PHYSICAL REVIEW D **94,** 055029 (2016)

Challenges for models with composite states

James M. Cline
Department of Physics, McGill University, 3600 Rue University, Montréal, Québec, Canada H3A 2T8

Weicong Huang
Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China and Department of Physics, McGill University, 3600 Rue University, Montréal, Québec, Canada H3A 2T8

Guy D. Moore
Institut für Kernphysik, Technische Universität Darmstadt Schlossgartenstrasse 2, D-64289 Darmstadt, Germany
(Received 2 August 2016; published 23 September 2016)

Composite states of electrically charged and QCD-colored hyperquarks (HQs) in a confining $SU(N_{HC})$ hypercolor gauge sector are a plausible extension of the standard model at the TeV scale and have been widely considered as an explanation for the tentative LHC diphoton excess. Additional new physics is required to avoid a stable charged hyperbaryon in such theories. We classify renormalizable models allowing the decay of this unwanted relic directly into standard model states, showing that they are significantly restricted if the new scalar states needed for UV completion are at the TeV scale. Alternatively, if hyperbaryon number is conserved, the charged relic can decay into a neutral hyperbaryon. Such theories are strongly constrained by direct detection, if the neutral constituent hyperquark carries color or weak isospin, and by LHC searches for leptoquarks if it is a color singlet. We show that the neutral hyperbaryon can have the observed relic abundance if the confinement scale and the hyperquark mass are above TeV scale, even in the absence of any hyperbaryon asymmetry.

DOI: 10.1103/PhysRevD.94.055029

I. INTRODUCTION

Hints of a small excess of events in the diphoton channel have been reported by ATLAS [1,2] and CMS [3,4] experiments during the 13 TeV run. Although they are probably a statistical fluctuation, it is intriguing that both experiments see the excess at the same invariant mass of the photon pair, at approximately $m_{\gamma\gamma} = 750$ GeV, and that there are further hints of an excess at higher invariant masses [2]. This has prompted numerous theoretical interpretations in terms of a spin-0 resonance decaying into photons. A plausible class of models considers the resonance to be composed of heavy constituents Ψ which we will call hyperquarks (HQs), bound by an $SU(N_{HC})$ hypercolor confining gauge theory. If the HQs carry electric charge then the pionlike or quarkoniumlike bound state assumed to be the 750 GeV resonance can decay into photons to explain the observed signal [5–17].

This kind of extension of the standard model seems natural since it simply enlarges the gauge symmetry group by an additional $SU(N_{HC})$ factor. One might expect that, similarly to the standard model, the HQs carry a new, possibly conserved charge, hyperbaryon (HB) number. In this work we explore the consequences of HB number being conserved, leading to a dark matter candidate but also potentially severe conflicts with observation.[1] We also consider models in which it is broken by renormalizable interactions, which turn out to be more constrained than one might at first think. Although the tentative excess of 750 GeV diphotons at LHC motivated our study, it could be of more general interest even if this signal does not persist, since it has now been established that new physics of this kind could be on the verge of discovery at LHC. In the following we will focus on models that predict a 750 GeV bound state $\tilde{\pi}$ that decays into photons, but our observations could obviously be adapted to other similar models.

We will assume that Ψ is vectorlike. If $m_\Psi \ll \Lambda_{HC}$ then the bound state $\tilde{\pi}$ is pionlike, with a mass scaling as $m_{\tilde{\pi}} \sim \sqrt{m_\Psi \Lambda_{HC}}$ due to an assumed approximate chiral symmetry, softly broken by m_Ψ. If $m_\Psi > \Lambda_{HC}$, the composite state would be more similar to charmonium. However the fact that the putative 750 GeV resonance is relatively narrow and distinct indicates that m_Ψ cannot be much greater than Λ_{HC}; otherwise one would expect to produce a series of closely spaced resonances with fractional mass splitting $\Delta m/m \sim (\Lambda_{HC}/m_\Psi)^{3/4}$, based upon a semiclassical model of bound states in a linear confining potential $V \sim \Lambda_{HC}^2 r$.

[1] For a discussion of more exotic hyperbaryons, consisting of bound states of hyperquarks in several different representations of the standard model gauge symmetries, see Ref. [18].

In these models, it is assumed that the HQs are also colored and can thus be produced by gluon-gluon fusion (ggF). Alternatively, it is possible to have sufficient production through photon-photon or vector boson fusion [19–23] if the HQ carries a large hypercharge ∼(3–4), indicating a Landau pole at a relatively low scale, barring additional states. However the modest growth of the photon parton distribution function (PDF) with energy puts this scenario in tension with the lack of any observed signal in the 8 TeV LHC run.[2] For these reasons models with ggF production are favored, and our focus will therefore be on HQs that carry QCD color. We will comment on colorless models in Sec. VI.

One possibility is that there is a conserved HQ number that leads to a stable hyperbaryon (HB) consisting of $N_{\rm HC}$ HQs. There are very stringent constraints on electrically charged relics, so that a realistic model should provide some way for these unwanted relics to decay. In some cases it is possible to write down a high-dimensional effective operator that would allow the charged HB to decay directly into standard model (SM) particles. However it is theoretically more satisfying to demonstrate the renormalizable interactions that would allow for the decays of the HB, either into purely SM particles, or into an electrically neutral HB that might be a viable dark matter candidate. One point of the present work is that there are relatively few categories of renormalizable models that lead to nonconserved HB number, and they are strongly constrained by collider searches if the new scalars that must be added are at the TeV scale. We survey these possibilities in Sec. II.

If HB number is conserved, the lightest HB must be electrically neutral, and is a dark matter candidate. *A priori*, it could carry $\mathrm{SU}(3)_c$ color, in which case it binds to ordinary quarks or antiquarks to make a color-neutral composite state, whose residual strong interactions give it a large cross section for scattering on nucleons. This is likely to be excluded at the same level as charged relics by searches for anomalous heavy isotopes, favoring models in which the lightest stable HQ is a color singlet. If it is an $\mathrm{SU}(2)_L$ doublet, the constraints from direct detection are less severe, but still quite significant. The safest case with constituents that are purely neutral under SM interactions turns out to have a relatively light leptoquark bound state, assuming that ordinary baryon number is still an accidental symmetry of the full theory; hence even this case comes under pressure from current LHC constraints. These scenarios are discussed in Sec. III.

The relic density of conserved HBs could be due to an asymmetry, analogous to the baryon asymmetry. Here we suppose that the mechanism for generating an HB asymmetry is weak or lacking, and focus on the symmetric component, which should be understood in any case before invoking an asymmetry. In Sec. V we compute the abundance for purely singlet HBs and for those that carry weak isospin, showing its dependence on the confinement scale $\Lambda_{\rm HC}$ and the mass of the neutral HQ. Even the purely singlet HB has electromagnetic interactions with protons through loops containing the charged HQ. This leads to weak constraints from direct detection that we derive in Sec. V. Models in which the charged HQ does not carry QCD color are much less restricted by the considerations of the previous sections. We briefly comment on them in Sec. VI, and give conclusions in Sec. VII.

II. HYPERBARYON NUMBER VIOLATION

First we deal with the possibility that hyperbaryon number is not a symmetry of the theory, and HBs can decay directly into SM particles. One might imagine further possibilities by allowing new dark matter particles in the final states, but we do not pursue this here.

Let us provisionally assume that the charged HQ is a fermion Ψ_A^a with HC index A, color index a, and weak hypercharge Y. In addition, there may be bosonic fields carrying fundamental HC indices. We can associate the global HB quantum number 1 to each fundamental HC index, and −1 to each antifundamental index. Thus a HC gauge boson, having one of each, has vanishing HB number. If we were only allowed to contract HC indices in fundamental/antifundamental pairs, then it would be impossible to violate HB number while respecting the gauge symmetry. However in $\mathrm{SU}(N)$ we also have the invariant tensor $\epsilon_{A_1,\ldots,A_N}$. This simple argument demonstrates that HB violation must involve the ϵ tensor.

We start by discussing a rather general class of models that lead to decays of the hyperbaryons into standard model quarks. Other types of models have a structure depending upon the value of $N_{\rm HC}$, so we consider the possibilities $N_{\rm HC} = 2, 3, 4$ in turn in the following.

A. Neutral scalar hyperquarks

There is a general class of models which have the same structure and are renormalizable for $N_{\rm HC} = 2, 3, 4$, requiring the presence of $N_{\rm HC}$ flavors of fundamental $\mathrm{SU}(N_{\rm HC})$ scalars $\Phi_{i,A}$, and that the hypercharge of Ψ matches that of the SM u_R or d_R quarks. Illustrating the former case where $Y_\Psi = 2/3$, it has the form

$$\sum_{i=1,j}\lambda_{ij}\bar\Psi_a^A\Phi_{i,A}u_{R,j}^a + \mu\epsilon^{A_1,\ldots,A_N}\Phi_{1,A_1}\ldots\Phi_{N,A_N} \qquad (1)$$

where μ has dimensions of $(\mathrm{mass})^{4-N}$ and $N = N_{\rm HC}$. If the Φ's are heavy they can be integrated out giving an operator schematically of the form

[2]However Ref. [21] points out that this is subject to uncertainties in the parton distribution functions and finds that the tension is not strong.

$$\frac{\mu\lambda^N}{m_\Phi^{2N}}(\bar{\Psi}u_R)^N \tag{2}$$

that allows the charged HB to decay into N up-type quarks.

