Unraveling the physical meaning of the Jaffe-Manohar decomposition of the nucleon spin

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A general consensus now is that there are two physically inequivalent complete decompositions of the nucleon spin, i.e. the decomposition of the canonical type and that of mechanical type. The well-known Jaffe-Manohar decomposition is of the former type. Unfortunately, there is a wide-spread misbelief that this decomposition matches the partonic picture, which states that motion of quarks in the nucleon is approximately free. In the present monograph, we reveal that this understanding is not necessarily correct and that the Jaffe-Manohar decomposition is not such a decomposition, which natively reflects the intrinsic (or static) orbital angular momentum structure of the nucleon.

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I. INTRODUCTION

Over the past few years, there have been intensive debates on the question of whether the gauge-invariant complete decomposition of the nucleon spin is possible or not. (See Refs. [1–3] for review.) One of the central issues of this debate was concerned with the significance of the concept of the physical component of the gauge field, which was first introduced by Chen *et al.* into the nucleon spin decomposition problem [4,5]. A consensus now is that the definition of the physical component of the gauge field $A^{\mu}_{\rm phys}$ is not unique. The ultimate reason is because $A^{\mu}_{\rm phys}$ cannot be defined independently of the choice of the Lorentz frame [6]. The original proposal for $A^{\mu}_{\rm phys}$ by Chen *et al.* amounts to a non-Abelian generalization of the familiar transverse-longitudinal decomposition of the photon field also called the Helmholtz decomposition. This Helmholtz decomposition works perfectly in the decomposition problem of the total photon angular momentum into its intrinsic spin and orbital parts [7–10]. The reason for it is twofold. First, in this problem, we are dealing with free photons, or more precisely, a wave packet of free photons. Second, the measurement of the photon spin and orbital angular momentum (OAM) is carried out in a fixed or prescribed Lorentz frame by making use of interactions with atoms, so that a particular choice of a Lorentz frame is nothing problematic [11].

Unfortunately but importantly, the situation is fairly different for the nucleon spin decomposition problem. Here, we must handle quarks and gluons tightly bound in the nucleon. To our present knowledge, the only way to probe the internal spin and OAM contents of such a composite particle is to use deep-inelastic scatterings (DIS). One important property of DIS observables (or quasiobservables) typified by parton distribution functions is the Lorentz-boost invariance along the direction of the momentum of the parent nucleon [12]. Accordingly, the definition of $A^{\mu}_{\rm phys}$, which is relevant for the DIS measurements of the nucleon spin contents, must also have this property [6]. (This is clear, for example, from the fact that the measurable gluon spin is the first moment of the longitudinally polarized gluon distribution function.) The Coulomb-gauge-motivated definition of $A^{\mu}_{\rm phys}$ proposed by Chen *et al.* does not satisfy this property [4,5]. The definition of $A^{\mu}_{\rm phys}$, which satisfies this property, is the light-cone-gauge motivated definition proposed by Hatta [13]. In this way, the claim that there can be infinitely many gauge-invariant decompositions of the nucleon spin loses its basis, once the importance of the boost-invariance requirement mentioned above is properly recognized [6].

Still, we are left with two physically inequivalent types of complete decompositions of the nucleon spin, which are now called the canonical-type decomposition and the mechanical-type (or kinetic-type) decomposition. Note that, from the physical viewpoint, the canonical decomposition is nothing different from the famous Jaffe-Manohar decomposition [14] later refined by Bashinsky and Jaffe [15]. In a series of papers [2,16–19], we have advocated a view which favors the mechanical decomposition rather than the canonical one as a natural decomposition of the nucleon spin. Unfortunately, there still remains a wide-spread misbelief in the DIS community that, as compared with the mechanical decomposition, the Jaffe-Manohar decomposition is more compatible with the familiar partonic picture of the quark motion inside the nucleon [15,20,21]. Undoubtedly, this misbelief comes from a careless extension of the parton model idea, which states that the motion of partons in the nucleon is free at the leading-twist approximation. Here is a pitfall, however. The partonic picture is certainly established for the collinear

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motion of constituents along the direction of the nucleon momentum. As a matter of course, however, the generation of the OAM component along the nucleon momentum requires motion of partons in the plane perpendicular to this direction. Whether this transverse motion of quarks is also partonic or not is a highly nontrivial question, which must be judged only after careful consideration. In fact, a more natural picture is that this motion of quarks, which generates the longitudinal component of the OAM, is a circular motion in the transverse plane. It seems obvious that such a circular motion cannot be a free motion in any sense.

Anyhow, the above consideration throws a strong doubt on the partonic interpretation of the Jaffe-Manohar decomposition of the nucleon spin. What is a correct physical interpretation of the quark and gluon OAM terms appearing in the Jaffe-Manohar decomposition, then? Is it really an observable decomposition? The purpose of the present paper is to answer these questions as clearly as possible. To this end, we think it very important to clearly understand the distinction between the canonical OAM and the mechanical OAM under the presence of the electromagnetic potential. The famous Landau problem is a quantum mechanics of a charged particle motion under the presence of uniform magnetic field [22]. In Sec. II, we concisely review the essence of these topics with the particular intention of unmasking the identities of the two types of OAMs, i.e. the canonical and mechanical OAMs. Next, in Sec. III, we demonstrate an important role of the non-Abelian Stokes theorem in the nucleon spin decomposition problem following the recent suggestion by Tiwari [23]. We explicitly show that the relation between the canonical and mechanical OAMs derived by Burkardt can more quickly be obtained by making use of this general theorem [24]. Next, after these preparations, we revisit in Sec. IV several fundamental questions of the gauge-invariant nucleon spin decomposition problem. Can one say that the complete decomposition of the nucleon spin based on the concept of the physical component of the gauge field is genuinely gauge invariant? Which of the types, canonical or mechanical, can be thought of as an observable decomposition? Next, in Sec. V, we reveal the physical meaning of the Jaffe-Manohar decomposition in a coherent fashion, to show why its partonic interpretation is not justified. Finally, in Sec. VI, we summarize what we have clarified in the present paper.

II. TWO ORBITAL ANGULAR MOMENTA IN THE LANDAU PROBLEM

In several previous publications, we repeatedly emphasized the fact that, under the presence of strong background of magnetic field, what describes the physical orbital motion of a charged particle is the mechanical (or kinetic) OAM, not the canonical one [2,16,17]. In view of the existence of strong color magnetic field inside the nucleon as a quark-gluon composite, this naturally implies that the physically favorable decomposition of the nucleon spin is the mechanical- (or kinetic-)type decomposition not the canonical-type one. Unfortunately, this reasonable claim of us is not necessarily accepted in the community of DIS physics. This is due to a blind belief of the parton picture, which states that the motion of quarks inside the nucleon must be approximately free at the leading order. To correct this misunderstanding, we think it useful to understand the essence of the famous Landau problem [22], i.e. the motion of a charged particle in a uniform magnetic field, especially by paying attention to the physical content of the two OAMs, i.e. the canonical and mechanical OAMs [25–27]. (A very comprehensible lecture note on the Landau problem can be found in Ref. [28].)

For simplicity, let us confine ourselves to the two-dimensional motion of a particle with charge e in the $x - y$ plane under uniform magnetic field $\boldsymbol{B} = B\boldsymbol{e}_z$ along the z axis. (Here, for clarity, the charge e of the particle is assumed to be positive.) In classical mechanics, the Lorentz force causes a circular motion of the charged particle. The balance equation between the centrifugal force and the Lorentz force reads as

$$\frac{mv^2}{r} = eBv. \tag{1}$$

(Here and hereafter, we use the natural unit $c = \hbar = 1$.) This gives the radius of the circular motion,

$$r = \frac{mv}{eB}, \tag{2}$$

which is called the Larmor radius or the cyclotron radius. The energy of the system is given by

$$E = \frac{1}{2} mv^2 = \frac{1}{2} mr^2\omega^2, \tag{3}$$

with $\omega = \frac{v}{r} = \frac{eB}{m}$ being the angular frequency of the cyclotron motion. In classical mechanics, the cyclotron radius as well as the velocity v can take any real values. In quantum mechanics, the orbit of the cyclotron motion as well as the energy are quantized. It can be seen already in the semi-classical treatment, which corresponds to imposing the so-called Bohr-Sommerfeld quantization condition as

$$\frac{1}{2\pi} \oint \boldsymbol{p} \cdot d\boldsymbol{r} = n + \frac{1}{2}. \tag{4}$$

With the use of the relation $\boldsymbol{p} = m\boldsymbol{v} + e\boldsymbol{A}$, where $\boldsymbol{A}$ is the gauge potential corresponding to the magnetic field $\boldsymbol{B}$, this leads to

$$n + \frac{1}{2} = \frac{1}{2\pi} \oint (m\boldsymbol{v} + e\boldsymbol{A}) \cdot d\boldsymbol{r}$$
$$= mvr - \frac{1}{2\pi} eB(\pi r^2) = mvr - \frac{1}{2} mvr = \frac{1}{2} mvr, \quad (5)$$

where we have used the relation $\omega = \frac{v}{r} = \frac{eB}{m}$. Here, use has been made of the Stokes theorem. [The origin of the minus sign in front of the second term on the rhs of Eq. (5) is that the cyclotron motion is clockwise for $eB > 0$, and in this case, the line integral of the vector potential gives the negative of the magnetic flux inside the Larmor radius.] Multiplying both sides with ω, this gives the quantized energy as

$$\left(n + \frac{1}{2}\right)\omega = \frac{1}{2} mv^2 = E, \quad (6)$$

with n being a non-negative integer. Accordingly, the cyclotron radius is also quantized as

$$r_n = \sqrt{\frac{1}{eB}} \cdot \sqrt{2n+1}. \quad (7)$$

As we shall see shortly, in quantum mechanics, the above discrete orbit with the radius r_n corresponds to a Landau state describing quantized cyclotron motion. However, we shall also see that each state has an infinite degeneracy originating from the fact that each state with a definite Landau quantum number n contains infinitely many states, which are characterized by another integer m, the eigenvalue of the canonical angular momentum operator $L_{\rm can}$.

As is well known, a quantum mechanical treatment of the cyclotron motion requires one to introduce the vector potential $\boldsymbol{A}$, which is defined through the relation $\nabla \times \boldsymbol{A} = \boldsymbol{B}$. The relevant Hamiltonian of the Landau problem is then given by

$$H = \frac{1}{2m} \Pi^2 = \frac{1}{2m} (\boldsymbol{p} - e\boldsymbol{A})^2. \quad (8)$$

The choice of $\boldsymbol{A}$, which gives the same $\boldsymbol{B}$, is not unique, but the physics must be independent of this choice. We say that the theory has a gauge invariance. To solve the quantum mechanical problem explicitly, however, we are forced to take some specific choice for the vector potential $\boldsymbol{A}$, which amounts to taking a particular gauge choice. Some of the popular choices are the rotationally symmetric gauge given by

$$\boldsymbol{A} = (A_x, A_y) = \frac{B}{2}(-y, x), \quad (9)$$

the gauge with the translational invariance along the x axis given as

$$\boldsymbol{A} = (A_x, A_y) = B(-y, 0), \quad (10)$$

and the gauge with the translational invariance along the y axis given as

$$\boldsymbol{A} = (A_x, A_y) = B(0, x). \quad (11)$$

The choice (10) is the gauge used by Landau in solving the problem for the first time, so that we call it the Landau gauge hereafter [22]. Although gauge-invariant quantities are independent of the gauge choice, the symmetric gauge is most convenient for understanding the relation between the canonical OAM and the mechanical OAM, so let us first work in this gauge.

