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Estimating $r_B^{D\pi}$ as an input to the determination of the CKM angle γ

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The interference between Cabibbo-favored and Cabibbo-suppressed $B \to D\pi$ decay amplitudes provides sensitivity to the CKM angle γ. The relative size of the interfering amplitudes is an important ingredient in the determination of γ. Using branching fractions from various $B \to Dh$ decays, and the measured value for r_B^{DK}, the magnitude of the amplitude ratio of $B^+ \to D^0\pi^+$ and $B^+ \to \bar{D}^0\pi^+$ decays is estimated to be $r_B^{D\pi} = 0.0053 \pm 0.0007$.

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I. INTRODUCTION

The CKM description of charged-current quark transitions has been experimentally scrutinized to an impressive accuracy. The CKM angle γ encapsulates the relative phase between $b \to c$ and $b \to u$ quark transitions, $\gamma \equiv \arg\left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right]$, and is determined with a precision of 7°, as compared to a precision below 3° deduced from indirect measurements [1,2].

The interference between $B^+ \to \bar{D}^0\pi^+$ and $B^+ \to D^0\pi^+$ decays, with the D^0 and $\bar{D}^0$ meson decaying to the same final state (charge conjugation is implied throughout), provides sensitivity to the relative weak phase γ [3–5]. Experimental determination of γ from $B \to D\pi$ like decays is influenced by the effect of the unknown hadronic parameters: $r_B^{D\pi}$, the relative magnitude of the Cabibbo-suppressed $B^+ \to D^0\pi^+$ amplitude compared to the Cabibbo-favored $B^+ \to \bar{D}^0\pi^+$ amplitude, and $\delta_B^{D\pi}$, the strong phase difference between the favored and suppressed modes. The ratio of amplitudes, $r_B^{D\pi}$, determines the size of the interference effect, and hence the sensitivity to the CKM angle γ.

A previous simultaneous determination of γ, $r_B^{D\pi}$ and $\delta_B^{D\pi}$ from the LHCb Collaboration, using $B \to D\pi$ like modes, found multiple solutions for $r_B^{D\pi}$ [6]. Consequently, an estimate of its magnitude can provide useful information to improve the determination of γ [7].

In this paper we estimate the ratio of amplitudes

$$r_B^{D\pi} \equiv A(B^+ \to D^0\pi^+)/A(B^+ \to \bar{D}^0\pi^+),$$

using branching fractions from various $B \to Dh$ decays, that proceed through similar decay topologies [8], and using the measured value of r_B^{DK} [7]. A similar approach was used to estimate the ratio of amplitudes for the decays $B^0 \to D^\pm\pi^\mp$ [9]. An overview of the decays used is given in Table I. The amplitudes of the decays that involve a kaon in the final state are denoted by primed symbols.

At tree level, the $B^+ \to \bar{D}^0\pi^+$ amplitude receives contributions from a color-allowed (T) and color-suppressed topology (C), whereas the $B^+ \to D^0\pi^+$ amplitude proceeds predominantly through the color-suppressed topology (C^{ub}) and also via the annihilation topology, as illustrated in Fig. 1, where the superscript ub indicates that the decay proceeds through a $b \to u$ transition.

The method to estimate $r_B^{D\pi}$ with $B^0 \to \bar{D}^0K^0$ decays is given in Sec. II A, whereas the use of $B^0 \to \bar{D}^0\pi^0$ decays is shown in Sec. II B. The effect of the annihilation diagram is estimated in Sec. II C.

II. ESTIMATING $r_B^{D\pi}$ FROM BRANCHING FRACTIONS

The expression for the branching fraction takes the following form:

TABLE I. The decays under study are listed, with the topologies contributing to the amplitude, the branching fraction, and the relevant CKM elements. T, C, E and A stand for color-allowed tree, color-suppressed tree, W-exchange and annihilation topologies, respectively. The primed symbols indicate the decays with a kaon as the bachelor particle in the final state. The factor $\sqrt{2}$ originates from the isospin decomposition of the neutral pion, $|\pi^0\rangle = (u\bar{u} - d\bar{d})/\sqrt{2}$.

