Understanding $B^- \to X(3823)K^-$ via rescattering mechanism and predicting $B^- \to \eta_{c2}(^1D_2)/\psi_3(^3D_3)K^-$

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We study the observed $B^- \to X(3823)K^-$ decay via the rescattering mechanism and show that this branching ratio is well reproduced by this mechanism. We further extend this theoretical framework to investigate the decays of $B^- \to \eta_{c2}(^1D_2)/\psi_3(^3D_3)K^-$, where the $\eta_{c2}(^1D_2)$ and the $\psi_3(^3D_3)$ are D-wave charmonium partners of the $X(3823)$. Our results show that the branching ratios, $B^- \to \eta_{c2}(^1D_2)K^-$ and $B^- \to \psi_3(^3D_3)K^-$, are of the order of 10^{-5}, which can be accessible at LHCb, Belle and forthcoming BelleII.

I. INTRODUCTION

In the past decade, abundant charmonium and charmonium-like states have been discovered. Some of them cannot fit into traditional quark model predictions and leave many puzzles. So it attracts great attention to explore their inner structure and interaction mechanism (see Refs. [1,2] for a review). An important feature is that many XYZ or charmonium states strongly couple to open charmed mesons, and as a result, it leads to a quite interesting phenomenon in many processes, which is called a rescattering effect. For example, in a hadronic transition process of a charmonium, instead of gluon emission, the charmonium can first decay into charmed and anticharmed mesons, and these two mesons rescatter each other into a charmonium plus a light meson. Such a rescattering effect has been extensively studied by many authors (see Refs. [3–11]). Their results indicate that rescattering effects can significantly change the line shapes of three-body decays and enhance the results of the Okubo-Zweig-Iizuka (OZI)-suppressed processes.

Another example is given in the situation that the rescattering effect is combined with nonleptonic B meson decays. As we will see later, such a rescattering effect even plays a dominant role. On the other hand, in a naive factorization approach which is normally adopted for nonleptonic processes, the amplitudes of some processes such as $B^- \to \chi_{c0}K^-$ vanish (see Sec. II). In Ref. [12], authors explained the large experimental branching fraction of the process $B^- \to \chi_{c0}K^-$ applying the rescattering

mechanism. Later, they also studied the $B^- \to h_c K^-$ process using the same mechanism and predicted its branching ratio [13]. In Ref. [14], authors systematically studied rescattering effects on nonleptonic B meson decays and their impact on direct CP violations. Reference [15] studied process $B^0 \to \eta_c K^*$ using the rescattering mechanism and reproduced the experimental data. We also notice that in an earlier time, authors in Refs. [16,17] have already applied a rescattering mechanism to study nonleptonic D meson decays.

In 2013, Belle observed a new charmonium-like state called $X(3823)$ in the $\chi_{c1}\gamma$ final state in the process $B^- \to X(3823)K^-$ [18] with measured mass $3823.1 \pm 1.8(\text{stat}) \pm 0.7(\text{syst})$ MeV and significance 3.8σ. Recently, BESIII confirmed $X(3823)$ in the process $e^+e^- \to \pi^+\pi^-\gamma\chi_{c1}$ with measured mass $3821.7 \pm 1.3(\text{stat}) \pm 0.7(\text{syst})$ MeV, width less than 16 MeV and significance 6.2σ [19]. $X(3823)$ is expected to be the long missing $\psi_2(1^3D_2)$ with $J^{PC} = 2^{--}$. This is because, first, the mass of $X(3823)$ is consistent with the quark model prediction [20,21]. Second, since the mass of $X(3823)$ is below any open charm threshold ($D\bar{D}$ channel is forbidden by parity conservation), the width is quite narrow as expected and as observed. $X(3823)$ largely decays to $\chi_{c1}\gamma$, which is the channel discovered in Belle and BESIII. Furthermore, the upper limit of the ratio $B(X(3823) \to \chi_{c2}\gamma)/B(X(3823) \to \chi_{c1}\gamma)$ was determined to be < 0.41 by Belle and < 0.42 by BESIII, which is consistent with theoretical predictions in Refs. [20–23]. Therefore $X(3823)$ is believed to be $\psi_2(^3D_2)$.

In Ref. [24], authors studied the OZI-suppressed process $X(3823) \to J\psi\pi\pi$ via a rescattering effect. Their calculation shows that since the mass of $X(3823)$ is close to the $D\bar{D}^*$ threshold, a rescattering effect can significantly change the line shape of the final $\pi\pi$ mass spectrum. In

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this work, we will focus on another aspect to investigate the rescattering effect on $X(3823)$, i.e., the $X(3823)$ production via a B meson decay. We will illustrate that the naive factorized amplitude of the process $B^- \to X(3823)K^-$ vanishes, and hence it provides us another good example to see how important the scattering effect is.

Besides $X(3823)$, two other D-wave low-lying charmonia are still missing, i.e., $\eta_{c2}(^1D_2)$ with $J^{PC} = 2^{-+}$ and $\psi_2(^3D_3)$ with $J^{PC} = 3^{--}$. Their predicted masses and decay properties are given in Refs. [20,21]. The naive factorized amplitudes for the processes $B^- \to \eta_{c2}K^-$ and $B^- \to \psi_2 K^-$ vanish, for which we will also apply the rescattering mechanism. Their production rates in the B decay will be a valuable information for experiments.

This paper is organized as follows. After the Introduction, we study the decay process $B^- \to X(3823)K^-$ through the rescattering mechanism in Sec. II. In Sec. III, we make predictions of the production rates for the processes $B^- \to \eta_{c2}K^-$ and $B^- \to \psi_2 K^-$. In the final section, we give discussions and the conclusion.

II. $B^- \to X(3823)K^-$ VIA RESCATTERING MECHANISM

First we will show that the naive factorization approach (see Ref. [25]) fails to describe our discussed processes. When studying $B^- \to X(3823)K^-$ in this approach, the effective weak Hamiltonian is written as

$$H_W = \frac{G_F}{\sqrt{2}} \left\{ V_{cb}V_{cs}^*[c_1(\mu)\mathcal{O}_1(\mu) + c_2(\mu)\mathcal{O}_2(\mu)] - V_{tb}V_{ts}^* \sum_{i=3}^{10} c_i(\mu)\mathcal{O}_i(\mu) \right\} + \text{H.c.}, \quad (1)$$

where the operators $\mathcal{O}_i$ read as

$$\mathcal{O}_1 = (\bar{s}_\alpha b_\beta)_{V-A}(\bar{c}_\beta c_\alpha)_{V-A},$$
$$\mathcal{O}_2 = (\bar{s}_\alpha b_\alpha)_{V-A}(\bar{c}_\beta c_\beta)_{V-A},$$
$$\mathcal{O}_{3(5)} = (\bar{s}_\alpha b_\alpha)_{V-A}\sum_q(\bar{q}_\beta q_\beta)_{V-A(V+A)},$$
$$\mathcal{O}_{4(6)} = (\bar{s}_\alpha b_\beta)_{V-A}\sum_q(\bar{q}_\beta q_\alpha)_{V-A(V+A)},$$
$$\mathcal{O}_{7(9)} = \frac{3}{2}(\bar{s}_\alpha b_\alpha)_{V-A}\sum_q e_q(\bar{q}_\beta q_\beta)_{V+A(V-A)},$$
$$\mathcal{O}_{8(10)} = \frac{3}{2}(\bar{s}_\alpha b_\beta)_{V-A}\sum_q e_q(\bar{q}_\beta q_\alpha)_{V+A(V-A)}.$$

Thus, the factorized amplitude of the process $B^- \to X(3823)K^-$ can be expressed as

$$\mathcal{M}(B^- \to X(3823)K^-)$$
$$= \frac{G_F}{\sqrt{2}}V_{cb}V_{cs}^*\left(a_2(\mu) + \sum_{i=3,5,7,9}a_i(\mu)\right)\langle K^-|(\bar{s}b)_{V-A}|B^-\rangle$$
$$\times \langle X(3823)|(\bar{c}c)_{V\mp A}|0\rangle \quad (2)$$

with $a_2 = c_2 + c_1/N_c$ and $a_i = c_i + c_{i+1}/N_c$. In this work, $X(3823)$ is treated as a D-wave charmonium with quantum numbers $J^{PC} = 2^{--}$. When checking the factorized amplitude in Eq. (2), we find the matrix element $\langle X(3823)|(\bar{c}c)_{V\mp A}|0\rangle = 0$ due to the Lorentz invariance. Hence this leads to vanishing of the branching ratio of $B^- \to X(3823)K^-$ in the naive factorization approach.