In the combination

$$\Pi = \boldsymbol{p} - e\boldsymbol{A}, \quad (12)$$

which enters the Hamiltonian, $\boldsymbol{p}$ is the standard canonical momentum, while Π is called the mechanical (or kinetic) momentum. At variance with the canonical momenta, the mechanical momenta do not commute with each other. Their commutation relation is

$$[\Pi_x, \Pi_y] = ieB. \quad (13)$$

To obtain the eigenvalues and eigenstates of the Landau Hamiltonian (8), we introduce the ladder (annihilation and creation) operators by

$$a = \sqrt{\frac{1}{2eB}}(\Pi_x + i\Pi_y), \qquad a^\dagger = \sqrt{\frac{1}{2eB}}(\Pi_x - i\Pi_y). \quad (14)$$

They satisfy the following commutation relations:

$$[a, a^\dagger] = 1, \qquad [a, a] = [a^\dagger, a^\dagger] = 0. \quad (15)$$

The Hamiltonian then reduces to

$$H = \frac{1}{2m}(\Pi_x^2 + \Pi_y^2) = \omega\left(a^\dagger a + \frac{1}{2}\right). \quad (16)$$

Since the last expression is nothing but the Hamiltonian of a one-dimentional harmonic oscillator, its eigenstates and eigenvalues are readily obtained as

$$H|n\rangle = E_n|n\rangle, \quad \text{with} \quad E_n = \left(n + \frac{1}{2}\right)\omega, \quad (17)$$

where

$$|n\rangle = \frac{(a^\dagger)^n}{\sqrt{n!}}|0\rangle. \quad (18)$$

Actually, it is a widely known fact that each Landau level with given n is infinitely degenerated. To understand this degeneracy, let us introduce the two operators X and Y, which have the meaning of the center of cyclotron motion:

$$X \equiv x + \frac{v_y}{\omega} = x + \frac{1}{eB}\Pi_y, \tag{19}$$

$$Y \equiv y - \frac{v_x}{\omega} = y - \frac{1}{eB}\Pi_x. \tag{20}$$

They satisfy the following commutation relation:

$$[X, Y] = -i\frac{1}{eB}. \tag{21}$$

Here, we introduce another ladder operator b and $b^\dagger$ by

$$b = \sqrt{\frac{eB}{2}}(X - iY), \qquad b^\dagger = \sqrt{\frac{eB}{2}}(X + iY). \tag{22}$$

It is an easy exercise to check that they satisfy the following commutation relations:

$$[b, b^\dagger] = 1, \qquad [b, b] = [b^\dagger, b^\dagger] = 0. \tag{23}$$

Furthermore, b and $b^\dagger$ commute with either of a and $a^\dagger$ as

$$[b, a] = [b, a^\dagger] = [b^\dagger, a] = [b^\dagger, a^\dagger] = 0. \tag{24}$$

Of our particular interest is the relation between the two orbital angular momenta, i.e. the canonical OAM and the mechanical OAM. Since the motion of the charged particle is confined in the $x - y$ plane, we have only to consider the z-component of the orbital angular momenta. The canonical OAM is given by

$$L_{\rm can} \equiv x p_y - y p_x, \tag{25}$$

whereas the mechanical OAM is given by

$$L_{\rm mech} \equiv m(x v_y - y v_x) = x\Pi_y - y\Pi_x. \tag{26}$$

In the symmetric gauge, the relation between these two OAMs is given as

$$L_{\rm can} = L_{\rm mech} + \frac{eB}{2}(x^2 + y^2). \tag{27}$$

It is interesting to point out that the difference between the canonical and mechanical OAMs is just given by the "potential angular momentum" introduced in Refs. [16,17],

$$L_{\rm pot} = e(\boldsymbol{r} \times \boldsymbol{A})_z = \frac{eB}{2}(x^2 + y^2), \tag{28}$$

which means that

$$L_{\rm can} = L_{\rm mech} + L_{\rm pot}. \tag{29}$$

As explained in Refs. [2,16,17], $L_{\rm pot}$ represents the angular momentum carried by the electromagnetic potential, which is the external magnetic field in the present problem. Equation (29) thus means that the canonical OAM represents the total OAM, that is, the *sum* of the particle OAM and the OAM carried by the electromagnetic field. (A support to this interpretation is also found in a recent paper [29].)

To proceed, we express $L_{\rm mech}$ and $L_{\rm can}$ in terms of the ladder operators $a, a^\dagger, b$, and $b^\dagger$. The answer is given by

$$L_{\rm mech} = i(b a^\dagger - b^\dagger a) - (a^\dagger a + a a^\dagger), \tag{30}$$

$$L_{\rm can} = \frac{1}{2}(b^\dagger b + b b^\dagger) - \frac{1}{2}(a^\dagger a + a a^\dagger). \tag{31}$$

[Remember that the Hamiltonian is already expressed as (16) only with a and $a^\dagger$.] Using the commutation relations of $a, a^\dagger, b$, and $b^\dagger$, one can easily verify that the canonical OAM operator $L_{\rm can}$ commutes with the Hamiltonian,

$$[L_{\rm can}, H] = 0, \tag{32}$$

although the mechanical OAM operator does not. This means that we can construct simultaneous eigenstates of H and $L_{\rm can}$, which are characterized by two harmonic oscillator quanta n and m as

$$H|n, n+m\rangle = \left(n + \frac{1}{2}\right)\omega|n, n+m\rangle, \tag{33}$$

$$L_{\rm can}|n, n+m\rangle = m|n, n+m\rangle, \tag{34}$$

where

$$|n, m\rangle = \frac{(a^\dagger)^n (b^\dagger)^m}{\sqrt{n! m!}}|0, 0\rangle. \tag{35}$$

Here, n are non-negative integers ($n = 0, 1, \cdots$) characterizing the Landau level, while m are integers satisfying the inequality $m \geq -n$. Thus, for a fixed Landau label n with the eigen-energy $E_n = (n + \frac{1}{2})\omega$, there are infinitely many states with exactly the same eigenenergy but different z-component of the canonical OAM.

To understand the physical content of the two OAMs, let us investigate the expectation value of the canonical and mechanical OAMs in the eigenstate $|n, n+m\rangle$, defined by

$$\langle O\rangle \equiv \langle n, n+m|O|n, n+m\rangle. \tag{36}$$

As can be easily checked, the expectation value of the mechanical OAM becomes

$$\langle L_{\rm mech}\rangle = -(2n + 1), \tag{37}$$

which is independent of m. The expectation value of the potential angular momentum can also be readily calculated as

$$\langle L_{\text{pot}} \rangle = \frac{eB}{2} \langle x^2 + y^2 \rangle = m + (2n + 1). \tag{38}$$

Adding up these two quantities, we find that

$$\langle L_{\text{can}} \rangle = \langle L_{\text{mech}} \rangle + \langle L_{\text{pot}} \rangle = m, \tag{39}$$

which naturally reproduces the eigenvalue of L_{can} in the state $|n, n + m\rangle$.

Somewhat surprisingly, the canonical OAM characterized by the quantum number m has little to do with the physical cyclotron motion of a charge particle in the magnetic field. This is reflected in the fact that the quantum number m does not appear in the (observable) energy $E_n = (n + \frac{1}{2})$ of the Landau problem, so that it is not a direct observable. On the other hand, the expectation value of the mechanical OAM is characterized by the Landau quantum number n, so that it is clearly an observable. Remember that the eigenenergy of the Landau level n is just consistent with the Bohr-Sommerfeld quantization condition corresponding to the semiclassical cyclotron motion.

Undoubtedly, this noticeable difference between the two OAMs is not unrelated to the fact that the canonical OAM is not a gauge-invariant quantity. To confirm it, let us investigate both OAMs in a different gauge from the symmetric gauge, for example, in the Landau gauge. The gauge transformation from the symmetric gauge $\boldsymbol{A}_S(\boldsymbol{r}) = \frac{B}{2}(-y, x, 0)$ to the Landau gauge $\boldsymbol{A}_L(\boldsymbol{r}) = B(-y, 0, 0)$ is given by

$$\boldsymbol{A}_L(\boldsymbol{r}) = \boldsymbol{A}_S(\boldsymbol{r}) + \nabla\chi(\boldsymbol{r}), \tag{40}$$

with the choice of the gauge function

$$\chi(\boldsymbol{r}) = -\frac{B}{2} xy. \tag{41}$$

Note that, in the Landau gauge, the mechanical momenta take the form

$$\Pi_x \equiv p_x - eA_x = p_x + eBy,$$
$$\Pi_y \equiv p_y - eA_y = p_y.$$

The Hamiltonian is given by

$$H = \frac{1}{2m}(\Pi_x^2 + \Pi_y^2) = \frac{1}{2m}\{(p_x + eBy)^2 + p_y^2\}. \tag{42}$$

Since this Hamiltonian does not contain the coordinate x, its eigenfunction is given as

$$\psi(x, y) \propto e^{ik_x x}\phi(y), \tag{43}$$

or in more abstract form as

$$|\psi\rangle = |k_x, \phi\rangle \equiv |k_x\rangle|\phi\rangle, \tag{44}$$

with $|k_x\rangle$ being the eigenstate of p_x:

$$p_x|k_x\rangle = k_x|k_x\rangle. \tag{45}$$

This leads to an effective Hamiltonian in the y-space as

$$H' = \frac{1}{2m}\{(k_x + eBy)^2 + p_y^2\}. \tag{46}$$

This is essentially the Hamiltonian of the one-dimensional Harmonic oscillator, so that its eigenvalues and eigenfunctions are easily be written down as

$$H'|n\rangle = \left(n + \frac{1}{2}\right)|n\rangle. \tag{47}$$

with

$$|n\rangle = \frac{(a^\dagger)^n}{n!}|0\rangle. \tag{48}$$

Here,

$$a = \sqrt{\frac{1}{2eB}}\{(k_x + eBy) + ip_y\}, \tag{49}$$

$$a^\dagger = \sqrt{\frac{1}{2eB}}\{(k_x + eBy) - ip_y\} \tag{50}$$

are the ladder operators in the Landau gauge.

To sum up, the eigenenergies and eigenstates of the original Hamiltonian are expressed as

$$|k_x, \phi\rangle = |k_x, n\rangle = |k_x\rangle|n\rangle, \tag{51}$$

with

$$\langle x|k_x\rangle = \frac{1}{\sqrt{L_x}} e^{ik_x x}. \tag{52}$$

Here, use the box normalization for the plane wave in the x-plane with large but finite length L_x.

