Generic instabilities of nonsingular cosmologies in Horndeski theory: A no-go theorem

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The null energy condition can be violated stably in generalized Galileon theories, which gives rise to the possibilities of healthy nonsingular cosmologies. However, it has been reported that in many cases cosmological solutions are plagued with instabilities or have some pathologies somewhere in the whole history of the universe. Recently, this was shown to be generically true in a certain subclass of the Horndeski theory. In this short paper, we extend this no-go argument to the full Horndeski theory and show that nonsingular models (with flat spatial sections) in general suffer from either gradient instabilities or some kind of pathology in the tensor sector. This implies that one must go beyond the Horndeski theory to implement healthy nonsingular cosmologies.

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I. INTRODUCTION

Inflation [1–3] is now the strongest candidate of the early universe scenario that explains current cosmological observations consistently. Nonetheless, alternative scenarios deserve to be considered as well. First, in order to be convinced that inflation indeed occurred in the early stage of the universe, all other possibilities must be ruled out. Second, even inflation cannot resolve the problem of the initial singularity [4]. It is therefore well motivated to study how good and how bad alternative possibilities are compared to inflation. Nonsingular stages in the early universe, such as contracting and bouncing phases [5], cannot only be something that replaces inflation, but also "early-time" completion of inflation just to get rid of the initial singularity. In this paper, we address whether healthy nonsingular cosmologies can be implemented in the framework of general scalar-tensor theories.

If gravity is described by general relativity and the energy-momentum tensor $T_{\mu\nu}$ of matter satisfies the null energy condition (NEC), that is, $T_{\mu\nu}k^\mu k^\nu \geq 0$ for every null vector k^μ, then (assuming flat spatial sections) it follows from the Einstein equations that $\mathrm{d}H/\mathrm{d}t \leq 0$, where H is the Hubble parameter. This implies that an expanding universe yields a singularity in the past, while NEC violation could lead to singularity-free cosmology. However, violating the NEC in a healthy manner turns out to be challenging. The NEC is by construction satisfied for a canonical scalar field, $T_{\mu\nu}k^\mu k^\nu = \dot{\phi}^2 \geq 0$. In a general noncanonical scalar-field theory whose Lagrangian is dependent on ϕ and its first derivative [6,7], the NEC can be violated, but NEC-violating cosmological solutions are unstable because the curvature perturbation has the wrong sign kinetic term.

Galileon theory [8] and its generalizations [9,10] involve the scalar field whose Lagrangian contains second derivatives of ϕ while maintaining the second-order nature of the equation of motion and thus erasing the Ostrogradsky instability. In contrast to the previous case, it was found that the NEC and the stability of cosmological solutions are uncorrelated in Galileon-type theories [11]. This fact gives rise to healthy NEC-violating models of Galilean genesis [11–17] and stable nonsingular bouncing solutions [18–20], as well as novel dark energy and inflation models with interesting phenomenology [21,22]. See also a recent review [23].

Although the Galileon-type theories do admit a stable early stage without an initial singularity, the genesis/bouncing universe must be interpolated to a subsequent (possibly conventional) stage and the stable early stage does not mean that the cosmological solution is stable at all times during the whole history. Several explicit examples [24–30] show that the sound speed squared of the curvature perturbation becomes negative at around the transition between the genesis/bouncing phase and the subsequent phase, leading to gradient instabilities. In some cases the universe can experience a healthy bounce, but then the solution has some kind of singularity in the past or future [19]. Although the gradient instabilities can be cured by introducing higher spatial derivative terms [29,30] and there are some models in which the strong coupling scale cuts off the instabilities [31], it would be preferable if the potential danger could be removed from the beginning. The next question to ask therefore is whether the appearance of instabilities is generic or a model-dependent nature. For general dilation invariant theories a no-go theorem was given in Ref. [32]. (A counterexample was presented in Ref. [33], but it has an initial singularity.) Recently, it was clearly shown in Ref. [34] that bouncing and genesis models suffer from instabilities or have singularities for the scalar-tensor theory whose Lagrangian is of the form

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$$\mathcal{L} = \frac{R}{2\kappa} + G_2(\phi, X) - G_3(\phi, X)\Box\phi,$$
$$X := -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi, \tag{1}$$

where R is the Ricci scalar. This Lagrangian is widely used in the attempt to obtain nonsingular stable cosmology.

