameter when ϕ would dominate the Universe: $H_{\rm dom} = \Gamma_{\rm inf}(\phi_i^2/6M_P^2)^2$ for $m_\phi > \Gamma_{\rm inf}$ and $H_{\rm dom} = m_\phi(\phi_i^2/6M_P^2)^2$ for $m_\phi < \Gamma_{\rm inf}$ when the exponent of the potential of ϕ is $n = 2$.

Suppose that χ is a massive noninteracting stable particle. Then, its abundance should be smaller than $\rho_\chi/s \lesssim 4\times 10^{-10}$ GeV to avoid the dark matter (DM) overproduction.[7] Next, suppose that χ is a moduli that has only Planck-suppressed interactions with SM fields. It is severely constrained from cosmology due to its longevity. If its mass is about $\mathcal{O}(1)$ TeV, big bang nucleosynthesis (BBN) gives a stringent bound on the χ abundance, $\rho_\chi/s \lesssim 10^{-14}$ GeV [21]. Thus, we roughly have

$$\frac{\rho_\chi}{s} \lesssim 10^{-14} - 4\times 10^{-10}\ \text{GeV} \tag{2.29}$$

depending on the mass, lifetime, decay modes, etc. Various cosmological constraints on massive particles in broad parameter space are found in Ref. [22]. There is no such constraint if χ decays well before BBN begins. Finally, if χ is (nearly) massless, like an axionlike particle, it contributes to dark radiation. In this case, however, the χ abundance as well as the gravitational wave abundance is so small that it does not affect observations.

Note also that χ can have either (dominantly) an adiabatic or isocurvature fluctuation depending on whether ϕ is massive or not during inflation. If ϕ remains light during inflation ($m_\phi \lesssim H_{\rm inf}$), it obtains long-wavelength quantum fluctuations and contributes to the curvature perturbation as

$$\zeta_\phi \simeq \frac{R_\phi}{3}\frac{H_{\rm inf}}{\pi\phi_i}, \tag{2.30}$$

where R_ϕ is the fraction of ϕ energy density at its decay, and it can act as the curvaton [3,4]. If ϕ is the dominant source of the curvature perturbation, χ produced by the inflaton oscillation has totally anticorrelated isocurvature perturbation, and it cannot be the dominant component of DM [23]. If the curvature perturbation is dominantly sourced by the inflaton, there is no significant constraint from the isocurvature perturbation. Also, if ϕ is heavy enough during inflation, there is no isocurvature perturbation.

Figure 1 shows contours of $Y_\chi \equiv n_\chi/s$ produced by the inflaton for $m_\phi = H_{\rm inf}/10$ (left) and $m_\phi = H_{\rm inf}/1000$ (right) on the plane of $(\phi_i, H_{\rm inf})$ for $n = 2$ and $w = 1/3$.[8] In this plot, we have assumed that ϕ decays via Planck-suppressed interaction: $\Gamma_\phi \simeq m_\phi^3/(128\pi M_P^2)$. We have also fixed the reheating temperature after inflation as $T_{\rm R} = 10^{10}$ GeV. We can deduce the cosmological constraint mentioned above by multiplying m_χ as $\rho_\chi/s = m_\chi Y_\chi$ for an arbitrary value of m_χ below the inflaton mass. The shaded region is excluded due to too large curvature perturbation if ϕ remains light during inflation. The above-mentioned cosmological constraints crucially depend on the mass and lifetime of χ, which is not fixed in the figure,

[7]Depending on m_χ and m_ϕ, the free-streaming length of χ can be so long that it fails to be a cold DM. In such case, its abundance must be well below the observed DM abundance. In order for χ to be cold, it should become nonrelativistic before the cosmic temperature drops to ∼1 keV.

[8]One can convert the quantity $\rho_\chi/s = m_\chi Y_\chi$ to the present density parameter $\Omega_\chi = \rho_\chi/\rho_{\rm cr}$ with a critical density $\rho_{\rm cr}$ through $\Omega_\chi h^2 \simeq 2.8\times 10^8 (m_\chi Y_\chi/\text{GeV})$ if χ is a stable and nonrelativistic particle. Here, $h(\sim 0.7)$ is the present Hubble parameter in units of 100 km/s/Mpc.

but we can easily infer ρ_χ/s (or $\Omega_\chi h^2$) from these plots once the mass is fixed and compare with various constraints. One can see that the cosmological constraints from the gravitational particle production are rather weak so that almost all parameter space is allowed.

III. $f(\phi)R$ MODEL

In this section, we consider gravitational particle production in $f(\phi)R$ models. One famous example of this class of models is the Higgs inflation [24–27], with $f(\phi) = \xi\phi^2/M_P^2$. Interestingly, this coupling $\xi\phi^2 R/2$ is inevitably generated by radiative corrections [28]. Here we analyze gravitational particle production for general $f(\phi)R$ models in the Jordan frame.

A. Background dynamics

Let us consider the action

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2}M_P^2 f(\phi)R - \frac{1}{2}(\partial\phi)^2 - V(\phi) + \mathcal{L}_M\right), \tag{3.1}$$

where $\mathcal{L}_M$ denotes the Lagrangian for matter other than the scalar ϕ. The equation of motion of ϕ is given by

$$\ddot{\phi} + 3H\dot{\phi} + V' - 3M_P^2(2H^2 + \dot{H})f' = 0. \tag{3.2}$$

It can be expressed as

$$\dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) - 3M_P^2(2H^2 + \dot{H})\dot{f} = 0, \tag{3.3}$$

where ρ_ϕ and p_ϕ are the same as before. The Einstein equation reads

$$3H^2 f + 3H\dot{f} = \frac{\rho_\phi + \rho_M}{M_P^2}, \tag{3.4}$$

$$\ddot{f} + 2H\dot{f} + (3H^2 + 2\dot{H})f = -\frac{p_\phi + p_M}{M_P^2}. \tag{3.5}$$

By using these equations, we also obtain

$$\dot{\rho}_M + 3H(\rho_M + p_M) = 0. \tag{3.6}$$

Hereafter, we again assume that the matter part satisfies the equation of state $p_M = w\rho_M$. This shows that ρ_M exactly scales as $\rho_M \propto a^{-3(1+w)}$. The cosmological setup we are considering is the same as the one in the previous section. We will estimate the oscillating part of the Hubble parameter or the scale factor induced by the coherent oscillation of ϕ which may or may not dominate the Universe. The equation of state of the matter part is taken as a free parameter.

In the following, we solve these equations of motion by the following perturbative expansion. We expand $f(\phi)$ as follows:

$$f(\phi) \equiv 1 + f_1(\phi) = 1 + c_1\frac{\phi}{M_P} + c_2\frac{\phi^2}{2M_P^2} + \dots \tag{3.7}$$

and regard f_1 as a small perturbation.[9] To be more precise, we require $|\ddot{f}| \ll H^2$. Other quantities are also expanded as

$$H = H_0 + H_1, \tag{3.8}$$

$$a = a_0 + a_1, \tag{3.9}$$

$$\rho_\phi = \rho_{\phi 0} + \rho_{\phi 1}, \tag{3.10}$$

$$\rho_M = \rho_{M0} + \rho_{M1}, \tag{3.11}$$

where the subscript 0 denotes solutions in the $f_1 \to 0$ limit, i.e., solutions in Einstein gravity. Since f_1 directly depends on ϕ and, hence, is a rapidly oscillating function, quantities such as H_1, $\rho_{\phi 1}$, ... are also expected to be rapidly oscillating.

Our goal is to express the oscillating part H_1, a_1, ... in terms of ϕ. We retain only first order in the oscillating parts induced by the nonminimal coupling in the following. In the equations of motion, the oscillating parts satisfy

$$2H_0H_1 = -H_0\dot{f}_1 - H_0^2 f_1 + \frac{\rho_{\phi 1} + \rho_{M1}}{3M_P^2}, \tag{3.12}$$

$$\begin{aligned}&\dot{\rho}_{\phi 1} + 3H_1(\rho_{\phi 0} + p_{\phi 0}) + 3H_0(\rho_{\phi 1} + p_{\phi 1})\\ &\quad = 3M_P^2(2H_0^2 + \dot{H}_0)\dot{f}_1.\end{aligned} \tag{3.13}$$

Noting that $\dot{\rho}_{\phi 1} \sim \mathcal{O}(m_\phi \rho_{\phi 1})$, we can neglect terms of $\sim O(H\rho_{\phi 1})$ in Eq. (3.13). Then we have

$$\dot{\rho}_{\phi 1} \simeq \left[3M_P^2(2H_0^2 + \dot{H}_0) + \frac{3}{2}(\rho_{\phi 0} + p_{\phi 0})\right]\dot{f}_1. \tag{3.14}$$

This implies $\rho_{\phi 1} \sim \mathcal{O}(\rho_{\rm tot} f_1)$. Also, ρ_{M1} is suppressed by m_ϕ since $\rho_M \propto a^{-3(1+w)}$ is exact. Thus, by noting that $\dot{f}_1 \sim \mathcal{O}(m_\phi f_1)$, we find that the second term and the third term on the rhs of Eq. (3.12) are safely neglected. As a result, we obtain a simple relation

$$H_1 \simeq -\frac{\dot{f}_1}{2} \simeq -\frac{1}{2}\left(c_1\frac{\dot{\phi}}{M_P} + \dots\right). \tag{3.15}$$

[9] Again, ϕ should be regarded as a deviation from the potential minimum $\phi = \phi_{\rm min}$. If $\phi_{\rm min} \neq 0$, the first term of (3.7) should be modified as $1 - c_1\phi_{\rm min}/M_P$.

