PHYSICAL REVIEW D **94,** 034003 (2016)

Tensor polarization dependent fragmentation functions and $e^+e^- \to V\pi X$ at high energies

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(Received 28 May 2016; published 3 August 2016)

We present the systematic results for three-dimensional fragmentation functions of spin-1 hadrons defined via the quark-quark correlator. There are totally 72 such fragmentation functions, among them 18 are twist-2, 36 are twist-3 and 18 are twist-4. We also present the relationships between the twist-3 parts and those defined via the quark-gluon-quark correlator obtained from the QCD equation of motion. We show that the two particle semi-inclusive hadron production process $e^+e^- \to V\pi X$ at high energies is one of the best places to study the three-dimensional tensor polarization dependent fragmentation functions. We present the general kinematic analysis of this process and show that the cross section should be expressed in terms of 81 independent structure functions. After that we present parton model results for the hadronic tensor, the structure functions, and the azimuthal and spin asymmetries in terms of these gauge invariant fragmentation functions at the leading order perturbative quantum chromodynamics up to twist-3.

DOI: 10.1103/PhysRevD.94.034003

I. INTRODUCTION

In describing high energy reactions, we need two sets of important quantities, the parton distribution functions (PDFs) and the fragmentation functions (FFs). The former are used to describe the hadron structure and the latter describe the hadronization process. In a quantum field theoretical formulation, both PDFs and FFs are defined via the corresponding quark-quark correlator. The quark-quark correlator is defined as a matrix in the Dirac space depending on the hadron states. It is then decomposed into different components expressed in terms of the basic Lorentz covariants and the scalar functions. These scalar functions contain the information of the hadron structure and/or hadronization mechanism and are called the corresponding PDFs or FFs. In many cases in the literature, specific PDFs and/or FFs are introduced whenever needed, sometimes with different conventions and/or notations. With the development of the related studies, it is necessary and useful to make a systematic study and present a complete set of such results. The results for three-dimensional PDFs of the nucleon defined in this way are presented in [1] in a systematical way. Since usually different types of hadrons with different flavors and spins are produced in a high energy reaction, FFs are therefore more involved and perhaps even more interesting but less studied yet. Specific recent discussions can also be found e.g. in [2–17]. A short summary can be found in a recent unpublished note and short reviews [18–20].

In this paper, we summarize the results for three-dimensional FFs defined via the quark-quark correlator for spin-1 hadrons in a systematical way. The FFs are divided into a spin-independent part, a vector polarization dependent part, and a tensor polarization dependent part. Formerly, the spin-independent part is the same as those for spin-0 hadrons and the vector polarization dependent part is the same as those for spin-$1/2$ hadrons. They are also similar to those for PDFs presented e.g. in [1] for the corresponding cases. We will pay special attention to the tensor polarization dependent part including higher twist contributions. In this connection, we will in particular show also FFs defined via the quark-gluon-quark correlator and their relationships to those defined via the quark-quark correlator obtained using the quantum chromodynamics (QCD) equation of motion.

The most convenient place to study the three-dimensional FFs of vector mesons is perhaps $e^+e^- \to V\pi X$. We present the results for the general kinematical analysis of this process and calculate the hadronic tensor and differential cross section up to twist-3 at leading order in perturbative (pQCD). We also present the results for the tensor polarizations of V in terms of the three-dimensional FFs.

The rest of the paper is organized as follows. After this introduction, we briefly summarize the general procedure of deriving the results of FFs from the quark-quark correlator and present results and relationships to those defined via the quark-gluon-quark correlator at twist-3 in Sec. II. We make a general kinematical analysis of $e^+e^- \to V\pi X$ in Sec. III. We calculate the hadronic tensor at leading order perturbative QCD up to twist-3 in Sec. IV. We present the results for the structure functions in Sec. V and those for azimuthal and spin asymmetries in Sec. VI. We make a summary and a discussion in Sec. VII. Since most of the equations are rather long, we will present the discussions in the corresponding sections but show most of the formulas and tables in the appendixes.

II. FRAGMENTATION FUNCTIONS DEFINED VIA THE QUARK-QUARK CORRELATOR

A systematic analysis is given in a recent unpublished note [18]. For completeness, we briefly summarize the basic ideas in this section and summarize the results in Appendix A. Similar to parton distribution and/or correlation functions, in quantum field theory, the quark fragmentation is defined via the quark-quark correlator given by

$$\hat{\Xi}^{(0)}_{ij}(k_F;p,S) = \frac{1}{2\pi}\sum_X \int d^4\xi e^{-ik_F\xi} \times \langle 0|\mathcal{L}^\dagger(0;\infty)\psi_i(0)|p,S;X\rangle \times \langle p,S;X|\bar{\psi}_j(\xi)\mathcal{L}(\xi;\infty)|0\rangle, \tag{2.1}$$

where k_F and p denote the 4-momenta of the quark and the hadron respectively, S denotes the spin of the hadron; $\mathcal{L}(\xi;\infty)$ is the gauge link that is given by

$$\begin{aligned}\mathcal{L}(\xi,\infty) &= Pe^{ig\int_{\xi^-}^{\infty} d\eta^- A^+(\eta^-;\xi^+,\vec{\xi}_\perp)} \\ &= 1 + ig\int_{\xi^-}^{\infty} d\eta^- A^+(\eta^-;\xi^+,\vec{\xi}_\perp) \\ &\quad + (ig)^2\int_{\xi^-}^{\infty} d\eta_1^- \int_{\xi^-}^{\eta_1^-} d\eta_2^- A^+(\eta_2^-;\xi^+,\vec{\xi}_\perp) \\ &\quad \times A^+(\eta_1^-;\xi^+,\vec{\xi}_\perp) + \cdots. \end{aligned}\tag{2.2}$$

The correlator given by Eq. (2.1) satisfies the following constraints imposed by Hermiticity and parity conservation, i.e.,

$$\hat{\Xi}^{\dagger(0)}(k_F;p,S) = \gamma^0\hat{\Xi}^{(0)}(k_F;p,S)\gamma^0, \tag{2.3}$$

$$\hat{\Xi}^{(0)}(k_F;p,S) = \gamma^0\hat{\Xi}^{(0)}(k_F^{\mathcal{P}};p^{\mathcal{P}},S^{\mathcal{P}})\gamma^0, \tag{2.4}$$

where a vector with the superscript $\mathcal{P}$ denotes the result after space reflection such as $p_\mu^{\mathcal{P}} = p^\mu$. Unlike that for the hadron structure, because of the presence of the gauge link and final state interactions between h and X, time-reversal invariance puts no such simple constraint on the correlator $\hat{\Xi}^{(0)}(k_F;p,S)$.

The three-dimensional or the transverse momentum dependent (TMD) FFs are defined via the three-dimensional quark-quark correlator $\hat{\Xi}^{(0)}(z,k_{F\perp};p,S)$ obtained from $\hat{\Xi}^{(0)}(k_F,p,S)$ by integrating over k_F^-, i.e.,

$$\hat{\Xi}^{(0)}(z,k_{F\perp};p,S) = \sum_X\int\frac{p^+d\xi^-}{2\pi}d^2\xi_\perp e^{-i(p^+\xi^-/z-\vec{k}_{F\perp}\cdot\vec{\xi}_\perp)} \times\langle 0|\mathcal{L}^\dagger(0;\infty)\psi(0)|p,S;X\rangle \times\langle p,S;X|\bar{\psi}(\xi)\mathcal{L}(\xi;\infty)|0\rangle, \tag{2.5}$$

where $z = p^+/k_F^+$ is the longitudinal momentum fraction defined in light-cone coordinates. Here we use the light-cone coordinate and define the light-cone unit vectors as $\bar{n} = (1,0,\vec{0}_\perp)$, $n = (0,1,\vec{0}_\perp)$, and $n_\perp = (0,0,\vec{1}_\perp)$. We choose the hadron's momentum as the z direction so that p is decomposed as $p^\mu = p^+\bar{n}^\mu + (M^2/2p^+)n^\mu$.

The FFs are obtained from $\hat{\Xi}^{(0)}(z,k_{F\perp};p,S)$ by decomposing it in the following two steps. First, we note that $\hat{\Xi}^{(0)}(z,k_{F\perp};p,S)$ is a matrix in Dirac space and expand it in terms of the Γ matrices, $\Gamma = \{\mathbf{I}, i\gamma_5, \gamma^\alpha, \gamma_5\gamma^\alpha, i\sigma^{\alpha\beta}\gamma_5\}$, i.e.,

$$\begin{aligned}\hat{\Xi}^{(0)}&(z,k_{F\perp};p,S) \\ &= \Xi^{(0)}(z,k_{F\perp};p,S) + i\gamma_5\tilde{\Xi}^{(0)}(z,k_{F\perp};p,S) \\ &\quad + \gamma^\alpha\Xi^{(0)}_\alpha(z,k_{F\perp};p,S) + \gamma_5\gamma^\alpha\tilde{\Xi}^{(0)}_\alpha(z,k_{F\perp};p,S) \\ &\quad + i\sigma^{\alpha\beta}\gamma_5\Xi^{(0)}_{\alpha\beta}(z,k_{F\perp};p,S).\end{aligned}\tag{2.6}$$

The coefficient functions are given by

$$\begin{aligned}\Xi^{(0)[\Gamma]}&(z,k_{F\perp};p,S) \\ &= \frac{1}{4}\sum_X\int\frac{p^+d\xi^-}{2\pi}d^2\xi_\perp e^{-i(p^+\xi^-/z-\vec{k}_{F\perp}\cdot\vec{\xi}_\perp)} \\ &\quad \times\langle p,S;X|\bar{\psi}(\xi)\mathcal{L}(\xi;\infty)|0\rangle \\ &\quad \times\Gamma\langle 0|\mathcal{L}^\dagger(0;\infty)\psi(0)|p,S;X\rangle,\end{aligned}\tag{2.7}$$

where $\Xi^{(0)[\Gamma]}$ represents respectively $\Xi^{(0)}$, $\tilde{\Xi}^{(0)}$, $\Xi^{(0)}_\alpha$, $\tilde{\Xi}^{(0)}_\alpha$, and $\Xi^{(0)}_{\alpha\beta}$ for different Γ's. Together with the demands imposed by the Hermiticity and parity invariance [Eqs. (2.3) and (2.4)], the Lorentz invariance demands that all the corresponding coefficient functions are real and are Lorentz scalar, pseudoscalar, vector, axial-vector, and tensor respectively. Furthermore, the tensor $\Xi^{(0)}_{\alpha\beta}$ is antisymmetric in Lorentz indices and odd under space reflection which implies that it can be made out of a vector and an axial vector.

Second, we expand these coefficient functions according to their respective Lorentz transformation properties in terms of the basic Lorentz covariants constructed from the basic variables at hand. They are expressed as the sum of the basic Lorentz covariants multiplied by scalar functions of z and $k_{F\perp}^2$. These scalar functions are the three-dimensional FFs. We note in particular that because of the Hermiticity given by Eq. (2.3), these FFs defined via the quark-quark correlator are real.

Clearly, the basic Lorentz covariants that we can construct depend on what basic variable(s) we have at hand. Besides the 4-momenta p and k_F, we have the variables describing the spin states. Such variables are different for hadrons with different spins. For spin-1 hadrons, the polarization is described by a 3×3 density matrix ρ, which, in the rest frame of the hadron, is usually decomposed as [21]

$$\rho = \frac{1}{3}(\mathbf{1} + \frac{3}{2}S^i\Sigma^i + 3T^{ij}\Sigma^{ij}), \tag{2.8}$$

where, Σ^i is the spin operator of the spin-1 particle, and $\Sigma^{ij} = \frac{1}{2}(\Sigma^i\Sigma^j + \Sigma^j\Sigma^i) - \frac{2}{3}\mathbf{1}\delta^{ij}$. The spin polarization tensor is $T^{ij} = \mathrm{Tr}(\rho\Sigma^{ij})$, and is parametrized as

$$\mathbf{T} = \frac{1}{2}\begin{pmatrix} -\frac{2}{3}S_{LL} + S_{TT}^{xx} & S_{TT}^{xy} & S_{LT}^{x} \\ S_{TT}^{xy} & -\frac{2}{3}S_{LL} - S_{TT}^{xx} & S_{LT}^{y} \\ S_{LT}^{x} & S_{LT}^{y} & \frac{4}{3}S_{LL} \end{pmatrix}. \tag{2.9}$$

Here, besides the polarization vector S, we also need a polarization tensor T. The polarization vector S is similar to that for spin-1/2 hadrons and the tensor T has five independent components that are given by a Lorentz scalar S_{LL}, a Lorentz vector $S_{LT}^{\mu} = (0, S_{LT}^x, S_{LT}^y, 0)$, and a Lorentz tensor $S_{TT}^{\mu\nu}$ that has two nonzero independent components $S_{TT}^{xx} = -S_{TT}^{yy}$ and $S_{TT}^{xy} = S_{TT}^{yx}$. In a covariant form, the polarization vector S is decomposed as

$$S^{\mu} = \lambda\frac{p^+}{M}\bar{n}^{\mu} + S_T^{\mu} - \lambda\frac{M}{2p^+}n^{\mu}, \tag{2.10}$$

where λ denotes the helicity and $S_T = (0, 0, \vec{S}_T)$ denotes the transverse polarization. The tensor polarization $T^{\mu\nu}$ is expressed as [21]

$$\begin{aligned} T^{\mu\nu} = \frac{1}{2}\Bigg[&\frac{4}{3}S_{LL}\left(\frac{p^+}{M}\right)^2\bar{n}^{\mu}\bar{n}^{\nu} + \frac{p^+}{M}n^{\{\mu}S_{LT}^{\nu\}} \\ &-\frac{2}{3}S_{LL}(\bar{n}^{\{\mu}n^{\nu\}} - g_{\perp}^{\mu\nu}) + S_{TT}^{\mu\nu} - \frac{M}{2p^+}\bar{n}^{\{\mu}S_{LT}^{\nu\}} \\ &+\frac{1}{3}S_{LL}\left(\frac{M}{p^+}\right)^2 n^{\mu}n^{\nu}\Bigg], \end{aligned} \tag{2.11}$$

where we used the anticommutation symbol $A^{\{\mu}B^{\nu\}} \equiv A^{\mu}B^{\nu} + A^{\nu}B^{\mu}$, and also in the following of this paper $A^{[\mu}B^{\nu]} \equiv A^{\mu}B^{\nu} - A^{\nu}B^{\mu}$ and $g_{\perp}^{\mu\nu} \equiv g^{\mu\nu} - \bar{n}^{\mu}n^{\nu} - n^{\mu}\bar{n}^{\nu}$.

Hence, for spin-1 hadrons, the quark-quark correlator $\hat{\Xi}^{(0)}$ can be written as the sum of a polarization independent part $\hat{\Xi}^{U(0)}$, a vector polarization dependent part $\hat{\Xi}^{V(0)}$, and a tensor polarization dependent part $\hat{\Xi}^{T(0)}$, i.e.,

$$\begin{aligned} \hat{\Xi}^{(0)}(z, k_{F\perp}; p, S) = \hat{\Xi}^{U(0)}(z, k_{F\perp}; p) + \hat{\Xi}^{V(0)}(z, k_{F\perp}; p, S) \\ + \hat{\Xi}^{T(0)}(z, k_{F\perp}; p, S). \end{aligned} \tag{2.12}$$

Since the polarization dependence is linear to the corresponding spin parameters, formally, the spin-independent part is exactly the same as that for spin-0 hadrons, and the vector polarization dependent part is the same as that for spin-1/2 hadrons. The tensor polarization dependent part is new and contributes only for spin-1 hadron production. We summarize them separately in the following.

Before we present the results, we describe the notation system for the FFs used throughout the paper. We will use D, G, and H for unpolarized, longitudinally polarized, and transversely polarized quarks. They correspond to those FFs obtained via decompositions of the vector, axial-vector, and tensor part of the correlator. Those defined via the scalar and the pseudoscalar are denoted by E. A number j in the subscripts specifies the twist: $j = 1$ for twist-2, null (no number) for twist-3, and $j = 3$ for twist-4. We will also use different symbols in the subscripts to denote the polarization of the produced hadron such as L and T in the vector polarization case and LL, LT, or TT in the tensor polarization case; a $\perp$ in the superscript denotes that the corresponding basic Lorentz covariant is $k_{F\perp}$ dependent.

If we decompose the quark field in Eq. (2.7) into the sum of the right- and left-handed parts, i.e., $\psi = \psi_R + \psi_L$ with $\psi_{R/L} \equiv \frac{1}{2}(1 \pm \gamma_5)\psi$, we see that for $\Gamma = \mathbf{I}$, $i\gamma_5$ and $i\sigma^{\alpha\beta}\gamma_5$, $\bar{\psi}_R\Gamma\psi_L$ and $\bar{\psi}_L\Gamma\psi_R$ are nonzero. So the terms related to them (i.e., the E and H terms) correspond to the helicity-flipped quark structure and are called chiral odd (χ odd). Similarly, for $\Gamma = \gamma^{\alpha}$ and $\gamma^5\gamma^{\alpha}$, $\bar{\psi}_L\Gamma\psi_L$ and $\bar{\psi}_R\Gamma\psi_R$ are nonzero. Hence, the terms related to them (i.e., the D's and the G's) do not flip the quark helicity and are χ even. We also recall the properties of the fermion bilinears under time reversal $\hat{\mathcal{T}}$, i.e.,

$$\begin{aligned} &\hat{\mathcal{T}}\{\bar{\psi}\psi, \bar{\psi}i\gamma_5\psi, \bar{\psi}\gamma_{\alpha}\psi, \bar{\psi}\gamma_5\gamma_{\alpha}\psi, \bar{\psi}i\sigma_{\alpha\beta}\gamma_5\psi\} \\ &\quad\Rightarrow \{\bar{\psi}\psi, -\bar{\psi}i\gamma_5\psi, \bar{\psi}\gamma^{\alpha}\psi, \bar{\psi}\gamma_5\gamma^{\alpha}\psi, \bar{\psi}i\sigma^{\alpha\beta}\gamma_5\psi\}. \end{aligned} \tag{2.13}$$

Using this, we can determine whether a component of FF defined via the quark-quark correlator is time-reversal even (T even) or odd (T odd) according to the time-reversal behavior of the corresponding basic Lorentz covariant. However, we should also note that they are usually referred as "naive T odd" or "naive T even" because the interactions between the produced hadron h and the rest X can destroy simple regularities so all of them can exist in a practical hadronization process.

A. Results of the decomposition and FFs

1. The unpolarized part

For the spin-independent part $\hat{\Xi}^{U(0)}(z, k_{F\perp}; p)$, the independent variables that can be used to construct the basic Lorentz covariants are p_{α}, $k_{F\perp\alpha}$, and n_{α}. The basic Lorentz covariants that we can construct from them are one Lorentz scalar $p^2 = M^2$, no pseudoscalar, three Lorentz vectors, p, $k_{F\perp}$, and n, one axial vector $\varepsilon_{\perp\rho\alpha}k_{F\perp}^{\rho} \equiv \tilde{k}_{F\alpha}$, and three antisymmetric and space reflection odd Lorentz tensors $p_{[\rho}\tilde{k}_{F\perp\alpha]}$, $\varepsilon_{\perp\rho\alpha}$, and $n_{[\rho}\tilde{k}_{F\perp\alpha]}$. Here

$\varepsilon_{\perp\rho\alpha} = \varepsilon_{\mu\nu\rho\alpha}\bar{n}^\mu n^\nu$ and $\varepsilon_{\mu\nu\rho\alpha}$ is the antisymmetric tensor. We also use the notation $\tilde{a}_{\perp\mu} \equiv \varepsilon_{\perp\rho\mu}a^\rho$ to denote the transverse vector perpendicular to $a_\perp$, and note in particular that $\tilde{a}_\perp \cdot b_\perp = \varepsilon_{\perp\rho\sigma}a^\rho b^\sigma = -a_\perp \cdot \tilde{b}_\perp$ and $\tilde{\tilde{a}}_\perp = -a_\perp$. The general decomposition of the spin-independent part of the quark-quark correlator is given by Eqs. (A1)–(A5) in Appendix A. We obtain eight unpolarized TMD FFs, two of them contribute at twist-2, four at twist-3, and the other two at the twist-4 level.

From Eqs. (A1)–(A5), we see in particular the existence of a leading twist FF $H_1^\perp(z, k_{F\perp})$ that leads to azimuthal asymmetry of the produced hadron in fragmentation of a transversely polarized quark. This was first introduced in [4] and is now known as the Collins function. We see also a twist-4 addendum to it described by $H_3^\perp(z, k_{F\perp})$.

If we integrate over $d^2k_{F\perp}$, we obtain the one-dimensional results as given by Eqs. (A6)–(A8) in Appendix A. We see that there are only four left and the number density $D_1(z)$ is the only leading twist, two of them contribute at twist-3 and the other one at twist-4.

We note in particular the direct one-to-one correspondence between the results obtained in this case for FFs and those obtained in [1] for PDFs. The only obvious difference is the existence of the naive time-reversal odd $H(z)$ due to final interaction between h and X while the corresponding term vanishes for PDFs.

2. The vector polarization dependent part

For the vector polarization dependent part, we have, besides p_α, $k_{F\perp\alpha}$, and n_α, the polarization vector S to use to construct the basic Lorentz covariants. The results obtained are given by Eqs. (A10)–(A14) in Appendix A. We see that there are 24 vector polarization dependent TMD FFs, six of them contribute at twist-2, 12 at twist-3, and the other six at twist-4 level. Among them, eight are naive T odd ($E_T^\perp$, E_L, $E_T'^\perp$, $D_{1T}^\perp$, $D_L^\perp$, D_T, $D_T^\perp$, and $D_{3T}^\perp$), and the other 16 are T even.

We also note that four of them ($E_L, G_{1L}, G_L^\perp, G_{3L}$) are for longitudinal (to longitudinal) spin transfer; six of them (H_{1T}, $H_{1T}^\perp$, $H_T^\perp$, $H_T'^\perp$, H_{3T}, $H_{3T}^\perp$) are for transverse (to transverse) spin transfer; five of them ($E_T'^\perp$, $G_{1T}^\perp$, G_T, $G_T^\perp$, G_{3T}) are for longitudinal to transverse spin transfer; three of them ($H_{1L}^\perp$, H_L, $H_{3L}^\perp$) are for transverse to longitudinal spin transfer; the other six ($E_T^\perp$, $D_{1T}^\perp$, D_T, $D_L^\perp$, $D_T^\perp$, $D_{3T}^\perp$) are for induced polarizations which leads to hadron polarizations in fragmentation of an unpolarized quark. At leading twist, we have a $D_{1T}^\perp$ for induced polarization, a longitudinal (G_{1L}), two transverse (H_{1T}, $H_{1T}^\perp$), a longitudinal to transverse ($G_{1T}^\perp$), and a transverse to longitudinal ($H_{1L}^\perp$) spin transfer.

We note in particular the induced polarization terms described by $E_T^\perp$ and the D's in fragmentation of an unpolarized quark. At leading twist, there is a Sivers-type [22] FF $D_{1T}^\perp$ describing polarization transverse to the production plane and corresponding to the transverse hyperon polarizations observed in high energy hadron-hadron and hadron-nucleus collisions [23]. Other higher twist FFs describe polarizations in longitudinal as well as two transverse directions.

If we integrate over $d^2k_{F\perp}$, we obtain the results given by Eqs. (A15)–(A19) in Appendix A. We see that only eight terms survive, which means that, in the one-dimensional case, for the vector polarization dependent part, we have totally eight FFs. We see also that two of them are leading twist, they are the longitudinal spin transfer $G_{1L}(z)$ and the transverse spin transfer $H_{1T}(z)$. We also have four twist-3 FFs that lead to induced polarization of the hadron and two twist-4 FFs that are addenda to the longitudinal and transverse spin transfer respectively. We also see that in this case induced polarization in the transverse direction exists at twist-3.

We note again the direct one-to-one correspondence between the results obtained in this case and those obtained in [1] for PDFs. The difference is the existence of the naive time-reversal odd $E_L(z)$ and $D_T(z)$ due to final state interactions between h and X while the corresponding term vanishes for PDFs. While $E_L(z)$ is an addendum to $G_{1L}(z)$, $D_T(z)$ leads to transverse polarization in fragmentation of the unpolarized quark. Both of them contribute at twist-3.

3. The tensor polarization dependent part

The general decomposition of the tensor polarization dependent part is given by Eqs. (A21)–(A25) in Appendix A which is obtained by constructing basic Lorentz covariants by using, besides p, $k_{F\perp}$, and n, the Lorentz scalar S_{LL}, Lorentz vector S_{LT}, and Lorentz tensor S_{TT}. We see that there are totally 40 tensor polarization dependent TMD FFs, ten contribute at twist-2, 20 at twist-3, and the other ten at twist-4. Among them, 24 (those related to $\tilde{\Xi}_\alpha^{T(0)}$ and $\Xi_{\rho\alpha}^{T(0)}$) are T odd and the other 16 are T even.

We emphasize in particular the similarities between the tensor polarization dependent terms given by Eqs. (A21)–(A25) in Appendix A and those unpolarized and vector polarization dependent terms given by Eqs. (A1)–(A14) in Appendix A.

(1) Since S_{LL} is a Lorentz scalar and thus has no influence on the basic Lorentz covariants, the S_{LL}-dependent terms have exactly one-to-one correspondence to the unpolarized terms.

(2) For the S_{LT}-dependent terms, because S_{LT} and S behave differently under space reflection, the S_{LT}-dependent terms are different from the S-dependent terms. Since S_{LT} has only two independent transverse components, we have one-to-one correspondence for S_{LT} to S_T terms with the replacement of $S_{T\alpha}$ by $\tilde{S}_{LT\alpha}$.

(3) Although there is no counterpart for the S_{TT}-dependent terms in other cases, however, there is no direct $S_{TT\rho\alpha}$ term contributing because $S_{TT\rho\alpha} = S_{TT\alpha\rho}$ is symmetric while $\Xi^{T(0)}_{\rho\alpha} = -\Xi^{T(0)}_{\alpha\rho}$ is antisymmetric. All the independent S_{TT} terms are in the form of $S_{TT\alpha\sigma}k^{\sigma}_{F\perp}$ which is denoted by $S^{k_F}_{TT\alpha}$. Because $S^{k_F}_{TT\alpha}$ has exactly the same Lorentz and space reflection behaviors as $S_{LT\alpha}$, we obtain a direct one-to-one correspondence between S_{LT}- and S_{TT}-dependent terms with the replacement of $S_{LT\alpha}$ by $S^{k_F}_{TT\alpha}$.

We note again the induced polarizations in the fragmentation of an unpolarized quark. We see that at leading twist an S_{LL}-dependent term exists and is described by D_{1LL}. There exist also terms depending on the other components of the tensor polarization at higher twists. We emphasize that, since they are independent of the polarization of the fragmenting quark, they might be much easier to study in experiments since no polarization in the initial state is needed.

We integrate over $d^2k_{F\perp}$ and obtain Eqs. (A26)–(A30) in Appendix A. We have totally eight terms, four of them are S_{LL} dependent and the other four are S_{LT} dependent. They have exact one-to-one correspondence to the unpolarized and S_T-dependent parts. We see that there are completely no S_{TT}-dependent terms that exist in the one-dimensional case. This means that no S_{TT}-dependent one-dimensional FF can be defined via the quark-quark correlator. The S_{TT}-dependent one-dimensional FFs can only be higher twists.

We list those twist-2 FFs in Table II, and those twist-3 FFs in Table III. The twist-4 FFs have the same structure of those at twist-2, so we will not make a separate table for them. We also list them according to chiral and time-reversal properties in Table IV.

We note in particular the S_{LL}-dependent terms exist also in the one-dimensional case. We see that the leading twist contribution D_{1LL} term survives the integration over $k_{F\perp}$ and the higher twist addenda such as E_{LL} and D_{3LL}. This means that it can be studied even in inclusive high energy reactions. In the case that the leading twist effect dominates, the results should be not very much dependent of energy. The energy dependence can be used as a sensitive test of higher twist contributions.

B. Relation to those defined via the quark-gluon-quark correlator at twist-3

Higher twist PDFs and FFs can also be defined via the corresponding quark-j-gluon-quark correlators ($j = 1, 2, \ldots$ represents the number of gluons) too [6–17]. However, because of QCD equation of motion $\gamma \cdot D(y)\psi(y) = 0$, the higher twist PDFs and FFs defined via these quark-j-gluon-quark correlators are often not independent. They are related to those defined via the quark-quark correlator by a set of equations derived using the equation of motion and can often be replaced by using these relationships when calculating the cross sections and other measurable quantities for different high energy reactions. In this section, we take twist-3 as an example to illustrate the results for FFs defined via the quark-j-gluon-quark correlator and their relationships to those defined via the quark-quark correlator.

Up to twist-3, we need to consider the quark-gluon-quark correlator defined as

$$\hat{\Xi}^{(1)}_{\rho,ij}(k_F; p, S) = \frac{1}{2\pi}\sum_X \int d^4\xi e^{-ik_F\xi}\langle p, S; X|\bar{\psi}_j(\xi)\mathcal{L}(\xi;\infty)|0\rangle \times \langle 0|\mathcal{L}^\dagger(0;\infty)D_\rho(0)\psi_i(0)|p, S; X\rangle, \tag{2.14}$$

where $D_\rho(y) \equiv -i\partial_\rho + gA_\rho(y)$ and $A_\rho(y)$ denotes the gluon field. Similar to the quark-quark correlator $\hat{\Xi}^{(0)}$, we decompose it as

$$\begin{aligned}\hat{\Xi}^{(1)}_{\rho}(z, k_{F\perp}; p, S) &= \Xi^{(1)}_{\rho}(z, k_{F\perp}; p, S) + i\gamma_5\tilde{\Xi}^{(1)}_{\rho}(z, k_{F\perp}; p, S) \\ &+ \gamma^\alpha\Xi^{(1)}_{\rho\alpha}(z, k_{F\perp}; p, S) + \gamma_5\gamma^\alpha\tilde{\Xi}^{(1)}_{\rho\alpha}(z, k_{F\perp}; p, S) \\ &+ i\sigma^{\alpha\beta}\gamma_5\Xi^{(1)}_{\rho\alpha\beta}(z, k_{F\perp}; p, S).\end{aligned} \tag{2.15}$$

Twist-3 components are the leading twist contributions that we obtain from $\hat{\Xi}^{(1)}_{\rho}$. There has to be one $\bar{n}$ involved in the basic Lorentz covariants and the other(s) are from the transverse components. Since the $\bar{n}$ component of the gluon field goes into the gauge link, we only have the other three components for D_ρ; thus no $\bar{n}_\rho$ component exists in the basic Lorentz covariants. We therefore do not have twist-3 contributions from $\Xi^{(1)}_{\rho}$ or $\tilde{\Xi}^{(1)}_{\rho}$. The twist-3 contributions are obtained from $\Xi^{(1)}_{\rho\alpha}$, $\tilde{\Xi}^{(1)}_{\rho\alpha}$, and $\Xi^{(1)}_{\rho\alpha\beta}$ and are given by Eqs. (A31)–(A39) in Appendix A. Here, we use a subscript d to specify that they are defined via the quark-gluon-quark correlator. A prime in the superscript before the $\perp$ denotes a different polarization situation, that after the $\perp$ specifies different FFs for the same polarization situation. We see that we have in total 36 FFs at twist-3 defined via the quark-gluon-quark correlator. This is just the same as what we obtained from the quark-quark correlator. Among them, 18 are χ even and the other 18 are χ odd; four contribute to the unpolarized part, 12 to the vector polarized part, and 20 to the tensor polarized part. We note in particular that the Hermiticity in this case does not demand that the FFs defined via the quark-gluon-quark correlator are real. They can have both real and imaginary parts.