The Lagrangian (1) forms a subclass of the most general scalar-tensor theory with second-order field equations, i.e., the Horndeski theory [35]. The goal of this short paper is to generalize the no-go argument of Ref. [34] to the *full* Horndeski theory.

II. NO-GO THEOREM

We consider the Horndeski theory [35] in its complete form,

$$S = \int d^4x\sqrt{-g}\mathcal{L}_H, \tag{2}$$

where

$$\begin{aligned}\mathcal{L}_H = {} & G_2(\phi, X) - G_3(\phi, X)\Box\phi \\ & + G_4(\phi, X)R + G_{4,X}[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2] \\ & + G_5(\phi, X)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi - \frac{1}{6}G_{5,X}[(\Box\phi)^3 \\ & - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3].\end{aligned} \tag{3}$$

(The Lagrangian here is written in the form of the generalized Galileon [10], but the two theories are in fact equivalent [36].) In the full Horndeski theory, we have four arbitrary functions of the scalar field ϕ and $X = -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi/2$. The scalar field is coupled to the Ricci scalar R and the Einstein tensor $G_{\mu\nu}$ in the particular way shown above. The structure of the Lagrangian (3) guarantees the second-order nature of the field equations.

The equations of motion governing the background cosmological evolution can be obtained by substituting $ds^2 = -N^2(t)dt^2 + a^2(t)\delta_{ij}dx^i dx^j$ and $\phi = \phi(t)$ to the Horndeski action and varying it with respect to N, a, and ϕ [36]. In this paper, we only consider a spatially flat universe.

Linear perturbations around a spatially flat Friedmann-Lemaître-Robertson-Walker spacetime in the Horndeski theory were studied in Ref. [36]. Taking the unitary gauge, $\delta\phi = 0$, the spatial part of the metric can be written as $\gamma_{ij} = a^2(t)e^{2\zeta}(e^h)_{ij}$, where ζ is the curvature perturbation and h_{ij} is the tensor perturbation. The quadratic actions for h_{ij} and ζ are given, respectively, by [36]

$$S_h^{(2)} = \frac{1}{8}\int dt d^3x a^3\left[\mathcal{G}_T\dot{h}_{ij}^2 - \frac{\mathcal{F}_T}{a^2}(\partial h_{ij})^2\right] \tag{4}$$

and

$$S_\zeta^{(2)} = \int dt d^3x a^3\left[\mathcal{G}_S\dot{\zeta}^2 - \frac{\mathcal{F}_S}{a^2}(\partial\zeta)^2\right]. \tag{5}$$

Here, the coefficients are written as

$$\mathcal{F}_T := 2[G_4 - X(\ddot{\phi}G_{5,X} + G_{5,\phi})], \tag{6}$$

$$\mathcal{G}_T := 2[G_4 - 2XG_{4,X} - X(H\dot{\phi}G_{5,X} - G_{5,\phi})], \tag{7}$$

where a dot denotes differentiation with respect to cosmic time t, while $\mathcal{F}_S$ and $\mathcal{G}_S$ have more complicated dependence on the functions G_2, G_3, G_4, and G_5, the explicit forms of which are found in Ref. [36]. It is reasonable to assume that $\mathcal{F}_T$, $\mathcal{G}_T$, $\mathcal{F}_S$, and $\mathcal{G}_S$ are smooth functions of time. To avoid ghost and gradient instabilities, we require that

$$\mathcal{F}_T > 0, \quad \mathcal{G}_T > 0, \quad \mathcal{F}_S > 0, \quad \mathcal{G}_S > 0. \tag{8}$$

If ϕ is minimally coupled to gravity, we have $G_4 = \text{const}$ and $G_5 = 0$, and hence $\mathcal{F}_T = \mathcal{G}_T = \text{const}$. In other words, the time evolution of $\mathcal{F}_T$ and $\mathcal{G}_T$ is caused by nonminimal coupling to gravity.

