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Zeros in the magic neutrino mass matrix

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We study the phenomenological implications of the presence of two zeros in a magic neutrino mass matrix. We find that only two such patterns of the neutrino mass matrix are experimentally acceptable. We express all the neutrino observables as functions of one unknown phase ϕ and two known parameters Δm^2_{12}, $r = \Delta m^2_{12}/\Delta m^2_{23}$. In particular, we find $\sin^2\theta_{13} = (2/3)r/(1+r)$. We also present a mass model for the allowed textures based upon the group A_4 using the type $\mathrm{I}+\mathrm{II}$ seesaw mechanism.

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I. INTRODUCTION

The observation of a nonzero reactor mixing angle (θ_{13}) [1] was an important landmark in neutrino physics as it excluded the possibility of the $\mu-\tau$ symmetry [2] as an exact symmetry of the neutrino mass matrix. Before this discovery, the tribimaximal (TBM) mixing [3] was an important feature in the neutrino mass models as it correctly predicted the solar mixing angle (θ_{12}) and the atmospheric mixing angle (θ_{23}). TBM mixing was thought to be a signature of some flavor symmetry in the Lagrangian that expresses itself as a residual symmetry in the neutrino mass matrix. However, TBM mixing is in itself a combination of the following two symmetries:

(1) *Magic symmetry*. The sum of elements in any row or column of the neutrino mass matrix is identical [4].

(2) *$\mu-\tau$ symmetry*. The neutrino mass matrix remains invariant after the interchange of the $\mu-\tau$ indices [2].

The neutrino mass matrix with $\mu-\tau$ symmetry implies a vanishing value of θ_{13} and a maximal value of θ_{23}. Such a mass matrix has the bimaximal eigenvector $v = (0 \frac{-1}{\sqrt{2}} \frac{1}{\sqrt{2}})^T$. After the measurement of a relatively large value of θ_{13}, the neutrino mass matrix cannot have exact $\mu-\tau$ symmetry. However, the neutrino mass matrix can still have the magic symmetry. The corresponding mixing pattern, called trimaximal (TM) mixing, has its middle column identical to that of TBM mixing. The other two columns are arbitrary within the unitarity constraints.

TM mixing has been intensively studied in the literature [5] and the corresponding magic mass matrix has been realized in many neutrino mass models [6]. The main limitation of the magic symmetry is that it is not very predictive. It predicts TM mixing that implies two sum rules: one between the mixing angles θ_{12} and θ_{13} and another between the mixing angle θ_{23} and the CP-violating Dirac phase δ. To make the magic symmetry more predictive, we can combine it with some additional constraint. The simplest constraint that could combine with magic symmetry was the $\mu-\tau$ symmetry. But, the observation of a nonvanishing θ_{13} has already ruled out this possibility. Another constraint can be the presence of zeros [7–9] in the magic neutrino mass matrix. In this work, we study this possibility.

In Sec. II, we highlight the salient features of the TBM mixing pattern and review its relation with TM mixing. We identify the phenomenologically allowed textures of two zeros in the magic neutrino mass matrix in Sec. III. Then, we study the phenomenology of the viable textures in Sec. IV and construct a mass model for them in Sec. V. Finally, we conclude in Sec. VI.

II. FROM TBM TO TM MIXING

The TBM mixing matrix is

$$U_{\mathrm{TBM}} = \begin{pmatrix} -\frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{1}$$

It is called the tribimaximal mixing matrix because the corresponding neutrino mass matrix

$$M_{\mathrm{TBM}} = U^*_{\mathrm{TBM}} M_{\mathrm{diag}} U^\dagger_{\mathrm{TBM}} \tag{2}$$

has a trimaximal eigenvector $u = (\frac{1}{\sqrt{3}} \frac{1}{\sqrt{3}} \frac{1}{\sqrt{3}})^T$ and a bimaximal eigenvector $v = (0 \frac{-1}{\sqrt{2}} \frac{1}{\sqrt{2}})^T$. Here,

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$$M_{\rm diag} = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & e^{2i\alpha}m_2 & 0 \\ 0 & 0 & e^{2i\beta}m_3 \end{pmatrix}, \qquad (3)$$

where m_1, m_2, and m_3 are the three neutrino masses and α and β are two Majorana phases. The TBM mass matrix $M_{\rm TBM}$ is invariant under the transformations G_u and G_v, i.e. $G_u^T M_{\rm TBM} G_u = M_{\rm TBM}$ and $G_v^T M_{\rm TBM} G_v = M_{\rm TBM}$, where $G_u = 1 - 2uu^T$ and $G_v = 1 - 2vv^T$. The transformation G_u corresponds to the magic symmetry and the transformation G_v corresponds to the $\mu - \tau$ symmetry. A diagonal charged lepton mass matrix will be invariant under the transformation $F = {\rm diag}(1, \omega, \omega^2)$, where $\omega = \exp(\frac{2\pi i}{3})$. In this way, the combined symmetry group generated by G_u, G_v, and F is S_4 [10]. Such neutrino mass models, where some of the generators of a symmetry group are directly preserved in the lepton sector, are called direct models. Another set of models, where the observed symmetry in the lepton sector emerges accidentally, are called indirect models. For a detailed discussion of this classification, see Refs. [11,12].

