Search for doubly-heavy dibaryons in a quark model

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We study the stability of hexaquark systems containing two heavy quarks and four light quarks within a simple quark model. No bound or metastable state is found. The reason stems from a delicate interplay between chromoelectric and chromomagnetic effects. Our calculation also provides information about anticharmed pentaquarks that are seemingly unbound in simple quark models.

I. INTRODUCTION

Many interesting hadrons have been discovered recently with hidden heavy flavor, including the XYZ mesons and the LHCb pentaquarks. Some reviews are available; the latest ones included the hidden-charm pentaquarks and a few of them also discussed doubly charmed hadrons [1–6].

The sector of doubly heavy flavor will certainly require major experimental activity, in particular for a confirmation of the long-awaited doubly charmed baryons [7] and the search for doubly charmed mesons [8,9] and other flavor-exotic states [10].

In the case of $(\bar{q}\,\bar{q}\,QQ)$ with spin parity $J^P = 1^+$ and isospin $I = 0$ (where Q stands for c or b and q stands for a light quark), there is the fortunate cooperation of two effects. First, the chromoelectric (CE) interaction—even if alone—gives stability below the $(\bar{q}Q) + (\bar{q}Q)$ threshold if the quark-to-antiquark mass ratio is large enough, as it takes advantage of the deeper binding of the QQ pair [11–14]. Second, the chromomagnetic (CM) interaction between the light quarks is also favorable [15,16]. Janc and Rosina predicted the stability of $(\bar{u}\,\bar{d}\,cc)$ using a quark model fitting ordinary hadrons [17]. Their calculation was confirmed and improved by Vijande *et al.* [18]. This configuration has also been identified as a good candidate for a stable exotic in other approaches, such as a DD^* molecule [19–23], lattice QCD [24–26], or QCD sum rules [27].

One hardly finds another configuration with both spin-independent and spin-dependent effects cooperating to provide a collective multiquark state rather than a splitting

into two hadrons. In this paper, we consider a multiquark system with two thresholds that might be nearly degenerate, and study to which extent the mixing of color and spin configurations can stabilize the multiquark. The system is $(qqqqQQ')$, where Q or Q' denotes a heavy quark and q stands for a light quark. The dissociation thresholds are either of the $(qqq) + (qQQ')$ type, which benefits from the QQ' CE interaction, or $(qqQ) + (qqQ')$ which can be shifted down by CM effects if qq is in a spin-singlet state. The question is whether $(qqqqQQ')$ can combine CE and CM dynamics coherently to build a bound state.

In writing down the formalism and discussing the results, some other states will be evoked, such as the H particle $(uuddss)$ in the limit of light Q and Q', or the pentaquark $(qqqq\bar{Q})$ which is very similar in the limit where Q and Q' are clustered in a compact diquark.

The paper is organized as follows. Section II contains the model and the method to calculate the relevant spin-color states and the matrix elements within this basis. In Sec. III we present the variational calculation and its application in the case of states suspected to be either bound or weakly bound. The results are presented and discussed in Sec. IV, while Sec. V is devoted to some conclusions and perspectives.

II. THE MODEL

We consider $(qqqqQQ')$, where Q and Q' are different heavy quarks (and hence no Pauli constraints apply), but they carry the same mass M for simplicity. Giving Q and Q' different masses would not change our conclusions. We also take the $SU(3)_F$ limit in the light sector, with the same mass m for $q = u$, d, or s. For each baryon involved in the threshold and for the dibaryon, we search the ground state of the Hamiltonian $H = T + V$, where T is the

