

Novel Randall-Sundrum model with S_3 flavor symmetryA. E. Cárcamo Hernández,^{1,*} I. de Medeiros Varzielas,^{2,†} and Nicolás A. Neill^{3,‡}¹*Universidad Técnica Federico Santa María and Centro Científico-Tecnológico de Valparaíso, Casilla 110-V, Valparaíso, Chile*²*School of Physics and Astronomy, University of Southampton, Southampton SO17 1BJ, United Kingdom*³*Physics Department, University of California–Davis, Davis, California 95616, USA*

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We propose a simple and predictive model of fermion masses and mixing in a warped extra dimension, with the smallest discrete non-Abelian group S_3 and the discrete symmetries $Z_2 \otimes Z_4$. Standard Model fields propagate in the bulk, and the mass hierarchies and mixing angles are accounted for the fermion zero modes localization profiles, similar to the Randall-Sundrum model. To the best of our knowledge, this model is the first implementation of an S_3 flavor symmetry in this type of warped extra dimension framework. Our model successfully describes the fermion masses and mixing pattern and is consistent with the current low energy fermion flavor data. The discrete flavor symmetry in our model leads to predictive mixing inspired textures, where the Cabibbo mixing arises from the down-type quark sector, whereas the up-type quark sector contributes to the remaining mixing angles.

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I. INTRODUCTION

The recent LHC discovery of a 126 GeV Higgs boson [1,2] confirms the great success of the Standard Model (SM) in describing electroweak phenomena, but the SM nevertheless remains with many unanswered issues [3]. One of them is the hierarchy problem that arises from the quadratic divergence of the Higgs mass, suggesting the presence of some underlying physics in the gauge symmetry breaking mechanism that is so far unknown. Another issue is that the SM does not specify the Yukawa structures, has no justification for the number of generations, and lacks an explanation for the large hierarchy of the fermion masses, spanning 5 orders of magnitude in the quark sector and a much wider range when neutrinos are included. The origin of fermion mixing and the size of CP violation in the quark and lepton sector is also a related issue. While the mixing angles in the quark sector are very small, in the lepton sector two of the mixing angles are large, and one mixing angle is small. Neutrino experiments have brought clear evidence of neutrino oscillations from the measured neutrino mass squared splittings. The three neutrino flavors mix, and at least two of the neutrinos have nonvanishing masses, which according to observations must be smaller than the SM charged fermion masses by many orders of magnitude.

This flavor puzzle, not addressed in the framework of the SM, gives motivation for extensions of the SM that explain the observed fermion mass spectrum and flavor mixings. With neutrino experiments increasingly constraining the

mixing angles in the leptonic sector, many models focus only on this sector, aiming to explain the near tri-bi-maximal structure of the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix through some non-Abelian symmetry. Furthermore, the fermion mass hierarchy can be described by assuming textures for the Yukawa matrices, as shown in Refs. [4–29]. Discrete flavor groups implemented in several models provide a very promising framework for describing the observed fermion mass and mixing hierarchy (recent reviews on discrete flavor groups can be found in Refs. [30–33]). The groups employed are quite diverse, mostly discrete subgroups of $SU(3)$ with triplet representations. Of note are the groups T_7 [34–42] and $\Delta(27)$ [43–54] as the smallest with distinct triplet and antitriplet representations. A_4 , the smallest group with a triplet representation [55–60], was one of the first groups explored and remains very popular after the measurement of θ_{13} [61–76], as does the group S_4 , which also has triplet representations [77–83].

Although it does not have a triplet irreducible representation, S_3 has been considerably studied in the literature. It is appealing as it is the smallest non-Abelian group and can lead to interesting mass structures for quarks, leptons, or both. It was used as a flavor symmetry for the first time by [84] and continues to be explored since θ_{13} was measured [85–98].

A scenario that remains relatively unexplored is that of combining discrete flavor symmetries with extra dimensions [57,80,99–104]. In this work we consider using S_3 in a modified Randall-Sundrum 1 (RS1) [105] model, where the Higgs doublet is in the bulk. In this implementation, the warped extra dimension controls the hierarchy problem between the electroweak (EW) scale and the Planck scale, and the hierarchy in fermion masses, while the S_3 flavor

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symmetry is responsible for making the flavor sector more predictive. This paper is organized as follows. In Sec. II we outline the proposed model. In Sec. III we discuss the implications of our model in quark masses and mixings. In Sec. IV we present our results on lepton masses and mixings. We conclude in Sec. V. In Appendix we present a brief description of the S_3 discrete group.