Since the neutrino oscillation experiments have measured a nonzero θ_{13}, the neutrino mass matrix M_ν cannot be invariant under the $\mu - \tau$ symmetry transformation G_v. However, M_ν can still be invariant under the magic symmetry transformation G_u. The magic symmetry is still allowed experimentally. The mixing matrix corresponding to the magic symmetry is called trimaximal mixing and is given by

$$U_{\rm TM} = \begin{pmatrix} \sqrt{\frac{2}{3}}\cos\theta & \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}}\sin\theta \\ -\frac{\cos\theta}{\sqrt{6}} + \frac{e^{-i\phi}\sin\theta}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -\frac{\sin\theta}{\sqrt{6}} - \frac{e^{-i\phi}\cos\theta}{\sqrt{2}} \\ -\frac{\cos\theta}{\sqrt{6}} - \frac{e^{-i\phi}\sin\theta}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -\frac{\sin\theta}{\sqrt{6}} + \frac{e^{-i\phi}\cos\theta}{\sqrt{2}} \end{pmatrix}. \qquad (4)$$

Since, the middle column of the TM mixing matrix is fixed to its TBM value (u), the mixing matrix still has two free parameters (θ and ϕ) after the unitarity constraints are taken into account. The corresponding neutrino mass matrix for TM mixing is called the magic mass matrix and is given as

$$M_{\rm magic} = U_{\rm TM}^* M_{\rm diag} U_{\rm TM}^\dagger. \qquad (5)$$

III. ZEROS OF THE MAGIC MASS MATRIX

In the basis where the charged lepton mass matrix is diagonal, there are seven mass matrices with two zeros [7,8] that are consistent with the current experimental data [13]. They have been further classified in the three classes which are depicted in Table I. When we combine the magic symmetry and the texture zeros, not all of the seven textures will be allowed.

TABLE I. Seven allowed mass matrices with two zeros classified into three classes.

Type	Constraining Equations
A_1	$M_{ee} = 0$, $M_{e\mu} = 0$
A_2	$M_{ee} = 0$, $M_{e\tau} = 0$
B_1	$M_{e\tau} = 0$, $M_{\mu\mu} = 0$
B_2	$M_{e\mu} = 0$, $M_{\tau\tau} = 0$
B_3	$M_{e\mu} = 0$, $M_{\mu\mu} = 0$
B_4	$M_{e\tau} = 0$, $M_{\tau\tau} = 0$
C	$M_{\mu\mu} = 0$, $M_{\tau\tau} = 0$

A most general magic mass matrix can be parametrized as [4]

$$M_{\rm magic} = \begin{pmatrix} a & b & c \\ b & d & a+c-d \\ c & a+c-d & b-c+d \end{pmatrix}. \qquad (6)$$

We can obtain the constraining equations for the various allowed textures of two zeros in the magic mass matrix by substituting the respective constraints from Table I into Eq. (6).

A. Class A

Magic neutrino mass matrices having textures A_1 and A_2 can be expressed as

$$M_{\rm magic}^{A_1} = \begin{pmatrix} 0 & 0 & c \\ 0 & d & c-d \\ c & c-d & -c+d \end{pmatrix} \qquad (7)$$

and

$$M_{\rm magic}^{A_2} = \begin{pmatrix} 0 & b & 0 \\ b & d & -d \\ 0 & -d & b+d \end{pmatrix}, \qquad (8)$$

respectively. The mass matrix for the magic A_1 texture can be rewritten as

$$M_{\rm magic}^{A_1} = \begin{pmatrix} 0 & 0 & c \\ 0 & c-\Delta & \Delta \\ c & \Delta & -\Delta \end{pmatrix}, \qquad (9)$$

where $\Delta = c - d$. This redefinition puts our representations of the textures A_1 and A_2 on equal footing. These two magic zero textures are allowed experimentally for normal hierarchy. Their phenomenology is studied in the Sec. IV.