This is the oscillating part of the Hubble parameter induced by the nonminimal coupling. Therefore, we arrive at

$$\frac{a_1}{a_0} \simeq 1 - \frac{1}{2} f_1, \tag{3.16}$$

and, hence, we finally find a relation between the oscillating part of the scale factor and ϕ.

Equation (3.15) is the same as the result obtained from the adiabatic invariant proposed in Ref. [29], though the proof given in Ref. [29] is applicable only to the cases where matter is subdominant. The point is that there is a so-called "adiabatic invariant" J[10]:

$$J \equiv -\frac{1}{6M_P^2} \frac{\partial L}{\partial H} = \frac{1}{2}(2Hf + \dot{f}) = \frac{1}{2a^2} \frac{\partial(a^2 f)}{\partial t}. \tag{3.17}$$

Here, we call a quantity Q an adiabatic invariant if it satisfies $\dot{Q} = \mathcal{O}(HQ)$. In the deeply oscillating regime, such a quantity is almost constant within one oscillation, and, hence, we can approximately view it as a conserved quantity. In Einstein gravity, the Hubble parameter, or, equivalently, the energy density of the scalar field, is obviously an adiabatic invariant, but in extended gravity models, the conserved quantity is nontrivial. Since J is almost constant within one oscillation, we can easily extract the oscillation part H_1 as

$$H_1 \simeq -\frac{\dot{f}_1}{2}. \tag{3.18}$$

B. Particle production rate

In the previous subsection, we obtained

$$a(t) \simeq \langle a(t) \rangle \left(1 - \frac{c_1}{2} \frac{\phi}{M_P}\right). \tag{3.19}$$

This expression is valid up to first order in ϕ. In contrast to Einstein gravity, here is a linear term in ϕ in the oscillating part of the scale factor, and it induces gravitational decay of ϕ. There also exist quadratic terms of the order of c_1^2 and c_2 in addition to the Einstein gravity contribution, which induce the gravitational annihilation of ϕ [14,15], although they are omitted in this expression. Note that the gravitational decay is possible only when the nonminimal coupling exists, while the gravitational annihilation takes place even in pure Einstein gravity. Below we consider the production of scalar particles and the graviton. The production of fermions and gauge bosons is suppressed by their masses and couplings as we explained before.

1. Scalar

First let us consider the particle production rate of a scalar minimally coupled with gravity, whose action is given by Eq. (2.20). The number density of χ particles produced during one Hubble time after ϕ begins to oscillate is given by

$$n_\chi(t) \simeq \frac{C}{32\pi H} \left(\frac{c_1 m_\phi^2 \Phi}{2M_P}\right)^2. \tag{3.20}$$

It can be interpreted as the decay of ϕ into the χ pair with the decay rate

$$\Gamma_{\phi\to\chi\chi} = C \frac{c_1^2}{128\pi} \frac{m_\phi^3}{M_P^2}. \tag{3.21}$$

This decay rate coincides with that calculated in the Einstein frame [30]. Contrary to the annihilation case, this effect becomes significant at late time for reasonable choices of w and n. Noting that each χ particle has the energy of $m_\phi/2$ at the production, we find

$$\frac{\rho_\chi(t)}{\rho_\phi(t)} \simeq \frac{C c_1^2 m_\phi^3}{128\pi M_P^2 H} = \frac{\Gamma_{\phi\to\chi\chi}}{H}. \tag{3.22}$$

Thus, ϕ completely decays into χ at $H \sim \Gamma_{\phi\to\chi\chi}$ if there is no other decay mode of ϕ.

2. Graviton

Next we apply our formalism to the graviton production. The graviton action is given by

$$S = \int d\tau d^3x a^2(t) f(\phi) \frac{M_P^2}{8} \left[\left(\frac{\partial h_{ij}}{\partial \tau}\right)^2 - (\partial_k h_{ij})^2\right]. \tag{3.23}$$

It should be noticed that the c_1 dependence vanishes in the overall coefficient $a^2 f(\phi)$. Hence, there is no gravitational decay of ϕ into the graviton pair, as opposed to the case of scalar particles [14]. Still there exists a gravitational annihilation of ϕ into the graviton pair, which exists even in Einstein gravity. The abundance of the graviton is similar to Eq. (2.24) except for the modification of $\mathcal{O}(c_2, c_1^2)$.

C. Cosmological implications

In the present case, the contribution from ϕ often becomes the dominant one. The abundance of a massive χ produced by the gravitational decay of ϕ is given by

$$\frac{\rho_\chi}{s} \simeq \Delta' \frac{3 m_\chi T_\phi}{2 m_\phi} \mathrm{Br}_{\phi\to\chi\chi}, \tag{3.24}$$

where $T_\phi \sim \sqrt{\Gamma_\phi M_P}$ is the decay temperature of ϕ with Γ_ϕ being the total decay width of ϕ, $\mathrm{Br}_{\phi\to\chi\chi} \equiv \Gamma_{\phi\to\chi\chi}/\Gamma_\phi$ is the branching ratio of ϕ into $\chi\chi$, and

[10]Here, integration by parts should be done to remove $\dot{H}$ in the Lagrangian.

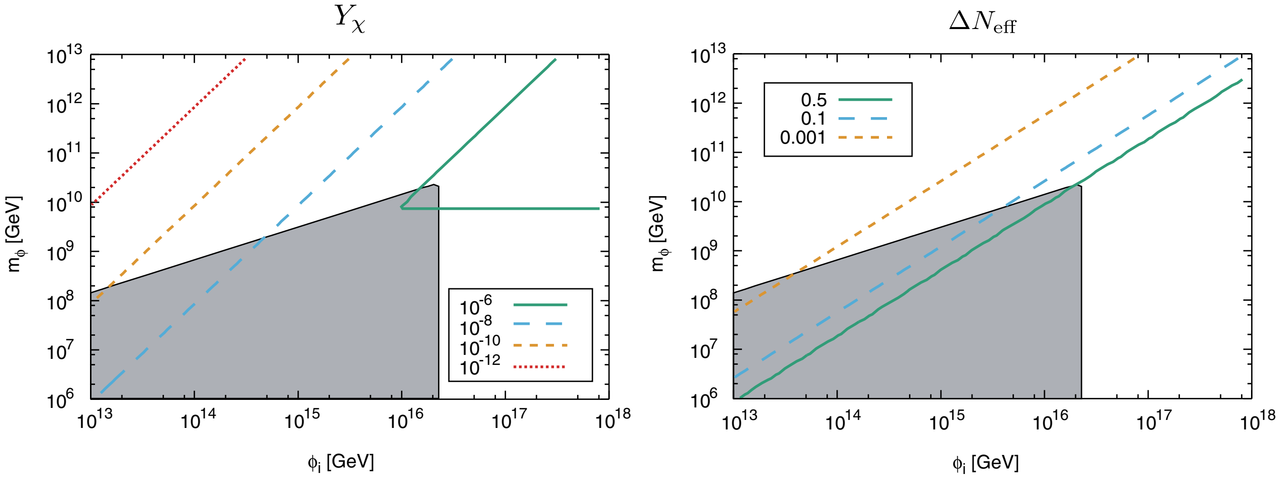

FIG. 2. Contour plot of Y_χ (left) and $\Delta N_{\rm eff}$ (right) on the plane of (ϕ_i, m_ϕ). The shaded region is excluded due to too large curvature perturbation for $H_{\rm inf} = 10^{13}$ GeV if ϕ remains light during inflation.

$$\Delta' = \min\left[1, \sqrt{H_{\rm dom}/\Gamma_\phi}\right], \qquad (3.25)$$

which roughly corresponds to the ratio $\rho_\phi/(\rho_\phi + \rho_M)$ at $H = \Gamma_\phi$. If the gravitational decay is the only decay mode, the branching ratio is $\mathcal{O}(1)$. In that case, using Eq. (3.21), we obtain

$$\frac{\rho_\chi}{s} \simeq 3 \times 10^{-8}\ \text{GeV} \frac{\Delta' c_1}{\sqrt{N+1}} \left(\frac{m_\phi}{10^6\ \text{GeV}}\right)^{1/2} \left(\frac{m_\chi}{1\ \text{GeV}}\right), \qquad (3.26)$$

where we have assumed that there are N light scalar fields other than χ that thermalize with SM degrees of freedom and, hence, $\text{Br}_{\phi\to\chi\chi} = 1/(N+1)$. Within the framework of the SM, we have $N = 4$ corresponding to the 4 real degrees of freedom of the Higgs boson. Strong constraints are imposed as discussed in Sec. II C: $\rho_\chi/s \lesssim 4 \times 10^{-10}$ GeV if χ is a noninteracting stable particle and $\rho_\chi/s \lesssim 10^{-14}$ GeV if χ is a late-decaying particlelike moduli.[11]

Then, suppose that χ is a (nearly) massless particle such as axionlike particles including the QCD axion. In this case, there is a danger of overproduction of dark radiation. It is convenient to express the abundance of dark radiation in terms of the effective number of neutrino species

$$\Delta N_{\rm eff} = \frac{43}{7} \left(\frac{10.75}{g_{*s}(T_\phi)}\right)^{1/3} \Delta' \text{Br}_{\phi\to\chi\chi} \sim \frac{3\Delta'}{N+1}. \qquad (3.27)$$

Therefore, if ϕ is a dominant component of the Universe at the decay (i.e., $\Delta' = 1$), we may need $N \gtrsim 5$ to satisfy the current constraint on the dark radiation [31], which is marginal for the SM.[12] The bound can be relaxed if ϕ has decay modes other than the gravitational decay mode.

