Holographic renormalization group flows in $N = 3$ Chern-Simons-Matter theory from $N = 3$ 4D gauged supergravity

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We study various supersymmetric renormalization group (RG) flows of $N = 3$ Chern-Simons-Matter theory in three dimensions by using four-dimensional $N = 3$ gauged supergravity coupled to eight vector multiplets with $SO(3) \times SU(3)$ gauge group. The AdS$_4$ critical point preserving the full $SO(3) \times SU(3)$ provides a gravity dual of $N = 3$ superconformal field theory with flavor symmetry $SU(3)$. We study the scalar potential and identify a new supersymmetric AdS$_4$ critical point preserving the full $N = 3$ supersymmetry and unbroken $SO(3) \times U(1)$ symmetry. An analytic RG flow solution interpolating between $SO(3) \times SU(3)$ and $SO(3) \times U(1)$ critical points is explicitly given. We then investigate possible RG flows from these AdS$_4$ critical points to nonconformal field theories in the IR. All of the singularities appearing in the IR turn out to be physically acceptable. Furthermore, we look for supersymmetric solutions of the form AdS$_2 \times \Sigma_2$ with Σ_2 being a two-sphere or a two-dimensional hyperbolic space and find a number of AdS$_2$ geometries preserving four supercharges with $SO(2) \times SO(2) \times SO(2)$ and $SO(2) \times SO(2)$ symmetries.

I. INTRODUCTION

AdS$_4$/CFT$_3$ correspondence is interesting in many aspects such as its applications in the study of M2-brane dynamics and in the holographic dual of condensed matter physics systems. There are a few examples of supersymmetric AdS$_4$ backgrounds with known M-theory origins. Apart from the maximally supersymmetic $N = 8$ AdS$_4 \times S^7$ compactification, there is an AdS$_4$ background with $N = 3$ supersymmetry arising from a compactification of M-theory on a tri-sasakian manifold N^{010} [1]. This is a unique solution for $2 < N < 8$ supersymmetry. The spectrum of the former example has been studied in [2] and the massless modes can be described by the maximally $SO(8)$ gauged supergravity constructed in [3]. The lowest modes of the latter are on the other hand encompassed in the gauged $N = 3$ supergravity coupled to eight vector multiplets constructed in [4], see also [5,6]. The holographic study of this background within the framework of $N = 8$ gauged supergravity and eleven-dimensional supergravity has appeared in many previous works, see for example [7–9].

The analysis of the complete spectrum of the Kaluza-Klein reduction of M-theory on AdS$_4 \times N^{010}$ has been carried out in [10], see also [11]. It has been argued that the compactification can be described by a four-dimensional effective theory in the form of $N = 3$ supergravity coupled to eight vector multiplets with $SO(3) \times SU(3)$ gauge group. From the AdS/CFT point of view, the $SO(3)$ and $SU(3)$ factors correspond respectively to the $SO(3)$

R-symmetry and $SU(3)$ flavor symmetry of the dual $N = 3$ superconformal field theory (SCFT) in three dimensions with the superconformal group $OSp(3|4) \times SU(3)$. The structure of $N = 3$ multiplets and some properties of the dual SCFT have been studied in [12–17]. Furthermore, a generalization to quiver gauge theories has been considered more recently in [18–23].

In the present work, we are interested in exploring possible supersymmetric solutions within four-dimensional $N = 3$ gauged supergravity. The $N = 3$ gauged supergravity coupled to n vector multiplets has been constructed in [4]. The theory contains $6n$ scalar fields parametrizing the $SU(3, n)/SU(3) \times SU(n) \times U(1)$ coset manifold. We will focus on the case of $n = 8$ which, together with the other three vectors from the supergravity multiplet, gives rise to eleven vector fields corresponding to a gauging of the $SO(3) \times SU(3)$ subgroup of the global symmetry group $SU(3, 8)$. The maximally supersymmetric AdS$_4$ critical point of the resulting gauged supergravity with all scalars vanishing is expected to describe the AdS$_4 \times N^{010}$ background of eleven-dimensional supergravity.

We will look for other possible supersymmetric AdS$_4$ critical points. According to the standard dictionary of the AdS/CFT correspondence, these should be dual to other conformal fixed points in the IR of the UV $N = 3$ SCFT with the $SU(3)$ flavor symmetry. We find that indeed there exists a nontrivial supersymmetric AdS$_4$ critical point with $SO(3) \times U(1)$ symmetry and unbroken $N = 3$ supersymmetry. We will also investigate holographic RG flows from the UV $N = 3$ SCFT to nonconformal field theories by looking for domain wall solutions interpolating between

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the AdS$_4$ critical points and some singular domain wall geometries in the IR.

Finally, we will look for supersymmetric AdS$_2 \times \Sigma_2$ solutions with Σ_2 being a Riemann surface. Like the higher-dimensional solutions, these solutions should be interpreted as twisted compactifications of the $N = 3$ SCFTs in three dimensions to one dimensional space-time. These results could be interesting both in the holography of three-dimensional SCFTs and in the context of AdS$_2$/CFT$_1$ correspondence which plays an important role in black hole physics, see for example [24] and [25]. Along this line, the topologically twisted indices for these theories on S^2 have been computed in [26,27]. These results can be used to find the microscopic entropy of AdS$_4$ black holes by following the approach of [28].

The paper is organized as follow. In Sec. II, we review $N = 3$ gauged supergravity in four dimensions coupled to eight vector multiplets. In Sec. III, we will give an explicit parametrization of $SU(3,8)/SU(3) \times SU(8) \times U(1)$ coset and study the scalar potential for the $SO(3)_{\mathrm{diag}}$ singlet scalars and identify possible supersymmetric vacua. An analytic renormalization group (RG) flow from the UV $SO(3) \times SU(3)$ SCFT to a new IR fixed point with residual symmetry $SO(3)_{\mathrm{diag}} \times U(1)$ is also given. We then move to possible supersymmetric RG flows to nonconformal field theories in Sec. IV. Supersymetric AdS$_2$ backgrounds obtained from twisted compactifications of AdS$_4$ on a Riemann surface are given in Sec. V. Some conclusions and comments on the results reported in this paper are presented in Sec. VI.

II. $N = 3$ GAUGED SUPERGRAVITY COUPLED TO VECTOR MULTIPLETS

In order to fix the notation and describe the relevant framework from which all the results are obtained, we will give a brief description of $N = 3$ gauged supergravity coupled to n vector multiplets and finally restrict ourselves to the case of $n = 8$. The theory has been constructed in [4] by using the geometric group manifold approach. For the present work, the space-time bosonic Lagrangian and supersymmetry transformations of fermionic component fields are sufficient. Therefore, we will focus only on these parts. The interested reader can find a more detailed construction in [4].

In four dimensions, the matter fields allowed in $N = 3$ supersymmetry are given by the fields in a vector multiplet with the following field content

$$(A_\mu, \lambda_A, \lambda, z_A).$$

Indices $A, B, \ldots = 1, 2, 3$ denote the fundamental representation of the $SU(3)_R$ part of the full $SU(3)_R \times U(1)_R$ R-symmetry. Each vector multiplet contains a vector field A_μ, four spinor fields λ and λ_A which are respectively

singlet and triplet of $SU(3)_R$, and three complex scalars z_A in the fundamental of $SU(3)_R$. For n vector multiplets, we use indices $i, j, \ldots = 1, \ldots, n$ to label each of them. Space-time and tangent space indices will be denoted by $\mu, \nu, \ldots$ and $a, b, \ldots$, respectively. In contrast to the construction in [4], we will use the metric signature $(- + + +)$ throughout this paper.

The $N = 3$ supergravity multiplet consists of the following fields

$$(e_\mu^a, \psi_{\mu A}, A_{\mu A}, \chi).$$

e_μ^a is the usual graviton, and $\psi_{\mu A}$ are three gravitini. The gravity multiplet also contains three vector fields $A_{\mu A}$ and an $SU(3)_R$ singlet spinor field χ. It should be noted that the fermions are subject to the chirality projection conditions

$$\psi_{\mu A} = \gamma_5 \psi_{\mu A}, \qquad \chi = \gamma_5 \chi, \qquad \lambda_A = \gamma_5 \lambda_A, \qquad \lambda = -\gamma_5 \lambda. \tag{1}$$

These also imply $\psi_\mu^A = -\gamma_5 \psi_\mu^A$ and $\lambda^A = -\gamma_5 \lambda^A$.

With n vector multiplets, there are $3n$ complex scalar fields $z_A{}^i$ living in the coset space $SU(3,n)/SU(3) \times SU(n) \times U(1)$. These scalars are conveniently parametrized by the coset representative $L(z)_\Lambda{}^\Sigma$. From now on, indices $\Lambda, \Sigma, \ldots$ will take the values $1, \ldots, n+3$. The coset representative transforms under the global $G = SU(3,n)$ and the local $H = SU(3) \times SU(n) \times U(1)$ symmetries by a left and right multiplications, respectively. It is convenient to split the index corresponding to H transformation as $\Sigma = (A, i)$, so we can write $L_\Lambda{}^\Sigma = (L_\Lambda{}^A, L_\Lambda{}^i)$.

Together with three vector fields from the gravity multiplet, there are $n + 3$ vectors which, accompanying by their magnetic dual, transform as the fundamental representation $\mathbf{n + 3}$ of the global symmetry $SU(3,n)$. These vector fields will be grouped together by a single notation $A_\Lambda = (A_A, A_i)$. From the result of [4], after gauging, a particular subgroup of $SO(3,n) \subset SU(3,n)$ becomes a local symmetry. The corresponding non-Abelian gauge field strengths are given by

$$F_\Lambda = dA_\Lambda + f_\Lambda{}^{\Sigma\Gamma} A_\Sigma \wedge A_\Gamma \tag{2}$$

where $f_{\Lambda\Sigma}{}^\Gamma$ denote the structure constants of the gauge group. The gauge generators T_Λ satisfy

$$[T_\Lambda, T_\Sigma] = f_{\Lambda\Sigma}{}^\Gamma T_\Gamma. \tag{3}$$

Indices on $f_{\Lambda\Sigma}{}^\Gamma$ are raised and lowered by the $SU(3,n)$ invariant tensor

$$J_{\Lambda\Sigma} = J^{\Lambda\Sigma} = (\delta_{AB}, -\delta_{ij}) \tag{4}$$

which will become the Killing form of the gauge group in the presence of gauging.