Decay	Topology	BR ($\times 10^{-4}$) [10]	CKM factor
$A(B^+ \to \bar{D}^0\pi^+)$	$T + C$	48.1 ± 1.5	$V_{cb}V_{ud}$
$A(B^0 \to \bar{D}^0\pi^0)$	$(C - E)/\sqrt{2}$	2.63 ± 0.14	$V_{cb}V_{ud}$
$A(B^+ \to \bar{D}^0K^+)$	$T' + C'$	3.70 ± 0.17	$V_{cb}V_{us}$
$A(B^0 \to \bar{D}^0K^0)$	C'	0.52 ± 0.07	$V_{cb}V_{us}$
$A(B^+ \to D_s^+\phi)$	A'	$0.017^{+0.012}_{-0.007}$	$V_{ub}V_{cs}$

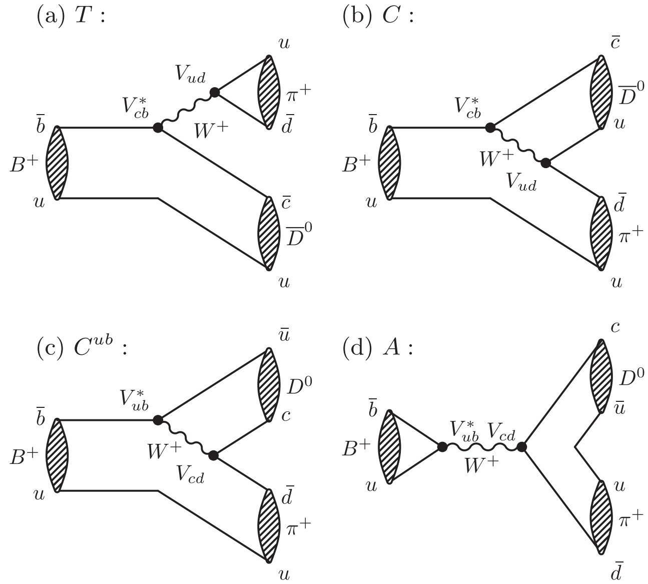

FIG. 1. (a), (b) The color-allowed (tree) (T) and color-suppressed (C) topologies contributing to the $B^+ \to \bar{D}^0\pi^+$ amplitude proceeding through V_{cb}, and (c), (d) the color-suppressed (C^{ub}) and annihilation topologies contributing to the $B^+ \to D^0\pi^+$ amplitude proceeding through V_{ub}.

$$\mathrm{BR}(B \to Dh) = |A(B \to Dh)|^2 \Phi^d_{Dh} \tau_B,$$

where h is a pion or a kaon, Φ^d_{Dh} is a phase-space factor, $A(B \to Dh)$ is the total amplitude (containing the CKM elements, form factors and decay constants) and τ_B is the lifetime of the B meson. Contributions from "rescattering" (like $B^+ \to D^-\pi^+ \to \bar{D}^0\pi^0$) are small, as shown within the framework of QCD factorization by Beneke *et al.* [11].

The following estimate of the ratio of amplitudes can be made:

$$r_B^{D\pi} = \frac{A(B^+ \to D^0\pi^+)}{A(B^+ \to \bar{D}^0\pi^+)} = \frac{|C^{ub}|}{|T + C|} = \left|\frac{V_{ub}V_{cd}}{V_{cb}V_{ud}}\right| \frac{z|C|}{|T + C|} \quad (1)$$

where z quantifies the ratio between the hadronic parts of the two color-suppressed tree diagrams proceeding through a $b \to c$ or $b \to u$ transition (shown in Fig. 1), $C^{ub} = zC \times (V_{ub}V_{cd})/(V_{cb}V_{ud})$. The contribution from the annihilation topology is also absorbed in the quantity z, and will be further discussed in Sec. II C.

We can estimate $|C|/(|T| + |C|)$ in two ways.

(A) $r_B^{D\pi} \sim A(B^0 \to \bar{D}^0K^0)/A(B^+ \to \bar{D}^0K^+)$, applying SU(3) symmetry, and correcting for the different CKM elements involved.

(B) $r_B^{D\pi} \sim A(B^0 \to \bar{D}^0\pi^0)/A(B^+ \to \bar{D}^0\pi^+)$, using external estimates for the contribution from W-exchange topologies (E) to the decay $B^0 \to \bar{D}^0\pi^0$.

TABLE II. Values of CKM elements used.

