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Preheating after technicolor inflation

Phongpichit Channuie*
Physics Division, School of Science, Walailak University, Nakhon Si Thammarat 80160, Thailand

Peeravit Koad†
Computational Science Program, School of Science, Walailak University, Nakhon Si Thammarat 80160, Thailand

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We investigate the particle production due to parametric resonances in a model of inflation where the lightest composite state stemming from the minimal walking technicolor theory plays the role of the inflaton. For this model of inflation, the effective theory couples nonminimally to gravity. Regarding the preheating, we study in detail a model of a composite inflaton field ϕ coupled to another scalar field χ with the interaction term $g^2\phi^2\chi^2$. Particularly, in Minkowski space, the stage of parametric resonances can be described by the Mathieu equation. Interestingly, we discover that broad resonances can be typically achieved and are potentially efficient in our model causing the particle number density in this process to exponentially increase.

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I. INTRODUCTION

Inflation [1–5] marks nowadays an inevitable ingredient when studying the very early evolution of the Universe. The reason stems from the fact that it solves most of the puzzles that plague the standard big bang theory, and simultaneously it is consistent with the observations [6,7]. In other words, it not only gives sensible explanations for the horizon, flatness, and relic abundant problems, but it also provides us with primordial density perturbations as seeds of the formation of large-scale structure in the Universe. Most of the inflationary scenarios proposed so far require the introduction of a new degree of freedom, e.g. a Higgs field [8], a gauge field [9], and so on, to successfully drive inflation. Yet there are other well-motivated inflationary models, e.g. 3-form field inflation [10,11] and multifield inflation [12]. However, the underlying nature of the theory of inflaton is still an open question. According to the cosmic frontier, the construction of inflationary models raises a lot of interest. Recent investigations revolutionized the possibility of model building in which the inflaton need not be an elementary degree of freedom [13–15].

In the standard picture of the early Universe, the Universe passes through the period of reheating. At this stage, (almost) all elementary particles populating the Universe were created. The instructive idea of a mechanism for reheating was proposed, for instance, by the author of Ref. [16] in which reheating occurs due to particle production by the oscillating scalar field. In the simplest version, this field is the inflaton that exponentially drives the expansion of the Universe. However, a phenomenological description of the reheating mechanism was first implemented in Ref. [17]. The authors added various friction terms to the equation of motion of the scalar field in order to imitate energy transfer from the inflaton to matter fields. However, it can be questioned what kind of terms should be practically added. The application of the theory of reheating to the new inflationary scenario was further developed in Ref. [18], in an application to R2 inflation in Ref. [19], and to noncommutative inflation in Ref. [20]. Here the treatment was based on perturbative theory where the knowledge of quantum field theory is mandatory.

Indeed, it has been recognized that in inflationary models the first stage of reheating occurs in a regime of a parametric amplification of a scalar field [21–24]. This preceding evolutionary phase is called the "preheating" stage in which particles are explosively produced due to the parametric resonance. There are many analytical works which examined the preheating mechanism, e.g. Refs. [25–27]. For instance, the authors of Ref. [28] have examined the properties of resonance with a nonminimally coupled scalar field χ in the preheating phase and have found that an effective resonance is possible only in the presence of a nonminimal coupling $\xi R\chi^2$ term with a sizable range of the parameter ξ. Moreover, the same authors of Ref. [29] later considered higher-curvature inflation models $(R + \alpha^n R^n)$ allowing them to study a parametric preheating of a scalar field coupled nonminimally to a spacetime curvature. Another interesting paradigm was proposed in Ref. [30]. In this scenario, they studied a preheating mechanism in which the standard model Higgs, strongly nonminimally coupled to gravity, plays the role of the inflaton. Consequently, they discovered that the Universe does reheat and thus perturbative and nonperturbative effects

*channuie@gmail.com
†harrykoad@gmail.com

are mixed. Moreover, the authors of Ref. [31] have investigated the production of particles due to parametric resonances in a 3-form field inflation and found that this process is more efficient compared to the result of the standard-scalar-field inflationary scenario (e.g. Refs. [22–24]), in which the broad resonance tends to disappear more quickly.

In this work, we study details of preheating, following the work presented in Ref. [28], in an inflationary scenario in which the inflaton is the lightest composite state, which are expected to be the most important ones for collider phenomenology, stemming from the minimal walking technicolor (MWT) theory which includes a new strong sector based on the SU(2) gauge group with two Dirac flavors transforming according to the adjoint representation. For model of composite inflation, the authors of Ref. [13] engaged the MWT effective Lagrangian with the standard slow-roll paradigm and here we call it "technicolor inflation" (TI). Interestingly, technicolor inflation not only provides a possible resolution to the well-known η problem[1] [32] of inflationary model building but it also allows for an inflationary phase in the early Universe [13].

This paper is organized as follows. In the next section, we briefly review our model of inflation in which the inflaton emerges as a composite state coupled nonminimally to a spacetime curvature. Adding a scalar field coupled minimally to gravity, in Sec. III, we then introduce an analytical approach to study the preheating for the model of composite inflation. In Sec. IV, we study parametric resonances of the model when a composite inflaton field ϕ is coupled to another scalar field χ with the interaction term $g^2\phi^2\chi^2$. Finally, we give our findings in the last section.