As pointed out in [4], one of the possible gauge groups takes the form of $SO(3) \times H_n$ with $SO(3) \subset SU(3)$ and H_n being an n-dimensional subgroup of $SO(n) \subset SU(n)$. In this case, only electric vector fields participate in the gauging. As a general requirement, gaugings consistent with supersymmetry impose the condition that $f_{\Lambda\Sigma\Gamma}$ obtained from the gauge structure constants via $f_{\Lambda\Sigma\Gamma} = f_{\Lambda\Sigma}{}^{\Gamma'} J_{\Gamma'\Gamma}$ are totally antisymmetric. In the present paper, we are interested only in this compact gauge group with a particular choice of $H_8 = SU(3)$ with $f_{\Lambda\Sigma}{}^{\Gamma} = (g_1\epsilon_{ABC}, g_2 f_{ijk})$. This choice clearly satisfies the consistency condition. f_{ijk} denote the $SU(3)$ structure constants while g_1 and g_2 are $SO(3) \times SU(3)$ gauge couplings. The independent, nonvanishing, components of f_{ijk} can be explicitly written as

$$f_{123} = 1, \qquad f_{147} = f_{246} = f_{257} = f_{345} = \frac{1}{2},$$

$$f_{156} = f_{367} = -\frac{1}{2}, \qquad f_{458} = f_{678} = \frac{\sqrt{3}}{2}. \tag{5}$$

Other possible gauge groups will be explored in the forthcoming paper [29].

The bosonic Lagrangian of the resulting gauged supergravity can be written as

$$e^{-1}\mathcal{L} = \frac{1}{4}R - \frac{1}{2}P_\mu^{iA}P^\mu_{Ai} - a^{\Lambda\Sigma}F^+_{\Lambda\mu\nu}F_\Sigma^{+\mu\nu} - \bar{a}^{\Lambda\Sigma}F^-_{\Lambda\mu\nu}F_\Sigma^{-\mu\nu}$$
$$- \frac{i}{2}e^{-1}\epsilon^{\mu\nu\rho\sigma}(a^{\Lambda\Sigma}F^+_{\Lambda\mu\nu} - \bar{a}^{\Lambda\Sigma}F^-_{\Lambda\mu\nu})F_{\Sigma\rho\sigma} - V. \tag{6}$$

We have translated the first order Lagrangian in the differential form language given in [4] to the usual space-time Lagrangian. In addition, we have multiplied the whole Lagrangian by a factor of 3. This results in a factor of 3 in the scalar potential compared to that given in [4].

Before giving the definitions of all quantities appearing in the above Lagrangian, we will present the fermionic supersymmetry transformations read off from the rheonomic parametrization of the fermionic curvatures as follow

$$\delta\psi_{\mu A} = D_\mu \epsilon_A - 2\epsilon_{ABC}G^B_{\mu\nu}\gamma^\nu\epsilon^C + S_{AB}\gamma_\mu\epsilon^B, \tag{7}$$

$$\delta\chi = -\frac{1}{2}G^A_{\mu\nu}\gamma^{\mu\nu}\epsilon_A + \mathcal{U}^A\epsilon_A, \tag{8}$$

$$\delta\lambda_i = -P_{i\mu}{}^A\gamma^\mu\epsilon_A + \mathcal{N}_{iA}\epsilon^A, \tag{9}$$

$$\delta\lambda_{iA} = -P_{i\mu}{}^B\gamma^\mu\epsilon_{ABC}\epsilon^C - G_{i\mu\nu}\gamma^{\mu\nu}\epsilon_A + \mathcal{M}_{iA}{}^B\epsilon_B. \tag{10}$$

From the coset representative, we can define the Mourer-Cartan one-form

$$\Omega_\Lambda{}^\Pi = (L^{-1})_\Lambda{}^\Sigma dL_\Sigma{}^\Pi + (L^{-1})_\Lambda{}^\Sigma f_\Sigma{}^{\Omega\Gamma}A_\Omega L_\Gamma{}^\Pi. \tag{11}$$

The inverse of $L_\Lambda{}^\Sigma$ is related to the coset representative via the following relation

$$(L^{-1})_\Lambda{}^\Sigma = J_{\Lambda\Pi}J^{\Sigma\Delta}(L_\Delta{}^\Pi)^*. \tag{12}$$

The component $\Omega_i{}^A = (\Omega_A{}^i)^*$ gives the vielbein $P_i{}^A$ of the $SU(3,n)/SU(3) \times SU(n) \times U(1)$ coset. Other components give the composite connections $(Q_A{}^B, Q_i{}^j, Q)$ for $SU(3) \times SU(n) \times U(1)$ symmetry

$$\Omega_A{}^B = Q_A{}^B - n\delta^B_A Q, \qquad \Omega_i{}^j = Q_i{}^j + 3\delta^j_i Q. \tag{13}$$

It should be noted that $Q_A{}^A = Q_i{}^i = 0$.

The covariant derivative for ϵ_A is defined by

$$D\epsilon_A = d\epsilon_A + \frac{1}{4}\omega^{ab}\gamma_{ab}\epsilon_A + Q_A{}^B\epsilon_B + \frac{1}{2}nQ. \tag{14}$$

The scalar matrices S_{AB}, $\mathcal{U}^A$, $\mathcal{N}_{iA}$ and $\mathcal{M}_{iA}{}^B$ are given in terms of the "boosted structure constants" $C^\Lambda{}_{\Pi\Gamma}$ as follow

$$S_{AB} = \frac{1}{4}(\epsilon_{BPQ}C_A{}^{PQ} + \epsilon_{ABC}C_M{}^{MC})$$

$$= \frac{1}{8}(C_A{}^{PQ}\epsilon_{BPQ} + C_B{}^{PQ}\epsilon_{APQ}),$$

$$\mathcal{U}^A = -\frac{1}{4}C_M{}^{MA}, \qquad \mathcal{N}_{iA} = -\frac{1}{2}\epsilon_{APQ}C_i{}^{PQ},$$

$$\mathcal{M}_{iA}{}^B = \frac{1}{2}(\delta^B_A C_{iM}{}^M - 2C_{iA}{}^B) \tag{15}$$

where

$$C^\Lambda{}_{\Pi\Gamma} = L_{\Lambda'}{}^\Lambda (L^{-1})_\Pi{}^{\Pi'}(L^{-1})_\Gamma{}^{\Gamma'} f_{\Pi'\Gamma'}{}^{\Lambda'} \quad \text{and}$$
$$C_\Lambda{}^{\Pi\Gamma} = J_{\Lambda\Lambda'}J^{\Pi\Pi'}J^{\Gamma\Gamma'}(C^{\Lambda'}{}_{\Pi'\Gamma'})^*. \tag{16}$$

With all these definitions, the scalar potential can be written as

$$V = -2S_{AC}S^{CM} + \frac{2}{3}\mathcal{U}_A\mathcal{U}^A + \frac{1}{6}\mathcal{N}_{iA}\mathcal{N}^{iA} + \frac{1}{6}\mathcal{M}^{iB}{}_A\mathcal{M}_{iB}{}^A$$

$$= \frac{1}{8}|C_{iA}{}^B|^2 + \frac{1}{8}|C_i{}^{PQ}|^2 - \frac{1}{4}(|C_A{}^{PQ}|^2 - |C_P|^2) \tag{17}$$

with $C_P = -C_{PM}{}^M$.

We now come to the gauge fields. The self-dual and antiself-dual field strengths are defined by

$$F^\pm_{\Lambda ab} = F_{\Lambda ab} \mp \frac{i}{2}\epsilon_{abcd}F_\Lambda^{cd} \tag{18}$$

with $\frac{1}{2}\epsilon_{abcd}F_\Lambda^{\pm cd} = \pm i F^\pm_{\Lambda ab}$ and $F^\pm_{\Lambda ab} = (F^\mp_{\Lambda ab})^*$. The explicit form of the symmetric matrix $a_{\Lambda\Sigma}$ in term of the coset representative is quite involved. We will not repeat it

here, but the interested reader can find a detailed discussion in the appendix of [4].

Finally, the field strengths appearing in the supersymmetry transformations are given in terms of $F^{\pm}_{\Lambda\mu\nu}$ by

$$G^i_{\mu\nu} = -\frac{1}{2}M^{ij}(L^{-1})_j{}^\Lambda F^-_{\Lambda\mu\nu},$$

$$G^A_{\mu\nu} = \frac{1}{2}M^{AB}(L^{-1})_B{}^\Lambda F^+_{\Lambda\mu\nu} \qquad (19)$$

where M^{ij} and M^{AB} are respectively inverse matrices of

$$M_{ij} = (L^{-1})_i{}^\Lambda(L^{-1})_j{}^\Pi J_{\Lambda\Pi} \quad \text{and}$$

$$M_{AB} = (L^{-1})_A{}^\Lambda(L^{-1})_B{}^\Pi J_{\Lambda\Pi}. \qquad (20)$$

In subsequent sections, we will study supersymmetric solutions to this gauged supergravity with $SO(3) \times SU(3)$ gauge group.

III. FLOWS TO $SO(3)_{\mathrm{diag}} \times U(1)$ IR FIXED POINT WITH $N = 3$ SUPERSYMMETRY

We now consider the case of $n = 8$ vector multiplets and $SO(3) \times SU(3)$ gauge group. There are 48 scalars transforming in $(\mathbf{3}, \bar{\mathbf{8}}) + (\bar{\mathbf{3}}, \mathbf{8})$ representation of the local symmetry $SU(3) \times SU(8)$. It is efficient and more convenient to study the scalar potential on a particular submanifold of the full $SU(3,8)/SU(3) \times SU(8) \times U(1)$ coset space. This submanifold consists of all scalars which are singlets under a particular subgroup of the full gauge group $SO(3) \times SU(3)$. All vacua found on this submanifold are guaranteed to be vacua on the full scalar manifold by a simple group theory argument [30].

A. Supersymmetric AdS$_4$ critical points

In terms of the dual $N = 3$ SCFT, the $SO(3)$ part of the full gauge group corresponds to the R-symmetry of $N = 3$ supersymmetry in three dimensions while the $SU(3)$ part plays the role of the global symmetry. There are no singlet scalars under the $SO(3)$ R-symmetry. In order to have $SO(3)$ symmetry, we then consider scalars invariant under a diagonal $SO(3)$ subgroup of $SO(3) \times SO(3) \subset SO(3) \times SU(3)$.