However, the Belle measurement [18] shows combined branching fraction $\mathcal{BR}(B^- \to X(3823)K^-) \times \mathcal{BR}(X(3823) \to \chi_{c1}\gamma) = (9.7 \pm 2.8 \pm 1.1) \times 10^{-6}$. To obtain the value of $\mathcal{BR}(B^- \to X(3823)K^-)$, we consider the theoretical partial widths of $X(3823)$ decaying into $\chi_{c1}\gamma$, $\chi_{c2}\gamma$, ggg, and $J/\psi\pi\pi$ which are given by $\Gamma(X(3823) \to \chi_{c1}\gamma) = 215$ keV [21], $\Gamma(X(3823) \to \chi_{c2}\gamma) = 59$ keV [21], $\Gamma(X(3823) \to ggg) = 36$ keV [20] and $\Gamma(X(3823) \to J\psi\pi\pi) \simeq 160$ keV [24], respectively. Summing up all the above partial widths, we can roughly estimate the total decay width of $X(3823)$ to be 470 keV, with which we get $\mathcal{BR}(X(3823) \to \chi_{c1}\gamma) = 46\%$. Then, we can extract

$$\mathcal{BR}(B^- \to X(3823)K^-) = (2.10 \pm 0.65) \times 10^{-5}, \quad (3)$$

where the error comes from the combined branching fraction of the Belle measurement. It shows that there exists a nonzero contribution to the $B^- \to X(3823)K^-$ decay.

To understand the discrepancy between the experimental data and theoretical estimate from the naive factorization approach, we study $B^- \to X(3823)K^-$ by introducing the rescattering mechanism, which was proposed in Ref. [12]. They indicated that such a nonleptonic process should have a large nonfactorizable contribution that comes from the rescattering mechanism. For the discussed $B^- \to X(3823)K^-$ process, B^- first decays into intermediate charmed and anticharmed meson pair, and then they transit into final states, $X(3823)$ and K^-. The typical diagram describing the rescattering effect on $B^- \to X(3823)K^-$ can be found in Fig. 1. In the following, we calculate these rescattering processes of $B^- \to X(3823)K^-$ to test whether the extracted branching ratio given by Eq. (3) can be understood under the rescattering mechanism.

In order to calculate these triangle diagrams at hadron level, we need to introduce the effective Lagrangians corresponding to each interaction vertex. As for the weak vertex $B^- \to D^{(*)0}D_s^{(*)-}$, we also assume the naive factorization of the amplitude. Neglecting the small contributions from the operators $\mathcal{O}_3 \sim \mathcal{O}_{10}$ in Eq. (1), the transition matrix element can be factorized as
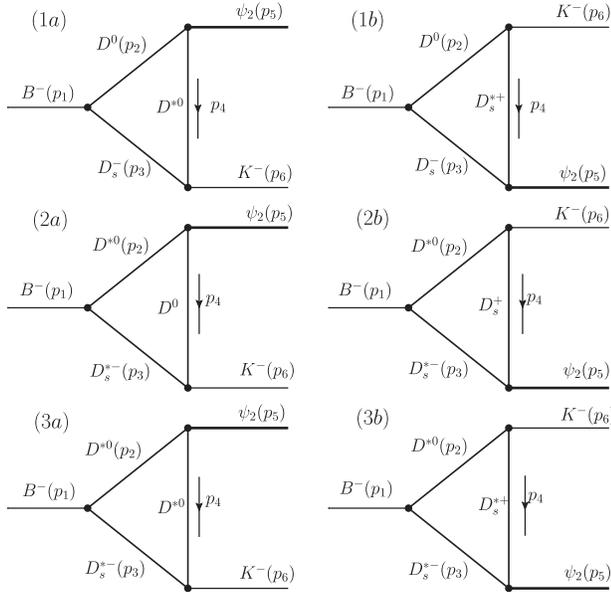
FIG. 1. The schematic diagrams for depicting the $B^- \rightarrow X(3823)K^-$ decay via the rescattering mechanism. Note that ψ_2 denotes $X(3823)$.

$$\langle D^{(*)0}D_s^{(*)-}|H_W|B^-\rangle = \frac{G_F}{\sqrt{2}}V_{cb}V_{cs}^*a_1\langle D^{(*)0}|V^\mu - A^\mu|B^-\rangle$$
$$\langle D_s^{(*)-}|V_\mu - A_\mu|0\rangle, \quad (4)$$

where $a_1 = c_1 + c_2/N_c$. One should notice that this naive factorization for the process $B^- \rightarrow D^{(*)0}D_s^{(*)-}$ had been shown to be a good approximation in Ref. [26]. The matrix element appearing in Eq. (4) can be simply written in terms of form factors and decay constants.

We use the following matrix elements that contain only one form factor ξ, i.e., Isgur-Wise function [27]:

$$\langle D^0(v')|V^\mu|B^-(v)\rangle = \sqrt{m_B m_D}\xi(v \cdot v')(v' + v)^\mu,$$
$$\langle D^{*0}(v',\epsilon)|V_\mu|B^-(v)\rangle = i\sqrt{m_B m_{D^*}}\xi(v \cdot v')\varepsilon_{\mu\nu\alpha\beta}\epsilon^{\nu*}v'^\alpha v^\beta,$$
$$\langle D^{*0}(v',\epsilon)|A^\mu|B^-(v)\rangle = \sqrt{m_B m_{D^*}}\xi(v \cdot v')((1 + v \cdot v')g^{\alpha\mu}$$
$$- v^\alpha v'^\mu)\epsilon_\alpha^*,$$
$$\langle 0|A^\mu|D_s(v)\rangle = f_{D_s}m_{D_s}v^\mu,$$
$$\langle 0|V^\mu|D_s^*(v,\epsilon)\rangle = f_{D_s^*}m_{D_s^*}\epsilon^\mu.$$