II. TECHNICOLOR INFLATION: A RECAP

The underlying gauge theory for the technicolor-inspired inflation is the SU(N) gauge group with $N_f = 2$ Dirac massless fermions. The two technifermions transform according to the adjoint representation of the SU(2) technicolor (TC) gauge group, called $\mathrm{SU(2)_{TC}}$. Here we engage the simplest model of technicolor known as the MWT theory [33–36] with the standard (slow-roll) inflationary paradigm as a template for composite inflation and name it, in short, the MCI model. In order to examine the symmetry properties of the theory, we arrange them by using the Weyl basis into a column vector, and the field contents in this case are

$$\mathcal{Q}^a = \begin{pmatrix} U_L^a \\ D_L^a \\ -i\sigma^2 U_R^{*a} \\ -i\sigma^2 D_R^{*a} \end{pmatrix}, \tag{1}$$

where U_L and D_L are the left-handed techni-up and techni-down respectively, U_R and D_R are the corresponding right-handed particles and the upper index $a = 1, 2, 3$ is the TC index indicating the three-dimensional adjoint representation. Since $\mathcal{Q}$ has four components, the technifermion fields are said to be in the fundamental representation of SU(4). With the standard breaking to the maximal diagonal subgroup, the SU(4) global symmetry spontaneously breaks to SO(4). Such a breaking is driven by the formation of the following condensate:

$$\langle \mathcal{Q}_i^\alpha \mathcal{Q}_j^\beta \epsilon_{\alpha\beta} \mathcal{E}^{ij} \rangle = -2\langle \overline{U}_R U_L + \overline{D}_R D_L \rangle, \tag{2}$$

where $i, j = 1, \ldots, 4$ denote the components of the tetraplet of $\mathcal{Q}$, and α, β indicate the ordinary spin. The 4×4 matrix $\mathcal{E}^{ij}$ is defined in terms of the two-dimensional identical matrix, $\mathbb{1}$, as

$$\mathcal{E} = \begin{pmatrix} \mathbb{0} & \mathbb{1} \\ \mathbb{1} & \mathbb{0} \end{pmatrix}, \tag{3}$$

with, for example, $\epsilon_{\alpha\beta} = -i\sigma^2_{\alpha\beta}$ and $\langle U_L^\alpha U_R^{*\beta} \epsilon_{\alpha\beta} \rangle = -\langle \overline{U}_R U_L \rangle$. The connection between the composite scalar fields and the underlying technifermions can be obtained from the transformation properties of SU(4). To this end, we observe that the elements of the matrix $\mathcal{M}$ transform like technifermion bilinears such that

$$\mathcal{M}_{ij} \sim \mathcal{Q}_i^\alpha \mathcal{Q}_j^\beta \epsilon_{\alpha\beta} \quad \text{with} \quad i, j = 1, \ldots, 4. \tag{4}$$

The composite action nonminimally coupled to gravity can be built in terms of the matrix $\mathcal{M}$ in the Jordan frame as [13]

$$\mathcal{S}_{\mathrm{MCI,J}} = \int d^4x \sqrt{-g} \bigg[-\frac{M_{\mathrm{P}}^2}{2} R - \frac{1}{2} \xi \mathrm{Tr}[\mathcal{M}\mathcal{M}^\dagger] R + \mathcal{L}_{\mathrm{MWT}} \bigg], \tag{5}$$

where $\mathcal{L}_{\mathrm{MWT}}$ is the Lagrangian density of the MWT sector; see Ref. [13] for more details. From the above action, the nonminimally coupled term corresponds at the fundamental level to a four-fermion interaction term coupled to the Ricci scalar in the following way:

$$\frac{1}{2} \xi \mathrm{Tr}[\mathcal{M}\mathcal{M}^\dagger] R = \frac{1}{2} \xi \frac{(\mathcal{Q}\mathcal{Q})^\dagger \mathcal{Q}\mathcal{Q}}{\Lambda_{\mathrm{Ex}}^4} R, \tag{6}$$

[1]This is the problem in which one of the inflationary slow-roll parameters, denoted by η, is proportional to the inflaton mass. Therefore, if the inflaton is a fundamental scalar, this parameter receives quantum corrections. As a result, such corrections would spoil the slow-roll approximation.

where $\Lambda_{\rm Ex.}$ is a new high-energy scale at which this operator generates. Here the nonminimal coupling is added at the fundamental level showing that the nonminimal coupling is well motivated at the level of the fundamental description. However, an instructive analysis of the generated coupling of a composite scalar field to gravity has been initiated in the Nambu–Jona-Lasinio model [37]. With this regard, the nonminimal coupling apparently seems rather natural. Using the renormalization group equation for the chiral condensate, we find

$$\langle \mathcal{Q}\mathcal{Q}\rangle_{\Lambda_{\rm Ex}} \sim \left(\frac{\Lambda_{\rm Ex}}{\Lambda_{\rm TI}}\right)^{\gamma} \langle \mathcal{Q}\mathcal{Q}\rangle_{\Lambda_{\rm TI}}, \tag{7}$$

where the subscripts indicate the energy scale at which the corresponding operators are evaluated, and basically $\Lambda_{\rm Ex} \gg \Lambda_{\rm TI}$. If we assume the fixed value of γ is around 2 the explicit dependence on the higher energy $\Lambda_{\rm Ex}$. disappears. This is because we have $\mathcal{M} \sim \langle \mathcal{Q}\mathcal{Q}\rangle_{\Lambda_{\rm TI}}/\Lambda_{\rm TI}^2$. According to this model in the effective description, the relevant effective theory consisting of a composite inflaton (ϕ) and its pseudoscalar partner (Θ), as well as nine pseudoscalar Goldstone bosons ($\Pi^{\mathbb{A}}$) and their scalar partners ($\tilde{\Pi}^{\mathbb{A}}$) can be assembled in matrix form such that

