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High energy resummation in dihadron production at the LHC

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We propose to study at the Large Hadron Collider (LHC) the inclusive production of a pair of hadrons (a "dihadron" system) in a kinematics where two detected hadrons with high transverse momenta are separated by a large interval of rapidity. This process has much in common with the widely discussed Mueller-Navelet jet production and can also be used to access the dynamics of hard proton-parton interactions in the Regge limit. For both processes large contributions enhanced by logarithms of energy can be resummed in perturbation theory within the Balitsky-Fadin-Kuraev-Lipatov (BFKL) formalism with next-to-leading logarithmic accuracy. The experimental study of dihadron production would provide an additional clear channel to test the BFKL dynamics. We present here the first theoretical predictions for cross sections and azimuthal angle correlations of the dihadrons produced with LHC kinematics.

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I. INTRODUCTION

The record energy of proton-proton collisions and the high luminosity of the LHC provide us with a unique opportunity to study the dynamics of strong interactions in an unexplored kinematic range. The production of two (the most forward and backward) jets, separated by a large interval of rapidity, was proposed by Mueller and Navelet [1] as a tool to access the dynamics of semihard parton interactions at a hadron collider. In theory such processes are described using the Balitsky-Fadin-Kuraev-Lipatov (BFKL) method [2], which allows one to resum to all orders the leading (LLA) and the next-to-leading terms (NLA) of the QCD perturbative series that are enhanced by powers of large energy logarithms. For the Mueller-Navelet jet production, the BFKL resummation with NLA accuracy relies on the combination of two ingredients: the NLA Green's function of the BFKL equation [3,4] and the NLA jet vertices [5–9]. NLA BFKL calculations of the cross sections for the Mueller-Navelet jet process and also predictions for the jet azimuthal angle correlations, observables suggested earlier in Refs. [10,11], can be found in Refs. [12–22]. Recently [23], the first measurements of the azimuthal correlation of the Mueller-Navelet jets at the LHC were presented by the CMS Collaboration at $\sqrt{s} = 7$ TeV. Further experimental studies of the Mueller-Navelet jets at higher LHC energies and larger rapidity separations are expected.

The important task of revealing the dynamical mechanisms behind partonic interactions in the Regge limit, $s \gg |t|$, by the comparison of theory predictions with data, can be better accomplished if some other observables, sensitive to the BFKL dynamics, are considered in the context of the LHC physics program. An interesting option—the detection of three jets and four jets, well separated in rapidity from each other—was recently suggested in Ref. [24] and in Ref. [25].

In this paper we want to suggest a novel possibility, i.e. the inclusive dihadron production

$$\mathrm{p}(p_1) + \mathrm{p}(p_2) \to \mathrm{h}_1(k_1) + \mathrm{h}_2(k_2) + \mathrm{X}, \qquad (1)$$

when the two detected hadrons have high transverse momenta and are separated by a large interval of rapidity. For this process, similarly to the Mueller-Navelet jet production, the BFKL resummation in the NLA is feasible, since the necessary item beyond the NLA BFKL Green's functions, i.e. the vertex describing the production of an identified hadron, was obtained with NLA in Ref. [26]. It was shown there that, after renormalization of the QCD coupling and the ensuing removal of the ultraviolet divergences, soft and virtual infrared divergences cancel each other, whereas the surviving infrared collinear ones are compensated by the collinear counterterms related to the renormalization of parton densities (PDFs) for the initial proton and parton fragmentation functions (FFs) describing the detected hadron in the final state within collinear factorization. All the theoretical requisites are thus fulfilled to write down infrared-safe NLA predictions, thus making this process an additional clear channel to test the BFKL dynamics at the LHC. The fact that hadrons can be detected at the LHC at much smaller values of the transverse momentum than jets, allows one to explore a kinematic range outside the reach of the Mueller-Navelet channel, so that the reaction (1) can be considered complementary to

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Mueller-Navelet jet production, although sharing the same theoretical framework.

We will give below the very first predictions for the cross sections and azimuthal angle correlations of the process (1). We will limit ourselves to presenting the main formulas, to guarantee the reproducibility of our results, and postpone a more detailed account about their derivation to a later publication.