Before going to the detail of an explicit parametrization, we first introduce an element of 11×11 matrices

$$(e_{\Lambda\Sigma})_{\Pi\Gamma} = \delta_{\Lambda\Pi}\delta_{\Sigma\Gamma}. \qquad (21)$$

The $SO(3) \times SU(3)$ gauge generators can be obtained from the structure constant $(T_\Lambda)_\Pi{}^\Gamma = f_{\Lambda\Pi}{}^\Gamma$. Accordingly, the $SO(3)$ part is generated by $(T^{(1)}_A)_\Pi{}^\Gamma = f_{A\Pi}{}^\Gamma$, $A = 1, 2, 3$, and the $SU(3)$ generators are given by $(T^{(2)}_i)_\Pi{}^\Gamma = f_{i+3,\Pi}{}^\Gamma$, $i = 1, \ldots, 8$. The $SO(3)_{\mathrm{diag}}$ is then generated by $(T^{(1)}_A)_\Pi{}^\Gamma + (T^{(2)}_A)_\Pi{}^\Gamma$.

Under $SU(3) \to SO(3) \times U(1)$, we have the branching

$$\mathbf{8} = \mathbf{3}_0 + \mathbf{1}_0 + \mathbf{2}_3 + \mathbf{2}_{-3}. \qquad (22)$$

This implies that the 48 scalars transform under $SO(3)_{\mathrm{diag}} \times U(1)$ as

$$2 \times [\mathbf{3}_0 \times (\mathbf{3}_0 + \mathbf{1}_0 + \mathbf{2}_3 + \mathbf{2}_{-3})]$$
$$= 2 \times (\mathbf{1}_0 + \mathbf{3}_0 + \mathbf{5}_0 + \mathbf{2}_3 + \mathbf{4}_3 + \mathbf{2}_{-3} + \mathbf{4}_{-3}). \quad (23)$$

A factor of 2 comes from the fact that both $(\mathbf{3}, \bar{\mathbf{8}})$ and $(\bar{\mathbf{3}}, \mathbf{8})$ of $SU(3) \times SU(8)$ become $(\mathbf{3}, \mathbf{8})$ under $SO(3) \times SU(3)$. We see that there are two $SO(3)_{\mathrm{diag}}$ singlets. These correspond to the $SU(3, 8)$ noncompact generators

$$\hat{Y}_1 = e_{14} + e_{41} + e_{25} + e_{52} + e_{36} + e_{63},$$
$$\hat{Y}_2 = -ie_{14} + ie_{41} - ie_{25} + ie_{52} - ie_{36} + ie_{63}. \qquad (24)$$

These two generators are noncompact generators of $SL(2, \mathbb{R}) \subset SU(3, 8)$ commuting with $SO(3)_{\mathrm{diag}}$. The $SO(2)$ compact generator of this $SL(2, \mathbb{R})$ is given by

$$J = \mathrm{diag}(2i\delta^{AB}, -2i\delta^{i+3,j+3}, 0, 0, 0, 0, 0), \quad i, j = 1, 2, 3. \qquad (25)$$

From (23), it should be noted that the two singlets are uncharged under the $U(1)$ factor from $SU(3)$. Therefore, the full symmetry of $\hat{Y}_{1,2}$ is in fact $SO(3)_{\mathrm{diag}} \times U(1)$.

By using a Euler angle parametrization of $SL(2, \mathbb{R})/SO(2) \sim SO(2, 1)/SO(2) \sim SU(1, 1)/U(1)$, we parametrize the coset representative by

$$L = e^{\varphi J}e^{\lambda\hat{Y}_1}e^{-\varphi J}. \qquad (26)$$

The resulting scalar potential can be written as

$$V = -\frac{3}{64}e^{-6\lambda}[(1 + e^{4\lambda})[(1 + e^{2\lambda})^4 g_1^2 + (e^{2\lambda} - 1)^4 g_2^2]$$
$$+ 2(e^{4\lambda} - 1)^3 \cos(4\varphi)g_1g_2]. \qquad (27)$$

The above potential admits two supersymmetric AdS$_4$ critical points. The first one is a trivial critical point, preserving the full $SO(3) \times SU(3)$ symmetry, with all scalars vanishing

$$\lambda = \varphi = 0, \qquad V_0 = -\frac{3}{2}g_1^2 \qquad (28)$$

where V_0 is the value of the potential at the critical point, the cosmological constant. This AdS$_4$ critical point should be identified with a compactification of M-theory on N^{010} manifold and dual to an $N = 3$ SCFT in three dimensions with $SU(3)$ flavor symmetry. In the present convention, the

AdS$_4$ radius L is related to the value of the cosmological constant by

$$L^2 = -\frac{3}{2V_0} = \frac{1}{g_1^2}. \quad (29)$$

At this critical point, all of the 48 scalars have $m^2 L^2 = -2$ in agreement with the spectrum of M-theory on AdS$_4 \times N^{010}$. These scalars are dual to operators of dimension $\Delta = 1, 2$ in the dual SCFT.

Another supersymmetric critical point is given by

$$\varphi = 0, \qquad \lambda = \frac{1}{2} \ln \left[\frac{g_2 - g_1}{g_2 + g_1} \right], \qquad V_0 = -\frac{3g_1^2 g_2^2}{2(g_2^2 - g_1^2)}. \quad (30)$$

This critical point is an AdS$_4$ critical point for $g_2^2 > g_1^2$ as required by the reality of λ. That this critical point preserves supersymmetry can be checked from the supersymmetry transformations given in the next subsection. The AdS$_4$ radius can be found to be

$$L^2 = \frac{g_2^2 - g_1^2}{g_1^2 g_2^2}. \quad (31)$$

More precisely, there are many critical points, equivalent to the one given above, with $\sin(4\varphi_0) = 0$ or $\varphi = \frac{n\pi}{4}$, $n \in \mathbb{Z}$. At this critical point, we can determine the full scalar masses as shown in Table I.

From the table, we see seven massless scalars corresponding to Goldstone bosons of the symmetry breaking of $SO(3) \times SU(3)$ to $SO(3)_{\text{diag}} \times U(1)$. The singlet scalar λ is dual to an irrelevant operator of dimension 4 at this critical point while φ is still dual to a relevant operator of dimension $\Delta = 1, 2$. It should also be noted that all the masses satisfy the Breitenlohner-Freedman bound as expected for a supersymmetric critical point.

TABLE I. Scalar masses at the $N = 3$ supersymmetric AdS$_4$ critical point with $SO(3)_{\text{diag}} \times U(1)$ symmetry and the corresponding dimensions of the dual operators.

$SO(3)_{\text{diag}} \times U(1)$ representations	$m^2 L^2$	Δ
$\mathbf{1}_0$	$4, -2$	$4, (1,2)$
$\mathbf{2}_3$	$0_{(\times 2)}, -2_{(\times 2)}$	$3, (1,2)$
$\mathbf{2}_{-3}$	$0_{(\times 2)}, -2_{(\times 2)}$	$3, (1,2)$
$\mathbf{3}_0$	$0_{(\times 3)}, -2_{(\times 3)}$	$3, (1,2)$
$\mathbf{4}_3$	$-\frac{9}{4}_{(\times 4)}, -2_{(\times 4)}$	$\frac{3}{2}, (1,2)$
$\mathbf{4}_{-3}$	$-\frac{9}{4}_{(\times 4)}, -2_{(\times 4)}$	$\frac{3}{2}, (1,2)$
$\mathbf{5}_0$	$-2_{(\times 10)}$	$(1,2)$

There is also a nonsupersymmetric critical point, but we will not give its location and value of the cosmological constant here due to its complexity.

B. A supersymmetric RG flow

In this subsection, we will find a supersymmetric domain wall solution interpolating between two AdS$_4$ critical points identified previously. In order to do this, we will set up the corresponding Bogomol'nyi–Prasad–Sommerfeld (BPS) equations by setting the supersymmetry transformations of fermions to zero. The nonvanishing bosonic fields are the metric and $SO(3)_{\text{diag}}$ singlet scalars.

We adopt the standard domain wall ansatz for the four-dimensional metric

$$ds^2 = e^{2A(r)} dx_{1,2}^2 + dr^2 \quad (32)$$

with $dx_{1,2}^2$ being the flat Minkowski metric in three dimensions. We will use the same convention as in [31]. All spinors will be written as chiral projected Majorana spinors. For example, we have

$$\epsilon_A = \frac{1}{2}(1 + \gamma_5)\tilde{\epsilon}^A, \qquad \epsilon^A = \frac{1}{2}(1 - \gamma_5)\tilde{\epsilon}^A \quad (33)$$

where $\tilde{\epsilon}^A$ is a Majorana spinor. In this Majorana representation, all of the gamma matrices γ^a are real while $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$ is purely imaginary. As a consequence, ϵ^A and ϵ_A are simply related by a complex conjugation, $\epsilon_A = (\epsilon^A)^*$.

In the present case, it turns out that $C_M{}^{MA} = 0$. Therefore, the variation $\delta\chi$ is identically zero. To satisfy the conditions $\delta\lambda_i = 0$ and $\delta\lambda_{iA} = 0$, we impose the following projector

$$\gamma^{\hat{r}} \epsilon_A = e^{i\Lambda} \epsilon^A \quad (34)$$

which implies $\gamma^{\hat{r}} \epsilon^A = e^{-i\Lambda} \epsilon_A$. With this projector, the conditions $\delta\psi_{\mu A} = 0$, for $\mu = 0, 1, 2$, reduce to a single condition

$$A' e^{i\Lambda} - \mathcal{W} = 0 \quad (35)$$

where $'$ is used to denote the r-derivative. The "superpotential" $\mathcal{W}$ is related to the eigenvalues of S_{AB}. It turns out that in the present case, S_{AB} is diagonal

$$S_{AB} = \frac{1}{2} \mathcal{W} \delta_{AB}. \quad (36)$$

This would imply unbroken $N = 3$ supersymmetry provided that the conditions $\delta\lambda_i = 0$ and $\delta\lambda_{iA} = 0$ can be satisfied. The explicit form of $\mathcal{W}$ is given by

$$\mathcal{W} = -\frac{1}{8}e^{-3\lambda}[[(1 + e^{2\lambda})^3 g_1 + (e^{2\lambda} - 1)^3 g_2]\cos(2\varphi)$$
$$+ i[(1 + e^{2\lambda})^3 g_1 - (e^{2\lambda} - 1)^3 g_2]\sin(2\varphi)]. \tag{37}$$

By writing $\mathcal{W} = |\mathcal{W}|e^{i\omega} \equiv We^{i\omega}$, the imaginary part of Eq. (35) gives rise to the relation

$$e^{i\Lambda} = \pm e^{i\omega}. \tag{38}$$

On the other hand, $\delta\lambda_i = 0$ and $\delta\lambda_{iA} = 0$ equations reduce to two independent equations that can be written as

$$\lambda' - \frac{1}{3}e^{-i\Lambda}\frac{\partial\mathcal{W}}{\partial\lambda} \pm ie^{-2\lambda}(e^{4\lambda} - 1)\varphi' = 0. \tag{39}$$

These two equations imply $\varphi' = 0$ or $\varphi = \varphi_0$ with φ_0 being a constant. It turns out that consistency with the field equations require $\sin(4\varphi_0) = 0$ or $\varphi_0 = \frac{n\pi}{4}$, $n \in \mathbb{Z}$. To make the solution interpolates between the two critical points, we will set $\varphi_0 = 0$.