Figure 2 shows contours of Y_χ (left) and $\Delta N_{\rm eff}$ (right) produced by ϕ for $c_1 = 1$ on the plane of (ϕ_i, m_ϕ) for $n = 2$ and $w = 1/3$. In this plot, we have assumed that ϕ decays only via the gravitational decay mode and $N = 4$. We have also fixed the reheating temperature as $T_{\rm R} = 10^{10}$ GeV. The shaded region is excluded due to too large curvature perturbation for $H_{\rm inf} = 10^{13}$ GeV if ϕ remains light during inflation. Again, we emphasize that the cosmological constraints crucially depend on the mass and lifetime of χ. Comparing with the typical constraint (2.29), one finds that the large parameter space of the present scenario is excluded if χ has a long lifetime. Of course, the constraints become weaker for small initial amplitude ϕ_i.

The results presented here are also applied to the inflaton decay by simply regarding ϕ as the inflaton and taking $\Delta' = 1$.

IV. $f(R)$ MODEL

In this section, we consider gravitational particle production in the $f(R)$ models [33]. In the $f(R)$ models, there is 1 additional degree of freedom in the metric sector, and it induces rapid oscillation of the scale factor. A famous example is the Starobinsky inflation [2], in which a scalar degree of freedom causes inflation and reheating [16–18,34–37].

[11]In the present model, it is likely that ϕ obtains a mass of Hubble scale during inflation from $f(\phi)R$ coupling. Then there is no DM/dark radiation isocurvature mode even if we consider the DM contribution from inflaton decay. Also, ϕ is displaced from the minimum of its potential $V(\phi)$ during inflation owing to the $f(\phi)R$ coupling. Thus, typically, the initial amplitude is close to the Planck scale, unless the potential becomes steeper than the quadratic for a large field value, like $V \sim \phi^{2n}$ $(n \geq 2)$.

[12]Note also that there may be a preference for $\Delta N_{\rm eff} \simeq 0.5$ according to a recent observation of the Hubble constant [32].

Here we analyze gravitational particle production for general $f(R)$ models in the Jordan frame.

A. Background dynamics

The action is given by

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2}M_P^2 f(R) + \mathcal{L}_M\right), \tag{4.1}$$

where $\mathcal{L}_M$ denotes the Lagrangian for matter. This model includes 1 scalar degree of freedom ("scalaron") if $F \equiv \partial f/\partial R \neq$ const. The background equations of motion are given by

$$3FH^2 = \frac{1}{2}(FR - f) - 3H\dot{F} + \frac{\rho_M}{M_P^2}, \tag{4.2}$$

$$\ddot{F} - H\dot{F} + 2F\dot{H} = -\frac{\rho_M + p_M}{M_P^2}. \tag{4.3}$$

Note that the second equation is derived from the first equation just by taking a time derivative if there is no matter sector.[13] This is natural because there is only one dynamical degree of freedom, i.e., the Hubble parameter, in the gravity sector. These two equations are combined to yield

$$\ddot{F} + 3H\dot{F} + \frac{1}{3}(2f - FR) = \frac{\rho_M - 3p_M}{3M_P^2}, \tag{4.4}$$

$$\dot{\rho}_M + 3H(\rho_M + p_M) = 0. \tag{4.5}$$

Hereafter, we assume that the matter satisfies the equation of state $p_M = w\rho_M$, which implies $\rho_M \propto a^{-3(1+w)}$.

In the following, we consider the case where $f(R)$ is given as

$$f(R) = R\left(1 + \frac{c}{n}\left(\frac{R}{M_P^2}\right)^{n-1}\right) \equiv R\left(1 + \frac{cF_1}{n}\right), \tag{4.6}$$

where c is a positive constant, and $n\,(\geq 2)$ is an even integer. The equation of motion of F_1 reads

$$\ddot{F}_1 + 3H\dot{F}_1 + \frac{\partial V_{F_1}}{\partial F_1} = 0, \tag{4.7}$$

where[14]

$$V_{F_1} = \frac{n-1}{n}\frac{M_P^2}{3c}|F_1|^{n/(n-1)}\left(1 - \frac{n-2}{2n-1}cF_1\right) - \frac{1-3w}{3cM_P^2}\rho_M F_1 + V_0. \tag{4.8}$$

Here we have included an F_1-independent term V_0 to make $V_{F_1} = 0$ at the minimum of the potential $F_1 = \langle F_1\rangle$:

$$V_0 = \frac{1-3w}{3ncM_P^2}\rho_M\langle F_1\rangle. \tag{4.9}$$

From Eq. (4.8), we easily find that the minimum of the potential is given by

$$\langle F_1\rangle = \left(\frac{R_0}{M_P^2}\right)^{n-1}, \tag{4.10}$$

for $c|F_1| \ll 1$, as expected. Equation (4.7) shows that F_1 exhibits a similar motion to the scalar field under the potential V_{F_1}.

Now let us consider the case $m_F^2 \equiv |(\partial V_{F_1}/\partial F_1)/F_1| \gg H^2$; i.e., F_1 is oscillating rapidly in the effective potential V_{F_1}. We also assume that the inequality

$$c|F_1| \ll 1 \tag{4.11}$$

is satisfied. In this case, we can expand the quantities as

$$R = R_0 + R_1, \tag{4.12}$$

$$H = H_0 + H_1, \tag{4.13}$$

$$\rho_M = \rho_{M0} + \rho_{M1} \tag{4.14}$$

to the first order in c. Here, the quantities with a subscript 0 correspond to those in the limit $c \to 0$:

$$H_0^2 = \frac{\rho_{M0}}{3M_P^2}, \qquad R_0 = (1-3w)\frac{\rho_{M0}}{M_P^2}. \tag{4.15}$$

If there is no matter ($\rho_M = 0$), F_1 oscillates around $F_1 = 0$. Otherwise, it oscillates around a finite expectation value. Thus, we further divide F_1 and H_1 into the oscillating part and nonoscillating part as

$$F_1 = \langle F_1\rangle + \delta F_1, \tag{4.16}$$

$$H_1 = \langle H_1\rangle + \delta H_1. \tag{4.17}$$

Note that for $w = 1/3$, $R_0 = 0$, and, hence, $\langle F_1\rangle = 0$.

Our goal is to express the oscillating part of the Hubble parameter or the scale factor in terms of F_1. When matter is subdominant, the adiabatic invariant J is useful for this purpose. In Appendix B, we show that J is given by

[13]Recall that the Ricci scalar is given by $R = 6(\dot{H} + 2H^2)$.

[14]The potential V_{F_1} is unbounded from below for $n > 2$. We only consider the region $c|F_1| \ll 1$ below so that the whole dynamics is described in the metastable region. Although there can be a quantum tunneling from the metastable vacuum to the deeper minimum, we do not discuss it here because higher order terms in $f(R)$ can easily change the structure of the potential for $c|F_1| \gg 1$.

$$J = HF + \frac{\dot{F}}{2}, \tag{4.18}$$

for the $f(R)$ models. Thus, by expanding with respect to c, we obtain the oscillating part of the Hubble parameter as

$$\delta H_1 \simeq -\frac{c}{2}\delta\dot{F}_1. \tag{4.19}$$

Below, we show that this is correct even when matter is non-negligible by solving the equations of motion directly. For completeness, we consider both of the cases where the scalaron is dominant and subdominant.

For later convenience, here we express H_1 in terms of F_1 by using Eq. (4.2):

$$H_1 \simeq -\frac{1}{2}c\dot{F}_1 + \sqrt{\frac{\rho_{F_1}}{3M_P^2} + \frac{\rho_M}{3M_P^2}\left(1 - cF_1 + \frac{1}{2}(1-3w)c\left(F_1 - \frac{1}{n}\langle F_1\rangle\right)\right)} - \sqrt{\frac{\rho_{M0}}{3M_P^2}}, \tag{4.20}$$

where we have kept only leading terms in $c|F_1|$ and defined the "energy density" of the scalaron F_1 as

$$\rho_{F_1} \equiv \frac{3}{2}c^2M_P^2\left(\frac{1}{2}\dot{F}_1{}^2 + V_{F_1}\right). \tag{4.21}$$

This is also a nonoscillating quantity.[15] We call this energy density because the Hubble parameter is given by $H^2 \sim \rho_{F_1}/(3M_P^2)$ for $\rho_{F_1} \gg \rho_M$ as we will show below. Actually, $\sqrt{3/2}cM_PF_1$ coincides with the canonical scalaron field in the Einstein frame for $c|F_1| \ll 1$. As one may see from Eqs. (4.19) and (4.20), and as we will see in the following, the dominant contribution to the oscillation mode of the Hubble parameter comes from the first term in Eq. (4.20).