On the other hand if the Φ's are lighter than m_Ψ, then the lightest HB is a scalar bound state Φ^N, which can decay via the operator $\mu\Phi^N$. Concretely, this would allow the HB to decay into $N-1$ hypermesons,

$$\Phi^N \to (N-1)\Phi^*\Phi \tag{3}$$

by converting one Φ into $N-1$ antihyperquarks (or fewer, if final state $\bar{\Phi}$ annihilate with Φ before hadronizing). If $N \geq 4$ there is generically enough phase space for the decay, even if the masses are dominated by the constituents, since $Nm_\Phi \geq 4m_\Phi$. For $N=3$ one would require the mesons to be pseudo-Goldstone bosons in order to overcome this restriction. [For $N_{\rm HC}=2$ there is no clear distinction between a meson and a baryon since the fundamental representation of SU(2) is pseudoreal.]

1. LHC constraints

If Φ is heavy, then in addition to the HB-violating operator (2), there is a dimension-6 HB-conserving interaction of the form

$$\frac{\lambda^2}{m_\Phi^2}|\bar{\Psi}u_R|^2. \tag{4}$$

Because of its chiral structure it would allow vector hypermesons to decay into u_R quarks plus a gluon, leading to three jets.[3] However there is no obstruction to making λ^2/m_Φ^2 sufficiently small so that the branching ratio for these decays is unimportant. A lower bound (depending upon $N_{\rm HC}$ and μ) can be placed on λ/m_Φ^2 by demanding that the charged HB decays before big bang nucleosynthesis (BBN), leading to $\lambda/m_\Phi^2 \gtrsim 10^{-4}/\mathrm{TeV}^2$ for the most restrictive case of $N_{\rm HC}=4$, consistent with a small coefficient $\sim 10^{-8}/\mathrm{TeV}^2$ for the effective operator (4), if $m_\Phi \sim$ TeV.

The case of light Φ is more interesting, since it leads to mass mixing between the SM quarks and the composite fermions $U_i \equiv \bar{\Psi}\Phi_i$ that have the same quantum numbers as u_R (or d_R in the alternate $Y_\Psi = -1/3$ models). The mixing comes from the Yukawa coupling $\lambda_{ij}\bar{\Psi}_a^A \Phi_{i,A} u_{R,j}^a$ leading to an off diagonal mass term of order $\lambda_{ij} f_{\tilde{\pi}} \bar{U}_i u_j$, where $f_{\tilde{\pi}}$ is the hypermeson decay constant.

The heavy composite U_i particles are constrained by LHC searches for heavy quarks. There will be Drell-Yan pair production of $\bar{U}_i U_i$, followed by decays $U_i \to u_j h$ where h is the Higgs boson, or $U_i \to W d_j$. The first decay comes from mixing of U_R with u_R, while the second is due to U_L–u_L mixing. We note that U is a Dirac fermion since we have implicitly assumed that Ψ is vectorlike in order to have a bare mass. The mass matrix takes the form

$$(\bar{u}_R \bar{U}_R)\begin{pmatrix} m_u & \lambda f_{\tilde{\pi}} \\ 0 & M_U \end{pmatrix}\begin{pmatrix} u_L \\ U_L \end{pmatrix} \tag{5}$$

where m_u is the SM quark mass matrix and $M_U \sim \delta_{ij}\sqrt{\Lambda_{\rm HC} m_\Psi} \lesssim 750$ GeV is the mass matrix of the composite states in the absence of U–u mixing. After diagonalization, the left- and right-handed states have mixing angles

$$\theta_L \sim \frac{\lambda f_{\tilde{\pi}} m_u}{M_U^2}, \qquad \theta_R \sim \frac{\lambda f_{\tilde{\pi}}}{M_U}. \tag{6}$$

The effective couplings for $U \to hu$ and $U \to Wd$ are thus of order

$$\frac{m_u}{v}\theta_R = \frac{\lambda\Lambda m_u}{vM_U}, \qquad g_2\theta_L = \frac{g_2\lambda\Lambda m_u}{M_U^2} \tag{7}$$

implying that $U \to hu$ is the dominant decay channel, as long as $m_U \gtrsim 300$ GeV.

This scenario has been considered by ATLAS [24] and CMS [25] (see also [26]) for the case of top partners decaying as $T \to ht$ and Wb. These searches constrain $m_T > 700$–900 GeV depending upon the respective branching ratios, with the strongest limit when $T \to ht$ dominates, as we expect here. This contradicts the assumption that $m_\Phi < m_\Psi$ which would imply that $m_T < 750$ GeV, hence ruling out a dominant coupling to top quarks.

2. Flavor constraints

The interaction (1) induces flavor changing neutral current decays $c \to u\gamma$ as shown in Fig. 1. The analogous diagram gives $b \to s\gamma$ in the related model with $Y_\Psi = -1/3$. Defining $t = m_\Psi^2/m_\Phi^2$ and writing the transition amplitude as $(m_b/\Lambda_b^2)\bar{s}_R\sigma^{\mu\nu}q_\nu\epsilon_\mu b_L$, we find [27]

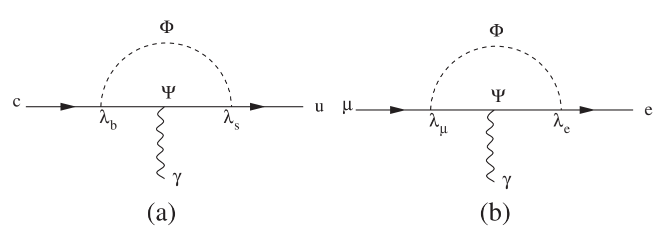

FIG. 1. (a) Contribution to $c \to u\gamma$ from the model Eq. (1). (b) Contribution to $\mu \to e\gamma$ in the HB-violating model of Eq. (11).

[3] A different dimension-6 operator of the form $(\bar{\Psi}\gamma_\mu\Psi)(\bar{u}_R\gamma^\mu u_R)$ would allow decays into two jets, but this operator is not induced by the heavy scalar Φ.

$$\frac{1}{\Lambda_b^2} = \frac{N_{\rm HC}(e/3)\lambda_b\lambda_s}{32\pi^2 m_\Psi^2} f(t) < \left(\frac{1}{55\ \mathrm{TeV}}\right)^2 \tag{8}$$

where $f(t) = t/(t-1)^4[(t-1)(t^2-5t-2)/6 + t\ln t] \sim 0.1$ for a typical value $t \sim 1.5$ and the experimental upper limit is inferred from Ref. [28] (specifically, $\Lambda_b^{-2} = 4\sqrt{2}G_F V_{tb}V_{ts}^* C_7' e/16\pi^2$ with $C_7' < 0.065$). Taking for example $m_\Psi = 400$ GeV and $N_{\rm HC} = 3$ we obtain a weak constraint, $\sqrt{|\lambda_b\lambda_s|} \lesssim 0.75$.

B. $N_{\rm HC} = 2$

We turn next to models that are specific to the value of $N_{\rm HC}$. For $N_{\rm HC} = 2$ we can construct the HB-violating dimension-6 operators

$$\epsilon_{abc}\epsilon_{AB}(\bar\Psi^c_{A,a}\Psi_{B,b})(\bar u^c_{R,c} l_R, \bar L^c_L Q_{L,c}, \bar d^c_{R,c} l_R) \tag{9}$$

where $\mathrm{SU}(2)_L$ indices are implicitly contracted with $\epsilon_{\alpha\beta}$ in the second operator. The first two require the electric charge of Ψ to be $q_\Psi = 1/6$ while the last one needs $q_\Psi = 2/3$. They can be UV-completed by introducing a color-triplet scalar with couplings

$$\epsilon_{abc}\epsilon_{AB}(\bar\Psi^c_{A,a}\Psi_{B,b})\Phi_c + \Phi^{*a}(\bar u^c_{R,a} l_R, \bar L^c_L Q_{L,a}, \bar d^c_{R,c} l_R) \tag{10}$$

(we omit writing the dimensionless coupling constants).

Φ therefore decays like a scalar leptoquark, which can be consistent with current constraints from LHC if $m_\Phi \gtrsim$ TeV [29–32]. However, the $\bar\Psi^c\Psi\Phi$ interaction makes it clear that there is a bound state $\tilde\Phi = \bar\Psi\Psi^c$ with the same quantum numbers as Φ. The $\bar\Psi^c\Psi\Phi$ operator becomes an off diagonal mass term $\sim f_{\tilde\pi}^2\tilde\Phi^*\tilde\Phi$ (where $f_{\tilde\pi}$ is the hypermeson decay constant) that causes mixing between the elementary and composite scalars. Therefore the experimental constraints on leptoquarks also apply to $\tilde\Phi$, assuming its production cross section is the same as that of Φ.

Generally, the production of $\tilde\Phi^*\tilde\Phi$ will be of the same order as that for $\bar\Psi\Psi$, depending upon the probability for $\bar\Psi\Psi$ to hadronize into $\tilde\Phi^*\tilde\Phi$ versus other hadronlike pairs. In the present case, there are only two ways to hadronize, either into mesons $\bar\Psi\Psi$ or baryons $\bar\Psi^c\Psi$, so we expect the production cross section to be about half of that for an elementary color triplet pair. Taking this into account, we can infer the CMS limits on the $\tilde\Phi$ mass to be $m_{\tilde\Phi} \gtrsim 680$ GeV if $\tilde\Phi \to \tau b$ predominantly [31] and $m_{\tilde\Phi} \gtrsim 650$ GeV $\tilde\Phi \to \mu q$ [30]. These are marginally compatible with the expected value $m_{\tilde\Phi} \sim 750$ GeV. The corresponding ATLAS limits are similar.