B. Class B

The four magic mass matrices of class **B** are

$$M_{\rm magic}^{B_1} = \begin{pmatrix} a & b & 0 \\ b & 0 & a \\ 0 & a & b \end{pmatrix}, \tag{10}$$

$$M_{\rm magic}^{B_2} = \begin{pmatrix} a & 0 & c \\ 0 & c & a \\ c & a & 0 \end{pmatrix}, \tag{11}$$

$$M_{\rm magic}^{B_3} = \begin{pmatrix} a & 0 & c \\ 0 & 0 & a+c \\ c & a+c & -c \end{pmatrix}, \tag{12}$$

and

$$M_{\rm magic}^{B_4} = \begin{pmatrix} a & b & 0 \\ b & -b & a+b \\ 0 & a+b & 0 \end{pmatrix}. \tag{13}$$

The magic mass matrices of type B_1 and B_2 are not allowed as they predict $m_1 = m_3$. The magic mass matrices of types B_3 and B_4 are not allowed because these textures predict a very large value for the ratio $r = \Delta m_{12}^2/\Delta m_{23}^2$ when θ_{13} is small. We illustrate this tension between r and θ_{13} for the magic mass matrices of types B_3 and B_4 in Sec. IV.

C. Class C

The magic mass matrix of class **C** is

$$M_{\rm magic}^{C} = \begin{pmatrix} a & b & b \\ b & 0 & a+b \\ b & a+b & 0 \end{pmatrix}. \tag{14}$$

This mass matrix has $\mu-\tau$ symmetry and implies $\theta_{13} = 0$. Hence, it is not allowed.

IV. PHENOMENOLOGICAL IMPLICATIONS

The phenomenology of the textures A_1 and A_2 is related: one can obtain the predictions for A_2 by making the transformations

$$\theta_{23} \to \frac{\pi}{2} - \theta_{23}, \qquad \delta = \pi - \delta \tag{15}$$

on the predictions of texture A_1. Hence, we study the phenomenological implications for texture A_1 only.

The above transformation [Eq. (15)] also relates the predictions for textures B_3 and B_4. So, we show the incompatibility of the magic mass matrix of type B_3 with the experimental data at the end of this section. Then, Eq. (15) automatically implies that the magic mass matrix of type B_4 is also inconsistent with the experimental data.

A. Diagonalization of a magic mass matrix

Any magic mass matrix M can be diagonalized by a trimaximal mixing matrix $U = U_{\rm TM}$ given in Eq. (4) using the relation

$$U^T M U = M_{\rm diag}, \tag{16}$$

where $M_{\rm diag}$ is the diagonal mass matrix given by Eq. (3).

The mixing angles can be calculated from U using the relations

$$s_{12}^2 = \frac{|U_{12}|^2}{1-|U_{13}|^2}, \quad s_{23}^2 = \frac{|U_{23}|^2}{1-|U_{13}|^2}, \quad \text{and} \quad s_{13}^2 = |U_{13}|^2. \tag{17}$$

Substituting the elements of the TM mixing matrix into the above equation, we get

$$s_{12}^2 = \frac{1}{3-2\sin^2\theta}, \tag{18}$$

$$s_{23}^2 = \frac{1}{2}\left(1 + \frac{\sqrt{3}\sin 2\theta\cos\phi}{3-2\sin^2\theta}\right), \tag{19}$$

and

$$s_{13}^2 = \frac{2}{3}\sin^2\theta. \tag{20}$$

The CP-violating phase δ can be calculated from the Jarlskog rephasing invariant measure of CP violation [14]

$$J = {\rm Im}(U_{11}U_{12}^* U_{21}^* U_{22}) \tag{21}$$

using the relation

$$J = s_{12}s_{23}s_{13}c_{12}c_{23}c_{13}^2 \sin\delta. \tag{22}$$

Substituting the elements of the TM mixing matrix into Eq. (21), we obtain

$$J = \frac{1}{6\sqrt{3}}\sin 2\theta\cos\phi. \tag{23}$$

From Eqs. (22) and (23), we get

$$\csc^2\delta = \csc^2\phi - \frac{3\sin^2 2\theta\cot^2\phi}{(3-2\sin^2\theta)^2}. \tag{24}$$

B. Analysis of class A_1

We reconstruct the magic neutrino mass matrix using Eq. (5), viz.,

$$M_\nu = U^* M_{\rm diag} U^\dagger, \tag{25}$$

where $M_\nu = M_{\rm magic}$ and $U = U_{\rm TM}$. To obtain the predictions for the neutrino mass matrix of the type A_1 given by Eq. (9), we have to solve the two complex equations $M_{\nu_{11}} = 0$ and $M_{\nu_{12}} = 0$.