Using these matrix elements, one further obtains the transition amplitudes:

$$\langle D^0(p_2)D_s^-(p_3)|B^-(v_1)\rangle$$
$$= \frac{G_F}{\sqrt{2}}V_{cb}V_{cs}^*a_1\sqrt{m_1 m_2}\xi(v \cdot v')\left(\frac{p_2^\mu}{m_2} + v_1^\mu\right)f_3 p_{3\mu}, \quad (5)$$

$$\langle D^{*0}(p_2)D_s^{*-}(p_3)|B^-(v_1)\rangle$$
$$= \frac{G_F}{\sqrt{2}}V_{cb}V_{cs}^*a_1\sqrt{m_1 m_2}\xi(v \cdot v')\left(i\epsilon_{\mu\nu\alpha\beta}\frac{p_2^\alpha}{m_2}v_1^\beta - (1 + \omega)g_{\nu\mu}\right.$$
$$\left. + v_{1\nu}\frac{p_{2\mu}}{m_2}\right)\epsilon_2^{*\nu}f_3 m_3 \epsilon_3^{*\mu}, \quad (6)$$

$$\langle D^0(p_2)D_s^{*-}(p_3)|B^-(v_1)\rangle$$
$$= \frac{G_F}{\sqrt{2}}V_{cb}V_{cs}^*a_1\sqrt{m_1 m_2}\xi(v \cdot v')\left(\frac{p_2^\mu}{m_2} + v_1^\mu\right)f_3 m_3 \epsilon_{3\mu}^*, \quad (7)$$

$$\langle D^{*0}(p_2)D_s^-(p_3)|B^-(v_1)\rangle$$
$$= \frac{G_F}{\sqrt{2}}V_{cb}V_{cs}^*a_1\sqrt{m_1 m_2}\xi(v \cdot v')\left(i\epsilon_{\mu\nu\alpha\beta}\frac{p_2^\alpha}{m_2}v_1^\beta - (1 + \omega)g_{\nu\mu}\right.$$
$$\left. + v_{1\nu}\frac{p_{2\mu}}{m_2}\right)\epsilon_2^{*\nu}f_3 p_3^\mu, \quad (8)$$

where m_1 and m_2 are the masses of B^- and $D^{(*)0}$, respectively, f_3 is a decay constant of the particle carrying a momentum p_3, and $a_1 = c_1 + c_2/N_c$ as defined in Eq. (4).

For the $D_s^{(*)}D^{(*)}K$ interactions, we adopt the effective Lagrangians respecting both the heavy quark symmetry and chiral symmetry. For a heavy-light meson system, there exist heavy quark spin symmetry and heavy quark flavor symmetry [28] in the heavy quark limit $m_Q \rightarrow \infty$. As a consequence, heavy-light mesons are degenerate and are classified into different multiplets, such as an H doublet $(0^-, 1^-)$ with quantum number of light degrees of freedom $j_\ell^P = \frac{1}{2}^-$. The multiplet can be described by an effective hadron field respecting the heavy quark symmetry. For example, the field of an H doublet (D, D^*) or (D_s, D_s^*) is given by

$$H_a = \left(\frac{1 + \rlap{/}v}{2}\right)(D_a^{*\mu}\gamma_\mu + iD_a\gamma_5) \quad (9)$$

with a the flavor index and v the meson velocity, where the fields $D_a^{*\mu}$ and D_a contain a normalization factor $\sqrt{m_M}$ and have dimension 3/2.

For an H field coupled with an octet chiral multiplet, the effective Lagrangian reads [29]

$$\mathcal{L}_H = ig_H\text{Tr}[H_b\gamma_\mu\gamma_5\mathcal{A}_{ba}^\mu\bar{H}_a], \quad (10)$$

where $\mathcal{A}_{ba}^\mu = i/f_\pi\partial^\mu M_{ba} + \cdots$ with M_{ba} the octet of light pseudoscalar mesons and $f_\pi = 132$ MeV. $\bar{H}_a$ satisfies the relation $\bar{H}_a = \gamma^0 H_a^\dagger\gamma^0$. By expanding the Lagrangian in Eq. (10), effective Lagrangians for the vertexes $D_s^{(*)}D^{(*)}K$ are explicitly given by

$$\mathcal{L}_{DD_s^*K} = ig_{DD_s^*K}\bar{D}_{s\mu}^*D\partial^\mu K, \quad (11)$$

$$\mathcal{L}_{D_s D^* K} = -ig_{D_s D^* K} \bar{D}_s D^*_\mu \partial^\mu K, \tag{12}$$

$$\mathcal{L}_{D^*_s D^* K} = -g_{D^*_s D^* K} \epsilon_{\mu\nu\alpha\beta} \partial^\mu \bar{D}^{*\nu}_s \partial^\alpha D^{*\beta} K, \tag{13}$$

where the coupling constants are related to g_H as

$$g_{DD^*_s K} = g_{D_s D^* K} = \sqrt{m_{D_s} m_D} \frac{2g_H}{f_\pi}, \tag{14}$$

$$g_{D^*_s D^* K} = \frac{\sqrt{m_{D^*_s} m_{D^*}}}{m_{D^*}} \frac{2g_H}{f_\pi}. \tag{15}$$

The vertexes $X(3823)D^{(*)}D^*$ and $X(3823)D^{(*)}_s D^*_s$ are additionally involved in our calculation, for which we also use the effective Lagrangians respecting the heavy quark symmetry. However, for a charmonium system, the heavy quark flavor symmetry does not hold, where only the heavy quark spin symmetry remains [30]. Thus, charmonia with the same orbital angular momentum L but with different total spin can form a multiplet. In our case, $X(3823)$ belongs to a D-wave multiplet [30,31], which is defined by

$$X^{\mu\nu} = \frac{1+\slashed{v}}{2} \left\{ \psi^{\mu\nu\alpha}_3 \gamma_\alpha + \frac{1}{\sqrt{6}} (\epsilon^{\mu\delta\alpha\beta} v_\delta \gamma_\alpha \psi^\nu_{2\beta} + \epsilon^{\nu\delta\alpha\beta} v_\delta \gamma_\alpha \psi^\mu_{2\beta}) \right.$$
$$+ \frac{\sqrt{15}}{10} ((\gamma^\mu - v^\mu)\psi^\nu + \psi^\mu(\gamma^\nu - v^\nu))$$
$$\left. - \frac{1}{\sqrt{15}} (g^{\mu\nu} - v^\mu v^\nu)\gamma_\alpha \psi^\alpha + \eta^{\mu\nu}_2 \gamma_5 \right\} \frac{1-\slashed{v}}{2}. \tag{16}$$

In the above expression, the fields ψ_3, ψ_2, ψ, and η_2 denote the charmonia with quantum numbers $J^{PC} = 3^{--}, 2^{--}, 1^{--}$ and 2^{-+}, respectively, where ψ_2 corresponds to the discussed $X(3823)$.

For the coupling of D-wave charmonium multiplet with charmed mesons, their effective Lagrangian reads [24]

$$\mathcal{L}_X = g_X \text{Tr}[X^{\mu\nu} \bar{H}_{\bar{Q}a}(\overleftarrow{\partial}_\mu - \overleftarrow{\partial}_\mu)\gamma_\nu \bar{H}_{Qa}], \tag{17}$$

where H_{Qa} is given by Eq. (9), and $H_{\bar{Q}a}$ is

$$H_{\bar{Q}a} = (D^{*\mu}_a \gamma_\mu + iD_a \gamma_5)\left(\frac{1-\slashed{v}}{2}\right), \tag{18}$$

which is obtained by the charge conjugation transformation. The fields $\bar{H}_{\bar{Q}a}$ and $\bar{H}_{Qa}$ appearing in Eq. (17) are defined as $\bar{H}_{\bar{Q}a} = \gamma^0 H^\dagger_{\bar{Q}a}\gamma^0$ and $\bar{H}_{Qa} = \gamma^0 H^\dagger_{Qa}\gamma^0$, respectively. Then, the explicit forms of the $X(3823)D^{(*)}D^*$ ($\psi_2 D^{(*)}D^*$) interactions can be obtained as