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kinetic-energy operator and V is the interaction. With $m = 0.4$ GeV, $M = 1.3$ GeV, and the potential

$$V = -\frac{3}{16}\sum_{i<j}\tilde{\lambda}_i.\tilde{\lambda}_j\left(-\frac{a}{r_{ij}} + br_{ij}\right.$$

$$\left. + \frac{c}{m_im_j}\left(\frac{\mu}{\pi}\right)^{3/2}\exp(-\mu r_{ij}^2)\sigma_i.\sigma_j\right), \qquad (1)$$

where $a = 0.4$, $b = 0.2$, $c = 2.0$, and $\mu = 1.0$, in appropriate powers of GeV, one obtains a satisfactory account for the baryon masses entering the thresholds, both in the $\mathrm{SU(3)_F}$ limit and with $\mathrm{SU(3)_F}$ broken. A large negative constant is omitted in the above potential, but it can be disregarded, as it has an equal effect on the thresholds and the multiquark energies. We do not elaborate here on the validity of a model with pairwise forces and color factors, which has been discussed already by several authors (see, e.g., Refs. [28,29]). As it is, this is the simplest tool for such an exploratory study.

There is already abundant literature on the wave functions of six-quark systems and the algebra of the spin, color, and spin-color operators entering the quark model [30–36], with a careful account for the antisymmetrization constraints. We thus restrict ourselves here to a brief summary of our notation. To construct the basis of color and spin states, we formally consider the system as a set of three two-quark subsystems, $(qq)(qq)(QQ')$, with color $\bar{3}$ or 6 and spin 0 or 1. We built the most general basis compatible with an overall color singlet and spin-0 state. The requirements of antisymmetrization are strictly enforced for all states shown.

For an overall scalar, one can combine either three spin-0 states, two spin-1 and one spin-0 state, or three spin-1 states, say

$$S_1 = (000), \qquad S_2 = (011), \qquad S_3 = (101),$$
$$S_4 = (110), \qquad S_5 = (111). \qquad (2)$$

The simplest color singlet is $(\bar{3}\,\bar{3}\,\bar{3})$, similar to any antibaryon made of three antiquarks. Another possibility consists of coupling two $\bar{3}$ diquarks into a color antisextet, and then getting an overall singlet with the third diquark being in a color sextet. The last possibility is to couple three 6 diquarks into a singlet. In short,

$$C_1 = (666), \qquad C_2 = (6\bar{3}\,\bar{3}), \qquad C_3 = (\bar{3}6\bar{3}),$$
$$C_4 = (\bar{3}\,\bar{3}6), \qquad C_5 = (\bar{3}\,\bar{3}\,\bar{3}). \qquad (3)$$

For the sake of cross-checking, the color states have been listed explicitly and rearranged using the SU(3) Clebsch-Gordan coefficients given in Ref. [37].

The matrix elements of the spin, color, and color-spin operators are obvious in this basis for the pairs (1,2), (3,4),

or (5,6). For the others, the crossing matrices corresponding to suited transpositions have been used. For instance, the spin crossing matrix corresponding to

$$(12)(23)(34) \leftrightarrow (13)(24)(56) \qquad (4)$$

is

$$\frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & \sqrt{3} & 0 \\ 0 & 1 & -1 & 0 & -\sqrt{2} \\ 0 & -1 & 1 & 0 & -\sqrt{2} \\ \sqrt{3} & 0 & 0 & -1 & 0 \\ 0 & -\sqrt{2} & -\sqrt{2} & 0 & 0 \end{pmatrix} \qquad (5)$$

and gives access to $\sigma_1.\sigma_3$ and $\sigma_2.\sigma_4$. Its color analogue,

$$\frac{1}{2}\begin{pmatrix} -1 & 0 & 0 & \sqrt{3} & 0 \\ 0 & -1 & 1 & 0 & -\sqrt{2} \\ 0 & 1 & -1 & 0 & -\sqrt{2} \\ \sqrt{3} & 0 & 0 & 1 & 0 \\ 0 & -\sqrt{2} & -\sqrt{2} & 0 & 0 \end{pmatrix}, \qquad (6)$$

allows for the calculation of $\tilde{\lambda}_1.\tilde{\lambda}_3$ and $\tilde{\lambda}_2.\tilde{\lambda}_4$.