C. $N_{\rm HC} = 3$

If $N_{\rm HC} = 3$ and Ψ carries hypercharge $Y = 1$, it can couple to right-handed leptons $e_{R,i}$ of generation i, and a neutral colored scalar hyperquark Φ,

$$\lambda_i \bar\Psi^A_a \Phi^a_A e_{R,i} + \mu\epsilon^{ABC}\epsilon_{abc}\Phi^a_A\Phi^b_B\Phi^c_C. \tag{11}$$

Supposing that the scalar is heavy, one can integrate it out to obtain a dimension-9 operator schematically of the form

$$\frac{\mu\lambda^3}{m_\Phi^6}(\bar\Psi e_R)^3 \tag{12}$$

that allows the charge-3 relic HB to decay into three leptons. This effective operator was pointed out in Ref. [9], where Ψ carried an extra flavor index $f = 1, 2$, necessitating the existence of all possible combinations of Ψ_1 and Ψ_2 in operators like (12) to deplete all the flavors of baryons. We note that the UV-completion solves another problem of their model, namely the overabundance of hyperpions of the form $\tilde\pi_{12} = \bar\Psi_1\Psi_2$ that were stable in the theory with only the effective operator (12), but become unstable to $\tilde\pi_{12} \to e_R\bar e_R$ by Φ exchange using interactions of the type (11).

1. Constraints on light Φ

If Φ is relatively light, then analogously to the discussion in Sec. II A 1, there will be a vectorlike composite state $E = \bar\Psi^c\Phi^*$ that has the same quantum numbers as $e_{R,i}$, and we get mass mixing between E and a linear combination of the SM leptons $e_{R,i}$. ATLAS has searched for the decays of a vectorlike lepton into Z and a SM lepton [33]. The constraints are not very restrictive, ruling out the mass ranges 129–176 GeV (114–168 GeV) if the mixing is primarily to electrons (muons), except for gaps 144–163 GeV (153–160 GeV) where a heavy lepton is still allowed.

It is also possible to derive constraints from the rare flavor-violating decays $Z \to \ell_i^+\bar\ell_j^-$. These arise because the unitary transformations U_L and U_R that diagonalize the 4×4 Dirac mass matrices do not act unitarily in the 3×3 subspace involving only the SM leptons, which couple to Z whereas the heavy E state does not. These flavor-changing couplings are proportional to

$$\mathcal{L}_Z = -\frac{g}{2c_W}\bar e_i[(U_L^\dagger P_3 U_L)_{ij}(-1+2s_W^2)P_L + (U_R^\dagger P_3 U_R)_{ij}(2s_W^2)P_R]e_j \tag{13}$$

where c_W, s_W are the weak mixing factors and P_3 projects onto the 3×3 subspace of the SM leptons. Taking $P_3 = 1 - P_4$ where P_4 projects onto the heavy state, one can express the nonstandard contributions to (13) as

$$\delta\mathcal{L}_Z = +\frac{g f_{\tilde\pi}^2}{2c_W m_E^2}\bar e_i \not{Z}[(m_\ell\lambda)_i(m_\ell\lambda)_j(-1+2s_W^2)P_L + \lambda_i\lambda_j(2s_W^2)P_R]e_j \tag{14}$$

where m_ℓ is the (diagonal) light lepton mass matrix and m_E is the composite state mass. Because of the m_ℓ-suppression of the U_L mixing, the right-handed couplings dominate. Assuming that $m_E \sim f_{\tilde{\pi}}$, the experimental upper limits on decays into $e\mu$, $e\tau$, $\mu\tau$ lead to $\sqrt{|\lambda_e \lambda_\mu|} \lesssim 0.08$, $\sqrt{|\lambda_e \lambda_\tau|}$, $\sqrt{|\lambda_\mu \lambda_\tau|} \lesssim 0.14$.

Somewhat stronger bounds arise from the diagonal contributions, which can induce flavor nonuniversality in flavor-conserving decays $Z \to \ell_i \bar{\ell}_i$ through interference with the SM amplitudes. Ignoring the small contribution from the left-handed couplings g_L, and assuming that $\lambda_\tau \gg \lambda_\mu$, λ_e, the fractional deviation $\Delta R_{\tau/e} = \mathrm{BR}(Z \to \tau\bar{\tau})/\mathrm{BR}(Z \to e\bar{e}) - 1$ is given by

$$\Delta R_{\tau/e} = 2\frac{g_R \delta g_R}{g_R^2 + g_L^2} = -\frac{8s_W^2}{1 - 4s_W^2 + 8s_W^4}\frac{\lambda_\tau^2 f_{\tilde{\pi}}^2}{m_E^2}. \tag{15}$$

The experimental limit (at 1σ) is $\Delta R_{\tau/e} > -0.0013$, implying $|\lambda_\tau| \lesssim 0.04$, again assuming that $m_E \sim f_{\tilde{\pi}}$.

Products $\lambda_i \lambda_j$ with $i \neq j$ are constrained by radiative flavor violating decays at one loop, illustrated by $\mu \to e\gamma$ in Fig. 1. Defining $t = m_\Psi^2/m_\Phi^2$ and writing the transition amplitude as $(m_\mu/\Lambda_\mu^2)\bar{e}_R \sigma^{\mu\nu} q_\nu \epsilon_\mu \mu_L$, we find [27]

$$\frac{1}{\Lambda_\mu^2} = \frac{9e\lambda_e\lambda_\mu}{32\pi^2 m_\Psi^2} f(t) < \left(\frac{1}{64\ \mathrm{TeV}}\right)^2 \tag{16}$$

where $f(t)$ is as in Eq. (8) and the experimental limit is inferred from Refs. [34,35]. Taking for example $m_\Psi \cong 400$ GeV and $f \sim 0.1$ leads to the limit $\sqrt{|\lambda_\mu \lambda_e|} < 0.2$, less restrictive than that from $Z \to \mu e$.

D. $N_{\mathrm{HC}} = 4$

For $N_{\mathrm{HC}} = 4$, it is also possible to violate HB with renormalizable interactions if there exists a colored scalar $\tilde{\Phi}_{AB,a}$ in the antisymmetric tensor representation of $\mathrm{SU}(N_{\mathrm{HC}})$, as well as a color-triplet fundamental Φ_a^A. One can then construct the interactions

$$\epsilon_{ABCD}\epsilon_{abc}\bar{\Psi}_{A,a}\Psi_{B,b}^c\tilde{\Phi}_{CD,c} + \mu\epsilon_{abc}\tilde{\Phi}_{CD,a}\Phi_b^C\Phi_c^D + \epsilon_{abc}\Phi_a^C\bar{\Psi}_{C,b}q_{R,c} \tag{17}$$

where q_R can be either u_R or d_R. Integrating out the scalars gives the dimension-9 operator

$$\frac{\mu}{M_\Phi^4 M_{\tilde{\Phi}}^2}\epsilon_{ABCD}\epsilon_{acd}\epsilon_{bef}(\bar{\Psi}_{A,a}\Psi_{B,b}^c)(\bar{\Psi}_{C,c}q_{R,d})(\bar{\Psi}_{D,e}q_{R,f}). \tag{18}$$

It allows four Ψ's to decay into two quarks and would mediate decay of the charged hyperbaryon Ψ^4, provided that Ψ has charge $1/3$ or $-1/6$. The collider phenomenology stemming from the $\Phi\bar{\Psi}q_R$ operator is similar to that of the model given by Eq. (1).

E. Summary

We have presented several renormalizable frameworks allowing for depletion of the unwanted charged relic hyperbaryon. Most of them require a charge-neutral scalar hyperquark Φ, that might also be colored if $N_{\mathrm{HC}} = 3$. If Φ is sufficiently heavy to avoid being produced at the LHC, these models are practically unconstrained. On the other hand, if $m_\Phi \lesssim$ TeV, Φ can form a mesonlike bound state with Ψ that has the quantum numbers as a SM quark or lepton. We showed that the first case is rather strongly constrained by ATLAS and CMS searches for heavy quarks (top partners).

If $N_{\mathrm{HC}} = 2$, another possibility is to introduce a scalar leptoquark Φ that is neutral under $\mathrm{SU}(N_{\mathrm{HC}})$. In this model, there is a bound state of $\Psi\Psi$ with the same quantum numbers that mixes with Φ and thus introduces a relatively light leptoquark state. This is strongly constrained by ATLAS and CMS, leaving little room for a model in which the $\bar{\Psi}\Psi$ hypermeson could be as light as 750 GeV.

These models typically also predict some level of quark or lepton flavor violation, but we find that the resulting constraints are typically weak. The new Yukawa couplings appearing in $\lambda_i\bar{\Psi}\Phi f_i$, where f_i is a SM fermion, need only be of order 0.1 in most cases. Flavor universality of $Z \to \ell_i\bar{\ell}_i$ decays gives the strongest such limit, $\lambda_\tau < 0.04$.