Solving the equation $M_{\nu_{11}} = 0$, we get

$$\frac{m_1}{m_2} = \frac{\sin 2(\alpha - \beta)}{2\sin 2\beta \cos^2\theta} \tag{26}$$

and

$$\frac{m_2}{m_3} = -\frac{2\sin 2\beta \sin^2\theta}{\sin 2\alpha}. \tag{27}$$

Using these two equations, we evaluate m_1/m_3 and invert the resulting relation to obtain

$$\cot 2\alpha = \cot 2\beta + \frac{m_1}{m_3}\csc 2\beta \cot^2\theta. \tag{28}$$

We note that the presence of a zero in the (1, 1) entry in a magic mass matrix, through Eqs. (26) and (27), implies a beautiful sum rule on neutrino masses:

$$\frac{\sin 2(\alpha-\beta)}{m_1} - \frac{2\sin 2\beta}{m_2} - \frac{\sin 2\alpha}{m_3} = 0. \tag{29}$$

The texture zero at the (1, 1) entry in a magic mass matrix also gives a nice prediction for the ratio $r = \Delta m^2_{12}/\Delta m^2_{23}$. From Eqs. (26) and (27), we obtain

$$r = \frac{-\sin^2 2(\alpha-\beta) + 4\cos^2\theta \sin^2 2\beta}{\cot^2\theta \sin^2 2\alpha - 4\cos^2\theta \sin^2 2\beta}. \tag{30}$$

Instead of solving the second equation $M_{\nu_{12}} = 0$, we solve the equivalent complex equation $M_{\nu_{11}} = M_{\nu_{12}}$ by equating the real and imaginary parts of the two sides. After a little algebra, we obtain

$$\frac{m_1}{m_3} = \frac{\sqrt{3}\sin 2\beta \tan\theta + \sin(2\beta - \phi)}{\sin\phi} \tag{31}$$

and

$$\tan 2\beta = -\frac{\sqrt{3}\sin\phi}{\sqrt{3}\cos 2\theta \cos\phi + \sin 2\theta}. \tag{32}$$

Using Eq. (31) to simplify Eq. (28), we obtain

$$\cot 2\alpha = \cot\phi + \frac{\cot\theta \csc\phi}{\sqrt{3}}. \tag{33}$$

Equations (32) and (33) express the two Majorana phases in terms of the two TM parameters (θ and ϕ). Substituting these two equations into Eq. (30), we obtain the most important result of this work as

$$r = \tan^2\theta. \tag{34}$$

It is interesting that r comes out to be independent of the phase ϕ.

We also substitute Eqs. (32) and (33) into the three mass ratios given by Eqs. (26), (31), and (27) to calculate the three neutrino masses. Finally, we express θ in terms of r everywhere using Eq. (34). Hence, we can express the three neutrino masses in terms of the three parameters: Δm^2_{12}, r, and ϕ. We obtain

$$m_1 = \sqrt{\Delta m^2_{12}}\sqrt{\frac{1+3r+2\sqrt{3}\sqrt{r}\cos\phi}{3-3r-2\sqrt{3}\sqrt{r}\cos\phi}}, \tag{35}$$

$$m_2 = \frac{2\sqrt{\Delta m^2_{12}}}{\sqrt{3-3r-2\sqrt{3}\sqrt{r}\cos\phi}}, \tag{36}$$

and

$$m_3 = \sqrt{\Delta m^2_{12}}\sqrt{\frac{3+r-2\sqrt{3}\sqrt{r}\cos\phi}{3-3r-2\sqrt{3}\sqrt{r}\cos\phi}}. \tag{37}$$

Now, we can use the experimental data [13] for Δm^2_{12} and Δm^2_{23}. Since, $\Delta m^2_{12} = (7.50 \pm 0.18) \times 10^{-5}$ eV^2 and $r = (3.149 \pm 0.098) \times 10^{-2}$ [13], the three masses are essentially functions of the phase ϕ (Fig. 1). We also depict the sum of the three neutrino masses $\sum_{i=1}^{3} m_i$ as a function of ϕ in Fig. 2.

The three mixing angles, calculated from Eqs. (18)–(20), are

$$\sin^2\theta_{12} = \frac{1+r}{3+r}, \tag{38}$$

$$\sin^2\theta_{23} = \frac{1}{2} + \frac{\sqrt{3}\sqrt{r}\cos\phi}{r+3}, \tag{39}$$

and

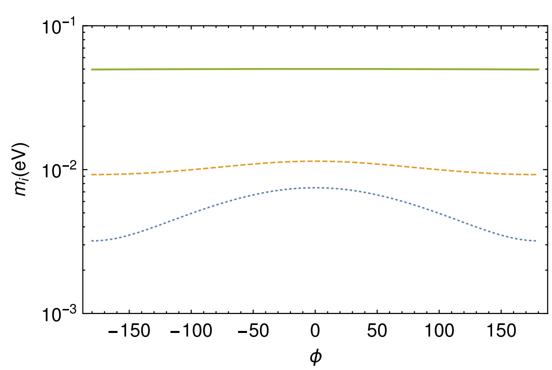

FIG. 1. The three neutrino masses m_1 (dotted line), m_2 (dashed line), and m_3 (solid line) in eV as functions of ϕ (in degrees).