For the 18 chiral even FFs (the D_d's and G_d's), the QCD equation of motion leads to rather simple relationships. They can be written in the following unified form, i.e.,

$$D^K_{dS}(z,k_\perp)+G^K_{dS}(z,k_\perp)=\frac{1}{z}[D^K_S(z,k_\perp)+iG^K_S(z,k_\perp)], \tag{2.16}$$

where the superscript K can be null (no superscript), a "$\perp$" or a "$'\perp$"; the subscript S specifies the polarization of the hadron and can be null (unpolarized), L, T, LL, LT, or TT. There are in fact totally nine such equations with the following combinations of K and S: $K=$ null and $S=T$ or LT; $K=\perp$ and $S=$ null, L, T, LL, LT, or TT; $K='\perp$ and $S=TT$. For the 18 chiral odd FFs, we have also nine equations in the form,

$$H^K_{dS}(z,k_\perp)+\frac{k^2_\perp}{2M^2}H^{K'}_{dS}(z,k_\perp) =\frac{1}{2z}[H^K_S(z,k_\perp)+\frac{i}{2}E^K_S(z,k_\perp)], \tag{2.17}$$

with the following combinations of K, K' and S: $(K,K')=(\text{null},\perp)$ and $S=$ null, L or LL; $(K,K')=(\perp,\perp')$ or $('\perp,'\perp')$ and $S=T$, LT, or TT. We note in particular that these 18 equations in fact represent 36 real equations which imply that all the 36 twist-3 FFs defined via the quark-quark correlator are given either by the real or imaginary part of those defined via the quark-gluon-quark correlator. We note also that there are of course different choices for the basic Lorentz covariants used here in defining these FFs via the quark-quark and/or quark-gluon-quark correlators. We choose them in a way so the defined FFs satisfy the relationships given by Eqs. (2.16) and (2.17).

These relationships reveal the physical essences of these FFs and also help us to choose correct conventions in defining FFs. It is also very interesting to observe that, although not generally proved, the final results obtained for the physical observables up to twist-3 are all expressed in terms of FFs defined via the quark-quark correlator [6–17]. The contributions from the quark-gluon-quark correlator can be replaced by using the relations given by Eqs. (2.16) and (2.17).

III. KINEMATIC ANALYSIS OF $e^+e^-\to V\pi X$

As mentioned in the Introduction, among all different high energy reactions, e^+e^- annihilation is most suitable for studying FFs. For one-dimensional FFs, the inclusive hadron production process $e^+e^-\to VX$ is the simplest case to study. In order to study transverse momentum dependence, we need at least two hadrons in the final state. Hence $e^+e^-\to V\pi X$ as illustrated in Fig. 1 is most suitable for studying the tensor polarization dependent part of the three-dimensional FFs. We now concentrate on this reaction and present the results for cross sections in this and the next sections.

For explicitness, we take $e^+e^-\to Z^0\to V\pi X$ as an example. The differential cross section is given by

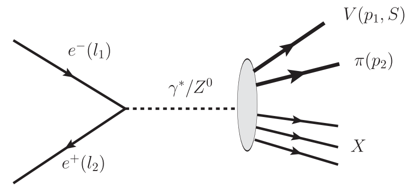

FIG. 1. Illustrating diagram for $e^+e^-\to V\pi X$.

$$\frac{2E_1E_2d\sigma}{d^3p_1d^3p_2}=\frac{\alpha^2\chi}{sQ^4}L_{\mu\nu}(l_1,l_2)W^{\mu\nu}(q,p_1,S,p_2). \tag{3.1}$$

Here we use the same notations as illustrated in Fig. 1: $\alpha=e^2/4\pi$, $\chi=Q^4/[(Q^2-M_Z^2)^2+\Gamma_Z^2M_Z^2]\sin^4 2\theta_W$, $Q^2=s=q^2$, θ_W is the Weinberg angle, M_Z is the Z-boson's mass, and Γ_Z is the decay width. The leptonic tensor is well known and is given by

$$L_{\mu\nu}(l_1,l_2)=c_1^e[l_{1\mu}l_{2\nu}+l_{1\nu}l_{2\mu}-(l_1\cdot l_2)g_{\mu\nu}] +ic_3^e\varepsilon_{\mu\nu\rho\sigma}l_1^\rho l_2^\sigma, \tag{3.2}$$

where $c_1^e=(c_V^e)^2+(c_A^e)^2$ and $c_3^e=2c_V^ec_A^e$; c_V^e and c_A^e are defined in the weak interaction current $\bar\psi\gamma^\mu(c_V^e-c_A^e\gamma^5)\psi$. Similar notations are also used for quarks. The hadronic tensors are defined as

$$W_{\mu\nu}(q,p_1,S,p_2)=\frac{1}{(2\pi)^4}\sum_X(2\pi)^4\delta^4(q-p_1-p_2-p_X) \times\langle 0|J_\nu(0)|p_1,S,p_2,X\rangle \times\langle p_1,S,p_2,X|J_\mu(0)|0\rangle, \tag{3.3}$$

where S denotes the polarization of the hadron and for the vector meson it includes both the vector and tensor polarization parts, $J_\mu(x)=\bar\psi(x)\Gamma_\mu\psi(x)$ and $\Gamma_\mu=\gamma^\mu(c_V^q-c_A^q\gamma^5)$.

Besides the Lorentz covariance, the hadronic tensor $W^{\mu\nu}$ satisfies the general constraints imposed by Hermiticity, current conservation, and parity conservation in the electromagnetic process, i.e.,

$$W^{*\mu\nu}(q,p_1,S,p_2)=W^{\nu\mu}(q,p_1,S,p_2), \tag{3.4}$$

$$q_\mu W^{\mu\nu}(q,p_1,S,p_2)=q_\nu W^{\mu\nu}(q,p_1,S,p_2)=0, \tag{3.5}$$

$$W^{\mu\nu}(q,p_1,S,p_2)=W_{\mu\nu}(q^{\mathcal{P}},p_1^{\mathcal{P}},S^{\mathcal{P}},p_2^{\mathcal{P}}). \tag{3.6}$$

We emphasize that parity conservation is not valid in the weak process via Z exchange.

A. The general structure of $W^{\mu\nu}(q,p_1,S,p_2)$

A systematic analysis of the hadronic tensor $W_{\mu\nu}$ for $e^+e^-\to h_1h_2X$ for the case that both h_1 and h_2 are spin-1/2 hadrons is presented in [14]. Here, we extend the analysis to $e^+e^- \to V\pi X$ including parity conserving as well as violating contributions. We present the results for the basic Lorentz tensors, the cross section and structure functions in the Lorentz invariant form, as well as in the form of azimuthal angular dependences in a particular Lorentz frame.

1. The basic Lorentz tensors for $W^{\mu\nu}(q,p_1,S,p_2)$

For the spin-independent and vector polarization dependent parts, the results can just be taken from [14]. We list them here for completeness and also for unification of notations that are more convenient to extend to including tensor polarization dependent parts.

First, the spin-independent (or unpolarized) part, we take the notations as

$$h_{Ui}^{S\mu\nu} = \left\{ g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}, p_{1q}^\mu p_{1q}^\nu, p_{1q}^{\{\mu} p_{2q}^{\nu\}}, p_{2q}^\mu p_{2q}^\nu \right\}, \quad (3.7)$$

$$\tilde{h}_{Ui}^{S\mu\nu} = \{\varepsilon^{\{\mu q p_1 p_2}(p_{1q}, p_{2q})^{\nu\}}\}, \quad (3.8)$$

$$h_U^{A\mu\nu} = p_{1q}^{[\mu} p_{2q}^{\nu]}, \quad (3.9)$$

$$\tilde{h}_{Ui}^{A\mu\nu} = \{\varepsilon^{\mu\nu q p_1}, \varepsilon^{\mu\nu q p_2}\}, \quad (3.10)$$

where h represents the parity conserved (space reflection P-even) tensors, i.e., those satisfying Eq. (3.6) or more precisely $h^{\mu\nu}(q^{\mathcal{P}}, p_1^{\mathcal{P}}, S^{\mathcal{P}}, p_2^{\mathcal{P}}) = h_{\mu\nu}(q, p_1, S, p_2)$ while $\tilde{h}$ represents those parity nonconserved (P odd), i.e., satisfying $\tilde{h}^{\mu\nu}(q^{\mathcal{P}}, p_1^{\mathcal{P}}, S^{\mathcal{P}}, p_2^{\mathcal{P}}) = -\tilde{h}_{\mu\nu}(q, p_1, S, p_2)$; the superscript S or A denotes symmetric or antisymmetric under exchange of $(\mu \leftrightarrow \nu)$, and the subscript U denotes the unpolarized part [24]. A 4-momentum p with a subscript q denotes $p_q \equiv p - q(p\cdot q)/q^2$ satisfying $p_q \cdot q = 0$. We use the shorthanded notations to make the expressions more concise such as $\varepsilon^{\mu q p_1 p_2} \equiv \varepsilon^{\mu\alpha\rho\sigma} q_\alpha p_{1\rho} p_{2\sigma}$, and $\varepsilon^{\{\mu q p_1 p_2}(p_{1q}, p_{2q})^{\nu\}}$ means $\varepsilon^{\{\mu q p_1 p_2} p_{1q}^{\nu\}}$ and $\varepsilon^{\{\mu q p_1 p_2} p_{2q}^{\nu\}}$. We see that there are totally nine such basic tensors in the unpolarized case.

For the vector polarization dependent part, we have

$$h_{Vi}^{S\mu\nu} = \{[(q\cdot S), (p_2\cdot S)]\tilde{h}_{Ui}^{S\mu\nu}, \varepsilon^{Sqp_1p_2} h_{Uj}^{S\mu\nu}\}, \quad (3.11)$$

$$\tilde{h}_{Vi}^{S\mu\nu} = \{[(q\cdot S), (p_2\cdot S)]h_{Ui}^{S\mu\nu}, \varepsilon^{Sqp_1p_2} \tilde{h}_{Uj}^{S\mu\nu}\}, \quad (3.12)$$

$$h_{Vi}^{A\mu\nu} = \{[(q\cdot S), (p_2\cdot S)]\tilde{h}_{Ui}^{A\mu\nu}, \varepsilon^{Sqp_1p_2} h_{U}^{A\mu\nu}\}, \quad (3.13)$$

$$\tilde{h}_{Vi}^{A\mu\nu} = \{[(q\cdot S), (p_2\cdot S)]h_{U}^{A\mu\nu}, \varepsilon^{Sqp_1p_2} \tilde{h}_{Uj}^{A\mu\nu}\}. \quad (3.14)$$

There are totally 27 such S-dependent basic tensors, three times as many as those for the unpolarized part, corresponding to three independent vector polarization modes.

For the tensor polarization dependent part, after some lengthy algebra, we find out that if we consider S_{LL}-, S_{LT}-, and S_{TT}-dependent parts separately, we obtained the following nice symmetric forms.

(1) *The S_{LL}-dependent part*. Since S_{LL} is a scalar, the S_{LL}-dependent part is very simple. The S_{LL}-dependent basic tensors are just given by the corresponding spin-independent tensors multiplied by S_{LL} such as $h_{LLi}^{S\mu\nu} = S_{LL} h_{Ui}^{S\mu\nu}$ and so on. We have therefore nine such tensors in this case.

(2) *The S_{LT}-dependent part*. In contrast to the axial-vector S, S_{LT} is a vector satisfying the constraint $S_{LT}\cdot p_1 = 0$, the S_{LT}-dependent part is thus different from the S-dependent part. Furthermore, both S_{LT} and S_{TT} have only two independent transverse components in the rest frame of the vector meson; this is guaranteed by demanding a further constraint $S_{LT}\cdot q = 0$ for S_{LT}. The basic S_{LT}-dependent Lorentz tensors are given by

$$h_{LTi}^{S\mu\nu} = \{(p_2\cdot S_{LT})h_{Ui}^{S\mu\nu}, \varepsilon^{S_{LT}qp_1p_2}\tilde{h}_{Uj}^{S\mu\nu}\}, \quad (3.15)$$

$$\tilde{h}_{LTi}^{S\mu\nu} = \{(p_2\cdot S_{LT})\tilde{h}_{Ui}^{S\mu\nu}, \varepsilon^{S_{LT}qp_1p_2}h_{Uj}^{S\mu\nu}\}, \quad (3.16)$$

$$h_{LTi}^{A\mu\nu} = \{(p_2\cdot S_{LT})h_{U}^{A\mu\nu}, \varepsilon^{S_{LT}qp_1p_2}\tilde{h}_{Uj}^{A\mu\nu}\}, \quad (3.17)$$

$$\tilde{h}_{LTi}^{A\mu\nu} = \{(p_2\cdot S_{LT})\tilde{h}_{Ui}^{A\mu\nu}, \varepsilon^{S_{LT}qp_1p_2}h_{U}^{A\mu\nu}\}. \quad (3.18)$$

There are totally 18 such tensors corresponding to the two independent S_{LT} components.

(3) *The S_{TT}-dependent part*. $S_{TT}^{\alpha\beta}$ is a tensor satisfying the constraints, $S_{TT}^{\alpha\beta} = S_{TT}^{\beta\alpha}$, $g_{\alpha\beta}S_{TT}^{\alpha\beta} = 0$, $S_{TT}^{p_1\beta}(\equiv S_{TT}^{\alpha\beta}p_{1\alpha}) = 0$, and $S_{TT}^{q\beta} = 0$. We have the S_{TT}-dependent basic Lorentz tensors as given by

$$h_{TTi}^{S\mu\nu} = \{S_{TT}^{p_2p_2}h_{Ui}^{S\mu\nu}, \varepsilon^{S_{TT}^{p_2}qp_1p_2}\tilde{h}_{Uj}^{S\mu\nu}\}, \quad (3.19)$$

$$\tilde{h}_{TTi}^{S\mu\nu} = \{S_{TT}^{p_2p_2}\tilde{h}_{Ui}^{S\mu\nu}, \varepsilon^{S_{TT}^{p_2}qp_1p_2}h_{Uj}^{S\mu\nu}\}, \quad (3.20)$$

$$h_{TTi}^{A\mu\nu} = \{S_{TT}^{p_2p_2}h_{U}^{A\mu\nu}, \varepsilon^{S_{TT}^{p_2}qp_1p_2}\tilde{h}_{Uj}^{A\mu\nu}\}, \quad (3.21)$$

$$\tilde{h}_{TTi}^{A\mu\nu} = \{S_{TT}^{p_2p_2}\tilde{h}_{Ui}^{A\mu\nu}, \varepsilon^{S_{TT}^{p_2}qp_1p_2}h_{U}^{A\mu\nu}\}. \quad (3.22)$$

There are also totally 18 S_{TT}-dependent basic Lorentz tensors. For $W_{\mu\nu}(q,p_1,S,p_2)$, we have totally 81 basic Lorentz tensors, 41 of them are space reflection even and 40 are odd.

2. General form of $W^{\mu\nu}(q,p_1,S,p_2)$

The hadronic tensor $W^{\mu\nu}(q,p_1,S,p_2)$ is in general expressed as a sum of all these basic Lorentz tensors multiplied by corresponding coefficients. The coefficients are real and functions of the Lorentz scalars q^2, $q\cdot p_1$,

$q\cdot p_2$, and $p_1\cdot p_2$, which can be replaced by $s=q^2$, $\xi_1=2q\cdot p_1/q^2$, $\xi_2=2q\cdot p_2/q^2$, and $\xi_{12}=s_{12}/s=(p_1+p_2)^2/s$. More precisely, we have

$$W^{\mu\nu}(q,p_1,S,p_2) = W^{S\mu\nu}(q,p_1,S,p_2)+iW^{A\mu\nu}(q,p_1,S,p_2), \quad (3.23)$$

$$W^{S\mu\nu}(q,p_1,S,p_2) = \sum_{\sigma,i} W^S_{\sigma i}(s,\xi_1,\xi_2,\xi_{12})h^{S\mu\nu}_{\sigma i} + \sum_{\sigma,j}\tilde{W}^S_{\sigma j}(s,\xi_1,\xi_2,\xi_{12})\tilde{h}^{S\mu\nu}_{\sigma j}, \quad (3.24)$$

$$W^{A\mu\nu}(q,p_1,S,p_2) = \sum_{\sigma,i} W^A_{\sigma i}(s,\xi_1,\xi_2,\xi_{12})h^{A\mu\nu}_{\sigma i} + \sum_{\sigma,j}\tilde{W}^A_{\sigma j}(s,\xi_1,\xi_2,\xi_{12})\tilde{h}^{A\mu\nu}_{\sigma j}, \quad (3.25)$$

where the subscript σ denotes U, V, LL, LT, and TT for different polarizations; all the coefficients W's are scalar functions of the Lorentz scalars s, ξ_1, ξ_2, and ξ_{12}.

B. The general structure for the cross section

Since the number of independent structure functions is rather large, in practice, it is often more convenient to write the cross section directly.

1. The Lorentz invariant form

Making the Lorentz contraction of $W^{\mu\nu}(q,p_1,S,p_2)$ with $L_{\mu\nu}(l_1,l_2)$, we obtain the general form of the cross section. For the unpolarized part, this is given by

$$\frac{2E_1E_2d\sigma^U}{d^3p_1d^3p_2} = \frac{\alpha^2\chi}{s^2}[\mathcal{F}_U(s,\xi_1,\xi_2,\xi_{12};y_1,y_2) + \tilde{\mathcal{F}}_U(s,\xi_1,\xi_2,\xi_{12};y_1,y_2,\tilde{y})], \quad (3.26)$$

where $\mathcal{F}_U$ and $\tilde{\mathcal{F}}_U$ represent the space reflection even and odd parts respectively and they have the structures as given by

$$\mathcal{F}_U = F^0_U + F^1_U y_1 + F^2_U y_2 + F^{11}_U y_1^2 + F^{22}_U y_2^2 + F^{12}_U y_1 y_2, \quad (3.27)$$

$$\tilde{\mathcal{F}}_U = \tilde{y}(\tilde{F}^0_U + \tilde{F}^1_U y_1 + \tilde{F}^2_U y_2), \quad (3.28)$$

where besides ξ_1, ξ_2, and ξ_{12} defined before, we introduced two new Lorentz scalars $y_1=2p_1\cdot l_1/q^2$, $y_2=2p_2\cdot l_1/q^2$, and one pseudoscalar $\tilde{y}=\varepsilon^{l_1qp_1p_2}/q^4$. The "structure functions" F's are all scalar functions depending on (s,ξ_1,ξ_2,ξ_{12}). We see also clearly that the six F's describe the parity conserved contributions while the three $\tilde{F}$'s represent the parity violated part. They are related to the W's by

$$F^0_U = -\frac{1}{2}c^e_1[2W^S_{U1} + (m_1^2W^S_{U2} + m_2^2W^S_{U3}) - (s\xi_{12}-m_1^2-m_2^2)W^S_{U4}] + \frac{1}{2}sc^e_3(\xi_1\tilde{W}^A_{U1} + \xi_2\tilde{W}^A_{U2}), \quad (3.29)$$

$$F^1_U = \frac{1}{2}c^e_1 s(\xi_1 W^S_{U2} + \xi_2 W^S_{U4}) - c^e_3 s\tilde{W}^A_{U1}, \quad (3.30)$$

$$F^2_U = \frac{1}{2}c^e_1 s(\xi_2 W^S_{U3} + \xi_1 W^S_{U4}) - c^e_3 s\tilde{W}^A_{U2}, \quad (3.31)$$

$$F^{11}_U = -\frac{1}{2}c^e_1 sW^S_{U2}, \quad (3.32)$$

$$F^{22}_U = -\frac{1}{2}c^e_1 sW^S_{U3}, \quad (3.33)$$

$$F^{12}_U = -c^e_1 sW^S_{U4}, \quad (3.34)$$

$$\tilde{F}^0_U = c^e_1 s^2(\xi_1\tilde{W}^S_{U1} + \xi_2\tilde{W}^S_{U2}) - 2c^e_3 sW^A_U, \quad (3.35)$$

$$\tilde{F}^1_U = -2c^e_1 s^2\tilde{W}^S_{U1}, \quad (3.36)$$

$$\tilde{F}^2_U = -2c^e_1 s^2\tilde{W}^S_{U2}. \quad (3.37)$$

We see here that although the F_{Ui}'s and $\tilde{F}_{Ui}$'s are all functions of s,ξ_1,ξ_2,ξ_{12}, they contain already information from the leptonic tensor due to the coefficients c^e_1 and c^e_3. We also see that the parity conserved parts come from parity conserved hadronic tensor terms (characterized by W's) contracted with parity conserved leptonic tensor terms (characterized by c^e_1) or parity violated hadronic tensor terms (characterized by $\tilde{W}$'s) contracted with the parity violated leptonic tensor term (characterized by c^e_3). We have six such F_{Ui}'s. Similarly we have three $\tilde{F}_{Ui}$'s for the parity violated parts obtained from Lorentz contractions of parity conserved leptonic tensor terms with parity violated hadronic tensor terms or parity violated leptonic tensor terms with parity conserved tensor terms.

The polarization dependent part has completely the same structure. For the vector polarization dependent part, from Eqs. (3.11)–(3.14), we obtain immediately that

$$\frac{2E_1E_2d\sigma^V}{d^3p_1d^3p_2} = \frac{\alpha^2}{s^2}\chi\{(q\cdot S)(\mathcal{F}_{V1}+\tilde{\mathcal{F}}_{V1}) + (p_2\cdot S)(\mathcal{F}_{V2}+\tilde{\mathcal{F}}_{V2}) + \varepsilon^{Sqp_1p_2}(\mathcal{F}_{V3}+\tilde{\mathcal{F}}_{V3})\}. \quad (3.38)$$

Here, we note that since $q\cdot S$ and $p_2\cdot S$ are space reflection odd, the parity conserved parts $\mathcal{F}_{V1}$ and $\mathcal{F}_{V2}$ take exactly the same form as $\tilde{\mathcal{F}}_U$ given by Eq. (3.28), while the parity violated parts $\tilde{\mathcal{F}}_{V1}$ and $\tilde{\mathcal{F}}_{V2}$ take the same form as $\mathcal{F}_U$

given by Eq. (3.27) with the subscript U replaced by $V1$ or $V2$. Since $\varepsilon^{Sqp_1p_2}$ is a scalar, $\mathcal{F}_{V3}$ and $\tilde{\mathcal{F}}_{V3}$ take exactly the same form as $\mathcal{F}_U$ and $\tilde{\mathcal{F}}_U$ given by Eqs. (3.27)–(3.37) respectively with the subscript U replaced by $V3$. We have three sets of F_{Vi} and $\tilde{F}_{Vi}$ because there are three independent components of vector polarization.

For the tensor polarization dependent part, we have

$$\frac{2E_1E_2d\sigma^{LL}}{d^3p_1d^3p_2} = \frac{\alpha^2}{s^2}\chi S_{LL}(\mathcal{F}_{LL} + \tilde{\mathcal{F}}_{LL}), \tag{3.39}$$

$$\frac{2E_1E_2d\sigma^{LT}}{d^3p_1d^3p_2} = \frac{\alpha^2}{s^2}\chi\{(p_2\cdot S_{LT})(\mathcal{F}_{LT1} + \tilde{\mathcal{F}}_{LT1}) + \varepsilon^{S_{LT}qp_1p_2}(\mathcal{F}_{LT2} + \tilde{\mathcal{F}}_{LT2})\}, \tag{3.40}$$

$$\frac{2E_1E_2d\sigma^{TT}}{d^3p_1d^3p_2} = \frac{\alpha^2}{s^2}\chi\{S_{TT}^{p_2p_2}(\mathcal{F}_{TT1} + \tilde{\mathcal{F}}_{TT1}) + \varepsilon^{S_{TT}^{p_2}qp_1p_2}(\mathcal{F}_{TT2} + \tilde{\mathcal{F}}_{TT2})\}. \tag{3.41}$$

Here S_{LL}, $p_2\cdot S_{LT}$, and $S_{TT}^{p_2p_2}$ are scalars, $\varepsilon^{S_{LT}qp_1p_2}$ and $\varepsilon^{S_{TT}^{p_2}qp_1p_2}$ are pseudoscalars. Hence, $\mathcal{F}_{LL}$, $\mathcal{F}_{LT1}$, $\mathcal{F}_{TT1}$, $\tilde{\mathcal{F}}_{LT2}$, and $\tilde{\mathcal{F}}_{TT2}$ take exactly the same form as $\mathcal{F}_U$ given by Eq. (3.27), while $\tilde{\mathcal{F}}_{LL}$, $\tilde{\mathcal{F}}_{LT1}$, $\tilde{\mathcal{F}}_{TT1}$, $\mathcal{F}_{LT2}$, and $\mathcal{F}_{TT2}$ take exactly the same form as $\tilde{\mathcal{F}}_U$ given by Eq. (3.28).

2. *In the helicity-GJ frame*

Going into a special reference frame, we can express the cross section in terms of angular dependences. The polarization of high energy particles is described and/or studied most conveniently in the helicity frame, i.e., where we choose the direction of motion of the particle as the z direction. Hence, to study polarization dependent FFs for V in $e^+e^- \to V\pi X$, we suggest to choose the following frame. We choose the center-of-mass frame of the e^+e^- system, and direction of motion of V, i.e., $\vec{p}_1$ as the z direction, and the lepton-hadron (vector meson) plane as the Oxz plane. This is a particular Gottfried-Jackson frame [25] which we will refer to as the "helicity-GJ frame" in the following of this paper. In this frame, we have

$$p_1 = (E_1, 0, 0, p_{1z}), \tag{3.42}$$

$$p_2 = (E_2, |\vec{p}_{2T}|\cos\varphi, |\vec{p}_{2T}|\sin\varphi, p_{2z}), \tag{3.43}$$

$$l_1 = \frac{Q}{2}(1, \sin\theta, 0, \cos\theta), \tag{3.44}$$

$$l_2 = \frac{Q}{2}(1, -\sin\theta, 0, -\cos\theta), \tag{3.45}$$

$$q = l_1 + l_2 = (Q, 0, 0, 0), \tag{3.46}$$

and we choose ξ_1, ξ_2, $|\vec{p}_{2T}|$, θ, or $y = l_2\cdot p_1/q\cdot p_1 \approx (1+\cos\theta)/2$ and φ as the independent variable set. The other variables are replaced. The basic volume element transforms as

$$\frac{d^3p_1d^3p_2}{E_1E_2} = \frac{\pi\xi_1}{\xi_2}s(1 - 4M_{2T}^2/s\xi_2^2)^{-1/2}d\xi_1d\xi_2dyd^2p_{2T}, \tag{3.47}$$

where $M_{2T}^2 = M_2^2 + \vec{p}_{2T}^2$ and $d^2p_{2T} = d\vec{p}_{2T}^2d\varphi/2$.

The structure functions.—For the unpolarized part, we have

$$\mathcal{F}_U = (1+\cos^2\theta)F_{1U} + \sin^2\theta F_{2U} + \cos\theta F_{3U} + \cos\varphi[\sin\theta F_{1U}^{\cos\varphi} + \sin 2\theta F_{2U}^{\cos\varphi}] + \cos 2\varphi \sin^2\theta F_U^{\cos 2\varphi}, \tag{3.48}$$

$$\tilde{\mathcal{F}}_U = \sin\varphi[\sin\theta\tilde{F}_{1U}^{\sin\varphi} + \sin 2\theta\tilde{F}_{2U}^{\sin\varphi}] + \sin 2\varphi\sin^2\theta\tilde{F}_U^{\sin 2\varphi}, \tag{3.49}$$

where F_{Ui} and $\tilde{F}_{Ui}$ are all scalar functions of s, ξ_1, ξ_2, and p_{2T}^2. We see also clearly that we have totally nine independent structure functions in the unpolarized case, six of them are denoted by F_U's and correspond to parity conserving terms and the other three are $\tilde{F}_U$'s describing the parity odd part of the cross section. This is just the same as those shown by Eqs. (3.29)–(3.37). We note in particular that the structure functions F_U's and $\tilde{F}_U$'s themselves are scalar functions of s, ξ_1, ξ_2, and p_{2T}^2 and are invariant under space reflection. But the angular dependent coefficients have the corresponding space reflection properties. The different basic Lorentz tensors $h_{Ui}^{\mu\nu}$'s and $\tilde{h}_{Ui}^{\mu\nu}$'s are transformed to different angular dependences. We also see that there are three azimuthal angle independent structure functions, three parity conserving, and three parity violating azimuthal angle dependent structure functions. They correspond to cos or sin asymmetries and are parity conserving and violating respectively.

Here we take the following conventions for the notations of structure functions, i.e., the superscript to denote the corresponding azimuthal angle φ dependence, the capital letter in the subscripts to denote the polarization, and the digital number in front of the capital letter to specify if we have more than one such structure function corresponding to the same azimuthal angle φ dependence but different θ or y dependences [26]. We also note that to replace θ by y we have

$$1+\cos^2\theta \approx 1 + (2y-1)^2 = 2A(y), \tag{3.50}$$

$$\cos\theta \approx -1 + 2y = -B(y), \tag{3.51}$$

$$\sin^2\theta \approx 1-(1-2y)^2 = 4y(1-y) = C(y), \quad (3.52)$$

that appear frequently in the expressions of the cross section.

For the vector polarized part, we note that

$$S = \left(\lambda \frac{p_{1z}}{M_1}, |\vec{S}_T|\cos\varphi_S, |\vec{S}_T|\sin\varphi_S, \lambda \frac{E_1}{M_1}\right). \quad (3.53)$$

The $(q\cdot S)$ and $\varepsilon^{Sqp_1p_2}$ terms in Eq. (3.38) contribute to longitudinal and transverse polarization separately, while the $(p_2\cdot S)$ terms contribute to both cases. The contributions to transverse polarization from the $(p_2\cdot S)$ and $\varepsilon^{Sqp_1p_2}$ terms are characterized by additional $\cos(\varphi_S-\varphi)$ and $\sin(\varphi_S-\varphi)$ dependence. We absorb the different kinematic factors into $\mathcal{F}$ and $\tilde{\mathcal{F}}$ and write the cross section as

$$\frac{2E_1E_2 d\sigma^V}{d^3p_1 d^3p_2} = \frac{\alpha^2}{s^2}\chi\{\lambda(\mathcal{F}_L+\tilde{\mathcal{F}}_L)+|\vec{S}_T|(\mathcal{F}_T+\tilde{\mathcal{F}}_T)\}. \quad (3.54)$$

Since λ changes sign under space reflection, the parity conserving $\mathcal{F}_L$ and parity violating $\tilde{\mathcal{F}}_L$ take exactly the same form as $\tilde{\mathcal{F}}_U$ and $\mathcal{F}_U$ respectively. We have three F_{iL}'s that have one-to-one correspondence to $\tilde{F}_{iU}$'s and six $\tilde{F}_{iL}$'s that have one-to-one correspondence to F_{iU}'s.