III. NEUTRAL HYPERBARYONS

A second possibility is that the charged HQ can decay into a lighter neutral HQ, which we denote by S^A, that is fundamental under $\mathrm{SU}(\mathrm{N_{HC}})$ and electrically neutral. In principle it could also carry QCD color or weak isospin, but as we will show, these options are generally disfavored by constraints from direct detection of the resulting hyperbaryon $S^{N_{\mathrm{HC}}}$. We introduce a scalar Φ that mediates the decay of Ψ to S plus standard model particles through an interaction of the form

$$\lambda\bar{S}^A\Phi\Psi_A \tag{19}$$

followed by decay of the mediator Φ into standard model particles. (Indices corresponding to any additional quantum numbers are suppressed here). Alternatively, Ψ and S could be in an $\mathrm{SU}(2)_L$ doublet, so that $\Psi \to SW$ by the SM weak interaction. In the following, we consider the different possible cases for additional quantum numbers carried by S.

A. Colored stable hyperquark

We first consider the case in which the neutral HQ S is colored. The baryonic state that is a singlet under $\mathrm{SU}(N_{\mathrm{HC}})$ is not color-neutral, if S is fermionic. The $\mathrm{SU}(N_{\mathrm{HC}})$ singlet operator $\epsilon_{A_1,\ldots,A_N}\bar{S}^{A_1,a_1}\ldots\bar{S}^{A_N,a_N}$ is symmetric under interchange of SU(3) indices, and can only be antisymmetric under spin if $N_{\rm HC} = 2$. To make a color singlet, it must bind with ordinary quarks,

$$B = \epsilon_{A_1,\ldots,A_N} \bar{S}^{A_1,a_1} \cdots \bar{S}^{A_N,a_N} q_{a_1} \ldots q_{a_N} \tag{20}$$

whose flavors and spins (or spatial configurations) are chosen so as to make the $q \ldots q$ part of the wave function totally antisymmetric, while maintaining charge neutrality. For example if $N_{\rm HC} = 3$, one can form the antisymmetric s-wave state of two down quarks and one up quark, whose flavor/spin wave function is

$$udd(\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow) + ddu(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow) + dud(\downarrow\uparrow\uparrow - \uparrow\uparrow\downarrow). \tag{21}$$

This bound state of $SSSudd$ could be expected to behave similarly to a heavy neutron in its scattering on ordinary baryonic matter.

The scattering properties of dark matter comprised of exotic baryoniclike bound states have been discussed in Ref. [36]. There it is noted that for low-energy nucleon-nucleon scattering, the scattering amplitude scales as $\mathcal{A} \sim 4\pi a/m_N$ where a is the scattering length and m_N is the nucleon mass. For scattering in a central potential, we can expect that m_N represents twice the reduced mass; hence the amplitude for scattering of a heavy neutron of mass m_B on a normal one of mass m_N should scale as $\mathcal{A} \sim 2\pi a/\mu$ where $\mu = m_B m_N/(m_B + m_N) \cong m_N$. Hence the cross section for B-N scattering is roughly 4 times smaller than that of N-N scattering. (The scattering length is determined by the pion mass and confinement scale, hence should not depend explicitly upon the mass m_B.)

Comparing to the experimentally measured neutron-proton cross section (see Fig. 6 of Ref. [36]), we can estimate the cross section for B-N scattering at center of mass energy $\sim m_N v^2 \sim 1$ keV appropriate for direct detection, namely $\sigma_{\rm BN} \sim 5$ b. This is many orders of magnitude higher than direct detection limits (spin-dependent or independent), but such strongly interacting dark matter would be stopped in the earth before reaching the underground detectors, making such limits inapplicable. High-altitude detectors do not suffer from this limitation [37–39], but are too weak to constrain our neutral HB having only spin-dependent interactions with baryons mediated by pion exchange. Instead one should consider the possibility that these particles will bind to ordinary matter, creating anomalously heavy isotopes for which stringent searches have been carried out.

It is impossible to know whether composite HB's containing ordinary quarks will bind to ordinary baryons to produce anomalous isotopes, without doing a non-perturbative calculation such as on the lattice. However if the HB interacts with nucleons in a similar manner as hyperons such as the Λ baryon, one could expect the analog of the hypertriton, the bound state consisting of Λ, p and n,

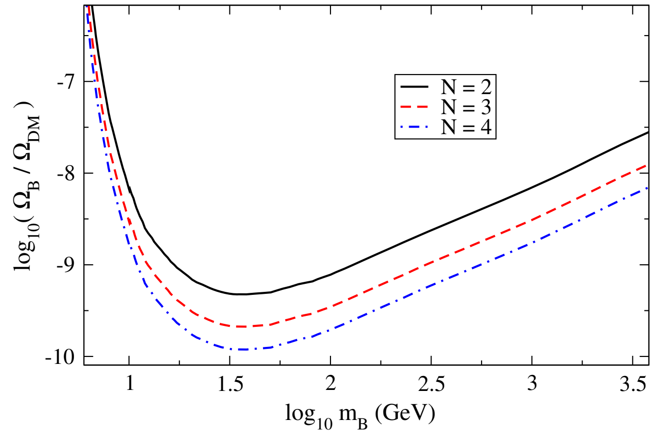

FIG. 2. LUX Limit [42,43] on fractional density of electroweakly interacting hyperbaryonic dark matter, rescaling predictions for Dirac neutralino scattering by Z exchange from Ref. [41].

which is known to exist. Moreover if the HB-baryon interaction is modeled by pion exchange, then there is always an attractive channel for fermionic HB's, since the interaction is spin-dependent.

If HB's do bind to protons, the abundance of such bound states relative to that of protons is given by $Y = (m_p/m_{\rm HB})(\Omega_{\rm HB}/\Omega_b)$ A search for anomalous hydrogen in sea water finds the limit $Y < 6 \times 10^{-15}$ [40], leading to the bound

$$\frac{\Omega_{\rm HB}}{\Omega_{\rm DM}} \lesssim 10^{-13}\left(\frac{m_{\rm HB}}{\rm TeV}\right). \tag{22}$$

In Sec. V we will show that the predicted relic density is far too large to satisfy this constraint.[4]

An exception is when $N_{\rm HC} = 3$ with colored scalar HQs, denoted by Φ. In that case the bosonic HB state

$$B = \epsilon_{ABC}\epsilon_{abc}\Phi^{A,a}\Phi^{B,b}\Phi^{C,c} \tag{24}$$

is neutral under all gauge symmetries and has the correct statistics. However this model does not have HB conservation as an accidental symmetry, since the superrenormalizable operator (24) can simply appear in the Lagrangian,

[4]To generalize the previous example to other values of $N_{\rm HC}$, one must admit nonzero values of the HQ electric charge since charge-neutral combinations of $N_{\rm HC}$ ordinary quarks do not generally exist. In this case it may be possible to dispense with Q altogether and find a neutral bound state of the form (20) with Ψ appearing in place of Q. For example with $N_{\rm HC} = 2$, one can form the state

$$B_\Psi = \epsilon_{AB}\bar{\Psi}^{A,\alpha}\bar{\Psi}^{B,\beta}u_\alpha u_\beta(\uparrow\downarrow - \downarrow\uparrow) \tag{23}$$

provided the charges of Ψ and u are equal. However regardless of these details, we expect that any bound state containing ordinary quarks will bind to protons, and the previous result will hold.

TABLE I. Possible hypercharges, weak isospin and electric charges of the colored scalar mediator Φ, the baryon-conserving operators leading to Φ decay into SM particles, and allowed baryon-violating operators. The last column is the ratio of branching ratios of a pionlike 750 GeV state into photons versus gluons, for constituents Ψ having the same SM quantum numbers as Φ.

y_Φ	$T_{3,\Phi}$	q_Φ	$\mathcal{L}_B$	$\mathcal{L}_{\not B}$	$\frac{\mathrm{BR}(\tilde\pi\to\gamma\gamma)}{\mathrm{BR}(\tilde\pi\to gg)}$
$+7/6$	$\pm 1/2$	$2/3, 5/3$	$\left\{\begin{matrix}\Phi_\alpha & \bar Q_L^\alpha & l_R \\ \Phi_\alpha & \bar u_R & L_L^\alpha\end{matrix}\right\}$	none	0.12
$+1/6$	$\pm 1/2$	$-1/3, 2/3$	$\Phi_\alpha \bar d_R L_L^\alpha$	none	2.7×10^{-3}
$-1/3$	0	$-1/3$	$\Phi \bar u_R l_R^c$	$\left\{\begin{matrix}\Phi^* & \bar Q_L & Q_L^c, \\ \Phi^* & \bar u_R & d_R^c\end{matrix}\right\}$	4.3×10^{-4}
$-4/3$	0	$-4/3$	$\Phi \bar d_R l_R^c$	$\Phi^* \bar u_R u_R^c$	0.11

like in the model of Eq. (11). We therefore consider it to be unnatural as an example of HB conservation.

B. Weakly interacting stable hyperquark

In models where the SM gauge indices of the hyperquarks are embedded in the fundamental of SU(5) for gauge unification [5–7,11,16,17], the colored HQ can be expected to decay into a doublet HQ by exchange of a heavy GUT gauge boson. The charged component of the doublet can then decay into the neutral S through weak interactions. In this case, the hyperbaryon $S^{N_{\mathrm{HC}}}$ will have weak interactions with ordinary matter through Z exchange, similar to a hypothetical heavy bound state containing N left-handed neutrinos. In terms of previously studied models, we expect the cross section for scattering on nucleons to be similar to that of Dirac Higgsino dark matter, neglecting the Higgs exchange contributions to the scattering that occur in that model and focusing only on Z exchange. This has been studied in Ref. [41], which finds that the spin-independent cross section for scattering on neutrons is

$$\sigma \cong 1.0\times 10^{-37}\ \mathrm{cm}^2 \tag{25}$$

This is $\sim(7–9)$ orders of magnitude above the current LUX limit [42,43], requiring that the relic density of such HBs be correspondingly depleted. The cross section for HB scattering is expected to be N_{HC}^2 times larger due to the number of constituents. We show the limit on the fractional abundance as a function of the HB mass in Fig. 2, including this dependence on N_{HC}^2.