$$\mathcal{L}_{\psi_2 DD^*} = g_{\psi_2 DD^*} \psi^{\mu\nu}_2 (\partial_\nu \bar{D} D^*_\mu - \bar{D} \partial_\nu D^*_\mu)$$
$$+ g_{\psi_2 DD^*} \psi^{\mu\nu}_2 (\bar{D}^*_\mu \partial_\nu D - \partial_\nu \bar{D}^*_\mu D), \tag{19}$$

$$\mathcal{L}_{\psi_2 D^* D^*} = ig_{\psi_2 D^* D^*} \varepsilon_{\mu\nu\alpha\beta} \partial^\mu \psi^{\nu\rho}_2 D^*_\rho \partial^\alpha \bar{D}^{*\beta}$$
$$+ ig_{\psi_2 D^* D^*} \varepsilon_{\mu\nu\alpha\beta} \partial^\mu \psi^{\nu\rho}_2 \partial^\alpha \bar{D}^*_\rho D^{*\beta}, \tag{20}$$

where

$$g_{\psi_2 DD^*} = \sqrt{6}g_X \sqrt{m_D m_{D^*} m_{\psi_2}}, \tag{21}$$

$$g_{\psi_2 D^* D^*} = \frac{-4g_X}{\sqrt{6}} \frac{\sqrt{m_{D^*} m_{D^*} m_{\psi_2}}}{m_{\psi_2}}. \tag{22}$$

The Lagrangians of vertexes $\psi_2 D^{(*)}_s D^*_s$ are similar to those shown in Eqs. (19) and (20), where the corresponding coupling constants satisfy $g_{\psi_2 D_s D^*_s} = g_{\psi_2 DD^*}$ and $g_{\psi_2 D^*_s D^*_s} = g_{\psi_2 D^* D^*}$ if the SU(3) flavor symmetry holds.

Applying the Cutkosky cutting rule [32], the imaginary parts of the decay amplitudes of $B^- \to X(3823)K^-$ can be obtained, for example, for the amplitude of diagram (1a) in Fig. 1 as

$$Abs^{B^- \to \psi_2 K^-}_{(1a)}$$
$$= \frac{|p_2|}{32\pi^2 m_1} \int d\Omega \frac{G_F}{\sqrt{2}} V_{cb} V^*_{cs} a_1 \sqrt{m_1 m_2} \xi(\omega) \left(v^\gamma_1 + \frac{p^\gamma_2}{m_2}\right) f_3 p_{3\gamma}$$
$$\times (-i)g_{\psi_2 DD^*} \epsilon^{*\mu\nu}_5 (p_{2\nu} + p_{4\nu})\left(-g_{\mu\alpha} + \frac{p_{4\mu} p_{4\alpha}}{m^2_4}\right) g_{D_s D^* K} p^\alpha_6$$
$$\times \frac{1}{p^2_4 - m^2_4} \mathfrak{F}^2(p^2_4). \tag{23}$$

Here m_i $(i = 1, 2, 3, \ldots)$ denotes the mass of the particle carrying momentum p_i in Fig. 1, and $\omega = v \cdot v'$. Other amplitudes are given in the Appendix. On the other hand, we need also introduce form factors to compensate the off-shell effect of the exchanged $D^{(*)}_{(s)}$ in Fig. 1. The concrete expression of the form factor is [4,14]

$$\mathfrak{F}(q^2) = \frac{\Lambda^2 - m^2}{\Lambda^2 - q^2}, \tag{24}$$

where the cutoff parameter Λ can be parametrized as

$$\Lambda = m + \alpha\Lambda_{\text{QCD}} \tag{25}$$

with $\Lambda_{\text{QCD}} = 220$ MeV. m denotes the mass of the exchanged meson.

The total absorptive part of the amplitude of the process $B^- \to X(3823)K^-$ is

$$\text{Abs}(\mathcal{M}[B^- \to X(3823)K^-]) = \sum_{i=1a,\ldots,3b} \text{Abs}^{B^- \to \psi_2 K^-}_{(i)},$$

with which we can estimate the decay width of the process $B^- \to X(3823)K^-$ as

$$\Gamma(B^- \to X(3823)K^-) = \frac{1}{8\pi} \frac{|\vec{p}|}{m_B^2} |\text{Abs}(\mathcal{M})|^2. \quad (26)$$

Here, $\vec{p}$ denotes the three-momentum of final states in the center of mass frame of B^- meson, and m_B is the mass of the B meson.

In principle, we may include the real part (dispersive part) of the scattering amplitude through the absorptive part:

$$\text{Dis}(\mathcal{M}(m_B^2)) = \frac{1}{\pi} \int_s^{+\infty} \frac{\text{Abs}(\mathcal{M}(s'))}{s' - m_b^2} ds'. \quad (27)$$

However, as discussed in Ref. [14], this real part has large uncertainties that come from a newly introduced cutoff parameter and integration itself. Furthermore, since the mass of the B meson is far from the D meson pair threshold, the imaginary part can largely increase and become dominant in full amplitude. Hence we assume the absorptive part is dominant as in Ref. [14], and ignore the dispersive part.

In order to obtain the results, the values of various parameters should be specified, which include the weak Fermi coupling constant $G_F = 1.16638 \times 10^{-5}$ GeV^{-2}, $V_{cb} = 0.04$ and $V_{cs} = 1.0$ [33], decay constants $f_{D_s} = f_{D_s^*} = 0.24$ GeV and Wilson coefficient $a_1 = 1.0$ [13]. As for the mass of $X(3823)$, we adopt the BESIII's result $m_{X(3823)} = 3.8217$ GeV [19] as an input. The strong coupling constants $g_H \simeq 0.57$ and $g_X \simeq 1.4$ GeV$^{-3/2}$ are given in Ref. [24]. As for the Isgur-wise function, we adopt the form calculated in Ref. [27]:

$$\xi(\omega) = 1 - 1.22(\omega - 1) + 0.85(\omega - 1)^2. \quad (28)$$

So far in our calculation the only unknown parameter left is α in Eq. (25). The rescattering mechanism becomes soft in the case of the B meson decay because of heaviness of the B meson mass. Since the rescattering mechanism as a long-distant contribution plays an important role to understand $B^- \to X(3823)K^-$, we try to reproduce the experimental branching ratio of $B^- \to X(3823)K^-$ shown in Eq. (3) by varying the parameter α to obtain $\alpha = 0.70 \pm 0.05$, where the error comes from Eq. (3). It is obvious that this is not the end of the whole story. This value of α can be applied to study similar processes like the productions of $\eta_{c2}(^1D_2)/\psi_3(^3D_3)$ plus a kaon via B meson decays, where $\eta_{c2}(^1D_2)$ and $\psi_3(^3D_3)$ are as the D-wave charmonium partners of $X(3823)$. In the next section, we illustrate the details of the corresponding deduction.

III. PREDICTION OF $B^- \to \eta_{c2}(^1D_2)K^-$ AND $B^- \to \psi_3(^3D_3)K^-$

After discussing the $B^- \to X(3823)K^-$ decay, in this section we further investigate the productions of two D-wave charmonia $\eta_{c2}(^1D_2)$ with $J^{PC} = 2^{-+}$ and

$\psi_3(^3D_3)$ with $J^{PC} = 3^{--}$ through similar B decay processes. Here, $\eta_{c2}(^1D_2)$ and $\psi_3(^3D_3)$ have not yet been observed in experiment, which also stimulates us to predict the production rates of $B^- \to \eta_{c2}K^-$ and $B^- \to \psi_3K^-$.