$$\mathcal{M} = \left[\frac{\phi + i\Theta}{2} + \sqrt{2}(i\Pi^{\mathbb{A}} + \tilde{\Pi}^{\mathbb{A}})X^{\mathbb{A}}\right]\mathcal{E}, \tag{8}$$

where $X^{\mathbb{A}}$'s, $\mathbb{A} = 1, \ldots, 9$, are the generators of the SU(4) gauge group which do not leave the vacuum expectation value of $\mathcal{M}$ invariant, i.e. $\langle \mathcal{M}\rangle = v\mathcal{E}/2$, $v \equiv \langle\phi\rangle$. Here the (composite) scale of the theory is identified by $\Lambda_{\rm TI} = 4\pi v$, where v is the scale of the (new) fermion condensate, implying that $\Lambda_{\rm Ex} \gtrsim 4\pi v$. In this model, the composite inflaton is the lightest state ϕ, and the remaining composite fields are massive. This provides a sensible possibility to consider the ϕ dynamics only. The relevant composite inflaton effective action (nonminimally coupled to gravity) is given by

$$\mathcal{S}_{\rm J} = \int d^4x\sqrt{-g}\left[-\frac{M_{\rm P}^2}{2}\Omega^2 R + \frac{1}{2}(\nabla\phi)^2 - V(\phi)\right], \tag{9}$$

where

$$\Omega^2 = \frac{(M_{\rm P}^2 + \xi\phi^2)}{M_{\rm P}^2}, \qquad V(\phi) = -\frac{1}{2}m_\phi^2\phi^2 + \frac{\lambda}{4}\phi^4 \tag{10}$$

where λ is a self-coupling and the inflaton mass is $m_{\rm TI}^2 = 2m_\phi^2$. In order to carry out our investigation below, we assume for $\phi \gg m_\phi/\sqrt{\lambda}$ that the inflaton mass term, $m_\phi^2\phi^2/2$, does not affect the frequency of oscillations of the inflaton field ϕ. Recently, the authors of Refs. [38,39] showed that its predictions, i.e. the spectral index of curvature perturbation n_s and the tensor-to-scalar ratio r, fit very well with the data from the Planck satellite [7] (see Fig. 1) and from the improved analysis by the BICEP2 and Keck Array cosmic microwave background polarization experiments [40].

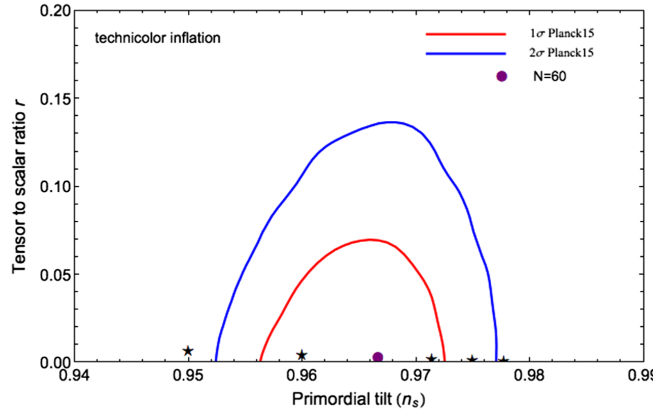

FIG. 1. The theoretical predictions in the $(r - n_s)$ plane for technicolor inflation with Planck 2015 results for TT, TE, EE, +lowP and assuming ΛCDM $+ r$ [7] for different values of e-folds $40 \leq N \leq 90$.

III. THE MODEL

In this work, we will examine the preheating process for a composite model of inflation involving nonminimal coupling between gravity and matter. Many attempts have been made to study inflationary models containing this kind of interaction term such as Higgs inflation [8] and pure $\lambda\phi^4$ theory [41–43]. In this section, we will follow the work proposed in Ref. [44] and focus on the two-field scenario. In addition, a simplified two-field model has been studied fairly often in the literature; e.g. see Refs. [45–47]. Moreover, the preheating effect of inflation for a multifield scenario has been investigated in Ref. [48]. In the following, we choose for our current investigation the four-dimensional action of our system in the Jordan (J) frame:

$$\mathcal{S}_{\rm J} = \int d^4x\sqrt{-g}\Big[-f(\Phi^i)R + \frac{1}{2}\delta_{ij}g^{\mu\nu}\nabla_\mu\Phi^i\nabla_\nu\Phi^j - V(\Phi^i)\Big], \tag{11}$$

where $i = 1, 2$ and $\Phi^i = (\phi, \chi)$ with χ being an additional scalar field. Let us consider a typical form of $f(\Phi^i)$ in our case in which the nonminimal couplings take the form

$$f(\Phi^i) = \frac{1}{2}(M_0^2 + \xi_{\Phi^i}(\Phi^i)^2), \tag{12}$$

where M_0 in this work is assigned to be the Planck constant $M_{\rm P}$ and the coupling strengths ξ_{Φ^i} are the couplings between curvature and matter fields.