To proceed, it is convenient to write the ladder operators in the form

$$a = \sqrt{\frac{1}{2eB}}\{eB(y - Y) + ip_y\}, \tag{53}$$

$$a^\dagger = \sqrt{\frac{1}{2eB}}\{eB(y - Y) - ip_y\}, \tag{54}$$

with

$$Y \equiv -\frac{k_x}{eB}. \tag{55}$$

Here, Y has the meaning of the center of cyclotron motion projected on the y axis. Using the equations

$$p_y = \frac{1}{i}\sqrt{\frac{eB}{2}}(a - a^\dagger), \qquad y - Y = \frac{1}{\sqrt{2eB}}(a + a^\dagger), \tag{56}$$

one can easily verify the following relations:

$$\langle n|p_y|n\rangle = 0, \tag{57}$$

$$\langle n|y - Y|n\rangle = 0, \tag{58}$$

$$\langle n|(y - Y)^2|n\rangle = \frac{1}{eB}(2n + 1). \tag{59}$$

Now, we are ready to evaluate the expectation values of the two OAM operators in the state $|k_x, n\rangle$. Note first that the expectation value of the mechanical OAM can be expressed as

$$\begin{aligned}\langle L_{\text{mech}}\rangle &= \langle k_x, n|x\Pi_y - y\Pi_x|k_x, n\rangle \\ &= \langle L_{\text{can}}\rangle - \langle L_{\text{pot}}\rangle,\end{aligned} \tag{60}$$

with

$$\langle L_{\text{can}}\rangle \equiv \langle k_x, n|xp_y - yp_x|k_x, n\rangle, \tag{61}$$

$$\langle L_{\text{pot}}\rangle \equiv eB\langle k_x, n|y^2|k_x, n\rangle. \tag{62}$$

Using the relation

$$\langle n|y^2|n\rangle = Y^2 + \frac{1}{eB}(2n + 1), \tag{63}$$

we find that

$$\langle L_{\text{pot}}\rangle = \frac{k_x^2}{eB} + (2n + 1). \tag{64}$$

On the other hand, we get

$$\langle L_{\text{can}}\rangle = \langle k_x|x|k_x\rangle\langle n|p_y|n\rangle - k_x\langle n|y|n\rangle = 0 - k_x Y = \frac{k_x^2}{eB}. \tag{65}$$

Here, we have used the relation

$$\langle n|p_y|n\rangle = 0. \tag{66}$$

Note that, although $\langle k_x|x|k_x\rangle$ diverges in the limit $L_x \to \infty$, this limit can be taken after using the relation $\langle n|p_y|n\rangle = 0$, or we can keep L_x a large but finite value.

One sees that the expectation value of the canonical OAM operator in the Landau gauge does not coincide with that in the symmetric gauge. (The same is true also for the potential angular momentum operator.) On the other hand, from Eq. (60), the expectation value of the mechanical OAM operator in the Landau gauge is given by

$$\langle L_{\text{mech}}\rangle = \frac{k_x^2}{eB} - \left\{\frac{k_x^2}{eB} + (2n + 1)\right\} = -(2n + 1), \tag{67}$$

which precisely reproduces the expectation value of the mechanical OAM operator in the symmetric gauge. The expectation value of the mechanical OAM operator is therefore gauge independent as expected. Undoubtedly, the demonstration above implies an unphysical nature of the canonical OAM, in spite of the fact that the canonical momentum as well as the canonical OAM are useful objects in solving the quantum mechanical problem. (More generally speaking, the canonical momentum is a fundamental element in the canonical formalism of quantum theory.) On the other hand, the mechanical OAM is gauge invariant, and it describes the physical cyclotron motion of a charged particle in the magnetic field. This analysis within a solvable system clearly shows the superiority of the mechanical OAM over the canonical OAM as a physical OAM of a charge particle under the presence of a strong magnetic field. In our opinion, it also throws slight doubts on the physical relevance or the observability of the canonical OAM of quarks, which appears in the Jaffe-Manohar decomposition of the nucleon. In the following sections, we shall investigate this QCD problem by keeping in mind the lesson learned from the Landau problem.

III. NON-ABELIAN STOKES THEOREM AND THE TWO TYPES OF QUARK OAMS IN THE NUCLEON

An important lesson learned from the Landau problem is that, under the presence of a strong magnetic field, one must pay the finest care regarding the physical difference between the two types of OAMs, i.e. the canonical one and the mechanical one. As first recognized by Burkardt [24], the existence of the two types of quark OAMs in the nucleon is deeply connected with the existence of a strong color-electromagnetic field inside the nucleon, which is generated by the QCD dynamics of bound quarks and gluons. As we shall see below, the essence of Burkardt's observation can more transparently be understood on the basis of the non-Abelian Stokes theorem as pointed out in a recent paper by Tiwari [23].

The non-Abelian Stokes theorem is an identity for the Wilson-loop operator

$$W(C) = \text{Tr}P\exp\left(ig\oint_C dz_\mu A^\mu(z)\right), \tag{68}$$

where C is a closed path in the four-dimensional space-time, Tr stands for the trace in color space, while P does the

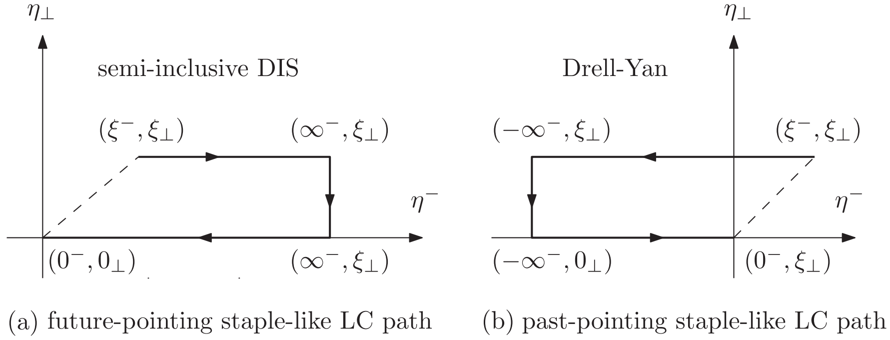

FIG. 1. Two gauge-link paths, which correspond to two DIS processes.

color-space path ordering operator. The theorem states that [30,31]

$$\mathrm{Tr}P\exp\left(ig\oint_C dz_\mu A^\mu(z)\right)=\mathrm{Tr}P\exp\left(ig\int_S d\sigma_{\mu\nu}\tilde{F}^{\mu\nu}(y)\right), \tag{69}$$

where

$$\tilde{F}^{\mu\nu}(y)=\mathcal{L}[a,y]F^{\mu\nu}(y)\mathcal{L}[y,a], \tag{70}$$

with $F^{\mu\nu}=\partial^\mu A^\nu-\partial^\nu A^\mu-ig[A^\mu,A^\nu]$ being the field-strength tensor for the non-Abelian gauge field, whereas

$$\mathcal{L}[y,x]=P\exp\left(ig\int_x^y dz_\mu A^\mu(z)\right) \tag{71}$$

is a gauge-link operator connecting the two space-time points x and y.

We apply this theorem to the average transverse momenta of quarks in the transversely polarized nucleon and also to the average longitudinal OAM of quarks in the longitudinally polarized nucleon, which were investigated by Burkardt in Ref. [24]. They are, respectively, defined by

$$\langle k_\perp^l\rangle^{\mathcal{L}}=\int dx\int d^2\boldsymbol{b}_\perp\int d^2\boldsymbol{k}_\perp k_\perp^l\rho^{\mathcal{L}}(x,\boldsymbol{b}_\perp,\boldsymbol{k}_\perp;S_\perp), \tag{72}$$

$$\langle L^3\rangle^{\mathcal{L}}=\int dx\int d^2\boldsymbol{b}_\perp\int d^2\boldsymbol{k}_\perp(\boldsymbol{b}\times\boldsymbol{k}_\perp)^3\rho^{\mathcal{L}}(x,\boldsymbol{b}_\perp,\boldsymbol{k}_\perp;S_\parallel), \tag{73}$$

where $l=1$, or 2. The Wigner distributions $\rho^{\mathcal{L}}$ appearing in the above equations are five-dimensional phase space distribution defined as

$$\begin{aligned}\rho^{\mathcal{L}}(x,\boldsymbol{b}_\perp,\boldsymbol{k}_\perp,S)=\frac{1}{2}\int\frac{d^2\Delta_\perp}{(2\pi)^2}\int\frac{d^2\boldsymbol{\xi}_\perp d\xi^-}{(2\pi)^3}\\ \times e^{-i\Delta_\perp\cdot\boldsymbol{b}_\perp}e^{i(xP^+\xi^--\boldsymbol{k}_\perp\cdot\boldsymbol{\xi}_\perp)}\\ \times\langle p',s'|\bar{\psi}(0)\gamma^+\mathcal{L}[0,\xi]\psi(\xi)|p,s\rangle,\end{aligned} \tag{74}$$

with $P=\frac{1}{2}(p'+p)$ and $p'-p=(0,\boldsymbol{\Delta}_\perp,0)$, while $S=\frac{1}{2}(s'+s)$ with s' and s denoting the polarization states of the final and initial nucleons. As is widely known, the Wigner distribution generally depends on the path of the gauge link $\mathcal{L}[0,\xi]$ connecting the two space-time points ξ and 0.

Two physically interesting choices of the gauge-link paths are the so-called future-pointing staplelike light-cone (LC) path denoted as $\mathcal{L}^{+LC}$ and the past-pointing staplelike LC path denoted as $\mathcal{L}^{-LC}$. They are, respectively, specified as (see Fig. 1)

$$\begin{aligned}\mathcal{L}^{\pm LC}[0,\xi]\equiv\mathcal{L}^{(st)}[0^-,\boldsymbol{0}_\perp;\pm\infty^-,\boldsymbol{0}_\perp]\\ \times\mathcal{L}^{(st)}[\pm\infty^-,\boldsymbol{0}_\perp;\pm\infty^-,\boldsymbol{\xi}_\perp]\\ \times\mathcal{L}^{(st)}[\pm\infty^-,\boldsymbol{\xi}_\perp;\xi^-,\boldsymbol{\xi}_\perp],\end{aligned} \tag{75}$$

where $\mathcal{L}^{(st)}[\xi,\eta]$ stands for a straight-line path directly connecting the two space-time points η and ξ. [In the following, the suffix (st) will be omitted for brevity, when there is no possibility of misunderstanding.] Remember that the above two choices of the gauge-link path correspond to the kinematics of semi-inclusive hadron productions and that of Drell-Yan processes, respectively. In fact, a future-pointing Wilson line appears in the semi-inclusive-deep-inelastic scattering (SIDIS) processes because the flow of color runs via an outgoing quark, whereas a past-pointing Wilson line appears because the flow of color runs via an incoming antiquark [32].

In addition to the above two paths, also physically important is the gauge-link path directly connecting the two space-time points ξ and 0. Although this choice of path does not directly correspond to the kinematics of the DIS processes, it is nevertheless important, since this choice in (72) and (73) is known to give manifestly gauge-invariant mechanical transverse momentum and mechanical longitudinal OAM of quarks in the nucleon [33].

Anyhow, an important fact is that, through the gauge-link path dependence of the Wigner distribution, the average transverse momentum as well as the average longitudinal OAM of quarks are generally path dependent. As pointed out by Tiwari, the reason of this path dependence can most transparently be understood on the basis of the non-Abelian Stokes theorem. Let us first consider the closed path C in the $(\eta^-, \boldsymbol{\eta}_\perp)$ plane as illustrated in Fig. 2. Because this closed gauge link is expressed as

$$\mathcal{L}^C[0,0] = \mathcal{L}^{+LC}[0,\xi]\mathcal{L}^{(st)}[\xi,0] = \mathcal{L}^{+LC}[0,\xi](\mathcal{L}^{(st)}[0,\xi])^{-1}, \tag{76}$$

we immediately obtain the following relation for the path-dependent average momenta of quarks,

$$\langle k_\perp^l \rangle^C = \langle k_\perp^l \rangle^{+LC} - \langle k_\perp^l \rangle^{\text{straight}}, \tag{77}$$

where

$$\langle k_\perp^l \rangle^C = \int dx \int d^2\boldsymbol{k}_\perp k_\perp^l \times \frac{1}{2}\int \frac{d^2\boldsymbol{\xi}_\perp d\xi^-}{(2\pi)^3} e^{i(xP^+\xi^- - \boldsymbol{k}_\perp\cdot\boldsymbol{\xi}_\perp)} \times \langle PS_\perp|\bar{\psi}(0)\gamma^+\mathcal{L}^C[0,0]\psi(0)|PS_\perp\rangle, \tag{78}$$

with

$$\mathcal{L}^C[0,0] = \mathrm{Tr}P\exp\left(ig\oint_C dz_\mu A^\mu(z)\right) \tag{79}$$

being the Wilson loop corresponding to the closed path C. By using the non-Abelian Stokes theorem, this Wilson loop can be rewritten as

$$\mathcal{L}^C[0,0] = \mathrm{Tr}P\exp\left(ig\int_S d\sigma_{\mu\nu}(\eta)\mathcal{L}[0,\eta]F^{\mu\nu}(\eta)\mathcal{L}[\eta,0]\right), \tag{80}$$

where S is an arbitrary surface with its boundary being the closed path C. The physical insight obtained from the non-Abelian Stokes theorem is simple but very important. If there is no color electromagnetic flux inside the nucleon, we would have $F^{\mu\nu}(\eta) = 0$ so that $\mathcal{L}^C[0,0] = 1$. In this case, one can easily verify that the difference between the two quantities $\langle k_\perp^l\rangle^{+LC}$ and $\langle k_\perp^l\rangle^{\text{straight}}$ vanishes identically. Conversely speaking, what generates the gauge-link path dependence of the two definitions of the average transverse momentum of quarks is the existence of the color electromagnetic field inside the nucleon.