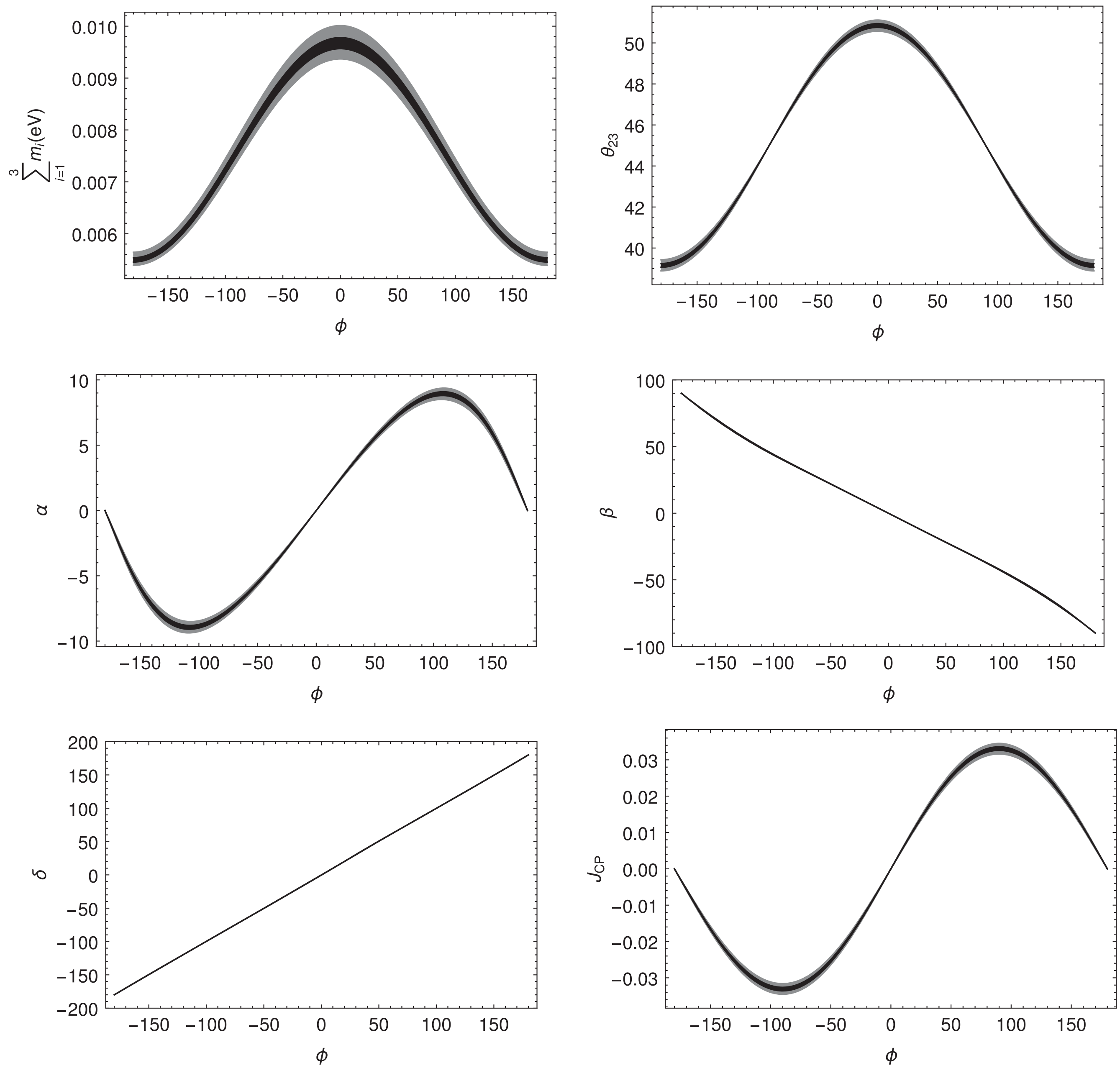

FIG. 2. The neutrino parameters $\sum_{i=1}^{3} m_i$, θ_{23}, α, β, δ, and J_{CP} as functions of ϕ. All phases and angles are in degrees. The dark (gray) bands depict the 1σ (3σ) allowed regions.

$$\sin^2\theta_{13} = \frac{2r}{3(r+1)}. \tag{40}$$

The two mixing angles θ_{12} and θ_{13} are functions of r only. Substituting the value of r, we obtain $\theta_{12} = 35.67° \pm 0.01°$ and $\theta_{13} = 8.20° \pm 0.12°$. For comparison, the experimental values are $\theta_{12} = 33.48° \pm 0.78°$ and $\theta_{13} = 8.50° \pm 0.21°$. The predicted and experimental values of θ_{12} become compatible at about 2.8σ C.L. This discrepancy is, however, a generic feature of TM mixing. One possible way to diffuse this tension with the data is to consider charged lepton corrections. We have presented our textures in a basis in which the charge lepton mass matrix is diagonal and the effective neutrino mass matrix is magic with two zeros. However, in a model realization of these textures, the charged lepton mass matrix can have small off-diagonal terms that will give corrections to the neutrino mixing angles. One can arrange these corrections to bring θ_{12} to its experimental value while keeping other two angles within the allowed ranges.