1. Matter-dominated case

First let us consider the matter-[16] dominated case in which $\rho_M \gg \rho_{F_1}$, i.e., $c|F_1| \ll 1$ and $c|\delta\dot{F}_1| \ll H_0$. For $\rho_{F1} \ll c|F_1|\rho_{M0}$ or $\delta\tilde{F}_1 \ll \langle F_1\rangle$ with $\delta\tilde{F}_1$ being an oscillation amplitude of δF_1, we obtain

$$\langle H_1\rangle \sim c\langle F_1\rangle H_0, \tag{4.22}$$

and for $\rho_{F_1} \gg c|F_1|\rho_{M0}$ or $\delta\tilde{F}_1 \gg \langle F_1\rangle$, we obtain

$$\langle H_1\rangle \sim H_0\frac{\rho_{F_1}}{\rho_{M0}}. \tag{4.23}$$

Note that the latter always holds if $w = 1/3$. In both cases, we have

$$\delta H_1 \simeq -\frac{c}{2}\delta\dot{F}_1, \tag{4.24}$$

since the oscillating part of ρ_{M1} is suppressed by m_ϕ as we discussed before. This implies $|\delta H_1| \ll H_0$ in the matter-dominated case. Thus, the scale factor a has also an oscillating part in this model as

$$a(t) \simeq \langle a(t)\rangle\left(1 - \frac{c}{2}\delta F_1\right). \tag{4.25}$$

We can estimate the gravitational particle production rate from this expression.

Let us see the evolution of ρ_{F_1} and ρ_M. From the equation of motion (4.7), we find that ρ_{F_1} scales as $\rho_{F_1} \propto a^{-6n/(3n-2)}$. This means that the amplitude scales as $\delta\tilde{F}_1 \propto a^{-6(n-1)/(3n-2)}$ while $\langle F_1\rangle \propto a^{-3(1+w)(n-1)}$. Therefore, as time goes on, the relative amplitude of $\delta\tilde{F}_1$ to the mean value $\langle F_1\rangle$ becomes larger for $n > 2(2+w)/3(1+w)$, which is satisfied for $n \geq 2$ and $w > -1/2$. It also tends to dominate the Universe at a later epoch. For $n = 2$, for example, we have $\rho_{F_1} \propto a^{-3}$, and it scales in the same way as the nonrelativistic matter. Thus, the oscillation energy density will dominate the Universe if the equation of state of background matter is $w > 0$. For $n > 2$, ρ_{F_1} decreases more slowly than the nonrelativistic matter, and, hence, the oscillation energy density eventually dominates the Universe even if $w = 0$, unless δF_1 decays before the domination due to the production of nonconformally coupled particles, as discussed later.

2. Oscillation-dominated case

Next, let us consider the opposite limit $\rho_M \ll \rho_{F_1}$. As we have seen above, ρ_{F_1} may eventually dominate the Universe at a later epoch even if we start with the matter-dominated Universe. In this case, the second term in the potential (4.8) can be neglected, and, hence, F_1 oscillates around zero: $\delta\tilde{F}_1 \gg \langle F_1\rangle$. From Eq. (4.20), we obtain

$$\langle H_1\rangle \simeq \sqrt{\frac{\rho_{F_1}}{3M_P^2}}, \tag{4.26}$$

hence, $\langle H_1\rangle \gg H_0$ and

[15] The contribution to ρ_{F_1} from the constant term V_0 is always smaller than ρ_M for $c\langle F_1\rangle \ll 1$.

[16] Again, this should not be confused with fluids with the nonrelativistic equation of state $w = 0$. We do not specify w in the following discussion.

$$\delta H_1 \simeq -\frac{c}{2}\delta \dot{F}_1. \quad (4.27)$$

This is the same expression as that of the previous case. The scale factor a can be expressed as

$$a(t) \simeq \langle a(t) \rangle \left(1 - \frac{c}{2}\delta F_1\right). \quad (4.28)$$

In this case, we have $|\delta H_1| \sim \langle H_1 \rangle \sim H$, and, hence, the Hubble parameter H violently oscillates.[17] Similar to the previous case, from the equation of motion Eq. (4.7), we find $\rho_{F_1} \propto a^{-6n/(3n-2)}$ and $\delta \tilde{F}_1 \propto a^{-6(n-1)/(3n-2)}$, while $\langle F_1 \rangle \propto a^{-3(1+w)(n-1)}$; hence, $\delta \tilde{F}_1 \gg \langle F_1 \rangle$ is always satisfied for $n \geq 2$ and $w > -1/2$ until δF_1 decays due to particle production, as discussed later. The Ricci curvature R oscillates rapidly around $R \sim 0$, and its amplitude decreases as $\tilde{R}_1 \propto a^{-6/(3n-2)}$. Thus, the Hubble parameter scales as $\langle H \rangle \simeq \langle H_1 \rangle \simeq (3n-2)/(3nt) \propto a^{-3n/(3n-2)}$.

In any case, the oscillation of F_1 or the oscillation of the scale factor leads to production of nonconformally coupled particles, and, hence, it decays. In the next subsection, we estimate the particle production rate.

B. Particle production rate

In the previous subsection, we obtained

$$a(t) \simeq \langle a(t) \rangle \left(1 - \frac{c}{2}\delta F_1\right). \quad (4.29)$$

As in the case of the $f(\phi)R$ models, here is also a linear term in the oscillating part of the scale factor. Thus, the gravitational decay of the scalaron occurs, and the scalaron can transfer its energy to other particles efficiently through this process. We also have terms which induce the gravitational annihilation, although omitted in this expression. Below we consider the production of minimally coupled scalar particles and the graviton. The production of fermions and gauge bosons is again suppressed by their masses and couplings.

1. Scalar

First let us consider the particle production rate of the minimally coupled scalar, whose action is given by Eq. (2.20). By noting that $\ddot{a}/a \simeq -(c/2)m_F^2 F_1$ and using Eq. (A24), we obtain the number density of χ created in one Hubble time as

$$n_\chi(t) \simeq \frac{C\tilde{R}^2}{1152\pi H}, \quad (4.30)$$

where $\tilde{R}$ is the amplitude of the Ricci scalar R. This expression does not depend on n except for the small dependence in the $\mathcal{O}(1)$ constant C. From this we can read off the effective "decay rate" of F_1 as

$$\Gamma_{F_1 \to \chi\chi} = \frac{C}{384\pi} \frac{n}{n-1} \frac{m_F^3}{M_P^2}, \quad (4.31)$$

which coincides with the decay rate of a canonical scalaron field calculated in the Einstein frame [34]. The ratio of the energy density of the created particles in each Hubble time to the scalaron energy density is given by

$$\frac{\rho_\chi(t)}{\rho_{F_1}(t)} \simeq \frac{C}{384\pi} \frac{n}{n-1} \frac{m_F^3}{M_P^2 H} = \frac{\Gamma_{F_1 \to \chi\chi}}{H}. \quad (4.32)$$

This ratio becomes $\mathcal{O}(1)$ at some epoch, even if it is initially much smaller since m_F is an increasing function of time. At that time, δF_1 completely "decays" into χ particles. If ρ_{F_1} dominates the Universe, it corresponds to the completion of the reheating. Actually, if χ is the SM Higgs boson, they are thermalized soon.

2. Graviton

The graviton action is given by

$$S = \int d\tau d^3x a^2(t) F(R) \frac{M_P^2}{8} \left[\left(\frac{\partial h_{ij}}{\partial \tau} \right)^2 - (\partial_k h_{ij})^2 \right]. \quad (4.33)$$

It is the combination $a^2(t)F(R)$ that determines the graviton production rate. It is estimated as

$$a^2(t) F(R) \simeq a_0^2 (1 + \mathcal{O}(c^2 F_1^2)). \quad (4.34)$$

Note that similar to the case of the $f(\phi)R$ models, the linear term in $c\delta F_1$ vanishes; hence, there is no decay of F_1 into the graviton pair. Compared with the scalar, the graviton abundance is suppressed by $c^2 F_1^2$:

$$n_h(t) \simeq \frac{C(cF_1\tilde{R})^2}{1152\pi H}. \quad (4.35)$$

The graviton production becomes less efficient as time goes on due to the time-dependent suppression factor F_1^2.[18] It corresponds to the gravitational annihilation of the oscillating scalaron field in the Einstein frame interpretation as written in Sec. II B 2.

[17] Although the oscillation amplitude of the Hubble parameter δH_1 and its averaged value $\langle H_1 \rangle$ are the same order, $H > 0$ is always ensured, as is easily checked by solving the Friedmann equation (4.2).

[18] This is inconsistent with Ref. [38]. Probably, they did not take into account $F(R)$ appearing in front of the graviton kinetic term.

C. Cosmological implications

Let us discuss the cosmological implications of gravitational particle production in $f(R)$ models. To be concrete, we take $n = 2$ in the following. If there is no matter initially, it can cause successful Starobinsky inflation, but here we concentrate on the cases where the inflation occurs in some other sector, and F_1 oscillation begins after inflation, which later becomes a dominant or subdominant component of the total energy density.