For the transverse (vector) polarization dependent part, due to φ_S dependence, the structure looks a bit different; they are given by

$$\begin{aligned}\mathcal{F}_T = {}& \sin\varphi_S[\sin\theta F_{1T}^{\sin\varphi_S} + \sin 2\theta F_{2T}^{\sin\varphi_S}] + \sin(\varphi_S+\varphi)\sin^2\theta F_T^{\sin(\varphi_S+\varphi)} + \sin(\varphi_S-\varphi)[(1+\cos^2\theta)F_{1T}^{\sin(\varphi_S-\varphi)} \\ & + \sin^2\theta F_{2T}^{\sin(\varphi_S-\varphi)} + \cos\theta F_{3T}^{\sin(\varphi_S-\varphi)}] + \sin(\varphi_S-2\varphi)[\sin\theta F_{1T}^{\sin(\varphi_S-2\varphi)} + \sin 2\theta F_{2T}^{\sin(\varphi_S-2\varphi)}] \\ & + \sin(\varphi_S-3\varphi)\sin^2\theta F_T^{\sin(\varphi_S-3\varphi)},\end{aligned} \quad (3.55)$$

$$\begin{aligned}\tilde{\mathcal{F}}_T = {}& \cos\varphi_S[\sin\theta \tilde{F}_{1T}^{\cos\varphi_S} + \sin 2\theta \tilde{F}_{2T}^{\cos\varphi_S}] + \cos(\varphi_S+\varphi)\sin^2\theta \tilde{F}_T^{\cos(\varphi_S+\varphi)} + \cos(\varphi_S-\varphi)[(1+\cos^2\theta)\tilde{F}_{1T}^{\cos(\varphi_S-\varphi)} \\ & + \sin^2\theta \tilde{F}_{2T}^{\cos(\varphi_S-\varphi)} + \cos\theta \tilde{F}_{3T}^{\cos(\varphi_S-\varphi)}] + \cos(\varphi_S-2\varphi)[\sin\theta \tilde{F}_{1T}^{\cos(\varphi_S-2\varphi)} + \sin 2\theta \tilde{F}_{2T}^{\cos(\varphi_S-2\varphi)}] \\ & + \cos(\varphi_S-3\varphi)\sin^2\theta \tilde{F}_T^{\cos(\varphi_S-3\varphi)}.\end{aligned} \quad (3.56)$$

There are 18 such transverse polarization dependent structure functions, nine of them are space reflection even and nine are space reflection odd. Totally we have 27 vector polarization dependent structure functions corresponding to the 27 independent basic Lorentz tensors $h_{Vi}^{\mu\nu}$'s for the hadronic tensor. Among them, 12 contribute to space reflection even terms in the cross section, the other 15 to space reflection odd terms. We note in particular the $\sin\varphi_S$ and $\cos\varphi_S$ terms correspond to single transverse spin asymmetries in deep-inelastic lepton-nucleon scattering $e^-h\to e^-X$ with respect to the leptonic plane. They are either parity or time reversal odd and do not exist in $e^-h\to e^-X$. In e^+e^- annihilation, they describe the transverse polarization in or transverse to the lepton-hadron plane.

The S_{LL}-dependent part is again completely the same as that for the unpolarized case; i.e., we have a one-to-one correspondence of F_{LL} to F_U and $\tilde{F}_{LL}$ to $\tilde{F}_U$.

For the S_{LT}-dependent part, we define

$$S_{LT}^x = |S_{LT}|\cos\varphi_{LT}, \quad (3.57)$$

$$S_{LT}^y = |S_{LT}|\sin\varphi_{LT}, \quad (3.58)$$

$$|S_{LT}| = \sqrt{(S_{LT}^x)^2 + (S_{LT}^y)^2}, \quad (3.59)$$

and we have

$$\frac{2E_1E_2 d\sigma^{LT}}{d^3p_1 d^3p_2} = \frac{\alpha^2}{s^2}\chi|S_{LT}|\{\mathcal{F}_{LT}+\tilde{\mathcal{F}}_{LT}\}. \quad (3.60)$$

Because S_{LT} behaves differently from S_T under space reflection, we obtain that $\mathcal{F}_{LT}$ takes exactly the same form as $\tilde{\mathcal{F}}_T$ and $\tilde{\mathcal{F}}_{LT}$ behaves in the same way as $\mathcal{F}_T$. More precisely, we obtain the results for $\mathcal{F}_{LT}$ by replacing φ_S with φ_{LT} and $\tilde{F}_{jT}$ with F_{jLT} in Eq. (3.56), and those for $\tilde{\mathcal{F}}_{LT}$ by replacing φ_S with φ_{LT} and F_{jT} with $\tilde{F}_{jLT}$ in Eq. (3.55). We have exactly one-to-one correspondence here.

For the S_{TT}-dependent part, we take

$$S_{TT}^{xx} = |S_{TT}|\cos 2\varphi_{TT}, \quad (3.61)$$

$$S_{TT}^{xy} = |S_{TT}|\sin 2\varphi_{TT}, \quad (3.62)$$

$$|S_{TT}| = \sqrt{(S_{TT}^{xx})^2 + (S_{TT}^{xy})^2}, \quad (3.63)$$

so that $S_{TT}^{p_2p_2}$ and $\varepsilon^{S_{TT}^{p_2}qp_1p_2}$ will contribute $\cos(2\varphi_{TT}-2\varphi)$ and $\sin(2\varphi_{TT}-2\varphi)$ terms. Compared with the S_T part, by changing $\varphi_S\to 2\varphi_{TT}-\varphi$, the S_{TT}-dependent part is classified into $\cos 2\varphi_{TT}$, $\cos(2\varphi_{TT}-\varphi)$, $\cos(2\varphi_{TT}-2\varphi)$, $\cos(2\varphi_{TT}-3\varphi)$, $\cos(2\varphi_{TT}-4\varphi)$, and the corresponding sin terms. More precisely, they are given by

$$\frac{2E_1E_2 d\sigma^{TT}}{d^3p_1 d^3p_2} = \frac{\alpha^2}{s^2}\chi|S_{TT}|\{\mathcal{F}_{TT}+\tilde{\mathcal{F}}_{TT}\}, \tag{3.64}$$

$$\begin{aligned}\mathcal{F}_{TT} &= \cos 2\varphi_{TT}\sin^2\theta F_{TT}^{\cos 2\varphi_{TT}} + \cos(2\varphi_{TT}-\varphi)[\sin\theta F_{1TT}^{\cos(2\varphi_{TT}-\varphi)} + \sin 2\theta F_{2TT}^{\cos(2\varphi_{TT}-\varphi)}] \\ &+ \cos(2\varphi_{TT}-2\varphi)[(1+\cos^2\theta)F_{1TT}^{\cos(2\varphi_{TT}-2\varphi)} + \sin^2\theta F_{2TT}^{\cos(2\varphi_{TT}-2\varphi)} + \cos\theta F_{3TT}^{\cos(2\varphi_{TT}-2\varphi)}] \\ &+ \cos(2\varphi_{TT}-3\varphi)[\sin\theta F_{1TT}^{\cos(2\varphi_{TT}-3\varphi)} + \sin 2\theta F_{2TT}^{\cos(2\varphi_{TT}-3\varphi)}] + \cos(2\varphi_{TT}-4\varphi)\sin^2\theta F_{TT}^{\cos(2\varphi_{TT}-4\varphi)},\end{aligned} \tag{3.65}$$

$$\begin{aligned}\tilde{\mathcal{F}}_{TT} &= \sin 2\varphi_{TT}\sin^2\theta \tilde{F}_{TT}^{\sin 2\varphi_{TT}} + \sin(2\varphi_{TT}-\varphi)(\sin\theta \tilde{F}_{1TT}^{\sin(2\varphi_{TT}-\varphi)} + \sin 2\theta \tilde{F}_{2TT}^{\sin(2\varphi_{TT}-\varphi)}) \\ &+ \sin(2\varphi_{TT}-2\varphi)[(1+\cos^2\theta)\tilde{F}_{1TT}^{\sin(2\varphi_{TT}-2\varphi)} + \sin^2\theta \tilde{F}_{2TT}^{\sin(2\varphi_{TT}-2\varphi)} + \cos\theta \tilde{F}_{3TT}^{\sin(2\varphi_{TT}-2\varphi)}] \\ &+ \sin(2\varphi_{TT}-3\varphi)(\sin\theta \tilde{F}_{1TT}^{\sin(2\varphi_{TT}-3\varphi)} + \sin 2\theta \tilde{F}_{2TT}^{\sin(2\varphi_{TT}-3\varphi)}) + \sin(2\varphi_{TT}-4\varphi)\sin^2\theta \tilde{F}_{TT}^{\sin(2\varphi_{TT}-4\varphi)}.\end{aligned} \tag{3.66}$$

To show the regularities we list all 81 structure functions together with the leading twist parton model results in a table. See Table I in Sec. V.

The azimuthal asymmetries.—From these equations, we can calculate the azimuthal asymmetries and different components of hadron polarization in a straightforward way. E.g.,

$$\langle\cos\varphi\rangle_U = (\sin\theta F_{1U}^{\cos\varphi} + \sin 2\theta F_{2U}^{\cos\varphi})/2F_{Ut}, \tag{3.67}$$

$$\langle\cos 2\varphi\rangle_U = \sin^2\theta F_U^{\cos 2\varphi}/2F_{Ut}, \tag{3.68}$$

$$\langle\sin\varphi\rangle_U = (\sin\theta \tilde{F}_{1U}^{\sin\varphi} + \sin 2\theta \tilde{F}_{2U}^{\sin\varphi})/2F_{Ut}, \tag{3.69}$$

$$\langle\sin 2\varphi\rangle_U = \sin^2\theta \tilde{F}_U^{\sin 2\varphi}/2F_{Ut}, \tag{3.70}$$

where F_{Ut} denotes the result of $\mathcal{F}_U+\tilde{\mathcal{F}}_U$ averaging over φ, i.e,

$$\begin{aligned}F_{Ut}(s,\xi_1,\xi_2,p_{2T},\theta) &\equiv \int\frac{d\varphi}{2\pi}(\mathcal{F}_U+\tilde{\mathcal{F}}_U) \\ &= (1+\cos^2\theta)F_{1U} \\ &\quad + \sin^2\theta F_{2U} + \cos\theta F_{3U}.\end{aligned} \tag{3.71}$$

We see that these azimuthal asymmetries just equal the corresponding structure functions divided by the azimuthal angle independent part. We also see that the cosine asymmetries correspond to the parity conserving part and the sin asymmetries correspond to the parity violating part of the cross section so the latter vanish in parity conserving processes.

The polarization of the vector meson V.—The average value of each component of the polarization is obtained from their correspondences to the probability differences in a different polarization such as $\bar{S}_{LL} = [1-3\mathcal{P}(0;0,0)]/2$ where $\mathcal{P}(m;\theta_n,\phi_n)$ is the probability for V to be in the eigenstate of Σ^n with the eigenvalue m [21]. For the five components describing the tensor polarization, we obtain

$$\bar{S}_{LL} = \frac{1}{2}\frac{\mathcal{F}_{LL}+\tilde{\mathcal{F}}_{LL}}{\mathcal{F}_U+\tilde{\mathcal{F}}_U}, \tag{3.72}$$

$$\bar{S}_{LT}^i = \frac{2}{3}\frac{\mathcal{F}_{LT}^i+\tilde{\mathcal{F}}_{LT}^i}{\mathcal{F}_U+\tilde{\mathcal{F}}_U}, \tag{3.73}$$

$$\bar{S}_{TT}^{xi} = \frac{2}{3}\frac{\mathcal{F}_{TT}^{xi}+\tilde{\mathcal{F}}_{TT}^{xi}}{\mathcal{F}_U+\tilde{\mathcal{F}}_U}, \tag{3.74}$$

where $i=x$ or y denotes different components of the polarization tensor. It is also interesting to see that the numerator $\mathcal{F}_{LT}^x$ and $\mathcal{F}_{LT}^y$ are equal to the $\cos\varphi_{LT}$ and $\sin\varphi_{LT}$ terms of $\mathcal{F}_{LT}$ respectively. They can be obtained as follows:

$$\mathcal{F}_{LT}^x = \int\frac{d\varphi_{LT}}{\pi}\cos\varphi_{LT}\mathcal{F}_{LT}, \tag{3.75}$$

$$\mathcal{F}_{LT}^y = \int\frac{d\varphi_{LT}}{\pi}\sin\varphi_{LT}\mathcal{F}_{LT}, \tag{3.76}$$

and similar for $\tilde{\mathcal{F}}_{LT}^i$. For $\mathcal{F}_{TT}^{xi}$, we have

$$\mathcal{F}_{TT}^{xx} = \int\frac{d\varphi_{TT}}{\pi}\cos 2\varphi_{TT}\mathcal{F}_{TT}, \tag{3.77}$$

$$\mathcal{F}_{TT}^{xy} = \int\frac{d\varphi_{TT}}{\pi}\sin 2\varphi_{TT}\mathcal{F}_{TT}, \tag{3.78}$$

and similar for $\tilde{\mathcal{F}}_{TT}^{xi}$. The explicit expressions can be obtained easily from those for the corresponding $\mathcal{F}_\sigma$ or $\tilde{\mathcal{F}}_\sigma$. We omit them here but simply emphasize that they are in general dependent on the variables s, ξ_1, ξ_2, p_{2T}, θ, and φ.

If we average over φ, we see that only the φ-independent terms in the expressions of $\mathcal{F}$'s and $\tilde{\mathcal{F}}$'s survive. We denote them as

$$\langle\mathcal{F}_\sigma\rangle = \int\frac{d\varphi}{2\pi}\mathcal{F}_\sigma, \tag{3.79}$$

and we obtain

$$\langle\mathcal{F}_U\rangle = (1+\cos^2\theta)F_{1U} + \sin^2\theta F_{2U} + \cos\theta F_{3U}, \tag{3.80}$$

$$\langle\tilde{\mathcal{F}}_U\rangle = 0, \tag{3.81}$$

$$\langle\mathcal{F}_L\rangle = 0, \tag{3.82}$$

$$\langle\tilde{\mathcal{F}}_L\rangle = (1+\cos^2\theta)\tilde{F}_{1L} + \sin^2\theta\tilde{F}_{2L} + \cos\theta\tilde{F}_{3L}, \tag{3.83}$$

$$\langle\mathcal{F}_T\rangle = \sin\varphi_S(\sin\theta F_{1T}^{\sin\varphi_S} + \sin 2\theta F_{2T}^{\sin\varphi_S}), \tag{3.84}$$

$$\langle\tilde{\mathcal{F}}_T\rangle = \cos\varphi_S(\sin\theta\tilde{F}_{1T}^{\cos\varphi_S} + \sin 2\theta\tilde{F}_{2T}^{\cos\varphi_S}), \tag{3.85}$$

$$\langle\mathcal{F}_{LT}\rangle = \cos\varphi_{LT}(\sin\theta F_{1LT}^{\cos\varphi_{LT}} + \sin 2\theta F_{2LT}^{\cos\varphi_{LT}}), \tag{3.86}$$

$$\langle\tilde{\mathcal{F}}_{LT}\rangle = \sin\varphi_{LT}(\sin\theta\tilde{F}_{1LT}^{\sin\varphi_{LT}} + \sin 2\theta\tilde{F}_{2LT}^{\sin\varphi_{LT}}), \tag{3.87}$$

$$\langle\mathcal{F}_{LL}\rangle = (1+\cos^2\theta)F_{1LL} + \sin^2\theta F_{2LL} + \cos\theta F_{3LL}, \tag{3.88}$$

$$\langle\tilde{\mathcal{F}}_{LL}\rangle = 0, \tag{3.89}$$

$$\langle\mathcal{F}_{TT}\rangle = \cos 2\varphi_{TT}\sin^2\theta F_{TT}^{\cos 2\varphi_{TT}}, \tag{3.90}$$

$$\langle\tilde{\mathcal{F}}_{TT}\rangle = \sin 2\varphi_{TT}\sin^2\theta\tilde{F}_{TT}^{\sin 2\varphi_{TT}}. \tag{3.91}$$

We see the similarities between different components and also the $\cos\varphi_\sigma$ or $\sin\varphi_\sigma$ term corresponding to the x or y component of the polarization. More precisely, in this case, we obtain

$$\langle\lambda\rangle = \frac{2}{3F_{Ut}}((1+\cos^2\theta)\tilde{F}_{1L} + \sin^2\theta\tilde{F}_{2L} + \cos\theta\tilde{F}_{3L}), \tag{3.92}$$

$$\langle S_{LL}\rangle = \frac{1}{2F_{Ut}}((1+\cos^2\theta)F_{1LL} + \sin^2\theta F_{2LL} + \cos\theta F_{3LL}), \tag{3.93}$$

$$\langle S_T^x\rangle = \frac{2}{3F_{Ut}}(\sin\theta\tilde{F}_{1T}^{\cos\varphi_S} + \sin 2\theta\tilde{F}_{2T}^{\cos\varphi_S}), \tag{3.94}$$

$$\langle S_T^y\rangle = \frac{2}{3F_{Ut}}(\sin\theta F_{1T}^{\sin\varphi_S} + \sin 2\theta F_{2T}^{\sin\varphi_S}), \tag{3.95}$$

$$\langle S_{LT}^x\rangle = \frac{2}{3F_{Ut}}(\sin\theta F_{1LT}^{\cos\varphi_{LT}} + \sin 2\theta F_{2LT}^{\cos\varphi_{LT}}), \tag{3.96}$$

$$\langle S_{LT}^y\rangle = \frac{2}{3F_{Ut}}(\sin\theta\tilde{F}_{1LT}^{\sin\varphi_{LT}} + \sin 2\theta\tilde{F}_{2LT}^{\sin\varphi_{LT}}), \tag{3.97}$$

$$\langle S_{TT}^{xx}\rangle = \frac{2}{3F_{Ut}}\sin^2\theta F_{TT}^{\cos 2\varphi_{TT}}, \tag{3.98}$$

$$\langle S_{TT}^{xy}\rangle = \frac{2}{3F_{Ut}}\sin^2\theta\tilde{F}_{TT}^{\sin 2\varphi_{TT}}. \tag{3.99}$$

We see that in this way we just pick up the corresponding φ-independent and, in the transverse polarization case, the $\cos\varphi_\sigma$ or $\sin\varphi_\sigma$ terms. These results are much simpler and can be used to study the corresponding components of the structure functions more conveniently. We also note that $\langle S_{LL}\rangle$, $\langle S_T^y\rangle$, $\langle S_{LT}^x\rangle$, and $\langle S_{TT}^{xx}\rangle$ are parity conserving, and the other components such as $\langle\lambda\rangle$, $\langle S_T^x\rangle$, $\langle S_{LT}^y\rangle$, and $\langle S_{TT}^{xy}\rangle$ are parity violating. This implies that if we consider parity conserving reactions, only the F terms survive and the $\tilde{F}_i$'s have to vanish. In this case we see that we have only nonzero $\langle S_{LL}\rangle$, $\langle S_T^y\rangle$, $\langle S_{LT}^x\rangle$, and $\langle S_{TT}^{xx}\rangle$. Other components such as $\langle\lambda\rangle$, $\langle S_T^x\rangle$, $\langle S_{LT}^y\rangle$, and $\langle S_{TT}^{xy}\rangle$ have to vanish.

In the case where transverse components are concerned, it is often useful to study different components with respect to the two transverse directions $\vec{e}_n$ and $\vec{e}_t$ defined as $\vec{e}_n = \vec{p}_1\times\vec{p}_2/|\vec{p}_1\times\vec{p}_2| = (-\sin\varphi,\cos\varphi)$ and $\vec{e}_t = \vec{p}_{2T}/|\vec{p}_{2T}| = (\cos\varphi,\sin\varphi)$, i.e., the normal and tangent of the hadron-hadron plane respectively. The corresponding components of the polarization are given by exactly the same equations such as Eqs. (3.73) and (3.74) with $i=n$ or t. It can easily be shown that such components can also be obtained from Eqs. (3.73) and (3.74) with φ_σ being replaced by $\varphi_\sigma - \varphi$ in the integrations given in Eqs. (3.75)–(3.78), e.g.,

$$\mathcal{F}_T^n = \int\frac{d\varphi_S}{\pi}\sin(\varphi_S-\varphi)\mathcal{F}_T, \tag{3.100}$$

$$\mathcal{F}_T^t = \int\frac{d\varphi_S}{\pi}\cos(\varphi_S-\varphi)\mathcal{F}_T, \tag{3.101}$$

$$\mathcal{F}_{LT}^n = \int\frac{d\varphi_{LT}}{\pi}\sin(\varphi_{LT}-\varphi)\mathcal{F}_{LT}, \tag{3.102}$$

$$\mathcal{F}_{LT}^t = \int\frac{d\varphi_{LT}}{\pi}\cos(\varphi_{LT}-\varphi)\mathcal{F}_{LT}, \tag{3.103}$$

$$\mathcal{F}_{TT}^{nn} = -\int \frac{d\varphi_{TT}}{\pi} \cos(2\varphi_{TT} - 2\varphi)\mathcal{F}_{TT}, \tag{3.104}$$

$$\mathcal{F}_{TT}^{nt} = \int \frac{d\varphi_{TT}}{\pi} \sin(2\varphi_{TT} - 2\varphi)\mathcal{F}_{TT}. \tag{3.105}$$

It will be also interesting to see the results after integrating over φ, we just pick the corresponding $\cos(\varphi_\sigma - \varphi)$ or $\sin(\varphi_\sigma - \varphi)$ terms. More precisely, we have

$$\langle S_T^n \rangle = \frac{2}{3F_{Ut}}[(1+\cos^2\theta)F_{1T}^{\sin(\varphi_S-\varphi)} + \sin^2\theta F_{2T}^{\sin(\varphi_S-\varphi)} + \cos\theta F_{3T}^{\sin(\varphi_S-\varphi)}], \tag{3.106}$$

$$\langle S_T^t \rangle = \frac{2}{3F_{Ut}}[(1+\cos^2\theta)\tilde{F}_{1T}^{\cos(\varphi_S-\varphi)} + \sin^2\theta \tilde{F}_{2T}^{\cos(\varphi_S-\varphi)} + \cos\theta \tilde{F}_{3T}^{\cos(\varphi_S-\varphi)}], \tag{3.107}$$

$$\langle S_{LT}^n \rangle = \frac{2}{3F_{Ut}}[(1+\cos^2\theta)\tilde{F}_{1LT}^{\sin(\varphi_{LT}-\varphi)} + \sin^2\theta \tilde{F}_{2LT}^{\sin(\varphi_{LT}-\varphi)} + \cos\theta \tilde{F}_{3LT}^{\sin(\varphi_{LT}-\varphi)}], \tag{3.108}$$

$$\langle S_{LT}^t \rangle = \frac{2}{3F_{Ut}}[(1+\cos^2\theta)F_{1LT}^{\cos(\varphi_{LT}-\varphi)} + \sin^2\theta F_{2LT}^{\cos(\varphi_{LT}-\varphi)} + \cos\theta F_{3LT}^{\cos(\varphi_{LT}-\varphi)}], \tag{3.109}$$

$$\langle S_{TT}^{nn} \rangle = \frac{-2}{3F_{Ut}}[(1+\cos^2\theta)F_{1TT}^{\cos(2\varphi_{TT}-2\varphi)} + \sin^2\theta F_{2TT}^{\cos(2\varphi_{TT}-2\varphi)} + \cos\theta F_{3TT}^{\cos(2\varphi_{TT}-2\varphi)}], \tag{3.110}$$

$$\langle S_{TT}^{nt} \rangle = \frac{2}{3F_{Ut}}[(1+\cos^2\theta)\tilde{F}_{1TT}^{\sin(2\varphi_{TT}-2\varphi)} + \sin^2\theta \tilde{F}_{2TT}^{\sin(2\varphi_{TT}-2\varphi)} + \cos\theta \tilde{F}_{3TT}^{\sin(2\varphi_{TT}-2\varphi)}]. \tag{3.111}$$

It is interesting to see that all the average transverse polarizations with respect to the hadron-hadron plane take a similar form in terms of the corresponding structure functions. We also see that in this case $\langle S_T^n \rangle$, $\langle S_{LT}^t \rangle$, and $\langle S_{TT}^{nn} \rangle$ are parity conserving while $\langle S_T^t \rangle$, $\langle S_{LT}^n \rangle$, and $\langle S_{TT}^{nt} \rangle$ are parity violating.

In experiments, it is usually very difficult to study azimuthal dependence and hadron polarization simultaneously. From the kinematic analysis given above, we see that we can either study the azimuthal asymmetries given by Eqs. (3.67)–(3.70) in the unpolarized case, or study the longitudinal hadron polarization in the helicity frame and transverse polarizations with respect to the lepton-hadron plane or the hadron-hadron plane averaged over the azimuthal angle φ to study the corresponding structure functions as given by Eqs. (3.94)–(3.99) or Eqs. (3.106)–(3.111).

C. Reduce to $e^+e^- \to VX$

It is also clear that if we consider the inclusive process $e^+e^- \to VX$, we should integrate over p_2, i.e., carrying out the integration $\int d^3p_2/(2E_2)$ to obtain the corresponding hadronic tensor and/or cross section. In this case, we obtain three for unpolarized, three for λ-, three for S_{LL}-, four for S_T-, four for S_{LT}-, and two for the S_{TT}-dependent parts. The basic Lorentz tensors for the hadronic tensor obtained in this case are given by

$$h_{Ui,in}^{S\mu\nu} = \left\{ g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}, p_{1q}^\mu p_{1q}^\nu \right\}, \tag{3.112}$$

$$\tilde{h}_{U,in}^{A\mu\nu} = \varepsilon^{\mu\nu q p_1}, \tag{3.113}$$

$$h_{V,in}^{S\mu\nu} = \varepsilon^{\{\mu q p_1 S} p_{1q}^{\nu\}}, \tag{3.114}$$

$$\tilde{h}_{Vi,in}^{S\mu\nu} = \{(q\cdot S)h_{Uj,in}^{S\mu\nu}, S_q^{\{\mu} p_{1q}^{\nu\}}\}, \tag{3.115}$$

$$h_{Vi,in}^{A\mu\nu} = \{(q\cdot S)\tilde{h}_{U,in}^{A\mu\nu}, \varepsilon^{[\mu q p_1 S} p_{1q}^{\nu]}\}, \tag{3.116}$$

$$\tilde{h}_{V,in}^{A\mu\nu} = S_q^{[\mu} p_{1q}^{\nu]}, \tag{3.117}$$

$$h_{LLi,in}^{S\mu\nu} = S_{LL} h_{Ui,in}^{S\mu\nu}, \tag{3.118}$$

$$\tilde{h}_{LL,in}^{A\mu\nu} = S_{LL} \tilde{h}_{U,in}^{A\mu\nu}, \tag{3.119}$$

$$h_{LT,in}^{S\mu\nu} = S_{LT}^{\{\mu} p_{1q}^{\nu\}}, \tag{3.120}$$

$$\tilde{h}_{LT,in}^{S\mu\nu} = \varepsilon^{\{\mu q p_1 S_{LT}} p_{1q}^{\nu\}}, \tag{3.121}$$

$$h_{LT,in}^{A\mu\nu} = S_{LT}^{[\mu} p_{1q}^{\nu]} \tag{3.122}$$

$$\tilde{h}_{LT,in}^{A\mu\nu} = \varepsilon^{[\mu q p_1 S_{LT}} p_{1q}^{\nu]}, \tag{3.123}$$

$$h_{TTi,in}^{S\mu\nu} = S_{TT}^{\mu\nu}, \tag{3.124}$$

$$\tilde{h}_{TT,in}^{S\mu\nu} = \varepsilon^{\{\mu\alpha q p_1} S_{TT\alpha}^{\nu\}}. \tag{3.125}$$

There are totally 19 such independent basic Lorentz tensors, ten of them are space reflection even and nine of them are space reflection odd. We note in particular the spin-dependent time-reversal odd term $h_{V,in}^{S\mu\nu} = \varepsilon^{\{\mu q p_1 S} p_{1q}^{\nu\}}$ novel to deep-inelastic lepton-nucleon scattering (DIS) as discussed in [27]. This corresponds to single transverse polarization of V with respect to the lepton-hadron plane. There could be also parity violating transverse polarization in the lepton-hadron plane described by the last one in Eq. (3.115), i.e., $\tilde{h}^{S\mu\nu}_{V3,in} = S_q^{\{\mu} p_{1q}^{\nu\}}$.

The inclusive process $e^+e^- \to VX$ can also be studied in the helicity-GJ frame. Formerly, the differential cross section for $e^+e^- \to VX$ takes exactly the same form as that for $e^+e^- \to V\pi X$ integrated over φ. The corresponding inclusive structure functions just have one-to-one correspondence to those given by Eqs. (3.80)–(3.91). They are just equal to the counterparts in Eqs. (3.80)–(3.91) integrated over ξ_2 and p_{2T}^2. In this case, we can study the longitudinal polarization and the transverse polarization with respect to the lepton-hadron plane that have similar expressions in terms of the structure functions as those given by Eqs. (3.92)–(3.99).

IV. HADRONIC TENSOR IN TERMS OF FFs

We now calculate the hadronic tensor and differential cross section in the partonic picture at leading order in pQCD but with leading and twist-3 contributions. In this section we present the results obtained for the hadronic tensor.

In the partonic picture at the leading order in pQCD, we need to consider the contributions from the diagrams shown in Figs. 2 and 3 just as in [7] where spin-1/2 hadrons are considered. We need to perform the collinear expansion and pick up the results up to the order $1/Q$ in order to get the twist-3 contributions. Collinear expansion was first proposed for the inclusive process [28,29] and has now been applied to all processes where one hadron is explicitly involved [15,17,30]. Systematic derivations have been given for such processes (for a recent short summary see, e.g., [20]). However, for processes with no less than two hadrons involved, systematic derivation for collinear expansion is still lacking. Usually, one just picks up terms up to $1/Q$ from these diagrams [6–13,16]. We do it in the same way in the following of this paper.

A. Hadronic tensor in the collinear frame

The leading power contribution from Fig. 2 gives us the leading twist contribution where no transverse gluon exchange is involved. The longitudinal gluon exchanges lead to the gauge link that is needed to keep the quark-quark correlator gauge invariant. Up to twist-3, we need the next-to-leading power contribution from Fig. 2 and also the leading power contributions from Fig. 3, where the quark-gluon-quark correlator is involved. We use the definition of the quark-gluon-quark correlator as given in Eq. (2.14), i.e., to use the covariant derivative D instead of A. This is not only to use the simple relationships as given by Eqs. (2.16) and (2.17) but also to be consistent to the cases of $e^+e^- \to V\bar{q}X$ and $e^+e^- \to VX$ where collinear expansion has already been systematically proven [15,17]. To do so, we need to pick up the corresponding $k_\perp$ terms from Fig. 2 and add them to those from Fig. 3. In this way, we obtain $W_{\mu\nu} = \tilde{W}^{(0)}_{\mu\nu} + \tilde{W}^{(1)}_{\mu\nu} - \Delta\tilde{W}^{(0)}_{\mu\nu}$. For the contribution $\tilde{W}^{(0)}_{\mu\nu}$ from Fig. 2, we have

$$\tilde{W}^{(0)}_{\mu\nu} = \frac{1}{p_1^+ p_2^-} \int \frac{d^2k_\perp}{(2\pi)^2} \frac{d^2k'_\perp}{(2\pi)^2} \delta^2(k_\perp + k'_\perp - q_\perp) \times \mathrm{Tr}[\Xi^{(0)}(z_1, k_\perp, p_1, S)\Gamma_\mu \bar{\Xi}^{(0)}(z_2, k'_\perp, p_2)\Gamma_\nu]. \tag{4.1}$$

Corresponding to Fig. 3(a), we have

$$\tilde{W}^{(1a)}_{\mu\nu} = \frac{-1}{\sqrt{2}Q p_1^+ p_2^-} \int \frac{d^2k_\perp}{(2\pi)^2} \frac{d^2k'_\perp}{(2\pi)^2} \delta^2(k_\perp + k'_\perp - q_\perp) \times \mathrm{Tr}[\Gamma_\mu \bar{\Xi}^{(0)}(z_2, k'_\perp, p_2)\gamma_\rho \bar{n}\Gamma_\nu \Xi^{(1)\rho}(z_1, k_\perp, p_1, S)], \tag{4.2}$$

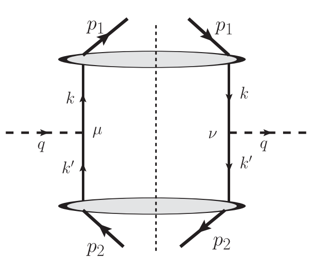

FIG. 2. Feynman diagram for $Z \to V\pi X$ without gluon exchange that contributes at leading and higher twists.