Now, we can proceed as follows. First, we rewrite (77) with (78) in the following form:

$$\langle k_\perp^l \rangle^{+LC} - \langle k_\perp^l \rangle^{\text{straight}} = \int dx \int d^2\boldsymbol{k}_\perp \frac{1}{2}\int \frac{d^2\boldsymbol{\xi}_\perp d\xi^-}{(2\pi)^3} e^{i(xP^+\xi^- - \boldsymbol{k}_\perp\cdot\boldsymbol{\xi}_\perp)} \times \frac{1}{i}\frac{\partial}{\partial\xi_\perp^l}\langle PS_\perp|\bar{\psi}(0)\gamma^+\mathcal{L}^C[0,0]\psi(0)|PS_\perp\rangle. \tag{81}$$

Here, by using the identities

$$\int dx e^{ixP^+\xi^-} = \frac{2\pi}{P^+}\delta(\xi^-), \tag{82}$$

$$\int d^2\boldsymbol{k}_\perp e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{\xi}_\perp} = (2\pi)^2\delta^2(\boldsymbol{\xi}_\perp), \tag{83}$$

it reduces to

$$\langle k_\perp^l \rangle^{+LC} - \langle k_\perp^l \rangle^{\text{straight}} = \frac{1}{2P^+}\times\frac{1}{i}\frac{\partial}{\partial\xi_\perp^l}\langle PS_\perp|\bar{\psi}(0)\gamma^+\mathcal{L}^C[0,0]\psi(0)|PS_\perp\rangle|_{\xi^-=0,\boldsymbol{\xi}_\perp=0}, \tag{84}$$

with $\mathcal{L}^C[0,0]$ given by (80).

Since the surface S in the integral (80) can be taken arbitrarily as long as its boundary is constrained to be the closed path C, we take it as a trapezoid in the $(\eta^-, \eta_\perp^m)$ plane as illustrated in Fig. 2. Then, we have

$$\int d\sigma_{\mu\nu}(\eta)\mathcal{L}[0,\eta]F^{\mu\nu}(\eta)\mathcal{L}[\eta,0] = -\left\{\int_0^{\xi^-} d\eta^- \int_0^{(\eta^-/\xi^-)\xi_\perp^m} d\eta_\perp^m \mathcal{L}[0,\eta]F^{+m}(\eta)\mathcal{L}[\eta,0] + \int_{\xi^-}^{+\infty} d\xi^- \int_0^{\xi_\perp^m} d\eta_\perp^m \mathcal{L}[0,\eta]F^{+m}(\eta)\mathcal{L}[\eta,0]\right\}. \tag{85}$$

This gives

$$\frac{1}{i}\frac{\partial}{\partial\xi_\perp^l}\exp\left(ig\int_S d\sigma_{\mu\nu}(\eta)\mathcal{L}[0,\eta]F^{\mu\nu}(\eta)\mathcal{L}[\eta,0]\right)\Big|_{\xi^-=0,\boldsymbol{\xi}_\perp=0} = -g\delta^{lm}\left\{\int_0^{\xi^-} d\eta^- \frac{\eta^-}{\xi^-}\mathcal{L}\left[0^-,\boldsymbol{0}_\perp;\eta^-,\frac{\eta^-}{\xi^-}\xi_\perp^m\right]F^{+m}\left(\eta^-,\frac{\eta^-}{\xi^-}\xi_\perp^m\right)\mathcal{L}\left[\eta^-,\frac{\eta^-}{\xi^-}\xi_\perp^m;0^-,\boldsymbol{0}_\perp\right] + \int_{\xi^-}^{+\infty} d\eta^- \mathcal{L}[0^-,\boldsymbol{0}_\perp;\eta^-,\xi_\perp^m]F^{+m}(\eta^-,\xi_\perp^m)\mathcal{L}[\eta^-,\xi_\perp^m;0^-,\boldsymbol{0}_\perp]\right\}\Big|_{\xi^-=0,\boldsymbol{\xi}_\perp=0}. \tag{86}$$

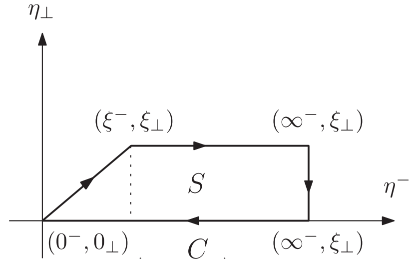

FIG. 2. The future-pointing staplelike LC path made closed to a loop.

It can be shown that the first term of the above equation vanishes, while the second term reduces to

$$-g\int_0^\infty d\eta^- \mathcal{L}[0^-,\mathbf{0}_\perp;\eta^-,\mathbf{0}_\perp]F^{+l}(\eta^-,\mathbf{0}_\perp)\mathcal{L}[\eta^-,\mathbf{0}_\perp;0^-,\mathbf{0}_\perp]$$
$$= -g\int_{-\infty}^{+\infty} d\eta^- \theta(\eta^-)\mathcal{L}[0,\eta^-]F^{+l}(\eta^-)\mathcal{L}[\eta^-,0], \tag{87}$$

[with $\theta(x)$ being the ordinary step function with the property $\theta(x)=1$ for $x>0$ and $\theta(x)=0$ for $x<0$] thereby leading to a simple relation,

$$\frac{1}{i}\frac{\partial}{\partial \xi_\perp^l}\exp\left(ig\int_S d\sigma_{\mu\nu}(\eta)\mathcal{L}[0,\eta]F^{\mu\nu}(\eta)\mathcal{L}[\eta,0]\right)\Bigg|_{\xi^-=0,\boldsymbol{\xi}_\perp=0}$$
$$= -g\int_{-\infty}^{+\infty} d\eta^- \theta(\eta^-)\mathcal{L}[0,\eta^-]F^{+l}(\eta^-)\mathcal{L}[\eta^-,0]. \tag{88}$$

Here, we recall the fact that the average transverse momentum corresponding to the straight-line path directly connecting ξ and 0 reduces to the mechanical transverse momentum [33]

$$\langle k_\perp^l\rangle^{\mathrm{straight}} = \langle k_\perp^l\rangle_{\mathrm{mech}}, \tag{89}$$

where

$$\langle k_\perp^l\rangle_{\mathrm{mech}} = \frac{1}{2P^+}\langle PS_\perp|\bar\psi(0)\gamma^+ D_\perp^l(0)\psi(0)|PS_\perp\rangle, \tag{90}$$

with $D_\perp^l = \partial^l - igA_\perp^l$ being the usual covariant derivative. In this way, we eventually arrive at a key relation,

$$\langle k_\perp^l\rangle^{+LC} - \langle k_\perp^l\rangle_{\mathrm{mech}}$$
$$= \frac{1}{2P^+}\langle PS_\perp|\bar\psi(0)\gamma^+\int_{-\infty}^{+\infty} d\eta^-(-\theta(\eta^-))$$
$$\times \mathcal{L}[0,\eta^-]gF^{+l}(\eta^-)\mathcal{L}[\eta^-,0]\psi(0)|PS_\perp\rangle. \tag{91}$$

According to Burkardt [24], the rhs of the above equation has a meaning of final-state interaction (FSI) in the SIDIS processes. In more detail, it represents the change of transverse momentum of the ejected quark due to the color Lorentz force caused by the residual target. In fact, in the LC gauge, the gauge link along the light-cone direction becomes unity, and the relevant component of the field-strength tensor reduces to

$$-\sqrt{2}gF^{+2} = -gF^{02} - gF^{32} = g[\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B}]^2, \tag{92}$$

which is nothing but the y-component of the color Lorentz force acting on a particle that moves with the light velocity $\boldsymbol{v} = (0,0,-1)$ in the $-z$ direction [24].

Repeating the same manipulation for the closed gauge link

$$\mathcal{L}^{C'}[0,0] = \mathcal{L}^{-LC}[0,\xi]\mathcal{L}^{(st)}[\xi,0]$$
$$= \mathcal{L}^{-LC}[0,\xi](\mathcal{L}^{(st)}[0,\xi])^{-1}, \tag{93}$$

containing the past-pointing staplelike LC path $\mathcal{L}^{-LC}$, we get an analogous relation for $\langle k_\perp^l\rangle^{-LC}$. Putting the two cases together, the answers can be summarized as

$$\langle k_\perp^l\rangle^{\pm LC} = \langle k_\perp^l\rangle_{\mathrm{mech}} + \langle k_\perp^l\rangle_{\mathrm{int}}^{\pm LC}, \tag{94}$$

with

$$\langle k_\perp^l\rangle_{\mathrm{int}}^{\pm LC} = \frac{1}{2P^+}\int_{-\infty}^{+\infty}(\mp\,\theta(\pm\eta^-))$$
$$\times \langle PS_\perp|\bar\psi(0)\gamma^+\mathcal{L}[0,\eta^-]gF^{+l}(\eta^-)$$
$$\times \mathcal{L}[\eta^-,0]\psi(0)|PS_\perp\rangle. \tag{95}$$

Here, $\langle k_\perp^l\rangle_{\mathrm{int}}^{+LC}$ represents the FSI in the SIDIS processes, while $\langle k_\perp^l\rangle_{\mathrm{int}}^{-LC}$ does the initial-state interaction (ISI) in the Drell-Yan processes. Note that Eq. (93) with Eq. (94) precisely reproduces the relations derived by Burkardt with a different method. We point out that essentially the same relations were also obtained by Boer *et al.*, although in a somewhat different form [34]. (See also Refs. [35,36].) To verify it, we recall mathematical identities

$$\int_{-\infty}^{+\infty} dx \frac{i}{x\mp i\epsilon}e^{i\lambda x} = \mp\, 2\pi\theta(\pm\lambda). \tag{96}$$

Using them, the above FSI or ISI term can also be expressed in the form

$$\langle k_\perp^l\rangle_{\mathrm{int}}^{\pm LC} = \frac{1}{2P^+}\int\frac{dx}{2\pi}\int_{-\infty}^{+\infty} d\eta^- \frac{i}{x\mp i\epsilon}e^{ixP^+\eta^-}$$
$$\times \langle PS_\perp|\bar\psi(0)\gamma^+\mathcal{L}[0,\eta^-]gF^{+l}(\eta^-)$$
$$\times \mathcal{L}[\eta^-,0]\psi(0)|PS_\perp\rangle, \tag{97}$$

which corresponds to the second term of rhs of Eq. (4) in Ref. [34].