The crucial point for the no-go argument is that $\mathcal{F}_S$ is generically of the form

$$\mathcal{F}_S = \frac{1}{a}\frac{d\xi}{dt} - \mathcal{F}_T, \tag{9}$$

where

$$\xi := \frac{a\mathcal{G}_T^2}{\Theta}, \tag{10}$$

with

$$\begin{aligned}\Theta := {} & -\dot{\phi}XG_{3,X} + 2HG_4 - 8HXG_{4,X} - 8HX^2G_{4,XX} \\ & + \dot{\phi}G_{4,\phi} + 2X\dot{\phi}G_{4,\phi X} + 2HX(3G_{5,\phi} + 2XG_{5,\phi X}) \\ & - H^2\dot{\phi}(5XG_{5,X} + 2X^2G_{5,XX}).\end{aligned} \tag{11}$$

Since Θ is something written in terms of ϕ and H, it is supposed to be a smooth function of time which is finite everywhere. This then implies that ξ can never vanish except at a singularity, $a = 0$. The absence of gradient instabilities is equivalent to

$$\frac{d\xi}{dt} > a\mathcal{F}_T > 0. \tag{12}$$

Integrating Eq. (12) from some t_i to t_f, we obtain

$$\xi_f - \xi_i > \int_{t_i}^{t_f} a\mathcal{F}_T dt. \tag{13}$$

This is the key equation for the following argument, and it was used to prove the no-go theorem in the subclass of the

Horndeski theory with $G_4 = \text{const}$ and $G_5 = 0$ in Ref. [34]. Remarkably, it turns out that essentially the same equation holds in the full Horndeski theory.

Now, consider a nonsingular universe which satisfies $a > \text{const}\ (> 0)$ for $t \to -\infty$ and is expanding for large t. The integral in the right hand side of Eq. (13) may be convergent or not as one takes $t_\mathrm{f} \to \infty$ and $t_\mathrm{i} \to -\infty$, depending on the asymptotic behavior of $\mathcal{F}_T$. To allow the integral to converge, it is necessary that $\mathcal{F}_T$ approaches zero sufficiently fast in the asymptotic past or future. For the moment let us focus on the case where the integral is not convergent.

Suppose that $\xi_\mathrm{i} < 0$. Equation (13) reads

$$-\xi_\mathrm{f} < |\xi_i| - \int_{t_\mathrm{i}}^{t_\mathrm{f}} a\mathcal{F}_T \mathrm{d}t. \tag{14}$$

Since the integral is an increasing function of t_f, the right hand side becomes negative for sufficiently large t_f. We therefore have $\xi_\mathrm{f} > 0$, which means that ξ crosses zero.[1] This is never possible in a nonsingular universe. It is therefore necessary to have $\xi > 0$ everywhere. Writing Eq. (13) as

$$-\xi_\mathrm{i} > -\xi_\mathrm{f} + \int_{t_\mathrm{i}}^{t_\mathrm{f}} a\mathcal{F}_T \mathrm{d}t, \tag{15}$$

we see that the right hand side will be positive for $t_\mathrm{i} \to -\infty$ and hence $\xi_\mathrm{i} < 0$. However, this is in contradiction to the assumption that ξ is always positive. Thus, we have generalized the no-go argument of Ref. [34] to the full Horndeski theory.

The same no-go theorem holds even in the presence of another field, provided at least that the field is described by

$$\mathcal{L}_\chi = P(\chi, Y), \qquad Y := -\frac{1}{2} g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi, \tag{16}$$

which is not coupled to the Horndeski field ϕ directly.

Now there are two degrees of freedom in the scalar sector of cosmological perturbations. In terms of

$$\vec{y} := \left(\zeta, \frac{\Theta}{\mathcal{G}_T}\frac{\delta\chi}{\dot{\chi}}\right), \tag{17}$$

the quadratic action can be written in the form [37–39]

$$S^{(2)} = \int \mathrm{d}t\mathrm{d}^3x a^3 \left[\dot{\vec{y}}\mathbf{G}\dot{\vec{y}} - \frac{1}{a^2}\partial\vec{y}\mathbf{F}\partial\vec{y} + \cdots\right], \tag{18}$$

where

$$\mathbf{G} = \begin{pmatrix} \mathcal{G}_S + Z & -Z \\ -Z & Z \end{pmatrix}, \quad \mathbf{F} = \begin{pmatrix} \mathcal{F}_S & -c_s^2 Z \\ -c_s^2 Z & c_s^2 Z \end{pmatrix}, \tag{19}$$

with

$$c_s^2 := \frac{P_{,Y}}{P_{,Y} + 2YP_{,YY}}, \quad Z := \left(\frac{\mathcal{G}_T}{\Theta}\right)^2 \frac{YP_{,Y}}{c_s^2}. \tag{20}$$

Here, $\mathcal{G}_S$ and $\mathcal{F}_S$ were defined previously and c_s is the sound speed of the χ field. We have the relation $2YP_{,Y} = \rho + P$, where ρ is the energy density of χ and P corresponds to the pressure of χ.