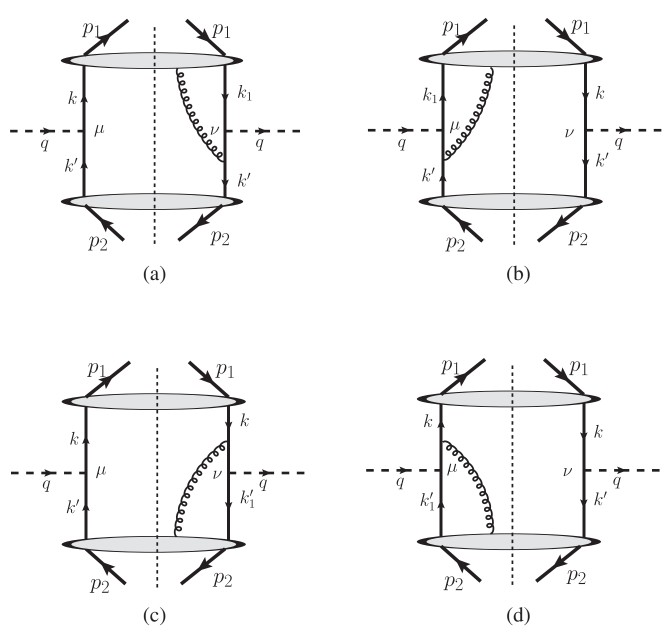

FIG. 3. Feynman diagrams for $Z \to V\pi X$ with one gluon exchange that contributes at twist-3 and higher twists. Here, in (a) and (b), we have quark-gluon-quark correlator for the fragmentation of the quark, and in (c) and (d) we have that of the anti-quark.

$$\Delta\tilde{W}^{(0a)}_{\mu\nu} = \frac{1}{\sqrt{2}Qp_1^+p_2^-}\int\frac{d^2k_\perp}{(2\pi)^2}\frac{d^2k'_\perp}{(2\pi)^2}\delta^2(k_\perp+k'_\perp-q_\perp) \times k^\rho_\perp \mathrm{Tr}[\Gamma_\mu\bar{\Xi}^{(0)}(z_2,k'_\perp,p_2)\gamma_\rho\bar{n}\Gamma_\nu\Xi^{(0)}(z_1,k_\perp,p_1,S)], \tag{4.3}$$

and similar for those from Figs. 3(b)–3(d). The transverse momentum dependent quark-quark or quark-gluon-quark correlators, $\Xi^{(0)}(z,k_\perp,p,S)$ or $\Xi^{(1)\rho}(z,k_\perp,p,S)$ are given by Eqs. (2.5) or (2.14) respectively. We use $\bar{\Xi}$ to denote that for antiquark fragmentation that differs from the corresponding one for the quark by exchanging ψ and $\bar{\psi}$ in the definition. Here as well as in the following of this paper, for explicitness, we consider only $q\to VX$ and $\bar{q}\to\pi X$. The complete results should be the sum of these contributions and those from $\bar{q}\to VX$ and $q\to\pi X$. The latter are just obtained simply by changing the Ξ's to the corresponding $\bar{\Xi}$'s and $\bar{\Xi}$'s to the corresponding Ξ's. Also a summation over the flavor of q is implicit.

We emphasize in particular that these expressions (4.1)–(4.3) are obtained from Figs. 2 and 3 and they are also straightforward extensions of the results obtained for $e^+e^-\to V\bar{q}X$ which is a special case by setting $|p_2,X\rangle$ as an antiquark final state $|k'\rangle$. In the latter case $\tilde{W}^{(0)}_{\mu\nu}-\Delta\tilde{W}^{(0)}_{\mu\nu}$ together reduces to the corresponding results of $\tilde{W}^{(0)}_{\mu\nu}$ while $\tilde{W}^{(1)}_{\mu\nu}$ reduces to the corresponding result directly.

To obtain the corresponding results for the hadronic tensors, we need to substitute the Lorentz decompositions of the quark-quark and quark-gluon-quark correlators as given by the equations in Appendix A into the above Eqs. (4.1)–(4.3) and carry out the traces. We note that all the decompositions of the quark-quark and quark-gluon-quark correlators are given in the collinear frame of the corresponding hadron; i.e., the direction of motion of the hadron is taken as the longitudinal direction. Hence, the most convenient frame to carry out the calculations of the hadronic tensor is the collinear frame of the hadron. Fortunately, in the case we discuss here, we have only two hadrons and we can make a Lorentz transformation into a frame where the two hadrons move in opposite directions. We call it the collinear frame of the two hadrons. We first present the results of the hadronic tensor in this frame and then transform them into the helicity-GJ frame.

1. Hadronic tensor at twist-2

The leading twist contribution to the hadronic tensor comes solely from $\tilde{W}^{(0)}_{\mu\nu}$ given by Eq. (4.1). To obtain the results, we insert the leading twist parts for the quark-quark correlator given in Appendix A. The unpolarized part and the vector polarization dependent parts are the same as those for spin-1/2 hadrons and can be found, e.g., in [14]. We present here for completeness and for unification of notations. First of all, the simplest case, i.e., the unpolarized part is given by

$$W^{(0)U}_{\mu\nu}(q,p_1,p_2) = \frac{4}{z_1z_2}\int\frac{d^2k_\perp}{(2\pi)^2}\frac{d^2k'_\perp}{(2\pi)^2}\delta^2(k_\perp+k'_\perp-q_\perp) \times\Big\{-(c_1^q g_{\perp\mu\nu}+ic_3^q\varepsilon_{\perp\mu\nu})D_1(z_1,k_\perp)\bar{D}_1(z_2,k'_\perp) + \frac{4c_2^q}{M_1M_2}(k_{\perp\{\mu}k'_{\perp\nu\}}-k_\perp\cdot k'_\perp g_{\perp\mu\nu}) \times H_1^\perp(z_1,k_\perp)\bar{H}_1^\perp(z_2,k'_\perp)\Big\}, \tag{4.4}$$

where $c_2^q=(c_V^q)^2-(c_A^q)^2$; $z_1\approx\xi_1$ and $z_2\approx\xi_2$ up to $1/Q$. To make the results look more concise and explicit, we introduce the basic Lorentz tensors similar to those defined in [17], i.e.,

$$c_{\perp\mu\nu}=c_1^q g_{\perp\mu\nu}+ic_3^q\varepsilon_{\perp\mu\nu}, \tag{4.5}$$

$$\tilde{c}_{\perp\mu\nu}=c_3^q g_{\perp\mu\nu}+ic_1^q\varepsilon_{\perp\mu\nu}, \tag{4.6}$$

$$\alpha_{\perp\mu\nu}(a,b)=a_{\perp\{\mu}b_{\perp\nu\}}-(a_\perp\cdot b_\perp)g_{\perp\mu\nu}, \tag{4.7}$$

for two Lorentz vectors a and b. We will also omit the arguments of FFs in the expressions in the following of this paper. Since we are considering only the case of $q\to VX$ and $\bar{q}\to\pi X$, this omission will not cause any ambiguity. The FFs defined via Ξ's, i.e., D's, G's, E's, and H's, are for $q\to VX$ and have the arguments $(z_1,k_\perp)$, while those defined via $\bar{\Xi}$'s, i.e., $\bar{D}$'s, $\bar{G}$'s, $\bar{E}$'s, and $\bar{H}$'s are for $\bar{q}\to\pi X$ and have the arguments $(z_2,k'_\perp)$. With such simplified notations, we have

$$W^{(0)U}_{\mu\nu} = \frac{4}{z_1z_2}\int\frac{d^2k_\perp}{(2\pi)^2}\frac{d^2k'_\perp}{(2\pi)^2}\delta^2(k_\perp+k'_\perp-q_\perp) \times\Big\{-c_{\perp\mu\nu}D_1\bar{D}_1+\frac{4c_2^q}{M_1M_2}\alpha_{\perp\mu\nu}(k,k')H_1^\perp\bar{H}_1^\perp\Big\}. \tag{4.8}$$

We see that for the unpolarized part at twist-2, we have chiral even contribution from D_1 convoluted with $\bar{D}_1$ and chiral odd contribution from $H_1^\perp$ convoluted with $\bar{H}_1^\perp$. We also note that for the chiral even contribution, there is a symmetric and an antisymmetric part. However for the chiral odd contribution, there is only a symmetric part.

For the vector polarization dependent part, we write the longitudinally and transversely polarized parts separately. For the longitudinally polarized part, we have

$$W^{(0)L}_{\mu\nu} = \frac{4\lambda}{z_1z_2}\int\frac{d^2k_\perp}{(2\pi)^2}\frac{d^2k'_\perp}{(2\pi)^2}\delta^2(k_\perp+k'_\perp-q_\perp) \times\Big\{\tilde{c}_{\perp\mu\nu}G_{1L}\bar{D}_1+\frac{4c_2^q}{M_1M_2}\alpha_{\perp\mu\nu}(\tilde{k}',k)H_{1L}^\perp\bar{H}_1^\perp\Big\}. \tag{4.9}$$

We see that besides the helicity λ factor, this takes a quite similar form as that for the unpolarized part. Here, we have contributions from G_{1L} convoluted with $\bar{D}_1$ and from $H_{1L}^{\perp}$ with $\bar{H}_1^{\perp}$. For the transverse polarization dependent part, we have

$$W_{\mu\nu}^{(0)T} = \frac{4}{z_1 z_2}\int \frac{d^2k_\perp}{(2\pi)^2}\frac{d^2k'_\perp}{(2\pi)^2}\delta^2(k_\perp + k'_\perp - q_\perp)\left\{\frac{k_\perp\cdot S_T}{M_1}\left[\tilde{c}_{\perp\mu\nu}G_{1T}^{\perp}\bar{D}_1 + \frac{4c_2^q}{M_1M_2}\alpha_{\perp\mu\nu}(\tilde{k}',k)H_{1T}^{\perp}\bar{H}_1^{\perp}\right]\right.$$
$$\left. - \frac{\tilde{k}_\perp\cdot S_\perp}{M_1}c_{\perp\mu\nu}D_{1T}^{\perp}\bar{D}_1 + \frac{4c_2^q}{M_2}\alpha_{\perp\mu\nu}(\tilde{k}',S)H_{1T}\bar{H}_1^{\perp}\right\}. \tag{4.10}$$

Because there are two transverse directions, this part looks more complicated. We see clearly that we have both contributions in $k_\perp$ or transverse to $k_\perp$ (i.e., in $\tilde{k}_\perp$) directions.

The S_{LL}-dependent part looks very much the same as the unpolarized part, i.e.,

$$W_{\mu\nu}^{(0)LL} = \frac{4S_{LL}}{z_1 z_2}\int \frac{d^2k_\perp}{(2\pi)^2}\frac{d^2k'_\perp}{(2\pi)^2}\delta^2(k_\perp + k'_\perp - q_\perp)\left\{-c_{\perp\mu\nu}D_{1LL}\bar{D}_1 + \frac{4c_2^q}{M_1M_2}\alpha_{\perp\mu\nu}(k,k')H_{1LL}^{\perp}\bar{H}_1^{\perp}\right\}, \tag{4.11}$$

where we have the chiral even contribution from D_{1LL} convoluted with $\bar{D}_1$ and chiral odd part from $H_{1LL}^{\perp}$ with $\bar{H}_1$.

For the S_{LT}- and S_{TT}-dependent part, we have

$$W_{\mu\nu}^{(0)LT} = \frac{4}{z_1 z_2}\int \frac{d^2k_\perp}{(2\pi)^2}\frac{d^2k'_\perp}{(2\pi)^2}\delta^2(k_\perp + k'_\perp - q_\perp)\left\{\frac{k_\perp\cdot S_{LT}}{M_1}\left[-c_{\perp\mu\nu}D_{1LT}^{\perp}\bar{D}_1 + \frac{4c_2^q}{M_1M_2}\alpha_{\perp\mu\nu}(k,k')H_{1LT}^{\perp}\bar{H}_1^{\perp}\right]\right.$$
$$\left. + \frac{\tilde{k}_\perp\cdot S_{LT}}{M_1}\tilde{c}_{\perp\mu\nu}G_{1LT}^{\perp}\bar{D}_1 + \frac{4c_2^q}{M_2}\alpha_{\perp\mu\nu}(k',S_{LT})H_{1LT}\bar{H}_1^{\perp}\right\}, \tag{4.12}$$

$$W_{\mu\nu}^{(0)TT} = \frac{4}{z_1 z_2}\int \frac{d^2k_\perp}{(2\pi)^2}\frac{d^2k'_\perp}{(2\pi)^2}\delta^2(k_\perp + k'_\perp - q_\perp)\left\{\frac{S_{TT}^{kk}}{M_1^2}\left[-c_{\perp\mu\nu}D_{1TT}^{\perp}\bar{D}_1 + \frac{4c_2^q}{M_1M_2}\alpha_{\perp\mu\nu}(k,k')H_{1TT}^{\perp}\bar{H}_1^{\perp}\right]\right.$$
$$\left. + \frac{S_{TT}^{\tilde{k}k}}{M_1^2}\tilde{c}_{\perp\mu\nu}G_{1TT}^{\perp}\bar{D}_1 + \frac{4c_2^q}{M_1M_2}\alpha_{\perp\mu\nu}(k',S_{TT}^{k})H_{1TT}'^{\perp}\bar{H}_1^{\perp}\right\}. \tag{4.13}$$

We see clearly the similarities and differences between them and the transverse polarization dependent part. We note once more that the chiral even contributions contain a symmetric and an antisymmetric part given by the basic tensor $c_{\perp\mu\nu}$ or $\tilde{c}_{\perp\mu\nu}$ while the chiral odd contributions are always characterized by c_2^q and have only symmetric tensor $\alpha_{\perp\mu\nu}$.

2. Hadronic tensor at twist-3

The twist-3 contribution to the hadronic tensor comes from both Eqs. (4.1) and (4.2). In Eq. (4.1), we either expand $\Xi^{(0)}$ to leading twist and $\bar{\Xi}^{(0)}$ to twist-3 or $\bar{\Xi}^{(0)}$ to leading twist and $\Xi^{(0)}$ to twist-3. In Eq. (4.2), we expand all the Ξ's to their leading twist contribution. The equations are a bit longer than those at leading twist, and we present as examples the results for the unpolarized and S_{LL}-dependent parts here but other parts in the Appendix,

$$W_{\mu\nu}^{(1)U} = \frac{4}{z_1 z_2}\int \frac{d^2k_\perp}{(2\pi)^2}\frac{d^2k'_\perp}{(2\pi)^2}\delta^2(k_\perp + k'_\perp - q_\perp)\left\{\frac{1}{p_1^+}[\omega_{\mu\nu}(k)D^{\perp} + \tilde{\omega}_{\mu\nu}(\tilde{k})G^{\perp}]\bar{D}_1 - \frac{1}{p_2^-}D_1[\omega_{\mu\nu}(k')\bar{D}^{\perp} + \tilde{\omega}_{\mu\nu}(\tilde{k}')\bar{G}^{\perp}]\right.$$
$$- \frac{2c_2^qM_2}{M_1p_2^-}H_1^{\perp}[2(k_n - k_{\bar{n}})_{\{\mu\nu\}}\bar{H} + i(k_n - k_{\bar{n}})_{[\mu\nu]}\bar{E}] + \frac{2c_2^qM_1}{M_2p_1^+}[2(k'_n - k'_{\bar{n}})_{\{\mu\nu\}}H + i(k'_n - k'_{\bar{n}})_{[\mu\nu]}E]\bar{H}_1^{\perp}$$
$$\left. + \frac{\sqrt{2}}{Q}\left[\omega_{\mu\nu}(k',k)D_1\bar{D}_1 - \frac{4c_2^q}{M_1M_2}\omega_{\mu\nu}^{(n)}(k,k')H_1^{\perp}\bar{H}_1^{\perp}\right]\right\}, \tag{4.14}$$

where we introduce the shorthanded notations defined as

$$a_{n\{\mu\nu\}} \equiv a_{\perp\{\mu}n_{\nu\}}, \qquad a_{n[\mu\nu]} \equiv a_{\perp[\mu}n_{\nu]}, \tag{4.15}$$

$$\omega_{\mu\nu}(a,b) = c_1^q(a_n + b_{\bar{n}})_{\{\mu\nu\}} - ic_3^q(\tilde{a}_n + \tilde{b}_{\bar{n}})_{[\mu\nu]}, \tag{4.16}$$

$$\tilde{\omega}_{\mu\nu}(a,b) = c_3^q(a_n + b_{\bar{n}})_{\{\mu\nu\}} - ic_1^q(\tilde{a}_n + \tilde{b}_{\bar{n}})_{[\mu\nu]}, \tag{4.17}$$

$$\omega_{\mu\nu}^{(n)}(a,b) = (a_\perp^2 b_{\bar{n}} + b_\perp^2 a_n)_{\{\mu\nu\}}, \tag{4.18}$$

and $\omega_{\mu\nu}(a) \equiv \omega_{\mu\nu}(a,-a)$, $\tilde{\omega}_{\mu\nu}(a) \equiv \tilde{\omega}_{\mu\nu}(a,-a)$.

The S_{LL}-dependent part looks very much similar, i.e.,

$$\begin{aligned} W_{\mu\nu}^{(1)LL} = \frac{4S_{LL}}{z_1 z_2}\int \frac{d^2k_\perp}{(2\pi)^2}\frac{d^2k'_\perp}{(2\pi)^2}\delta^2(k_\perp + k'_\perp - q_\perp)\Bigg\{\frac{1}{p_1^+}[\omega_{\mu\nu}(k)D_{LL}^\perp + \tilde{\omega}_{\mu\nu}(\tilde{k})G_{LL}^\perp]\bar{D}_1 - \frac{1}{p_2^-}D_{1LL}[\omega_{\mu\nu}(k')\bar{D}^\perp + \tilde{\omega}_{\mu\nu}(\tilde{k}')\bar{G}^\perp] \\ - \frac{2c_2^q M_2}{M_1 p_2^-}H_{1LL}^\perp[2(k_n - k_{\bar{n}})_{\{\mu\nu\}}\bar{H} + i(k_n - k_{\bar{n}})_{[\mu\nu]}\bar{E}] + \frac{2c_2^q M_1}{M_2 p_1^+}[2(k'_n - k'_{\bar{n}})_{\{\mu\nu\}}H_{LL} + i(k'_n - k'_{\bar{n}})_{[\mu\nu]}E_{LL}]\bar{H}_1^\perp \\ + \frac{\sqrt{2}}{Q}\left[\omega_{\mu\nu}(k',k)D_{1LL}\bar{D}_1 - \frac{4c_2^q}{M_1M_2}\omega_{\mu\nu}^{(n)}(k,k')H_{1LL}^\perp\bar{H}_1^\perp\right]\Bigg\}. \end{aligned} \tag{4.19}$$

B. Transform into the helicity-GJ frame

We now transform the hadronic tensor into helicity-GJ frame as described in Sec. III B 2. Since our goal is to express the hadronic tensor by FFs that are usually defined in the collinear way, we should just keep the FFs defined this way and transform the coefficients into the helicity-GJ frame of the vector meson V. This is achieved by replacing the vectors and tensors in the hadronic tensor by their expressions in the helicity-GJ frame. Up to $1/Q$, we have [7,13]

$$(k_{\perp\mu})_{\rm coll} = k_{\perp\mu} - \sqrt{2}q_\perp\cdot k_\perp \bar{n}_\mu/Q + \cdots, \tag{4.20}$$

$$(g_{\perp\mu\nu})_{\rm coll} = g_{\perp\mu\nu} - \sqrt{2}q_{\bar{n}\{\mu\nu\}}/Q + \cdots, \tag{4.21}$$

$$(\varepsilon_{\perp\mu\nu})_{\rm coll} = \varepsilon_{\perp\mu\nu} + \sqrt{2}\tilde{q}_{\bar{n}[\mu\nu]}/Q + \cdots, \tag{4.22}$$

and $q_\perp = -p_{2T}/z_2 + \cdots$, where $\cdots$ are higher power suppressed terms. We see that the differences are all higher twist. It implies that the leading twist part is unchanged but there are additional twist-3 terms generated by transforming the twist-2 parts. E.g., for the unpolarized part, we have

$$\begin{aligned} \delta W_{\mu\nu}^{(1)U} = \frac{4\sqrt{2}}{z_1 z_2 Q}\int \frac{d^2k_\perp}{(2\pi)^2}\frac{d^2k'_\perp}{(2\pi)^2}\delta^2(k_\perp + k'_\perp - q_\perp) \\ \times\Bigg\{-(c_1^q q_{\bar{n}\{\mu\nu\}} - ic_3^q\tilde{q}_{\bar{n}[\mu\nu]})D_1\bar{D}_1 \\ + \frac{4c_2^q}{M_1M_2}(k_\perp^2 k'_{\bar{n}} + k'^2_\perp k_{\bar{n}})_{\{\mu\nu\}}H_1^\perp\bar{H}_1^\perp\Bigg\}. \end{aligned} \tag{4.23}$$

Others are given in Appendix B.

V. STRUCTURE FUNCTIONS IN TERMS OF FFs

Making a Lorentz contraction with the leptonic tensor, we obtain the cross section and the structure functions. The parton model results for the structure functions are given as the convolution of the gauge invariant TMD FFs in the form,

$$\begin{aligned} \mathcal{C}[wD\bar{D}] = \frac{1}{z_1 z_2}\int \frac{d^2k_\perp}{(2\pi)^2}\frac{d^2k'_\perp}{(2\pi)^2}\delta^2(k_\perp + k'_\perp - q_\perp) \\ \times w(k_\perp, k'_\perp)D(z_1,k_\perp)\bar{D}(z_2,k'_\perp). \end{aligned} \tag{5.1}$$

The weight w is a scalar function of $k_\perp$ and $k'_\perp$. As in [14], we introduce the following dimensionless scalars,

$$w_0 = -k_\perp^2/M_1^2, \tag{5.2}$$

$$\bar{w}_0 = -k'^2_\perp/M_2^2, \tag{5.3}$$

$$w_1 = -p_{2T}\cdot k_\perp/M_1|\vec{p}_{2T}|, \tag{5.4}$$

$$\bar{w}_1 = -p_{2T}\cdot k'_\perp/M_2|\vec{p}_{2T}|, \tag{5.5}$$

$$w_2 = -k_\perp\cdot k'_\perp/M_1M_2. \tag{5.6}$$

Others are just functions of them and are given when needed.

A. Structure functions at twist-2

We note that the twist-2 results presented here are for leading order in pQCD. Formally they just correspond to the results obtained from the naive or intuitive parton model.

We introduce a second digital in the subscript to specify the contributions at twist level, e.g., $F_{jTi}^{\sin(\varphi_S-\varphi)}$, and $i=1,2,3,\ldots$ to specify the twist-$(i+1)$ contributions. The unpolarized and vector polarization dependent parts can be derived from those given in, e.g., [14]. We list them here for completeness and comparison. We list only those nonzero structure functions. Those not listed are zero at twist-2.

For the unpolarized part, we have

$$F_{1U1} = 2c_1^e c_1^q \mathcal{C}[D_1 \bar{D}_1], \tag{5.7}$$

$$F_{3U1} = 4c_3^e c_3^q \mathcal{C}[D_1 \bar{D}_1], \tag{5.8}$$

$$F_{U1}^{\cos 2\varphi} = -8c_1^e c_2^q \mathcal{C}[w_{hh} H_1^\perp \bar{H}_1^\perp], \tag{5.9}$$

where $w_{hh} = 2w_1\bar{w}_1 - w_2$. The other six F_U's are zero at twist-2. We note in particular that there is a twist-2 contribution to $\cos 2\varphi$ due to the Collins function [4] but no such contribution to $\cos\varphi$ or $\sin\varphi$.

The longitudinal polarization dependent part is very much the same as the unpolarized part. There are three nonzero F_L's at twist-2, they are given by

$$\tilde{F}_{1L1} = -2c_1^e c_3^q \mathcal{C}[G_{1L} \bar{D}_1], \tag{5.10}$$

$$\tilde{F}_{3L1} = -4c_3^e c_1^q \mathcal{C}[G_{1L} \bar{D}_1], \tag{5.11}$$

$$F_{L1}^{\sin 2\varphi} = -8c_1^e c_2^q \mathcal{C}[w_{hh} H_{1L}^\perp \bar{H}_1^\perp]. \tag{5.12}$$

We see a one-to-one correspondence to the unpolarized terms. More precisely we have that $\tilde{F}_{jL1}$ just corresponds to F_{jU1} upon exchange of D_1 to G_{1L} and $F_{jL1}^{\sin 2\varphi}$ just corresponds to $F_{U1}^{\cos 2\varphi}$ upon exchange of $H_1^\perp$ to $H_{1L}^\perp$.

For the transverse polarization dependent part, we have

$$F_{1T1}^{\sin(\varphi_S-\varphi)} = 2c_1^e c_1^q \mathcal{C}[w_1 D_{1T}^\perp \bar{D}_1], \tag{5.13}$$

$$F_{3T1}^{\sin(\varphi_S-\varphi)} = 4c_3^e c_3^q \mathcal{C}[w_1 D_{1T}^\perp \bar{D}_1], \tag{5.14}$$

$$\tilde{F}_{1T1}^{\cos(\varphi_S-\varphi)} = 2c_1^e c_3^q \mathcal{C}[w_1 G_{1T}^\perp \bar{D}_1], \tag{5.15}$$

$$\tilde{F}_{3T1}^{\cos(\varphi_S-\varphi)} = 4c_3^e c_1^q \mathcal{C}[w_1 G_{1T}^\perp \bar{D}_1], \tag{5.16}$$

$$F_{T1}^{\sin(\varphi_S+\varphi)} = -8c_1^e c_2^q \mathcal{C}[\bar{w}_1 \mathcal{H}_{1T}^\perp \bar{H}_1^\perp], \tag{5.17}$$

$$F_{T1}^{\sin(\varphi_S-3\varphi)} = -8c_1^e c_2^q \mathcal{C}[w_{hh}^t H_{1T}^\perp \bar{H}_1^\perp], \tag{5.18}$$

where $w_{hh}^t = w_1 w_{hh} - w_0\bar{w}_1/2$, $\mathcal{H}_{1T}^\perp$ is defined by Eq. (A20). We see that there are six nonzero transverse polarization dependent structure functions (F_T or $\tilde{F}_T$) at twist-2, four of them are parity conserving and the other two are parity violating.

We see that among the 36 spin-independent and vector polarization dependent structure functions, 12 of them have twist-2 contributions while the other 24 are zero at twist-2. Among these 12 are nonzero F's, eight are parity conserving, four are parity violating, and eight of them correspond to azimuthal asymmetries.

For the tensor polarization dependent part, the results are much similar. First the S_{LL}-dependent part looks very much the same as the unpolarized part. There are only three nonzero F_{LL}'s at twist-2, they are given by

$$F_{1LL1} = 2c_1^e c_1^q \mathcal{C}[D_{1LL} \bar{D}_1], \tag{5.19}$$

$$F_{3LL1} = 4c_3^e c_3^q \mathcal{C}[D_{1LL} \bar{D}_1], \tag{5.20}$$

$$F_{LL1}^{\cos 2\varphi} = -8c_1^e c_2^q \mathcal{C}[w_{hh} H_{1LL}^\perp \bar{H}_1^\perp]. \tag{5.21}$$

The S_{LT}-dependent part is very much similar to the S_T part. The six nonzeros are given by

$$F_{1LT1}^{\cos(\varphi_{LT}-\varphi)} = -2c_1^e c_1^q \mathcal{C}[w_1 D_{1LT}^\perp \bar{D}_1], \tag{5.22}$$

$$F_{3LT1}^{\cos(\varphi_{LT}-\varphi)} = -4c_3^e c_3^q \mathcal{C}[w_1 D_{1LT}^\perp \bar{D}_1], \tag{5.23}$$

$$\tilde{F}_{1LT1}^{\sin(\varphi_{LT}-\varphi)} = -2c_1^e c_3^q \mathcal{C}[w_1 G_{1LT}^\perp \bar{D}_1], \tag{5.24}$$

$$\tilde{F}_{3LT1}^{\sin(\varphi_{LT}-\varphi)} = -4c_3^e c_1^q \mathcal{C}[w_1 G_{1LT}^\perp \bar{D}_1], \tag{5.25}$$

$$F_{LT1}^{\cos(\varphi_{LT}+\varphi)} = -8c_1^e c_2^q \mathcal{C}[\bar{w}_1 \mathcal{H}_{1LT}^\perp \bar{H}_1^\perp], \tag{5.26}$$

$$F_{LT1}^{\cos(\varphi_{LT}-3\varphi)} = 8c_1^e c_2^q \mathcal{C}[w_{hh}^t H_{1LT}^\perp \bar{H}_1^\perp]. \tag{5.27}$$

The S_{TT}-dependent part is similar to the S_T part but the weights are different,

$$F_{1TT1}^{\cos(2\varphi_{TT}-2\varphi)} = 2c_1^e c_1^q \mathcal{C}[w_{dd}^{tt} D_{1TT}^\perp \bar{D}_1], \tag{5.28}$$

$$F_{3TT1}^{\cos(2\varphi_{TT}-2\varphi)} = 4c_3^e c_3^q \mathcal{C}[w_{dd}^{tt} D_{1TT}^\perp \bar{D}_1], \tag{5.29}$$

$$\tilde{F}_{1TT1}^{\sin(2\varphi_{TT}-2\varphi)} = 2c_1^e c_3^q \mathcal{C}[w_{dd}^{tt} G_{1TT}^\perp \bar{D}_1], \tag{5.30}$$

$$\tilde{F}_{3TT1}^{\sin(2\varphi_{TT}-2\varphi)} = 4c_3^e c_1^q \mathcal{C}[w_{dd}^{tt} G_{1TT}^\perp \bar{D}_1], \tag{5.31}$$

$$F_{TT1}^{\cos(2\varphi_{TT}-4\varphi)} = -4c_1^e c_2^q \mathcal{C}[w_{hh}^{tt} H_{1TT}^\perp \bar{H}_1^\perp], \tag{5.32}$$

$$F_{TT1}^{\cos 2\varphi_{TT}} = 8c_1^e c_2^q \mathcal{C}[w_2 H_{1TT}^{\perp\prime} \bar{H}_1^\perp], \tag{5.33}$$

where $w_{dd}^{tt} = 2w_1 - w_0$, $w_{hh}^{tt} = w_0 w_2 - 4w_0 w_1 \bar{w}_1 + 4w_1^2 w_2 + 8w_1^3 \bar{w}_1$, and $H_{1TT}^{\perp\prime} \equiv H_{1TT}^{\prime\perp} + [k_\perp^2 + 8(k_\perp \cdot p_{2T})^2/p_{2T}^2] H_{1TT}^\perp / 2M_1^2$.

B. Discussion about the twist-2 results

As we mentioned earlier in this paper, the twist-2 results presented here just correspond to the results obtained from the intuitive parton model with FFs defined in the gauge invariant form. Just as for the structure functions in inclusive DIS obtained using the original intuitive parton model, at the leading order (LO) in pQCD and twist-2, the results exhibit a number of simple regularities (symmetries) such as the Callan-Gross relation. To see these regularities more clearly, we list the leading twist results in Table I.

Indeed, from these results, we see that although there are 81 independent structure functions, a large part of them vanish at twist-2. Totally 27 of them are nonzero, among them 19 are parity conserved and eight are parity violated. Furthermore we see the following regularities.

(1) Among the 27 nonzero structure functions, five with $c_1^e c_1^q$, five with $c_3^e c_3^q$, and nine with $c_1^e c_2^q$ are parity even, and four with $c_1^e c_3^q$ and four with $c_3^e c_1^q$ are parity odd. This can be understood easily since from Eq. (3.2) we see that c_1^e symbolizes the symmetric parity conserving part and c_3^e the antisymmetric parity violating part of the tensor.