It is known that the inclusion of NLA terms has a very large effect on the theory predictions for the Mueller-Navelet jet cross sections and the jet azimuthal angle distributions. Similar features are expected also for our case of inclusive dihadron production. This results in a large dependence of predictions on the choice of the renormalization scale μ_R and the factorization scale μ_F. Here we will take them equal, $\mu_R = \mu_F$, and adopt the Brodsky-Lepage-Mackenzie (BLM) scheme [27] for the renormalization scale setting. In BLM the renormalization scale ambiguity is eliminated by absorbing the nonconformal terms proportional to the QCD β_0 function into the running coupling. Such an approach was successfully used (first in Ref. [16]) for a satisfactory description of the LHC data on the azimuthal correlations of Mueller-Navelet jets [23], obtained by the CMS Collaboration.

II. BFKL WITH BLM OPTIMIZATION

We consider the production, in high-energy proton-proton collisions, of a pair of identified hadrons with large transverse momenta, $\vec{k}_1^2 \sim \vec{k}_2^2 \gg \Lambda_{\rm QCD}^2$ and large separation in rapidity.

In collinear factorization we neglect power-suppressed contributions, and therefore the proton mass can be taken to be vanishing and the Sudakov vectors can be chosen to coincide with the proton momenta p_1 and p_2, satisfying $p_1^2 = p_2^2 = 0$ and $2p_1 \cdot p_2 = s$. Then, the momentum of each identified hadron can be decomposed as

$$k_{1,2} = \alpha_{1,2} p_{1,2} + \frac{\vec{k}_{1,2}^2}{\alpha_{1,2} s} p_{2,1} + k_{1,2\perp}, \qquad k_{1,2\perp}^2 = -\vec{k}_{1,2}^2. \qquad (2)$$

In the center-of-mass system, the longitudinal fractions $\alpha_{1,2}$ are related to the hadron rapidities by $y_1 = \frac{1}{2}\ln\frac{\alpha_1^2 s}{\vec{k}_1^2}$ and $y_2 = \frac{1}{2}\ln\frac{\vec{k}_2^2}{\alpha_2^2 s}$, which imply $dy_1 = \frac{d\alpha_1}{\alpha_1}$ and $dy_2 = -\frac{d\alpha_2}{\alpha_2}$, if the space part of the four-vector p_1 is taken positive. The differential cross section of the process (1) can be written as follows:

$$\frac{d\sigma}{dy_1 dy_2 d|\vec{k}_1| d|\vec{k}_2| d\phi_1 d\phi_2} = \frac{1}{(2\pi)^2}\left[\mathcal{C}_0 + \sum_{n=1}^{\infty} 2\cos(n\phi)\mathcal{C}_n\right], \qquad (3)$$

where $\phi = \phi_1 - \phi_2 - \pi$, and $\phi_{1,2}$ are the two hadrons' azimuthal angles, while $y_{1,2}$ and $\vec{k}_{1,2}$ are their rapidities and transverse momenta, respectively. The eigenvalues of the kernel of the BFKL equation and the expressions for the hadron vertices are needed to calculate this cross section. In LLA the BFKL eigenvalues, parametrized by the continuous ν variable and the integer conformal spin parameter n, read

$$\chi(n,\nu) = 2\psi(1) - \psi\left(\frac{n}{2} + \frac{1}{2} + i\nu\right) - \psi\left(\frac{n}{2} + \frac{1}{2} - i\nu\right),$$

and the LLA hadron vertices,

$$c_1(n,\nu,|\vec{k}_1|,\alpha_1) = \frac{4}{3}(\vec{k}_1^2)^{i\nu-1/2}\int_{\alpha_1}^1 \frac{dx}{x}\left(\frac{x}{\alpha_1}\right)^{2i\nu-1} \times \left[\frac{C_A}{C_F} f_g(x) D_g^h\left(\frac{\alpha_1}{x}\right) + \sum_{a=q,\bar{q}} f_a(x) D_a^h\left(\frac{\alpha_1}{x}\right)\right], \qquad (4)$$

$$c_2(n,\nu,|\vec{k}_2|,\alpha_2) = [c_1(n,\nu,|\vec{k}_2|,\alpha_2)]^*, \qquad (5)$$