With this choice, $\mathcal{W}$ is real, and the phase factor $e^{i\Lambda}$ is simply given by

$$e^{i\Lambda} = \pm 1. \tag{40}$$

We can finally write down all the relevant BPS equations as

$$\lambda' = \mp\frac{1}{8}e^{-3\lambda}(e^{4\lambda} - 1)[(1 + e^{2\lambda})g_1 + (e^{2\lambda} - 1)g_2], \tag{41}$$

$$A' = \pm\frac{1}{8}e^{-3\lambda}[(1 + e^{2\lambda})^3 g_1 + (e^{2\lambda} - 1)^3 g_2]. \tag{42}$$

In what follows, we will choose the upper signs in order to identify the trivial critical point with the UV fixed point of the RG flow.

As in other cases, $W = |\mathcal{W}|$ provides the "real superpotential" in term of which the scalar potential can be written as

$$V = -\frac{1}{6}\left(\frac{\partial W}{\partial\lambda}\right)^2 - \frac{3}{2}W^2. \tag{43}$$

In the present case, the scalar kinetic terms are given by

$$-\frac{1}{2}P^{iA}_\mu P^\mu_{Ai} = -\frac{3}{2}e^{-4\lambda}(e^{4\lambda} - 1)^2\varphi'^2 - \frac{3}{2}\lambda'^2. \tag{44}$$

With all these results, it can be verified that the second order field equations are satisfied by the first order BPS equations (41) and (42).

We now solve for the RG flow solution. Equation (41) clearly admits two fixed points at $\lambda = 0$ and $\lambda = \frac{1}{2}\ln[\frac{g_2-g_1}{g_2+g_1}]$. These are supersymmetric critical points identified previously. The solution for Eq. (41) is given by

$$g_1 g_2 r = C_1 + 2g_1\tan^{-1}e^\lambda - 2\sqrt{g_2^2 - g_1^2}\tanh^{-1}\left[e^\lambda\sqrt{\frac{g_2 + g_1}{g_2 - g_1}}\right]$$
$$+ g_2\ln\left[\frac{1 + e^\lambda}{1 - e^\lambda}\right] \tag{45}$$

where the constant C_1 can be set to zero by shifting the r coordinate. By choosing $g_1, g_2 > 0$, it can be seen that as $\lambda \to 0$, we find $r \to \infty$, and $r \to -\infty$ as $\lambda \to \frac{1}{2}\ln[\frac{g_2-g_1}{g_2+g_1}]$. These correspond to the UV and IR fixed points of the RG flow, respectively. Near the two critical points, we find

$$\text{UV: } \lambda \sim e^{-g_1 r} \sim e^{-\frac{r}{L_{\text{UV}}}}$$
$$\text{IR: } \lambda \sim e^{\frac{g_1 g_2}{\sqrt{g_2^2 - g_1^2}}r} \sim e^{\frac{r}{L_{\text{IR}}}}. \tag{46}$$

Therefore, the flow is driven by an operator of dimension $\Delta = 1, 2$, and this operator becomes irrelevant in the IR with the corresponding scaling dimension $\Delta = 4$.

Finally, by combining Eqs. (41) and (42), we obtain

$$\frac{dA}{d\lambda} = -\frac{(1 + e^{2\lambda})^3 g_1 + (e^{2\lambda} - 1)^3 g_2}{(e^{4\lambda} - 1)[(1 + e^{2\lambda})g_1 + (e^{2\lambda} - 1)g_2]} \tag{47}$$

whose solution is given by

$$A = C_2 + \lambda - \ln(1 - e^{4\lambda}) + \ln[g_1(1 + e^{2\lambda}) + g_2(e^{2\lambda} - 1)]. \tag{48}$$

The integration constant C_2 can be neglected by rescaling the coordinates of $dx_{1,2}^2$. It can readily be verified that $A \to \frac{r}{L}$ when $\lambda \to 0, \frac{1}{2}\ln[\frac{g_2-g_1}{g_2+g_1}]$ as expected for the two conformal fixed points.

We now identify a possible dual operator driving this flow. From the results of [11,12], the eight vector multiplets in the $N = 3$ gauged supergravity correspond to the global $SU(3)$ flavor current given, in terms of the $N = 2$ language, by the superfield

$$\Sigma^i_j = \frac{1}{\sqrt{2}}\text{Tr}(U^i\bar{U}_j + \bar{V}^i V_j) - \text{flavor trace}. \tag{49}$$

The trace (Tr) above is over the gauge group $SU(N) \times SU(N)$ under which U^i and V_i transform as a bifundamental. The hypermultiplets $(U^i, i\bar{V}^i)$ form a doublet of $SU(2)_R$ and transform in a fundamental representation the $SU(3)$ flavor. The flow given above is driven by scalar fields in the vector multiplets, and these scalars arise from the eleven-dimensional metric rather than the three-form field [10]. According to the UV behavior in (46), we then expect that the flow is driven by turning on an $SO(3) \times U(1)$ invariant combination of the scalar mass terms within Σ^i_j.

IV. FLOWS TO NONCONFORMAL FIELD THEORIES

In this section, we consider RG flows to nonconformal field theories. The supergravity solutions will interpolate between UV AdS$_4$ critical points and domain walls in the IR.

A. Flows within $SO(2) \times SO(2) \times SO(2)$ singlet scalars

We first consider scalars invariant under $SO(2) \times SO(2) \times SO(2)$ symmetry. The first $SO(2)$ is embedded in the $SO(3)_R$ such that $\mathbf{3} \rightarrow \mathbf{2} + \mathbf{1}$. From the branching of $\mathbf{3}_0 + \mathbf{1}_0$ in (22) under $SO(2) \subset SO(3) \subset SU(3)$, we find $\mathbf{2}_0 + \mathbf{1}_0 + \mathbf{1}_0$. Combining the two decompositions together, we finally obtain the relevant scalar representations under $SO(2) \times SO(2) \times SO(2)$

$$2 \times [(\mathbf{3}_0, \mathbf{3}_0 + \mathbf{1}_0)] = 2 \times [2(\mathbf{1}, \mathbf{1})_0, (\mathbf{1}, \mathbf{2})_0, 2(\mathbf{2}, \mathbf{1})_0, (\mathbf{2}, \mathbf{2})_0]. \tag{50}$$

There are accordingly four singlets corresponding to the noncompact generators

$$\tilde{Y}_1 = e_{3,11} + e_{11,3}, \qquad \tilde{Y}_2 = ie_{11,3} - ie_{3,11},$$
$$\tilde{Y}_3 = e_{3,6} + e_{6,3}, \qquad \tilde{Y}_4 = ie_{6,3} - ie_{3,6}. \tag{51}$$

It should be noted that $\tilde{Y}_{1,2}$ are invariant under a bigger symmetry $SO(2) \times SU(2) \times U(1)$. The above four singlets correspond to noncompact directions of $SU(2,1) \subset SU(3,8)$. We then effectively need to parametrize the $SU(2,1)/SU(2) \times U(1)$ coset manifold. It is more convenient to adopt a parametrization using $SU(2)$ Euler angles. The $SU(2) \times U(1)$ compact subgroup of the $SU(2,1)$ group is generated by

$$J_1 = \frac{i}{2}(e_{11,11} - e_{66}), \qquad J_2 = \frac{1}{2}(e_{6,11} - e_{11,6}),$$
$$J_3 = -\frac{i}{2}(e_{6,11} + e_{11,6}), \qquad \hat{J} = \frac{i}{2\sqrt{3}}(2e_{33} - e_{66} - e_{11,11}) \tag{52}$$

with $[J_\alpha, J_\beta] = \epsilon_{\alpha\beta\gamma}J_\gamma$ and $\hat{J}$ corresponding to the $U(1)$.

The coset representative for $SO(2) \times SO(2) \times SO(2)$ invariant scalars is accordingly parametrized by

$$L = e^{\varphi_1 J_1} e^{\varphi_2 J_2} e^{\varphi_3 J_3} e^{\Phi \tilde{Y}_1} e^{-\varphi_3 J_3} e^{-\varphi_2 J_2} e^{-\varphi_1 J_1}. \tag{53}$$

The scalar potential turns out to be independent of all the φ_i

$$V = -\frac{1}{2}g_1^2[1 + 2\cosh(2\Phi)] \tag{54}$$

which clearly has only the trivial critical point at $\Phi = 0$.

The matrix S_{AB} in this case is diagonal

$$S_{AB} = \frac{1}{2}g_1 \cosh \Phi \delta_{AB} \tag{55}$$

implying that the maximal $N = 3$ supersymmetry is preserved if the conditions $\delta\lambda_i = 0$ and $\delta\lambda_{iA} = 0$ can be satisfied. This is similar to solutions studied in the maximal $N = 8$ gauged supergravity in [32].

We can proceed as in the previous section to analyze other BPS equations. Since $\mathcal{W}$ is real in this case, we simply have $\omega = 0$ and $e^{i\Lambda} = \pm1$. Generally, the flow equations for a scalar ϕ_i is, up to a numerical factor, given by $G^{ij} \frac{\partial W}{\partial \phi^j}$ in which G^{ij} being the inverse of the scalar matrix appearing in the scalar kinetic terms. The above superpotential depending only on Φ will immediately give $\varphi_i' = 0$. Remarkably, this precisely agrees with the results from solving $\delta\lambda_i = 0$ and $\delta\lambda_{iA} = 0$ equations. This is another consistency check for our results.