II. THE MODEL

We consider the RS1 model based on a warped extra dimension compactified on an S^1/Z_2 orbifold, which corresponds to the interval $[0, \pi R]$, and with a metric of anti-de Sitter (AdS) type given by [105]

$$ds^2 = e^{-2ky} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2. \quad (1)$$

Here $\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$ is the four-dimensional Minkowski metric and k the AdS curvature. Since the Planck mass $M_P \sim 10^{19}$ GeV is the fundamental scale, a natural theory should have $k \sim M_P$. The TeV scale can be generated at the brane located at $y = \pi R$ if the compactification radius R is such that $ke^{-\pi kR} = 1$ TeV, which in turn means $kR \simeq 12$, also a rather natural number. The electroweak gauge symmetry in the bulk has to be extended to $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ in order to avoid a too large violation of the custodial symmetry caused by $U(1)_Y$ Kaluza-Klein (KK) modes [99,106–115]. Since the custodial symmetry is preserved in our model, the T parameter is consistent with the experimental data. Because of the preserved custodial symmetry in the bulk, the lower bound on the KK masses will mainly arise from the S parameter constraint and is close to about 3 TeV [99,107]. An alternative approach to suppress contributions to the T parameter is to consider a modified class of metrics that depart from AdS in the IR region [116].

Our model is an extension of the SM with seven EW scalar singlets and two heavy Majorana neutrinos, embedded in a warped extra dimension, with the inclusion of the S_3 discrete flavor symmetry. The inclusion of the discrete S_3 symmetry serves to reduce the number of parameters in the Yukawa sector of our model making it more predictive. We assume that 3 EW scalar singlets and two heavy Majorana neutrinos are located at the Planck brane, whereas the remaining EW scalar singlets are set at the TeV brane. The Higgs doublet as well as the three generations of fermions propagate in the five-dimensional (5D) bulk.

Boundary conditions at the $y = 0$ and $y = \pi R$ branes determine a tower of KK modes, of which the zero modes are assumed to be the SM fields in four dimensions (4D). Since there is no chirality in five dimensions, the left- and right-handed fermions in 4D correspond to zero modes of

different fields in the bulk, conventionally called Ψ_R and Ψ_L , which obey the conditions $\Psi_{R,L} = \pm \gamma_5 \Psi_{R,L}$. The masses of the zero modes in 4D appear when the electroweak symmetry is spontaneously broken, as in the SM. Fermion fields in the bulk of 5D space, generically denoted as $\Psi(x, y)$, obey an action of the form

$$S_5^{(\Psi)} = \int d^4x \int_0^{\pi R} dy \sqrt{-g} \left(i \bar{\Psi} \Gamma^M \nabla_M \Psi + im_\Psi \bar{\Psi} \Psi + \frac{\lambda_{ij}}{M_5^2} \bar{\Psi}_L H \Psi_R \right), \quad (2)$$

where capital indices run over the five coordinates, $\Gamma^M = (e^{ky} \gamma^\mu, \gamma^5)$ are the Gamma matrices in the 5D curved spacetime, and ∇_M is the covariant derivative that includes the interaction with the gauge fields. The fermion masses in 5D have natural values of the order of the Planck mass,

$$m_\Psi = kd_\Psi, \quad (3)$$

where k is the AdS curvature and d_Ψ are parameters of order unity. These parameters will determine the localization of the fermion profiles along the fifth dimension. The masses of fermions in the TeV brane (the observable 4D space) will be a consequence of this localization after electroweak symmetry breaking. M_5 is the 5D Planck mass, which is related to M_P , the Planck mass in 4D, through the following relation:

$$M_P^2 = \frac{M_5^3}{k} (1 - e^{-2\pi kR}). \quad (4)$$

Since the exponential term is highly suppressed, M_5 , M_P , and k are all of the same order of magnitude.