V_{CKM}	Ref.
$\lvert V_{ud}\rvert = 0.97425 \pm 0.00022$	[10]
$\lvert V_{us}\rvert = 0.2253 \pm 0.0008$	[10]
$\lvert V_{ub}\rvert = (3.72 \pm 0.16) \times 10^{-3}$	[12]
$\lvert V_{cd}\rvert = 0.225 \pm 0.008$	[10]
$\lvert V_{cs}\rvert = 0.986 \pm 0.016$	[10]
$\lvert V_{cb}\rvert = (41.1 \pm 1.3) \times 10^{-3}$	[10]

The magnitude of z will be estimated in Sec. III by comparing the result of the amplitude ratio of the decays $B^+ \to D^0K^+$, r_B^{DK}, to the measured value by LHCb [7]. For the numerical values of the CKM elements, we use the values listed in Table II.

A. Estimating $r_B^{D\pi}$ from $B^0 \to \bar{D}^0K^0$

The decays $B \to DK$ can be used to estimate the contributions of various $B \to D\pi$ decay topologies, assuming SU(3) symmetry.

The validity of this assumption was probed by comparing the $D^{(*)}K$ and $D^{(*)}\pi$ decay rates, correcting for differences in phase space, CKM elements, form factors and decay constants [8]. This assures that the decays $B^0 \to \bar{D}^0K^0$ and $B^+ \to \bar{D}^0K^+$ can be used to estimate a value for the amplitude ratio, $|C|/|T + C| = |C'|/|T' + C'|$, where

$$\left|\frac{C'}{T' + C'}\right| = \sqrt{\frac{\alpha \mathrm{BR}(B^0 \to \bar{D}^0K^0)}{\mathrm{BR}(B^+ \to \bar{D}^0K^+)}}. \quad (2)$$

The factor α quantifies a correction to the quoted value of $\mathrm{BR}(B^0 \to \bar{D}^0K^0)$ from the Particle Data Group [10]. The measured branching fraction by the *BABAR* [13] and Belle [14] collaborations is obtained from the sum over the charge-conjugate final states, and therefore the quoted branching fraction represents the sum of the $B^0 \to \bar{D}^0K^0$ and $B^0 \to D^0K^0$ branching fractions. Recently LHCb also performed an analysis of the decays $B^0_{(s)} \to \bar{D}^0K^0$ [15].

The quoted branching fraction can thus be expressed as the sum of the squares of the two color-suppressed tree amplitudes,

$$\begin{aligned}\mathrm{BR}(B^0 \to \overset{(-)}{D}{}^0K^0) &= A(B^0 \to \bar{D}^0K^0)^2 + A(B^0 \to D^0K^0)^2 \\ &= |C'|^2 + |C'^{ub}|^2 \\ &= \left(1 + z'\left|\frac{V_{ub}V_{cs}}{V_{cb}V_{us}}\right|^2\right) \times |C'|^2 \\ &= (1 + 0.156z') \times |C'|^2, \end{aligned} \quad (3)$$

where z' quantifies the ratio between the hadronic parts of the two color-suppressed tree diagrams proceeding through the $b \to u$ and $b \to c$ transitions,

$|C'^{ub}| = z'C' \times |V_{ub}V_{cs}/V_{cb}V_{us}|$. Hence, we need to correct the quoted branching fraction of the decay $B^0 \to \bar{D}^0 K^0$ to yield an estimate of the amplitude of C', relative to $|T' + C'|$ with $\alpha = 1/(1 + 0.156z')$, to obtain

$$r_B^{D\pi} = \left|\frac{V_{ub}V_{cd}}{V_{cb}V_{ud}}\right| \frac{z'}{1 + 0.156z'} \sqrt{\frac{\mathrm{BR}(B^0 \to \bar{D}^0 K^0)}{\mathrm{BR}(B^+ \to \bar{D}^0 K^+)}}. \qquad (4)$$