In order to bring the gravitational portion of the action into the canonical Einstein-Hilbert form, we perform a conformal transformation by rescaling $\tilde{g}_{\mu\nu} = \Omega^2(x) g_{\mu\nu}$. Here we can relate the conformal factor $\Omega^2(x)$ to the nonminimal-coupling sector via

$$\Omega^2(x) = \frac{2}{M_{\rm P}^2} f(\Phi^i(x)). \tag{13}$$

By applying the conformal transformation given above, we can eliminate the nonminimal-coupling sector and obtain the resulting action in the Einstein frame [44]

$$\mathcal{S}_{\rm E} = \int d^4x\sqrt{-g}\bigg[-\frac{M_{\rm P}^2}{2}R + \frac{1}{2}\mathcal{G}_{ij} g^{\mu\nu}\nabla_\mu\Phi^i\nabla_\nu\Phi^j - \mathcal{U}(\Phi^i)\bigg], \tag{14}$$

with $\mathcal{U}(\Phi^i) \equiv V(\Phi^i)/\Omega^4$. Here we have dropped the tildes for convenience and $\mathcal{G}_{ij}$ is given by

$$\mathcal{G}_{ij} = \frac{M_{\rm P}^2}{2f}\delta_{ij} + \frac{3}{2}\frac{M_{\rm P}^2}{f^2} f_{,i} f_{,j}, \tag{15}$$

where $f_{,i} = \partial f/\partial\Phi^i$. In our case, the above quantity can be explicitly recast in terms of the fields (ϕ,χ) as

$$\mathcal{G}_{\phi\phi} = \frac{M_{\rm P}^2}{2f}\left(1 + \frac{3\xi_\phi \phi^2}{f}\right), \tag{16}$$

$$\mathcal{G}_{\phi\chi} = \mathcal{G}_{\chi\phi} = \frac{M_{\rm P}^2}{2f}\left(\frac{3\xi_\phi\xi_\chi\phi\chi}{f}\right), \tag{17}$$

$$\mathcal{G}_{\chi\chi} = \frac{M_{\rm P}^2}{2f}\left(1 + \frac{3\xi_\chi \phi^2}{f}\right). \tag{18}$$

The action in terms of the fields (ϕ,χ) takes the form

$$\begin{aligned}\mathcal{S}_{\rm E} = \int d^4x\sqrt{-g}\bigg[&-\frac{M_{\rm P}^2}{2}R \\ &+ \frac{1}{2}\frac{M_{\rm P}^2}{2f}\left(1+\frac{3\xi_\phi\phi^2}{f}\right) g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi \\ &+ \frac{1}{2}\frac{M_{\rm P}^2}{f}\left(\frac{3\xi_\phi\xi_\chi\phi\chi}{f}\right) g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\chi \\ &+ \frac{1}{2}\frac{M_{\rm P}^2}{2f}\left(1+\frac{3\xi_\chi\chi^2}{f}\right) g^{\mu\nu}\nabla_\mu\chi\nabla_\nu\chi - \mathcal{U}(\phi,\chi)\bigg]. \end{aligned} \tag{19}$$

Notice that the model being currently investigated coincides with that of the two-field scenario in which each of them is nonminimally coupled to the spacetime curvature. In such a case, there is no conformal transformation that can bring both the gravitational sector and the kinetics terms of the scalar sector into the canonical form. As a result, the fields would not behave as they would in a minimally coupled scenario.

However, in the present work we suppose that the field ϕ is nonminimally coupled to gravity and we set $\xi_\phi = \xi$, while only the field χ is minimally coupled to gravity, i.e. $\xi_\chi = 0$. Therefore, the action in terms of the fields (ϕ,χ) becomes

$$\begin{aligned}\mathcal{S}_{\rm E} = \int d^4x\sqrt{-g}\bigg[&-\frac{M_{\rm P}^2}{2}R \\ &+ \frac{1}{2}\frac{M_{\rm P}^2}{2F}\left(1+\frac{3\xi\phi^2}{F}\right) g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi \\ &+ \frac{1}{2}\frac{M_{\rm P}^2}{2F} g^{\mu\nu}\nabla_\mu\chi\nabla_\nu\chi - \mathcal{U}(\phi,\chi)\bigg], \end{aligned} \tag{20}$$

where $F \equiv f_{\xi_\chi=0}$. Notice that the resulting action can then be translated to a canonical form by imposing the rescaled fields. This implementation can be achieved by introducing the rescaled fields $\hat{\phi}(\phi,\chi)$ and $\hat{\chi}(\phi,\chi)$ such that

$$\frac{\partial\hat{\phi}}{\partial\phi} = \sqrt{\frac{M_{\rm P}^2}{2F}\left(1+\frac{3\xi\phi^2}{F}\right)} \quad \text{and} \quad \frac{\partial\hat{\chi}}{\partial\chi} = \sqrt{\frac{M_{\rm P}^2}{2F}}. \tag{21}$$

In terms of the new fields, the action takes the form

$$\mathcal{S}_{\rm E} = \int d^4x\sqrt{-g}\bigg[-\frac{M_{\rm P}^2}{2}R + \frac{1}{2}(\nabla\hat{\phi})^2 + \frac{1}{2}(\nabla\hat{\chi})^2 - \mathcal{U}(\hat{\phi}(\phi,\chi),\hat{\chi}(\phi,\chi))\bigg]. \tag{22}$$

In this case, the second term of Eq. (21) can then be integrated to yield

$$\chi = \left(1 + \frac{\xi\phi^2}{M_{\rm P}^2}\right)^{1/2}\hat{\chi}. \tag{23}$$