We now give the flow equations after choosing a choice of signs such that the $SO(3) \times SU(3)$ AdS$_4$ critical point is identified with $r \rightarrow \infty$

$$\Phi' = -g_1 \sinh \Phi, \qquad \varphi_i' = 0, \qquad i = 1, 2, 3,$$
$$A' = g_1 \cosh \Phi. \tag{56}$$

A solution to the above equations can be readily obtained

$$\Phi = \pm \ln\left[\frac{e^{g_1 r} - e^C}{e^{g_1 r} + e^C}\right],$$
$$A = \Phi - \ln(1 - e^{2\Phi}) + C'. \tag{57}$$

As $r \rightarrow \infty$, the solution approaches the UV AdS$_4$ critical point with $\Phi \sim e^{-g_1 r}$ and $A \sim g_1 r$. At $g_1 r \sim C$, there is a singularity with Φ becoming infinite

$$\Phi \sim \pm \ln(g_1 r - C). \tag{58}$$

Both of the signs give rise to the same domain wall metric in the IR

$$ds^2 = (g_1 r - C)^2 dx_{1,2}^2 + dr^2. \tag{59}$$

It can also be checked that the potential (54) is bounded above for $\Phi \rightarrow \pm\infty$ namely $V(\Phi \rightarrow \pm\infty) \rightarrow -\infty$. The singularity is then physical according to the criterion of [33]. Therefore, the solution should be interpreted as an RG flow from the UV $N = 3$ SCFT to an $N = 3$ nonconformal field theory in the IR.

B. Flows within $SO(2)_{\mathbf{diag}} \times SO(2)$ singlet scalars

The solutions considered in the previous subsection describe RG flows from the trivial $N = 3$ critical point. These solutions do not connect to the nontrivial AdS$_4$

critical point identified in Sec. III. We now consider another class of flow solutions describing RG flows from both the trivial and nontrivial critical points to IR gauge theories with $SO(2) \times SO(2)$ symmetry.

We will consider scalars which are singlets under $SO(2)_{\text{diag}} \times SO(2) \subset SO(2) \times SO(2) \times SO(2)$ symmetry. Further decomposing the scalar representations gives eight singlets under this symmetry. These correspond to the following $SU(3,8)$ noncompact generators

$$\bar{Y}_1 = e_{36} + e_{63}, \quad \bar{Y}_2 = -ie_{36} + ie_{63},$$

$$\bar{Y}_3 = e_{25} + e_{52} + e_{14} + e_{41}, \quad \bar{Y}_4 = -ie_{25} + ie_{52} - ie_{14} + ie_{41},$$

$$\bar{Y}_5 = e_{15} + e_{51} - e_{24} - e_{42}, \quad \bar{Y}_6 = -ie_{15} + ie_{51} + ie_{24} - ie_{42},$$

$$\bar{Y}_7 = e_{3,11} + e_{11,3}, \quad \bar{Y}_8 = -ie_{3,11} + ie_{11,3}. \quad (60)$$

In this case, using Euler parametrization does not simplify the result to any useful extent. We then simply parametrize the coset representative in a straightforward way

$$L = e^{\Phi_1 \bar{Y}_1} e^{\Phi_2 \bar{Y}_2} e^{\Phi_3 \bar{Y}_3} e^{\Phi_4 \bar{Y}_4} e^{\Phi_5 \bar{Y}_5} e^{\Phi_6 \bar{Y}_6} e^{\Phi_7 \bar{Y}_7} e^{\Phi_8 \bar{Y}_8}. \quad (61)$$

The resulting scalar potential and BPS equations are much more complicated than the previous cases. We refrain from giving their explicit form here.

However, there are some interesting truncations. We will simply consider these and give the full result within these truncations. With only Φ_7 and Φ_8 nonvanishing, the residual symmetry is enhanced to $SO(2) \times SU(2) \times SO(2)$. Furthermore, if one of these two scalars is set to zero, we recover the result obtained in the previous subsection. A new deformation arises from Φ_7 and Φ_8 both being nonzero. In this case, the $N = 3$ supersymmetry is broken to $N = 1$.

The matrix S_{AB} is diagonal with two different eigenvalues, with $S_{11} = S_{22}$. It turns out that the third eigenvalue gives the true superpotential

$$\mathcal{W} = 2S_{33} = g_1 \cosh \Phi_7 \cosh \Phi_8 + ig_1 \sinh \Phi_7 \sinh \Phi_8 \quad (62)$$

in terms of which the scalar potential can be written as

$$V = \frac{1}{2} G^{ij} \frac{\partial W}{\partial \Phi^i} \frac{\partial W}{\partial \Phi^j} - \frac{3}{2} W^2$$

$$= -\frac{1}{2} g_1^2 [1 + 2 \cosh(2\Phi_7) \cosh(2\Phi_8)] \quad (63)$$

where the real superpotential is given by

$$W = |\mathcal{W}| = \frac{1}{\sqrt{2}} g_1 \sqrt{1 + \cosh(2\Phi_7) \cosh(2\Phi_8)}. \quad (64)$$

In the above result, we have used the scalar kinetic term

$$-\frac{1}{2} G_{ij} \partial_\mu \Phi^i \partial^\mu \Phi^j = -\frac{1}{2} P_\mu^{Ai} P_{iA}^\mu$$

$$= -\frac{1}{2} \cosh^2(2\Phi_8) \Phi_7'^2 - \frac{1}{2} \Phi_8'^2 \quad (65)$$

which gives G_{ij}, $i, j = 7, 8$. The inverse G^{ij} can readily be read off. The supersymmetry transformations of $\psi_{\mu A}$ corresponding to $\epsilon_{1,2}$ can be satisfied by setting $\epsilon_{1,2} = 0$. Accompanied by the usual γ^r projection, the unbroken supersymmetry is then $N = 1$ Poincaré supersymmetry in three dimensions.

The BPS equations coming from $\delta\lambda_{iA} = 0$ has no components along ϵ_3. They are accordingly automatically satisfied with $\epsilon_{1,2} = 0$. $\delta\lambda_i = 0$ equations become

$$e^{i\Lambda}[\cosh(2\Phi_8) \Phi_7' + i\Phi_8'] = g_1 \cosh \Phi_8 \sinh \Phi_7$$
$$+ ig_1 \cosh \Phi_7 \sinh \Phi_8. \quad (66)$$

By a similar analysis as in the previous section, we find $e^{i\Lambda} = \pm e^{i\omega}$ with $e^{i\omega} = \frac{\mathcal{W}}{|\mathcal{W}|}$. The above equations can be solved by

$$\Phi_7' = \mp \frac{g_1 \sinh(2\Phi_7)\text{sech}(2\Phi_8)}{\sqrt{2 + 2\cosh(2\Phi_7)\cosh(2\Phi_8)}}, \quad (67)$$

$$\Phi_8' = \mp \frac{g_1 \sinh(2\Phi_8)\cosh(2\Phi_7)}{\sqrt{2 + 2\cosh(2\Phi_7)\cosh(2\Phi_8)}}. \quad (68)$$

Together with $A' = \pm W$, these form the full set of flow equations.

By combining these equations, we can solve for Φ_8 and A as a function of Φ_7

$$\coth(2\Phi_8) = \text{csch}(2\Phi_7), \quad (69)$$

$$A = -\frac{1}{2}\tanh^{-1}\left[\frac{\sqrt{2}\cosh(2\Phi_7)}{\sqrt{3 - \cosh(4\Phi_7)}}\right] - \frac{1}{4}\ln[\cosh(4\Phi_7) - 3]$$

$$+ \frac{1}{2}\ln\sinh(2\Phi_7). \quad (70)$$

In principle, we can put the solution for Φ_8 in Φ_7' equation and solve for $\Phi_7(r)$. However, we have not found the full analytic solution for $\Phi_7(r)$. In the following, we simply study the Φ_7 behaviors near the UV AdS$_4$ critical point and near the IR singularity. As $r \to \infty$, we find

$$\Phi_7 \sim \Phi_8 \sim e^{-g_1 r}, \qquad A \sim g_1 r. \quad (71)$$

At large $|\Phi_7|$, we find

$$\Phi_7 \sim \pm\frac{1}{3}\ln(g_1 r), \qquad \Phi_8 \sim \text{constant},$$

$$ds^2 = (g_1 r)^{\frac{2}{3}}dx_{1,2}^2 + dr^2, \tag{72}$$

where we have put the singularity at $r = 0$ by choosing an integration constant. These singularities are also physical. The solution preserves two supercharges and describes an

RG flow from $N = 3$ SCFT to a nonconformal field theory in the IR with $N = 1$ supersymmetry.

We will now make another truncation by setting $\Phi_i = 0$ for $i = 2, 4, 6, 8$. This can be verified to be consistent with both the BPS equations and the second order field equations. In this truncation, the scalar potential is given by

$$
\begin{aligned}
V = -\frac{1}{64}&[(1 + \cosh(2\Phi_3)\cosh(2\Phi_5))[4\cosh(2\Phi_1) - 4 + 3\cosh[2(\Phi_1 - \Phi_3 - \Phi_5)] \\
&+ 2\cosh[2(\Phi_1 + \Phi_3 - \Phi_5)] + 3\cosh[2(\Phi_1 - \Phi_3 + \Phi_5)] + 2\cosh[2(\Phi_3 + \Phi_5)] \\
&+ 3\cosh[2(\Phi_1 + \Phi_3 + \Phi_5)] + 8\cosh^2\Phi_1[1 + 3\cosh(2\Phi_3)\cosh(2\Phi_5)]\cosh(2\Phi_7)]g_1^2 \\
&- 12(\cosh(4\Phi_3) + 2\cosh^2(2\Phi_3)\cosh(4\Phi_5) - 3)\cosh^2\Phi_7\sinh(2\Phi_1)g_1g_2 \\
&+ [\cosh(2\Phi_3)\cosh(2\Phi_5) - 1][4 + 4\cosh(2\Phi_1) - 3\cosh[2(\Phi_1 - \Phi_3 - \Phi_5)] \\
&+ 2\cosh[2(\Phi_3 - \Phi_5)] - 3\cosh[2(\Phi_1 + \Phi_3 - \Phi_5)] - 3\cosh[2(\Phi_1 - \Phi_3 + \Phi_5)] \\
&+ 2\cosh[2(\Phi_3 + \Phi_5)] - 3\cosh[2(\Phi_1 + \Phi_3 + \Phi_5)] \\
&+ 8[1 - 3\cosh(2\Phi_3)\cosh(2\Phi_5)]\cosh(2\Phi_7)\sinh^2\Phi_1]g_2^2].
\end{aligned}
\tag{73}
$$