are given as an integral in the parton fraction x, containing the PDFs of the gluon and of the different quark/antiquark flavors in the proton, and the FFs of the detected hadron (for more details, see Ref. [26]). It is known [28], that in the BLM approach applied to semihard processes, we need to perform a finite renormalization from the modified minimal subtraction ($\overline{\rm MS}$) scheme to the physical momentum (MOM) one:

$$\alpha_s^{\overline{\rm MS}} = \alpha_s^{\rm MOM}\left(1 + \frac{\alpha_s^{\rm MOM}}{\pi} T\right), \qquad (6)$$

with $T = T^\beta + T^{\rm conf}$,

$$T^\beta = -\frac{\beta_0}{2}\left(1 + \frac{2}{3}I\right),$$
$$T^{\rm conf} = \frac{3}{8}\left[\frac{17}{2}I + \frac{3}{2}(I-1)\xi + \left(1 - \frac{1}{3}I\right)\xi^2 - \frac{1}{6}\xi^3\right], \qquad (7)$$

where $I = -2\int_0^1 dx \frac{\ln(x)}{x^2 - x + 1} \simeq 2.3439$ and ξ is the gauge parameter of the MOM scheme, which is fixed at zero in the following. The optimal scale $\mu_R^{\rm BLM}$ is the value of μ_R that makes the β_0-dependent part in the expression for the observable of interest vanish. In Ref. [20] some of us showed that terms proportional to the QCD β_0 function are present not only in the NLA BFKL kernel, but also in the expressions for the NLA vertices (called "impact factors" in the BFKL jargon). This leads to a nonuniversality of the BLM scale and to its dependence on the energy of the process. It was also found [20] that contributions proportional to the NLA impact factors are universally expressed in terms of the LLA impact factors of the considered process, through the function $f(\nu)$, defined as follows:

$$i\frac{d}{d\nu}\ln\left(\frac{c_1}{c_2}\right) \equiv 2[f(\nu) - \ln(|\vec{k}_1||\vec{k}_2|)]. \tag{8}$$

Finally, the condition for the BLM scale setting was found to be

$$C_n^\beta \propto \int d(\text{P.S.}) \int_{-\infty}^{\infty} d\nu e^{Y\bar{\alpha}_s^{\text{MOM}}(\mu_R^{\text{BLM}})\chi(n,\nu)} c_1(n,\nu)c_2(n,\nu) \times \left[\frac{5}{3} + \ln\frac{(\mu_R^{\text{BLM}})^2}{|\vec{k}_1||\vec{k}_2|} + f(\nu) - 2\left(1+\frac{2}{3}I\right) + \bar{\alpha}_s^{\text{MOM}}(\mu_R^{\text{BLM}})Y\frac{\chi(n,\nu)}{2}\left(-\frac{\chi(n,\nu)}{2} + \frac{5}{3} + \ln\frac{(\mu_R^{\text{BLM}})^2}{|\vec{k}_1||\vec{k}_2|} + f(\nu) - 2\left(1+\frac{2}{3}I\right)\right)\right] = 0, \tag{9}$$

where $\bar{\alpha}_s = 3\alpha_s/\pi$; the first term on the rhs of Eq. (9) originates from the NLA corrections to the hadron vertices and the second one from the NLA part of the kernel. Here Y is the rapidity separation of two detected hadrons, $Y = y_1 - y_2$. We consider the *coefficients* integrated over the phase space for two final-state hadrons,

$$C_n = \int d(\text{P.S.})\mathcal{C}_n(y_1, y_2, k_1, k_2), \tag{10}$$

where

$$\int d(\text{P.S.}) = \int_{k_{1,\min}}^{\infty} d|\vec{k}_1| \int_{k_{2,\min}}^{\infty} d|\vec{k}_2| \int_{y_{1,\min}}^{y_{1,\max}} dy_1 \times \int_{y_{2,\min}}^{y_{2,\max}} dy_2 \delta(y_1 - y_2 - Y). \tag{11}$$