A similar analysis can also be carried out for the average longitudinal OAM of quarks in the longitudinally polarized nucleon. The answer is given as

$$\langle L^3\rangle^{\pm LC} = \langle L^3\rangle_{\mathrm{mech}} + \langle L^3\rangle_{\mathrm{int}}^{\pm LC}, \tag{98}$$

where

$$\langle L^3\rangle_{\text{mech}} = \mathcal{N}\int d^2\boldsymbol{b}_\perp \epsilon_\perp^{ij} b_\perp^i \langle PS_\parallel|\bar{\psi}(0^-,\boldsymbol{b}_\perp)\gamma^+ \frac{1}{i} D_\perp^j(0^-,\boldsymbol{b}_\perp)\psi(0^-,\boldsymbol{b}_\perp)|PS_\parallel\rangle, \tag{99}$$

with $\mathcal{N} = 1/(2P^+ \int d^2\boldsymbol{b}_\perp)$ is the manifestly gauge-invariant mechanical OAM, while

$$\begin{aligned}\langle L^3\rangle_{\text{int}}^{\pm LC} = \mathcal{N}\int d^2\boldsymbol{b}_\perp \epsilon_\perp^{ij} b_\perp^i \int_{-\infty}^{+\infty} d\eta^-(\mp\,\theta(\pm\eta^-))\\ \times \langle PS_\parallel|\bar{\psi}(0^-,\boldsymbol{b}_\perp)\gamma^+\mathcal{L}[0^-,\boldsymbol{b}_\perp;\eta^-,\boldsymbol{b}_\perp]gF^{+j}(\eta^-,\boldsymbol{b}_\perp)\mathcal{L}[\eta^-,\boldsymbol{b}_\perp,0^-;\boldsymbol{b}_\perp]\psi(0^-,\boldsymbol{b}_\perp)|PS_\parallel\rangle\end{aligned} \tag{100}$$

is the FSI or ISI term. [Here, $\epsilon_\perp^{ij}(i,j=1,2)$ is the antisymmetric tensor in the transverse plane with the convention $\epsilon_\perp^{12} = +1$.] Again, this precisely reproduces the relation derived by Burkardt [24]. Alternatively, by using the identities (96), the FSI or ISI term can also be expressed in the following form:

$$\begin{aligned}\langle L^3\rangle_{\text{int}}^{\pm LC} = \mathcal{N}\int \frac{dx}{2\pi}\int d^2\boldsymbol{b}_\perp \epsilon_\perp^{ij} b_\perp^i \int_{-\infty}^{+\infty} d\eta^- \frac{i}{x\pm i\epsilon} e^{ixP^+\eta^-}\\ \times \langle PS_\parallel|\bar{\psi}(0^-,\boldsymbol{b}_\perp)\gamma^+\mathcal{L}[0^-,\boldsymbol{b}_\perp;\eta^-,\boldsymbol{b}_\perp]gF^{+j}(\eta^-,\boldsymbol{b}_\perp)\mathcal{L}[\eta^-,\boldsymbol{b}_\perp;0^-,\boldsymbol{b}_\perp]\psi(0^-,\boldsymbol{b}_\perp)|PS_\parallel\rangle.\end{aligned} \tag{101}$$

The physical interpretation of the above relations are essentially the same as the average transverse momentum case. The term $\langle L^3\rangle_{\text{int}}^{+LC}$ represents the FSI in the SIDIS processes, while $\langle L^3\rangle_{\text{int}}^{-LC}$ represents the ISI in the Drell-Yan processes. The only change from the previous case is that the role of color Lorentz force is now replaced by the torque of it given by

$$T^z = g[\boldsymbol{b}_\perp \times (\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B})]^3. \tag{102}$$

IV. ON THE IDEA OF PHYSICAL COMPONENT OF THE GAUGE FIELD

It is important to recognize the fact that the theoretical formulation so far is absolutely independent of the issue of a proper definition of the physical component of the gauge field, which brought about a lot of controversies in the nucleon spin decomposition problem. Note that each term on the rhs of the relations (94) and (98) has clear and unambiguous physical meaning. Namely, the first terms of (94) and (98) represent the manifestly gauge-invariant mechanical momentum and the mechanical OAM, respectively, whereas the second terms in the same equations stand for the FSI in the SIDIS processes or the ISI in the Drell-Yan processes. Unfortunately, there is some delicacy in the interpretation of the lhs. In particular, if one wants to relate Eq. (98) to the problem of gauge-invariant complete decomposition of the nucleon spin, one cannot stay out of the idea of the physical component of the gauge field A^μ_{phys}. According to Hatta [37], the original proposal for A^μ_{phys} by Chen *et al.* [4,5] based on the non-Abelian generalization of the transverse component of the photon field is not acceptable, because it does not correspond to observable decomposition of the nucleon spin probed by DIS measurements. Instead, he proposed three candidates for the proper definition of A^μ_{phys}, given as

$$\begin{aligned}A^j_{\text{phys}}(0) &= \int_{-\infty}^{+\infty} d\eta^-(-\theta(+\eta^-))\mathcal{L}[0,\eta^-]F^{+j}(\eta^-)\mathcal{L}[\eta^-,0]\\ &= \int \frac{dx}{2\pi}\int_{-\infty}^{+\infty} d\eta^- \frac{i}{x-i\epsilon} e^{ixP^+\eta^-}\\ &\quad\times \mathcal{L}[0,\eta^-]F^{+j}(\eta^-)\mathcal{L}[\eta^-,0],\end{aligned} \tag{103}$$

which will be called the postform here, or as

$$\begin{aligned}A^j_{\text{phys}}(0) &= \int_{-\infty}^{+\infty} d\eta^-(+\theta(-\eta^-))\mathcal{L}[0,\eta^-]F^{+j}(\eta^-)\mathcal{L}[\eta^-,0]\\ &= \int \frac{dx}{2\pi}\int_{-\infty}^{+\infty} d\eta^- \frac{i}{x+i\epsilon} e^{ixP^+\eta^-}\\ &\quad\times \mathcal{L}[0,\eta^-]F^{+j}(\eta^-)\mathcal{L}[\eta^-,0],\end{aligned} \tag{104}$$

called the prior form, or

$$\begin{aligned}A^j_{\text{phys}}(0) &= -\frac{1}{2}\int_{-\infty}^{+\infty} d\eta^-\epsilon(\eta^-)\mathcal{L}[0,\eta^-]F^{+j}(\eta^-)\mathcal{L}[\eta^-,0]\\ &= \int \frac{dx}{2\pi}\int_{-\infty}^{+\infty} d\eta^- P\frac{i}{x} e^{ixP^+\eta^-}\\ &\quad\times \mathcal{L}[0,\eta^-]F^{+j}(\eta^-)\mathcal{L}[\eta^-,0],\end{aligned} \tag{105}$$

called the principle-value form. As pointed out by Hatta, for any of the above three choices, the parity and time-reversal (PT) symmetries ensures that the FSI and ISI terms in (98) precisely coincide and reduce to the following form [37],

$$\langle L^3\rangle_{\rm int}^{+LC} = \langle L^3\rangle_{\rm int}^{-LC} = \mathcal{N}\int d^2\boldsymbol{b}_\perp \epsilon_\perp^{ij} b_\perp^i \langle PS_\parallel|\bar{\psi}(\boldsymbol{b}_\perp)\gamma^+ gA^j_{\rm phys}(\boldsymbol{b}_\perp)\psi(\boldsymbol{b}_\perp)|PS_\parallel\rangle, \tag{106}$$

which can be identified with the so-called potential angular momentum term $\langle L^3\rangle_{\rm pot}$ according to the terminology in Refs. [16,17]. Inserting it into (98), we therefore get the relation

$$\langle L^3\rangle^{\pm LC} = \langle L^3\rangle_{\rm mech} + \langle L^3\rangle_{\rm pot}. \tag{107}$$

Here, the sum of the mechanical OAM and the potential OAM reduces to

$$\langle L^3\rangle_{\text{“can”}} = \mathcal{N}\int d^2\boldsymbol{b}_\perp \epsilon_\perp^{ij} b_\perp^i \langle PS_\parallel|\bar{\psi}(0^-,\boldsymbol{b}_\perp) \times\gamma^+\frac{1}{i}D^j_{\rm pure,\perp}(0^-,\boldsymbol{b}_\perp)\psi(0^-,\boldsymbol{b}_\perp)|PS_\parallel\rangle, \tag{108}$$

with the definition of the so-called pure-gauge-covariant derivative as

$$D^j_{\rm pure,\perp}(0^-,\boldsymbol{b}_\perp) = \partial^j_\perp - gA^j_{\rm pure,\perp}(0^-,\boldsymbol{b}_\perp). \tag{109}$$

Equation (108) is nothing but the gauge-invariant canonical OAM. In this way, the average longitudinal OAM defined through the Wigner distribution with the future-pointing LC path as well as with the past-pointing LC path just coincide, and both reduce to the gauge-invariant canonical OAM,

$$\langle L^3\rangle^{+LC} = \langle L^3\rangle^{-LC} = \langle L^3\rangle_{\text{“can”}}, \tag{110}$$

which is physically equivalent to the canonical OAM appearing in the Jaffe-Manohar decomposition of the nucleon spin [14,15].

As emphasized in our previous paper [6], however, the situation is considerably different for the case of average transverse momentum of quarks in the transversally polarized nucleon. In fact, if we adopt the postform definition (103) of $A^l_{\rm phys}$, the average transverse momentum corresponding to the SIDIS processes reduces to

$$\langle k^l_\perp\rangle^{+LC} = \langle k^l_\perp\rangle_{\rm mech} + \frac{1}{2P^+}\langle PS_\perp|\bar{\psi}(0)\gamma^+ gA^l_{\rm phys}(0)\psi(0)|PS_\perp\rangle, \tag{111}$$

which formally takes the form of gauge-invariant canonical momentum. On the other hand, if we use the prior-form definition (104) of $A^l_{\rm phys}$, the average transverse momentum corresponding to the Drell-Yan processes becomes

$$\langle k^l_\perp\rangle^{-LC} = \langle k^l_\perp\rangle_{\rm mech} + \frac{1}{2P^+}\langle PS_\perp|\bar{\psi}(0)\gamma^+ gA^l_{\rm phys}(0)\psi(0)|PS_\perp\rangle, \tag{112}$$

which also takes the form of gauge-invariant canonical momentum. However, we already know the fact that the average transverse momentum corresponding to the SIDIS processes and that corresponding to the Drell-Yan processes have opposite signs [38],

$$\langle k^l_\perp\rangle^{-LC} = -\langle k^l_\perp\rangle^{+LC}. \tag{113}$$

This means that, at least for the average transverse momentum case, neither the postform definition nor the prior-form definition of $A^l_{\rm phys}$ is acceptable as a concept with universal or process-independent meaning.

An important lesson learned from the above consideration is that, while it is certainly true that the gauge-link structure of the average transverse momentum as well as the average longitudinal OAM is determined by the kinematics of DIS processes, the definition of the physical component $A^l_{\rm phys}$ still has some sort of arbitrariness. As pointed out in Ref. [6], the most natural choice of $A^l_{\rm phys}$, which holds universally in both the average transverse momentum case and the average longitudinal OAM case, would be to use the principle-value prescription for $A^l_{\rm phys}$ given by (105). In fact, the principle-value prescription for avoiding $1/x$-type singularity of the parton distributions is nothing uncommon [39]. It is widely used in other situations, too. Especially relevant to our present problem is the definition of the longitudinally polarized gluon distribution.