Similar to the process $B^- \to X(3823)K^-$, the processes $B^- \to \eta_{c2}K^-$ and $B^- \to \psi_3K^-$ are also forbidden if simply considering the naive factorization contribution, since $\langle \eta_{c2}(2^{-+})|(\bar{c}c)_{V\mp A}|0\rangle = 0$ and $\langle \psi_3(3^{--})|(\bar{c}c)_{V\mp A}|0\rangle = 0$. According to the former experience of study of $B^- \to X(3823)K^-$, we need to introduce the rescattering mechanism to estimate the decay rates of these two processes.

In order to calculate the processes $B^- \to \eta_{c2}(^1D_2)K^-$ and $B^- \to \psi_3(^3D_3)K^-$, one needs to have the effective Lagrangian given in Eq. (17). For the vertexes $\eta_{c2}DD^*$ and $\eta_{c2}D^*D^*$, the corresponding Lagrangians read

$$\mathcal{L}_{\eta_{c2}DD^*} = ig_{\eta_{c2}DD^*}\eta_{c2}^{\mu\nu}(\partial_\nu \bar{D}D_\mu^* - \bar{D}\partial_\nu D_\mu^*)$$
$$- ig_{\eta_{c2}DD^*}\eta_{c2}^{\mu\nu}(\bar{D}_\mu^*\partial_\nu D - \partial_\nu \bar{D}_\mu^*D), \quad (29)$$

$$\mathcal{L}_{\eta_{c2}D^*D^*} = g_{\eta_{c2}D^*D^*}\varepsilon_{\mu\nu\alpha\beta}\partial^\mu\eta_{c2}^{\rho\nu}D^{*\alpha}\partial_\rho\bar{D}^{*\beta} \quad (30)$$

with

$$g_{\eta_{c2}DD^*} = 2g_X\sqrt{m_D m_{D^*} m_{\eta_{c2}}} \quad (31)$$

$$g_{\eta_{c2}D^*D^*} = 4g_X\frac{\sqrt{m_{D^*}m_{D^*}m_{\eta_{c2}}}}{m_{\eta_{c2}}}. \quad (32)$$

For the vertex $\psi_3D^*D^*$, the Lagrangian is

$$\mathcal{L}_{\psi_3D^*D^*} = g_{\psi_3D^*D^*}\psi_3^{\mu\nu\alpha}(\partial_\mu\bar{D}_\nu^*D_\alpha^* - \bar{D}_\nu^*\partial_\mu D_\alpha^*) \quad (33)$$

with

$$g_{\psi_3D^*D^*} = 4g_X\sqrt{m_{D^*}m_{D^*}m_{\psi_3}}. \quad (34)$$

The Lagrangians for the vertexes $\eta_{c2}D_s^{(*)}D_s^*$ and $\psi_3D_s^*D_s^*$ have the same form as those shown in Eqs. (29), (30), and (33), where we only need to have the relations among the involved coupling constants, i.e., $g_{\eta_{c2}D_sD_s^*} = g_{\eta_{c2}DD^*}$, $g_{\eta_{c2}D_s^*D_s^*} = g_{\eta_{c2}D^*D^*}$ and $g_{\psi_3D_s^*D_s^*} = g_{\psi_3D^*D^*}$, which are obtained by assuming the $SU(3)$ flavor symmetry.

For $B^- \to \eta_{c2}K^-$, the corresponding diagrams are the same as those of $B^- \to X(3823)K^-$, where we only need to make a replacement $X(3823) \to \eta_{c2}$ in the diagrams shown in Fig. 1. With the above preparation, the absorptive parts of the amplitudes in the process $B^- \to \eta_{c2}K^-$ can be obtained, for example, for the amplitude of diagram (1a) in Fig. 1 as

$\text{Abs}_{(1a)}^{B^- \to \eta_{c2} K^-}$

$$= \frac{|p_2|}{32\pi^2 m_1} \int d\Omega \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* a_1 \sqrt{m_1 m_2} \xi(\omega) \left(v_1^\gamma + \frac{p_2^\gamma}{m_2} \right) f_3 p_{3\gamma}$$

$$\times (-1) g_{\eta_{c2} D D^*} \epsilon_5^{*\mu\nu} (p_{2\nu} + p_{4\nu}) \left(-g_{\mu\alpha} + \frac{p_{4\mu} p_{4\alpha}}{m_4^2} \right) g_{D_s D^* K} p_6^\alpha$$

$$\times \frac{1}{p_4^2 - m_4^2} \mathfrak{F}^2(p_4^2), \tag{35}$$

where m_i ($i = 1, 2, 3, \ldots$) denotes the mass of the particle carrying momentum p_i in Fig. 1. The rest of the amplitudes are given in the Appendix.

The total absorptive part of the amplitude of the process $B^- \to \eta_{c2} K^-$ is

$$\text{Abs}(\mathcal{M}[B^- \to \eta_{c2} K^-]) = \sum_{i=1a,\ldots,3b} \text{Abs}_{(i)}^{B^- \to \eta_{c2} K^-}.$$

For $B^- \to \psi_3 K^-$, the relevant diagrams are shown in Fig. 2. The absorptive parts of the amplitudes of the process $B^- \to \psi_3 K^-$ can be obtained, for example, for the amplitude of diagram (1a) in Fig. 2 as

$\text{Abs}_{(1a)}^{B^- \to \psi_3 K^-}$

$$= \frac{|p_2|}{32\pi^2 m_1} \int d\Omega \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* a_1 \sqrt{m_1 m_2} \xi(\omega) \left(i\epsilon_{\gamma\delta\eta\beta} \frac{p_2^\eta}{m_2} v_1^\beta \right.$$

$$\left. - (1+\omega) g_{\delta\gamma} + v_{1\delta} \frac{p_{2\gamma}}{m_2} \right) f_3 p_3^\gamma \left(-g_\alpha^\delta + \frac{p_2^\delta p_{2\alpha}}{m_2^2} \right) i g_{\psi_3 D^* D^*}$$

$$\times \epsilon_5^{*\mu\nu\alpha} (p_{4\mu} + p_{2\mu}) \left(-g_{\nu\theta} + \frac{p_{4\nu} p_{4\theta}}{m_4^2} \right) g_{D^* D_s K} p_6^\theta \frac{1}{p_4^2 - m_4^2}$$

$$\times \mathfrak{F}^2(p_4^2). \tag{36}$$

Note that m_i ($i = 1, 2, 3, \ldots$) denotes the mass of the particle carrying momentum p_i in Fig. 2. The rest of the amplitudes are given in the Appendix.
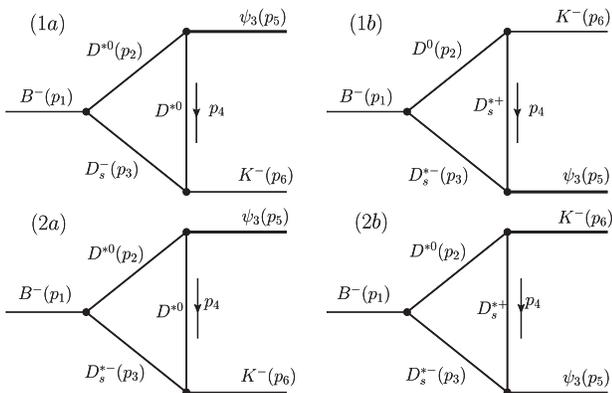

FIG. 2. The schematic diagrams for the decay $B^- \to \psi_3 K^-$ via the rescattering mechanism.