C. Singlet stable hyperquark

An interesting possibility is that the scalar mediator carries away the charge and color of the Ψ hyperquark, leaving S^A charged only under $\mathrm{SU}(N_{\mathrm{HC}})$. Then the possible couplings of Φ allowing decays to SM particles, while maintaining ordinary baryon number as an accidental symmetry, are limited to the first two cases shown in table I, where the hypercharge y_Φ can take values $7/6$ or $1/6$. The last two, with $y_\Phi = -1/3$ or $-4/3$, are disfavored because they allow for baryon violation by marginal operators, leading to rapid proton decay. In the favored models, baryon number can be consistently assigned to all fields, such that B coincides with HB number.

In the preferred models with $y_\Phi = 7/6, 1/6$, the scalar Φ (and therefore Ψ) is an $\mathrm{SU}(2)_L$ doublet. These models are viable for producing the 750 GeV diphoton signal in the pion-like regime, for which the ratio of branching ratios $R \equiv \mathrm{BR}(\tilde\pi\to\gamma\gamma)/\mathrm{BR}(\tilde\pi\to gg)$ shown in the last column of Table I is relevant; this ratio is computed following Ref. [9]. If R is too small, the observed diphoton rate would require the width for decays into gluons to be so large (in order to compensate for the small BR into photons) that they would exceed the ATLAS bound on dijets [44], $\sigma(\tilde\pi\to gg) < 2.5$ pb at $\sqrt{s} = 8$ TeV. We find the lower limit $R > 1.6\times 10^{-4}$ to satisfy this constraint; details are given in Appendix A. All the models in Table I are consistent with this constraint.

However these models suffer from another constraint, namely searches for leptoquarks at the LHC. Even though Φ may be very heavy, it mixes with bound states of $\tilde\Phi = \bar\Psi S$ that have the same quantum numbers as Φ. These hypermesons will be pair produced at LHC and their masses must be less than 750 GeV since $m_S < m_\Psi$; $m_{\tilde\Phi}$ can only be decreased by mixing with Φ. They can decay only into quarks and leptons since they have the quantum numbers of leptoquarks. ATLAS and CMS find lower bounds on scalar leptoquarks decaying into jets and electrons or muons such that $m_{\tilde\Phi} \gtrsim 1$ TeV for branching ratio $\beta = 100\%$ into one of those channels, and $m_{\tilde\Phi} \gtrsim 800$ GeV for $\beta = 30\%$ [29–32].[5] Even if Φ decays mostly into τ and third generation quarks, the limit ranges from $m_{\tilde\Phi} > 500–740$ GeV for $\beta = 50\%–100\%$, from the run I data. Thus there is very little parameter space in which to hide an expected leptoquark with $m_\Phi < 750$ GeV, making it difficult to accommodate these models.

A conceivable way out might be to choose small dimensionless couplings for (19) and the interactions of Table I such that the composite leptoquark is metastable and decays outside of the detector (but with a lifetime still

[5]CMS reports a slight excess of $eejj$ events corresponding to $m_\Phi = 650$, $\beta = 0.015$.

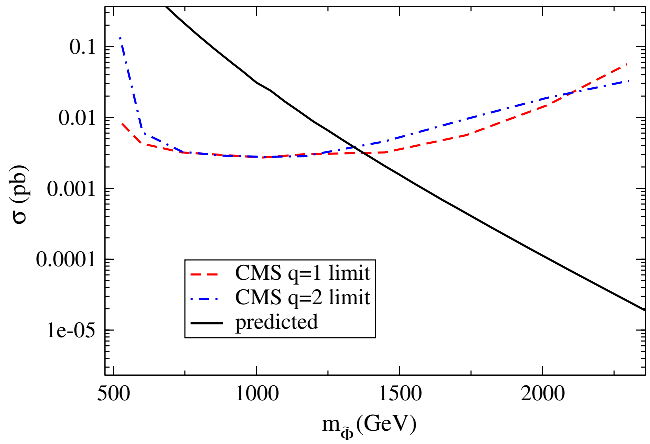

FIG. 3. Predicted LHC production cross section at $\sqrt{s} = 13$ TeV for $\bar\Psi\Psi$ pairs that hadronize into leptoquarklike bound states $\tilde\Phi = \bar\Psi S$, and CMS upper limits for heavy stable particles of charge 1 and 2.

below 1 s to avoid problems with BBN). CMS has searched for such long-lived charged particles. [6]Reference [45] from run I directly constrains charged hypermesons, which however are assumed to be produced by Drell-Yan rather than gg fusion. Reference [46] does a similar analysis in run II, considering DY-produced particles of charge 1 and 2, obtaining limits on the production cross section that we reproduce in Fig. 3. Our prediction for the production of $\tilde\Phi\tilde\Phi^*$ pairs (assuming they originate from $gg \to \bar\Psi\Psi$ and $q\bar q \to \bar\Psi\Psi$ that hadronize mainly into $\tilde\Phi\tilde\Phi^*$) is also shown there.[7] We expect the leptoquark states of charge 2/3 and 5/3 to be constrained at a similar level to the charge 1 and 2 particles considered in [46], leading to a limit of $m_{\tilde\Phi} \gtrsim 1.35$ TeV, again in contradiction to the premises of the present model.

IV. RELIC DENSITY OF NEUTRAL HYPERBARYONS

If hyperbaryon number is conserved and results in a HB that is neutral under SM gauge interactions, it could be a viable dark matter candidate. If it carries weak isospin, then the considerations of Sec. III B show that it can only be a very subdominant component of the total dark matter. In either case, it is interesting to know what the minimum abundance can be as a result of thermal freeze-out. Because it has a conserved charge, it is also possible to have a larger abundance through generation of an asymmetry. We leave aside this possibility and consider here the abundance of the symmetric component, assuming at first that the S hyperquark has only HC interactions.

A. $SU(2)_L$ singlet hyperbaryons

The relic HB abundance is sensitive to the ratio Λ_{HC}/m_S, the HC confinement scale over the neutral HQ mass. If $m_S > \Lambda_{HC}$, there is depletion of the initial S density through annihilations before confinement, whereas if $m_S < \Lambda_{HC}$, the S hyperquarks have a thermal abundance at the confinement phase transition. Once confinement begins, a given S has a roughly equal probability of forming a hypercolor flux string with a neighboring S or $\bar S$, leading to roughly equal numbers of hypermesons (that quickly decay away) and HBs. Following the confinement phase transition, there can be further depletion of the HBs by their annihilation.

We estimate the abundance of HBs by solving the Boltzmann equation in the different regimes of temperature described above. The annihilation cross sections for $S\bar S \to GG$ (annihilation of HQs into hypergluons) and of HBs with their antiparticles are needed. Using Refs. [47,48], we find that the first one is

$$\langle\sigma v\rangle_{S\bar S\to GG} = \frac{\pi\alpha_{HC}^2(m_S)}{4m_S^2 N_{HC}^3}(N_{HC}^2 - 1)(N_{HC}^2 - 2). \tag{26}$$

For the gauge coupling we take the four-loop approximation of ref. [49] with $n_f = 0$ flavors, since we are interested in running only up to the scale m_S, presumed to be the lightest HQ mass in the theory.

For the annihilation of HBs, it is difficult to estimate the cross section due to the strong dynamics and the fact that the HBs are composite states. One possibility is to use the geometric cross section

$$\langle\sigma v\rangle_{\rm geo} = \frac{\pi}{\mu_*^2} \tag{27}$$

where μ_* is the inverse Bohr radius of the HB, estimated along the lines of Ref. [50].[8] To extend their method to the regime of strong coupling, we add a linear confining potential $\sum_{i<j} c\Lambda_{HC}^2 r_{ij}$ to the Coulomb-like term, to obtain the expectation value of the Hamiltonian for a hydrogenlike wave function $\psi \sim e^{-\mu_* r/2}$ as

$$\frac{\langle H\rangle}{N_{HC}} = \frac{\mu_*^2}{8m_S} - \frac{5(N_{HC} - N_{HC}^{-1})}{64}\alpha_{HC}(\mu_*)\mu_* + \frac{35c(N_{HC}-1)\Lambda_{HC}^2}{16\mu_*} \tag{28}$$

where the value $c = 1.9$ is inferred from Refs. [51–53] for the case of $N_{HC} = 3$. Minimizing (28) with respect to μ_* gives an implicit equation for μ_* that can be solved by

[6]Similar searches by ATLAS are difficult to interpret in the context of the present model.
[7]We thank Grace Dupuis for computing this using MADGRAPH.

[8]We disagree with their numerical coefficient for the expectation value of $1/r_{ij}$ for the Coulomb-like contribution to the potential.