Using the same procedure as before, we find the full set of the BPS equations within this particular truncation

$$
\begin{aligned}
\Phi_1' = -\frac{1}{8}&\frac{e^{-\Phi_1 - 2(\Phi_3 + \Phi_5 + \Phi_7)}}{1 + e^{2\Phi_7}}[(e^{2\Phi_1} - 1)(1 + e^{4\Phi_3} + e^{4\Phi_5} + 4e^{2(\Phi_3 + \Phi_5)} + e^{4(\Phi_3 + \Phi_5)})g_1 \\
&+ (1 + e^{2\Phi_1})(1 + e^{4\Phi_3} + e^{4\Phi_5} - 4e^{2(\Phi_3 + \Phi_5)} + e^{4(\Phi_3 + \Phi_5)})g_2],
\end{aligned}
\tag{74}
$$

$$
\Phi_3' = -\frac{e^{-\Phi_1 - 2\Phi_3 + 2\Phi_5 - \Phi_7}}{8(1 + e^{4\Phi_5})}(e^{4\Phi_3} - 1)(1 + e^{2\Phi_7})[(1 + e^{2\Phi_1})g_1 + (e^{2\Phi_1} - 1)g_2],
\tag{75}
$$

$$
\Phi_5' = -\frac{1}{32}e^{-\Phi_1 - 2\Phi_3 - 2\Phi_5 - \Phi_7}(e^{4\Phi_3} + 1)(1 + e^{2\Phi_7})(e^{4\Phi_5} - 1)[(1 + e^{2\Phi_1})g_1 + (e^{2\Phi_1} - 1)g_2],
\tag{76}
$$

$$
\begin{aligned}
\Phi_7' = -\frac{1}{32}&e^{-\Phi_1 - 2\Phi_3 - 2\Phi_5 - \Phi_7}(1 - e^{2\Phi_7})[(e^{2\Phi_1} - 1)[1 + e^{4\Phi_3} + e^{4\Phi_5} + 4e^{2(\Phi_3 + \Phi_5)} \\
&+ e^{4(\Phi_3 + \Phi_5)}]g_1 + (1 + e^{2\Phi_1})[1 + e^{4\Phi_3} + e^{4\Phi_5} - 4e^{2(\Phi_3 + \Phi_5)} + e^{4(\Phi_3 + \Phi_5)}]g_2],
\end{aligned}
\tag{77}
$$

$$
\begin{aligned}
A' = \frac{1}{32}&e^{-\Phi_1 - 2\Phi_3 - 2\Phi_5 - \Phi_7}(1 + e^{2\Phi_7})[(e^{2\Phi_1} - 1)[1 + e^{4\Phi_3} + e^{4\Phi_5} + 4e^{2(\Phi_3 + \Phi_5)} \\
&+ e^{4(\Phi_3 + \Phi_5)}]g_1 + (1 + e^{2\Phi_1})[1 + e^{4\Phi_3} + e^{4\Phi_5} - 4e^{2(\Phi_3 + \Phi_5)} + e^{4(\Phi_3 + \Phi_5)}]g_2].
\end{aligned}
\tag{78}
$$

Due to the γ_r projector, the solutions will preserve six supercharges or $N = 3$ supersymmetry in three dimensions. When $\Phi_3 = \Phi_1$ and $\Phi_5 = \Phi_7 = 0$, the above equations reduce to those considered in Sec. III. These equations do not admit any nontrivial AdS$_4$ fixed points apart from the $N = 3$ $SO(3)_{\text{diag}} \times U(1)$ critical point already identified in Sec. III. This agrees with the remark given in [6] in which partial supersymmetry breaking has been shown to be impossible.

We are now in a position to consider various possible RG flows from the UV $N = 3$ SCFTs. In this case, we have not

found any possible analytic solutions. Therefore, numerical solutions will be needed in order to obtain the full flow solutions. Although these solutions always exist and can be found by imposing suitable boundary conditions, we will not give them here. Instead, we will give the behavior near the IR singularity which can be put to $r = 0$ by choosing appropriate constants of integration. This is similar to the analysis given in [34]. Note also that, from the above equations, setting $\Phi_5 = 0$ and $\Phi_7 = 0$ is also a consistent truncation.

We will now consider RG flows to the IR with infinite values of scalar fields. From the above equations, as $\Phi_3 \to \pm\infty$, we find that $\Phi'_5 \to 0$. Since both of the AdS$_4$ critical points have $\Phi_5 = 0$, we will set $\Phi_5 = 0$ throughout the analysis.

At the trivial $N = 3$ AdS$_4$ critical point, all scalars are dual to relevant operator of dimensions $\Delta = 1, 2$. For $\Phi_3 > 0$, there are flows with the IR behavior

$$\Phi_1 \sim \phi_0, \qquad \Phi_7 \sim \Phi_3, \qquad \Phi_3 \sim -\frac{1}{3}\ln\left[\frac{3}{8}\tilde{g}r\right],$$
$$ds^2 = r^{\frac{2}{3}}dx^2_{1,2} + dr^2 \tag{79}$$

where ϕ_0 is a constant and $\tilde{g} = g_1\cosh\phi_0 + g_2\sinh\phi_0$. There is also another flow with asymptotic behavior

$$\Phi_1 \sim \phi_0, \qquad \Phi_7 \sim -2\Phi_3, \qquad \Phi_3 \sim -\frac{1}{4}\ln\left[\frac{1}{2}\tilde{g}r\right],$$
$$ds^2 = r^{\frac{1}{2}}dx^2_{1,2} + dr^2. \tag{80}$$

For $\Phi_3 < 0$, we have flows with

$$\Phi_1 \sim \phi_0, \qquad \Phi_7 \sim \pm\Phi_3, \qquad \Phi_3 \sim -\frac{1}{3}\ln\left[\frac{3}{8}\tilde{g}r\right],$$
$$ds^2 = r^{\frac{2}{3}}dx^2_{1,2} + dr^2. \tag{81}$$

It should be noted that when $\Phi_7 \neq 0$, we always have constant Φ_1 in the IR. This is however not the case when $\Phi_7 = 0$. An example of this flow is given by

$$\Phi_1 \sim -2\Phi_3, \qquad \Phi_7 = 0, \qquad \Phi_3 \sim -\frac{1}{4}\ln\left[\frac{1}{2}(g_1 - g_2)r\right],$$
$$ds^2 = r^{\frac{1}{2}}dx^2_{1,2} + dr^2. \tag{82}$$

Remarkably, all of these flows are physical according to the criterion of [33] as can be checked from (73) that all the flows give $V \to -\infty$.

The nontrivial AdS$_4$ critical point can be approached by setting $\Phi_1 = \pm\Phi_3 = \Phi_0 = \frac{1}{2}\ln[\frac{g_2-g_1}{g_2+g_1}]$ in the UV with different signs corresponding to different combinations of $SO(3) \times SO(3)$ generators in forming $SO(3)_{\text{diag}}$. We will additionally set $\Phi_7 = \Phi_5 = 0$ in the following analysis.

For $\Phi_3 > \Phi_0$, there is a flow with asymptotic behavior

$$\Phi_1 \sim \Phi_3 \sim -\frac{1}{3}\ln\left[\frac{3}{8}(g_1 + g_2)r\right],$$
$$ds^2 = r^{\frac{2}{3}}dx^2_{1,2} + dr^2. \tag{83}$$

For $\Phi_3 < \Phi_0$, we have flows with the IR behavior

$$\Phi_1 \sim \pm\Phi_3, \qquad \Phi_3 \sim \frac{1}{3}\ln\left[\frac{3}{8}(g_1 \mp g_2)r\right],$$
$$ds^2 = r^{\frac{2}{3}}dx^2_{1,2} + dr^2. \tag{84}$$

All of these flows are also physical with $V \to -\infty$ near the IR singularity.

V. FLOWS TO LOWER DIMENSIONS

In this section, we consider supersymmetric solutions of the form AdS$_2 \times \Sigma_2$ in which Σ_2 is a Riemann surface in the form of a two-sphere S^2 or a two-dimensional hyperbolic space H^2. Domain wall solutions interpolating between AdS$_4$ critical points and these geometries should be interpreted as RG flows to lower dimensional superconformal field theories. In the present case, the lower dimensional SCFTs would be described by twisted compactifications of the $N = 3$ SCFTs in three dimensions resulting in one-dimensional SCFTs. We will look for supersymmetric AdS$_2$ solutions with $SO(2) \times SO(2) \times SO(2)$ and $SO(2) \times SO(2)$ symmetries within $N = 3$ $SO(3) \times SU(3)$ gauged supergravity.

A. AdS$_2$ critical points with $SO(2) \times SO(2) \times SO(2)$ symmetry

We begin with the BPS equations relevant for the present analysis. The gauge fields are now nonvanishing. We adopt the twist procedure in order to preserve some amount of supersymmetry. This involves turning on some gauge field to cancel the spin connection along the Σ_2 directions. We will primarily consider the case of curved Σ_2 in the form of S^2 and H^2.

The four-dimensional metric is taken to be

$$ds^2_4 = -e^{2A(r)}dt^2 + dr^2 + e^{2B(r)}ds^2(\Sigma_2) \tag{85}$$

where $ds^2(\Sigma_2)$ is the metric on Σ_2. Its explicit form can be written as

$$ds^2(S^2) = d\theta^2 + \sin^2\theta d\phi^2 \quad \text{and} \quad ds^2(H^2) = \frac{1}{y^2}(dx^2 + dy^2) \tag{86}$$

for the S^2 and H^2 cases, respectively. In the following, we will only give the detail of the S^2 case. The H^2 case can be done in a similar way.