B. Estimating $r_B^{D\pi}$ from $B^0 \to \bar{D}^0\pi^0$

A second estimate of $r_B^{D\pi}$ can be obtained using the decay $B^0 \to \bar{D}^0\pi^0$. The decay $B^0 \to \bar{D}^0\pi^0$ receives contributions from the color-suppressed tree diagram (C) and from the W-exchange diagram (E). The comparison of the $B^0 \to \bar{D}^0\pi^0$ and $B^+ \to \bar{D}^0\pi^+$ decay rates gives [8]

$$\left|\frac{C - E}{T + C}\right| = \sqrt{2}\sqrt{\frac{\mathrm{BR}(B^0 \to \bar{D}^0\pi^0)}{\mathrm{BR}(B^+ \to \bar{D}^0\pi^+)}} = 0.331 \pm 0.010(\mathrm{BR}) \qquad (5)$$

again assuming that CKM elements, form factors, decay constants and phase space factors cancel in the ratio. The uncertainty originates from the uncertainty on the measured branching fractions. The factor $\sqrt{2}$ originates from the isospin decomposition of the neutral pion. Although the branching fraction $\mathrm{BR}(B^0 \to \bar{D}^0\pi^0)$ is determined as the sum of the D^0 and $\bar{D}^0$ final states, the $b \to u$ color-suppressed tree amplitude is negligible compared to the $b \to c$ amplitude, unlike the situation of Eq. (2).

The color-suppressed tree diagram is expected to dominate the total transition amplitude with respect to the W-exchange topology E, which is supported by the comparison of the branching fractions of $B^0 \to \bar{D}^0\pi^0$ and $B^0 \to \bar{D}^0 K^0$ [8], leading to

$$\left|\frac{C - E}{C}\right| = 0.913 \pm 0.074. \qquad (6)$$

To obtain an independent estimate of $r_B^{D\pi}$ with respect to Eq. (4), i.e. without reusing information on the branching fraction of $B^0 \to \bar{D}^0 K^0$, the size of the W-exchange amplitude can be estimated from the decay $B^0 \to D_s^- K^+$ [16], resulting in the following value [8]:

$$\left|\frac{E}{T + C}\right| = 0.056 \pm 0.004. \qquad (7)$$

Without assuming any value for the relative phase between the W-exchange (E) and color-suppressed (C) amplitudes, we assign the full contribution of the W-exchange amplitude as uncertainty to the estimate of $|C/|T + C|$,

$$\left|\frac{C}{T + C}\right| = 0.331 \pm 0.010(\mathrm{BR}) \pm 0.056(E). \qquad (8)$$

The resulting expression for $r_B^{D\pi}$ then becomes

$$r_B^{D\pi} = \left|\frac{V_{ub}V_{cd}}{V_{cb}V_{ud}}\right| \sqrt{2}z \sqrt{\frac{\mathrm{BR}(B^0 \to \bar{D}^0\pi^0)}{\mathrm{BR}(B^+ \to \bar{D}^0\pi^+)}}. \qquad (9)$$

C. Effect of annihilation topology

The relative contribution from the annihilation topology with respect to the color-suppressed tree topology for the $B^+ \to D^0\pi^+$ amplitude, is estimated using the measured branching fraction of the decay $B^+ \to D_s^+\phi$ [17], relative to the decay $B^0 \to D^0 K^0$. At lowest order the $B^+ \to D_s^+\phi$ decay proceeds purely through the annihilation topology.

The estimate of $|A/C|$ for the $B^+ \to D^0\pi^+$ case can be directly obtained from the branching ratios, when corrected for by the appropriate CKM elements and decay constants f_X,

$$|A/C| = \sqrt{\frac{\mathrm{BR}(B^+ \to D_s^+\phi)}{\mathrm{BR}(B^0 \to D^0 K^0)}} \left(\frac{V_{cb}V_{us}}{V_{ub}V_{cs}}\right) \left(\frac{f_D f_K}{f_{Ds} f_\phi}\right) \sim 0.25$$

with a large uncertainty from the branching fraction measurement of $B^+ \to D_s^+\phi$; see Table I. It is also noted that the branching fraction $\mathrm{BR}(B^+ \to \bar{D}^0 D_s^+)$ deviates from $\mathrm{BR}(B^0 \to D^- D_s^+)$, where the main difference is expected to arise from the annihilation contribution [18]. Possible contributions to these final states from rescattering processes are discussed in Ref. [19]. The relative phase between the annihilation and color-suppressed tree topology is unknown, so the annihilation contribution can enhance or reduce the value of $r_B^{D\pi}$. Assuming SU(3) symmetry, this contribution is expected to be equal in the $B^+ \to D^0 K^+$ system, and will thus be accounted for in the determination of z from r_B^{DK} in the next section.