Several checks can be made on the matrix elements $\sigma_i.\sigma_j$, $\tilde{\lambda}_i.\tilde{\lambda}_j$ and $\tilde{\lambda}_i.\tilde{\lambda}_j\sigma_i.\sigma_j$. For instance, the Casimir operators such as $\sum\sigma_i.\sigma_j$ or $\sum\tilde{\lambda}_i.\tilde{\lambda}_j$ depend only on the overall spin or color value. In the case of the H, the maximal value $\sum\tilde{\lambda}_i.\tilde{\lambda}_j\sigma_i.\sigma_j = 24$ is recovered, which exceeds the value 16 corresponding to the $\Lambda\Lambda$ threshold, as first shown by Jaffe [38]. Similarly, if the sum is restricted to the light sector, the maximal value $\sum\tilde{\lambda}_i.\tilde{\lambda}_j\sigma_i.\sigma_j = 16$ is obtained as in Refs. [39,40], corresponding to more attraction than the value 8 of the single baryon entering the lowest threshold.

We note in Eq. (1) the smearing of the spin-spin interaction, instead of a mere delta function when this term is treated at first order. Here the smearing parameter μ is the same for all pairs, unlike some more elaborate models where it depends on the masses [41]. For consistency, the stability is discussed with respect to the threshold computed within the same model.

III. VARIATIONAL CALCULATION

We solved the six-body problem using a Gaussian expansion,

$$\phi(\tilde{x}) = \sum_i\gamma_i\exp[-(\tilde{x}^\dagger.A^{(i)}.\tilde{x})/2], \qquad (7)$$

where $\tilde{x} = \{x_1, ...x_5\}$ is a set of five Jacobi variables describing the relative motion, $x_1 = r_2 - r_1$, etc., and $A^{(i)}$ is

a 5×5 positive-definite matrix. For a given choice of the matrices $A^{(i)}$, the weight factors γ_i are given by a generalized eigenvalue problem. The overall variational energy is obtained by a numerical minimization over $A^{(i)}$.

The calculation was started using a single spin-color channel, say

$$\psi_{a,b} = \phi_{a,b}|C_a\rangle|S_b\rangle, \qquad (8)$$

with $\phi_{a,b}$ given by Eq. (7). It was later extended to account for the coupling among the spin-color states, mainly due to the CM term, i.e.,

$$\Psi = \sum_{a,b} \psi_{a,b}, \qquad (9)$$

where the summation is extended to the states sharing the same symmetry properties.

For estimating the energy of a deeply bound state, a straightforward strategy consists of first choosing a few diagonal matrices $A_{a,b}^{(i)}$ containing the range parameters, and then adding some nondiagonal terms and increasing the number of matrices.

If binding does not show up, an alternative strategy consists of choosing Gaussians that describe two baryons times a relative function, for instance,

$$\Psi = \phi_{123}\phi_{456}\sum_j \delta_j \exp(-\eta_j r_{123-456}^2/2), \qquad (10)$$

where ϕ_{ijk} is a Gaussian approximation to the baryon containing the $\{i, j, k\}$ quarks and $r_{123-456}$ links the centers of mass of the two baryons. Then the partitioning can be extended to other baryon-baryon configurations, say, in an obvious notation,

$$\Psi = \sum_{\substack{a=\{i,j,k\} \\ b=\{i',j',k'\}}} \phi_a\phi_b \sum_j \delta_j^{(a,b)} \exp(-\eta_j^{(a,b)} r_{a-b}^2/2). \qquad (11)$$

This is similar to the method of Kamimura et al. [42], which has been applied successfully to a variety of few-body systems.

IV. RESULTS AND DISCUSSION

We first show the energies of the baryons constituting the thresholds in Table I.

We show in Table II the results for the scalar state $J^P = 0^+$ with isospin $I = 1/2, 3/2$, that would be degenerate because the potential in Eq. (1) does not depend on the total isospin. This would stand, for example, for a flavor content $(uudscc)$.[1] In this case 13 different color-spin vectors are

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[1]Other channels and flavor contents have been studied with similar results.

TABLE I. Energy (in GeV) of the baryons involved in the thresholds within the model (1). Σ and Λ stand for baryons where the first two quarks are in a spin-1 or spin-0 state, respectively.