For the integrations over rapidities we use the limits, $y_{1,\min} = -y_{2,\max} = -2.4$, $y_{1,\max} = -y_{2,\min} = 2.4$, that are typical for the identified hadron detection at the LHC. As minimum transverse momenta we choose $k_{1,\min} = k_{2,\min} = 5$ GeV, which are also realistic values for the LHC. We observe that the minimum transverse momentum in the CMS analysis [23] of Mueller-Navelet jet production is much larger, $k_{\text{jet,min}} = 35$ GeV. In our calculations we use the PDF set MSTW 2008 NLO [29] with two different next-to-leading-order (NLO) parametrizations for hadron FFs: AKK [30] and HKNS [31]. We considered also the DSS parametrization [32], but do not show the related results here, since they would be hardly distinguishable from those with the HKNS parametrization. In the results presented below we sum over the production of charged light hadrons: $\pi^\pm, K^\pm, p, \bar{p}$.

In order to find the values of the BLM scales, we introduce the ratios of the BLM to the "natural" scale suggested by the kinematics of the process, $\mu_N = \sqrt{|\vec{k}_1||\vec{k}_2|}$, so that $m_R = \mu_R^{\text{BLM}}/\mu_N$, and look for the values of m_R such that Eq. (9) is satisfied. Results are presented in Fig. 1 as functions of Y for the first few values of n and for the two values of the LHC center-of-mass energy. Then we plug these scales into our expression for the integrated coefficients in the BLM scheme (for the derivation see Ref. [20]):

$$C_n = \int d(\text{P.S.}) \int_{-\infty}^{\infty} d\nu \times \frac{e^Y}{s} e^{Y\bar{\alpha}_s^{\text{MOM}}(\mu_R^{\text{BLM}})[\chi(n,\nu)+\bar{\alpha}_s^{\text{MOM}}(\mu_R^{\text{BLM}})(\bar{\chi}(n,\nu)+\frac{T^{\text{conf}}}{3}\chi(n,\nu))]} \times (\alpha_s^{\text{MOM}}(\mu_R^{\text{BLM}}))^2 c_1(n,\nu)c_2(n,\nu)\left[1 + \bar{\alpha}_s^{\text{MOM}}(\mu_R^{\text{BLM}}) \times \left\{\frac{\bar{c}_1^{(1)}(n,\nu)}{c_1(n,\nu)} + \frac{\bar{c}_2^{(1)}(n,\nu)}{c_2(n,\nu)} + \frac{2T^{\text{conf}}}{3}\right\}\right]. \tag{12}$$

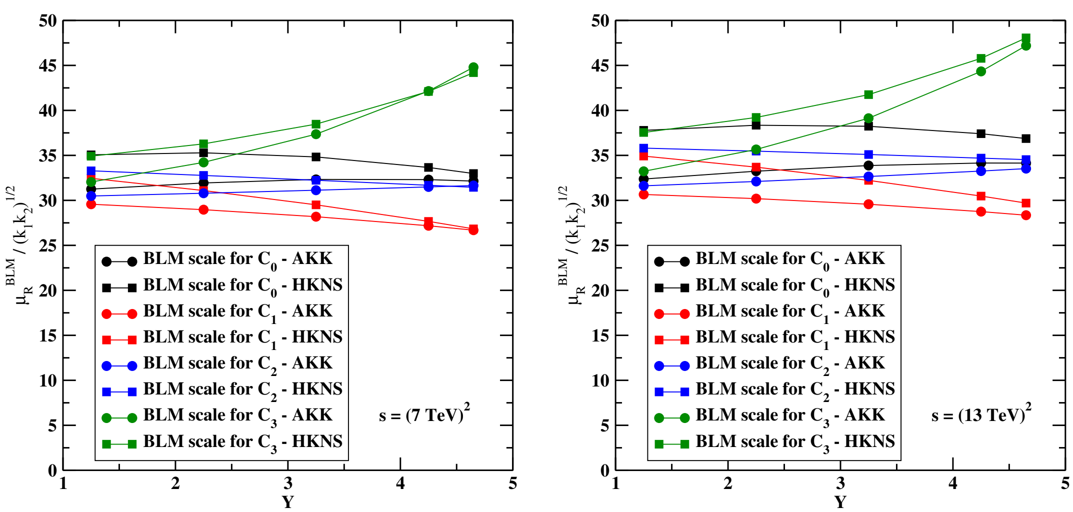

FIG. 1. BLM scales for the dihadron production versus the rapidity interval Y for the two parametrizations of FFs and for all the observables considered in this work.