To find the explicit form of the potential in terms of a new variable $\hat{\phi}$, we must verify the expression of ϕ in terms of $\hat{\phi}$. As suggested in Ref. [30], this can be simply implemented by integrating the first term of Eq. (21), whose general solution is given by

$$\frac{\sqrt{\xi}}{M_{\rm P}}\hat{\phi}(\phi) = \sqrt{1+6\xi}\sinh^{-1}\left(\sqrt{1+6\xi}\,\mathbb{u}\right) - \sqrt{6\xi}\sinh^{-1}\left(\sqrt{6\xi}\frac{\mathbb{u}}{\sqrt{1+\mathbb{u}^2}}\right), \tag{24}$$

where $\mathbb{u} \equiv \sqrt{\xi}\phi/M_{\rm P}$. It is worth noting that in our case since $\xi \gg 1$ and using the identity $\sinh^{-1}\mathbb{x} = \ln(\mathbb{x} + \sqrt{\mathbb{x}+1})$ for $-\infty < \mathbb{x} < \infty$, this therefore allows us to rewrite the above general solution as

$$\frac{\sqrt{\xi}}{M_{\rm P}}\hat{\phi}(\phi) \approx \sqrt{6\xi}\ln\left(1+\frac{\xi\phi^2}{M_{\rm P}^2}\right)^{1/2}, \tag{25}$$

or, equivalently,

$$\frac{\xi}{M_{\rm P}^2}\phi^2(\hat{\phi}) \approx 1-\exp\left(\sqrt{\frac{2}{3}}\frac{\hat{\phi}}{M_{\rm P}}\right). \tag{26}$$

From the above relations, we also find

$$\Omega^2(\hat{\phi}) = \exp\left(\sqrt{\frac{2}{3}}\frac{\hat{\phi}}{M_{\rm P}}\right). \tag{27}$$

Notice that the field $\hat{\phi}$ is directly related to the conformal transformation factor Ω. With the above machinery, the resulting action in the Einstein frame yields the following equations of motion for ϕ and χ, respectively:

$$\ddot{\hat{\phi}} + 3\frac{\dot{a}}{a}\dot{\hat{\phi}} - \frac{1}{a^2}\nabla^2\hat{\phi} + \frac{\partial\mathcal{U}}{\partial\hat{\phi}} = 0, \tag{28}$$

$$\ddot{\hat{\chi}} + 3\frac{\dot{a}}{a}\dot{\hat{\chi}} - \frac{1}{a^2}\nabla^2\hat{\chi} + \frac{\partial\mathcal{U}}{\partial\hat{\chi}} = 0. \tag{29}$$

Here we are interested in a preheating effect after inflation. To begin with, we shall assume that the spacetime and the inflaton ϕ give a classical background and the scalar field χ is treated as a quantum field on that background. In the next section, we will examine the preheating mechanism of the inflationary model that has been proposed in the literature [13].

IV. PARAMETRIC RESONANCE

In terms of the component fields, after performing the conformal (Weyl) transformation, the transformed potential, $U(\hat{\phi},\hat{\chi})$, is given by

$$U(\hat{\phi},\hat{\chi}) = \exp\left(-2\sqrt{\frac{2}{3}}\frac{\hat{\phi}}{M_{\rm P}}\right)\left[\frac{\lambda}{4}\phi^4 - \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}g^2\phi^2\chi^2 - \frac{1}{2}m_\chi^2\chi^2\right]. \tag{30}$$

After relabeling $\hat{\phi}$ as ϕ, $\hat{\chi}$ as χ, etc., the resulting potential takes the form

$$U(\phi,\chi) \simeq \frac{\lambda M_{\rm P}^2}{4\xi^2}(1-e^{-\sqrt{2/3}\phi/M_{\rm P}})^2 - \frac{1}{2}m_\chi^2 e^{-2\sqrt{2/3}\phi/M_{\rm P}}\chi^2 + \frac{g^2M_{\rm P}^2}{2\xi}e^{-2\sqrt{2/3}\phi/M_{\rm P}}(e^{\sqrt{2/3}\phi/M_{\rm P}}-1)\chi^2, \tag{31}$$

where, in accord with our examination, we have neglected the inflaton mass term. The Klein-Gordon equation for the inflaton field is approximately given by

$$\ddot{\phi} + 3H\dot{\phi} + \mathcal{M}^2\phi = 0, \quad \text{with} \quad \mathcal{M}^2 \equiv \frac{\lambda M_{\rm P}^2}{3\xi^2}. \tag{32}$$

It was mentioned in Ref. [30] that the backreaction of the χ particle into the dynamics of the inflaton field will only be relevant once its occupation numbers have grown sufficiently and can in principle suppress the resonant particle production. However, in this investigation, we will assume that the process of parametric resonance is at the stage in which the backreaction of the created particles can be neglected. Nevertheless, we will carefully examine the backreaction effect on the model by closely following the work studied in Ref. [22] and will leave this for a future investigation. In order to obtain the solution of the above equation, we use for a power-law evolution $a \propto t^p$ and the equation of motion becomes

$$t^2\ddot{\phi} + 3pt\dot{\phi} + t^2\mathcal{M}^2\phi = 0. \tag{33}$$

The general solution of the effective equation of ϕ can be expressed in terms of the Bessel functions as

$$\phi(t) = \frac{1}{(\mathcal{M}t)^\nu}[AJ_{+\nu}(\mathcal{M}t) + BJ_{-\nu}(\mathcal{M}t)], \tag{34}$$