Fermion, gauge boson fields, and scalars can be expanded in their respective KK modes as follows:

$$\Psi_{L,R}(x, y) = \sum_{n=0}^{\infty} \Psi_{L,R}^{(n)}(x) f_{L,R}^{(n)}(y), \quad (5)$$

$$A_\mu^a(x, y) = \sum_{n=0}^{\infty} A_\mu^{a(n)}(x) \chi^{(n)}(y), \quad (6)$$

$$H(x, y) = \sum_{n=0}^{\infty} H_\mu^{(n)}(x) h^{(n)}(y), \quad (7)$$

where their profiles in the fifth dimension are, respectively, given by [111]

$$f_{L,R}^{(n)}(y) = \begin{cases} \sqrt{\frac{k(1-2d_{L,R})}{e^{(1-2d_{L,R})k\pi R}-1}} e^{(2-d_{L,R})ky}, & n = 0, \\ N_{\Psi_{L,R}}^{(n)} e^{\frac{5}{2}ky} \left[J_{d_{L,R}\pm\frac{1}{2}}\left(\frac{m_{\Psi}^{(n)}}{k} e^{ky}\right) + \alpha_{\Psi_{L,R}}^{(n)} Y_{d_{L,R}\pm\frac{1}{2}}\left(\frac{m_{\Psi}^{(n)}}{k} e^{ky}\right) \right], & n > 0, \end{cases} \quad (8)$$

$$\chi^{(n)}(y) = \begin{cases} \frac{1}{\sqrt{\pi R}}, & n = 0, \\ N_A^{(n)} e^{ky} \left[J_1\left(\frac{m_A^{(n)}}{k} e^{ky}\right) + \alpha_A^{(n)} Y_1\left(\frac{m_A^{(n)}}{k} e^{ky}\right) \right], & n > 0. \end{cases} \quad (9)$$

$$h^{(n)}(y) = \begin{cases} \sqrt{\frac{2k(1\pm\sqrt{4+a})}{[e^{2(1\pm\sqrt{4+a})k\pi R}-1]}} e^{(2\pm\sqrt{4+a})ky}, & n = 0, \\ N_H^{(n)} e^{2ky} \left[J_{\pm\sqrt{4+a}}\left(\frac{m_H^{(n)}}{k} e^{ky}\right) + \alpha_H^{(n)} Y_{\pm\sqrt{4+a}}\left(\frac{m_H^{(n)}}{k} e^{ky}\right) \right], & n > 0. \end{cases} \quad (10)$$

Here $\chi^{(n)}(y)$ are given for the gauge where $A_5 = 0$. The functions J_ρ , Y_ρ are the first and second kind of Bessel functions, respectively, and $N_{\Psi_{L,R}}^{(n)}$, $N_A^{(n)}$ are normalization constants computed from the following orthonormality relations:

$$\begin{aligned} \int_0^{\pi R} dy e^{-3ky} f_{L,R}^{(n)}(y) f_{L,R}^{(m)}(y) &= \delta_{nm}, \\ \int_0^{\pi R} dy \chi^{(n)}(y) \chi^{(m)}(y) &= \delta_{nm}, \\ \int_0^{\pi R} dy e^{-2ky} h^{(n)}(y) h^{(m)}(y) &= \delta_{nm}. \end{aligned} \quad (11)$$

The coefficients $\alpha_{\Psi_{L,R}}^{(n)}$, $\alpha_A^{(n)}$, and $\alpha_H^{(n)}$ are determined by the boundary conditions on the branes, resulting in the following relations:

$$\alpha_{\Psi_{L,R}}^{(n)} = -\frac{J_{\pm d_{L,R}\pm\frac{1}{2}}\left(\frac{m_{\Psi}^{(n)}}{k}\right)}{Y_{\pm d_{L,R}\pm\frac{1}{2}}\left(\frac{m_{\Psi}^{(n)}}{k}\right)} = -\frac{J_{\pm d_{L,R}\pm\frac{1}{2}}\left(\frac{m_{\Psi}^{(n)}}{k} e^{\pi k R}\right)}{Y_{\pm d_{L,R}\pm\frac{1}{2}}\left(\frac{m_{\Psi}^{(n)}}{k} e^{\pi k R}\right)}, \quad (12)$$