The total absorptive part of the amplitude of the process $B^- \to \psi_3 K^-$ is

$$\text{Abs}(\mathcal{M}[B^- \to \psi_3 K^-]) = \sum_{i=1a,\ldots,2b} \text{Abs}_{(i)}^{B^- \to \psi_3 K^-}.$$

Other input parameters are the masses of two unobserved charmonia η_{c2} and ψ_3, which are given by $m_{\eta_{c2}(^1D_2)} = 3.811$ GeV and $m_{\psi_3(^3D_3)} = 3.815$ GeV [21]. We vary ± 50 MeV to account for the uncertainties of these predicted masses. When taking $\alpha = 0.70 \pm 0.05$, the same value as that for $B^- \to X(3823)K^-$, we obtain the branching fractions for the processes $B^- \to \eta_{c2} K^-$ and $B^- \to \psi_3 K^-$,

$$\mathcal{BR}(B^- \to \eta_{c2}(^1D_2)K^-) = (1.72 \pm 0.47) \times 10^{-5}, \tag{37}$$

$$\mathcal{BR}(B^- \to \psi_3(^3D_3)K^-) = (0.80 \pm 0.21) \times 10^{-5}, \tag{38}$$

where the errors come from the uncertainties of α and the masses of η_{c2} and ψ_3. The results are sizable and are the same order of magnitude as $B^- \to X(3823)K^-$, which means that these two decay channels can be accessible in future experiments.

There are some remarks on our theoretical uncertainties. The uncertainties come from three parts, the lack of real parts of the amplitudes, the weak vertexes and the strong vertexes in loops. As for the real parts of the amplitudes, we assume they are not dominant as in Ref. [14]. As for the weak vertexes, there are actually much smaller uncertainties since either the naive factorization assumption for $B \to D^{(*)} \bar{D}_s^{(*)}$ [Eq. (4)] or the form factor of the matrix element [Eq. (28)] has been proven to have a good agreement with experiment. The dominant uncertainties come from strong vertexes: the coupling constants, the cutoff parameter α and the predicted masses of $\eta_{c2}(^1D_2)$ and $\psi_3(^3D_3)$. Since the coupling constants g_H in Eq. (10) and g_X in Eq. (17) appear in all the amplitudes as global factors, after fitting to the process $B^- \to X(3823)K^-$, the uncertainties caused by g_H and g_X are just canceled when calculating the processes $B^- \to \eta_{c2}(^1D_2)K^-$ and $B^- \to \psi_3(^3D_3)K^-$. The uncertainties from cutoff α and the predicted masses of $\eta_{c2}(^1D_2)$ and $\psi_3(^3D_3)$ have already been considered in the text. As a whole, we stress that these uncertainties do not influence our main conclusion.

IV. CONCLUSIONS AND DISCUSSION

The rescattering mechanism has been widely applied to the studies involved in hadronic transitions [4–11] and B decays [12–17]. As a long distant contribution, the rescattering mechanism is a typical nonperturbative QCD effect. Stimulated by the observation of $B^- \to X(3823)K^-$ [18], we study the contribution from the rescattering mechanism to $B^- \to X(3823)K^-$ since the naive factorization

contribution to $B^- \to X(3823)K^-$ vanishes. With a reasonable cutoff parameter, we can reproduce the experimental branching ratio of $B^- \to X(3823)K^-$. Under the same theoretical framework and with fitted parameters, we further investigate the processes $B^- \to \eta_{c2}(^1D_2)K^-$ and $B^- \to \psi_3(^3D_3)K^-$. Our results show $\mathcal{BR}(B^- \to \eta_{c2}(^1D_2)K^-) = 1.7 \times 10^{-5}$ and $\mathcal{BR}(B^- \to \psi_3(^3D_3)K^-) = 0.8 \times 10^{-5}$, which are comparable to $\mathcal{BR}(B^- \to X(3823)K^-) \simeq 2.1 \times 10^{-5}$ extracted from experimental data. Our study shows that nonfactorizable contributions to $B^- \to X(3823)K^-$, $B^- \to \eta_{c2}(^1D_2)K^-$ and $B^- \to \psi_3(^3D_3)K^-$ are sizable. Thus, experimental exploration of $B^- \to \eta_{c2}(^1D_2)K^-$ and $B^- \to \psi_3(^3D_3)K^-$ becomes possible at future experiments like LHCb, Belle, and the forthcoming BelleII. We also expect that our predictions of $B^- \to \eta_{c2}(^1D_2)K^-$ and $B^- \to \psi_3(^3D_3)K^-$ can be confirmed in experiments.

So far, two D-wave charmonia $\eta_{c2}(^1D_2)$ and $\psi_3(^3D_3)$ are still missing in experiments. When exploring $B^- \to \eta_{c2}(^1D_2)K^-$ and $B^- \to \psi_3(^3D_3)K^-$, a key point is how to identify $\eta_{c2}(^1D_2)$ and $\psi_3(^3D_3)$ experimentally, whose task is full of challenges faced by experimentalists, since the present study shows that $B^- \to \eta_{c2}(^1D_2)K^-$ and $B^- \to \psi_3(^3D_3)K^-$ have sizable branching ratios. These two processes are also ideal channels to search for $\eta_{c2}(^1D_2)$ and $\psi_3(^3D_3)$. If future experiments can find these predicted decays, it will not only make our knowledge of B meson decays become more abundant, but also be helpful in establishing the charmonium family.

In summary, experimental study of $B^- \to \eta_{c2}(^1D_2)K^-$ and $B^- \to \psi_3(^3D_3)K^-$ will be a potential issue in the near future. If these channels can be confirmed in experiments, the role of the rescattering mechanism in $B^- \to \eta_{c2}(^1D_2)K^-$ and $B^- \to \psi_3(^3D_3)K^-$ can be further identified, which will deepen our understanding of nonperturbative QCD behavior.

APPENDIX: THE REST OF THE AMPLITUDES OF THE PROCESSES $B^- \to X(3823)K^-$, $B^- \to \eta_{c2}K^-$ AND $B^- \to \psi_3K^-$

The amplitudes of the process $B^- \to X(3823)K^-$ depicted in diagrams (1b)–(3b) in Fig. 1 are

$$
\begin{aligned}
&\mathrm{Abs}_{(1b)}^{B^- \to \psi_2 K^-} \\
&= \frac{|p_2|}{32\pi^2 m_1} \int d\Omega \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* a_1 \sqrt{m_1 m_2} \xi(\omega) \left(v_1^\gamma + \frac{p_2^\gamma}{m_2} \right) f_3 p_{3\gamma} \\
&\quad \times (-1) g_{DD_s^*K} p_6^\alpha \left(-g_{\alpha\mu} + \frac{p_{4\alpha} p_{4\mu}}{m_4^2} \right) (-i) g_{\psi_2 D_s D_s^*} \epsilon_5^{*\mu\nu} \\
&\quad \times (p_{3\nu} - p_{4\nu}) \frac{1}{p_4^2 - m_4^2} \mathfrak{F}^2(p_4^2),
\end{aligned}
\tag{A1}
$$

$$
\begin{aligned}
&\mathrm{Abs}_{(2a)}^{B^- \to \psi_2 K^-} \\
&= \frac{|p_2|}{32\pi^2 m_1} \int d\Omega \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* a_1 \sqrt{m_1 m_2} \xi(\omega) \left(i\varepsilon_{\gamma\delta\alpha\beta} \frac{p_2^\alpha}{m_2} v_1^\beta \right. \\
&\quad \left. - (1+\omega) g_{\delta\gamma} + v_{1\delta} \frac{p_{2\gamma}}{m_2} \right) f_3 m_3 \left(-g_\mu^\delta + \frac{p_2^\delta p_{2\mu}}{m_2^2} \right) i g_{\psi_2 DD^*} \epsilon_5^{*\mu\nu} \\
&\quad \times (p_{4\nu} + p_{2\nu})(-g_{DD_s^*K}) p_{6\theta} \left(-g^{\theta\gamma} + \frac{p_3^\theta p_3^\gamma}{m_3^2} \right) \frac{1}{p_4^2 - m_4^2} \\
&\quad \times \mathfrak{F}^2(p_4^2),
\end{aligned}
\tag{A2}
$$