Ghost instabilities can be evaded if $\mathbf{G}$ is a positive definite matrix. The condition amounts to

$$\mathcal{G}_S > 0, \qquad \frac{YP_{,Y}}{c_s^2} > 0. \tag{21}$$

The propagation speed v can be determined by solving

$$\det(v^2\mathbf{G} - \mathbf{F}) = 0, \tag{22}$$

yielding the condition for the absence of gradient instabilities,

$$c_s^2 > 0, \qquad \frac{\mathcal{F}_S - c_s^2 Z}{\mathcal{G}_S} > 0. \tag{23}$$

We thus have the inequality

$$\mathcal{F}_S > \frac{1}{2}\left(\frac{\mathcal{G}_T}{\Theta}\right)^2 (\rho + P) > 0, \tag{24}$$

and taking the same way we can show the no-go theorem for the Horndeski $+k$-essence (or a perfect fluid) system.

The no-go theorem we have thus established can be circumvented only if $\mathcal{F}_T$ approaches zero sufficiently fast either in the asymptotic past or the future, given the assumption that the evolution of the scale factor is nonsingular.[2] The normalization of vacuum quantum fluctuations tells us that they would grow and diverge as $\mathcal{F}_T \to 0$, and hence the tensor sector is pathological in the asymptotic past or future.[3] In the next section, we will demonstrate that, in contrast to the cases in Refs. [29,30], it is indeed possible to construct a model that exhibits a stable transition from the Galilean genesis to inflation by allowing for some kind of pathology in the tensor sector due to vanishing $\mathcal{F}_T$.

III. STABLE TRANSITION FROM GENESIS TO DE SITTER WITH PATHOLOGIES IN THE PAST

Let us turn to study a specific setup as an example: Galilean genesis followed by inflation. Such an expansion

[1]We do not allow for discontinuity in ξ because $\mathcal{F}_S$ is supposed to be smooth. (This means that Θ cannot cross zero.)

[2]The "modified genesis" model proposed in Ref. [34] evades the no-go theorem by the use of the vanishing scale factor in the asymptotic past. In contrast, we are assuming that the expansion history is nonsingular everywhere, i.e., $a \geq \text{const}$.

[3]One could resolve this issue by the particular, fine-tuned evolution of $\mathcal{G}_T$, which would offer a loophole.

history was proposed in Refs. [29,30] as early-time completion of the inflationary universe, and there it was pointed out that the sound speed squared (or more specifically $\mathcal{F}_S$) becomes negative at the transition from the genesis phase to inflation. This is consistent with the no-go theorem, because in the genesis phase we have $a \to \text{const}$ as $t \to -\infty$ and $\mathcal{F}_T = \text{const}$. The resultant gradient instability is cured by the introduction of higher order spatial derivatives arising in the effective field theory approach [29] or in theories beyond Horndeski [30,40,41].

Working within the second-order theory, i.e., the Horndeski theory, we are going to show in this section that the stable transition is indeed possible if $\mathcal{F}_T \to 0$ as $t \to -\infty$ so that the integral in Eq. (13) is convergent. To do so it is more convenient to use the Arnowitt-Deser-Misner (ADM) form of the action rather than the original covariant one [30]. The ADM decomposition of the Horndeski Lagrangian leads to [40]

$$\begin{aligned}\mathcal{L} &= A_2(t,N) + A_3(t,N)K + A_4(t,N)(K^2 - K_{ij}^2)\\ &\quad + A_5(t,N)(K^3 - 3KK_{ij}^2 + 2K_{ij}^3)\\ &\quad + B_4(t,N)R^{(3)} + B_5(t,N)K^{ij}G_{ij}^{(3)},\end{aligned} \tag{25}$$

where $\phi = \text{const}$ hypersurfaces are taken to be constant time hypersurfaces, and K_{ij}, $R_{ij}^{(3)}$, and $G_{ij}^{(3)}$ are the extrinsic curvature, the Ricci tensor, and the Einstein tensor of the spatial slices. The functions of ϕ and X in the covariant Lagrangian are now the functions of t and the lapse function N. Two of the six functions in the ADM Lagrangian (25) are subject to the constraints

$$A_4 = -B_4 - N\frac{\partial B_4}{\partial N}, \qquad A_5 = \frac{N}{6}\frac{\partial B_5}{\partial N}, \tag{26}$$

in accordance with the fact that there are four arbitrary functions in the Horndeski theory.