The component of the spin connection on S^2 that needs to be canceled is given by

$$\omega^{\hat{\phi}\hat{\theta}} = e^{-B}\cot\theta e^{\hat{\phi}}. \tag{87}$$

This appears in the $\delta\psi_{\phi A}$ variation. To cancel this contribution, we turn on some of the gauge fields $A_{\mu A}$ appearing in the $SU(3)$ composite connection $Q_A{}^B$. We will choose the nonvanishing gauge field to be

$$A_3 = a\cos\theta d\phi \tag{88}$$

which gives rise to the nonvanishing components of the composite connection

$$Q_1{}^2 = -Q_2{}^1 = -a_1 g_1 \quad \text{or} \quad Q_{AB} = -g_1 \epsilon_{ABC} A^C. \tag{89}$$

The cancellation is achieved by imposing the following twist and projection conditions

$$a_1 g_1 = \frac{1}{2}, \qquad \gamma_{\hat\phi\hat\theta}\epsilon_a = i\sigma_{2a}{}^b \epsilon_b, \qquad a,b = 1,2. \tag{90}$$

In the above equation, $\sigma_{2a}{}^b$ denotes the usual second Pauli matrix. We have split the index A into $(a,3)$ such that $\epsilon^A = (\epsilon^a, \epsilon^3)$. It should be noted that with only A_3 nonvanishing, the supersymmetry corresponding to ϵ^3 cannot be preserved, so we will set $\epsilon^3 = 0$. Eventually, there are only four unbroken supercharges corresponding to ϵ^a that are subject to the $\gamma_{\hat\phi\hat\theta}$ projection.

In addition, there are other two gauge fields that can be turned on along with A_3. These correspond to the $SO(2) \times SO(2) \subset SU(3)$ symmetry and are given by

$$A^6 = b\cos\theta d\phi \quad \text{and} \quad A^{11} = c\cos\theta d\phi. \tag{91}$$

All other gauge fields are zero. The field strengths of (A^3, A^6, A^{11}) are given by

$$F_\Lambda = -a_\Lambda e^{-2B} e^{\hat\theta} \wedge e^{\hat\phi} \tag{92}$$

with nonvanishing $a_\Lambda = (a_3, a_6, a_{11}) = (a,b,c)$. With the convention $\epsilon^{\hat t\hat r\hat\theta\hat\phi} = 1$, we find the dual field strength

$$\tilde F_\Lambda = a_\Lambda e^{-2B} e^{\hat t} \wedge e^{\hat r}. \tag{93}$$

The four-dimensional chirality on ϵ_A relates the $\gamma_{\hat\phi\hat\theta}$ to the $\gamma_{\hat t\hat r}$ as follows

$$\gamma_5 \epsilon_a = i\gamma_{\hat t}\gamma_{\hat r}\gamma_{\hat\theta}\gamma_{\hat\phi}\epsilon_a = \epsilon_a \tag{94}$$

implying that

$$\gamma_{\hat t\hat r}\epsilon_a = \sigma_{2a}{}^b \epsilon_b. \tag{95}$$

We are now in a position to set up the BPS equations by using all of the above conditions and the formulas given in Sec. II. In the presence of gauge fields, unlike the solutions considered in Sec. III, it turns out that the parametrization of the coset representative for $SO(2) \times SO(2) \times SO(2)$ invariant scalars using $SU(2)$ Euler angles does not simplify the resulting equations to any appreciable degree. We will rather choose to parametrize the coset representative in the form of

$$L = e^{\tilde Y_1 \Phi_1} e^{\tilde Y_2 \Phi_2} e^{\tilde Y_3 \Phi_3} e^{\tilde Y_4 \Phi_4}. \tag{96}$$

Furthermore, we will make a truncation $\Phi_2 = \Phi_4 = 0$ to make things more manageable. This can also be verified to be consistent with all of the BPS equations as well as the corresponding field equations.

As in the previous cases, the equations coming from $\delta\chi = 0$ are identically satisfied since $C_M{}^{MA} = 0$, and the particular ansatz for the gauge fields given above gives $G^A_{\mu\nu}\gamma^{\mu\nu} = 0$. In addition, $\delta\lambda_i = 0$ equations are identically satisfied provided that we set $\epsilon^3 = 0$. In our particular truncation, $\mathcal{W}$ is real, so we can impose the $\gamma_{\hat r}$ projection simply as $\gamma_{\hat r}\epsilon_a = \pm\epsilon_a$. With the usual choice of signs chosen, the independent BPS equations coming from $\delta\lambda_{Ai} = 0$ are given by

$$\Phi_1' = \frac{1}{4} e^{-\Phi_1 - \Phi_3 - 2B}[4ce^{\Phi_3}(1 + e^{2\Phi_1}) + 2b(e^{2\Phi_1} - 1)(e^{2\Phi_3} - 1)$$
$$- 2a(e^{2\Phi_1} - 1)(1 + e^{2\Phi_3}) + g_1 e^{2B}(1 - e^{2\Phi_1}) + g_1 e^{2\Phi_3 + 2B}(1 - e^{2\Phi_1})], \tag{97}$$

$$\Phi_3' = -\frac{e^{\Phi_1 - \Phi_3 - 2B}}{1 + e^{2\Phi_1}}[2a(e^{2\Phi_3} - 1) - 2b(1 + e^{2\Phi_3}) + g_1 e^{2B}(e^{2\Phi_3} - 1)]. \tag{98}$$

With the twist conditions (90), $\delta\psi_{\hat\phi A} = 0$ equations are the same as $\delta\psi_{\hat\theta A} = 0$ equations. All of these conditions reduce to a single equation for the function B while the conditions $\delta\psi_{\mu A}$, for $\mu = t$, give an equation for the function A. These are given by

$$B' = -\frac{1}{4} e^{-\Phi_1 - \Phi_3 - 2B}[2ce^{\Phi_3}(1 - e^{2\Phi_1}) - b(1 + e^{2\Phi_1})(e^{2\Phi_3} - 1)$$
$$+ a(1 + e^{2\Phi_1})(1 + e^{2\Phi_3}) - g_1 e^{2B}(1 + e^{2\Phi_1}) - g_1 e^{2\Phi_3 + 2B}(1 + e^{2\Phi_1})], \tag{99}$$

$$A' = -\frac{1}{4} e^{-\Phi_1 - \Phi_3 - 2B}[-2ce^{\Phi_3}(1 - e^{2\Phi_1}) + b(1 + e^{2\Phi_1})(e^{2\Phi_3} - 1)$$
$$- a(1 + e^{2\Phi_1})(1 + e^{2\Phi_3}) - g_1 e^{2B}(1 + e^{2\Phi_1}) - g_1 e^{2\Phi_3 + 2B}(1 + e^{2\Phi_1})]. \tag{100}$$

For the H^2 case, a similar analysis can be carried out. The result is the same as the above equations with (a, b, c) replaced by $(-a, -b, -c)$.

An $AdS_2 \times \Sigma_2$ geometry is given by a fixed point of the above equations satisfying $\Phi_1' = \Phi_3' = B' = 0$ and $A' = \frac{1}{L_{AdS_2}}$. We find a class of solutions given by

$$
B = \frac{1}{2} \ln \left[\frac{2[a(1 - e^{2\Phi_3}) + b(1 + e^{2\Phi_3})]}{(e^{2\Phi_3} - 1)g_1} \right],
$$

$$
\Phi_1 = \frac{1}{2} \ln \left[\frac{c(1 - e^{2\Phi_3}) - 2be^{\Phi_3}}{c(e^{2\Phi_3} - 1) - 2be^{\Phi_3}} \right],
$$

$$
\Phi_3 = \frac{1}{2} \ln \left[\frac{b^2 - c^2 \pm \sqrt{b^2[9a^2 - 8(b^2 + c^2)]}}{3ab - 3b^2 - c^2} \right]. \tag{101}
$$

The expression for the AdS_2 radius is much more complicated. We will not give it here, but in any case this can be obtained by substituting the values of B, Φ_1 and Φ_3 in the A' equation.

B. AdS$_2$ critical points with $SO(2) \times SO(2)$ symmetry

We now look for AdS_2 solutions that can be obtained from twisted compactifications of the non-trivial AdS_4 critical point. As in Sec. IV, we consider $SO(2)_{\mathrm{diag}} \times SO(2)$ invariant scalars. The coset representative is still given by (61). The ansatze for the gauge fields are similar to the previous case but with $b = \frac{g_1}{g_2} a$ to implement the gauge field of $SO(2)_{\mathrm{diag}}$.

Following the same procedure as in the previous subsection, we obtain a set of BPS equations, again in a consistent truncation with $\Phi_i = 0$, for $i = 2, 4, 6, 8$,

$$
\Phi_1' = -\frac{2e^{\Phi_7}}{1 + e^{2\Phi_7}} \left[-\frac{a}{g_2} e^{-\Phi_1 - 2B}[(1 + e^{2\Phi_1})g_1 + (1 - e^{2\Phi_1})g_2] \right.
$$
$$
+ \frac{1}{16} e^{-\Phi_1 - 2\Phi_3 - 2\Phi_5}[(e^{2\Phi_1} - 1)(1 + e^{4\Phi_3} + e^{4\Phi_5} + 4e^{2(\Phi_3 + \Phi_5)}
$$
$$
\left. + e^{4(\Phi_3 + \Phi_5)})g_1 + (1 + e^{2\Phi_1})(1 + e^{4\Phi_3} + e^{4\Phi_5} - 4e^{2(\Phi_3 + \Phi_5)} + e^{4(\Phi_3 + \Phi_5)})g_2] \right], \tag{102}
$$

$$
\Phi_3' = -\frac{1}{8} \left[\frac{1 + e^{2\Phi_7}}{1 + e^{4\Phi_5}} \right] e^{-\Phi_1 - 2\Phi_3 + 2\Phi_5 - \Phi_7}(e^{4\Phi_3} - 1) \times [(1 + e^{2\Phi_1})g_1 + (e^{2\Phi_1} - 1)g_2], \tag{103}
$$

$$
\Phi_5' = -\frac{1}{32} e^{-\Phi_1 - 2\Phi_3 - 2\Phi_5 - \Phi_7}(1 + e^{4\Phi_3})(e^{4\Phi_5} - 1)(1 + e^{2\Phi_7}) \times [(1 + e^{2\Phi_1})g_1 + (e^{2\Phi_1} - 1)g_2], \tag{104}
$$

$$
\Phi_7' = \frac{1}{32} e^{-\Phi_1 - 2\Phi_3 - 2\Phi_5 - \Phi_7}(1 - e^{2\Phi_7})[(1 + e^{2\Phi_1})[1 + e^{4\Phi_3} + e^{4\Phi_5} + 4e^{2(\Phi_3 + \Phi_5)}
$$
$$
+ e^{4(\Phi_3 + \Phi_5)}]g_1 + (e^{2\Phi_1} - 1)(1 + e^{4\Phi_3} + e^{4\Phi_5} - 4e^{2(\Phi_3 + \Phi_5)} + e^{4(\Phi_3 + \Phi_5)})g_2]
$$
$$
+ \frac{1}{2g_2} e^{-\Phi_1 - \Phi_7 - 2B}[2ce^{\Phi_1}(1 + e^{2\Phi_7})g_2 + a(e^{2\Phi_7} - 1)[(e^{2\Phi_1} - 1)g_1 - (1 + e^{2\Phi_1})g_2]], \tag{105}
$$