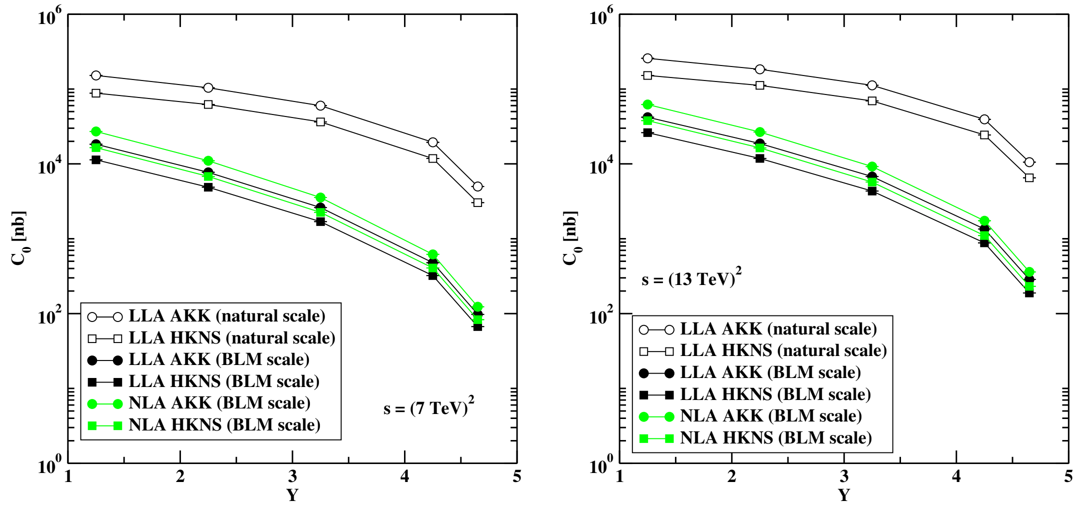

FIG. 2. Cross sections of the dihadron production at the LHC versus the rapidity interval Y for the two parametrizations of FFs considered in this work: a) $\sqrt{s} = 7$ TeV, b) $\sqrt{s} = 13$ TeV. See the text for the definition of “natural” and “BLM” scales.