(2) The nonvanishing structure functions are associated with either $1+\cos^2\theta$, or $\cos\theta$ or $\sin^2\theta$.

For those associated with $1+\cos^2\theta$ or $\cos\theta$, there are five with coefficient $c_1^e c_1^q$ and five with $c_3^e c_3^q$. They are all from $\mathcal{C}[D\bar{D}]$, i.e., fragmentations of the unpolarized quark and are parity conserving. There are also four with coefficient $c_1^e c_3^q$ and four with $c_3^e c_1^q$. They are all from $\mathcal{C}[G\bar{D}]$, i.e., fragmentations of the longitudinally polarized quark and unpolarized antiquark and are parity violating.

Those associated with $\sin^2\theta$ all have coefficient $c_1^e c_2^q$ and are from $\mathcal{C}[H\bar{H}]$, i.e., transversely polarized quark and antiquark.

To understand such regularities, we recall the result for the basic weak process $e^+e^- \to Z \to q\bar{q}$. We recall that the differential cross section is [31]

$$\frac{d\hat{\sigma}}{d\Omega} = \frac{\alpha^2}{4s}\chi[c_1^e c_1^q(1+\cos^2\theta) + 2c_3^e c_3^q \cos\theta], \quad (5.34)$$

and the produced quark (antiquark) is longitudinally polarized and the polarization is given by

$$P_q(\theta) = -\frac{c_1^e c_3^q(1+\cos^2\theta) + 2c_3^e c_1^q \cos\theta}{c_1^e c_1^q(1+\cos^2\theta) + 2c_3^e c_3^q \cos\theta}. \quad (5.35)$$

Furthermore, although the quark (antiquark) is not transversely polarized, their transverse spin components are correlated. We define

$$c_{nn}^q = \frac{|M_{n++}|^2 + |M_{n--}|^2 - |M_{n+-}|^2 - |M_{n-+}|^2}{|M_{n++}|^2 + |M_{n--}|^2 + |M_{n+-}|^2 + |M_{n-+}|^2}, \quad (5.36)$$

where M is the scattering amplitude, $+$ or $-$ denote that the quark or antiquark is in the $s_n = 1/2$ or $-1/2$ state. We obtain that, for $\vec{n}$ in the normal of the production plane,

$$c_{nn}^q(\theta) = \frac{c_1^e c_2^q \sin^2\theta}{c_1^e c_1^q(1+\cos^2\theta) + 2c_3^e c_3^q \cos\theta}, \quad (5.37)$$

which is in fact also true for any transverse direction $\vec{n}$ if we replace $\sin^2\theta$ in the numerator by $\sin^2\theta\cos 2\varphi_n$ where φ_n is the azimuthal angle between $\vec{n}$ and the normal of the production plane. In terms of $y = (1+\cos\theta)/2$, we have

$$\frac{d\hat{\sigma}}{d\Omega} = \frac{\alpha^2}{2s}\chi T_0^q(y), \quad (5.38)$$

$$P_q(y) = T_1^q(y)/T_0^q(y), \quad (5.39)$$

$$c_{nn}^q(y) = c_1^e c_2^q C(y)/2T_0^q(y), \quad (5.40)$$

where $T_0^q(y) = c_1^e c_1^q A(y) - c_3^e c_3^q B(y)$ is the relative production weight for flavor q, $T_1^q(y) = -c_1^e c_3^q A(y) + c_3^e c_1^q B(y)$; $A(y)$, $B(y)$, and $C(y)$ are given in Sec. III B 2 by Eqs. (3.50)–(3.52). We see clearly why we have the regularities for the structure functions mentioned at the beginning of this point.

(3) It is also clear that if we consider $e^+e^- \to \gamma^* \to q\bar{q}$, i.e., the electromagnetic process, we have $T_0^{q(em)}(y) = e_q^2 A(y)$, $P_q^{(em)} = 0$. The quark transverse spin correlation $c_{nn}^{q(em)}(y) = C(y)/2A(y)$ is independent of the flavor of the quark. In this case, we will not have $\mathcal{C}[G\bar{D}]$ terms but $\mathcal{C}[D\bar{D}]$ and $\mathcal{C}[H\bar{H}]$ terms.

(4) If we integrate over p_2, we obtain the results for the inclusive process $e^+e^- \to Z \to VX$. The nonvanishing structure functions are

$$z_1 F_{1U1,in} = 2c_1^e c_1^q D_1(z_1), \quad (5.41)$$

$$z_1 F_{3U1,in} = 4c_3^e c_3^q D_1(z_1), \quad (5.42)$$

$$z_1 \tilde{F}_{1L1,in} = -2c_1^e c_3^q G_{1L}(z_1), \quad (5.43)$$

$$z_1 \tilde{F}_{3L1,in} = -4c_3^e c_1^q G_{1L}(z_1), \quad (5.44)$$

$$z_1 F_{1LL1,in} = 2c_1^e c_1^q D_{1LL}(z_1), \quad (5.45)$$

$$z_1 F_{3LL1,in} = 4c_3^e c_3^q D_{1LL}(z_1). \quad (5.46)$$

All the others vanish at twist-2. This is consistent with the results obtained in [15]. We emphasize in particular that the Callan-Gross relation in DIS now is replaced by $F_{2U1,in} = 0$, and all the structure functions associated with the transverse spin components vanish at leading twist.

TABLE I. The 81 structure functions and their twist-2 results. The capital letters U, L, LL, and so on in the first column denote the polarization of V. The θ dependence is common for all lines so we list it only in the first line. For the U, L, and LL polarized parts, the space reflection behaviors of first three columns are different from the other six. For the others, i.e., T, LT, and TT polarized parts, the whole line has the same reflection behavior and we use PC and PV to denote party conserved and parity violated respectively.

θ dependence	$\sin\theta$	$\sin 2\theta$	$\sin^2\theta$	$1+\cos^2\theta$	$\sin^2\theta$	$\cos\theta$	$\sin\theta$	$\sin 2\theta$	$\sin^2\theta$
φ dependence	$\sin\varphi$		$\sin 2\varphi$		1		$\cos\varphi$		$\cos 2\varphi$
U	$\tilde F_{1U}^{\sin\varphi}$	$\tilde F_{2U}^{\sin\varphi}$	$\tilde F_{U}^{\sin 2\varphi}$	$F_{1U}/2c_1^ec_1^q$	F_{2U}	$F_{3U}/4c_3^ec_3^q$	$F_{1U}^{\cos\varphi}$	$F_{2U}^{\cos\varphi}$	$F_U^{\cos 2\varphi}/8c_1^ec_2^q$
twist-2	0	0	0	$\mathcal{C}[D_1\bar D_1]$	0	$\mathcal{C}[D_1\bar D_1]$	0	0	$-\mathcal{C}[w_{hh}H_1^\perp\bar H_1^\perp]$
L	$F_{1L}^{\sin\varphi}$	$F_{2L}^{\sin\varphi}$	$F_L^{\sin 2\varphi}/8c_1^ec_2^q$	$\tilde F_{1L}/2c_1^ec_3^q$	$\tilde F_{2L}$	$\tilde F_{3L}/4c_3^ec_1^q$	$\tilde F_{1L}^{\cos\varphi}$	$\tilde F_{2L}^{\cos\varphi}$	$\tilde F_L^{\cos 2\varphi}$
twist-2	0	0	$-\mathcal{C}[w_{hh}H_{1L}^\perp\bar H_1^\perp]$	$-\mathcal{C}[G_{1L}\bar D_1]$	0	$-\mathcal{C}[G_{1L}\bar D_1]$	0	0	0
LL	$\tilde F_{1LL}^{\sin\varphi}$	$\tilde F_{2LL}^{\sin\varphi}$	$\tilde F_{LL}^{\sin 2\varphi}$	$F_{1LL}/2c_1^ec_1^q$	F_{2LL}	$F_{3LL}/4c_3^ec_3^q$	$F_{1LL}^{\cos\varphi}$	$F_{2LL}^{\cos\varphi}$	$F_{LL}^{\cos 2\varphi}/8c_1^ec_2^q$
twist-2	0	0	0	$\mathcal{C}[D_{1LL}\bar D_1]$	0	$\mathcal{C}[D_{1LL}\bar D_1]$	0	0	$-\mathcal{C}[w_{hh}H_{1LL}^\perp\bar H_1^\perp]$
T-PC	$F_{1T}^{\sin\varphi_S}$	$F_{2T}^{\sin\varphi_S}$	$F_T^{\sin(\varphi_S+\varphi)}/8c_1^ec_2^q$	$F_{1T}^{\sin(\varphi_S-\varphi)}/2c_1^ec_1^q$	$F_{2T}^{\sin(\varphi_S-\varphi)}$	$F_{3T}^{\sin(\varphi_S-\varphi)}/4c_3^ec_3^q$	$F_{1T}^{\sin(\varphi_S-2\varphi)}$	$F_{2T}^{\sin(\varphi_S-2\varphi)}$	$F_T^{\sin(\varphi_S-3\varphi)}/8c_1^ec_2^q$
φ dependence	$\sin\varphi_S$		$\sin(\varphi_S+\varphi)$		$\sin(\varphi_S-\varphi)$		$\sin(\varphi_S-2\varphi)$		$\sin(\varphi_S-3\varphi)$
twist-2	0	0	$-\mathcal{C}[\bar w_1\mathcal{H}_{1T}^\perp\bar H_1^\perp]$	$\mathcal{C}[w_1D_{1T}^\perp\bar D_1]$	0	$\mathcal{C}[w_1D_{1T}^\perp\bar D_1]$	0	0	$-\mathcal{C}[w_{hh}^tH_{1T}^\perp\bar H_1^\perp]$
T-PV	$\tilde F_{1T}^{\cos\varphi_S}$	$\tilde F_{2T}^{\cos\varphi_S}$	$\tilde F_T^{\cos(\varphi_S+\varphi)}$	$\tilde F_{1T}^{\cos(\varphi_S-\varphi)}/2c_1^ec_3^q$	$\tilde F_{2T}^{\cos(\varphi_S-\varphi)}$	$\tilde F_{3T}^{\cos(\varphi_S-\varphi)}/4c_3^ec_1^q$	$\tilde F_{1T}^{\cos(\varphi_S-2\varphi)}$	$\tilde F_{2T}^{\cos(\varphi_S-2\varphi)}$	$\tilde F_T^{\cos(\varphi_S-3\varphi)}$
φ dependence	$\cos\varphi_S$		$\cos(\varphi_S+\varphi)$		$\cos(\varphi_S-\varphi)$		$\cos(\varphi_S-2\varphi)$		$\cos(\varphi_S-3\varphi)$
twist-2	0	0	0	$\mathcal{C}[w_1G_{1T}^\perp\bar D_1]$	0	$\mathcal{C}[w_1G_{1T}^\perp\bar D_1]$	0	0	0
LT-PC	$F_{1LT}^{\cos\varphi_{LT}}$	$F_{2LT}^{\cos\varphi_{LT}}$	$F_{LT}^{\cos(\varphi_{LT}+\varphi)}/8c_1^ec_2^q$	$F_{1LT}^{\cos(\varphi_{LT}-\varphi)}/2c_1^ec_1^q$	$F_{2LT}^{\cos(\varphi_{LT}-\varphi)}$	$F_{3LT}^{\cos(\varphi_{LT}-\varphi)}/4c_3^ec_3^q$	$F_{1LT}^{\cos(\varphi_{LT}-2\varphi)}$	$F_{2LT}^{\cos(\varphi_{LT}-2\varphi)}$	$F_{LT}^{\cos(\varphi_{LT}-3\varphi)}/8c_1^ec_2^q$
φ dependence	$\cos\varphi_{LT}$		$\cos(\varphi_{LT}+\varphi)$		$\cos(\varphi_{LT}-\varphi)$		$\cos(\varphi_{LT}-2\varphi)$		$\cos(\varphi_{LT}-3\varphi)$
twist-2	0	0	$-\mathcal{C}[\bar w_1\mathcal{H}_{1LT}^\perp\bar H_1^\perp]$	$-\mathcal{C}[w_1D_{1LT}^\perp\bar D_1]$	0	$-\mathcal{C}[w_1D_{1LT}^\perp\bar D_1]$	0	0	$\mathcal{C}[w_{hh}^tH_{1LT}^\perp\bar H_1^\perp]$
LT-PV	$\tilde F_{1LT}^{\sin\varphi_{LT}}$	$\tilde F_{2LT}^{\sin\varphi_{LT}}$	$\tilde F_{LT}^{\sin(\varphi_{LT}+\varphi)}$	$\tilde F_{1LT}^{\sin(\varphi_{LT}-\varphi)}/2c_1^ec_3^q$	$\tilde F_{2LT}^{\sin(\varphi_{LT}-\varphi)}$	$\tilde F_{3LT}^{\sin(\varphi_{LT}-\varphi)}/4c_3^ec_1^q$	$\tilde F_{1LT}^{\sin(\varphi_{LT}-2\varphi)}$	$\tilde F_{2LT}^{\sin(\varphi_{LT}-2\varphi)}$	$\tilde F_{LT}^{\sin(\varphi_{LT}-3\varphi)}$
φ dependence	$\sin\varphi_{LT}$		$\sin(\varphi_{LT}+\varphi)$		$\sin(\varphi_{LT}-\varphi)$		$\sin(\varphi_{LT}-2\varphi)$		$\sin(\varphi_{LT}-3\varphi)$
twist-2	0	0	0	$-\mathcal{C}[w_1G_{1LT}^\perp\bar D_1]$	0	$-\mathcal{C}[w_1G_{1LT}^\perp\bar D_1]$	0	0	0
TT-PC	$F_{1TT}^{\cos(2\varphi_{TT}-\varphi)}$	$F_{2TT}^{\cos(2\varphi_{TT}-\varphi)}$	$F_{TT}^{\cos 2\varphi_{TT}}/8c_1^ec_2^q$	$F_{1TT}^{\cos(2\varphi_{TT}-2\varphi)}/2c_1^ec_1^q$	$F_{2TT}^{\cos(2\varphi_{TT}-2\varphi)}$	$F_{3TT}^{\cos(2\varphi_{TT}-2\varphi)}/4c_3^ec_3^q$	$F_{1TT}^{\cos(2\varphi_{TT}-3\varphi)}$	$F_{2TT}^{\cos(2\varphi_{TT}-3\varphi)}$	$F_{TT}^{\cos(2\varphi_{TT}-4\varphi)}/4c_1^ec_2^q$
φ dependence	$\cos(2\varphi_{TT}-\varphi)$		$\cos 2\varphi_{TT}$		$\cos(2\varphi_{TT}-2\varphi)$		$\cos(2\varphi_{TT}-3\varphi)$		$\cos(2\varphi_{TT}-4\varphi)$
twist-2	0	0	$\mathcal{C}[w_2\mathcal{H}_{1TT}^\perp\bar H_1^\perp]$	$\mathcal{C}[w_{dd}^{tt}D_{1TT}^\perp\bar D_1]$	0	$\mathcal{C}[w_{dd}^{tt}D_{1TT}^\perp\bar D_1]$	0	0	$-\mathcal{C}[w_{hh}^{tt}H_{1TT}^\perp\bar H_1^\perp]$
TT-PV	$\tilde F_{1TT}^{\sin(2\varphi_{TT}-\varphi)}$	$\tilde F_{2TT}^{\sin(2\varphi_{TT}-\varphi)}$	$\tilde F_{TT}^{\sin 2\varphi_{TT}}$	$\tilde F_{1TT}^{\sin(2\varphi_{TT}-2\varphi)}/2c_1^ec_3^q$	$\tilde F_{2TT}^{\sin(2\varphi_{TT}-2\varphi)}$	$\tilde F_{3TT}^{\sin(2\varphi_{TT}-2\varphi)}/4c_3^ec_1^q$	$\tilde F_{TT}^{\sin(2\varphi_{TT}-3\varphi)}$	$\tilde F_{2TT}^{\sin(2\varphi_{TT}-3\varphi)}$	$\tilde F_{TT}^{\sin(2\varphi_{TT}-4\varphi)}$
φ dependence	$\sin(2\varphi_{TT}-\varphi)$		$\sin 2\varphi_{TT}$		$\sin(2\varphi_{TT}-2\varphi)$		$\sin(2\varphi_{TT}-3\varphi)$		$\sin(2\varphi_{TT}-4\varphi)$
twist-2	0	0	0	$\mathcal{C}[w_{dd}^{tt}G_{1TT}^\perp\bar D_1]$	0	$\mathcal{C}[w_{dd}^{tt}G_{1TT}^\perp\bar D_1]$	0	0	0

C. Twist-3 contributions

Among the 54 structure functions that vanish at twist-2, 36 have twist-3 contributions as the leading power contributions. The results are a bit lengthy so we present them in Appendix C. We see that all 36 structure functions associated with $\sin\theta$ and $\sin 2\theta$ have twist-3 contributions as leading power contributions. Besides others, we have $F_{1U2}^{\cos\varphi}$, $F_{2U2}^{\cos\varphi}$, $\tilde{F}_{1U2}^{\sin\varphi}$, and $\tilde{F}_{2U2}^{\sin\varphi}$ in the unpolarized part, also $F_{1T2}^{\sin\varphi_S}$, $F_{2T2}^{\sin\varphi_S}$, $\tilde{F}_{1T2}^{\cos\varphi_S}$, and $\tilde{F}_{2T2}^{\cos\varphi_S}$ in the vector polarization dependent part. This means that at the twist-3 level there should be parity conserved azimuthal asymmetry $\langle\cos\varphi\rangle_U$ and parity violated asymmetry $\langle\sin\varphi\rangle_U$ in the unpolarized case and parity conserved transverse polarization in the normal direction of the lepton-hadron plane and parity violated component in the plane. We will discuss this more in the next section.

VI. AZIMUTHAL ASYMMETRIES AND HADRON POLARIZATIONS

A. Azimuthal asymmetries

At leading twist and for unpolarized V (i.e., polarization is not measured), there is only one azimuthal asymmetry as given by Eq. (3.68), i.e.,

$$\langle\cos 2\varphi\rangle_U^{(0)} = -\frac{C(y)\sum_q c_1^e c_2^q \mathcal{C}[w_{hh}H_1^\perp \bar{H}_1^\perp]}{\sum_q T_0^q(y)\mathcal{C}[D_1\bar{D}_1]}. \tag{6.1}$$

This is the only leading twist azimuthal asymmetry in the unpolarized case due to the Collins effect [4] and transverse spin correlation c_{nn}^q given by Eq. (5.37) for $q\bar{q}$ produced via e^+e^- annihilation. Here, as well as in the following of this paper, when writing the expressions for azimuthal asymmetries and/or polarizations in terms of FFs, to avoid confusion, we include the summation over q explicitly but still keep the $q \leftrightarrow \bar{q}$ terms implicitly and omit the flavor indices for the FFs.

If we could consider the polarization and azimuthal asymmetry simultaneously, we would have

$$\langle\cos 2\varphi\rangle_{LL}^{(0)} = -\frac{C(y)\sum_q c_1^e c_2^q \mathcal{C}[w_{hh}(H_1^\perp + S_{LL}H_{1LL}^\perp)\bar{H}_1^\perp]}{\sum_q T_0^q(y)\mathcal{C}[(D_1 + S_{LL}D_{1LL})\bar{D}_1]}, \tag{6.2}$$

$$\langle\sin 2\varphi\rangle_L^{(0)} = -\frac{\lambda C(y)\sum_q c_1^e c_2^q \mathcal{C}[w_{hh}H_{1L}^\perp \bar{H}_1^\perp]}{\sum_q T_0^q(y)\mathcal{C}(D_1 - \lambda G_{1L})\bar{D}_1}. \tag{6.3}$$

Although it is academic since it will be very difficult to measure this asymmetry, it is interesting to see the existence of such asymmetry.

Up to twist-3, we have another two azimuthal asymmetries in the unpolarized case, i.e.,

$$\langle\cos\varphi\rangle_U^{(1)} = -\frac{8D(y)}{z_1 z_2 Q F_{Ut}^{(0)}}\sum_q\{T_2^q(y)(M_1\mathcal{C}[w_1 D^\perp z_2\bar{D}_1] + M_2\mathcal{C}[\bar{w}_1 z_1 D_1\bar{D}^{\perp\prime}]) + T_4^q(y)(M_1\mathcal{C}[\bar{w}_1 H z_2\bar{H}_1^\perp] + M_2\mathcal{C}[w_1 z_1 H_1^\perp \bar{H}^{\perp\prime}])\}, \tag{6.4}$$

$$\langle\sin\varphi\rangle_U^{(1)} = \frac{8D(y)}{z_1 z_2 Q F_{Ut}^{(0)}}\sum_q\{T_3^q(y)(M_1\mathcal{C}[w_1 G^\perp z_2\bar{D}_1] - M_2\mathcal{C}[\bar{w}_1 z_1 D_1\bar{G}^\perp]) + 2c_3^e c_2^q(M_1\mathcal{C}[\bar{w}_1 E z_2\bar{H}_1^\perp] - M_2\mathcal{C}[w_1 z_1 H_1^\perp\bar{E}])\}, \tag{6.5}$$

where $D(y) = \sqrt{y(1-y)}$, $T_2^q(y) = -c_3^e c_3^q + c_1^e c_1^q B(y)$, $T_3^q(y) = c_3^e c_1^q - c_1^e c_3^q B(y)$, $T_4^q(y) = 4c_1^e c_2^q B(y)$, and $F_{Ut}^{(0)}$ is the twist-2 contribution to F_{Ut} and is given by

$$F_{Ut}^{(0)} = 4\sum_q T_0^q(y)\mathcal{C}[D_1\bar{D}_1]. \tag{6.6}$$

We see that they depend on several twist-3 FFs.

If we consider $e^+e^- \to \gamma^* \to V\pi X$, we have

$$\langle\cos 2\varphi\rangle_U^{(0,em)} = -\frac{C(y)}{A(y)}\frac{\sum_q e_q^2\mathcal{C}[w_{hh}H_1^\perp\bar{H}_1^\perp]}{\sum_q e_q^2\mathcal{C}[D_1\bar{D}_1]}, \tag{6.7}$$

$$\langle \cos\varphi\rangle_U^{(1,em)} = -\frac{2\tilde{B}(y)}{A(y)}\frac{1}{z_1 z_2 Q\sum_q e_q^2 \mathcal{C}[D_1\bar{D}_1]}\sum_q e_q^2\{M_1\mathcal{C}[w_1 D^\perp z_2 \bar{D}_1 + 4\bar{w}_1 H z_2 \bar{H}_1^\perp] + M_2\mathcal{C}[\bar{w}_1 z_1 D_1 \bar{D}^{\perp\prime} + 4 w_1 z_1 H_1^\perp \bar{H}^{\perp\prime}]\}, \tag{6.8}$$

and $\langle \sin\varphi\rangle_U^{(1,em)} = 0$, where $\tilde{B}(y) = \sqrt{y(1-y)}B(y)$. In this case we have a nonzero azimuthal asymmetry $\langle \cos 2\varphi\rangle_U^{(0,em)}$ at leading twist due to the Collins effect [4] and a twist-3 asymmetry $\langle \cos\varphi\rangle_U^{(0,em)}$ similar to the Cahn effect [32] in deep-inelastic lepton-nucleon scattering.

B. Hadron polarizations at twist-2

The polarization is in general dependent on φ. Experimentally it is much easier to consider the case where φ is integrated. In this case, at the leading twist, we have for the longitudinal polarization,

$$\langle \lambda\rangle^{(0)} = \frac{2}{3}\frac{\sum_q P_q(y) T_0^q(y)\mathcal{C}[G_{1L}\bar{D}_1]}{\sum_q T_0^q(y)\mathcal{C}[D_1\bar{D}_1]}, \tag{6.9}$$

$$\langle S_{LL}\rangle^{(0)} = \frac{1}{2}\frac{\sum_q T_0^q(y)\mathcal{C}[D_{1LL}\bar{D}_1]}{\sum_q T_0^q(y)\mathcal{C}[D_1\bar{D}_1]}. \tag{6.10}$$

For transverse dependent components with respect to the hadron-hadron plane, we have

$$\langle S_T^n\rangle^{(0)} = \frac{2}{3}\frac{\sum_q T_0^q(y)\mathcal{C}[w_1 D_{1T}^\perp \bar{D}_1]}{\sum_q T_0^q(y)\mathcal{C}[D_1\bar{D}_1]}, \tag{6.11}$$

$$\langle S_T^t\rangle^{(0)} = -\frac{2}{3}\frac{\sum_q P_q(y) T_0^q(y)\mathcal{C}[w_1 G_{1T}^\perp \bar{D}_1]}{\sum_q T_0^q(y)\mathcal{C}[D_1\bar{D}_1]}, \tag{6.12}$$

$$\langle S_{LT}^n\rangle^{(0)} = \frac{2}{3}\frac{\sum_q P_q(y) T_0^q(y)\mathcal{C}[w_1 G_{1LT}^\perp \bar{D}_1]}{\sum_q T_0^q(y)\mathcal{C}[D_1\bar{D}_1]}, \tag{6.13}$$

$$\langle S_{LT}^t\rangle^{(0)} = -\frac{2}{3}\frac{\sum_q T_0^q(y)\mathcal{C}[w_1 D_{1LT}^\perp \bar{D}_1]}{\sum_q T_0^q(y)\mathcal{C}[D_1\bar{D}_1]}, \tag{6.14}$$

$$\langle S_{TT}^{nn}\rangle^{(0)} = -\frac{2}{3}\frac{\sum_q T_0^q(y)\mathcal{C}[w_{dd}^{tt} D_{1TT}^\perp \bar{D}_1]}{\sum_q T_0^q(y)\mathcal{C}[D_1\bar{D}_1]}, \tag{6.15}$$

$$\langle S_{TT}^{nt}\rangle^{(0)} = -\frac{2}{3}\frac{\sum_q P_q(y) T_0^q(y)\mathcal{C}[w_{dd}^{tt} G_{1TT}^\perp \bar{D}_1]}{\sum_q T_0^q(y)\mathcal{C}[D_1\bar{D}_1]}. \tag{6.16}$$

The transverse components with respect to the lepton-hadron plane are zero at the leading twist in the φ integrated case.

If we consider $e^+e^- \to \gamma^* \to V\pi X$, i.e., annihilate via electromagnetic interaction only, we have

$$\langle S_{LL}\rangle^{(0,em)} = \frac{1}{2}\frac{\sum_q e_q^2\mathcal{C}[D_{1LL}\bar{D}_1]}{\sum_q e_q^2\mathcal{C}[D_1\bar{D}_1]}, \tag{6.17}$$

$$\langle S_T^n\rangle^{(0,em)} = \frac{2}{3}\frac{\sum_q e_q^2\mathcal{C}[w_1 D_{1T}^\perp \bar{D}_1]}{\sum_q e_q^2\mathcal{C}[D_1\bar{D}_1]}, \tag{6.18}$$

$$\langle S_{LT}^t\rangle^{(0,em)} = \frac{2}{3}\frac{\sum_q e_q^2\mathcal{C}[w_1 D_{1LT}^\perp \bar{D}_1]}{\sum_q e_q^2\mathcal{C}[D_1\bar{D}_1]}, \tag{6.19}$$

$$\langle S_{TT}^{nn}\rangle^{(0,em)} = -\frac{2}{3}\frac{\sum_q e_q^2\mathcal{C}[w_{dd}^{tt} D_{1TT}^\perp \bar{D}_1]}{3\sum_q e_q^2\mathcal{C}[D_1\bar{D}_1]}, \tag{6.20}$$

while the parity violating components,

$$\langle \lambda\rangle^{(0,em)} = \langle S_T^t\rangle^{(0,em)} = \langle S_{LT}^n\rangle^{(0,em)} = \langle S_{TT}^{nt}\rangle^{(0,em)} = 0. \tag{6.21}$$

We see in particular that the S_{LL} component is nonzero at leading twist also in the parity conserved case. Parity conserving transverse components exist due to Sivers-type FFs such as $D_{1T}^\perp$, $D_{1LT}^\perp$, and $D_{1TT}^\perp$ similar to the Sivers function $f_{1T}^\perp$ in three-dimensional PDFs [22].

For the inclusive process $e^+e^- \to Z \to VX$, we have

$$\langle \lambda\rangle_{in}^{(0)} = \sum_q 2P_q(y) T_0^q(y) G_{1L}(z_1)/\sum_q 3T_0^q(y) D_1(z_1), \tag{6.22}$$

$$\langle S_{LL}\rangle_{in}^{(0)} = \sum_q T_0^q(y) D_{1LL}(z_1)/\sum_q 2T_0^q(y) D_1(z_1), \tag{6.23}$$

while all the transverse components such as $\langle S_T^i\rangle_{in}^{(0)}$, $\langle S_{LT}^i\rangle_{in}^{(0)}$, and $\langle S_{TT}^{ij}\rangle_{in}^{(0)}$ ($i,j = x$ or y) vanish at twist-2. We also see that $\langle S_{LL}\rangle_{in}^{(0)}$ is nonzero also in parity conserved reactions while $\langle \lambda\rangle_{in}^{(0)}$ exists only in the parity violated case.

C. Transverse polarizations with respect to the lepton-hadron plane at twist-3

As mentioned in Sec. V C, the twist-3 contribution exists only for those structure functions that are zero at twist-2. They are the leading power contributions for the corresponding structure functions. In particular we see that there is no twist-3 contribution to the transverse components with respect to the hadron-hadron plane discussed in last subsection. However, for the transverse components with respect to the lepton-hadron plane, four of them, i.e., $\langle S_T^x\rangle$, $\langle S_T^y\rangle$, $\langle S_{LT}^x\rangle$, and $\langle S_{LT}^y\rangle$ have twist-3 contributions. They are determined by $F_{jT2}^{\sin\varphi_S}$, $\tilde{F}_{jT2}^{\cos\varphi_S}$, $\tilde{F}_{jLT2}^{\sin\varphi_S}$, and $F_{jLT2}^{\cos\varphi_S}$ given in Appendix C respectively. The expressions can easily be obtained by inserting these results into Eqs. (3.94)–(3.97) but are a bit lengthy so we omit them here. However we emphasize that if we consider $e^+e^-\to\gamma^*\to V\pi X$, the parity violating parts vanish and we have only the following two components,

$$\langle S_T^y\rangle^{(1,em)} = \frac{8M_1\tilde{B}(y)}{3z_1z_2QA(y)\sum_q e_q^2\mathcal{C}[D_1\bar{D}_1]} \times\sum_q e_q^2\Big\{z_2\mathcal{C}\Big[\mathcal{D}_T^\perp\bar{D}_1-2\frac{w_2}{M_1}H_T^{\perp-}\bar{H}_1^\perp\Big] -\frac{z_1M_2}{2M_1}\mathcal{C}[w_2(D_{1T}^\perp\bar{D}^{\perp\prime}-G_{1T}^\perp\bar{G}^\perp)-8\mathcal{H}_{1T}^\perp\bar{H}_1^{\perp\prime}]\Big\}, \tag{6.24}$$

$$\langle S_{LT}^x\rangle^{(1,em)} = -\frac{8M_1\tilde{B}(y)}{3z_1z_2QA(y)\sum_q e_q^2\mathcal{C}[D_1\bar{D}_1]} \times\sum_q e_q^2\Big\{z_2\mathcal{C}\Big[\mathcal{D}_{LT}^\perp\bar{D}_1-2\frac{w_2}{M_1}H_{LT}^{\perp+}\bar{H}_1^\perp\Big] +\frac{z_1M_2}{2M_1}\mathcal{C}[w_2(D_{1LT}^\perp\bar{D}^{\perp\prime}+G_{1LT}^\perp\bar{G}^\perp) -8\mathcal{H}_{1LT}^\perp\bar{H}_1^{\perp\prime}]\Big\}. \tag{6.25}$$

It is also interesting to see that these transverse components are defined with respect to the lepton-hadron plane and exist also in the inclusive process. For $e^+e^-\to Z\to VX$, we have

$$\langle S_T^x\rangle_{in}^{(1)} = -\frac{8M_1D(y)}{3z_1Q}\frac{\sum_q T_3^q(y)G_T}{\sum_q T_0^q(y)D_1}, \tag{6.26}$$

$$\langle S_T^y\rangle_{in}^{(1)} = \frac{8M_1D(y)}{3z_1Q}\frac{\sum_q T_2^q(y)D_T}{\sum_q T_0^q(y)D_1}, \tag{6.27}$$

$$\langle S_{LT}^x\rangle_{in}^{(1)} = -\frac{8M_1D(y)}{3z_1Q}\frac{\sum_q T_2^q(y)D_{LT}}{\sum_q T_0^q(y)D_1}, \tag{6.28}$$

$$\langle S_{LT}^y\rangle_{in}^{(1)} = \frac{8M_1D(y)}{3z_1Q}\frac{\sum_q T_3^q(y)G_{LT}}{\sum_q T_0^q(y)D_1}. \tag{6.29}$$

We recall that $\langle S_T^y\rangle$ is P even and naive T odd, $\langle S_T^x\rangle$ is P odd and naive T even, and $\langle S_{LT}^y\rangle$ is P odd and naive T odd. Neither of these three can exist in deep-inelastic scattering such as $e^-N\to e^-X$. The only existing one is $\langle S_{LT}^x\rangle$ which is both P and T even. We see also from Table IV whether the corresponding FFs are T odd or T even which is consistent with the structure functions and/or the polarizations.