The specific example we are going to study is given by the functions of the form

$$\begin{aligned}&A_2 = f^{-2(\alpha+1)-\delta}a_2(N), \qquad A_3 = f^{-2\alpha-1-\delta}a_3(N),\\ &A_4 = -B_4 = -f^{-2\alpha}, \qquad A_5 = B_5 = 0,\end{aligned} \tag{27}$$

where $f = f(t)$ is dependent only on t, and α and δ are constant parameters satisfying $2\alpha > 1 + \delta > 1$. This class of models is similar to but different from that in Ref. [30]. The covariant form of the Lagrangian can be recovered by reintroducing the scalar field, e.g., through $-t = e^{-\phi}$ and $N^{-1} = e^{-\phi}\sqrt{2X}$ and using the Gauss-Codazzi equations. In terms of $G_2(\phi,X)$, $G_3(\phi,X)$, …, the Lagrangian is written in a slightly more complicated form [42]. Without moving to the covariant description, one can derive the equations of motion for the homogeneous background directly from variation of the ADM action with respect to N and the scale factor a.

The evolution of the Hubble parameter, $H \coloneqq N^{-1}\mathrm{d}\ln a/\mathrm{d}t$, is dependent crucially on the choice of $f(t)$, and to describe the genesis to de Sitter transition we take $f(t)$ such that $f \sim c(-t) \gg 1$ $(c > 0)$ in the past and $f \sim \text{const}$ in the future. In the early time, we have an approximate solution of the form

$$H \simeq \frac{\text{const}}{(-t)^{1+\delta}}, \tag{28}$$

and hence the universe starts expanding from Minkowski,

$$a \simeq 1 + \frac{\text{const}}{(-t)^{\delta}}, \tag{29}$$

with $N \simeq \text{const}$. In the late time where $f \simeq \text{const}$, we have an inflationary solution $H \simeq \text{const}$, again with $N \simeq \text{const}$. For all the models described by (27), we have

$$\mathcal{F}_T = \mathcal{G}_T = f^{-2\alpha} > 0, \tag{30}$$

and hence the stability conditions for the tensor modes are fulfilled. Since $a\mathcal{F}_T \sim (-t)^{-2\alpha}$ with $2\alpha > 1$ as $t \to -\infty$, $\mathcal{F}_T$ possesses the desired property to evade the no-go theorem.

As a concrete example, we consider

$$a_2 = -\frac{1}{N^2} + \frac{1}{3N^4}, \qquad a_3 = \frac{1}{4N^3}, \tag{31}$$

with $\alpha = 1$, $\delta = 1/2$, and

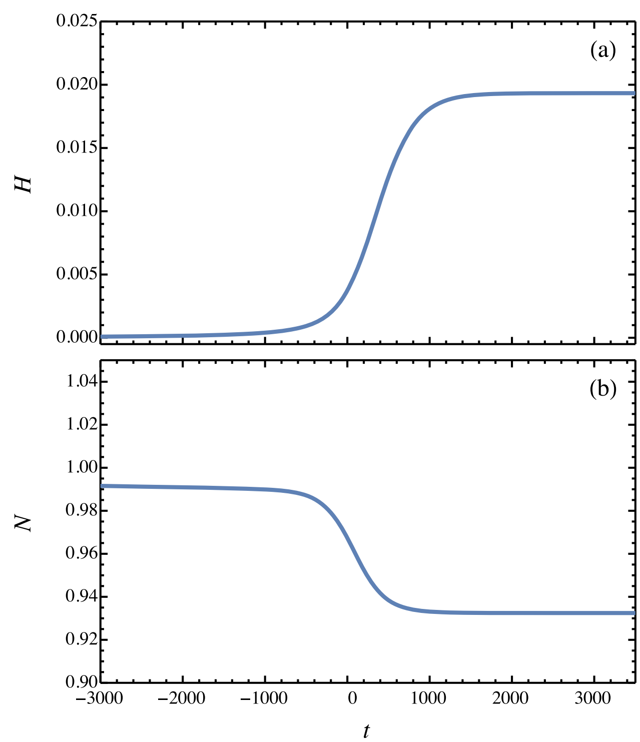

FIG. 1. Evolution of the Hubble parameter and the lapse function around the genesis–de Sitter transition.