The effects of gravitational particle production in the $f(R)$ model are similar to the case of the $f(\phi)R$ model with $c_1 \neq 0$ studied in Sec. III C. The massive χ abundance produced by the gravitational F_1 decay is given by

$$\frac{\rho_\chi}{s} \simeq \Delta' \frac{3m_\chi T_F}{2m_F} \mathrm{Br}_{F_1 \to \chi\chi}, \tag{4.36}$$

where $T_F \sim \sqrt{\Gamma_{F_1} M_P}$ is the decay temperature of F_1, and $\mathrm{Br}_{F_1\to\chi\chi} \equiv \Gamma_{F_1\to\chi\chi}/\Gamma_{F_1}$ is the branching ratio of F_1 into $\chi\chi$ with Γ_{F_1} being the total decay width of F_1 and

$$\Delta' = \min\left[1, \sqrt{H_{\mathrm{dom}}/\Gamma_{F_1}}\right]. \tag{4.37}$$

Here, H_{dom} is the Hubble parameter at which F_1 would dominate the Universe, and Δ' roughly corresponds to the ratio $\rho_{F1}/(\rho_{F_1} + \rho_M)$ at $H = \Gamma_{F_1}$. Writing the initial condition of F_1 as F_{1i}, we obtain $H_{\mathrm{dom}} = \Gamma_{\mathrm{inf}}(F_{1i}^2/6M_P^2)^2$ for $m_F > \Gamma_{\mathrm{inf}}$ and $H_{\mathrm{dom}} = m_F(F_{1i}^2/6M_P^2)^2$ for $m_F < \Gamma_{\mathrm{inf}}$, respectively. The energy density of χ is then given by

$$\frac{\rho_\chi}{s} \simeq 2\times 10^{-8}\ \mathrm{GeV}\frac{\Delta'}{\sqrt{N+1}}\left(\frac{m_F}{10^6\ \mathrm{GeV}}\right)^{1/2}\left(\frac{m_\chi}{1\ \mathrm{GeV}}\right). \tag{4.38}$$

This is severely constrained if χ is a stable noninteracting particle, or if it is a late-decaying moduli as shown in Sec. II C. In our setup, F_1 remains light during inflation, and it obtains long-wavelength quantum fluctuation. Whether χ has (large) isocurvature perturbation or not depends on the dominant source of the curvature perturbation: if it is the inflaton, the fluctuation of χ is mostly an uncorrelated isocurvature and cannot be a dominant DM, while if it is F_1, there is essentially no isocurvature mode except for a (small) contribution from the inflaton oscillation. Also, there is no isocurvature mode if F_1 itself is the inflaton.

If χ is a practically massless noninteracting particle, we have

$$\Delta N_{\mathrm{eff}} = \frac{43}{7}\left(\frac{10.75}{g_{*s}(T_F)}\right)^{1/3}\Delta'\mathrm{Br}_{F_1\to\chi\chi} \sim \frac{3\Delta'}{N+1}. \tag{4.39}$$

Again we have a stringent constraint.[19] The constraints are similar to the case of the $f(\phi)R$ model with $c_1 = 1$ after ϕ_i is replaced with F_{1i}, and readers are referred to Fig. 2.

[19]Dilaton dark radiation from the decay of scalaron field in the R^2 model was discussed in Ref. [39].

These results can be applied to the reheating of the Starobinsky inflation model once we take $\Delta' = 1$ and $m_F \simeq 3\times 10^{13}$ GeV. It is noticeable that in the Starobinsky model with a minimal extension of the QCD axion, we have $N = 4$ (corresponding to the SM Higgs boson), and the axion dark radiation may be detectable in a future CMB experiment.[20] Such axion dark radiation can also have an isocurvature mode depending on the origin of the dominant curvature perturbation.

V. $G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ MODEL

Finally, we study a scalar field with a nonminimal derivative coupling to gravity, namely, $L \sim G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ with $G^{\mu\nu}$ being the Einstein tensor. An example with such a coupling is the new Higgs inflation model [40]. This class of model has an advantage in that it does not introduce an additional degree of freedom, although the action itself contains higher derivatives. In fact, it is the simplest version of the G_5-type (or the G_4 type involving the kinetic term) models in the context of the Horndeski or generalized Galileon theories [41–43].

A. Background dynamics

We consider the following action

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_P^2 R - \frac{1}{2}\left(g^{\mu\nu} - \frac{G^{\mu\nu}}{M^2}\right) \times \partial_\mu\phi\partial_\nu\phi - V(\phi) + \mathcal{L}_M\right]. \tag{5.1}$$

The background equation of motion of ϕ is given by

$$\left(1+\frac{3H^2}{M^2}\right)\ddot{\phi} + 3H\left(1+\frac{3H^2+2\dot{H}}{M^2}\right)\dot{\phi} + V' = 0. \tag{5.2}$$

The Friedmann equation reads

$$3H^2 = \frac{\rho_\phi + \rho_M}{M_P^2}, \qquad \rho_\phi \equiv \left(1+\frac{9H^2}{M^2}\right)\frac{\dot{\phi}^2}{2} + V, \tag{5.3}$$

$$3H^2 + 2\dot{H} = -\frac{p_\phi + p_M}{M_P^2}, \quad p_\phi \equiv \left(1-\frac{3H^2}{M^2}\right)\frac{\dot{\phi}^2}{2} - V - \frac{1}{M^2}\frac{d}{dt}(H\dot{\phi}^2), \tag{5.4}$$

where ρ_M and p_M are the same as before. From these equations, we obtain

[20]If the radial component of the Peccei-Quinn scalar is lighter than the inflaton, we have $N = 5$. But it dominantly decays into the axion pair, and the axion dark radiation becomes even more abundant.

$$\dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) = 0, \tag{5.5}$$

$$\dot{\rho}_M + 3H(\rho_M + p_M) = 0. \tag{5.6}$$

The oscillating regime of this system without the matter ($\rho_M = 0$) was extensively studied in Refs. [44,45]. It is found that this system has a so-called gradient instability in the oscillating epoch if ϕ dominates the Universe and the nonminimal kinetic term dominates over the standard one ($H \gtrsim M$) [45].[21] The gradient instability indicates that the sound speed squared of the scalar perturbation becomes negative for a finite period during one oscillation, which means that the scalar fluctuations are exponentially enhanced. In particular, the enhancement rate is larger for higher momentum modes. The system soon becomes nonlinear, and it is quite difficult to follow the dynamics at least analytically. In order to avoid this instability when ϕ dominates the Universe, the nonminimal kinetic term must be so small that the model effectively reduces to just a canonical scalar field with Einstein gravity. Therefore, we limit ourselves to the case where ρ_M dominates the Universe and the ϕ oscillation is a subdominant component and also require that there is no gradient instability.

For later convenience, we define the effective mass of the scalar as

$$m_{\rm eff} \equiv \min\left[1, \frac{M}{H}\right] \times \sqrt{\frac{V'}{\phi}}\Bigg|_{\phi=\Phi}, \tag{5.7}$$

where Φ denotes the oscillation amplitude of ϕ. This roughly corresponds to the scalar oscillation frequency. Note that the energy conservation (5.5) immediately means that

$$\dot{\rho}_\phi \sim \begin{cases} m_{\rm eff}\rho_\phi & \text{for } H \gtrsim M, \\ \frac{H^2}{M^2} m_{\rm eff}\rho_\phi & \text{for } M^2/m_{\rm eff} \lesssim H \lesssim M, \\ H\rho_\phi & \text{for } H \lesssim M^2/m_{\rm eff}. \end{cases} \tag{5.8}$$

This implies that ρ_ϕ is a rapidly oscillating quantity for $H \gg M$. The relative amplitude of the oscillating part of ρ_ϕ is estimated as

$$\frac{\delta\rho_\phi}{\rho_{\phi 0}} \sim \begin{cases} \mathcal{O}(1) & \text{for } H \gtrsim M, \\ \mathcal{O}(\frac{H^2}{M^2}) & \text{for } M^2/m_{\rm eff} \lesssim H \lesssim M, \\ \mathcal{O}(\frac{H}{m_{\rm eff}}) & \text{for } H \lesssim M^2/m_{\rm eff}. \end{cases} \tag{5.9}$$

The last case is the same as that of the canonical scalar with Einstein gravity. On the other hand, as usual, ρ_M just scales as $a^{-3(1+w)}$, and, therefore, its relative oscillation amplitude is small: $\delta\rho_M/\rho_M \sim \delta H/m_{\rm eff}$. Therefore, the oscillating part of the Hubble parameter is expressed as

$$\frac{\delta H}{H_0} \simeq \frac{\delta\rho_\phi}{2\rho_M}. \tag{5.10}$$

The scale factor can also be expanded as

$$a(t) \simeq a_0\left(1 + \mathcal{O}\left(\frac{\delta H}{m_{\rm eff}}\right)\right). \tag{5.11}$$

We can calculate the particle production using these expressions.

Let us make an order estimation on the condition to avoid the gradient instability. The sound speed squared of the scalar field is given by [45][22]

$$c_s^2 \sim \begin{cases} 1 + \mathcal{O}(\frac{\dot{H}}{H^2}) & \text{for } H \gg M, \\ 1 + \mathcal{O}(\frac{\dot{H}}{M^2}) & \text{for } H \ll M. \end{cases} \tag{5.12}$$

If H is violently oscillating, the sound speed squared may be negatively large, which leads to a gradient instability. Thus, we require $\min[|\dot{H}/H^2|, |\dot{H}/M^2|] \lesssim 1$ to avoid the instability. From Eq. (5.3), one can see that this condition is written as

$$\min\left[1, \frac{H^4}{M^4}\right] \frac{m_{\rm eff}}{H}\frac{\rho_\phi}{\rho_M} \lesssim 1 \quad \text{for } M^2/m_{\rm eff} \lesssim H. \tag{5.13}$$

No condition is required for $H \lesssim M^2/m_{\rm eff}$. Hereafter, we assume that this inequality is always satisfied. From this expression it is clear that if ϕ is the dominant component of the Universe, we must have $H \ll M$ to avoid the instability, as stated above.