$$
\begin{aligned}
&\mathrm{Abs}_{(2b)}^{B^- \to \psi_2 K^-} \\
&= \frac{|p_2|}{32\pi^2 m_1} \int d\Omega \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* a_1 \sqrt{m_1 m_2} \xi(\omega) \left(i\varepsilon_{\gamma\delta\alpha\beta} \frac{p_2^\alpha}{m_2} v_1^\beta \right. \\
&\quad \left. - (1+\omega) g_{\delta\gamma} + v_{1\delta} \frac{p_{2\gamma}}{m_2} \right) f_3 m_3 \left(-g_\theta^\delta + \frac{p_2^\delta p_{2\theta}}{m_2^2} \right) g_{D_s D^* K} p_6^\theta (-i) \\
&\quad \times g_{\psi_2 D_s D_s^*} \epsilon_5^{*\mu\nu} (p_{4\nu} - p_{3\nu}) \left(-g_\mu^\gamma + \frac{p_{3\mu} p_3^\gamma}{m_3^2} \right) \frac{1}{p_4^2 - m_4^2} \\
&\quad \times \mathfrak{F}^2(p_4^2),
\end{aligned}
\tag{A3}
$$

$$
\begin{aligned}
&\mathrm{Abs}_{(3a)}^{B^- \to \psi_2 K^-} \\
&= \frac{|p_2|}{32\pi^2 m_1} \int d\Omega \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* a_1 \sqrt{m_1 m_2} \xi(\omega) \left(i\varepsilon_{\gamma\delta\zeta\eta} \frac{p_2^\zeta}{m_2} v_1^\eta \right. \\
&\quad \left. - (1+\omega) g_{\delta\gamma} + v_{1\delta} \frac{p_{2\gamma}}{m_2} \right) f_3 m_3 (-i) g_{\psi_2 D^* D^*} \left[\left(-g_\nu^\delta + \frac{p_2^\delta p_{2\nu}}{m_2^2} \right) \right. \\
&\quad \times \varepsilon_{\mu\rho\alpha\beta} \epsilon_5^{*\mu\nu} p_4^\alpha p_5^\beta \left(-g^{\rho\sigma} + \frac{p_4^\rho p_4^\sigma}{m_4^2} \right) + \left(-g^{\delta\rho} + \frac{p_2^\delta p_2^\rho}{m_2^2} \right) \varepsilon_{\mu\rho\alpha\beta} \\
&\quad \left. \times \epsilon_5^{*\mu\nu} p_4^\alpha p_5^\beta \left(-g_\nu^\sigma + \frac{p_{4\nu} p_4^\sigma}{m_4^2} \right) \right] g_{D^* D_s^* K} \varepsilon_{\tau\sigma\kappa\lambda} p_4^\kappa p_6^\lambda \\
&\quad \times \left(-g^{\tau\gamma} + \frac{p_3^\tau p_3^\gamma}{m_3^2} \right) \frac{1}{p_4^2 - m_4^2} \mathfrak{F}^2(p_4^2),
\end{aligned}
\tag{A4}
$$

$\text{Abs}^{B^-\to\psi_2 K^-}_{(3b)}$

$$=\frac{|p_2|}{32\pi^2 m_1}\int d\Omega \frac{G_F}{\sqrt{2}}V_{cb}V^*_{cs}a_1\sqrt{m_1 m_2}\xi(\omega)\left(i\varepsilon_{\gamma\delta\zeta\eta}\frac{p_2^\zeta}{m_2}v_1^\eta\right.$$

$$-(1+\omega)g_{\delta\gamma}+v_{1\delta}\frac{p_{2\gamma}}{m_2}\bigg)f_3 m_3\left(-g^{\delta\theta}+\frac{p_2^\delta p_2^\theta}{m_2^2}\right)g_{D^* D^*_s K}\varepsilon_{\sigma\theta\tau\kappa}$$

$$\times p_2^\tau p_6^\kappa i g_{\psi_2 D^*_s D^*_s}\left[\left(-g_\nu^\sigma+\frac{p_4^\sigma p_{4\nu}}{m_4^2}\right)\varepsilon_{\mu\rho\alpha\beta}\epsilon_5^{*\mu\nu}p_3^\alpha p_5^\beta\right.$$

$$\times\left(-g^{\rho\gamma}+\frac{p_3^\rho p_3^\gamma}{m_3^2}\right)+\left(-g^{\sigma\rho}+\frac{p_4^\sigma p_4^\rho}{m_4^2}\right)\varepsilon_{\mu\rho\alpha\beta}\epsilon_5^{*\mu\nu}p_3^\alpha p_5^\beta$$

$$\times\left.\left(-g^\gamma_\nu+\frac{p_{3\nu}p_3^\gamma}{m_3^2}\right)\right]\frac{1}{p_4^2-m_4^2}\mathfrak{F}^2(p_4^2). \qquad (A5)$$

The amplitudes of the process $B^-\to\eta_{c2}K^-$ depicted in diagrams (1b)–(3b) in Fig. 1 are

$\text{Abs}^{B^-\to\eta_{c2}K^-}_{(1b)}$

$$=\frac{|p_2|}{32\pi^2 m_1}\int d\Omega \frac{G_F}{\sqrt{2}}V_{cb}V^*_{cs}a_1\sqrt{m_1 m_2}\xi(\omega)\left(v_1^\gamma+\frac{p_2^\gamma}{m_2}\right)f_3 p_{3\gamma}$$

$$\times(-1)g_{DD^*_s K}p_6^\alpha\left(-g_{\alpha\mu}+\frac{p_{4\alpha}p_{4\mu}}{m_4^2}\right)g_{\eta_{c2}D_s D^*_s}\epsilon_5^{*\mu\nu}(p_{3\nu}-p_{4\nu})$$

$$\times\frac{1}{p_4^2-m_4^2}\mathfrak{F}^2(p_4^2), \qquad (A6)$$

$\text{Abs}^{B^-\to\eta_{c2}K^-}_{(2a)}$

$$=\frac{|p_2|}{32\pi^2 m_1}\int d\Omega \frac{G_F}{\sqrt{2}}V_{cb}V^*_{cs}a_1\sqrt{m_1 m_2}\xi(\omega)\left(i\varepsilon_{\gamma\delta\alpha\beta}\frac{p_2^\alpha}{m_2}v_1^\beta\right.$$

$$-(1+\omega)g_{\delta\gamma}+v_{1\delta}\frac{p_{2\gamma}}{m_2}\bigg)f_3 m_3\left(-g^\delta_\mu+\frac{p_2^\delta p_{2\mu}}{m_2^2}\right)(-1)g_{\eta_{c2}DD^*}$$

$$\times\epsilon_5^{*\mu\nu}(p_{4\nu}+p_{2\nu})(-g_{DD^*_s K})p_{6\theta}\left(-g^{\theta\gamma}+\frac{p_3^\theta p_3^\gamma}{m_3^2}\right)\frac{1}{p_4^2-m_4^2}$$

$$\times\mathfrak{F}^2(p_4^2), \qquad (A7)$$

$\text{Abs}^{B^-\to\eta_{c2}K^-}_{(2b)}$

$$=\frac{|p_2|}{32\pi^2 m_1}\int d\Omega \frac{G_F}{\sqrt{2}}V_{cb}V^*_{cs}a_1\sqrt{m_1 m_2}\xi(\omega)\left(i\varepsilon_{\gamma\delta\alpha\beta}\frac{p_2^\alpha}{m_2}v_1^\beta\right.$$