The mixing angle θ_{23} is a function of the phase ϕ after substituting for r. We depict the mixing angle θ_{23} as a function of the phase ϕ in Fig. 2.

We can calculate the three CP-violating phases from Eqs. (33), (32), and (24). We obtain

$$\cot 2\alpha = \cot\phi + \frac{\csc\phi}{\sqrt{3}\sqrt{r}}, \tag{41}$$

$$\tan 2\beta = -\frac{\sqrt{3}(1+r)\sin\phi}{2\sqrt{r}+\sqrt{3}(1-r)\cos\phi}, \tag{42}$$

and

$$\tan\delta = \frac{3+r}{3-r}\tan\phi. \tag{43}$$

The Jarlskog invariant J, calculated from Eq. (23), is

$$J = \frac{\sqrt{r}\sin\phi}{3\sqrt{3}(1+r)}. \tag{44}$$

The three CP-violating phases (α, β, and δ) depend upon the ratio r and the unknown phase ϕ. Therefore, we can plot α, β, δ, and J as functions of ϕ by just plugging in one experimental number r (Fig. 2).

This high level of predictability makes these textures good candidates for model building. It is rarely seen that a neutrino mass model can predict the nine neutrino parameters using just two inputs from the experiments: Δm^2_{12} and Δm^2_{23}. We present an A_4-based model for these two textures in the next section.

C. Inconsistency of class B_3

The magic mass matrix of type B_3 has zeros in the (1, 2) and (2, 2) entries. This implies the following two complex equations:

$$\frac{m_1}{m_2}e^{2i\alpha} = \frac{2(\sqrt{3}e^{-i\phi}\sin^2\theta + \sqrt{3}e^{i\phi}\cos^2\theta + 2\sin 2\theta)}{(1-3e^{2i\phi})\sin 2\theta + 2\sqrt{3}e^{i\phi}\cos 2\theta} \tag{45}$$

and

$$\frac{m_2}{m_3}e^{2i\beta} = \frac{\sqrt{3}+3e^{i\phi}\cot\theta}{\sqrt{3}-3e^{-i\phi}\cot\theta}. \tag{46}$$

Using absolute squares of these ratios, we can calculate the ratio r as

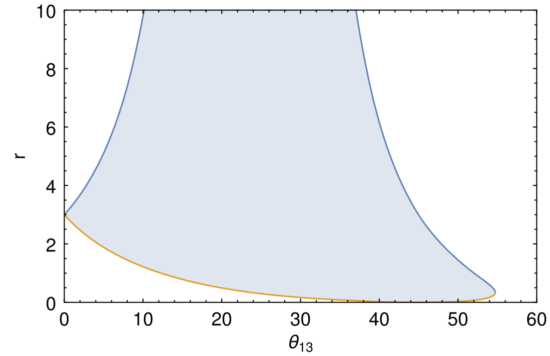

FIG. 3. The ratio $r = \Delta m^2_{12}/\Delta m^2_{23}$ as a function of θ_{13} (degrees) for a magic mass matrix of type B_3.

$$r = \frac{1-|\frac{m_1}{m_2}e^{2i\alpha}|^2}{(|\frac{m_2}{m_3}e^{2i\beta}|)^{-1}-1}. \tag{47}$$

Using these expressions, we express r as a function of θ_{13} (Fig. 3) by substituting the value of θ in terms of θ_{13} from Eq. (20). We find that r has a minimum value $r = 0$ at the point ($\theta_{13} = \pi/4$, $\phi = \pi$). We obtain the experimental value of r only in a small interval around this point for $\theta_{13} \in [40°, 50°]$. As θ_{13} decreases, the minimum value of r increases. It is clear that we cannot have both r and θ_{13} in their experimentally allowed ranges simultaneously. Hence, this texture is inconsistent with the experimental data.