$qqQ(\Sigma)$	$qqQ'(\Lambda)$	$qqq(\Sigma)$	$QQ'q(\Sigma)$
1.372	1.258	1.461	1.109

TABLE II. Six-quark energies of the different color-spin vectors contributing to the $J^P = 0^+$ state, together with the coupled-channel result and the energies of the allowed thresholds.

Color-spin vector	E (GeV)
$C_1 S_1$	3.079
$C_2 S_1$	2.829
$C_3 S_4$	2.831
$C_2 S_2$	3.030
$C_3 S_3$	3.030
$C_3 S_5$	2.908
$C_1 S_2$	2.995
$C_4 S_3$	2.835
$C_4 S_4$	3.080
$C_4 S_5$	3.016
$C_5 S_3$	2.891
$C_5 S_4$	2.997
$C_5 S_5$	3.034
Coupled	2.767
Thresholds	2.570, 2.630

allowed by antisymmetry requirements; with the notation of Eqs. (2) and (3) they will be $C_1 S_1$, $C_2 S_1$, $C_3 S_4$, $C_2 S_2$, $C_3 S_3$, $C_3 S_5$, $C_1 S_2$, $C_4 S_3$, $C_4 S_4$, $C_4 S_5$, $C_5 S_3$, $C_5 S_4$, and $C_5 S_5$. The two thresholds allowed for the dissociation of the $J^P = 0^+$ six-quark state would have energies $qqQ(\Sigma)+qqQ'(\Lambda)=2.630\,\text{GeV}$ and $QQ'q(\Sigma)+qqq(\Sigma)=2.570\,\text{GeV}$. We also give in Table III the probabilities of the different channels contributing to the coupled-channel calculation (those that are not listed have probabilities smaller than 10^{-6}).

The calculations using the variational wave function (8) for a single channel, and Eq. (9) for the case of coupled channels always give values above the threshold, which go down very slowly when the Gaussian basis is augmented. As already said, this is the sign of either the absence of a bound state or, at most, a very tiny binding. This is confirmed by the use of the alternative bases—Eq. (10) or Eq. (11)—where one always finds a six-body energy equal to the sum of the two baryon energies, obtained in the

TABLE III. Probabilities of the different six-body channels contributing to the $J^P = 0^+$ six-quark state.

Channel	$C_1 S_2$	$C_2 S_1$	$C_3 S_4$	$C_4 S_3$
Probability	0.004	0.539	0.456	0.001

approximation of the Gaussian expansion ϕ_{ijk} and $\phi_{i'j'k'}$. This means that neither the residual color-singlet exchange between the two clusters nor the coupling of the different baryon-baryon thresholds is sufficient to bind the system.

We have checked that in the infinite-mass limit for the mass of the heavy quarks the system gets binding with respect to the upper threshold, $(qqQ) + (qqQ')$, but it is always above the lowest one $(QQ'q) + (qqq)$. For example, for $M = 10$ GeV and $m = 0.4$ GeV we get 2.326 GeV for the energy of the six-quark state in the coupled-channel calculation, while the thresholds are given by $E(QQ'q) + E(qqq) = 2.162$ GeV and $E(qqQ) + E(qqQ') = 2.477$ GeV. The six-quark state—because it is now in between the two thresholds—is described by the same color-spin vectors shown in Table III, C_2S_1 and C_3S_4, where the two-heavy quarks are in a $\bar{3}$ color state [see Eq. (3)] that would split into the lowest threshold. In other words, the two heavy quarks control the mass of the six-body state in the infinite-mass limit. As mentioned above, by making use of the variational wave function of Eq. (10) or Eq. (11) one obtains exactly the two-baryon mass in the six-body calculation.

We now try to explain why these results are plausible. The CM part is already well documented, including discussions concerning the H dibaryon or the 1987-vintage pentaquark (see, for instance, Refs. [43–45]). The effect of $SU(3)_F$ breaking—a different mass for the strange quark—tends to spoil the promises of binding based on the sole spin-color algebra, and, more importantly, the short-range correlation factors [the expectation values of $\exp(-\mu r_{ij}^2)$ in our model] are significantly smaller in a multiquark than in baryons.