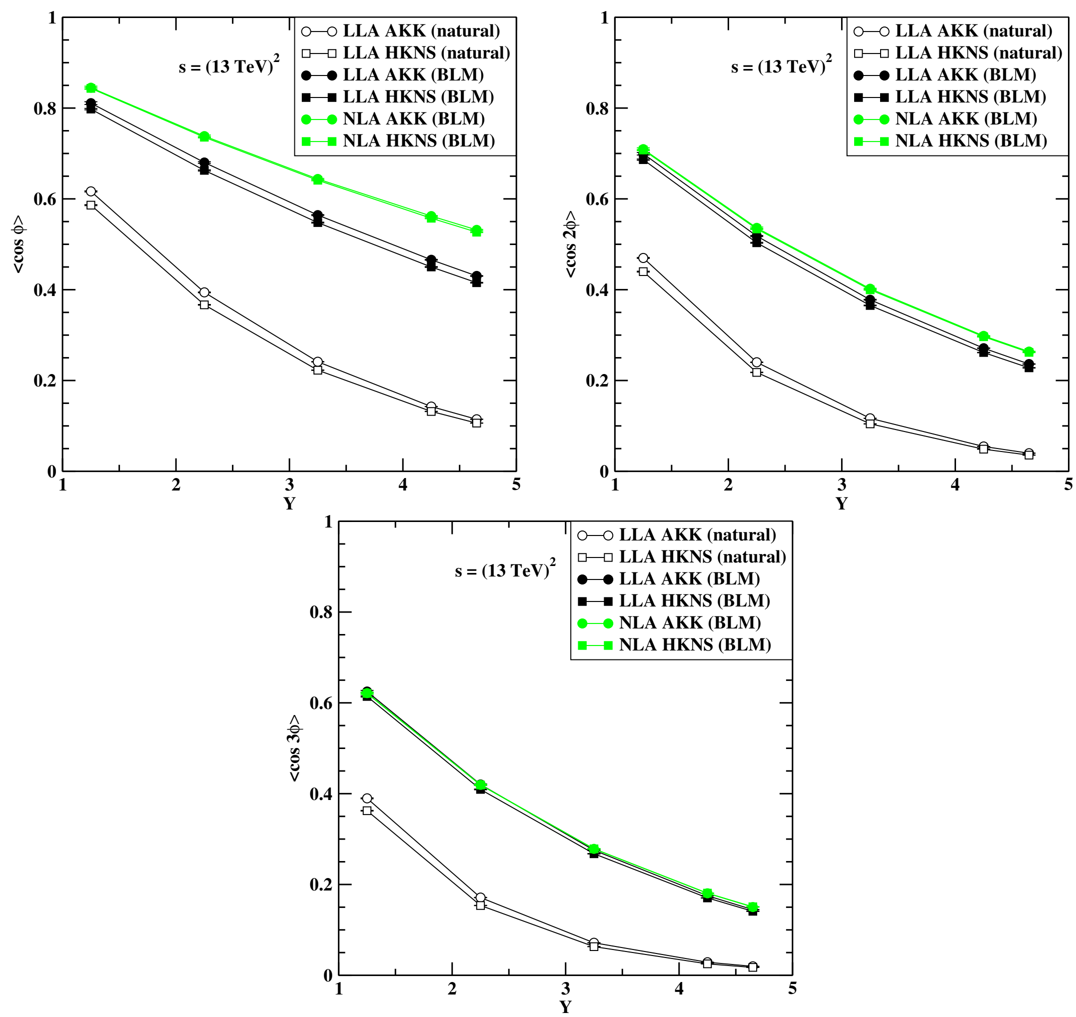

FIG. 3. $\langle \cos\phi \rangle$, $\langle \cos 2\phi \rangle$ and $\langle \cos 3\phi \rangle$ for dihadron production at $\sqrt{s} = 13$ TeV for the two parametrizations of FFs considered in this work. See the text for the definition of “natural” and “BLM” scales.

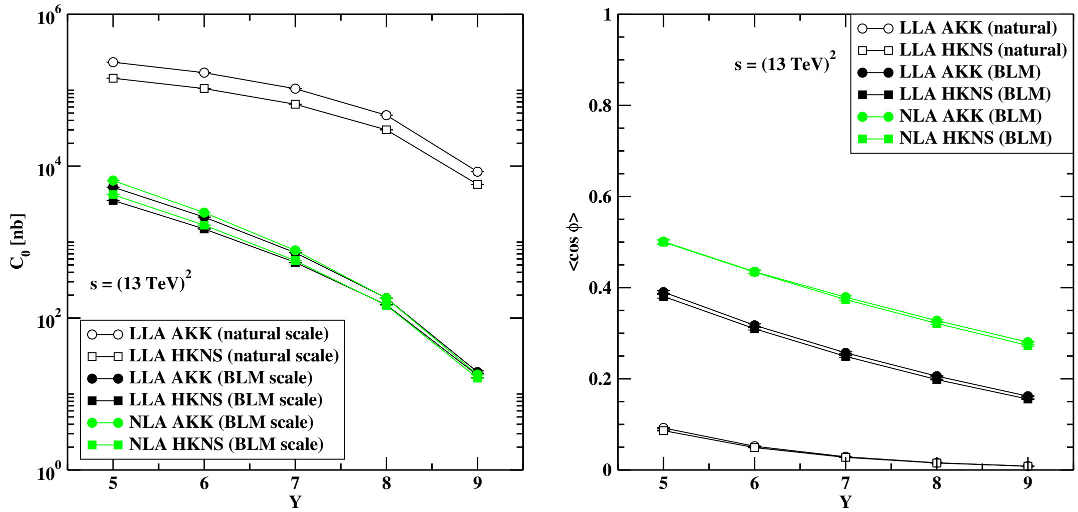

FIG. 4. Cross sections and $\langle\cos\phi\rangle$, for dihadron production at $\sqrt{s} = 13$ TeV and larger rapidity intervals Y.