where A and B are constants depending on the initial conditions at the end of inflation, and $J_{\pm\nu}(\mathcal{M}t)$ are Bessel functions of order $\pm\nu$, with $\nu = (3p-1)/2$. Note that for a reasonable power index, we use $p = 2/3$ for matter and $p = 1/2$ for radiation. Moreover, the second term on the right-hand side of Eq. (34) diverges in the limit $\mathcal{M}t \to 0$ and therefore should be neglected on physical grounds. The physical solution to this equation is then simply given by

$$\phi(t) = A(\mathcal{M}t)^{\frac{-(3p-1)}{2}}J_{\frac{(3p-1)}{2}}(\mathcal{M}t), \tag{35}$$

where we have used the large argument expansion of fractional Bessel functions [49] such that $\mathcal{M}t \gg 1$. For the large argument expansion, the physical solution can be approximated by a cosinusoidal function [30]:

$$\phi(t) \simeq A(\mathcal{M}t)^{\frac{-(3p)}{2}}\cos\left(\mathcal{M}(t-t_{\rm os}) - (3p\pi/4)\right). \tag{36}$$

Here we can choose a constant A by considering the oscillatory behavior which starts just at the end of inflation, i.e. $\phi(t=t_0) = \phi_{\rm end} = \sqrt{6}M_{\rm P}\log(4/3)^{1/4}$. However, before further studying Eq. (36), it would be of great interest to examine the effective equation of state during the preheating phase. As already suggested in Ref. [48] by using the virial theorem, the equation of state parameter (ω) for the system to background (φ^I) order is given by

$$\omega \equiv \frac{\mathbb{P}}{\rho} = \frac{\dot{\varphi}^2 - 2\mathcal{U}}{\dot{\varphi}^2 + 2\mathcal{U}}, \tag{37}$$

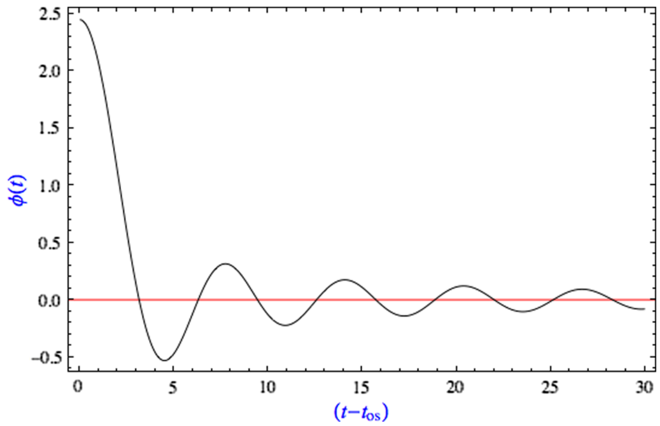

FIG. 2. We plot the approximate solution of the field $\phi(t)$ as given in Eq. (38). The value of the scalar field here is measured in units of M_P and time is measured in units of $\mathcal{M}^{-1}$.

where $\mathbb{p}$ and ρ are the pressure and the energy density, and $\dot{\psi}^2 \equiv \mathcal{G}_{IJ}\dot{\varphi}^I\dot{\varphi}^J$. Here we will estimate the parameter ω when the background fields begin to oscillate. As investigated in Ref. [48], the actual dynamics of the system interpolates over the first few oscillations between the matter-dominated and radiation-dominated effective equation of state.

More precisely, for large nonminimal coupling, the averaged parameter $\omega_{\rm avg}$ spends more time around $\omega_{\rm avg} \approx 0$ corresponding to $p = 2/3$ for matter-dominated behavior as the Universe continues to expand, while at late times the system behaves like radiation, that is to say $\omega_{\rm avg} \approx 1/3$ corresponding to $p = 1/2$ for this case. In the present examination, we will study the dynamics of the oscillation by considering $p = 2/3$ during the beginning of the preheating phase. In this case, hence, we obtain the physical solution of Eq. (36) as

$$\phi(t) = \frac{\phi_{\rm end}}{\mathcal{M}(t-t_{\rm os})}\sin(\mathcal{M}(t-t_{\rm os})), \tag{38}$$

where $t_{\rm os}$ is a time when the oscillating phase begins. In Fig. 2, we plot the evolution of $\phi(t)$ as governed by the approximate equation of motion of Eq. (33) with $p = 2/3$, and we see that the amplitude of the first oscillation drops by a factor of 10 during the first oscillation. From Eq. (29), the equation of motion for the field χ becomes

$$\ddot{\chi} + 3H\dot{\chi} - \frac{1}{a^2}\nabla^2\chi + \left[m_\chi^2 + \sqrt{\frac{2}{3}}\frac{g^2M_{\rm P}\phi}{\xi}\right]\chi = 0. \tag{39}$$

Expanding the scalar fields χ in terms of the Heisenberg representation as

$$\chi(t,\mathbf{x}) \sim \int (a_k\chi_k(t)e^{-i\mathbf{k}\cdot\mathbf{x}} + a_k^\dagger\chi_k^*(t)e^{i\mathbf{k}\cdot\mathbf{x}})d^3\mathbf{k}, \tag{40}$$

where a_k and $a_k^\dagger$ are annihilation and creation operators, we find that χ_k obeys the following equation of motion:

$$\ddot{\chi}_k + 3H\dot{\chi}_k + \left[\frac{k^2}{a^2} + m_\chi^2 + \sqrt{\frac{2}{3}}\frac{g^2M_{\rm P}\phi}{\xi}\right]\chi_k = 0. \tag{41}$$