As for the CE part, a superficial analysis would argue that—as soon as $-\sum \tilde{\lambda}_i.\tilde{\lambda}_j$ is locked to 16 in any spin-color channel $|C_a\rangle|S_b\rangle$—the CE part of the binding will remain basically untouched, independent of the combination of the $|C_a\rangle|S_b\rangle$ dictated by the CM part. However, this is not the case. For equal masses, the deepest CE binding is obtained when the distribution of CE strength factors $\{\tilde{\lambda}_i.\tilde{\lambda}_j\}$ is the most asymmetric [46], which favors the threshold against a compact multiquark. For a mass distribution such as $(qqqqQQ')$, CE dynamics favors a color $\bar{3}$ state for the QQ' two-quark state. Once this is enforced, the best CE energy is obtained when the Qq and $Q'q$ pairs receive the largest strength, and they come with a larger reduced mass than qq. This can be checked explicitly in a simple solvable model with an interaction proportional to $-\tilde{\lambda}_i.\tilde{\lambda}_j r_{ij}^2$. However, CM effects are optimized when the light sector receives the largest color strengths. Hence, there is somewhat of a conflict between CE and CM effects, and this explains the lack of bound states in our model.

V. SUMMARY AND OUTLOOK

In this paper, we have used a simple quark model with CE and CM components to search for possible bound states

of $(qqqqQQ')$ configurations below their lowest threshold. The answer is negative: no bound state is found, nor any metastable state in the continuum with a mass below the highest threshold and a suppressed decay to the lowest threshold.

Our calculation also provides some information about the anticharmed pentaquark, or the beauty analogue $P = (qqqq\bar{Q})$. In the limit of large Q and Q' masses, the QQ' pair in $(qqqqQQ')$ behaves as a single antiquark. From our results, $(qqqq\bar{Q})$ is seemingly unbound in simple quark models, while a simple CM counting suggests that this configuration is bound [39,40]. While the H has been studied extensively (in particular within lattice QCD [47–51]), the P has received less attention.

It is worth emphasizing how our study illustrates the difficulty of getting multiquark bound states in constituent quark models. Other approaches have suggested some ways out, such as the following:

(i) Another spin-dependent part for our potential (1): For instance, a chiral quark model was considered in Ref. [36] for the H and for $(uuddsQ)$, but no bound state was found.

(ii) Multibody potentials, generalizing the Y-shape potential for baryons, provide more attraction than the color-additive model (1) [52], but in the minimization of the flux tube configurations several color states are mixed, and this is delicate in the case of identical quarks, where color is constrained by the requirement of antisymmetry [53].

(iii) Diquarks, whose clustering is motivated (but not really demonstrated) by CM effects, might lead to dibaryon states [54,55].

(iv) String dynamics *à la* Rossi-Veneziano suggests the existence of states containing more junctions than the lowest thresholds, and thus states that are metastable, as the internal annihilation of junctions is suppressed by an extension of the Zweig rule [56].

(v) Molecular dynamics: Doubly heavy dibaryons have been also recently approached in a molecular picture [57–62]. The main motivation for these studies originates from the reduction of the kinetic energy due to a large reduced mass as compared to systems made of light baryons. However, such molecular states—which have been intriguing objects of investigations and speculations for many years—are usually the concatenation of several effects and not just a fairly attractive interaction. The coupling between close channels or the contribution of non-central forces used to play a key role in their existence. When comparing to similar problems in the strange sector the mass difference between the two competing channels $(qqQ) + (qqQ')$ and $(qqq) + (qQQ')$ increases, making the coupled-channel effect less important. Thus, without the strong transition potentials reported in the quark

delocalization color screening model of Ref. [61] or the strong tensor couplings occurring in the hadronic one-pion exchange models of Refs. [58,59], it seems difficult to get a molecular bound state of two heavy baryons, as was recently reported in Ref. [62].

On the experimental side, the search for doubly charmed dibaryons can be made together with the search for doubly charmed baryons [63–65] and doubly charmed exotic mesons [8–10] as they share some triggers.

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