Let us start here with the popular definition of the longitudinally polarized gluon distribution given in the paper by Manohar [40,41] (see also Refs. [39,42])

$$x\Delta g(x) = \frac{i}{4P^+}\int\frac{d\xi^-}{2\pi}e^{ixP^+\xi^-} \times\{\langle PS_\parallel|\tilde{F}^{+,a}_\lambda(0)\mathcal{L}^b_a[0,\xi^-]F^{+\lambda}_b(\xi^-)|PS_\parallel\rangle - \langle PS_\parallel|\tilde{F}^{+,a}_\lambda(\xi^-)\mathcal{L}^b_a[\xi^-,0]F^{+\lambda}_b(0)|PS_\parallel\rangle\}, \tag{114}$$

where $\mathcal{L}^b_a[0,\xi^-]$ represents the gauge link in the adjoint representation. Using the gauge link in the fundamental representation, the same quantity can also be expressed as

$$x\Delta g(x) = \frac{i}{4P^+}\int\frac{d\xi^-}{2\pi}e^{ixP^+\xi^-} \times\{\langle PS_\parallel|2{\rm Tr}(\tilde{F}^+_\lambda(0)\mathcal{L}[0,\xi^-]F^{+\lambda}(\xi^-)\mathcal{L}[\xi^-,0])|PS_\parallel\rangle - \langle PS_\parallel|2{\rm Tr}(\tilde{F}^+_\lambda(\xi^-)\mathcal{L}[\xi^-,0]F^{+\lambda}(0)\mathcal{L}[0,\xi^-])|PS_\parallel\rangle\}. \tag{115}$$

Rewriting the second term by utilizing the translational invariance together with the variable change $\xi^- \to -\xi^-$, one can rewrite the above equation as

$$x\Delta g(x) = \frac{i}{4P^+}\int \frac{d\xi^-}{2\pi}(e^{ixP^+\xi^-} - e^{-ixP^+\xi^-}) \times \langle PS_\parallel | 2\mathrm{Tr}(\tilde{F}^+_\lambda(0)\mathcal{L}[0,\xi^-]F^{+\lambda}(\xi^-)\mathcal{L}[\xi^-,0])|PS_\parallel\rangle. \tag{116}$$

Since the above expression shows that the distribution $\Delta g(x)$ has $1/x$-type singularity, there is a danger that the first moment of $\Delta g(x)$ depends on how to avoid this singularity. Fortunately, we do not need to worry about it [13]. As is clear from the consideration of the average transverse momentum as well as the average longitudinal OAM, physics-motivated choices would be given by the replacements

$$\frac{1}{x} \to \frac{1}{x \mp i\epsilon}, \tag{117}$$

which correspond to the post- and prior-form prescriptions, respectively, relevant for the DIS processes and the Drell-Yan processes. However, because of the identity

$$\begin{aligned} \frac{1}{x \mp i\epsilon}&(e^{ixP^+\xi^-} - e^{-xP^+\xi^-}) \\ &= P\frac{1}{x}(e^{ixP^+\xi^-} - e^{-xP^+\xi^-}) \pm i\pi\delta(x)(e^{ixP^+\xi^-} - e^{-xP^+\xi^-}) \\ &= P\frac{1}{x}(e^{ixP^+\xi^-} - e^{-xP^+\xi^-}), \end{aligned} \tag{118}$$

only the principle-value parts survive in both cases. We thus obtain

$$\Delta g(x) = \frac{1}{4P^+}\int \frac{d\xi^-}{2\pi} P\frac{i}{x}(e^{ixP^+\xi^-} - e^{-xP^+\xi^-}) \times \langle PS_\parallel | 2\mathrm{Tr}(\tilde{F}^+_\lambda(0)\mathcal{L}[0,\xi^-]F^{+\lambda}[\xi^-,0])|PS_\parallel\rangle. \tag{119}$$

This ensures that the longitudinally polarized gluon distribution measured in the DIS processes and that in the Drell-Yan processes are just the same [36]. Clearly, this is related to the PT-even nature of the longitudinally polarized gluon distribution defined by (114). Now, by using the identity

$$\int_{-\infty}^{+\infty} dx P\frac{i}{x}(e^{ixP^+\xi^-} - e^{-xP^+\xi^-}) = -2\pi\epsilon(\xi^-), \tag{120}$$

the first moment of $\Delta g(x)$ can be written as

$$\int \Delta g(x)dx = -\frac{1}{4P^+}\int_{-\infty}^{+\infty} \epsilon(\xi^-)\langle PS_\parallel | 2\mathrm{Tr}(\tilde{F}^+_\lambda(0) \times \mathcal{L}[0,\xi^-]F^{+\lambda}[\xi^-,0])|PS_\parallel\rangle. \tag{121}$$

We recall that this is just the form given in the paper [43] by Jaffe. Now, if we introduce the physical component of the gluon field by the equation,

$$\begin{aligned} A^\lambda_{\mathrm{phys}}(0) &= -\frac{1}{2}\int_{-\infty}^{+\infty} d\xi^-\epsilon(\xi^-)\mathcal{L}[0,\xi^-]F^{+\lambda}(\xi^-)\mathcal{L}[\xi^-,0] \\ &= \int\frac{dx}{2\pi}\int_{-\infty}^{+\infty} P\frac{i}{x}e^{ixP^+\xi^-}\mathcal{L}[0,\xi^-]F^{+\lambda}\mathcal{L}[\xi^-,0], \end{aligned} \tag{122}$$

the first moment of $\Delta g(x)$ just reduces to the familiar form

$$\int \Delta g(x)dx = \frac{1}{2P^+}\langle PS_\parallel | 2\mathrm{Tr}(\tilde{F}^+_\lambda(0)A^\lambda_{\mathrm{phys}}(0))|PS_\parallel\rangle. \tag{123}$$

In any case, we confirm that, once we define the physical component of the gluon field by Eq. (105), a gauge-invariant complete decomposition of the nucleon spin including the gluon intrinsic spin term is possible. A delicate question is whether it is a gauge-invariant decomposition in a standard sense. From a formal standpoint, the rhs of the definition (105) for the physical component of the gluon looks completely gauge invariant, since it contains only the field-strength tensor. Furthermore, although this definition is motivated by the LC gauge, it does not prevent us from working in other gauges including the covariant gauges like the Feynman gauge. However, we also know that this definition of the physical component is path dependent and there are many indications that the path dependence after all means gauge dependence [44–47]. Lorcé argued that the above definition of the physical component is gauge invariant but it is not invariant under what he called the Stückelberg transformation [48–50]. According to him, if some quantity is gauge invariant but Stückelberg variant, such a quantity is said to have only weak gauge invariance. The gauge-invariant canonical quark OAM is typical of such quantities. On the other hand, the mechanical quark OAM is Stückelberg invariant as well as gauge invariant. Such a quantity is said to have strong gauge invariance. Admitting the existence of two forms of gauge symmetry, an immediate question is the relation with the gauge principle of physics, especially the relation between the observability and the two types of gauge symmetry. Lorcé argued that a strong form of gauge symmetry is a sufficient condition of observability but it is not a necessary condition. The weak form of gauge invariance is enough for observability. Based on these considerations, he proposed to classify measurable quantities into two categories as follows [48]:

(i) *Observables*, which are gauge-invariant quantities in a strong sense;
(ii) *Quasiobservables*, which are gauge-invariant quantities in a weak sense.

When he refers to quasiobservables, what is in his mind are principally the parton distribution functions (PDFs). To provide a supplementary explanation, we first recall that the nucleon structure functions are genuine observables, because they appear directly in the cross section formulas of DIS reactions. On the other hand, the PDFs are not, since they are theoretical concepts, which generally depend on the factorization scheme within the framework of the perturbative QCD. Despite this theoretical-scheme dependence, the PDFs are approximately (i.e. at the leading order of twist expansion) equal to the corresponding structure functions. In this sense, the PDFs are sometimes called quasiobservables.

One might think that the above classification of an observable is roughly to the point. However, there remains some question for admitting it as a general rule. In fact, according to Lorcé, the standard transverse-longitudinal decomposition (or the Helmholtz decomposition) of the photon field is also gauge invariant but Stückelberg variant. The transverse-longitudinal decomposition is therefore gauge invariant only in a weak sense. As we have repeatedly emphasized, the reason why the transverse-longitudinal decomposition has only weak gauge invariance can be explained by using a more familiar concept of physics. As far as we are working in a fixed Lorentz frame of reference, there is no doubt that the transverse (or physical) component of the photon is gauge invariant [7,8,51]. Still, this invariance cannot be a strong gauge invariance, because the concept of transversality is necessarily Lorentz-frame dependent. Importantly, however, the measurements of the spin and OAM of the photon are carried out in a prescribed Lorentz frame by making use of interactions with atoms. Thus, even though the spin and OAM decomposition of the photon are only weakly gauge invariant, several concrete experiments carried out in the past definitely show that they are genuine observables, not quasiobservables [52,53]. In our opinion, what ensures the observability of a given quantity is whether there is an external current or a probe that couples to the quantity in question. A typical example is electroweak current, which can be used to probe the internal electroweak structure of hadrons. In the photon spin and OAM measurements, interactions with atoms play the role of external probes. Turning back to the general rule of Lorcé, the canonical OAMs of quarks as well as the gluon spin are quasiobservables, not because they are weakly gauge-invariant quantities. This is obvious from the fact that even the manifestly gauge-invariant OAM of quarks, which is related to the generalized parton distributions (GPDs), is also a quasiobservable. The quasiobservability is rather related to the fact that we are dealing with the bound state, not free photons, and that, for extracting the information on the internal quark-gluon structure of the nucleon, we need a special theoretical framework of perturbative QCD.

To sum up, we agree that the observability does not necessarily require strong gauge-invariance. The weak gauge invariance is enough for observability. Still, it would not be so easy to make a simple and clear-cut statement on the relation between the observability and the weak gauge invariance. Only a statement we can make at the present moment would be the following. To the best of our belief, it highly improbable that some general principle like the Noether theorem is able to give an unambiguous answer to our intricate question whether a quantity with a weak gauge invariance only is observable or not. After all, for obtaining a definite answer, we cannot avoid to discuss a concrete measuring method of the quantity in question. Anyhow, with the understanding gained from the above general consideration in mind, we compare the following four decompositions of the nucleon spin. They are the Ji decomposition (I) [54,55]

$$\frac{1}{2} = J^q + J^G; \tag{124}$$

the Ji decomposition (II) [54]

$$\frac{1}{2} = L^q_{\rm mech} + \frac{1}{2}\Delta\Sigma + J^G; \tag{125}$$

the mechanical decomposition proposed in Refs. [16,17],

$$\frac{1}{2} = L^q_{\rm mech} + \frac{1}{2}\Delta\Sigma + L^G_{\rm mech} + \Delta G; \tag{126}$$

and the canonical decomposition, which is equivalent to the Jaffe-Manohar decomposition [14,15],

$$\frac{1}{2} = L^q_{\text{“can”}} + \frac{1}{2}\Delta\Sigma + L^G_{\text{“can”}} + \Delta G, \tag{127}$$

where the quark and gluon OAMs in the decomposition (126) and the canonical decomposition (127) are related by the following equations:

$$L^q_{\text{“can”}} = L^q_{\rm mech} + L_{\rm pot}, \tag{128}$$

$$L^G_{\text{“can”}} = L^G_{\rm mech} - L_{\rm pot}. \tag{129}$$

Among these four decomposition, manifestly gauge-invariant decompositions are the first two. The last two decompositions, which provide us with complete decompositions of the nucleon spin, requires the concept of the physical component of the gluon field.