Since we have imposed the condition (5.13), the evolution of ϕ is greatly simplified. By noting $\dot{H} \simeq -3(1+w)H^2/2$, the equation of motion is approximated as

$$\ddot{\phi} - 3Hw\dot{\phi} + \frac{M^2}{3H^2}V' = 0 \quad \text{for } H \gg M. \tag{5.14}$$

For $H \ll M$, the equation of motion is the same as that of the canonical scalar field. Using the Virial theorem, we find

[21] References [46,47] argued a subtlety on the gauge choice $\delta\phi = 0$ around the end point of the field oscillation $\dot{\phi} = 0$ in analyzing the perturbation of the scalar field oscillation. It does not matter, however, for the discussion here. This is because the gradient instability occurs in the time scale much shorter than the one scalar oscillation period: the relevant wave number for the instability is $|c_s|k \gg m_{\rm eff}$ with c_s being the sound speed.

[22] It can be estimated as $c_s^2 \sim (1 - a^2G^{ij}/M^2)/(1 + G^{00}/M^2)$ with $G^{00} = 3H^2$ and $G^{ij} = -a^{-2}(3H^2 + 2\dot{H})\delta^{ij}$.

$$\Phi \propto \begin{cases} a^{-3(1-w)/(n+2)} & \text{for } H \gg M, \\ a^{-6/(n+2)} & \text{for } H \ll M, \end{cases} \tag{5.15}$$

where we have assumed $V \propto \phi^n$.

B. Particle production rate

In the previous subsection, we have seen that

$$a(t) \simeq \langle a(t) \rangle \left(1 + \mathcal{O}\left(\frac{\delta H}{m_{\rm eff}}\right)\right), \tag{5.16}$$

with the oscillation part of the Hubble parameter δH given by Eqs. (5.9) and (5.10). Thus, we can view the particle production as the gravitational annihilation in the present case. In addition, there is a direct coupling between the graviton and the scalar field induced by the nonminimal derivative coupling to gravity, and it can also cause the graviton production. Below we analyze the gravitational particle production of a minimally coupled scalar field and graviton. We do not discuss fermions and vector bosons since they are classically Weyl invariant in the massless limit.

1. Scalar

Now we evaluate the production rate of a minimally coupled scalar (2.20). The number density of the produced particles per one Hubble time is estimated by Eq. (A24) as

$$n_\chi(t) \sim \begin{cases} \frac{C}{512\pi H}\left(\frac{H m_{\rm eff}^3 \Phi^2}{M^2 M_P^2}\right)^2 & \text{for } M^2/m_{\rm eff} \lesssim H, \\ \frac{C}{512\pi H}\left(\frac{m_{\rm eff}^2 \Phi^2}{M_P^2}\right)^2 & \text{for } H \lesssim M^2/m_{\rm eff}. \end{cases} \tag{5.17}$$

From this, we can deduce the effective "annihilation rate" of ϕ into the χ pair, as

$$\Gamma_{\phi\phi\to\chi\chi} \sim \begin{cases} \frac{C}{512\pi}\frac{\Phi^2 m_{\rm eff}^5}{M^2 M_P^4} & \text{for } H \gtrsim M, \\ \frac{C}{512\pi}\frac{H^2 \Phi^2 m_{\rm eff}^5}{M^4 M_P^4} & \text{for } M^2/m_{\rm eff} \lesssim H \lesssim M, \\ \frac{C}{512\pi}\frac{\Phi^2 m_{\rm eff}^3}{M_P^4} & \text{for } H \lesssim M^2/m_{\rm eff}. \end{cases} \tag{5.18}$$

To obtain these results, we have defined the number density of ϕ as $n_\phi \equiv \rho_\phi/m_{\rm eff}$. It is soon realized that $\Gamma_{\phi\phi\to\chi\chi}/H$ is the nondecreasing function of time during $H \gtrsim M$ for $5/2 + 9/2(1+6w) \geq n$ (i.e., $2 \leq n \leq 4$ for $w \leq 1/3$). It is easily shown that $\Gamma_{\phi\phi\to\chi\chi}$ never exceeds H under the condition (5.13).

2. Graviton

For the graviton production, in addition to the "usual" gravitational production similar to Eq. (5.17), there is a contribution coming from the direct coupling between ϕ and the graviton through the nonminimal kinetic term. The former is the same as that of the scalar field, and, hence, we concentrate on the latter here. As shown in Ref. [45], the graviton kinetic term is written as

$$S \sim \int dt' d^3x \frac{1}{2}\sqrt{1 - \left(\frac{(d\phi/dt')^2}{2M_P^2 M^2}\right)^2}\left[\left(\frac{\partial h_{ij}}{\partial t'}\right)^2 - (\partial_l h_{ij})^2\right], \tag{5.19}$$

where

$$dt' \equiv \left(\frac{1 + \frac{\dot{\phi}^2}{2M^2 M_P^2}}{1 - \frac{\dot{\phi}^2}{2M^2 M_P^2}}\right)^{1/2} dt, \tag{5.20}$$

and we have omitted the scale factor here. The effective annihilation rate of ϕ into the graviton pair is

$$\frac{\Gamma_{\phi\phi\to hh}}{\Gamma_{\phi\phi\to\chi\chi}} \sim \begin{cases} \left(\frac{m_{\rm eff}}{H}\right)^2 \left(\frac{\rho_\phi}{\rho_M}\right)^2 & \text{for } H \gtrsim M, \\ \frac{H^4}{M^4}\left(\frac{m_{\rm eff}}{H}\right)^2 \left(\frac{\rho_\phi}{\rho_M}\right)^2 & \text{for } M^2/m_{\rm eff} \lesssim H \lesssim M, \\ \left(\frac{H}{M^2/m_{\rm eff}}\right)^4 \left(\frac{\rho_\phi}{\rho_M}\right)^2 & \text{for } H \lesssim M^2/m_{\rm eff}. \end{cases} \tag{5.21}$$

Therefore, this annihilation mode cannot exceed the ordinary gravitational production if we prohibit the gradient instability.[23]

C. Cosmological implications

Now we discuss the cosmological implications of the gravitational particle production. To be concrete, we take $n = 2$ and $w = 1/3$. The dominant contribution to the abundance of the minimally coupled scalar comes from $H \sim M$, since $\Gamma_{\phi\phi\to\chi\chi}/H$ is an increasing function of time for $H \gtrsim M$, while it is decreasing at $H \lesssim M$. We obtain

$$Y_\chi|_{H>M} \sim \frac{\alpha m_{\rm eff}^2 T}{M_P^2 H}\left(\frac{\rho_\phi}{\rho_M}\right)^2, \tag{5.22}$$

for $H \gtrsim M$, where $T \sim \rho_M/s$ is the "temperature" of the Universe, and $\alpha \sim 10^{-3}$ is a numerical coefficient. As an extreme case, let us assume that the inequality (5.13) is almost saturated at $H \sim M$. Then we have

$$\frac{\rho_\chi}{s} \lesssim \frac{\alpha m_\chi M^{3/2}}{M_P^{3/2}} \sim 3 \times 10^{-10}\ \text{GeV}\left(\frac{\alpha}{10^{-3}}\right) \times \left(\frac{m_\chi}{10^6\ \text{GeV}}\right)\left(\frac{M}{10^{10}\ \text{GeV}}\right)^{3/2}. \tag{5.23}$$

[23] If we allow the gradient instability to occur, the graviton (or gravitational wave) signal would be much more stronger, although the precise analysis is difficult to perform.

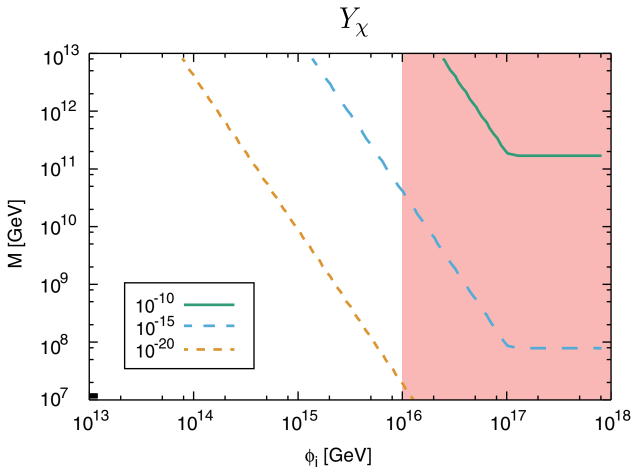

FIG. 3. Contour plot of Y_χ (left) and $\Delta N_{\rm eff}$ (right) for $m_\phi = 100M$ on the plane of (ϕ_i, M). In the shaded region there is a gradient instability.