For $e^+e^-\to\gamma^*\to VX$, we have

$$\langle S_T^y\rangle_{in}^{(1,em)} = \frac{8M_1\tilde{B}(y)}{3z_1QA(y)}\frac{\sum_q e_q^2D_T}{\sum_q e_q^2D_1}, \tag{6.30}$$

$$\langle S_{LT}^x\rangle_{in}^{(1,em)} = -\frac{8M_1\tilde{B}(y)}{3z_1QA(y)}\frac{\sum_q e_q^2D_{LT}}{\sum_q e_q^2D_1}, \tag{6.31}$$

and other two parity violating components are zero.

VII. SUMMARY AND DISCUSSION

Three parts were presented in this paper: A summary of results of a general decomposition of the quark-quark correlator that leads to the operator definition of TMD FFs, a general kinematical analysis for $e^+e^-\to V\pi X$, and a complete twist-3 calculation based on the partonic picture at leading order in pQCD. We summarize the main results in the following.

(1) We presented the results of general decomposition of the quark-quark correlator for fragmentation of the quark to the spin-1 hadron. The correlator is expressed as a sum of a spin-independent, a vector polarization dependent, and a tensor polarization dependent part. Formally, the spin-independent part is identical to that for spin-0 hadrons, the vector polarization dependent part is the same as that for spin-1/2 hadrons, while the tensor polarization dependent part is novel for spin-1 hadrons. The decomposition leads to totally 72 TMD FFs, eight for spin-independent, 24 for the vector polarization dependent, and the other 40 for the tensor polarization dependent part. Among them, 18 contribute at leading twist, 36 at twist-3, and the other 18 at twist-4; half of them (36) are T odd, the other half are T even; also half are χ odd and the other half are χ even.

(2) These TMD FFs are used in describing the semi-inclusive high energy reaction (see, e.g., [17]). We note that usually for a complete description of a semi-inclusive reaction, the quark-quark correlator is not sufficient. One usually needs the quark-j-gluon-quark correlator, too ($j=1,2,\ldots$ represents the number of gluons). They contribute at higher twist starting at twist-$(j+2)$. For example, to make a complete calculation up to twist-3, besides the quark-quark correlator, one needs the quark-gluon-quark correlator. These contributions should be taken into account simultaneously. It is also important to note that because of the QCD equation of motion, they are often not independent and

relationships obtained from QCD equation of motion should be used.

(3) We presented also the results for a general kinematic analysis for $e^+e^- \to V\pi X$. This process is in general described by 81 structure functions, 42 are parity conserving and 39 are parity violating. The azimuthal asymmetries and hadron polarizations are in general coupled with each other and are described by the corresponding structure functions. In practice, it is much simpler to study the azimuthal asymmetries in the unpolarized case and hadron polarizations averaged over the azimuthal angle φ. For unpolarized hadrons, there are four azimuthal asymmetries, i.e., $\langle\cos\varphi\rangle_U$, $\langle\sin\varphi\rangle_U$, $\langle\cos 2\varphi\rangle_U$, and $\langle\sin 2\varphi\rangle_U$. The two cos asymmetries are parity conserving while the two sin asymmetries are parity violating.

(4) The hadron polarizations are most conveniently studied in the helicity Gottfried-Jackson frame. Here, we have two longitudinal components $\langle\lambda\rangle$ and $\langle S_{LL}\rangle$ defined in the helicity basis, and six transverse components that can be defined either with respect to the lepton-hadron plane, i.e., $\langle S_T^x\rangle$, $\langle S_T^y\rangle$, $\langle S_{LT}^x\rangle$, $\langle S_{LT}^y\rangle$, $\langle S_{TT}^{xx}\rangle$, and $\langle S_{TT}^{xy}\rangle$, or with respect to the hadron-hadron plane, i.e., $\langle S_T^n\rangle$, $\langle S_T^t\rangle$, $\langle S_{LT}^n\rangle$, $\langle S_{LT}^t\rangle$, $\langle S_{TT}^{nn}\rangle$, and $\langle S_{TT}^{nt}\rangle$. In the case of averaging over φ, they correspond to different structure functions as given by Eqs. (3.94)–(3.99) and Eqs. (3.106)–(3.111) respectively. Half of them are parity conserving while the other half are parity violating.

(5) The results obtained in the partonic picture at LO pQCD up to twist-3 were also presented in terms of the gauge invariant FFs. These results showed that at leading twist there are 27 nonvanishing structure functions, 19 correspond to parity conserving and eight are parity violating. We have also 36 structure functions that have twist-3 as leading power contributions.

(6) For unpolarized hadrons, there is only one azimuthal asymmetry $\langle\cos 2\varphi\rangle$ at leading twist due to the Collins effect [4] in fragmentation and transverse spin correlation c_{nn}^q given by Eq. (5.37) in e^+e^- annihilations, and two twist-3 asymmetries $\langle\cos\varphi\rangle$ and $\langle\sin\varphi\rangle$, the former is similar to the Cahn effect [32] in DIS and the latter exists only in parity violating reactions.

(7) Longitudinal components of hadron polarization $\langle\lambda\rangle$ and $\langle S_{LL}\rangle$ exist at leading twist as given by Eqs. (6.9) and (6.10). While the former depends on the initial polarization P_q of the quark produced at the e^+e^- annihilation vertex and exists only in weak interaction processes, the latter is independent of P_q and exists also in electromagnetic processes.

(8) Transverse components $\langle S_T^n\rangle$, $\langle S_T^t\rangle$, $\langle S_{LT}^n\rangle$, $\langle S_{LT}^t\rangle$, $\langle S_{TT}^{nn}\rangle$, and $\langle S_{TT}^{nt}\rangle$ with respect to the hadron-hadron plane exist at leading twist given by Eqs. (6.11)–(6.16). Among them $\langle S_T^n\rangle$, $\langle S_{LT}^t\rangle$, and $\langle S_{TT}^{nn}\rangle$ are parity conserving and $\langle S_T^t\rangle$, $\langle S_{LT}^n\rangle$, and $\langle S_{TT}^{nt}\rangle$ are parity violating.

(9) There are also twist-3 transverse components $\langle S_T^x\rangle$, $\langle S_T^y\rangle$, $\langle S_{LT}^x\rangle$, $\langle S_{LT}^y\rangle$, $\langle S_{TT}^{xx}\rangle$, and $\langle S_{TT}^{xy}\rangle$ with respect to the lepton-hadron plane. They are determined by the corresponding twist-3 FFs as given by Eqs. (6.26)–(6.29). Similarly, $\langle S_T^y\rangle$, $\langle S_{LT}^x\rangle$, and $\langle S_{TT}^{xx}\rangle$ are parity conserving and $\langle S_T^x\rangle$, $\langle S_{LT}^y\rangle$, and $\langle S_{TT}^{xy}\rangle$ are parity violating.

(10) For inclusive reaction $e^+e^- \to VX$, we can only study z dependence. Kinematically, the hadronic tensor and/or cross section take the same form as that of the semi-inclusive reaction $e^+e^- \to V\pi X$ averaged over φ. We have two longitudinal components of polarization, i.e., $\langle\lambda\rangle$ and $\langle S_{LL}\rangle$ at leading twist. In particular we have four transverse components $\langle S_T^x\rangle$, $\langle S_T^y\rangle$, $\langle S_{LT}^x\rangle$, $\langle S_{LT}^y\rangle$ at twist-3. Three of them are either T odd or P odd and do not exist in deep-inelastic scattering such as $e^-h \to e^-X$. The only one that is both P and T even is $\langle S_{LT}^x\rangle$.

Finally, we would like to emphasize in particular that, in experiments, different components of the (vector) polarizations of octet hyperons such as Λ, $\Sigma^\pm$, and $\Xi^{0,-}$ and those of the tensor polarizations of vector mesons such as ρ and K^* can be measured in a conceptually simple way. Polarizations of these hyperons can be measured by studying the angular distributions of the decay products of their spin self-analyzing parity violating decays. All the five independent components of the tensor polarization, S_{LL}, S_{LT}^x, S_{LT}^y, S_{TT}^{xx}, and S_{TT}^{xy}, of these vector mesons can also be measured via the angular distributions in their strong decays into two pseudoscalar mesons [21]. Such measurements have also been carried out in the past in different high energy reactions. Transverse polarizations of different hyperons have been observed in unpolarized hadron-hadron, hadron-nucleus collisions [23], in e^+e^- annihilations [33] and lepton-hadron reactions [34] that correspond to the Sivers-type FF $D_{1T}^\perp$ and higher twist addenda to it. We see in particular that in experiments with e^+e^- annihilation at high energies where FFs can be best studied, measurements have been carried out, e.g., at the LEP on longitudinal polarization of Λ hyperon production [35,36] by the ALEPH and OPAL Collaborations, and also on the spin alignment $\rho_{00} = (1 - 2S_{LL})/3$ for vector mesons such as K^*, ρ, and so on [37–39]. Results for z dependences have been obtained in both cases. Even nondiagonal components (corresponds to higher twist contributions only) have also been measured [37–39]. The data available are definitely still far from enough to limit the precise forms of the FFs involved. They have however provided important hints for the corresponding components and have attracted much attention theoretically. Many phenomenological model studies have been carried out in the last few years [40–58].

Recent measurements have been carried out on azimuthal asymmetries for two hadron production by the Belle, *BABAR*, and BESIII Collaborations [59–63]. They provide useful constraints on the Collins function [64,65]. Presently, related measurements can be and are being carried out, e.g., in pp collisions by STAR at RHIC, and in the existing e^+e^- colliders such as Belle at KEK and BES at BEPC [66]. They can certainly also be studied in future e^+e^- colliders at high energies, electron-ion colliders discussed in the community [67]. We would in particular like to note that usually the production rates of vector mesons are much higher than hyperons in high energy reactions. Hence, we expect that studies of vector meson tensor polarization might provide us a more sensitive window to study polarization effects in fragmentation process in particular and to develop QCD theory in general.

ACKNOWLEDGMENTS

We thank Yu-kun Song and Long Chen for helpful discussions. This work was supported in part by the National Natural Science Foundation of China (Grant No. 11375104), the Major State Basic Research Development Program in China (Grant No. 2014CB845406), and the CAS Center for Excellence in Particle Physics.

APPENDIX A: FRAGMENTATION FUNCTIONS DEFINED VIA THE QUARK-QUARK CORRELATOR

We make a full list of the TMD FFs defined via the quark-quark correlator in this appendix.

1. The spin-independent part

The general decomposition of the spin-independent part of the quark-quark correlator is given by

$$z\Xi^{U(0)}(z,k_{F\perp};p) = ME(z,k_{F\perp}), \tag{A1}$$

$$z\tilde{\Xi}^{U(0)}(z,k_{F\perp};p) = 0, \tag{A2}$$

$$z\Xi^{U(0)}_{\alpha}(z,k_{F\perp};p) = p^+\bar{n}_\alpha D_1(z,k_{F\perp}) + k_{F\perp\alpha}D^\perp(z,k_{F\perp}) + \frac{M^2}{p^+}n_\alpha D_3(z,k_{F\perp}), \tag{A3}$$

$$z\tilde{\Xi}^{U(0)}_{\alpha}(z,k_{F\perp};p) = -\tilde{k}_{F\perp\alpha}G^\perp(z,k_{F\perp}), \tag{A4}$$

$$z\Xi^{U(0)}_{\rho\alpha}(z,k_{F\perp};p) = -\frac{p^+}{M}\bar{n}_{[\rho}\tilde{k}_{F\perp\alpha]}H_1^\perp(z,k_{F\perp}) + M\varepsilon_{\perp\rho\alpha}H(z,k_{F\perp}) - \frac{M}{p^+}n_{[\rho}\tilde{k}_{F\perp\alpha]}H_3^\perp(z,k_{F\perp}). \tag{A5}$$

Here, we note in particular that, compared with the corresponding $\bar{n}$ component, the $n_\perp$ and n components are suppressed by M/p^+ and $(M/p^+)^2$ and contribute at twist-3 and twist-4 respectively. If we integrate over $d^2k_{F\perp}$, terms with $k_{F\perp}$ odd Lorentz structures vanish and we obtain

$$z\Xi^{U(0)}(z;p) = ME(z),$$
$$z\tilde{\Xi}^{U(0)}(z;p) = 0, \tag{A6}$$

$$z\Xi^{U(0)}_{\alpha}(z;p) = p^+\bar{n}_\alpha D_1(z) + \frac{M^2}{p^+}n_\alpha D_3(z),$$
$$z\tilde{\Xi}^{U(0)}_{\alpha}(z;p) = 0, \tag{A7}$$

$$z\Xi^{U(0)}_{\rho\alpha}(z;p) = M\varepsilon_{\perp\rho\alpha}H(z), \tag{A8}$$

where the one-dimensional FF is just equal to the corresponding three-dimensional one integrated over $d^2k_{F\perp}$ such as

$$D_1(z) = \int\frac{d^2k_{F\perp}}{(2\pi)^2}D_1(z,k_{F\perp}) = z\sum_X\int\frac{d\xi^-}{2\pi}e^{-ip^+\xi^-/z}\langle p,S;X|\bar{\psi}(\xi^-)\mathcal{L}(\xi^-;\infty)|0\rangle \times\frac{\gamma^+}{4}\langle 0|\mathcal{L}^\dagger(0;\infty)\psi(0)|p,S;X\rangle. \tag{A9}$$

The factor z before $\Xi^{(0)}$ on the left-hand side of Eqs. (A1)–(A5) is needed so that $D_1(z)$ obtained this way is the number density for a quark fragmentation into a specified hadron. However, when polarization is involved, we note the difference: While for phenomenologically defined $D_1(z)$, a sum over spin of h and an average over the spin of the quark is understood; for $D_1(z)$ defined via the quark-quark correlator as given by Eq. (A9), we have an average over the hadron spin and a sum over the quark spin. Hence $D_1(z)$ is identical in the two cases only for spin-1/2 hadrons.

2. Vector polarization dependent part

We build the S-dependent basic Lorentz covariants with the corresponding properties under space reflection as demanded and obtain the general decomposition of the S-dependent part of the quark-quark correlator as

$$z\Xi^{V(0)}(z,k_{F\perp};p,S) = (\tilde{k}_{F\perp}\cdot S_T)E_T^\perp(z,k_{F\perp}), \tag{A10}$$

$$z\tilde{\Xi}^{V(0)}(z,k_{F\perp};p,S) = M\left[\lambda E_L(z,k_{F\perp}) + \frac{k_{F\perp}\cdot S_T}{M}E_T^{\prime\perp}(z,k_{F\perp})\right], \tag{A11}$$

$$z\Xi_\alpha^{V(0)}(z,k_{F\perp};p,S) = p^+\bar{n}_\alpha \frac{\tilde{k}_{F\perp}\cdot S_T}{M} D_{1T}^\perp(z,k_{F\perp}) - M\tilde{S}_{T\alpha} D_T(z,k_{F\perp}) - \tilde{k}_{F\perp\alpha}\left[\lambda D_L^\perp(z,k_{F\perp}) + \frac{k_{F\perp}\cdot S_T}{M} D_T^\perp(z,k_{F\perp})\right] + \frac{M}{p^+} n_\alpha(\tilde{k}_{F\perp}\cdot S_T) D_{3T}^\perp(z,k_{F\perp}), \tag{A12}$$

$$z\tilde{\Xi}_\alpha^{V(0)}(z,k_{F\perp};p,S) = p^+\bar{n}_\alpha\left[\lambda G_{1L}(z,k_{F\perp}) + \frac{k_{F\perp}\cdot S_T}{M} G_{1T}^\perp(z,k_{F\perp})\right] - MS_{T\alpha}G_T(z,k_{F\perp}) - k_{F\perp\alpha}\left[\lambda G_L^\perp(z,k_{F\perp}) + \frac{k_{F\perp}\cdot S_T}{M} G_T^\perp(z,k_{F\perp})\right] + \frac{M^2}{p^+} n_\alpha\left[\lambda G_{3L}(z,k_{F\perp}) + \frac{k_{F\perp}\cdot S_T}{M} G_{3T}^\perp(z,k_{F\perp})\right], \tag{A13}$$

$$z\Xi_{\rho\alpha}^{V(0)}(z,k_{F\perp};p,S) = p^+\bar{n}_{[\rho}S_{T\alpha]}H_{1T}(z,k_{F\perp}) + \frac{p^+}{M}\bar{n}_{[\rho}k_{F\perp\alpha]}\left[\lambda H_{1L}^\perp(z,k_{F\perp}) + \frac{k_{F\perp}\cdot S_T}{M} H_{1T}^\perp(z,k_{F\perp})\right] + k_{F\perp[\rho}S_{T\alpha]}H_T^\perp(z,k_{F\perp}) + M\bar{n}_{[\rho}n_{\alpha]}\left[\lambda H_L(z,k_{F\perp}) + \frac{k_{F\perp}\cdot S_T}{M} H_T'^\perp(z,k_{F\perp})\right] + \frac{M^2}{p^+} n_{[\rho}S_{T\alpha]}H_{3T}(z,k_{F\perp}) + \frac{M}{p^+} n_{[\rho}k_{F\perp\alpha]}\left[\lambda H_{3L}^\perp(z,k_{F\perp}) + \frac{k_{F\perp}\cdot S_T}{M} H_{3T}^\perp(z,k_{F\perp})\right]. \tag{A14}$$

If we integrate over $d^2k_{F\perp}$, only eight terms survive, i.e.,

$$z\Xi^{V(0)}(z;p,S) = 0, \tag{A15}$$

$$z\tilde{\Xi}^{V(0)}(z;p,S) = \lambda M E_L(z), \tag{A16}$$

$$z\Xi_\alpha^{V(0)}(z;p,S) = -M\tilde{S}_{T\alpha}D_T(z), \tag{A17}$$

$$z\tilde{\Xi}_\alpha^{V(0)}(z;p,S) = \lambda p^+\bar{n}_\alpha G_{1L}(z) - MS_{T\alpha}G_T(z) + \lambda\frac{M^2}{p^+} n_\alpha G_{3L}(z), \tag{A18}$$

$$z\Xi_{\rho\alpha}^{V(0)}(z;p,S) = p^+\bar{n}_{[\rho}S_{T\alpha]}H_{1T}(z) - \lambda M\bar{n}_{[\rho}n_{\alpha]}H_L(z) + \frac{M^2}{p^+} n_{[\rho}S_{T\alpha]}H_{3T}(z), \tag{A19}$$

where the one-dimensional FF in the longitudinally polarized case is just equal to the corresponding three-dimensional FF integrated over $d^2k_{F\perp}$, while in the transversely polarized case, we have

$$K_T(z) = \int \frac{d^2k_{F\perp}}{(2\pi)^2}\mathcal{K}_T^\perp(z,k_{F\perp}), \qquad \mathcal{K}_T^\perp(z,k_{F\perp}) \equiv K_T(z,k_{F\perp}) + \frac{k_{F\perp}^2}{2M^2}K_T^\perp(z,k_{F\perp}), \tag{A20}$$

for the transverse polarization dependent FFs such as $K_T = D_T$, G_T, H_{1T}, or H_{3T}, and similar for the S_{LT}-dependent part in the following.

3. Tensor polarization dependent part

The most general decomposition for the tensor polarization dependent part is given by

$$z\Xi^{T(0)}(z,k_{F\perp};p,S) = M\left[S_{LL}E_{LL}(z,k_{F\perp}) + \frac{k_{F\perp}\cdot S_{LT}}{M} E_{LT}^\perp(z,k_{F\perp}) + \frac{S_{TT}^{k_F k_F}}{M^2} E_{TT}^\perp(z,k_{F\perp})\right], \tag{A21}$$

$$z\tilde{\Xi}^{T(0)}(z,k_{F\perp};p,S) = M\left[\frac{\tilde{k}_{F\perp}\cdot S_{LT}}{M} E_{LT}'^\perp(z,k_{F\perp}) + \frac{S_{TT}^{\tilde{k}_F k_F}}{M^2} E_{TT}'^\perp(z,k_{F\perp})\right], \tag{A22}$$

$$\begin{aligned}
z\Xi_\alpha^{T(0)}(z,k_{F\perp};p,S) = {}& p^+\bar{n}_\alpha\left[S_{LL}D_{1LL}(z,k_{F\perp}) + \frac{k_{F\perp}\cdot S_{LT}}{M}D_{1LT}^\perp(z,k_{F\perp}) + \frac{S_{TT}^{k_Fk_F}}{M^2}D_{1TT}^\perp(z,k_{F\perp})\right]\\
&+ MS_{LT\alpha}D_{LT}(z,k_{F\perp}) + S_{TT\alpha}^{k_F}D_{TT}'^\perp(z,k_{F\perp})\\
&+ k_{F\perp\alpha}\left[S_{LL}D_{LL}^\perp(z,k_{F\perp}) + \frac{k_{F\perp}\cdot S_{LT}}{M}D_{LT}^\perp(z,k_{F\perp}) + \frac{S_{TT}^{k_Fk_F}}{M^2}D_{TT}^\perp(z,k_{F\perp})\right]\\
&+ \frac{M^2}{p^+}n_\alpha\left[S_{LL}D_{3LL}(z,k_{F\perp}) + \frac{k_{F\perp}\cdot S_{LT}}{M}D_{3LT}^\perp(z,k_{F\perp}) + \frac{S_{TT}^{k_Fk_F}}{M^2}D_{3TT}^\perp(z,k_{F\perp})\right],
\end{aligned} \tag{A23}$$

$$\begin{aligned}
z\tilde{\Xi}_\alpha^{T(0)}(z,k_{F\perp};p,S) = {}& p^+\bar{n}_\alpha\left[\frac{\tilde{k}_{F\perp}\cdot S_{LT}}{M}G_{1LT}^\perp(z,k_{F\perp}) + \frac{S_{TT}^{\tilde{k}_Fk_F}}{M^2}G_{1TT}^\perp(z,k_{F\perp})\right] - M\tilde{S}_{LT\alpha}G_{LT}(z,k_{F\perp}) - \tilde{S}_{TT\alpha}^{k_F}G_{TT}'^\perp(z,k_{F\perp})\\
&- \tilde{k}_{F\perp\alpha}\left[S_{LL}G_{LL}^\perp(z,k_{F\perp}) + \frac{k_{F\perp}\cdot S_{LT}}{M}G_{LT}^\perp(z,k_{F\perp}) + \frac{S_{TT}^{k_Fk_F}}{M^2}G_{TT}^\perp(z,k_{F\perp})\right]\\
&+ \frac{M^2}{p^+}n_\alpha\left[\frac{\tilde{k}_{F\perp}\cdot S_{LT}}{M}G_{3LT}^\perp(z,k_{F\perp}) + \frac{S_{TT}^{\tilde{k}_Fk_F}}{M^2}G_{3TT}^\perp(z,k_{F\perp})\right],
\end{aligned} \tag{A24}$$

$$\begin{aligned}
z\Xi_{\rho\alpha}^{T(0)}(z,k_{F\perp};p,S) = {}& -p^+\bar{n}_{[\rho}\tilde{S}_{LT\alpha]}H_{1LT}(z,k_{F\perp}) - \frac{p^+}{M}\bar{n}_{[\rho}\tilde{S}_{TT\alpha]}^{k_F}H_{1TT}'^\perp(z,k_{F\perp})\\
&- \frac{p^+}{M}\bar{n}_{[\rho}\tilde{k}_{F\perp\alpha]}\left[S_{LL}H_{1LL}^\perp(z,k_{F\perp}) + \frac{k_{F\perp}\cdot S_{LT}}{M}H_{1LT}^\perp(z,k_{F\perp}) + \frac{S_{TT}^{k_Fk_F}}{M^2}H_{1TT}^\perp(z,k_{F\perp})\right]\\
&+ M\varepsilon_{\perp\rho\alpha}\left[S_{LL}H_{LL}(z,k_{F\perp}) + \frac{k_{F\perp}\cdot S_{LT}}{M}H_{LT}^\perp(z,k_{F\perp}) + \frac{S_{TT}^{k_Fk_F}}{M^2}H_{TT}^\perp(z,k_{F\perp})\right]\\
&+ \bar{n}_{[\rho}n_{\alpha]}\left[(\tilde{k}_{F\perp}\cdot S_{LT})H_{LT}'^\perp(z,k_{F\perp}) + \frac{S_{TT}^{\tilde{k}_Fk_F}}{M}H_{TT}'^\perp(z,k_{F\perp})\right]\\
&- \frac{M}{p^+}n_{[\rho}\tilde{k}_{F\perp\alpha]}\left[S_{LL}H_{3LL}^\perp(z,k_{F\perp}) + \frac{k_{F\perp}\cdot S_{LT}}{M}H_{3LT}^\perp(z,k_{F\perp}) + \frac{S_{TT}^{k_Fk_F}}{M^2}H_{3TT}^\perp(z,k_{F\perp})\right]\\
&- \frac{M}{p^+}n_{[\rho}[M\tilde{S}_{LT\alpha]}H_{3LT}(z,k_{F\perp}) + \tilde{S}_{TT\alpha]}^{k_F}H_{3TT}'^\perp(z,k_{F\perp})].
\end{aligned} \tag{A25}$$

We integrate over $d^2k_{F\perp}$ and obtain

$$z\Xi^{T(0)}(z;p,S) = MS_{LL}E_{LL}(z), \tag{A26}$$

$$z\tilde{\Xi}^{T(0)}(z;p,S) = 0, \tag{A27}$$

$$z\Xi_\alpha^{T(0)}(z;p,S) = p^+\bar{n}_\alpha S_{LL}D_{1LL}(z) + MS_{LT\alpha}D_{LT}(z) + \frac{M^2}{p^+}n_\alpha S_{LL}D_{3LL}(z), \tag{A28}$$

$$z\tilde{\Xi}_\alpha^{T(0)}(z;p,S) = -M\tilde{S}_{LT\alpha}G_{LT}(z), \tag{A29}$$

$$z\Xi_{\rho\alpha}^{T(0)}(z;p,S) = -\,p^+\bar{n}_{[\rho}\tilde{S}_{LT\alpha]}H_{1LT}(z) + M\varepsilon_{\perp\rho\alpha}S_{LL}H_{LL}(z) - \frac{M^2}{p^+}n_{[\rho}\tilde{S}_{LT\alpha]}H_{3LT}(z). \tag{A30}$$

Again, the four S_{LL}-dependent one-dimensional FFs are just equal to the corresponding three-dimensional FFs integrated over $d^2k_{F\perp}$, while the four S_{LT}-dependent FFs are given by Eq. (A20) for $K_T = D_{LT}$, G_{LT}, H_{1LT}, and H_{3LT}.

We list those twist-2 FFs in Table II, and those twist-3 FFs in Table III. The twist-4 FFs have the same structure of those at twist-2, so we do not make a separate table. We also list them according to chiral and time-reversal properties in Table IV.

TABLE II. The 18 leading twist components of the FFs for quark fragments to spin-1 hadrons. The symbol × means that the corresponding FF disappears after the integration over transverse momentum.

Quark polarization	Hadron polarization	TMD FFs	Integrated over $\vec{k}_{F\perp}$	Name
U	U	$D_1(z, k_{F\perp})$	$D_1(z)$	Number density
	T	$D_{1T}^{\perp}(z, k_{F\perp})$	×	
	LL	$D_{1LL}(z, k_{F\perp})$	$D_{1LL}(z)$	Spin alignment
	LT	$D_{1LT}^{\perp}(z, k_{F\perp})$	×	
	TT	$D_{1TT}^{\perp}(z, k_{F\perp})$	×	
L	L	$G_{1L}(z, k_{F\perp})$	$G_{1L}(z)$	Spin transfer (longitudinal)
	T	$G_{1T}^{\perp}(z, k_{F\perp})$	×	
	LT	$G_{1LT}^{\perp}(z, k_{F\perp})$	×	
	TT	$G_{1TT}^{\perp}(z, k_{F\perp})$	×	
T	U	$H_1^{\perp}(z, k_{F\perp})$	×	Collins function
	$T(\parallel)$	$H_{1T}(z, k_{F\perp})$	$H_{1T}(z)$	Spin transfer (transverse)
	$T(\perp)$	$H_{1T}^{\perp}(z, k_{F\perp})$		
	L	$H_{1L}^{\perp}(z, k_{F\perp})$	×	
	LL	$H_{1LL}^{\perp}(z, k_{F\perp})$	×	
	LT	$H_{1LT}(z, k_{F\perp}), H_{1LT}^{\perp}(z, k_{F\perp})$	$H_{1LT}(z)$	
	TT	$H_{1TT}^{\perp}(z, k_{F\perp}), H_{1TT}^{\prime\perp}(z, k_{F\perp})$	×, ×	

TABLE III. The 36 twist-3 components of the FFs for quark fragments to spin-1 hadrons. The symbol × means that the corresponding FF disappears after the integration over transverse momentum.