Fourier transforming this equation and rescaling the field by $Y_k = a^{3/2}\chi_k$ yields

$$\ddot{Y}_k + \omega_k^2 Y_k = 0, \tag{42}$$

where a time-dependent frequency of Y_k is given by

$$\omega_k^2 = \frac{k^2}{a^2} + m_\chi^2 + \left[\sqrt{\frac{2}{3}}\frac{g^2\phi_{\rm end}M_{\rm P}}{\xi}\frac{1}{T(t)}\sin(T(t))\right], \tag{43}$$

with $T(t) = \mathcal{M}(t-t_{\rm os})$. Certainly, Eq. (42) describes an oscillator with a periodically changing frequency $\omega_k = k^2/a^2 + m_\chi^2 + [\sqrt{\frac{2}{3}}\frac{g^2\phi_{\rm end}M_{\rm P}}{\xi}\frac{1}{T(t)}\sin(T(t))]$. With $a = 1$, the physical momentum $\mathbf{p}$ coincides with $\mathbf{k}$ for Minkowski space such that $k = \sqrt{\mathbf{k}^2}$. The periodicity of Eq. (42) may lead to the parametric resonance for modes with certain values of k. In order to examine this behavior, we will introduce a new variable, z, defined by $\mathcal{M}(t-t_{\rm os}) = 2z - \pi/2$. In the Minkowski space for which we neglect the expansion of the Universe taking $a(t) = 1$, Eq. (42) becomes the standard Mathieu equation [50]:

$$\frac{d^2Y_k}{dz^2} + (A_k - 2q\cos(2z))Y_k = 0, \tag{44}$$

where

$$A_k = \frac{4}{\mathcal{M}^2}(k^2 + m_\chi^2), \qquad q = \frac{6\sqrt{2}g^2}{\sqrt{3}\beta}\frac{\phi_{\rm end}}{T(t)}, \tag{45}$$

where $\beta = \lambda M_{\rm P}/\xi$. In general, the parameters A_k and q control the strength of parametric resonance. This can be described by a stability-instability chart of the Mathieu equation; see for example Ref. [22]. As already mentioned above, the parameters A_k and q are constant in the Minkowski space. An important feature of the solution of Eq. (42) is the existence of an exponential instability $Y_k \propto \exp(\mu_k^{(n)}z)$ implying that Y_k grows exponentially with a growth index μ_k. In order to guarantee enough efficiency for the particle production, the Mathieu equation's parameters should satisfy the broad-resonance condition, i.e. $q \gg 1$. If this is the case, a broad resonance can possibly occur for a wide range of the parameter spaces and momentum modes.

Therefore, in order to satisfy a broad resonance condition, we discover that $q \sim 12g^2\xi/\lambda \gg 1$ implying $g^2/\lambda \gg 8.33\times10^{-6}$. For a strongly coupled theory we expect λ to be of the order of unity and in order to obtain successful inflation $\xi \sim \mathcal{O}(10^4)$ is initially required. In this case, we can estimate the coupling $g \gg 3.0\times10^{-3}$.

The existence of an exponential instability $\chi_k \propto \exp(\mu_k^{(n)} z)$ causes an exponential growth of occupation numbers of quantum fluctuations $n_k(t) \propto \exp(2\mu_k^{(n)} z)$ that may be interpreted as particle production.

Likewise, the growth of the modes Y_k leads to the growth of the occupation numbers of the produced particles n_k. As suggested in Ref. [22], the number density of particles $n_k(t)$ with momentum $\mathbf{k}$ can be evaluated as the energy of that mode $\frac{1}{2}|\dot{Y}_k|^2 + \frac{1}{2}\omega_k^2|Y_k|^2$ divided by the energy ω_k of each particle:

$$n_k(z) = \frac{\omega_k}{2}\left(\frac{|\dot{Y}_k|^2}{\omega_k^2} + |Y_k|^2\right) - \frac{1}{2}. \tag{46}$$

Typically, in the simplest inflationary scenario including the one we are considering now, the value of the Hubble constant at the end of inflation is of the same order as (but somewhat smaller than) the inflaton (effective) mass, $\mathcal{M}$. In order to clarify this, we can be even more concrete by evaluating the Hubble constant during the first oscillation. As shown in Fig. 2, during the first period of oscillation, the amplitude of the field $\phi(t)$ drops to around $1/2$ of the reduced Planck mass, $M_{\rm P}$. During this early phase of oscillation we may also expect the field's kinetic energy to be roughly equal to its potential energy, and hence we can estimate the energy density of the field to be $\rho \sim \mathcal{M}^2\phi^2 \sim \frac{1}{4}\mathcal{M}^2 M_{\rm P}^2$. This approximation allows us to further estimate the Hubble constant and we find that the Hubble rate would then be $H = \sqrt{\frac{1}{3M_{\rm P}^2}\rho} \sim \mathcal{M}/(2/\sqrt{3}) \sim 0.3\mathcal{M}$. Notice that the estimate for $H/\mathcal{M} \sim 0.3$ is consistent with the exact treatment found for this exact same approach in Fig. 8 of Ref. [48] in the limit $\xi_\phi \gg 1$.