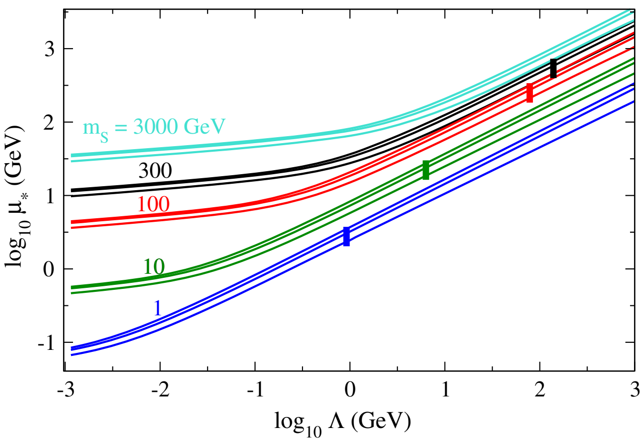

FIG. 4. Inverse Bohr radius of the hyperbaryon bound state versus $\Lambda_{\rm HC}$, for several values of the HQ mass $m_S = 3000$, 300, 100, 10, 1 GeV and $N = 2, 3, 4$ (smaller N gives lower curve for a given mass). Vertical bars show where the perturbative treatment breaks down and extrapolation of low-Λ behavior is used, for a given m_S.

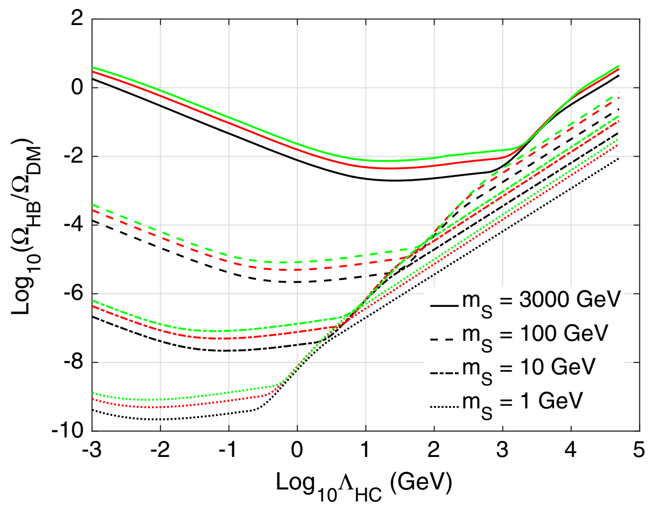

FIG. 5. Estimated relic abundance of the lightest neutral hyperbaryon versus confinement scale $\Lambda_{\rm HC}$, for neutral HQ masses $m_S = 1$, 10, 100, 3000 GeV (from bottom to top), and $N_{\rm HC} = 2, 3, 4$ (from bottom to top for each m_S).

iteration. The resulting μ_* as a function of $\Lambda_{\rm HC}$ is shown for several values of m_S in Fig. 4. This procedure breaks down when $\Lambda \gtrsim m_S$ because the gauge coupling becomes nonperturbative and the middle term in (28) diverges to large negative values as $\mu^* \to \Lambda_{\rm HC}$ from above. The approximate values of $\Lambda_{\rm HC}$ where this starts to occur are indicated by heavy dots in Fig. 4. Since the dependence of $\log_{10}\mu_*$ on $\log_{10}\Lambda_{\rm HC}$ is very linear (corresponding to the power law $\mu_* \sim \Lambda_{\rm HC}^{0.63}$) in the regions below the dots, we use linear extrapolation to extend our predictions to higher values of $\Lambda_{\rm HC}$. The available final states for HB annihilation almost always include the hypermesons $\tilde{\pi}\tilde{\pi}$, even when they are more quarkoniumlike than pionlike (the regime of $m_S \gg \Lambda_{\rm HC}$). The only exception is when $N = 2$ so that mesons and baryons have the same number of constituents.

To compute the relic HB abundance, we numerically solve the Boltzmann equation starting from high temperatures using the $S\bar{S} \to GG$ cross section, evolving down to the confinement temperature $T = \Lambda_{\rm HC}$. We assume the confinement transition occurs rapidly and that the initial abundance of HBs at $T = \Lambda_{\rm HC}$ is related to that of HQs by $Y_B = Y_Q/(2N_{\rm HC})$. Taking this as the initial condition for the Boltzmann equation using the HB annihilation cross section, we evolve Y_B to its freeze-out temperature. This approach is generally necessary, rather than the usual analytic approximations, because of the unusual thermal history that often occurs: at the confinement transition, HBs can often be produced starting with a density far exceeding the equilibrium abundance (since the HB mass is $\sim N_{\rm HC}$ times m_S). Then the HB annihilations can be in equilibrium, even though a naive treatment would imply that they froze out already at an earlier temperature.

The results are shown in Fig. 5, where the fractional abundance of HBs relative to the total observed dark matter is plotted as a function of $\Lambda_{\rm HC}$ for several HQ masses and $N_{\rm HC} = 2, 3, 4$. From conventional thermal freeze-out one expects that $\Omega_{HB} \sim 1/(\sigma v)$ as a function of $\Lambda_{\rm HC}$. This explains the simple power-law behavior of $\Omega_{\rm HB}$ in Fig. 5(a), since $\mu_* \sim \Lambda_{\rm HC}^{0.63}$ at large $\Lambda_{\rm HC}$. At small $\Lambda_{\rm HC}$ the trend is different because at the confinement temperature $T_c = \Lambda_{\rm HC}$, the initial HB abundance is much higher than the equilibrium abundance. In this situation the conventional dependence is not applicable, and we find the different behavior $\Omega_{HB} \sim 1/(\Lambda_{\rm HC}\sigma v)$ in Fig. 5.

We find that unless m_S is near the TeV scale (hence not relevant for a diphoton signal at 750 GeV), the symmetric HB component can provide only a subdominant contribution to the total dark matter density. Although the observed density is obtained at low $\Lambda_{\rm HC} \sim 3$ MeV, this region is not viable because of the presence of very light and long-lived glueballs that will disrupt big bang nucleosynthesis for $\Lambda_{\rm HC} \lesssim$ several GeV [22]. But at large $\Lambda_{\rm HC}$, it is possible to obtain the observed abundance. For $m_S \cong 3$ TeV and $\Lambda_{\rm HC} \cong 20$ TeV for example, we find that HBs can constitute all the dark matter, even with no asymmetry. We expect the geometric cross section to provide a lower limit on the true annihilation cross section, which might require a dedicated lattice study to determine with greater certainty. Hence the actual abundance might be smaller than our estimate, although we expect the qualitative dependences to hold. Even a highly subdominant component of HB dark matter could still lead to observable consequences if the HQs have standard model weak interactions, as we describe next.

B. $SU(2)_L$ doublet hyperquarks

As mentioned in Sec. III B, one way in which the charged HQ Ψ could decay into the neutral one S is by embedding Ψ and S into a fundamental of $SU(5)$ for GUT.

This leads to strong constraints on the relic abundance from direct detection. But the same weak interactions could in principle have an impact on the abundance through the annihilations into SM particles. The possible two-body final states include ZZ, WW, Zh and $f\bar{f}$, depending on the mass of the HQs or HBs. More details on annihilation cross sections are given in Appendix C.

However we find that these extra annihilation channels have a' negligible effect on the HB abundance, being much weaker than the hypercolor interactions; we can therefore infer the densities from Fig. 5. Comparing to the direct detection constraints shown in Fig. 2, it can be seen that very light HBs made from HQs with mass $m_S \sim 1$ GeV $\gtrsim \Lambda_{\rm HC}$ can be compatible with the constraints, but heavier ones are ruled out by several orders of magnitude.

V. ELECTROMAGNETIC INTERACTIONS OF NEUTRAL RELICS

Even if the neutral hyperbaryons have no residual strong or weak interactions with nuclei, as would states of the form (20), they inevitably have electromagnetic interactions through the diagrams shown in Fig. 6. Although this turns out to be unimportant for direct detection, for completeness we discuss their effects. We estimate these diagrams as giving magnetic dipole moments for S of order

$$\mu_S = \frac{e\lambda^2 f}{16\pi^2 m_\Psi}, \qquad \frac{e\alpha_{\rm HC}^3(m_\Psi)}{1152\pi^3 m_\Psi} \tag{29}$$

where f is a function of mass ratios, of order -0.2 for models of interest here. Details are given in Appendix B. The magnetic moment of the hyperbaryon is then $\mu_B \cong N_{\rm HC}\mu_S$. The above estimates assume that Ψ is the heaviest particle in the loop. In the pionlike regime where $m_\Psi \ll \Lambda$, the quark model shows that m_Ψ should be interpreted as the constituent quark mass rather than the current quark mass. For our estimates we thus take it to be $\sim$375 GeV.

The limit on the magnetic dipole moment from direct detection can be parameterized by writing it in terms of the gyromagnetic ratio g_M,

$$\mu_B = \frac{g_M e}{4 m_B}. \tag{30}$$

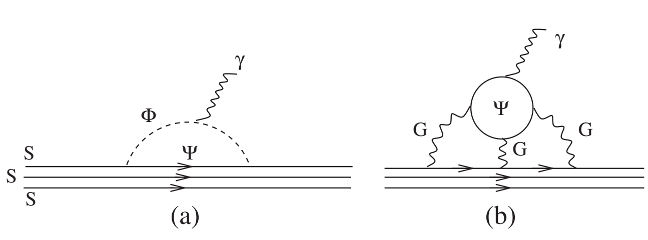

FIG. 6. (a,b) Diagrams that induce a magnetic dipole for the neutral hyperbaryon. G denotes the hypergluon in (b).