Quark polarization	Hadron polarization	TMD FFs	Integrated over $\vec{k}_{F\perp}$
U	U	$E(z, k_{F\perp})$, $D^{\perp}(z, k_{F\perp})$	$E(z)$, ×
	L	$D_L^{\perp}(z, k_{F\perp})$	×
	T	$E_T^{\perp}(z, k_{F\perp})$, $D_T(z, k_{F\perp})$, $D_T^{\perp}(z, k_{F\perp})$	×, $D_T(z)$
	LL	$E_{LL}(z, k_{F\perp})$, $D_{LL}^{\perp}(z, k_{F\perp})$	$E_{LL}(z)$, ×
	LT	$E_{LT}^{\perp}(z, k_{F\perp})$, $D_{LT}(z, k_{F\perp})$, $D_{LT}^{\perp}(z, k_{F\perp})$	×, $D_{LT}(z)$
	TT	$E_{TT}^{\perp}(z, k_{F\perp})$, $D_{TT}^{\perp}(z, k_{F\perp})$, $D_{TT}^{\prime\perp}(z, k_{F\perp})$	×, ×, ×
L	U	$G^{\perp}(z, k_{F\perp})$	×
	L	$E_L(z, k_{F\perp})$, $G_L^{\perp}(z, k_{F\perp})$	$E_L(z)$, ×
	T	$E_T^{\prime\perp}(z, k_{F\perp})$, $G_T(z, k_{F\perp})$, $G_T^{\perp}(z, k_{F\perp})$	×, $G_T(z)$
	LL	$G_{LL}^{\perp}(z, k_{F\perp})$	×
	LT	$E_{LT}^{\prime\perp}(z, k_{F\perp})$, $G_{LT}(z, k_{F\perp})$, $G_{LT}^{\perp}(z, k_{F\perp})$	×, $G_{LT}(z)$
	TT	$E_{TT}^{\prime\perp}(z, k_{F\perp})$, $G_{TT}^{\perp}(z, k_{F\perp})$, $G_{TT}^{\prime\perp}(z, k_{F\perp})$	×, ×, ×
T	U	$H(z, k_{F\perp})$	$H(z)$
	L	$H_L(z, k_{F\perp})$	$H_L(z)$
	$T(\parallel)$	$H_T^{\perp}(z, k_{F\perp})$	×
	$T(\perp)$	$H_T^{\prime\perp}(z, k_{F\perp})$	×
	LL	$H_{LL}(z, k_{F\perp})$	$H_{LL}(z)$
	LT	$H_{LT}^{\perp}(z, k_{F\perp})$, $H_{LT}^{\prime\perp}(z, k_{F\perp})$	×, ×
	TT	$H_{TT}^{\perp}(z, k_{F\perp})$, $H_{TT}^{\prime\perp}(z, k_{F\perp})$	×, ×

4. Twist-3 FFs defined via the quark-gluon-quark correlator

Twist-3 components are the leading twist contributions that we obtain from $\hat{\Xi}_\rho^{(1)}$. There has to be one $\bar{n}$ involved in the basic Lorentz covariants and the other(s) are from the transverse components. Since the $\bar{n}$ component of the gluon field goes into the gauge link, we only have the other three components for D_ρ; thus no $\bar{n}_\rho$ component exists in the

TABLE IV. Chiral and time-reversal properties of TMD FFs from the quark-quark correlator.

		Chiral even		Chiral odd	
Quark polarization	Hadron polarization	T even	T odd	T even	T odd
U	U	D_1, $D^{\perp}$, D_3		E	
	L		$D_L^{\perp}$		
	T		$D_{1T}^{\perp}$, D_T, $D_T^{\perp}$, $D_{3T}^{\perp}$		$E_T^{\perp}$
	LL	D_{1LL}, $D_{LL}^{\perp}$, D_{3LL}		E_{LL}	
	LT	$D_{1LT}^{\perp}$, D_{LT}, $D_{LT}^{\perp}$, $D_{3LT}^{\perp}$		$E_{LT}^{\perp}$	
	TT	$D_{1TT}^{\perp}$, $D_{TT}^{\perp}$, $D_{TT}'^{\perp}$, $D_{3TT}^{\perp}$		$E_{TT}^{\perp}$	
L	U		$G^{\perp}$		
	L	G_{1L}, $G_L^{\perp}$, G_{3L}			E_L
	T	$G_{1T}^{\perp}$, G_T, $G_T^{\perp}$, $G_{3T}^{\perp}$			$E_T'^{\perp}$
	LL		$G_{LL}^{\perp}$		
	LT		$G_{1LT}^{\perp}$, G_{LT}, $G_{LT}^{\perp}$, $G_{3LT}^{\perp}$	$E_{LT}'^{\perp}$	
	TT		$G_{1TT}^{\perp}$, $G_{TT}^{\perp}$, $G_{TT}'^{\perp}$, $G_{3TT}^{\perp}$	$E_{TT}'^{\perp}$	
T	U				$H_1^{\perp}$, H, $H_3^{\perp}$
	L			$H_{1L}^{\perp}$, H_L, $H_{3L}^{\perp}$	
	$T(\Vert)$			H_{1T}, $H_T^{\perp}$, H_{3T}	
	$T(\perp)$			$H_{1T}^{\perp}$, $H_T'^{\perp}$, $H_{3T}^{\perp}$	
	LL				$H_{1LL}^{\perp}$, H_{LL}, $H_{3LL}^{\perp}$
	LT				H_{1LT}, $H_{1LT}^{\perp}$, $H_{LT}^{\perp}$, $H_{LT}'^{\perp}$, H_{3LT}, $H_{3LT}^{\perp}$
	TT				$H_{1TT}^{\perp}$, $H_{1TT}'^{\perp}$, $H_{TT}^{\perp}$, $H_{TT}'^{\perp}$, $H_{3TT}^{\perp}$, $H_{3TT}'^{\perp}$

basic Lorentz covariants. We therefore do not have twist-3 contributions from $\Xi_{\rho}^{(1)}$ or $\tilde{\Xi}_{\rho}^{(1)}$. The twist-3 contributions are obtained from $\Xi_{\rho\alpha}^{(1)}$, $\tilde{\Xi}_{\rho\alpha}^{(1)}$, and $\Xi_{\rho\alpha\beta}^{(1)}$ and are given in the following.

For the unpolarized part, we have

$$z\Xi_{\rho\alpha}^{U(1)}(z, k_{F\perp}; p) = -p^{+}\bar{n}_{\alpha}k_{F\perp\rho}D_{d}^{\perp}(z, k_{F\perp}) + \cdots, \tag{A31}$$

$$z\tilde{\Xi}_{\rho\alpha}^{U(1)}(z, k_{F\perp}; p) = -ip^{+}\bar{n}_{\alpha}\tilde{k}_{F\perp\rho}G_{d}^{\perp}(z, k_{F\perp}) + \cdots, \tag{A32}$$

$$z\Xi_{\rho\alpha\beta}^{U(1)}(z, k_{F\perp}; p) = -p^{+}\left[M\varepsilon_{\perp\rho[\alpha}\bar{n}_{\beta]}H_{d}(z, k_{F\perp}) - \frac{1}{M}\tilde{k}_{F\perp\rho}k_{F\perp[\alpha}\bar{n}_{\beta]}H_{d}^{\perp}(z, k_{F\perp})\right] + \cdots. \tag{A33}$$

For the vector polarization dependent part, we have

$$z\Xi_{\rho\alpha}^{V(1)}(z, k_{F\perp}; p, S) = p^{+}\bar{n}_{\alpha}\left\{M\tilde{S}_{T\rho}D_{dT}(z, k_{F\perp}) + \tilde{k}_{F\perp\rho}\left[\lambda D_{dL}^{\perp}(z, k_{F\perp}) + \frac{k_{F\perp}\cdot S_{T}}{M}D_{dT}^{\perp}(z, k_{F\perp})\right]\right\} + \cdots, \tag{A34}$$

$$z\tilde{\Xi}_{\rho\alpha}^{V(1)}(z, k_{F\perp}; p, S) = -ip^{+}\bar{n}_{\alpha}\left\{MS_{T\rho}G_{dT}(z, k_{F\perp}) + k_{\perp\rho}\left[\lambda G_{dL}^{\perp}(z, k_{F\perp}) + \frac{k_{F\perp}\cdot S_{T}}{M}G_{dT}^{\perp}(z, k_{F\perp})\right]\right\} + \cdots, \tag{A35}$$

$$z\Xi^{V(1)}_{\rho\alpha\beta}(z,k_{F\perp};p,S) = p^+\Big\{\lambda\Big[Mg_{\perp\rho[\alpha}\bar{n}_{\beta]}H_{dL}(z,k_{F\perp}) + \frac{1}{M}k_{F\perp\rho}k_{F\perp[\alpha}\bar{n}_{\beta]}H^{\perp}_{dL}(z,k_{F\perp})\Big]$$
$$- (\tilde{k}_{F\perp}\cdot S_T)\Big[\varepsilon_{\perp\rho[\alpha}\bar{n}_{\beta]}H^{\perp}_{dT}(z,k_{F\perp}) - \frac{1}{M^2}\tilde{k}_{F\perp\rho}k_{F\perp[\alpha}\bar{n}_{\beta]}H^{\perp\prime}_{dT}(z,k_{F\perp})\Big]$$
$$+ (k_{F\perp}\cdot S_T)\Big[g_{\perp\rho[\alpha}\bar{n}_{\beta]}H^{\prime\perp}_{dT}(z,k_{F\perp}) + \frac{1}{M^2}k_{F\perp\rho}k_{F\perp[\alpha}\bar{n}_{\beta]}H^{\prime\perp\prime}_{dT}(z,k_{F\perp})\Big]\Big\} + \cdots. \tag{A36}$$

For tensor polarization dependent part, we have

$$z\Xi^{T(1)}_{\rho\alpha}(z,k_{F\perp};p,S) = -p^+\bar{n}_\alpha\Big[k_{F\perp\rho}S_{LL}D^{\perp}_{dLL}(z,k_{F\perp}) + MS_{LT\rho}D_{dLT}(z,k_{F\perp}) + k_{F\perp\rho}\frac{k_{F\perp}\cdot S_{LT}}{M}D^{\perp}_{dLT}(z,k_{F\perp})$$
$$+ S^{k_F}_{TT\rho}D^{\prime\perp}_{dTT}(z,k_{F\perp}) + k_{F\perp\rho}\frac{S^{k_Fk_F}_{TT}}{M^2}D^{\perp}_{dTT}(z,k_{F\perp})\Big] + \cdots, \tag{A37}$$

$$z\tilde{\Xi}^{(1)}_{\rho\alpha}(z,k_{F\perp};p,S) = -ip^+\bar{n}_\alpha\Big[\tilde{k}_{F\perp\rho}S_{LL}G^{\perp}_{dLL}(z,k_{F\perp}) + M\tilde{S}_{LT\rho}G_{dLT}(z,k_{F\perp}) + \frac{1}{M}\tilde{k}_{F\perp\rho}k_{F\perp}\cdot S_{LT}G^{\perp}_{dLT}(z,k_{F\perp})$$
$$+ \tilde{S}^{k_F}_{TT\rho}G^{\prime\perp}_{dTT}(z,k_{F\perp}) + \tilde{k}_{F\perp\rho}\frac{S^{k_Fk_F}_{TT}}{M^2}G^{\perp}_{dTT}(z,k_{F\perp})\Big] + \cdots, \tag{A38}$$

$$z\Xi^{T(1)}_{\rho\alpha\beta}(z,k_{F\perp};p,S) = p^+\Big\{S_{LL}\Big[M\varepsilon_{\perp\rho[\alpha}\bar{n}_{\beta]}H_{dLL}(z,k_{F\perp}) - \frac{1}{M}\tilde{k}_{F\perp\rho}k_{F\perp[\alpha}\bar{n}_{\beta]}H^{\perp}_{dLL}(z,k_{F\perp})\Big]$$
$$+ (k_{F\perp}\cdot S_{LT})\Big[\varepsilon_{\perp\rho[\alpha}\bar{n}_{\beta]}H^{\perp}_{dLT}(z,k_{F\perp}) - \frac{1}{M^2}\tilde{k}_{F\perp\rho}k_{F\perp[\alpha}\bar{n}_{\beta]}H^{\perp\prime}_{dLT}(z,k_{F\perp})\Big]$$
$$- (\tilde{k}_{F\perp}\cdot S_{LT})\Big[g_{\perp\rho[\alpha}\bar{n}_{\beta]}H^{\prime\perp}_{dLT}(z,k_{F\perp}) + \frac{1}{M^2}k_{F\perp\rho}k_{F\perp[\alpha}\bar{n}_{\beta]}H^{\prime\perp\prime}_{dLT}(z,k_{F\perp})\Big]$$
$$+ \frac{S^{k_Fk_F}_{TT}}{M}\Big[\varepsilon_{\perp\rho[\alpha}\bar{n}_{\beta]}H^{\perp}_{dTT}(z,k_{F\perp}) - \frac{1}{M^2}\tilde{k}_{F\perp\rho}k_{F\perp[\alpha}\bar{n}_{\beta]}H^{\perp\prime}_{dTT}(z,k_{F\perp})\Big]$$
$$+ \frac{S^{\tilde{k}_Fk_F}_{TT}}{M}\Big[g_{\perp\rho[\alpha}\bar{n}_{\beta]}H^{\prime\perp}_{dTT}(z,k_{F\perp}) + \frac{1}{M^2}k_{F\perp\rho}k_{F\perp[\alpha}\bar{n}_{\beta]}H^{\prime\perp\prime}_{dTT}(z,k_{F\perp})\Big]\Big\} + \cdots. \tag{A39}$$

Here, we use a subscript d to specify that they are defined via the quark-gluon-quark correlator. A prime in the superscript before the $\perp$ denotes different polarization situation, that after the $\perp$ specifies different FF for the same polarization situation. We see that we have totally 36 FFs at twist-3 defined via the quark-gluon-quark correlator. This is just the same as what we obtained from the quark-quark correlator. Among them, 18 are χ even and the other 18 are χ odd; four contribute to the unpolarized part, 12 to vector polarized part, and 20 to the tensor polarized part. We note in particular that the Hermiticity in this case does not demand that the FFs defined via the quark-gluon-quark correlator are real. They can have both real and imaginary parts.

APPENDIX B: TWIST-3 CONTRIBUTIONS TO THE HADRONIC TENSOR

In the two-hadron-collinear frame, the twist-3 contributions to other parts of the hadronic tensor besides $W^{(1)U}_{\mu\nu}$ and $W^{(1)LL}_{\mu\nu}$ given by Eqs. (4.14) and (4.19) are given by

$$
\begin{aligned}
W_{\mu\nu}^{(1)L} = {} & \frac{4\lambda}{z_1 z_2}\int \frac{d^2k_\perp}{(2\pi)^2}\frac{d^2k'_\perp}{(2\pi)^2}\delta^2(k_\perp + k'_\perp - q_\perp)\left\{\frac{1}{p_1^+}[-\omega_{\mu\nu}(\tilde{k})D_L^\perp + \tilde{\omega}_{\mu\nu}(k)G_L^\perp]\bar{D}_1\right. \\
& + \frac{2M_1 c_2^q}{M_2 p_1^+}[2(\tilde{k}'_n - \tilde{k}'_{\bar{n}})_{\{\mu\nu\}}H_L + i(\tilde{k}'_n - \tilde{k}'_{\bar{n}})_{[\mu\nu]}E_L]\bar{H}_1^\perp + \frac{1}{p_2^-}G_{1L}[\omega_{\mu\nu}(k')\bar{D}^\perp + \tilde{\omega}_{\mu\nu}(\tilde{k}')\bar{G}^\perp] \\
& - \frac{2M_2 c_2^q}{M_1 p_2^-}H_{1L}^\perp[2(\tilde{k}_n - \tilde{k}_{\bar{n}})_{\{\mu\nu\}}\bar{H} + i(\tilde{k}_n - \tilde{k}_{\bar{n}})_{[\mu\nu]}\bar{E}] - \frac{\sqrt{2}}{Q}\left[\tilde{\omega}_{\mu\nu}(k',k)G_{1L}\bar{D}_1\right. \\
& \left.\left. + \frac{4c_2^q}{M_1 M_2}(k_\perp^2\tilde{k}'_{\bar{n}} + \tilde{k}_\perp\cdot k'_\perp k'_n + k'_\perp\cdot k_\perp k'_n)_{\{\mu\nu\}}H_{1L}^\perp\bar{H}_1^\perp\right]\right\},
\end{aligned}
\tag{B1}
$$

$$
\begin{aligned}
W_{\mu\nu}^{(1)T} = {} & \frac{4}{z_1 z_2}\int \frac{d^2k_\perp}{(2\pi)^2}\frac{d^2k'_\perp}{(2\pi)^2}\delta^2(k_\perp + k'_\perp - q_\perp)\left\{\frac{k_\perp\cdot S_T}{M_1}\left(\frac{1}{p_1^+}[\omega_{\mu\nu}(-\tilde{k},\tilde{k})D_T^\perp + \tilde{\omega}_{\mu\nu}(k)G_T^\perp]\bar{D}_1\right.\right. \\
& + \frac{1}{p_2^-}G_{1T}[\tilde{\omega}_{\mu\nu}(k')\bar{D}^\perp + \tilde{\omega}_{\mu\nu}(\tilde{k}')\bar{G}^\perp] + \frac{2M_1 c_2^q}{M_2 p_1^+}[2(\tilde{k}'_n - \tilde{k}'_{\bar{n}})_{\{\mu\nu\}}H_T'^\perp + i(\tilde{k}'_n - \tilde{k}'_{\bar{n}})_{[\mu\nu]}E_T'^\perp]\bar{H}_1^\perp \\
& - \frac{2M_2 c_2^q}{M_1 p_2^-}H_{1T}^\perp[2(\tilde{k}_n - \tilde{k}_{\bar{n}})_{\{\mu\nu\}}\bar{H} + i(\tilde{k}_n - \tilde{k}_{\bar{n}})_{[\mu\nu]}\bar{E}] - \frac{\sqrt{2}}{Q}\left[\tilde{\omega}_{\mu\nu}(k',k)G_{1T}^\perp\bar{D}_1\right. \\
& \left.\left. + \frac{4c_2^q}{M_1 M_2}(k_\perp^2\tilde{k}'_{\bar{n}} + \tilde{k}_\perp\cdot k'_\perp k'_n + k'_\perp\cdot k_\perp k'_n)_{\{\mu\nu\}}H_{1T}^\perp\bar{H}_1^\perp\right]\right) + \frac{\tilde{k}_\perp\cdot S_T}{M_1}\left(\frac{1}{p_2^-}D_{1T}^\perp[-\omega_{\mu\nu}(k')\bar{D}^\perp - \tilde{\omega}_{\mu\nu}(\tilde{k}')\bar{G}^\perp]\right. \\
& \left. + \frac{2M_1 c_2^q}{M_2 p_1^+}[2(k'_n - k'_{\bar{n}})_{\{\mu\nu\}}H_T^\perp + i(k'_n - k'_{\bar{n}})_{[\mu\nu]}E_T^\perp]\bar{H}_1^\perp + \frac{\sqrt{2}}{Q}\omega_{\mu\nu}(k',k)D_{1T}^\perp\bar{D}_1\right) \\
& - \frac{2M_2 c_2^q}{p_2^-}H_{1T}[2(\tilde{S}_n - \tilde{S}_{\bar{n}})_{\{\mu\nu\}}\bar{H} + i(\tilde{S}_n - \tilde{S}_{\bar{n}})_{[\mu\nu]}\bar{E}] + \frac{M_1}{p_1^+}[-\omega_{\mu\nu}(\tilde{S})D_T + \tilde{\omega}_{\mu\nu}(S)G_T]\bar{D}_1 \\
& \left. - \frac{4\sqrt{2}c_2^q}{M_2 Q}[-\tilde{k}'_\perp\cdot S_\perp(k_{\bar{n}} + k'_n)_{\{\mu\nu\}} + \tilde{k}'_\perp\cdot k_\perp S_{\bar{n}\{\mu\nu\}} + k_\perp\cdot S_\perp\tilde{k}'_{\bar{n}\{\mu\nu\}} + k'_\perp\cdot S_\perp\tilde{k}'_{n\{\mu\nu\}}]H_{1T}\bar{H}_1^\perp\right\},
\end{aligned}
\tag{B2}
$$

$$
\begin{aligned}
W_{\mu\nu}^{(1)LT} = {} & \frac{4}{z_1 z_2}\int \frac{d^2k_\perp}{(2\pi)^2}\frac{d^2k'_\perp}{(2\pi)^2}\delta^2(k_\perp + k'_\perp - q_\perp)\left\{\frac{k_\perp\cdot S_{LT}}{M_1}\left(\frac{1}{p_1^+}[\omega_{\mu\nu}(k)D_{LT}^\perp + \tilde{\omega}_{\mu\nu}(\tilde{k})G_{LT}^\perp]\bar{D}_1\right.\right. \\
& - \frac{1}{p_2^-}[\omega_{\mu\nu}(k')D_{1LT}^\perp\bar{D}^\perp + \tilde{\omega}_{\mu\nu}(\tilde{k}')D_{1LT}^\perp\bar{G}^\perp] + \frac{2M_1 c_2^q}{M_2 p_1^+}[2(k'_n - k'_{\bar{n}})_{\{\mu\nu\}}H_{LT}^\perp + i(k'_n - k'_{\bar{n}})_{[\mu\nu]}E_{LT}^\perp]\bar{H}_1^\perp \\
& \left. - \frac{2M_2 c_2^q}{M_1 p_2^-}[2(k_n - k_{\bar{n}})_{\{\mu\nu\}}H_{1LT}^\perp\bar{H} + i(k_n - k_{\bar{n}})_{[\mu\nu]}H_{1LT}^\perp\bar{E}] + \frac{\sqrt{2}}{Q}\left[\omega_{\mu\nu}(k',k)D_{1LT}^\perp\bar{D}_1 - \frac{4}{M_1 M_2}\omega_{\mu\nu}^{(n)}(k,k')H_{1LT}^\perp\bar{H}_1^\perp\right]\right) \\
& + \frac{\tilde{k}_\perp\cdot S_{LT}}{M_1}\left(\frac{1}{p_2^-}G_{1LT}^\perp[\tilde{\omega}_{\mu\nu}(k')\bar{D}^\perp + \omega_{\mu\nu}(\tilde{k}')\bar{G}^\perp] + \frac{2M_1 c_2^q}{M_2 p_1^+}[2(\tilde{k}'_n - \tilde{k}'_{\bar{n}})_{\{\mu\nu\}}H_{LT}'^\perp + i(\tilde{k}'_n - \tilde{k}'_{\bar{n}})_{[\mu\nu]}E_{LT}'^\perp]\bar{H}_1^\perp\right. \\
& \left. - \frac{\sqrt{2}}{Q}\tilde{\omega}_{\mu\nu}(k',k)G_{1LT}^\perp\bar{D}_1\right) + \frac{M_1}{p_1^+}[\omega_{\mu\nu}(S_{LT})D_{LT} + \tilde{\omega}_{\mu\nu}(\tilde{S}_{LT})G_{LT}]\bar{D}_1 \\
& - \frac{2M_2 c_2^q}{p_2^-}[2(S_{LTn} - S_{LT\bar{n}})_{\{\mu\nu\}}H_{1LT}\bar{H} + i(S_{LTn} - S_{LT\bar{n}})_{[\mu\nu]}H_{1LT}\bar{E}] \\
& \left. - \frac{4\sqrt{2}c_2^q}{M_2 Q}(k_\perp\cdot S_{LT}k'_{\bar{n}} - k'_\perp\cdot S_{LT}k_{\bar{n}} + k_\perp\cdot k'_\perp S_{LT\bar{n}} + k_\perp'^2 S_{LTn})_{\{\mu\nu\}}H_{1LT}\bar{H}_1^\perp\right\},
\end{aligned}
\tag{B3}
$$

$$W_{\mu\nu}^{(1)TT} = \frac{4}{z_1 z_2} \int \frac{d^2k_\perp}{(2\pi)^2} \frac{d^2k'_\perp}{(2\pi)^2} \delta^2(k_\perp + k'_\perp - q_\perp) \left\{ \frac{S_{TT}^{kk}}{M_1^2} \left(\frac{1}{p_1^+} [\omega_{\mu\nu}(k) D_{TT}^\perp + \tilde{\omega}_{\mu\nu}(\tilde{k}) G_{TT}^\perp] \bar{D}_1 \right.\right.$$
$$- \frac{1}{p_2^-} [\omega_{\mu\nu}(k') D_{1TT}^\perp \bar{D}^\perp + \tilde{\omega}_{\mu\nu}(\tilde{k}') D_{1TT}^\perp \bar{G}^\perp] + \frac{2M_1 c_2^q}{M_2 p_1^+} [2(k'_n - k'_{\bar{n}})_{\{\mu\nu\}} H_{TT}^\perp + i(k'_n - k'_{\bar{n}})_{[\mu\nu]} E_{TT}^\perp] \bar{H}_1^\perp$$
$$- \frac{2M_2 c_2^q}{M_1 p_2^-} [2(k_n - k_{\bar{n}})_{\{\mu\nu\}} H_{1TT}^\perp \bar{H} + i(k_n - k_{\bar{n}})_{[\mu\nu]} S_{TT}^{kk} H_{1TT}^\perp \bar{E}] + \frac{\sqrt{2}}{Q} \left[\omega_{\mu\nu}(k', k) D_{1TT}^\perp \bar{D}_1 \right.$$
$$\left.\left. - \frac{4}{M_1 M_2} \omega_{\mu\nu}^{(n)}(k, k') H_{1TT}^\perp \bar{H}_1^\perp \right] \right) + \frac{S_{TT}^{k\tilde{k}}}{M_1^2} \left(\frac{1}{p_2^-} G_{1TT}^\perp [\tilde{\omega}_{\mu\nu}(k') \bar{D}^\perp + \omega_{\mu\nu}(\tilde{k}') \bar{G}^\perp] \right.$$
$$\left. + \frac{2M_1 c_2^q}{M_2 p_1^+} [2(\tilde{k}'_n - \tilde{k}'_{\bar{n}})_{\{\mu\nu\}} H_{TT}'^\perp + i(\tilde{k}'_n - \tilde{k}'_{\bar{n}})_{[\mu\nu]} E_{TT}'^\perp] \bar{H}_1^\perp - \frac{\sqrt{2}}{Q} \tilde{\omega}_{\mu\nu}(k', k) G_{1TT}^\perp \bar{D}_1 \right)$$
$$+ \frac{1}{p_1^+} [\omega_{\mu\nu}(S_{TT}^k) D_{TT}'^\perp + \tilde{\omega}_{\mu\nu}(\tilde{S}_{TT}^k) G_{TT}'^\perp] \bar{D}_1 - \frac{2M_2 c_2^q}{p_2^- M_1} [2(S_{TTn}^k - S_{TT\bar{n}}^k)_{\{\mu\nu\}} H_{1TT}'^\perp \bar{H} + i(S_{TTn}^k - S_{TT\bar{n}}^k)_{[\mu\nu]} H_{1TT}'^\perp \bar{E}]$$
$$\left. - \frac{4\sqrt{2} c_2^q M_1^2}{M_2 Q} (S_{TT}^{kk} k'_{\bar{n}} - S_{TT}^{kk'} k_{\bar{n}} + k_\perp \cdot k'_\perp S_{TT\bar{n}}^k + k_\perp'^2 S_{TTn}^k)_{\{\mu\nu\}} H_{1TT}'^\perp \bar{H}_1^\perp \right\}. \tag{B4}$$

Transforming them into the helicity-GJ frame, we obtain from Eqs. (B1)–(B4) the contributions at twist-3 and they take exactly the same form as given in these equations. However, we obtain also additional twist-3 contributions from the twist-2 parts given by Eqs. (4.4)–(4.13). The corresponding terms for the unpolarized part are given by Eq. (4.23). Other parts are given in the following:

$$\delta W_{\mu\nu}^{(1)L} = \frac{4\sqrt{2}}{z_1 z_2 Q} \int \frac{d^2k_\perp}{(2\pi)^2} \frac{d^2k'_\perp}{(2\pi)^2} \delta^2(k_\perp + k'_\perp - q_\perp) \lambda \left\{ (c_3^q q_{\bar{n}\{\mu\nu\}} - i c_1^q \tilde{q}_{\bar{n}[\mu\nu]}) G_{1L} \bar{D}_1 + \frac{4c_2^q}{M_1 M_2} k_\perp \cdot (q_\perp - \tilde{k}'_\perp) k'_{\bar{n}\{\mu\nu\}} H_{1L}^\perp \bar{H}_1^\perp \right\}, \tag{B5}$$

$$\delta W_{\mu\nu}^{(1)T} = \frac{4\sqrt{2}}{z_1 z_2 Q} \int \frac{d^2k_\perp}{(2\pi)^2} \frac{d^2k'_\perp}{(2\pi)^2} \delta^2(k_\perp + k'_\perp - q_\perp)$$
$$\times \left\{ \frac{k_\perp \cdot S_\perp}{M_1} \left[(c_3^q q_{\bar{n}\{\mu\nu\}} - i c_1^q \tilde{q}_{\bar{n}[\mu\nu]}) G_{1T}^\perp \bar{D}_1 + \frac{4c_2^q}{M_1 M_2} k_\perp \cdot (q_\perp - \tilde{k}'_\perp) k'_{\bar{n}\{\mu\nu\}} H_{1T}^\perp \bar{H}_1^\perp \right] \right.$$
$$\left. - \frac{\tilde{k}_\perp \cdot S_\perp}{M_1} (c_1^q q_{\bar{n}\{\mu\nu\}} - i c_3^q \tilde{q}_{\bar{n}[\mu\nu]}) D_{1T}^\perp \bar{D}_1 + \frac{4c_2^q}{M_2} (k_\perp \cdot S_\perp \tilde{k}'_{\bar{n}} + k_\perp \cdot \tilde{k}'_\perp S_{\bar{n}} - \tilde{k}'_\perp \cdot S_\perp k_{\bar{n}})_{\{\mu\nu\}} H_{1T} \bar{H}_1^\perp \right\}, \tag{B6}$$

$$\delta W_{\mu\nu}^{(1)LL} = \frac{4\sqrt{2}}{z_1 z_2 Q} \int \frac{d^2k_\perp}{(2\pi)^2} \frac{d^2k'_\perp}{(2\pi)^2} \delta^2(k_\perp + k'_\perp - q_\perp)$$
$$\times S_{LL} \left\{ -(c_1^q q_{\bar{n}\{\mu\nu\}} - i c_3^q \tilde{q}_{\bar{n}[\mu\nu]}) D_{1LL} \bar{D}_1 + \frac{4c_2^q}{M_1 M_2} (k_\perp^2 k'_{\bar{n}} + k_\perp'^2 k_{\bar{n}})_{\{\mu\nu\}} H_{1LL}^\perp \bar{H}_1^\perp \right\}, \tag{B7}$$

$$\delta W_{\mu\nu}^{(1)LT} = \frac{4\sqrt{2}}{z_1 z_2 Q} \int \frac{d^2k_\perp}{(2\pi)^2} \frac{d^2k'_\perp}{(2\pi)^2} \delta^2(k_\perp + k'_\perp - q_\perp)$$
$$\times \left\{ \frac{k_\perp \cdot S_{LT}}{M_1} \left[-(c_1^q q_{\bar{n}\{\mu\nu\}} - i c_3^q \tilde{q}_{\bar{n}[\mu\nu]}) D_{1LT}^\perp \bar{D}_1 + \frac{4c_2^q}{M_1 M_2} (k_\perp^2 k'_{\bar{n}} + k_\perp'^2 k_{\bar{n}})_{\{\mu\nu\}} H_{1LT}^\perp \bar{H}_1^\perp \right] \right.$$
$$\left. + \frac{\tilde{k}_\perp \cdot S_{LT}}{M_1} (c_3^q q_{\bar{n}\{\mu\nu\}} - i c_1^q \tilde{q}_{\bar{n}[\mu\nu]}) G_{1LT}^\perp \bar{D}_1 + \frac{4c_2^q}{M_2} (q_\perp \cdot k'_\perp S_{LT\bar{n}} + k_\perp \cdot S_{LT} k'_{\bar{n}} - k'_\perp \cdot S_{LT} k_{\bar{n}})_{\{\mu\nu\}} H_{1LT} \bar{H}_1^\perp \right\}, \tag{B8}$$

$$\delta W_{\mu\nu}^{(1)TT} = \frac{4\sqrt{2}}{z_1 z_2 Q} \int \frac{d^2k_\perp}{(2\pi)^2} \frac{d^2k'_\perp}{(2\pi)^2} \delta^2(k_\perp + k'_\perp - q_\perp) \left\{ \frac{S_{TT}^{kk}}{M_1^2} [-(c_1^q q_{\bar{n}\{\mu\nu\}} - i c_3^q \tilde{q}_{\bar{n}[\mu\nu]}) D_{1TT}^\perp \bar{D}_1 + (k_\perp^2 k'_{\bar{n}} + k_\perp'^2 k_{\bar{n}})_{\{\mu\nu\}} H_{1TT}^\perp \bar{H}_1^\perp] \right.$$
$$\left. + \frac{S_{TT}^{k\tilde{k}}}{M_1^2} (c_3^q q_{\bar{n}\{\mu\nu\}} - i c_1^q \tilde{q}_{\bar{n}[\mu\nu]}) G_{1LT}^\perp \bar{D}_1 + \frac{4c_2^q}{M_1 M_2} (q_\perp \cdot k'_\perp S_{TT\bar{n}}^k + S_{TT}^{kk} k'_{\bar{n}} - S_{TT}^{kk'} k_{\bar{n}})_{\{\mu\nu\}} H_{1TT} \bar{H}_1^\perp \right\}. \tag{B9}$$