As is widely known, the total angular momenta of quarks and gluons can be related to the second moments of the GPDs $H^{q/G}(x,\xi,t)$ and $E^{q/G}(x,\xi,t)$, or equivalently the forward limits of the so-called generalized (or gravitational) form factors $A^{q/G}(t)$ and $B^{q/G}(t)$ as [54,55]

$$J^q = \frac{1}{2}\int dx x[H^q(x,0,0) + E^q(x,0,0)] = \frac{1}{2}[A^q(0) + B^q(0)], \quad (130)$$

$$J_G = \frac{1}{2}\int dx x[H^G(x,0,0) + E^G(x,0,0)] = \frac{1}{2}[A^G(0) + B^G(0)]. \quad (131)$$

Although the GPDs $H^{q/G}(x,\xi,t)$ and $E^{q/G}(x,\xi,t)$ are quasiobservables just like the PDFs, the gravitational form factors $A^{q/G}(t)$ and $B^{q/G}(t)$ can in principle be extracted frame independently by carrying out gedanken graviton-nucleon scattering experiments, so that J^q and J^G may be thought of as genuine observables as emphasized in Ref. [56]. (Of course, the graviton-nucleon scattering measurement is practically impossible.) Turning to the Ji decomposition (II), the quark spin term $\Delta\Sigma$ is usually believed to be observable. To be more strict, it is a quasiobservable, since it is just the first moment of the longitudinally polarized distribution function of quarks. In fact, the definition of $\Delta\Sigma$ is known to be factorization-scheme dependent. The two popular choices of factorization schemes are the standard $\overline{\text{MS}}$ scheme and the so-called Adler-Bardeen (AB) scheme [57]. However, the current understanding is that there is no compelling reason to choose the AB scheme, which breaks gauge invariance at the cost of chiral symmetry. Once the $\overline{\text{MS}}$ scheme is chosen, $\Delta\Sigma$ can be identified with the forward limit of the flavor-singlet axial form factor of the nucleon, which will be extracted in the near-future measurements of the neutrino-nucleon scatterings [58]. We may then be able to say that the Ji decomposition (II) is also an observable decomposition.

As repeatedly emphasized, the last two gauge-invariant complete decompositions of the nucleon spin require the idea of the physical component of the gluon field. Still, we emphasize that there is a big difference between these two decompositions from the observational point of view. The gluon spin term ΔG in the mechanical decomposition is certainly a quasiobservable. (Note that the same ΔG appears also in the canonical decomposition.) There is no form factor measurement, which can be used to extract ΔG. Nevertheless, within the theoretical formulation of DIS scatterings, ΔG and $\Delta\Sigma$ appear on equal footing [59]. Although the extraction of ΔG is far more difficult than that of $\Delta\Sigma$, great progress is under way, and there is no doubt that it will be determined more precisely in the near future [60]. Once $\Delta\Sigma$ and ΔG are known, the quark and gluon OAM terms in the mechanical decomposition can be extracted from the relations

$$L^q_{\text{mech}} = J^q - \frac{1}{2}\Delta\Sigma, \quad (132)$$

$$L^G_{\text{mech}} = J^G - \Delta G. \quad (133)$$

Even a direct extraction of L_q might be possible through the known relation

$$L^q_{\text{mech}} = -\int dx x G_2(x,0,0), \quad (134)$$

where G_2 is one of the twist-3 GPDs [61–64]. We would thus conclude that the mechanical decomposition is an experimentally accessible decomposition of the nucleon spin.

Let us now turn to the last decomposition, i.e. the canonical decomposition or the Jaffe-Manohar decomposition. We emphasize that the quark and gluon OAMs in this decomposition cannot be extracted from the knowledge of $\Delta\Sigma$ and ΔG supplemented with that of J^q and J^G, since [16,17]

$$L^q_{\text{“can”}} \neq J^q - \frac{1}{2}\Delta\Sigma, \quad (135)$$

$$L^G_{\text{“can”}} \neq J^G - \Delta G. \quad (136)$$

Some years ago, Lorce and Pasquini pointed out that the canonical quark OAM L'_q appearing in the Jaffe-Manohar decomposition can be related to a moment of a Wigner distribution F_{14} as [65] (see also Ref. [37])

$$L^q_{\text{“can”}} = -\int dx \int d^2k_\perp \frac{\boldsymbol{k}_\perp^2}{M_N^2} F_{14} \times (x, \xi = 0, \boldsymbol{k}_\perp^2, \boldsymbol{k}_\perp \cdot \boldsymbol{\Delta}_\perp = 0, \boldsymbol{\Delta}_\perp^2 = 0). \quad (137)$$

Soon after, however, Courtoy *et al.* pointed out that this Wigner function F_{14} disappears in both the GPD and transverse-momentum-dependent distribution (TMD) factorization schemes [66]. Since the appearance in the factorization scheme or in the cross section formula is a necessary condition of observability or quasiobservability, we must say that F_{14} is not even a quasiobservable, at least within our limited knowledge of DIS measurements. One might suspect that the fact that the canonical OAM is not observable would be connected with the fact that it is not gauge-invariant in a strong sense. However, the gauge-invariant definition of the gluon spin ΔG also needs the idea of the physical component, while it appears in the cross section formula within the standard collinear factorization scheme. The underlying reason of this difference between the canonical quark OAM and the gluon spin is still unexplained.

V. PHYSICAL INTERPRETATION OF THE TWO OAMS OF QUARKS

In the previous section, we have demonstrated that the principle-value prescription (105) would be the most natural choice for defining the physical component of the gluon field. Once we accept this choice, the FSI and the ISI terms of the average transverse momenta can be written as

$$\langle k_\perp^l \rangle_{\rm int}^{\pm LC} = \langle k_\perp^l \rangle_{\rm pot} + \langle k_\perp^l \rangle_{\rm gluon-pole}, \tag{138}$$

where

$$\langle k_\perp^l \rangle_{\rm pot} = \frac{1}{2P^+} \langle PS_\perp | \bar{\psi}(0) \gamma^+ g A^l_{\rm phys}(0) \psi(0) | PS_\perp \rangle \tag{139}$$

corresponds to the potential momentum, while

$$\langle k_\perp^l \rangle_{\rm gluon-pole} = \mp \frac{1}{4P^+} \int_{-\infty}^{+\infty} d\eta^- \langle PS_\perp | \bar{\psi}(0) \gamma^+ \times \mathcal{L}[0,\eta^-] g F^{+l}(\eta^-) \mathcal{L}[\eta^-, 0] \psi(0) | PS_\perp \rangle, \tag{140}$$

is the so-called gluon-pole term of the Efremov-Teryaev-Qui-Stermann (ETQS) quark-gluon correlation function $\Psi_F(x, x')$ [67–69], i.e.

$$\langle k_\perp^l \rangle_{\rm gluon-pole} = \frac{1}{2} \epsilon_\perp^{ij} S_\perp^j (\mp \pi) \int dx \Psi_F(x,x). \tag{141}$$

(We recall that $\langle k_\perp^l \rangle_{\rm int}^{\pm LC}$ can also be related to a moment of the T-odd TMD called the Sivers function [70,71].) Since it holds that

$$\langle k_\perp^l \rangle_{\rm mech} + \langle k_\perp^l \rangle_{\rm pot} = \langle k_\perp^l \rangle_{\text{“can”}}, \tag{142}$$

the average transverse momenta $\langle k_\perp^l \rangle^{\pm LC}$ in (94) can be expressed in either of the following two forms:

$$\langle k_\perp^l \rangle^{\pm LC} = \langle k_\perp^l \rangle_{\rm mech} + \langle k_\perp^l \rangle_{\rm int}^{\pm LC} \tag{143}$$

or

$$\langle k_\perp^l \rangle^{\pm LC} = \langle k_\perp^l \rangle_{\rm can} + \langle k_\perp^l \rangle_{\rm gluon-pole}^{\pm LC}. \tag{144}$$

There is no inconsistency between these two expressions, since the PT symmetry dictates that

$$\langle k_\perp^l \rangle_{\rm mech} = \langle k_\perp^l \rangle_{\rm can} = 0 \tag{145}$$

and that

$$\langle k_\perp^l \rangle_{\rm int}^{\pm LC} = \langle k_\perp^l \rangle_{\rm gluon-pole}. \tag{146}$$

We stress that $\langle k_\perp^l \rangle^{\pm LC}$ coincide with *neither* the canonical momentum nor the mechanical one. Since this is the case, one may conclude that the idea of a physical component (or the concept of canonical momentum) plays no practically useful role in the case of average transverse momentum.

It is therefore convenient to return to the original gauge-invariant relation (94), which is independent of the idea of the physical component of the gauge field. For clarity, we consider below the case of the SIDIS processes. The relation in this case is written as

$$\langle k_\perp^l \rangle^{+LC} = \langle k_\perp^l \rangle_{\rm mech} + \langle k_\perp^l \rangle_{\rm int}^{+LC}, \tag{147}$$

with the additional information that $\langle k_\perp^l \rangle_{\rm mech} = 0$. The physical interpretation of this relation should be obvious by now. Initially, the average transverse momentum of quarks *inside* the nucleon is given by $\langle k_\perp^l \rangle_{\rm mech}$, which is actually zero due to the PT symmetry. Through the FSI $\langle k_\perp^l \rangle_{\rm int}^{+LC}$ in the SIDIS processes, the quark ejected by the virtual photon acquires nonzero transverse momentum. The lhs of the relation (147) can therefore be interpreted as the transverse momentum of the quark at the asymptotic distance, or that well outside the nucleon.

Exactly the same interpretation must hold also for the average longitudinal OAM. For clarity, we again confine to the case of the future-pointing staplelike LC path $\mathcal{L} = +LC$ corresponding to the SIDIS processes. In this case, we have the relation

$$\langle L^3 \rangle^{+LC} = \langle L^3 \rangle_{\rm mech} + \langle L^3 \rangle_{\rm int}^{+LC}. \tag{148}$$

We already know that the FSI term $\langle L^3 \rangle_{\rm int}^{+LC}$ coincides with the potential angular momentum $\langle L^3 \rangle_{\rm pot}$, so that we can also write

$$\langle L^3 \rangle^{+LC} = \langle L^3 \rangle_{\rm mech} + \langle L^3 \rangle_{\rm pot} = \langle L^3 \rangle_{\rm can}, \tag{149}$$

where the rhs is the so-called gauge-invariant canonical OAM. (Note that it is gauge invariant only in a weak sense.) A natural interpretation of the above relation deduced from the average transverse momentum case is as follows. Initially, the average OAM of quarks inside the nucleon is obviously the manifestly gauge-invariant mechanical OAM $\langle L^3 \rangle_{\rm mech}$, which is generally nonzero. Through the FSI caused by the torque of color Lorentz force, the ejected quark acquires an additional OAM, i.e. the potential angular momentum $\langle L^3 \rangle_{\rm pot}$, which was originally stored in the gluon OAM part appearing in the mechanical decomposition of the nucleon spin. Consequently, the final OAM of the ejected quark is converted into the canonical OAM. We emphasize that this interpretation is just consistent with our previous observation in the Landau problem that the canonical OAM represents the total OAM, i.e. the sum of the mechanical OAM of a particle and the OAM carried by the electromagnetic potential. Now, the reason why the relation $\langle L^3 \rangle^{+LC} = \langle L^3 \rangle_{\rm can}$ holds should be clear. For, according to our general rule, the average longitudinal OAM $\langle L^3 \rangle^{+LC}$, defined by the Wigner distribution with the gauge-link path $\mathcal{L} = +LC$, must represent the asymptotic OAM of the ejected quark after leaving the spectator in the SIDIS processes. It is only natural that this OAM of a quark well separated from the original nucleon center reduces to the seemingly free canonical OAM, since there is no background of the color electromagnetic field in this asymptotic distance. It is also clear that this canonical OAM is not an intrinsic OAM carried by the quarks inside the nucleon. Stated differently, the canonical OAM of the Jaffe-Manohar decomposition is not an intrinsic (or static) property of the nucleon.