V. THE A_4 MODEL

We present an A_4 model in the framework of the type I + II seesaw mechanism [15,16] to obtain the neutrino mass matrices studied in this work. Apart from the three left-handed lepton doublets D_{l_L} and three right-handed charged leptons l_R (where $l = e, \mu$, and τ), we introduce six $SU(2)_L$ doublet Higgs fields ψ_i and φ_i, (where $i = 1, 2$ and 3) and a $SU(2)_L$ triplet Higgs field Δ. We depict the transformation properties of the fields present in our model in Table II. In addition to A_4 symmetry, we also need a Z_2 symmetry to prevent the coupling of the charged leptons (neutrinos) with scalars φ_i (ψ_i). These transformation properties lead to the following Lagrangian for the leptons that is invariant under A_4 and Z_2:

$$\begin{aligned} -\mathcal{L} = {} & y_1(\bar{D}_{e_L}\psi_1 + \bar{D}_{\mu_L}\psi_2 + \bar{D}_{\tau_L}\psi_3)_{\underline{1}}e_{R_1} + y_2(\bar{D}_{e_L}\psi_1 + \omega^2\bar{D}_{\mu_L}\psi_2 + \omega\bar{D}_{\tau_L}\psi_3)_{\underline{1}'}\tau_{R_{1''}} \\ & + y_3(\bar{D}_{e_L}\psi_1 + \omega\bar{D}_{\mu_L}\psi_2 + \omega^2\bar{D}_{\tau_L}\psi_3)_{\underline{1}''}\mu_{R_{1'}} + y_4(\bar{D}_{e_L}\tilde{\varphi}_1 + \bar{D}_{\mu_L}\tilde{\varphi}_2 + \bar{D}_{\tau_L}\tilde{\varphi}_3)_{\underline{1}}\nu_{R_1} \\ & - y_\Delta(D^T_{e_L}C^{-1}D_{e_L} + \omega^2 D^T_{\mu_L}C^{-1}D_{\mu_L} + \omega D^T_{\tau_L}C^{-1}D_{\tau_L})_{\underline{1}''}i\tau_2\Delta_{1'} - m_R(\nu^T_R C^{-1}\nu_R) + \text{H.c.}, \end{aligned} \tag{48}$$

where $\tilde{\varphi} = i\tau_2\varphi^*$.

We assume that the vacuum expectation values (VEVs) of the Higgs fields are $\langle\psi\rangle_o = v_\psi(1,1,1)^T$, which leads to the charged lepton mass matrix

TABLE II. Transformation properties of various fields: $D_{l_L}(D_{e_L}, D_{\mu_L}, D_{\tau_L})^T$, $l_R(e_R, \mu_R, \tau_R)^T$, ν_{l_R}, $\psi(\psi_1, \psi_2, \psi_3)^T$, $\varphi(\varphi_1, \varphi_2, \varphi_3)^T$, and Δ.

Fields	D_{l_L}	l_R	ν_R	ψ	φ	Δ
$SU(2)_L$	2	1	1	2	2	3
A_4	3	1, 1′, 1″	1	3	3	1′
Z_2	1	1	−1	1	−1	1

$$m_l = \begin{pmatrix} y_1 v_\psi & y_2 v_\psi & y_3 v_\psi \\ y_1 v_\psi & y_2 \omega v_\psi & y_3 \omega^2 v_\psi \\ y_1 v_\psi & y_2 \omega^2 v_\psi & y_3 \omega v_\psi \end{pmatrix}. \tag{49}$$

For the type I see saw contribution, we assume that φ_i develop VEVs along the direction $\langle\varphi\rangle_o = v_\varphi(0, -1, 1)^T$. Such a vacuum alignment has been obtained in Ref. [17] for $SU(2)_L$ and A_4 triplet scalars by allowing specific terms in the scalar potential which break A_4 softly. This choice of VEVs leads to the following Dirac neutrino mass matrix:

$$m_D = y_4 v_\varphi (0, -1, 1)^T. \tag{50}$$

We have only one right-handed neutrino with mass m_R. Using the type I seesaw mechanism, the effective neutrino mass matrix is $m_\nu^I \approx m_D m_R^{-1} m_D^T$,

$$m_\nu^I = c \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}, \tag{51}$$

where $c = y_4^2 v_\psi^2 / m_R$. When the $SU(2)_L$ triplet Higgs acquires a nonzero and small VEV, we get the following type II seesaw contribution to the effective neutrino mass matrix:

$$m_\nu^{II} = \Delta \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}, \tag{52}$$

where $\Delta = y_\Delta v_\Delta$. The combined effective neutrino mass matrix $m_\nu = m_\nu^I + m_\nu^{II}$ from the type I + II seesaw mechanism becomes

$$m_\nu = \begin{pmatrix} \Delta & 0 & 0 \\ 0 & c + \omega^2\Delta & -c \\ 0 & -c & c + \omega\Delta \end{pmatrix}. \tag{53}$$

In the symmetry basis, the charged lepton mass matrix m_l is not diagonal. We make a transformation to the basis where the charge lepton mass matrix is diagonal with the transformation $M_l = U_L^\dagger m_l U_R$, where

$$U_L = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}, \tag{54}$$

and U_R is a unit matrix. In this basis where M_l is diagonal, the effective neutrino mass matrix becomes

$$M_\nu = \begin{pmatrix} 0 & 0 & c \\ 0 & c - \Delta & \Delta \\ c & \Delta & -\Delta \end{pmatrix}. \tag{55}$$

This is the mass matrix of type A_1 having magic symmetry and two texture zeros.