$$
B' = -\frac{1}{32} e^{-\Phi_1 - \Phi_7 - 2B} \left[\frac{8a}{g_2}(1 + e^{2\Phi_7})[(1 - e^{2\Phi_1})g_1 + (1 + e^{2\Phi_1})g_2] - e^{-2(\Phi_3 + \Phi_5)}[16ce^{\Phi_1 + 2\Phi_3 + 2\Phi_5}(e^{2\Phi_7} - 1) \right.
$$
$$
+ e^{2B}(1 + e^{2\Phi_7})[(1 + e^{2\Phi_1})(1 + e^{4\Phi_3} + e^{4\Phi_5} + 4e^{2(\Phi_3 + \Phi_5)} + e^{4(\Phi_3 + \Phi_5)})g_1
$$
$$
\left. + (e^{2\Phi_1} - 1)(1 + e^{4\Phi_3} + e^{4\Phi_5} - 4e^{2(\Phi_3 + \Phi_5)} + e^{4(\Phi_3 + \Phi_5)})g_2] \right], \tag{106}
$$

$$
A' = \frac{1}{32} e^{-\Phi_1 - \Phi_7 - 2B} \left[\frac{8a}{g_2}(1 + e^{2\Phi_7})[(1 - e^{2\Phi_1})g_1 + (1 + e^{2\Phi_1})g_2] - e^{-2(\Phi_3 + \Phi_5)}[16ce^{\Phi_1 + 2\Phi_3 + 2\Phi_5}(e^{2\Phi_7} - 1) \right.
$$
$$
- e^{2B}(1 + e^{2\Phi_7})[(1 + e^{2\Phi_1})(1 + e^{4\Phi_3} + e^{4\Phi_5} + 4e^{2(\Phi_3 + \Phi_5)} + e^{4(\Phi_3 + \Phi_5)})g_1
$$
$$
\left. + (e^{2\Phi_1} - 1)(1 + e^{4\Phi_3} + e^{4\Phi_5} - 4e^{2(\Phi_3 + \Phi_5)} + e^{4(\Phi_3 + \Phi_5)})g_2] \right]. \tag{107}
$$

From these equations, we find a number of $AdS_2 \times \Sigma_2$ solutions given below.

(1) For $\Phi_3 = \Phi_5 = 0$, we find a critical point

$$B = \frac{1}{2}\ln\left[\frac{2a[(1 + e^{2\Phi_1})g_1 + (1 - e^{2\Phi_1})g_2]}{(e^{2\Phi_1} - 1)g_1 g_2}\right],$$

$$\Phi_7 = \frac{1}{2}\ln\left[\frac{2ae^{\Phi_1}g_1 + c(e^{2\Phi_1} - 1)g_2}{2ae^{\Phi_1}g_1 - c(e^{2\Phi_1} - 1)g_2}\right],$$

$$\Phi_1 = \frac{1}{2}\ln\left[\frac{a^2 g_1^2 - c^2 g_2^2 \pm \sqrt{a^2 g_1^2[a^2(9g_2^2 - 8g_1^2) - 8c^2 g_2^2]}}{3a^2 g_1(g_2 - g_1) - c^2 g_2^2}\right]. \tag{108}$$

(2) For $c = 0$, Φ_7 can be consistently set to zero. If we further set $\Phi_3 = 0$, we find the following critical point

$$\Phi_1 = \frac{1}{2}\ln\left[\frac{g_1 \pm \sqrt{9g_2^2 - 8g_1^2}}{3(g_2 - g_1)}\right],$$

$$B = \frac{1}{2}\ln\left[\frac{2a(g_2 - g_1)[2g_1 + 3g_2 \mp \sqrt{9g_2^2 - 8g_1^2}]}{g_1 g_2[4g_1 - 3g_2 \pm \sqrt{9g_2^2 - 8g_1^2}]}\right],$$

$$L_{\text{AdS}_2} = \frac{2g_1 + 3g_2 \mp \sqrt{9g_2^2 - 8g_1^2}}{4g_1^2}\sqrt{\frac{3(g_2 - g_1)}{g_1 \pm \sqrt{9g_2^2 - 8g_1^2}}}. \tag{109}$$

(3) For $c = 0$ and $\Phi_7 = 0$ but $\Phi_3 \neq 0$, we find a critical point

$$\Phi_1 = \frac{1}{2}\ln\left[\frac{g_2 - g_1}{g_2 + g_1}\right],$$

$$B = \frac{1}{2}\ln\left[\frac{a}{g_1} + \frac{ag_1}{g_2^2}\right],$$

$$\Phi_3 = \frac{1}{2}\ln\left[\frac{g_1^4 + 10g_1^2 g_2^2 + g_2^4 - 2\sqrt{5g_1^6 g_2^2 + 26g_1^4 g_2^4 + 5g_1^2 g_2^6}}{g_2^4 - g_1^4}\right],$$

$$L_{\text{AdS}_2} = \frac{\sqrt{g_2^2 - g_1^2}}{2g_1 g_2}. \tag{110}$$

It can be checked that all of the above solutions are valid by choosing suitable choices of the two coupling (g_1, g_2) and the parameters (a, c) in a manner that is consistent with the twist condition $2g_1 a = 1$. For example, taking $b = 2c$ and $a = 5c$ in the solution (101) leads to

$$B = \ln\left[0.927441\sqrt{\frac{a}{g_1}}\right],$$

$$\Phi_1 = 0.146711,$$

$$\Phi_3 = 0.287363. \tag{111}$$

There might be more critical points, but we have not found any other real solutions.

We end this section with a remark on $\text{AdS}_2 \times T^2$ solutions. Since T^2 is flat, the twist is not needed. We will set $A_3 = 0$ or equivalently $a = 0$. From the above two cases, we have not found any valid $\text{AdS}_2 \times T^2$ solutions.

VI. CONCLUSIONS

In this paper, we have studied $N = 3$ gauged supergravity in four dimensions with $SO(3) \times SU(3)$ gauge group. We have found a new supersymmetric AdS_4 critical point, with $SO(3) \times U(1)$ symmetry and unbroken $N = 3$ supersymmetry, and given the full mass spectrum of all 48 scalars at this critical point. An analytic RG flow interpolating between this new critical point and the trivial UV fixed point has also been explicitly given. The flow

describes a supersymmetric deformation by a relevant operator of dimension $\Delta = 1, 2$. It would be of particular interest to precisely identify the dual operator that drives the flow in the dual $N = 3$ SCFT. This result provides another example of supersymmetric deformations of $N = 3$ Chern-Simons-Matter gauge theories which might be useful in the holographic study of Aharony-Bergman-Jafferis-Maldacena (ABJM)-type theories coupled to matter multiplets.

In addition, we have studied RG flows to non-conformal $N = 3$ gauge theories in three dimensions with $SO(2) \times SU(2) \times U(1)$ and $SO(2)_{\text{diag}} \times SO(2)$ symmetries. In the former class of solutions, we have found $N = 3$ supersymmetric deformations in the absence of the "pseudoscalars" corresponding to the imaginary part of the complex scalars. When a pseudoscalar is turned on, the corresponding deformation breaks supersymmetry to $N = 1$. The latter class includes supersymmetric deformations that break conformal symmetry of the $SO(3) \times U(1)$ $N = 3$ SCFT dual to the nontrivial AdS$_4$ critical point. Remarkably, all of these solutions have physically acceptable IR singularities. This is due to the particular form of the scalar potential which is always bounded above in the scalar sectors considered in this paper. This is very similar to the solution studied in [32]. These results would be of particular interest in describing world volume theory of M2-branes and hopefully in condensed matter physics systems along the line of [35].

The last result of this paper consists of supersymmetric AdS$_2 \times \Sigma_2$ solutions preserving four supercharges or $N = 2$ Poincaré supersymmetry in three dimensions. We have given AdS$_2$ solutions with $SO(2) \times SO(2) \times SO(2)$ and $SO(2)_{\text{diag}} \times SO(2)$ symmetries. In the context of twisted field theories, these solutions describe possible twisted compactifications of $N = 3$ SCFTs dual to the two AdS$_4$ critical points mentioned above. These should be useful in the context of AdS$_2$/CFT$_1$ correspondence and black hole physics. It should also be noted that there is no AdS$_2 \times T^2$ solutions within the scalar submanifolds considered here.

There are many possible future directions to investigate. First, it is interesting to find whether the new $SO(3) \times U(1)$ critical point and the corresponding RG flows can be uplifted to eleven dimensions. This would give a geometric interpretation to the solutions obtained here in the context of M-theory in much the same way as the recent work

for the $N = 8$ gauged supergravity in [36]. The complete truncation of eleven-dimensional supergravity on N^{010} keeping only $SU(3)$ singlet fields is given in [37]. However, the result of [37] obviously cannot be used to uplift the AdS$_4$ critical point and the RG flows given in this paper since the scalars that transform nontrivially under the flavor group $SU(3)$ are also turned on.

It should be remarked here about the condition $g_2^2 > g_1^2$ related to the existence of the $SO(3) \times U(1)$ critical point. Within the four-dimensional framework, the two coupling constants are completely free. The consistency of the gauging does not impose any relation between them. On the other hand, from the eleven-dimensional point of view, the ratio between g_1 and g_2 should be fixed since there is no continuous parameter in N^{010}. This might indicate that the $SO(3) \times U(1)$ critical point in eleven dimensions does not exist if the condition $g_2^2 > g_1^2$ is not satisfied. Alternatively, this critical point might arise from a more complicated compactification. It would be interesting to investigate these issues in more detail.

In finding AdS$_2 \times \Sigma_2$ solutions, we have truncated out the pseudoscalars. It would be interesting to investigate their role in AdS$_2 \times \Sigma_2$ backgrounds as well as in the holographic AdS$_2$/CFT$_1$ context. In particular, finding black hole solutions interpolating between $N = 3$ AdS$_4$ and these AdS$_2 \times \Sigma_2$ geometries and comparing the black hole entropy with the result from superconformal indices in the dual $N = 3$ SCFT, as in the AdS$_4 \times S^7$ case studied in [28], would provide an example of this study in a less supersymmetric case. The solutions found here would also be useful in this context. We leave all these issues for future investigations.

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