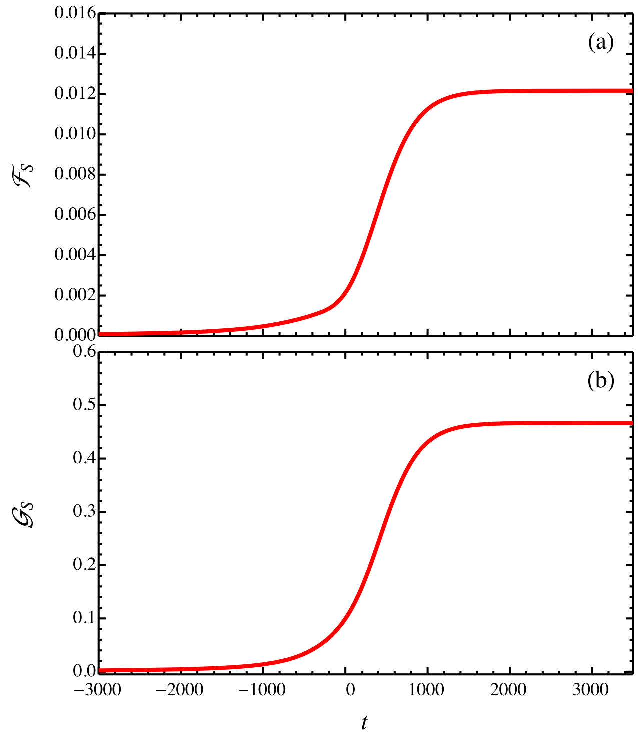

FIG. 2. $\mathcal{F}_S$ and $\mathcal{G}_S$ around the genesis–de Sitter transition.

$$f(t) = \frac{c}{2}\left[-t + \frac{\ln(2\cosh(st))}{s}\right] + f_1, \qquad (32)$$

where the parameters are taken to be $c = 10^{-1}$, $f_1 = 10$, and $s = 2 \times 10^{-3}$. The background equations are solved numerically to give the evolution of H and N as shown in Fig. 1. It can be seen that the universe indeed undergoes the genesis phase followed by inflation. For this background solution we evaluate $\mathcal{F}_S$ and $\mathcal{G}_S$ numerically to judge its stability. As presented in Fig. 2, we find that $\mathcal{F}_S$ and $\mathcal{G}_S$ remain positive in the whole expansion history. This is in contrast to the similar example in Refs. [29,30] which has $\mathcal{F}_S < 0$ around the transition.

Although the present model can circumvent the gradient instability at the genesis–de Sitter transition, some pathologies arise in the $t \to -\infty$ limit. We see that $\mathcal{F}_T, \mathcal{G}_T \sim (-t)^{-2\alpha}$ and $\mathcal{F}_S, \mathcal{G}_S \sim (-t)^{-2\alpha+\delta}$ in the genesis phase, leading to the vanishing quadratic action for tensor and scalar fluctuations in the $t \to -\infty$ limit. This implies that the validity of the perturbative expansion is questionable early in the genesis phase, which is, in fact, worse than what is required for violating the no-go theorem, i.e., $\mathcal{F}_T \to 0$ as $t \to -\infty$.

IV. SUMMARY

In this paper, we have generalized the no-go argument of Ref. [34] to the full Horndeski theory and shown that nonsingular cosmological models with flat spatial sections are in general plagued with gradient instabilities or some pathological behavior of tensor perturbations. We have presented an explicit example which is free from singularities and instabilities but has a vanishing quadratic action for the tensor perturbations (and for the curvature perturbation as well) in the asymptotic past. To circumvent the no-go theorem, it is therefore necessary to go beyond the Horndeski theory. One direction is to consider a (yet unknown) multifield extension of the Horndeski theory [39,43–47] in which scalar fields are coupled nontrivially to each other. Another is extending further the single-field Horndeski theory as has been done recently, e.g., in Refs. [40,41,48–53]. It would be interesting to explore to what extent the no-go argument for nonsingular cosmologies can be generalized.

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