The coefficient C_0 gives the total cross sections and the ratios $C_n/C_0 = \langle\cos(n\phi)\rangle$ determine the values of the mean cosines, or azimuthal correlations, of the produced hadrons. In Eq. (12), $\bar{\chi}(n,\nu)$ is the eigenvalue of the NLA BFKL kernel [33] and its expression is given, e.g. in Eq. (23) of Ref. [14], whereas $\bar{c}^{(1)}_{1,2}$ are the NLA parts of the hadron vertices [26]. The evaluation of Eq. (12) requires a complicated eight-dimensional numerical integration [the expressions for $\bar{c}^{(1)}_{1,2}$ contain an additional longitudinal fraction integral in comparison to the formulas for the LLA vertices, given in Eqs. (4) and (5)]. Since the main aim of this work is to stress the potential relevance of the process we are proposing, rather than to give a full NLA prediction, we will present our first results neglecting the NLA parts of hadron vertices, i.e. putting $\bar{c}^{(1)}_{1,2} = 0$. This reduces the expression on the rhs of Eq. (12) to a six-dimensional integral, which is manageable for numerical calculations using a FORTRAN code. In Fig. 2 we present results for total cross sections at two values of the center-of-mass LHC energy: $\sqrt{s} = 7$ TeV and $\sqrt{s} = 13$ TeV. Figure 3 shows our predictions for the azimuthal correlations at $\sqrt{s} = 13$ TeV.

For comparison, we considered also larger values of Y, similar to those used in the CMS Mueller-Navelet jets analysis. In Fig. 4 we present the cross section, C_0, and $\langle\cos\phi\rangle$ in this larger Y interval, calculated at a center-of-mass energy of $\sqrt{s} = 13$ TeV and with the other settings as in Fig. 3. These results may be of future reference for CMS, LHCb or other experiments.

III. DISCUSSION AND OUTLOOK

We checked that in our numerical analysis the essential values of x are not too small, $x \sim [10^{-3} \div 10^{-2}]$, and even bigger in the case of the larger Y intervals presented in Fig. 4. This justifies our use of PDFs with the standard DGLAP evolution. Note that our process is not a low-x one, and similarly to the Mueller-Navelet jet production, we are dealing with a dilute partonic system. Therefore possible saturation effects are not important here, and the BFKL dynamics appears only through resummation effects in the hard scattering subprocesses, without influencing the PDFs' evolution.

Our results are obtained using both the AKK and HKNS parametrizations for the hadron FFs. We see in Figs. 1 and 2 the sizable difference between predictions in these two cases, which means that the FFs are not well constrained in the required kinematic region. In a similar range the difference between $\pi^{\pm}$ and $K^{\pm}$ AKK and HKNS FFs was discussed recently in Ref. [34]. Our calculation with the AKK FFs gives bigger cross sections, whereas the difference between AKK and HKNS in Fig. 3 is small, since the FF uncertainties are largely canceled in the coefficient ratios describing the azimuthal angle correlations. We do not present separate plots for azimuthal correlations at $\sqrt{s} = 7$ TeV because we found that the difference between our predictions for these observables at two LHC energies, $\sqrt{s} = 7$ TeV and $\sqrt{s} = 13$ TeV, is not larger than 3%.