III. CORRECTION USING r_B^{DK}

To quantify the ratio z between the hadronic parts of the $b \to u$ and $b \to c$ color-suppressed tree diagrams, $C'^{ub} = z'C' \times (V_{ub}V_{cs}/V_{cb}V_{us})$, an estimate for r_B^{DK} can be obtained in a similar way, and be compared to the fitted value for r_B^{DK} from the LHCb fit [7]. The quantity z also contains the correction due to contributions from the annihilation topology; see Fig. 1. We obtain the following expression for r_B^{DK}:

$$r_B^{DK} = \left|\frac{V_{ub}V_{cs}}{V_{cb}V_{us}}\right| \frac{z'}{1 + 0.156z'} \sqrt{\frac{\mathrm{BR}(B^0 \to \bar{D}^0 K^0)}{\mathrm{BR}(B^+ \to \bar{D}^0 K^+)}},$$

which differs from Eq. (4) by the different CKM elements involved. Inserting the value for r_B^{DK} obtained from the

LHCb fit, $r_B^{DK} = 0.101 \pm 0.006$ [7], the following estimate for the ratio of the hadronic parts of the color-suppressed amplitudes is obtained:

$$\frac{z'}{1 + 0.156z'} = 0.68 \pm 0.05 \Rightarrow z' = 0.76 \pm 0.07. \quad (10)$$

The fact that the value of z' is close to unity, indicates that the hadronic parts of the two color-suppressed tree diagrams are of similar magnitude, in particular if the annihilation topology negatively interferes with the color-suppressed tree topology, i.e. if the relative strong phase is close to 180°, which would lead to a value $z' \sim 0.75$. We assume that the deviation from unity is equal for the $D\pi$ case, with an uncertainty of 10% from SU(3) symmetry breaking effects, $z = 0.76 \pm 0.07(\mathrm{BR}) \pm 0.02(\mathrm{SU}(3))$.

Inserting the numerical values in Eq. (4) and Eq. (9) leads to the following estimates of $r_B^{D\pi}$:

$$r_B^{D\pi}(D^0K^0) = 0.0053 \pm 0.0002(\mathrm{V_{CKM}}) \pm 0.0004(\mathrm{BR}) \pm 0.0005(\mathrm{SU}(3)), \quad (11)$$

$$r_B^{D\pi}(D^0\pi^0) = 0.0053 \pm 0.0002(\mathrm{V_{CKM}}) \pm 0.0002(\mathrm{BR}) \pm 0.0009(\mathrm{E}) \pm 0.0005(z) \quad (12)$$

which are in good agreement, albeit with a large uncertainty from the W-exchange contribution to the $B^0 \to \bar{D}^0\pi^0$ decay rate. The agreement shows the internal consistency of the approach presented here. An additional 10% uncertainty from SU(3) symmetry is assumed in the estimate of Eq. (11), based on the agreement of the relative contributions of the various decay topologies to the $B \to DK$ and $B \to D\pi$ decays [8]. Given the correlated systematic uncertainties between the two results, the following combined estimate is obtained:

$$r_B^{D\pi} = 0.0053 \pm 0.0007.$$

IV. CONCLUSIONS

The estimate for the value of the amplitude ratio $r_B^{D\pi}$ that is presented here provides a valuable input to the discussion of the measurement of $r_B^{D\pi}$ at LHCb. The actual measurement of $r_B^{D\pi}$ can be achieved either by a combination of indirect measurements, as presented in Refs. [6,7], or by direct measurement using semileptonic decays of the form $B^+ \to D^0\pi^+$, where $D^0 \to K^-\mu^+\nu_\mu$, and the charge of the kaon and muon can unambiguously tag the D^0 flavor. Future determinations of $r_B^{D\pi}$ can be compared to the estimate presented here, to assess the validity of the assumptions on rescattering and SU(3) symmetry as used in this paper. The LHCb Collaboration foresees accumulating a 4 times larger data set by the end of 2018, and a 5 times smaller uncertainty at the end of the LHCb upgrade, which will result in an experimental uncertainty of the measured value of $r_B^{D\pi}$ that is smaller than the one presented here.

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