Because our conclusion is fairly different from the naive picture believed by quite a few researchers in the DIS physics community, some more explanation would be mandatory. After all, what makes our problem delicate and complicated is the FSI or ISI, which comes into the game through the transverse gauge link. This can be easily understood if one inspects the average longitudinal momentum defined through the Wigner distribution:

$$\langle x\rangle^{\mathcal{L}} = \int dx \int d^2\boldsymbol{b}_\perp \int d^2\boldsymbol{k}_\perp x \rho^{\mathcal{L}}(x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp). \tag{150}$$

In this case, the integration over $\boldsymbol{b}_\perp$ and $\boldsymbol{k}_\perp$ is trivial (the contribution from the transverse gauge link vanishes), and the gauge-link path dependence essentially disappears, thereby leading to the familiar result,

$$\langle x\rangle = \frac{1}{2P^+}\langle PS|\bar{\psi}(0)\gamma^+ \frac{1}{i} D^+(0)\psi(0)|PS\rangle = \langle x\rangle_{\text{mech}}. \tag{151}$$

This is nothing but the manifestly gauge-invariant mechanical quark momentum $\langle x\rangle_{\text{mech}}$. At first glance, it appears to contradict our general rule that the average longitudinal momentum of quarks defined through the Wigner distribution should represent the asymptotic quark momentum. There is no discrepancy, however, since we generally get

$$\begin{aligned}\langle x\rangle_{\text{mech}} &= \frac{1}{2P^+}\langle PS|\bar{\psi}(0)\gamma^+ \frac{1}{i} D^+\psi(0)|PS\rangle \\ &= \frac{1}{2P^+}\langle PS|\bar{\psi}(0)\gamma^+ \frac{1}{i} D^+_{\text{pure}}\psi(0)|PS\rangle \\ &\quad - \frac{1}{2P^+}\langle PS|\bar{\psi}(0)\gamma^+ A^+_{\text{phys}}(0)\psi(0)|PS\rangle \\ &= \langle x\rangle_{\text{“can”}} - \langle x\rangle_{\text{pot}},\end{aligned} \tag{152}$$

and since we know that the FSI or the potential momentum term vanishes identically, i.e. $\langle x\rangle_{\text{pot}} = 0$. (This is manifest in the LC gauge $A^+ = A^+_{\text{phys}} = 0$, and it is true also in the general gauge [72].) Namely, due to the cancellation of the FSI for the collinear momentum case, there is no difference between the canonical and mechanical momenta,

$$\langle x\rangle_{\text{mech}} = \langle x\rangle_{\text{“can”}}. \tag{153}$$

In this case, one is therefore allowed to say that either of the canonical or mechanical momentum is partonic and at the same time either represents the intrinsic property of the nucleon.

As explained above, this is clearly not the case for the OAM of quarks in the nucleon. What would be an underlying physical reason for this difference? It can be easily understood from our consideration of the cyclotron motion of a charged particle in Sec. II. A generation of nonzero orbital angular momentum in the stationary nucleon state necessarily requires the circular motion of quarks. This circular motion cannot be a free (or translational) motion in any sense. One might say that this is certainly true for the mechanical OAM but that the same argument does not apply to the canonical OAM, since the latter looks like the OAM of free quarks. However, what meaning does it have to say that such an orbital angular momentum well outside the nucleon is partonic?

After all, a natural conclusion is that neither the canonical OAM nor the mechanical OAM cannot be interpreted as partonic. Both are intrinsically twist-3 quantities. To convince the statement above, we recall the following relation derived by Hatta and Yoshida [64],

$$\Phi_D(x_1, x_2) = P\frac{1}{x_1 - x_2}\Phi_F(x_1, x_2) + \delta(x_1 - x_2) L^q_{\text{can}}(x_1), \tag{154}$$

where $\Phi_D(x_1, x_2)$ is the D-type quark-gluon correlation function of twist 3 defined by

$$\int \frac{d\lambda}{2\pi}\int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2 - x_1)}\langle p', s'|\bar{\psi}(0)\gamma^+ \mathcal{L}[0,\mu] D^i(\mu)\mathcal{L}[\mu,\lambda]\psi(\lambda)|p, s\rangle = \epsilon^{+i\rho\sigma} S_\rho \Delta_{\perp,\sigma}\Phi_D(x_1, x_2) + \cdots, \tag{155}$$

while $\Phi_F(x_1, x_2)$ is the F-type quark-gluon correlation functions defined by

$$\begin{aligned}&\int \frac{d\lambda}{2\pi}\int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2 - x_1)}\langle p', s'|\bar{\psi}(0)\gamma^+ \mathcal{L}[0,\mu] g F^{+i}(\mu)\mathcal{L}[\mu,\lambda]\psi(\lambda)|p, s\rangle \\ &\quad = P^+\epsilon^{ij}_\perp S^j_\perp \Psi_F(x_1, x_2) + \epsilon^{ij}_\perp \Delta^j_\perp S^+ \Phi_F(x_1, x_2) + \cdots.\end{aligned} \tag{156}$$

[Note that $\Psi_F(x_1, x_2)$ here is the more familiar ETQS function, while another correlation function $\Phi_F(x_1, x_2)$ appears in (154).] $L^q_{\rm can}(x_1)$ in (154) is the canonical OAM density given as [64]

$$L_{\rm can}(x) = x\int_x^{\epsilon(x)}\frac{dx'}{x'}(H_q(x') + E_q(x')) - s\int_x^{\epsilon(x)}\frac{dx'}{x'^2}\tilde{H}_q(x') - x\int_x^{\epsilon(x)} dx_1\int_{-1}^{1} dx_2 \Phi_F(x_1, x_2)\frac{3x_1 - x_2}{x_1^2(x_1 - x_2)^2} - x\int_x^{\epsilon(x)} dx_1\int_{-1}^{1} dx_2 \tilde{\Phi}_F(x_1, x_2) P\frac{1}{x_1^2(x_1 - x_2)}, \tag{157}$$

with $\tilde{\Phi}_F(x_1, x_2)$ being an F-type quark-gluon correlation function defined by

$$\int\frac{d\lambda}{2\pi}\int\frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2 - x_1)}\langle p', s'|\bar{\psi}(0)\gamma^+\gamma_5 \times \mathcal{L}[0,\mu] g F^{+i}(\mu)\mathcal{L}[\mu,\lambda]\psi(\lambda)|p, s\rangle = P^+\epsilon_\perp^{ij} S_\perp^j \tilde{\Psi}_F(x_1, x_2) + \epsilon_\perp^{ij}\Delta_\perp^j S^+ \tilde{\Phi}_F(x_1, x_2) + \cdots. \tag{158}$$

Our interest here is only the integrated OAMs, since we think that the density level decomposition needs more satisfactory understanding of the role of surface terms, which we do not believe has been cleared up yet. Then, using the relations

$$L^q_{\rm mech} = \int dx_1\int dx_2 \Phi_D(x_1, x_2), \tag{159}$$

$$L_{\rm pot} = \int dx_1\int dx_2 P\frac{1}{x_1 - x_2}\Phi_F(x_1, x_2) \tag{160}$$

as well as the symmetries of the correlation functions $\Phi_F(x_1, x_2)$ and $\tilde{\Phi}_F(x_1, x_2)$,

$$\Phi(x_1, x_2) = \Phi(x_2, x_1), \quad \tilde{\Phi}(x_1, x_2) = -\tilde{\Phi}(x_2, x_1), \tag{161}$$

one readily obtains

$$L_{\rm can} = \int dx_1\int dx_2 \delta(x_1 - x_2) L_{\rm can}(x_1) = \frac{1}{2}\int dx x(H_q(x) + E_q(x)) - \frac{1}{2}\int dx\tilde{H}_q(x) + L_{\rm pot}. \tag{162}$$

The fact that the potential OAM $L_{\rm pot}$ is related to the genuine twist-3 quark-gluon $\Phi_F(x_1, x_2)$ correlation function is nothing surprising, since we already explained our interpretation that $L_{\rm pot}$ is just the FSI in the SIDIS processes or ISI in the Drell-Yan processes. A noteworthy fact here is that the genuine twist-3 piece of $L_{\rm can}$ is precisely canceled by that of $L_{\rm pot}$ in the combination $L_{\rm mech} = L_{\rm can} - L_{\rm pot}$. This result could be anticipated from the famous Ji sum rule [55], which is given only with the twist-2 quantities as

$$L_{\rm mech} = \frac{1}{2}\int dx x(H_q(x) + E_q(x)) - \frac{1}{2}\int dx\tilde{H}_q(x). \tag{163}$$

Still interesting is the fact that this cancellation reminds us of the observation in the Landau problem that the quantum number m dependence of the canonical OAM and that of the potential angular momentum are just canceled and the mechanical OAM is independent of this unphysical quantum number m, the eigenvalue of the canonical OAM operator.

VI. CONCLUDING REMARKS

The main objective of the present paper is to get a clear understanding on the physical meaning of the two existing decompositions of the nucleon spin, i.e. the canonical and mechanical decompositions. Needless to say, when one talks about the decomposition of the nucleon spin, one is tacitly supposing in mind the intrinsic spin structure of the nucleon. As we have shown, what meets this requirement is the mechanical decomposition, not the canonical decomposition also known as the Jaffe-Manohar decomposition. In fact, the canonical quark OAM represents the OAM of an ejected quark in the SIDIS processes. Putting this in other words, it stands for the OAM of a quark well *outside* the nucleon. How can one think of it as representing an intrinsic (static) structure of the nucleon?

There is wide-spread misbelief in the DIS physics community that the canonical OAM just matches the partonic picture of quark motion in the nucleon. This misunderstanding partially comes from the fact that, for the collinear quark (or gluon) momentum fraction, there is no difference between the canonical and mechanical momenta due to the cancellation of the final-state interaction in the inclusive DIS processes. In this case, one can say that either the canonical or mechanical momentum is partonic and besides either represents the intrinsic property of the nucleon. This is not the case for the orbital angular momentum of quarks, however. The reason for it is clear by now from our present analysis. The generation of nonzero orbital angular momentum inside the stationary nucleon state necessarily requires the circular motion of a particle. The point is that this circular motion cannot be a free motion in any sense. In fact, we showed that neither the canonical OAM nor the mechanical OAM can be partonic. They are intrinsically twist-3 objects. Still, one should pay close attention to the vital difference between these two OAMs. An obvious superiority of the mechanical OAMs is that they are observables (or at least a quasiobservables) within the framework of the GPD factorization scheme.

On the other hand, the F_{14} sum rule, which was once believed to provide us with a hope to experimentally access the canonical OAM of quarks, is questioned now since the Wigner distribution F_{14} does not appear in either of the GPD (collinear) or TMD factorization schemes. In that sense, one might be able to say that it is not even a quasiobservable, at least according to our present knowledge of the method of measurement based on the perturbative QCD framework. In our opinion, this proves the validity of our claim of long years, which advocates the superiority of the mechanical-type decomposition of the nucleon spin over the canonical one either from the physical viewpoint or from the observational viewpoint.

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