The general features of our predictions for dihadron production are rather similar to those obtained earlier for the Mueller-Navelet jet process. Although the BFKL resummation leads to the growth with energy of the partonic subprocess cross sections, the convolution of the latter with the proton PDFs makes the net effect of a decrease with Y of our predictions in Fig. 2. This is due to the fact that, at larger values of Y, PDFs are probed effectively at larger values of x, where they fall very fast. For the dihadron azimuthal correlations we predict in Fig. 3 a decreasing behavior with Y. That originates from the increasing amount of hard undetected parton radiation in the final state allowed by the growth of the partonic subprocess energy. The values of the BLM scales we found are much larger than μ_N, the scale suggested by the kinematics of the process. For the BLM-to-natural scale ratios we obtain rather large numbers, $m_R \sim 35$. These values are larger than those obtained previously for similar scale ratios in the case of the Mueller-Navelet jet production process. The difference may be attributed to the fact that, in the case of dihadron production, we have an additional branching of the parton momenta (described by the detected hadron FFs), and typical transverse momenta of the partons participating in the hard scattering turn out to be considerably larger than $|\vec{k}_{1,2}|$, the momenta of the hadrons detected in the final state. We found that a typical value of the fragmentation fraction is $z = \alpha_h/x \sim 0.4$, which explains the main source of the difference between the values of the BLM scales in the case of dijet and dihadron production. Another source is related to the difference in the function $f(\nu)$, defined in Eq. (8), which appears in the expression for the jet and hadron vertex in these two reactions, and enters also the definition of the BLM scale: $f(\nu)$ is zero for the jets and nonzero in the dihadron case.

Our predictions for dihadron production calculated in LLA with the use of the natural scale μ_N and our NLA results obtained with the BLM scale setting are different: with NLA BLM we got much lower values of the cross sections (see Fig. 2) and considerably larger predictions for the $\langle \cos n\phi \rangle$ (see Fig. 3). For comparison, in Figs. 2 and 3 we show our NLA BLM predictions together with the results we obtained in LLA, but using the large values of the scales as determined with the BLM setting. The plots of Figs. 2 and 3 show that the LLA results with BLM scales lie closer to the NLA BLM ones than LLA results with natural scales. The difference between NLA BLM and LLA with BLM scale predictions is due to the account of NLA corrections to the BFKL kernel in the former. In this paper we did not include the known results for the NLA corrections to the hadron vertices, and therefore our NLA analysis is approximate. As the next stage, we plan to incorporate the terms $\bar{c}^{(1)}_{1,2}(n,\nu)$ in our numerical code. At present we can only rely on the experience gained with the analysis of the similar Mueller-Navelet jet production process, where it was shown that NLA effects coming from the corrections to the BFKL kernel and to the jet vertices are equally important. Therefore the difference between our incomplete NLA BLM and LLA with BLM scale results could be considered as a rough estimate of the uncertainty of the present analysis.

The rapidity range we focused on here, $Y \leq 4.8$, may seem to be not large enough for the dominance of BFKL dynamics. But we see, however, that in this range there are large NLA BFKL corrections, thus indicating that the BFKL resummation is playing here a nontrivial role. To clarify the issue it would be very interesting to confront our predictions with the results of fixed-order NLO DGLAP calculations. But this would require new numerical analysis in our semihard kinematic range, because the existing NLO DGLAP results cover the hard kinematic range for the energies of fixed target experiments; see for instance Refs. [35,36].

In our calculation we adopted, somewhat arbitrarily, the limit $|\vec{k}_{1,2}| \geq 5$ GeV for hadron transverse momenta. With this choice we obtained large values of the process cross sections, presented in Fig. 2. This makes us confident that the inclusive production of two detected hadrons separated by large rapidity intervals could be considered in forthcoming analyses at the LHC.

Considering a region of lower hadron transverse momenta, say $|\vec{k}_{1,2}| \geq 2$ GeV, would lead to even larger values of the cross sections. But it should be noted that in our calculation we use the BFKL method together with leading-twist collinear factorization, which means that we are systematically neglecting power-suppressed corrections. Therefore, by going to smaller transverse momenta we would enter a region where higher-twist effects must be important. The applicability border for our approach could be established either by comparing our predictions with future data or by confronting it with some other theoretical predictions which do include higher-twist effects. For the last point, one can consider an alternative, higher-twist production mechanism, related to multiparton interactions in QCD (for a review, see Ref. [37]). The double-parton scattering contribution to the Mueller-Navelet jet production was considered in the papers [19] and [38], using different approaches. It would be very interesting if similar estimates were done also for the case of dihadron production.

In conclusion, we believe that, even within the approximation adopted in our calculation and the systematic uncertainties related to the scale-setting procedure, we have provided enough evidence that the study of the dihadron production can be successfully included in the program of future analyses at the LHC and can improve our knowledge about the dynamics of strong interactions in the Regge limit.

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