$$-(1+\omega)g_{\delta\gamma}+v_{1\delta}\frac{p_{2\gamma}}{m_2}\bigg)f_3 m_3\left(-g^\delta_\theta+\frac{p_2^\delta p_{2\theta}}{m_2^2}\right)g_{D_s D^* K}p_6^\theta(-1)$$

$$\times g_{\eta_{c2}D_s D^*_s}\epsilon_5^{*\mu\nu}(p_{4\nu}-p_{3\nu})\left(-g^\gamma_\mu+\frac{p_{3\mu}p_3^\gamma}{m_3^2}\right)\frac{1}{p_4^2-m_4^2}$$

$$\times\mathfrak{F}^2(p_4^2), \qquad (A8)$$

$\text{Abs}^{B^-\to\eta_{c2}K^-}_{(3a)}$

$$=\frac{|p_2|}{32\pi^2 m_1}\int d\Omega \frac{G_F}{\sqrt{2}}V_{cb}V^*_{cs}a_1\sqrt{m_1 m_2}\xi(\omega)\left(i\varepsilon_{\gamma\delta\zeta\eta}\frac{p_2^\zeta}{m_2}v_1^\eta\right.$$

$$-(1+\omega)g_{\delta\gamma}+v_{1\delta}\frac{p_{2\gamma}}{m_2}\bigg)f_3 m_3\left(-g^{\delta\alpha}+\frac{p_2^\delta p_2^\alpha}{m_2^2}\right)(-1)g_{\eta_{c2}D^* D^*}$$

$$\times\varepsilon_{\rho\nu\alpha\beta}p_5^\rho\epsilon_5^{*\mu\nu}p_{4\mu}\left(-g^{\beta\sigma}+\frac{p_4^\beta p_4^\sigma}{m_4^2}\right)g_{D^* D^*_s K}\varepsilon_{\tau\sigma\kappa\lambda}p_4^\kappa p_6^\lambda$$

$$\times\left(-g^{\tau\gamma}+\frac{p_3^\tau p_3^\gamma}{m_3^2}\right)\frac{1}{p_4^2-m_4^2}\mathfrak{F}^2(p_4^2), \qquad (A9)$$

$\text{Abs}^{B^-\to\eta_{c2}K^-}_{(3b)}$

$$=\frac{|p_2|}{32\pi^2 m_1}\int d\Omega \frac{G_F}{\sqrt{2}}V_{cb}V^*_{cs}a_1\sqrt{m_1 m_2}\xi(\omega)\left(i\varepsilon_{\gamma\delta\zeta\eta}\frac{p_2^\zeta}{m_2}v_1^\eta\right.$$

$$-(1+\omega)g_{\delta\gamma}+v_{1\delta}\frac{p_{2\gamma}}{m_2}\bigg)f_3 m_3\left(-g^{\delta\theta}+\frac{p_2^\delta p_2^\theta}{m_2^2}\right)g_{D^* D^*_s K}\varepsilon_{\sigma\theta\tau\kappa}$$

$$\times p_2^\tau p_6^\kappa\left(-g^{\sigma\alpha}+\frac{p_4^\sigma p_4^\alpha}{m_4^2}\right)g_{\eta_{c2}D^*_s D^*_s}\varepsilon_{\rho\nu\alpha\beta}p_5^\rho\epsilon_5^{*\mu\nu}p_{3\mu}$$

$$\times\left(-g^{\beta\gamma}+\frac{p_3^\beta p_3^\gamma}{m_3^2}\right)\frac{1}{p_4^2-m_4^2}\mathfrak{F}^2(p_4^2). \qquad (A10)$$

The amplitudes of the process $B^-\to\psi_3 K^-$ depicted in diagrams (1b)–(2b) in Fig. 2 are

$\text{Abs}^{B^-\to\psi_3 K^-}_{(1b)}$

$$=\frac{|p_2|}{32\pi^2 m_1}\int d\Omega \frac{G_F}{\sqrt{2}}V_{cb}V^*_{cs}a_1\sqrt{m_1 m_2}\xi(\omega)\left(\frac{p_2^\beta}{m_2}+v_1^\beta\right)f_3 m_3$$

$$(-1)g_{DD^*_s K}p_6^\theta\left(-g_{\theta\alpha}+\frac{p_{4\theta}p_{4\alpha}}{m_4^2}\right)ig_{\psi_3 D^*_s D^*_s}\epsilon_5^{*\mu\nu\alpha}(p_{4\mu}-p_{3\mu})$$

$$\times\left(-g_{\nu\beta}+\frac{p_{3\nu}p_{3\beta}}{m_3^2}\right)\frac{1}{p_4^2-m_4^2}\mathfrak{F}^2(p_4^2), \qquad (A11)$$

$\text{Abs}^{B^-\to\psi_3 K^-}_{(2a)}$

$$=\frac{|p_2|}{32\pi^2 m_1}\int d\Omega \frac{G_F}{\sqrt{2}}V_{cb}V^*_{cs}a_1\sqrt{m_1 m_2}\xi(\omega)\left(i\varepsilon_{\gamma\delta\zeta\eta}\frac{p_2^\zeta}{m_2}v_1^\eta\right.$$

$$-(1+\omega)g_{\delta\gamma}+v_{1\delta}\frac{p_{2\gamma}}{m_2}\bigg)f_3 m_3\left(-g^\delta_\alpha+\frac{p_2^\delta p_{2\alpha}}{m_2^2}\right)ig_{\psi_3 D^* D^*}\epsilon_5^{*\mu\nu\alpha}$$

$$\times(p_{4\mu}+p_{2\mu})\left(-g^\tau_\nu+\frac{p_{4\nu}p_4^\tau}{m_4^2}\right)g_{D^* D^*_s K}\varepsilon_{\sigma\tau\rho\beta}p_4^\rho p_6^\beta$$

$$\times\left(-g^{\sigma\gamma}+\frac{p_3^\sigma p_3^\gamma}{m_3^2}\right)\frac{1}{p_4^2-m_4^2}\mathfrak{F}^2(p_4^2), \qquad (A12)$$

$$\text{Abs}_{(2b)}^{B^- \to \psi_3 K^-}$$

$$= \frac{|p_2|}{32\pi^2 m_1} \int d\Omega \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* a_1 \sqrt{m_1 m_2} \xi(\omega) \left(i\varepsilon_{\gamma\delta\zeta\eta} \frac{p_2^\zeta}{m_2} v_1^\eta \right.$$

$$\left. - (1+\omega)g_{\delta\gamma} + v_{1\delta}\frac{p_{2\gamma}}{m_2} \right) f_3 m_3 \left(-g^{\delta\tau} + \frac{p_2^\delta p_2^\tau}{m_2^2} \right) g_{D^* D_s^* K} \varepsilon_{\sigma\tau\rho\beta}$$

$$\times p_2^\rho p_6^\beta \left(-g_\alpha^\sigma + \frac{p_4^\sigma p_{4\alpha}}{m_4^2} \right) i g_{\psi_3 D_s^* D_s^*} \epsilon_5^{*\mu\nu\alpha}(p_{4\mu} - p_{3\mu})$$

$$\times \left(-g_\nu^\gamma + \frac{p_{3\nu} p_3^\gamma}{m_3^2} \right) \frac{1}{p_4^2 - m_4^2} \mathfrak{F}^2(p_4^2). \tag{A13}$$

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