A similar mechanism with a $SU(2)_L$ triplet Higgs Δ transforming as 1″ instead of 1′ will give the neutrino mass matrix

$$M_\nu = \begin{pmatrix} 0 & b & 0 \\ b & -a & a \\ 0 & a & b - a \end{pmatrix}. \tag{56}$$

This is the mass matrix of type A_2 having magic symmetry and two texture zeros.

Our model requires six Higgs doublets, three of which couple to charged leptons (Table II). In such multi-Higgs models, the flavor-changing neutral currents can contribute to charged lepton flavor-violating decays. However, an explicit calculation is beyond the scope of the present work due to the complexity of the Higgs sector of our model. Nevertheless, there exist models in the literature (e.g., Ref. [18]) where the charged lepton Yukawa Lagrangian (including the A_4 assignments of the charged lepton and scalar fields) is similar to our model. The flavor-violating decays of leptons for our model can be studied in a manner similar to that in Ref. [18].

VI. CONCLUSIONS

We studied the phenomenological implications of two texture zeros in the magic neutrino mass matrix. In the absence of magic symmetry, there are seven allowed patterns for the presence of two zeros in the neutrino mass matrix. The additional constraint of magic symmetry disallows five of these patterns. The two allowed patterns are of the types A_1 and A_2. The combination of magic symmetry and texture zeros make these classes very predictive. We can express all nine neutrino observables (the three masses, the three mixing angles, and the three CP-violating phases) as functions of ϕ by plugging in just two experimental parameters (Δm_{12}^2 and Δm_{23}^2). In particular, θ_{12} and θ_{13} do not even depend on the phase ϕ and can be expressed as functions of the ratio $r = \Delta m_{12}^2/\Delta m_{23}^2$ as $\sin^2\theta_{12} = \frac{1+r}{3+r}$ and $\sin^2\theta_{13} = \frac{2r}{3(r+1)}$. Finally, we have derived these highly predictive mass matrices from a neutrino mass model based upon the symmetry group A_4.

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APPENDIX: THE GROUP A_4

A_4 is the group of even permutations of four objects having 12 elements. Geometrically, it can be viewed as the group of rotational symmetries of the tetrahedron. A_4 has four inequivalent irreducible representations (IRs) which are three singlets $\mathbf{1}$, $\mathbf{1}'$, and $\mathbf{1}''$, and one triplet $\mathbf{3}$. The group A_4 is generated by two generators S and T such that

$$S^2 = T^3 = (ST)^3 = 1. \tag{A1}$$

The one-dimensional unitary IRs are

$$\mathbf{1}\, S = 1\ T = 1, \qquad \mathbf{1}'\, S = 1\ T = \omega, \qquad \mathbf{1}''\, S = 1\ T = \omega^2. \tag{A2}$$

The three-dimensional unitary IR is

$$S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \qquad T = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}. \tag{A3}$$

The multiplication rules of the IRs are as follows:

$$\mathbf{1}' \otimes \mathbf{1}' = \mathbf{1}'', \qquad \mathbf{1}'' \otimes \mathbf{1}'' = \mathbf{1}', \qquad \mathbf{1}' \otimes \mathbf{1}'' = \mathbf{1}. \tag{A4}$$

The product of two $\mathbf{3}$'s gives

$$\mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{1}'' \oplus \mathbf{3}_s \oplus \mathbf{3}_a, \tag{A5}$$

where $s(a)$ denotes the symmetric (antisymmetric) product. Let (x_1, x_2, x_3) and (y_1, y_2, y_3) denote the basis vectors of two $\mathbf{3}$'s. Then the IRs obtained from their products are

$$(\mathbf{3} \otimes \mathbf{3})_{\mathbf{1}} = x_1y_1 + x_2y_2 + x_3y_3, \tag{A6}$$

$$(\mathbf{3} \otimes \mathbf{3})_{\mathbf{1}'} = x_1y_1 + \omega x_2y_2 + \omega^2 x_3y_3, \tag{A7}$$

$$(\mathbf{3} \otimes \mathbf{3})_{\mathbf{1}''} = x_1y_1 + \omega^2 x_2y_2 + \omega x_3y_3, \tag{A8}$$

$$(\mathbf{3} \otimes \mathbf{3})_{\mathbf{3}_s} = (x_2y_3 + x_3y_2, x_3y_1 + x_1y_3, x_1y_2 + x_2y_1), \tag{A9}$$

$$(\mathbf{3} \otimes \mathbf{3})_{\mathbf{3}_a} = (x_2y_3 - x_3y_2, x_3y_1 - x_1y_3, x_1y_2 - x_2y_1). \tag{A10}$$

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