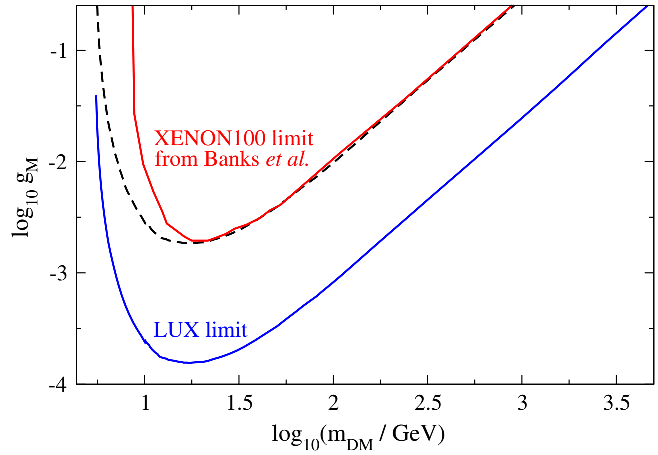

FIG. 7. Upper limit from direct detection on gyromagnetic ratio for magnetic dipolar dark matter, found by updating results of Ref. [54].

The limit on g_M was found in Ref. [54] using data of XENON100 [55]. Updating this limit using the current LUX bounds [42,43], we find the result shown in Fig. 7. To rescale the limit from XENON100 to LUX, we notice that the sensitivity of both experiments scales with dark matter mass in the same way for $m_{\rm DM} > 20$ GeV, while LUX has greater relative sensitivity at lower masses, as shown by the dashed curve in Fig. 7. In the large mass region, the LUX limit on the spin-independent scattering cross section is 140 times lower than that of XENON100, so we rescale the limit on g_M by a factor of $1/\sqrt{140}$, taking into account the greater sensitivity of LUX at lower masses.

The limit on g_M assumes that the dark matter candidate has the full relic density, which as we have seen in Sec. V need not be the case. For the three-loop contribution, we can use this to constrain $\Omega_{\rm HB}/\Omega_{\rm DM}$ as a function of Λ and m_S since all the quantities entering into μ_S are determined. However the resulting upper limit is always greater than unity, so this provides no meaningful constraint. For the one-loop contribution, we can insert the minimum fractions of the total DM density found in Sec. V to get an upper bound on the coupling λ that induces decay of Ψ to S plus SM particles. Again, the upper limits on λ are hardly constraining, being greater than unity in all cases.

VI. UNCOLORED MODELS

Although not favored by the compatibility of $\sqrt{s} = 8$ TeV versus 13 TeV LHC data, a number of authors have shown that the diphoton signal in the 13 TeV data can be accommodated by purely electromagnetic production through photon fusion. The charged hyperquark Ψ is then neutral under $SU(3)_c$. If HB number is conserved, then it is compulsory to have neutral hypercolored particles to prevent charged stable hyperbaryons; so we will assume the model is extended with a neutral spinor S and a scalar Φ

TABLE II. Possible combinations of SM fermions coupling to the bosonic mediator Φ, and their charges, for models with uncolored hyperquarks. L and l stand for $SU(2)_L$ doublet and singlet leptons, respectively.

$\bar{f}_i f_j$	$\epsilon_{ab}\bar{L}_a L_b^c$	$\bar{l}_R l_R^c$
q	1	2

as previously. Then Ψ can decay by via $\Psi \to S f_i \bar{f}_j$ through interactions of the form

$$\lambda \bar{S}^A \Phi \Psi_A + \lambda_{ij} \bar{f}_i \Phi f_j \tag{31}$$

where $f_{i,j}$ are standard model fermions. There are two ways of choosing combinations of SM fermions consistent with gauge invariance, listed in Table II. They correspond to electric charges of 1 or 2 for the mediator and hence of the Ψ, and they imply that Φ carries lepton number 2. We can consistently assign the same lepton number to S so that overall lepton number is conserved by the new interactions. Depending upon the generational structure of the couplings λ_{ij} however, there could be violations of individual flavor conservation, or of lepton flavor universality. For example the considerations of Sec. II C 1 give $\sqrt{|\lambda_{\mu\mu}\lambda_{\mu e}|} < 0.6$, for the same choices of mass spectrum, weaker by the factor of $(3N_{\mathrm{HC}})^{1/2}$ for the lack of color/hypercolor in the loop.

These models are similar to the favored ones discussed in Sec. III C in terms of constraints on the relic HB; they are cosmologically safe, with subdominant relic densities as predicted by Fig. 5 and unimportant interactions with normal matter through their small loop-induced magnetic moments. Since the scalar mediator couples to lepton pairs instead of being a leptoquark, pairs of charged mesons $\bar{\Psi}S$ decaying to monoleptons (in the case of the $\bar{L}L^c$ coupling where one of the particles is a neutrino) or same-sign dileptons would be a collider signature at partonic center of mass energies below 1.5 TeV.

The single-lepton signal is constrained by ATLAS and CMS searches for events with one lepton and missing transverse energy [56,57]. The more recent ATLAS result limits $m_S \gtrsim 4$ TeV if the couplings λ_{ei} or $\lambda_{\mu i}$ are of the same order as the SU(2)L gauge coupling. The dilepton channel is relatively unconstrained, since ATLAS and CMS searches for same-sign dileptons have so far also required the presence of jets.

VII. CONCLUSIONS

An additional $SU(N_{\mathrm{HC}})$ gauge group factor is a plausible and economical extension of the standard model. If new matter fields transform in the fundamental of $SU(N_{\mathrm{HC}})$, the analog of baryon number in the new sector is an issue: if it is conserved then the properties of relic particles must be considered, while if it is broken through renormalizable interactions, interesting constraints can arise from LHC searches for decays associated with these new interactions.

Our considerations are most relevant for models similar to those that can explain the tentative LHC diphoton excess, where we assumed that a charged hyperquark in the fundamental of $SU(N_{\mathrm{HC}})$ also carries QCD color. Such a particle must decay into standard model states and possibly a neutral state that allows for hyperbaryon number to be conserved, and which is a dark matter candidate. In the case that HB is not conserved, we identified a limited range of renormalizable models, that are summarized in Sec. II E. Even if the 750 GeV diphoton excess at LHC is only a statistical fluctuation, models consistent with it provide a benchmark for what could be close to the current sensitivity of ATLAS and CMS.

If HB is conserved, the renormalizable models consistent with direct detection constraints and normal baryon conservation are also limited, and turn out to be significantly constrained by LHC leptoquark searches. The viable models have a charged hyperquark Ψ and a scalar mediator Φ with quantum numbers $(3,2,7/6)$ or $(3,2,1/6)$ under $SU(3)_c \times SU(2)_L \times U(1)_y$, while the neutral hyperquark S is a singlet. An interesting feature of these models is the presence of composite $\bar{\Psi}S$ leptoquarks (in addition to the heavy fundamental scalar leptoquark Φ), that must have masses $\gtrsim$1 TeV to satisfy LHC constraints.

An aspect of our work that transcends diphoton signals is the more general possibility that dark matter is a baryonlike state of a new confining sector. The relic density computation for the symmetric component is complicated by the first-order confinement phase transition of the $SU(N_{\mathrm{HC}})$ sector. We find that hyperquark masses and confinement scales below a TeV, the density is generally much smaller than the observed dark matter density, but for $m_S \sim 3$ TeV, $\Lambda_{\mathrm{HC}} \sim 20$ TeV, it could account for all of the dark matter. Searches for anomalous isotopes strongly disfavor the S hyperquark from being colored under QCD, and if it is part of an $SU(2)_L$ doublet, direct dark matter searches limit its abundance to be $\lesssim 10^{-8}$ of the observed DM density, depending upon the hyperbaryon mass. Otherwise the interactions of hyperbaryons with nuclei arise only through loops and give very weak constraints on the model parameters. Such models would be probed more directly through the collider constraints as discussed above.

ACKNOWLEDGMENTS

This work was performed in part at the Aspen Center for Physics, which is supported by National Science Foundation Grant No. PHY-1066293. We thank B. Cyr for collaboration at an early stage, J. Cornell, C. Kilic, and J. Berger for helpful discussions, and G. Dupuis for assistance with MADGRAPH. W. C. H. is supported by the Postgraduate Scholarship Program of China Scholarship Council (Program No. 201504910563). J. C. acknowledges support from NSERC (Natural Sciences and Engineering Research Council of Canada) and FRQNT (Fonds de recherche du Québec—Nature et technologies). The research of G. D. M. is supported by Land Hessen.

Note added.—Recently, Ref. [58] appeared, which treats astrophysical constraints on models similar to those we have considered, but in the case where the charged HQ Ψ is stable and binds with ordinary quarks to make a neutral relic. According to our analysis in Sec. III A, such relics are disfavored by anomalous isotope searches, which were not considered in Ref. [58].