The observational upper bound is $\rho_\chi/s \lesssim 4\times 10^{-10}$ GeV for a stable noninteracting χ field and $\rho_\chi/s \lesssim 10^{-14}$ GeV for χ as massive moduli. Note again that if ϕ remains light during inflation, the χ particle produced in this way has isocurvature fluctuation and, hence, cannot be a dominant component of DM. In the present model, ϕ cannot dominantly contribute to the curvature perturbation because the energy density of ϕ must be sufficiently small to avoid the gradient instability, and such a subdominant curvaton would lead to too large non-Gaussianity.

The graviton abundance is also the same as that of the light scalar field. The corresponding peak frequency is estimated as

$$f_{\rm GW} \sim 2\times 10^9\ {\rm Hz}\left(\frac{m_\phi}{10^{13}\ {\rm GeV}}\right)\left(\frac{10^{10}\ {\rm GeV}}{M}\right)^{1/2}. \qquad (5.24)$$

Around this frequency range, the gravitational wave abundance is too small to detect.

Figure 3 shows contours of Y_χ (left) and $\Delta N_{\rm eff}$ (right) for $m_\phi = 100M$ on the plane of (ϕ_i, M) for $n = 2$ and $w = 1/3$. We have implicitly assumed that the inflation scale $H_{\rm inf}$ satisfies $H_{\rm inf} > (M/H_{\rm inf})m_\phi$ ($H_{\rm inf} > 10M$ for $m_\phi = 100M$), and ϕ decays into radiation after $H \sim M$ but before the domination. In the red shaded region, there is a gradient instability. From this figure, it is seen that once we avoid the gradient instability, which would otherwise invalidate the reheating analysis, cosmological constraints are not so stringent [compared with a typical constraint for a massive long-lived particle (2.29)].

VI. CONCLUSIONS AND DISCUSSION

In this paper, we have studied the gravitational particle production caused by a coherently oscillating scalar field in the Universe. We have treated the Einstein gravity, $f(\phi)R$ gravity, $f(R)$ gravity, and $G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ gravity theories where ϕ is the scalar field, R is the Ricci scalar, and $G^{\mu\nu}$ is the Einstein tensor, respectively.

We have estimated the particle production rate for such a broad class of models in a unified framework. In particular, we have paid attention to an oscillating part of the scale factor, which makes manifest how the background oscillation produces nonconformally coupled particles. A coherently oscillating scalar field, no matter if it is dominant or subdominant, induces an oscillating feature of the scale factor. It exists even in the Einstein gravity theory and is more violent for the extended gravity theories. All particles couple to the scale factor unless they are Weyl invariant and feel the oscillation of the scale factor. Thus, gravitational particle production by the scalar field occurs through its oscillation. In the previous paper [14], we considered only the case where the scalar field dominates the Universe. In this paper, we have extended our study so that it can be applied to a subdominant scalar field as well. We have also treated a broader class of gravity theories systematically. For the Einstein gravity theory, the production caused by the inflaton is larger than any other subdominant scalar fields. However, in the extended gravity theories, the contribution from the subdominant scalar field, other than inflaton, can be the dominant one.

An interesting feature of our viewpoint is that, once we express the oscillating part of the scale factor by the coherently oscillating scalar field, we can easily deduce effective couplings between the scalar field and other particles mediated by the gravity from the Lagrangian. In the Einstein and $G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ theories, the oscillating part of the scale factor depends quadratically on the scalar field, and, hence, we can view it as gravitational annihilation. In the $f(\phi)R$ and $f(R)$ theories, it depends linearly on the scalar field in general, and, hence, we can view it as gravitational decay. We can easily estimate the production rate, which coincides with that obtained from more rigorous calculations. For example, in our viewpoint, it is clear that the scalar field (or the scalaron) does not decay into the gravitons in the $f(\phi)R$ and $f(R)$ theories. Indeed, we have explicitly seen that the direct coupling cancels with the oscillating part of the scale factor, resulting in no effective coupling between the scalar field and the graviton. It is consistent with the results in the Einstein frame.

We have also discussed the cosmological implications of the gravitational particle production. All particles whose masses are smaller than that of the oscillating scalar field are produced by the gravitational particle production if they are not Weyl invariant. Thus, it is possible that the daughter particle itself is quite massive. If it is stable and heavy enough, it can serve a sizable contribution to the dark matter abundance. Alternatively, if it is a long-lived particle such as moduli, a severe constraint on the abundance is obtained from the observation of big bang nucleosynthesis. If it is massless, on the other hand, it can contribute to the dark radiation that is constrained by the cosmic microwave background observation. One of the well-motivated examples of such a light particle is the axion. For example, if the theory is described solely by the standard model, the Peccei-Quinn sector and the Starobinsky R^2 inflation, it may produce an observable amount of axion dark radiation. A detailed study on this respect may be interesting, which we leave as a future work.

ACKNOWLEDGMENTS

This work was supported by the Grant-in-Aid for Scientific Research on Scientific Research A (Grant No. 26247042 [K. N.]), Young Scientists B (Grant No. 26800121 [K. N.]), and Innovative Areas (Grants No. 26104009 and No. 15H05888 [K. N.]). This work was supported by World Premier International Research Center Initiative (WPI Initiative), MEXT, Japan. The work of Y. E., R. J., and K. M. was supported in part by JSPS Research Fellowships for Young Scientists. The work of Y. E. was also supported in part by the Program for Leading Graduate Schools, MEXT, Japan.

APPENDIX A: PARTICLE PRODUCTION RATE IN OSCILLATING BACKGROUND

We consider a real scalar field χ with time-dependent mass:

$$S = \int d^4x \left(-\frac{1}{2}(\partial\chi)^2 - \frac{1}{2}m_\chi^2(t)\chi^2 \right). \tag{A1}$$

Let us estimate the production rate of the χ particle. Typically, $m_\chi(t)$ is proportional to powers of another coherently oscillating scalar field $\phi(t)$ whose mass scale is m_ϕ. Hereafter, we do not assume a specific form of $m_\chi(t)$ but only assume that it is an oscillating function with frequency of Ω.

1. Quantization

Let us expand χ as

$$\chi = \int \frac{d^3k}{(2\pi)^3} \chi_{\vec{k}} e^{i\vec{k}\cdot\vec{x}}. \tag{A2}$$

From the reality condition $\chi^* = \chi$, we have $\chi_{\vec{k}}^* = \chi_{-\vec{k}}$. The equation of motion of the Fourier mode is given by

$$\ddot{\chi}_{\vec{k}} + \omega_k^2(t)\chi_{\vec{k}} = 0, \tag{A3}$$

where $\omega_k^2 \equiv k^2 + m_\chi^2(t)$. Now we write χ_k in terms of the ladder operator as

$$\chi_{\vec{k}} = a_{\vec{k}} v_{\vec{k}}(t) + a_{-\vec{k}}^\dagger v_{\vec{k}}^*(t), \tag{A4}$$

where $v_{\vec{k}}(t)$ and $v_{\vec{k}}^*(t)$ are independent solutions of (A3). Note that we should have $v_{\vec{k}} = v_{-\vec{k}}$ to satisfy the reality condition. By using the freedom to choose the overall normalization of $v_{\vec{k}}(t)$ and $v_{\vec{k}}^*(t)$, we can take $a_{\vec{k}}$ and $a_{\vec{k}}^\dagger$ so that they satisfy the following commutation relation,

$$[a_{\vec{k}}, a_{\vec{k}'}^\dagger] = (2\pi)^3\delta(\vec{k} - \vec{k}'), \qquad [a_{\vec{k}}, a_{\vec{k}'}] = [a_{\vec{k}}^\dagger, a_{\vec{k}'}^\dagger] = 0. \tag{A5}$$

On the other hand, we must have the following canonical commutation relation:

$$[\chi(\vec{x}), \dot{\chi}(\vec{x}')] = i\delta(\vec{x} - \vec{x}'). \tag{A6}$$

From this, we obtain

$$v_{\vec{k}}\dot{v}_{\vec{k}}^* - v_{\vec{k}}^*\dot{v}_{\vec{k}} = i. \tag{A7}$$

Now let us assume the solution of the form

$$v_{\vec{k}}(t) = \frac{1}{\sqrt{2\omega_k}}\left[\alpha_{\vec{k}}(t)e^{-i\int_0^t dt'\omega_k(t')} + \beta_{\vec{k}}(t)e^{i\int_0^t dt'\omega_k(t')}\right]. \tag{A8}$$

There is a functional degree of freedom to impose an arbitrary condition between $\alpha_{\vec{k}}(t)$ and $\beta_{\vec{k}}(t)$. We choose it as

$$\dot{\alpha}_{\vec{k}} = \frac{\dot{\omega}_k}{2\omega_k} e^{2i\int_0^t dt'\omega_k(t')}\beta_{\vec{k}}, \qquad \dot{\beta}_{\vec{k}} = \frac{\dot{\omega}_k}{2\omega_k} e^{-2i\int_0^t dt'\omega_k(t')}\alpha_{\vec{k}}, \tag{A9}$$

with $\alpha_{\vec{k}}(0) = 1$ and $\beta_{\vec{k}}(0) = 0$ to satisfy the initial condition

$$v_{\vec{k}}(t \to 0) \simeq \frac{1}{\sqrt{2\omega_k}} e^{-i\omega_k t}, \qquad \dot{v}_{\vec{k}}(t \to 0) \simeq -i\sqrt{\frac{\omega_k}{2}} e^{-i\omega_k t}. \tag{A10}$$

Under these conditions, $\dot{v}_{\vec{k}}$ is expressed as

$$\dot{v}_{\vec{k}}(t) = -i\sqrt{\frac{\omega_k}{2}}[\alpha_{\vec{k}}(t) e^{-i\int_0^t dt'\omega_k(t')} - \beta_{\vec{k}}(t) e^{i\int_0^t dt'\omega_k(t')}]. \tag{A11}$$

Note that (A7) requires the following normalization condition,

$$|\alpha_{\vec{k}}|^2 - |\beta_{\vec{k}}|^2 = 1, \tag{A12}$$

which is automatically satisfied at all times once we impose the condition (A9).