APPENDIX C: TWIST-3 CONTRIBUTIONS TO THE STRUCTURE FUNCTIONS

In the partonic picture at the LO pQCD, 36 of the 81 structure functions for $e^+e^- \to V\pi X$ have twist-3 contributions. We list the results in this appendix in the following.

$$F_{1U2}^{\cos\varphi} = \frac{8c_3^e c_3^q}{z_1 z_2 Q}\mathcal{C}[M_1 w_1 D^\perp z_2 \bar{D}_1 + M_2 \bar{w}_1 z_1 D_1 \bar{D}^{\perp\prime}], \tag{C1}$$

$$F_{2U2}^{\cos\varphi} = \frac{4c_1^e}{z_1 z_2 Q}\{c_1^q \mathcal{C}[M_1 w_1 D^\perp z_2 \bar{D}_1 + M_2 \bar{w}_1 z_1 D_1 \bar{D}^{\perp\prime}] + 4c_2^q \mathcal{C}[M_1 \bar{w}_1 H z_2 \bar{H}_1^\perp - M_2 w_1 z_1 H_1^\perp \bar{H}^{\perp\prime}]\}, \tag{C2}$$

$$\tilde{F}_{1U2}^{\sin\varphi} = \frac{8c_3^e}{z_1 z_2 Q}\{c_1^q \mathcal{C}[(M_1 w_1 G^\perp z_2 \bar{D}_1 - M_2 \bar{w}_1 z_1 D_1 \bar{G}^\perp)] + 2c_2^q \mathcal{C}[(M_1 \bar{w}_1 E z_2 \bar{H}_1^\perp - M_2 w_1 z_1 H_1^\perp \bar{E})]\}, \tag{C3}$$

$$\tilde{F}_{2U2}^{\sin\varphi} = \frac{4c_1^e c_3^q}{z_1 z_2 Q}\mathcal{C}[M_1 w_1 G^\perp z_2 \bar{D}_1 - M_2 \bar{w}_1 z_1 D_1 \bar{G}^\perp], \tag{C4}$$

$$\tilde{F}_{1L2}^{\cos\varphi} = \frac{8c_3^e}{z_1 z_2 Q}\{c_1^q \mathcal{C}[M_1 w_1 G_L^\perp z_2 \bar{D}_1 - M_2 \bar{w}_1 z_1 G_{1L} \bar{D}^{\perp\prime}] + 2c_2^q \mathcal{C}[-M_1 \bar{w}_1 E_L z_2 \bar{H}_1^\perp + M_2 w_1 z_1 H_{1L}^\perp \bar{E}]\}, \tag{C5}$$

$$\tilde{F}_{2L2}^{\cos\varphi} = \frac{4c_1^e c_3^q}{z_1 z_2 Q}\mathcal{C}[M_1 w_1 G_L^\perp z_2 \bar{D}_1 - M_2 \bar{w}_1 z_1 G_{1L} \bar{D}^{\perp\prime}], \tag{C6}$$

$$F_{1L2}^{\sin\varphi} = \frac{8c_3^e c_3^q}{z_1 z_2 Q}\mathcal{C}[-M_1 w_1 D_L^\perp z_2 \bar{D}_1 + M_2 \bar{w}_1 z_1 G_{1L} \bar{G}^\perp], \tag{C7}$$

$$F_{2L2}^{\sin\varphi} = \frac{4c_1^e}{z_1 z_2 Q}\{c_1^q \mathcal{C}[-M_1 w_1 D_L^\perp z_2 \bar{D}_1 + M_2 \bar{w}_1 z_1 G_{1L} \bar{G}^\perp] + 4c_2^q \mathcal{C}[M_1 \bar{w}_1 H_L z_2 \bar{H}_1^\perp - M_2 w_1 z_1 H_{1L}^\perp \bar{H}^{\perp\prime}]\}, \tag{C8}$$

$$\tilde{F}_{1T2}^{\cos\varphi_S} = \frac{4c_3^e}{z_1 z_2 Q}\{c_1^q \mathcal{C}[2M_1 \mathcal{G}_T^\perp z_2 \bar{D}_1 + M_2 w_2 z_1 (G_{1T}^\perp \bar{D}^{\perp\prime} + D_{1T}^\perp \bar{G}^\perp)] + c_2^q \mathcal{C}[-2M_1 w_2 E_T^{\perp-} z_2 \bar{H}_1^\perp + M_2 z_1 \mathcal{H}_{1T}^\perp \bar{E}]\}, \tag{C9}$$

$$\tilde{F}_{2T2}^{\cos\varphi_S} = \frac{4c_1^e c_3^q}{z_1 z_2 Q}\mathcal{C}\left[M_1 \mathcal{G}_T^\perp z_2 \bar{D}_1 + M_2 \frac{w_2}{2} z_1 (G_{1T}^\perp \bar{D}^{\perp\prime} + D_{1T}^\perp \bar{G}^\perp)\right], \tag{C10}$$

$$F_{1T2}^{\sin\varphi_S} = \frac{8c_3^e c_3^q}{z_1 z_2 Q}\mathcal{C}\left[-M_1 \mathcal{D}_T^\perp z_2 \bar{D}_1 + M_2 \frac{w_2}{2} z_1 (D_{1T}^\perp \bar{D}^{\perp\prime} - G_{1T}^\perp \bar{G}^\perp)\right], \tag{C11}$$

$$F_{2T2}^{\sin\varphi_S} = \frac{4c_1^e}{z_1 z_2 Q}\left\{c_1^q \mathcal{C}\left[-M_1 \mathcal{D}_T^\perp z_2 \bar{D}_1 + M_2 \frac{w_2}{2} z_1 (D_{1T}^\perp \bar{D}^{\perp\prime} - G_{1T}^\perp \bar{G}^\perp)\right] + 4c_2^q \mathcal{C}\left[M_1 \frac{w_2}{2} H_T^{\perp-} z_2 \bar{H}_1^\perp - M_2 z_1 \mathcal{H}_{1T}^\perp \bar{H}_1^{\perp\prime}\right]\right\}, \tag{C12}$$

$$\tilde{F}_{1T2}^{\cos(\varphi_S-2\varphi)} = \frac{8c_3^e}{z_1 z_2 Q}\{c_1^q \mathcal{C}[M_1 w_3 G_T^\perp z_2 \bar{D}_1 - M_2 w_4 z_1 (G_{1T}^\perp \bar{D}^{\perp\prime} - D_{1T}^\perp \bar{G}^\perp)] + 2c_2^q \mathcal{C}[-M_1 w_4 E_T^{\perp+} z_2 \bar{H}_1^\perp + M_2 w_3 z_1 H_{1T}^\perp \bar{E}]\}, \tag{C13}$$

$$\tilde{F}_{2T2}^{\cos(\varphi_S-2\varphi)} = \frac{4c_1^e c_3^q}{z_1 z_2 Q}\mathcal{C}[M_1 w_3 G_T^\perp z_2 \bar{D}_1 - M_2 w_4 z_1 (G_{1T}^\perp \bar{D}^{\perp\prime} - D_{1T}^\perp \bar{G}^\perp)], \tag{C14}$$

$$F_{1T2}^{\sin(\varphi_S-2\varphi)} = \frac{8c_3^e c_3^q}{z_1 z_2 Q}\mathcal{C}[M_1 w_3 D_T^\perp z_2 \bar{D}_1 - M_2 w_4 z_1 (D_{1T}^\perp \bar{D}^{\perp\prime} + G_{1T}^\perp \bar{G}^\perp)], \tag{C15}$$

$$F_{2T2}^{\sin(\varphi_S-2\varphi)} = \frac{4c_1^e}{z_1z_2Q}\{c_1^q\mathcal{C}[M_1w_3D_T^\perp z_2\bar{D}_1 - M_2w_4z_1(D_{1T}^\perp\bar{D}^{\perp\prime} + G_{1T}^\perp\bar{G}^\perp)] + 4c_2^q\mathcal{C}[-M_1w_4H_T^{\perp+}z_2\bar{H}_1^\perp + M_2w_3z_1H_{1T}^\perp\bar{H}^{\perp\prime}]\}, \tag{C16}$$

$$F_{1LL2}^{\cos\varphi} = \frac{8c_3^ec_3^q}{z_1z_2Q}\mathcal{C}[M_1w_1D_{LL}^\perp z_2\bar{D}_1 + M_2\bar{w}_1z_1D_{1LL}\bar{D}^{\perp\prime}], \tag{C17}$$

$$F_{2LL2}^{\cos\varphi} = \frac{4c_1^e}{z_1z_2Q}\{c_1^q\mathcal{C}[M_1w_1D_{LL}^\perp z_2\bar{D}_1 + M_2\bar{w}_1z_1D_{1LL}\bar{D}^{\perp\prime}] + 4c_2^q\mathcal{C}[M_1\bar{w}_1H_{LL}z_2\bar{H}_1^\perp - M_2w_1z_1H_{1LL}^\perp\bar{H}^{\perp\prime}]\}, \tag{C18}$$

$$\tilde{F}_{2LL2}^{\sin\varphi} = \frac{4c_1^ec_3^q}{z_1z_2Q}\mathcal{C}[M_1w_1G_{LL}^\perp z_2\bar{D}_1 - M_2\bar{w}_1z_1D_{1LL}\bar{G}^\perp], \tag{C19}$$

$$\tilde{F}_{1LL2}^{\sin\varphi} = \frac{8c_3^e}{z_1z_2Q}\{c_1^q\mathcal{C}[M_1w_1G_{LL}^\perp z_2\bar{D}_1 - M_2\bar{w}_1z_1D_{1LL}\bar{G}^\perp] + 2c_2^q\mathcal{C}[M_1\bar{w}_1E_{LL}z_2\bar{H}_1^\perp - M_2w_1z_1H_{1LL}^\perp\bar{E}]\}, \tag{C20}$$

$$F_{1LT2}^{\cos\varphi_{LT}} = \frac{8c_3^ec_3^q}{z_1z_2Q}\mathcal{C}\left[M_1\mathcal{D}_{LT}^\perp z_2\bar{D}_1 - M_2\frac{w_2}{2}z_1(D_{1LT}^\perp\bar{D}^{\perp\prime} + G_{1LT}^\perp\bar{G}^\perp)\right], \tag{C21}$$

$$F_{2LT2}^{\cos\varphi_{LT}} = \frac{4c_1^e}{z_1z_2Q}\left\{c_1^q\mathcal{C}\left[M_1\mathcal{D}_{LT}^\perp z_2\bar{D}_1 - M_2\frac{w_2}{2}z_1(D_{1LT}^\perp\bar{D}^{\perp\prime} + G_{1LT}^\perp\bar{G}^\perp)\right] - 4c_2^q\mathcal{C}\left[M_1\frac{w_2}{2}H_{LT}^{\perp+}z_2\bar{H}_1^\perp + M_2z_1\mathcal{H}_{1LT}^\perp\bar{H}_1^{\perp\prime}\right]\right\}, \tag{C22}$$

$$\tilde{F}_{1LT2}^{\sin\varphi_{LT}} = \frac{8c_3^e}{z_1z_2Q}\left\{c_1^q\mathcal{C}\left[M_1\mathcal{G}_{LT}^\perp z_2\bar{D}_1 - M_2\frac{w_2}{2}z_1(G_{1LT}^\perp\bar{D}^{\perp\prime} - D_{1LT}^\perp\bar{G}^\perp)\right] + c_2^q\mathcal{C}[-M_1w_2E_{LT}^{\perp+}z_2\bar{H}_1^\perp - 2M_2z_1\mathcal{H}_{1LT}^\perp\bar{E}]\right\}, \tag{C23}$$

$$\tilde{F}_{2LT2}^{\sin\varphi_{LT}} = \frac{4c_1^ec_3^q}{z_1z_2Q}\mathcal{C}\left[M_1\mathcal{G}_{LT}^\perp z_2\bar{D}_1 - M_2\frac{w_2}{2}z_1(G_{1LT}^\perp\bar{D}^{\perp\prime} - D_{1LT}^\perp\bar{G}^\perp)\right], \tag{C24}$$

$$F_{1LT2}^{\cos(\varphi_{LT}-2\varphi)} = \frac{8c_3^ec_3^q}{z_1z_2Q}\mathcal{C}[M_1w_3D_{LT}^\perp z_2\bar{D}_1 + M_2w_4z_1(D_{1LT}^\perp\bar{D}^{\perp\prime} - G_{1LT}^\perp\bar{G}^\perp)], \tag{C25}$$

$$F_{2LT2}^{\cos(\varphi_{LT}-2\varphi)} = \frac{4c_1^e}{z_1z_2Q}\{c_1^q\mathcal{C}[M_1w_3D_{LT}^\perp z_2\bar{D}_1 + M_2w_4z_1(D_{1LT}^\perp\bar{D}^{\perp\prime} - G_{1LT}^\perp\bar{G}^\perp)] + 4c_2^q\mathcal{C}[M_1w_4H_{LT}^{\perp-}z_2\bar{H}_1^\perp - M_2w_3z_1H_{1LT}^\perp\bar{H}^{\perp\prime}]\}. \tag{C26}$$

$$\tilde{F}_{1LT2}^{\sin(\varphi_{LT}-2\varphi)} = \frac{8c_3^e}{z_1z_2Q}\{c_1^q\mathcal{C}[-M_1w_3G_{LT}^\perp z_2\bar{D}_1 + M_2w_4z_1(G_{1LT}^\perp\bar{D}^{\perp\prime} + D_{1LT}^\perp\bar{G}^\perp)] - 2c_2^q\mathcal{C}[M_1w_4E_{LT}^{\perp-}z_2\bar{H}_1^\perp - M_2w_3z_1H_{1LT}^\perp\bar{E}]\}, \tag{C27}$$

$$\tilde{F}_{2LT2}^{\sin(\varphi_{LT}-2\varphi)} = \frac{4c_1^ec_3^q}{z_1z_2Q}\mathcal{C}[-M_1w_3G_{LT}^\perp z_2\bar{D}_1 + M_2w_4z_1(G_{1LT}^\perp\bar{D}^{\perp\prime} + D_{1LT}^\perp\bar{G}^\perp)], \tag{C28}$$

$$F_{1TT2}^{\cos(2\varphi_{TT}-\varphi)} = \frac{8c_3^ec_3^q}{z_1z_2Q}\mathcal{C}[-M_1w_1\mathcal{D}_{TT}^\perp z_2\bar{D}_1 - M_2z_1(w_3\bar{w}_1D_{1TT}^\perp\bar{D}^{\perp\prime} - w_5G_{1TT}^\perp\bar{G}^\perp)], \tag{C29}$$

$$\begin{aligned} F_{2TT2}^{\cos(2\varphi_{TT}-\varphi)} = \frac{4c_1^e}{z_1z_2Q}&\{c_1^q\mathcal{C}[-M_1w_1\mathcal{D}_{TT}^\perp z_2\bar{D}_1 - M_2z_1(w_3\bar{w}_1D_{1TT}^\perp\bar{D}^{\perp\prime} - w_5G_{1TT}^\perp\bar{G}^\perp)] \\ &+ 4c_2^q\mathcal{C}[M_1(w_6H_{TT}^\perp z_2\bar{H}_1^\perp - w_8H_{TT}^{\prime\perp}\bar{H}_1^\perp) + M_2w_1z_1\mathcal{H}_{1TT}^\perp\bar{H}^{\perp\prime}]\}, \end{aligned} \tag{C30}$$

$$\tilde{F}_{1TT2}^{\sin(2\varphi_{TT}-\varphi)} = \frac{8c_3^e}{z_1 z_2 Q}\{c_1^q \mathcal{C}[-M_1 w_1 \mathcal{G}_{TT}^{\perp} z_2 \bar{D}_1 + M_2 z_1 (w_6 G_{1TT}^{\perp} \bar{D}^{\perp\prime} + w_3 \bar{w}_1 D_{1TT}^{\perp} \bar{G}^{\perp})]$$
$$+ 2c_2^q \mathcal{C}[M_1 z_2 (w_6 E_{TT}^{\perp} \bar{H}_1^{\perp} - w_8 E_{TT}^{\prime\perp} \bar{H}_1^{\perp}) + M_2 w_1 z_1 \mathcal{H}_{1TT}^{\perp} \bar{E}]\}, \tag{C31}$$

$$\tilde{F}_{2TT2}^{\sin(2\varphi_{TT}-\varphi)} = \frac{4c_1^e c_3^q}{z_1 z_2 Q}\mathcal{C}[-M_1 w_1 \mathcal{G}_{TT}^{\perp} z_2 \bar{D}_1 + M_2 z_1 (w_6 G_{1TT}^{\perp} \bar{D}^{\perp\prime} + w_3 \bar{w}_1 D_{1TT}^{\perp} \bar{G}^{\perp})], \tag{C32}$$

$$F_{1TT2}^{\cos(2\varphi_{TT}-3\varphi)} = \frac{8c_3^e c_3^q}{z_1 z_2 Q}\mathcal{C}\left[M_1 w_9 D_{TT}^{\perp} z_2 \bar{D}_1 + M_2 \frac{w_7}{2} z_1 (D_{1TT}^{\perp} \bar{D}^{\perp\prime} - G_{1TT}^{\perp} \bar{G}^{\perp})\right], \tag{C33}$$

$$F_{2TT2}^{\cos(2\varphi_{TT}-3\varphi)} = \frac{4c_1^e}{z_1 z_2 Q}\left\{c_1^q \mathcal{C}\left[M_1 w_9 D_{TT}^{\perp} z_2 \bar{D}_1 + M_2 \frac{w_7}{2} z_1 (D_{1TT}^{\perp} \bar{D}^{\perp\prime} - G_{1TT}^{\perp} \bar{G}^{\perp})\right]\right.$$
$$\left. + 4c_2^q \mathcal{C}\left[M_1 \frac{w_7}{2} H_{TT}^{\perp-} z_2 \bar{H}_1^{\perp} - M_2 w_9 z_1 H_{1TT}^{\perp} \bar{H}^{\perp\prime}\right]\right\}, \tag{C34}$$

$$\tilde{F}_{1TT2}^{\sin(2\varphi_{TT}-3\varphi)} = \frac{8c_3^e}{z_1 z_2 Q}\left\{c_1^q \mathcal{C}\left[-M_1 w_9 G_{TT}^{\perp} z_2 \bar{D}_1 + M_2 \frac{w_7}{2} z_1 (G_{1TT}^{\perp} \bar{D}^{\perp\prime} + D_{1TT}^{\perp} \bar{G}^{\perp})\right]\right.$$
$$\left. - c_2^q \mathcal{C}[M_1 w_7 E_{TT}^{\perp-} z_2 \bar{H}_1^{\perp} - 2M_2 w_9 z_1 H_{1TT}^{\perp} \bar{E}]\right\}, \tag{C35}$$

$$\tilde{F}_{2TT2}^{\sin(2\varphi_{TT}-3\varphi)} = \frac{4c_1^e c_3^q}{z_1 z_2 Q}\mathcal{C}\left[-M_1 w_9 G_{TT}^{\perp} z_2 \bar{D}_1 + M_2 \frac{w_7}{2} z_1 D_{1TT}^{\perp} \bar{G}^{\perp}\right]. \tag{C36}$$

Here, just as for the S_T- and S_{LT}-dependent FFs given by Eq. (A20), for S_{TT}-dependent K, we define

$$\mathcal{K}_{TT}^{\perp}(z, k_\perp) = K_{TT}^{\prime\perp}(z, k_\perp) + \frac{k_\perp^2}{2M_1^2} K_{TT}^{\perp}(z, k_\perp), \tag{C37}$$

for $K = D$, G, or H. Also, $K_\sigma^{\perp\pm} = K_\sigma^{\perp} \pm K_\sigma^{\prime\perp}$, for all different K's and polarization σ's, and for the leading twist involved combinations,

$$\bar{D}^{\perp\prime} = z_2 \bar{D}_1 - \bar{D}^{\perp}, \quad \bar{H}^{\perp\prime} = \bar{H} - \bar{w}_0 z_2 \bar{H}_1^{\perp}. \tag{C38}$$

Besides the w's given by Eqs. (5.2)–(5.6) and in the text in Sec. V, we have also introduced the scalar weights defined as

$$w_3 = \frac{1}{2} w_0 - w_1^2, \quad w_4 = \frac{1}{2} w_2 - w_1 \bar{w}_1, \quad w_5 = w_1 w_2 - w_0 w_2 + \frac{1}{2} w_0 w_1, \tag{C39}$$

$$w_6 = w_1 w_2 - \frac{1}{2} w_0 \bar{w}_1, \quad w_7 = 4w_1^2 \bar{w}_1 - 2w_1 w_2 - w_0 \bar{w}_1, \quad w_8 = 4w_1^2 \bar{w}_1 - 2w_1 w_2 - \frac{1}{2} w_0 \bar{w}_1, \quad w_9 = \left(2w_1^2 - \frac{3}{2} w_0\right) w_1. \tag{C40}$$

They are all scalar functions of $k_\perp$, $k'_\perp$, and p_{2T}.

[1] K. Goeke, A. Metz, and M. Schlegel, Phys. Lett. B **618**, 90 (2005).
[2] K. A. Olive *et al.* (Particle Data Group Collaboration), Chin. Phys. C **38**, 090001 (2014).
[3] For a recent review, see, for example, S. Albino, Rev. Mod. Phys. **82**, 2489 (2010).
[4] J. C. Collins, Nucl. Phys. **B396**, 161 (1993).
[5] X. D. Ji, Phys. Rev. D **49**, 114 (1994).
[6] P. J. Mulders and R. D. Tangerman, Nucl. Phys. **B461**, 197 (1996); **B484**, 538(E) (1997).
[7] D. Boer, R. Jakob, and P. J. Mulders, Nucl. Phys. **B504**, 345 (1997).
[8] D. Boer and P. J. Mulders, Phys. Rev. D **57**, 5780 (1998).
[9] D. Boer, R. Jakob, and P. J. Mulders, Phys. Lett. B **424**, 143 (1998).
[10] D. Boer, R. Jakob, and P. J. Mulders, Nucl. Phys. **B564**, 471 (2000).
[11] K. Goeke, A. Metz, P. V. Pobylitsa, and M. V. Polyakov, Phys. Lett. B **567**, 27 (2003).
[12] A. Bacchetta, M. Diehl, K. Goeke, A. Metz, P. J. Mulders, and M. Schlegel, J. High Energy Phys. 02 (2007) 093.
[13] D. Boer, Nucl. Phys. **B806**, 23 (2009).
[14] D. Pitonyak, M. Schlegel, and A. Metz, Phys. Rev. D **89**, 054032 (2014).
[15] S.-y. Wei, Y.-k. Song, and Z.-t. Liang, Phys. Rev. D **89**, 014024 (2014).
[16] K. Kanazawa and Y. Koike, Phys. Rev. D **88**, 074022 (2013).
[17] S. Y. Wei, K. b. Chen, Y. k. Song, and Z. t. Liang, Phys. Rev. D **91**, 034015 (2015).
[18] K. b. Chen, S. y. Wei, W. h. Yang, and Z. t. Liang, arXiv:1505.02856.
[19] Z. t. Liang, Int. J. Mod. Phys. Conf. Ser. **40**, 1660008 (2016).
[20] K. b. Chen, S. y. Wei, and Z. t. Liang, Front. Phys. **10**, 101204 (2015).
[21] A. Bacchetta and P. J. Mulders, Phys. Rev. D **62**, 114004 (2000).
[22] D. W. Sivers, Phys. Rev. D **41**, 83 (1990); **43**, 261 (1991).
[23] See, e. g., A. Lesnik, D. M. Schwartz, I. Ambats, E. Hayes, W. T. Meyer, C. E. W. Ward, T. M. Knasel, E. G. Swallow, R. Winston, and T. A. Romanowski, Phys. Rev. Lett. **35**, 770 (1975); G. Bunce *et al.*, Phys. Rev. Lett. **36**, 1113 (1976); J. Bensinger *et al.*, Phys. Rev. Lett. **50**, 313 (1983); S. A. Gourlay, H. L. Melanson, M. A. Abolins, K. W. Edwards, W. R. Francis, H. G. E. Kobrak, D. P. Owen, R. A. Swanson, and P. M. Yager, Phys. Rev. Lett. **56**, 2244 (1986); and a recent review in A. D. Krisch, Eur. Phys. J. A **31**, 417 (2007), and the reference given there.
[24] One can similarly define whether a basic Lorentz tensor $h^{\mu\nu}$ is time-reversal T even, $h_{\mu\nu}(q^{\mathcal{T}}, p_1^{\mathcal{T}}, S^{\mathcal{T}}, p_2^{\mathcal{T}}) = h^{\mu\nu}(q, p_1, S, p_2)$, or T odd, $h_{\mu\nu}(q^{\mathcal{T}}, p_1^{\mathcal{T}}, S^{\mathcal{T}}, p_2^{\mathcal{T}}) = -h^{\mu\nu}(q, p_1, S, p_2)$, where $a^{\mathcal{T}}$ denotes the a under time reversal. We note that for $a = p$, S_{LL}, S_{LT}, or S_{TT}, $a^{\mathcal{T}} = a^{\mathcal{P}}$, while for the polarization vector S, $S^{\mathcal{T}} = -S^{\mathcal{P}}$. Hence for the symmetric tensors, the unpolarized, S_{LL}-, S_{LT}-, and S_{TT}-dependent parts have the same P and T behavior, while the S-dependent part has different P and T behaviors. For the antisymmetric parts, the unpolarized, S_{LL}-, S_{LT}-, and S_{TT}-dependent parts have different P and T behaviors, while the S-dependent part has the same P and T behaviors.
[25] K. Gottfried and J. D. Jackson, Nuovo Cimento **33**, 309 (1964).
[26] Our naming system for the structure functions is slightly different from that in [14]. We use it this way for two reasons: (1) it is more convenient to calculate hadron polarizations with respect to the lepton-hadron as well as hadron-hadron plane; (2) it is convenient to extend it to include the tensor polarization.
[27] W. Lu, Phys. Rev. D **51**, 5305 (1995).
[28] R. K. Ellis, W. Furmanski, and R. Petronzio, Nucl. Phys. **B207**, 1 (1982); **B212**, 29 (1983).
[29] J.-w. Qiu and G. F. Sterman, Nucl. Phys. **B353**, 105 (1991); **B353**, 137 (1991).
[30] Z. t. Liang and X. N. Wang, Phys. Rev. D **75**, 094002 (2007).
[31] The angle θ here is the angle between the incident electron and the produced quark for $e^+e^- \to q\bar{q}$. Up to $1/Q$ it is the same as that defined in the helicity-GJ frame for $e^+e^- \to V\pi X$.
[32] R. N. Cahn, Phys. Lett. **78B**, 269 (1978).
[33] M. Althoff *et al.* (TASSO Collaboration), Z. Phys. C **27**, 27 (1985).
[34] M. I. Adamovich *et al.* (WA89 Collaboration), Z. Phys. A **350**, 379 (1995).
[35] D. Buskulic *et al.* (ALEPH Collaboration), Phys. Lett. B **374**, 319 (1996).
[36] K. Ackerstaff *et al.* (OPAL Collaboration), Eur. Phys. J. C **2**, 49 (1998).
[37] K. Ackerstaff *et al.* (OPAL Collaboration), Phys. Lett. B **412**, 210 (1997).
[38] K. Ackerstaff *et al.* (OPAL Collaboration), Z. Phys. C **74**, 437 (1997).
[39] P. Abreu *et al.* (DELPHI Collaboration), Phys. Lett. B **406**, 271 (1997).
[40] G. Gustafson and J. Hakkinen, Phys. Lett. B **303**, 350 (1993).
[41] C. Boros and Z.-t. Liang, Phys. Rev. D **57**, 4491 (1998).
[42] C.-x. Liu and Z.-t. Liang, Phys. Rev. D **62**, 094001 (2000).
[43] C.-x. Liu, Q.-h. Xu, and Z.-t. Liang, Phys. Rev. D **64**, 073004 (2001).
[44] Z.-t. Liang and C.-x. Liu, Phys. Rev. D **66**, 057302 (2002).
[45] Q.-h. Xu, C.-x. Liu, and Z.-t. Liang, Phys. Rev. D **65**, 114008 (2002).
[46] B. Q. Ma and J. Soffer, Phys. Rev. Lett. **82**, 2250 (1999).
[47] B. Q. Ma, I. Schmidt, and J. J. Yang, Phys. Lett. B **477**, 107 (2000).
[48] B. Q. Ma, I. Schmidt, and J. J. Yang, Phys. Rev. D **61**, 034017 (2000).
[49] B. Q. Ma, I. Schmidt, and J. J. Yang, Nucl. Phys. **B574**, 331 (2000).
[50] B. Q. Ma, I. Schmidt, J. Soffer, and J. J. Yang, Eur. Phys. J. C **16**, 657 (2000).
[51] B. Q. Ma, I. Schmidt, J. Soffer, and J. J. Yang, Phys. Rev. D **62**, 114009 (2000).
[52] Y. Chi and B. Q. Ma, Phys. Lett. B **726**, 737 (2013).
[53] J. R. Ellis, A. Kotzinian, and D. V. Naumov, Eur. Phys. J. C **25**, 603 (2002).
[54] M. Anselmino, M. Bertini, F. Murgia, and P. Quintairos, Eur. Phys. J. C **2**, 539 (1998).

[55] M. Anselmino, M. Bertini, F. Murgia, and B. Pire, Phys. Lett. B **438**, 347 (1998).
[56] M. Anselmino, M. Bertini, F. Caruso, F. Murgia, and P. Quintairos, Eur. Phys. J. C **11**, 529 (1999).
[57] Q. h. Xu, C. x. Liu, and Z. t. Liang, Phys. Rev. D **63**, 111301 (2001).
[58] Q. h. Xu and Z. t. Liang, Phys. Rev. D **67**, 114013 (2003).
[59] K. Abe *et al.* (Belle Collaboration), Phys. Rev. Lett. **96**, 232002 (2006).
[60] R. Seidl *et al.* (Belle Collaboration), Phys. Rev. D **78**, 032011 (2008); **86**, 039905(E) (2012).
[61] A. Vossen *et al.* (Belle Collaboration), Phys. Rev. Lett. **107**, 072004 (2011).
[62] J. P. Lees *et al.* (*BABAR* Collaboration), Phys. Rev. D **90**, 052003 (2014).
[63] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. Lett. **116**, 042001 (2016).
[64] M. Anselmino, M. Boglione, U. D'Alesio, A. Kotzinian, F. Murgia, A. Prokudin, and C. Turk, Phys. Rev. D **75**, 054032 (2007).
[65] M. Anselmino, M. Boglione, U. D'Alesio, S. Melis, F. Murgia, and A. Prokudin, Phys. Rev. D **87**, 094019 (2013).
[66] Chang-zheng Yuan, Xin-ping Xu, and Qing-hua Xu (private communications).
[67] See, e.g., J. L. Rosner *et al.*, arXiv:1401.6075; A. Accardi *et al.*, arXiv:1212.1701; M. Bicer *et al.* (TLEP Design Study Working Group Collaboration), J. High Energy Phys. 01 (2014) 164; J. Tang *et al.*, arXiv:1507.03224; Zhang ZhaoXi, Sci. China Phys. Mech. Astron. **42**, 716 (2012), and the references given there.