APPENDIX A: DIJET CONSTRAINT

Here we derive a lower bound on the ratio of branching ratios $R = \mathrm{BR}(\tilde{\pi} \to \gamma\gamma)/\mathrm{BR}(\tilde{\pi} \to gg)$ from the observed LHC diphoton excess and the upper limit on dijet production. The total cross section for $pp \to \tilde{\pi}$ by gluon fusion is [59]

$$\sigma(pp \to \tilde{\pi}) = \frac{1}{s}\frac{\Gamma_{\tilde{\pi}}}{m_{\tilde{\pi}}} \times \begin{cases} 174, & \sqrt{s} = 8\ \mathrm{TeV} \\ 2137, & \sqrt{s} = 13\ \mathrm{TeV} \end{cases} \tag{A1}$$

while the cross section for $pp \to \tilde{\pi} \to \gamma\gamma$ at 13 TeV is

$$\sigma(pp \to \tilde{\pi} \to \gamma\gamma) = 5 \times 10^6\ \mathrm{fb} \cdot \frac{\Gamma(\tilde{\pi} \to \gamma\gamma)}{m_{\tilde{\pi}}} \cong 5\ \mathrm{fb} \tag{A2}$$

to match the central experimental value. Similarly, the cross section for $pp \to \tilde{\pi} \to gg$ at 8 TeV is

$$\sigma(pp \to \tilde{\pi} \to gg) = 4 \times 10^5\ \mathrm{fb} \cdot \frac{\Gamma(\tilde{\pi} \to gg)}{m_{\tilde{\pi}}} < 2.5\ \mathrm{pb} \tag{A3}$$

taking account of the 174/2137 reduction in gluon luminosity at 8 TeV, and quoting the experimental dijet limit of 2.5 pb [44]. Taking the ratio of (A2) and (A3) gives the lower limit $R > 1.6 \times 10^{-4}$.

APPENDIX B: DIPOLE MOMENTS

In this Appendix we estimate the loop-induced interactions of neutral hyperbaryons with photons, relevant for direct detection. The interactions are shown in Fig. 6. The one-loop diagram can be computed exactly, giving

$$\mu_B = \frac{q_\Phi e|\lambda|^2 f}{32\pi^2 m_\Psi} \tag{B1}$$

where f is the loop function,

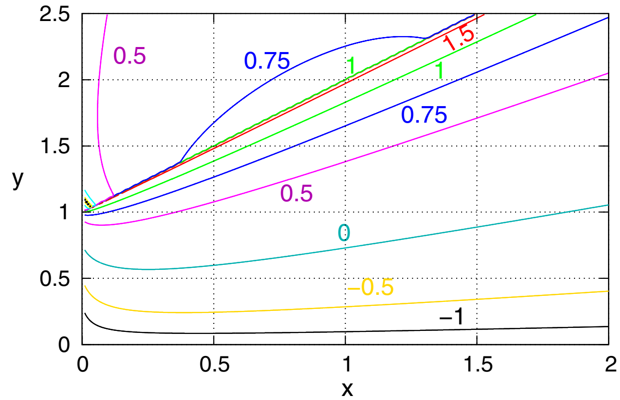

FIG. 8. Contours of $\log_{10}|f|$ for the loop function f from Eq. (B2) for the dipole moment of neutral hypequarks.

$$f = 2 + \frac{1 - x(x+y)}{y^2}\log x^2 - \frac{2[(x+y)^2 - 1](1 - x^2 + xy)}{y^2 r}\ln\frac{2x}{x^2 - y^2 + 1 - r},$$
$$r = \sqrt{[(x-y)^2 - 1][(x+y)^2 - 1]} \tag{B2}$$

with q_Φ the electric charge of Φ, $x = m_\Psi/m_\Phi$ and, $y = m_S/m_\Phi$. We plot f in Fig. 8.

To estimate the three-loop diagram, we start by integrating out the Ψ hyperquark to obtain an Euler-Heisenberg-like effective Lagrangian for the photon-hypergluons vertex [60]

$$\mathcal{L} = \frac{e g_H^3 d_{abc}}{180 m_\Psi^4 (4\pi)^2}[14\mathrm{tr}(FG^aG^bG^c) - 5\mathrm{tr}(FG^a)\mathrm{tr}(G^bG^c)] \tag{B3}$$

where $F_{\mu\nu}$, $G^a_{\mu\nu}$ are field strength tensors for photon and hypergluon respectively, $d_{abc} = 2\mathrm{tr}(\{t_a, t_b\}t_c)$ is the totally symmetric structure constant for $SU(N_{HC})$, and the traces are taken with respect to the Lorentz indices. Then the color factor for the dipole moment diagram is $d_{abc}\mathrm{tr}(t^at^bt^c)/N_{HC} = 10/9$ if $N_{HC} = 3$. We roughly estimate the effect of the Lorentz structure and the additional two loops as giving a factor of $9/(16\pi^2)^2$ to the magnetic moment,

$$\mu_B \sim \frac{e g_H^6}{180 m_\Psi^4 (4\pi)^2}\cdot\frac{10}{9}\cdot\frac{9}{(16\pi^2)^2} = \frac{e\alpha_H^3}{1152\pi^3 m_\Psi}. \tag{B4}$$

Since the limits from direct detection on this operator are very weak, it is unlikely that a more accurate computation would change our Conclusions.

APPENDIX C: ANNIHILATION CROSS SECTION FOR DOUBLET S

If the neutral hyperquark S is in an $\mathrm{SU}(2)_L$ doublet, it has additional channels for annihilation into SM states ZZ, WW, Zh and $f\bar{f}$, with purely vectorial couplings to gauge bosons due to the vectorlike nature we assumed here. The relevant s-wave cross sections are

$$\langle \sigma v\rangle_{ZZ} = \frac{g^4}{64\pi c_W^4 m_S^2}\frac{(1-x_Z^2)^{\frac{3}{2}}}{(2-x_Z^2)^2} \tag{C1}$$

$$\begin{aligned}\langle \sigma v\rangle_{WW} = \frac{g^4}{64\pi m_S^2}(1-x_W^2)^{\frac{3}{2}}\Bigg[&\frac{4(4+20x_W^2+3x_W^4)}{(4-x_Z^2)^2} \\ &-\frac{4(4x_\Psi+10x_W^2(1+x_\Psi)+3x_W^4)}{(1+x_\Psi^2-x_W^2)(4-x_Z^2)} \\ &+\frac{4x_W^2(x_\Psi^2+3x_\Psi+1)+4x_\Psi^2+5x_W^4}{(1+x_\Psi^2-x_W^2)^2}\Bigg]\end{aligned} \tag{C2}$$

$$\begin{aligned}\langle \sigma v\rangle_{Zh} = &\frac{g^4[(4-x_h^2)^2+2x_Z^2(20-x_h^2)+x_Z^4]}{4096\pi c_W^4 m_S^2(4-x_Z^2)^2} \\ &\times\sqrt{(4-x_Z^2)^2-2x_h^2(4+x_Z^2)+x_h^4}\end{aligned} \tag{C3}$$

$$\begin{aligned}\langle \sigma v\rangle_{ff} = &\frac{g^4}{8\pi c_W^4 m_S^2}\frac{(1-x_f^2)^{\frac{1}{2}}}{(4-x_Z^2)^2} \\ &\times[g_{Vf}^2(2+x_f^2)+2g_{Af}^2(1-x_f^2)]\end{aligned} \tag{C4}$$

where $c_W \equiv \cos\theta_W$, $x_i = m_i/m_S (i = Z, W, h, f, \Psi)$ and $g_{Vf}(g_{Af})$ is the (axial) vector coupling of the fermion f to Z boson. For hyperquarks in fundamental of $\mathrm{SU}(N_{\mathrm{HC}})$, there is an extra factor of $1/N_{\mathrm{HC}}$ for the above formulas taking account of averaging initial degrees of freedom.

For a hyperbaryon with N_{HC} hyperquarks, we expect that there is a coherent enhancement factor of N_{HC} for each gauge interaction vertex of the HB. Then the s-wave annihilation cross section for HB can be rescaled from Eqs. (C1)–(C4) as

$$\langle \sigma v\rangle_{ZZ}^B = \frac{4}{(N_{\mathrm{HC}}+1)^2}N_{\mathrm{HC}}^4\langle \sigma v\rangle_{ZZ} \tag{C5}$$

$$\langle \sigma v\rangle_{Zh}^B = \frac{4}{(N_{HC}+1)^2}N_{HC}^2\langle \sigma v\rangle_{Zh} \tag{C6}$$

$$\langle \sigma v\rangle_{ff}^B = \frac{4}{(N_{\mathrm{HC}}+1)^2}N_{\mathrm{HC}}^2\langle \sigma v\rangle_{ff} \tag{C7}$$

with m_S replaced by m_B and x_i by $y_i = m_i/m_B (i = Z, W, h, f, B_\Psi)$. The factor of $4/(N_{\mathrm{HC}}+1)^2$ corrects for the averaging over spin degrees of freedom of the HB. For the WW final state, the rescaling is more complicated because of interference between the s- and t-channel annihilations,

$$\begin{aligned}\langle \sigma v\rangle_{WW}^B = &\frac{4N_{\mathrm{HC}}^4}{(N_{\mathrm{HC}}+1)^2}\frac{g^4}{64\pi m_B^2}(1-y_W^2)^{\frac{3}{2}} \\ &\times\Bigg[\frac{4y_W^2(y_\Psi^2+3y_\Psi+1)+4y_\Psi^2+5y_W^4}{(1+y_\Psi^2-y_W^2)^2} \\ &-\frac{4(4y_\Psi+10y_W^2(1+y_\Psi)+3y_W^4)}{N_{\mathrm{HC}}(1+y_\Psi^2-y_W^2)(4-y_Z^2)} \\ &+\frac{4(4+20y_W^2+3y_W^4)}{N_{\mathrm{HC}}^2(4-y_Z^2)^2}\Bigg].\end{aligned} \tag{C8}$$

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