2. Production rate

The occupation number or the phase space distribution of χ is given by

$$f_\chi(k) = \frac{1}{2\omega_k}(|\dot{v}_k|^2 + \omega_k^2 |v_k|^2) - \frac{1}{2} = |\beta_{\vec{k}}|^2. \tag{A13}$$

Thus, $f_\chi(k) = 0$ at $t \to 0$, but it grows after that. The total number density is given by

$$n_\chi(t) = \int \frac{d^3k}{(2\pi)^3} f_\chi(k). \tag{A14}$$

Thus, the remaining task is to derive time evolution of $\beta_{\vec{k}}$. It is easily calculated from (A9) as long as $\alpha_{\vec{k}} \simeq 1$ and $|\beta_{\vec{k}}| \ll 1$ hold. In this case, we have

$$\begin{aligned} \beta_k(t) &\simeq \int_0^t dt' \frac{\dot{\omega}_k}{2\omega_k} e^{-2i\int_0^{t'} dt''\omega_k(t'')} \\ &= \int_0^t dt' \frac{m_\chi \dot{m}_\chi}{2\omega_k^2} e^{-2i\int_0^{t'} dt''\omega_k(t'')}. \end{aligned} \tag{A15}$$

Recall that $m_\chi(t')$ is an oscillating function with frequency of Ω. It is not hard to imagine that the time integral in (A15) cancels out if Ω and ω_k are much different from each other. However, if $\omega_k \simeq \Omega$, the time integral gives a linearly growing result with t.

To see only the time growing part, we perform integration by parts and assume $k^2 \gg m_\chi^2$ to rewrite (A15) as

$$\beta_k(t) \simeq \frac{i}{2\omega_k} \int_0^t dt' m_\chi^2(t') e^{-2i\omega_k t'}. \tag{A16}$$

Now we consider a frequency range

$$\Omega - \Delta\Omega \lesssim \omega_k \lesssim \Omega + \Delta\Omega. \tag{A17}$$

At $t \lesssim 1/\Delta\Omega$, the phase of $m_\chi^2(t)$ and $e^{-2i\omega_k t}$ roughly cancel with each other, and, hence, β_k in this frequency range linearly grows with t. After that, however, the oscillation feature forbids further growth. Conversely, for fixed t, the frequency range with $\Delta\Omega \simeq 1/t$ experienced a linear growth. Therefore, we have

$$f_{\vec{k}}(t) \simeq \frac{\tilde{m}_\chi^4}{4\omega_k^2} t^2 \quad \text{for } \Omega - \frac{1}{t} \lesssim \omega_k \lesssim \Omega + \frac{1}{t}. \tag{A18}$$

Here, $\tilde{m}_\chi$ stands for the amplitude of $m_\chi(t)$. This expression is valid as long as $f_k \ll 1$, i.e., $t \lesssim 1/(q\Omega)$ with $q \equiv \tilde{m}_\chi^2/\Omega^2 (\ll 1)$. The total number density linearly grows with t as[24]

$$n_\chi(t) \simeq C \frac{\tilde{m}_\chi^4}{32\pi} t. \tag{A19}$$

This expression does not refer to the parent field ϕ. We only assumed that $m_\chi(t)$ is an oscillating function with frequency Ω.

This result is easily understood in terms of ρ_ϕ and Γ_ϕ, if the coherent oscillation of ϕ is responsible for the oscillating $m_\chi(t)$. Assuming that ϕ is canonically normalized, the perturbative decay rate of ϕ into the χ pair is given by (notice that $\Omega \sim m_\phi$).[25]

$$\Gamma_\phi \sim \frac{C}{32\pi} \frac{\tilde{m}_\chi^4}{\Phi^2 m_\phi} \sim \frac{C}{32\pi} \frac{q^2 m_\phi^3}{\Phi^2}, \tag{A20}$$

with Φ being the amplitude of ϕ. Since the energy density of ϕ is given by $\rho_\phi \simeq m_\phi^2 \Phi^2/2$, we obtain

$$n_\chi(t) \simeq 2C \frac{\rho_\phi \Gamma_\phi}{m_\phi} t \sim C \frac{\tilde{m}_\chi^4}{32\pi} t. \tag{A21}$$

3. Gravitational production rate

Now let us consider the production of the χ field which couples to ϕ gravitationally:

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2} f(\chi) R - \frac{1}{2} (\partial\chi)^2 - \frac{1}{2} m^2 \chi^2 \right). \tag{A22}$$

[24] In the case of the three-point interaction as $m_\chi^2 = \mu\phi$ (hence, $\Omega = m_\phi/2$), we can explicitly calculate (A16) and find (A19) with a numerical coefficient $C = 1$. For the other types of interactions, C slightly deviates from 1.

[25] Again, in the case of the three-point interaction as $m_\chi^2 = \mu\phi$, we find that the perturbative decay rate $\phi \to \chi\chi$ is given by (A20) with a numerical coefficient $C = 1$. For the other types of interactions, C slightly deviates from 1.

Here, m is constant and assumed to be smaller than the Hubble scale so that we can neglect it, and $f(\chi)$ is a function of χ. In the minimal case, we have $f(\chi) = M_P^2$, and, hence, χ feels the background oscillation only through the Hubble parameter or the scale factor. By using the conformal time $d\tau = dt/a$ and defining $\tilde{\chi} \equiv a\chi$, it is rewritten as

$$S = \int d\tau d^3x \frac{1}{2}\left[\tilde{\chi}'^2 - (\partial_i\tilde{\chi})^2 + \frac{a''}{a}(\tilde{\chi}^2 + 6a^2 f(\chi))\right], \tag{A23}$$

where we have dropped the mass term because we consider the case $m_\phi \gg m$ from now. It is seen that $\tilde{\chi}$ generally obtains a mass of $\sim a''/a (= a^2R/6)$, and it is an oscillating function if there is a coherently oscillating scalar field as repeatedly shown in the main text, which leads to $\tilde{\chi}$ particle production. Note that the scale factor dependence vanishes in the conformal coupling $f(\chi) = -\chi^2/6 + M_P^2$. Therefore, there is no particle production in this case.

Below we consider the minimal case: $f(\chi) = M_P^2$. Then we can apply the formula (A19) as a number density created within one Hubble time by interpreting $m_\chi^2(\tau) = a''/a$. Thus,

$$\frac{d[a^3 n_\chi]}{d\tau} \simeq C\frac{(a''/a)^2}{32\pi} \to \frac{dn_\chi}{dt} \simeq \frac{C}{32\pi}\left[\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2\right]^2. \tag{A24}$$

Here we estimate a''/a with its amplitude. In the second similarity, we have omitted terms from the cosmic expansion. If one can express a in terms of ϕ as

$$a(t) = \langle a(t)\rangle\left[1 - \frac{c_n}{n}\frac{\phi^n - \langle\phi^n\rangle}{M_P^n}\right], \tag{A25}$$

we have the χ number density produced in a time interval $t = H^{-1}$ as

$$n_\chi(t) \simeq \frac{C}{32\pi H}\left(\frac{c_n m_\phi^2 \Phi^n}{M_P^n}\right)^2. \tag{A26}$$

APPENDIX B: ADIABATIC INVARIANT IN $f(R)$ THEORIES

In Ref. [29], we introduced an adiabatic invariant J for the generalized Galileon theories. This quantity satisfies $\dot{J} \sim \mathcal{O}(HJ)$ even when H oscillates rapidly as $\dot{H} \sim \mathcal{O}(m_{\mathrm{eff}}H)$. In this appendix we generalize this quantity to $f(R)$ theories.

We consider the action (4.1) in the absence of matter. Using an auxiliary field ϕ, this system is rewritten as

$$S = \int d^4x\sqrt{-g}\frac{M_P^2}{2}[f(\phi) + F(\phi)(R - \phi)]. \tag{B1}$$

Using integration by parts, we have

$$S = \int d^4x a^3 \frac{M_P^2}{2}[f(\phi) - F(\phi)\phi - 6F(\phi)H^2 - 6\dot{F}(\phi)H], \tag{B2}$$

which now has the form of the generalized Galileon action. The adiabatic invariant can be derived by taking derivative with respect to H:

$$J \equiv -\frac{1}{6M_P^2}\frac{\partial\mathcal{L}}{\partial H} = FH + \frac{1}{2}\dot{F}. \tag{B3}$$

Since we have $\phi = R$ from the action (B2), F in Eq. (B3) is understood as $F(R)$ with R being the background value $R = 12H^2 + 6\dot{H}$.

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