Regarding the Mathieu equation, we note that there are instability bands in which the modes Y_k grow exponentially with the growth index $\mu_k = q/2$. As we have mentioned above, these instability bands depend on the parameters A_k and q. In order to ensure that the particle production is efficient, the Mathieu equation must satisfy the broad-resonance condition for which the conditions $A_k \simeq l^2$ where $l^2 = 1, 2, 3, \ldots,$ and $q \gg 1$ should be satisfied. Nevertheless, in general, the parameter q decreases with time. Therefore it must take a large enough initial value.

Finally, we consider in this section the limiting case when the parameter q takes a large value and make a plot of the time evolution of the fluctuations Y_k. We consider the typical resonance of particle production by taking $k = 5\mathcal{M}$ and $q \simeq 10^2$. The upper plot of Fig. 3 shows the amplification of the real part of the eigenmode $Y_k(z)$. Here the exponents show the order of magnitude for each given mode of fluctuations. Apparently, the amplitude of the fluctuation for the second mode is much larger than that of the first one, while the third one is much larger than those of the first two modes, and so forth. Moreover, the lower plot shows the logarithm of the comoving particle number density, $n_k(z)$, calculated with Eq. (46). The basic finding is that the number of particles grows exponentially, $\ln n_k \approx 2\mu_k z$. Here z is measured in units of $2\pi/\mathcal{M}$. The jump of the particle number density occurs only near $z = z_*$ when the amplitude of the inflaton field crosses zero, i.e. $\phi(t = t_*) = 0$.

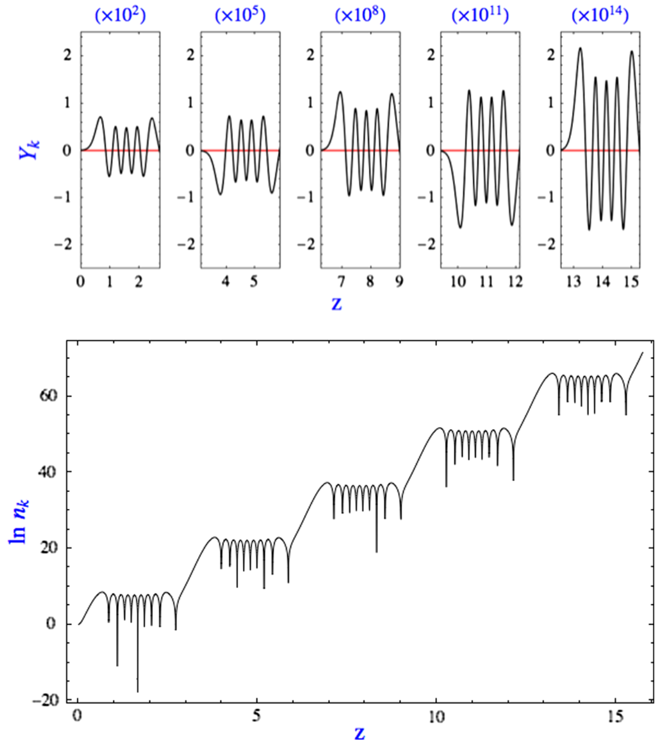

FIG. 3. By taking $k = 5\mathcal{M}(= m_\chi), q \sim 10^2$, the upper plot shows the amplification of the real part of the eigenmode $Y_k(z)$. The exponents show the order of magnitude for each given mode of fluctuations. We also see for each oscillation of the field $\phi(t)$ that the field Y_k oscillates many times. The lower plot shows the logarithm of the comoving particle number density, $n_k(z)$, calculated with Eq. (46). The basic finding is that the number of particles grows exponentially, $\ln n_k \approx 2\mu_k z$. Here z is measured in units of $2\pi/\mathcal{M}$.

V. DISCUSSION AND OUTLOOK

We have investigated the production of particles due to parametric resonances in a model of inflation in which the lightest composite states, which are expected to be the most important ones for collider phenomenology, stem from the MWT theory. We studied a model of a composite inflaton field ϕ coupled to another scalar field χ with the interaction term $g^2\phi^2\chi^2$. Particularly, in Minkowski space, the stage of parametric resonance can be simply described by the Mathieu equation. Furthermore, we discovered that broad resonances can be typically achieved and potentially efficient in our model.

In order to satisfy a broad resonance $q \gg 1$, we discover that $q \sim 12g^2\xi/\lambda$ implying $g^2/\lambda \gg 8.33 \times 10^{-6}$. For a strongly coupled theory we expect λ to be of the order of unity and in order to obtain successful inflation $\xi \sim \mathcal{O}(10^4)$ is initially required. In this case, we can estimate the coupling $g \gg 3.0 \times 10^{-3}$. Quite surprisingly, the condition for a broad resonance in our model is very similar to that of the $f(R)$ gravity model [51] in the sense that the nonminimal coupling must take a large value.

From Fig. 3, we discover that particle production in our model is potentially efficient causing the number of particles n_k in this process to increase. Using $q \simeq 10^2$, evidently for each oscillation of the field $\phi(t)$, the basic finding is that the number of particles grows exponentially, $\ln n_k \approx 2\mu kz$. Note that we here neglected the expansion of the Universe by taking $a = 1$ in Eq. (44) and assumed that $k = 5\mathcal{M} = m_\chi$.

Once again, in this exploratory study our approach is minimalistic rather than being aimed at great generality since during the parametric resonance stage, the produced particles are far away from equilibrium. As a result, the study of the thermalization at the end of the parametric resonance regime is also of interest, and yet the back-reaction effect of other particles into the dynamics of the inflaton is also interesting. However, we will leave these interesting topics for our future project.

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