$$\alpha_A^{(n)} = -\frac{J_0\left(\frac{m_A^{(n)}}{k}\right)}{Y_0\left(\frac{m_A^{(n)}}{k}\right)} = -\frac{J_0\left(\frac{m_A^{(n)}}{k} e^{\pi k R}\right)}{Y_0\left(\frac{m_A^{(n)}}{k} e^{\pi k R}\right)}, \quad (13)$$

$$\alpha_H^{(n)} = -\frac{J_0\left(\frac{m_H^{(n)}}{k}\right)}{Y_0\left(\frac{m_H^{(n)}}{k}\right)} = -\frac{J_0\left(\frac{m_H^{(n)}}{k} e^{\pi k R}\right)}{Y_0\left(\frac{m_H^{(n)}}{k} e^{\pi k R}\right)}. \quad (14)$$

In turn, these relations determine the masses $m_{\Psi}^{(n)}$, $m_A^{(n)}$, and $m_H^{(n)}$ of the nonzero modes. These modes correspond to heavy particles beyond the spectrum of the SM. On the other hand, the zero modes, which are identified with the

SM fermions, remain massless at this level and become massive only after electroweak symmetry breaking.

Quark and lepton fields are assigned into two S_3 doublets and several S_3 singlets, as follows:

$$\begin{aligned} Q_L &= (q_{1L}, q_{2L}) \sim \mathbf{2}, & q_{3L} &\sim \mathbf{1}', & u_{1R} &\sim \mathbf{1}, \\ u_{2R} &\sim \mathbf{1}', & u_{3R} &\sim \mathbf{1}', \\ d_{1R} &\sim \mathbf{1}, & d_{2R} &\sim \mathbf{1}, & d_{3R} &\sim \mathbf{1}', \end{aligned} \quad (15)$$

$$\begin{aligned} l_R &= (l_{1R}, l_{2R}) \sim \mathbf{2}, & l_{3R} &\sim \mathbf{1}', & l_{1L} &\sim \mathbf{1}, \\ l_{2L} &\sim \mathbf{1}', & l_{3L} &\sim \mathbf{1}', \\ \nu_{1R} &\sim \mathbf{1}', & \nu_{2R} &\sim \mathbf{1}'. \end{aligned} \quad (16)$$

The $SU(2)_L$ scalar doublet H is assigned as a trivial S_3 singlet, whereas the different EW scalar singlets are grouped into two S_3 doublets, one S_3 trivial singlet, and two S_3 nontrivial singlets. The scalar assignments under the S_3 flavor symmetry are

$$H \sim \mathbf{1}, \quad \zeta \sim \mathbf{1}', \quad \eta \sim \mathbf{1}, \quad \rho \sim \mathbf{1}', \quad (17)$$

$$\chi = (\chi_1, \chi_2) \sim \mathbf{2}, \quad \xi = (\xi_1, \xi_2) \sim \mathbf{2}. \quad (18)$$

We assume that χ and ζ are located in the TeV brane, whereas the remaining scalar singlets, i.e., η , ρ , and ξ are set at the Planck brane.

The minimization equations for the scalar potentials for a single S_3 scalar doublet can be seen in Ref. [98]. With ξ and χ being in different branes, their minimization can proceed independently (similar to the strategy of [57]). They acquire the following vacuum expectation value (VEV) pattern:

$$\langle \xi \rangle = v_\xi(1, 0), \quad \langle \chi \rangle = v_\chi(1, 0), \quad (19)$$

so that the VEVs are both pointing in the $(1,0)$ S_3 direction.

Furthermore, we include a discrete $Z_2 \otimes Z_4$ symmetry. The Z_2 symmetry decouples the bottom quark from the

down and strange quarks. The fields charged under Z_2 transform as follows:

$$\begin{aligned} Q_L &\rightarrow -Q_L, & u_{1R} &\rightarrow -u_{1R}, & u_{2R} &\rightarrow -u_{2R}, \\ d_{1R} &\rightarrow -d_{1R}, & d_{2R} &\rightarrow -d_{2R}, & \chi &\rightarrow -\chi, \end{aligned} \quad (20)$$

while the fields charged under Z_4 have the following nontrivial transformation properties:

$$d_{1R} \rightarrow id_{1R}, \quad d_{2R} \rightarrow -d_{2R}, \quad d_{3R} \rightarrow id_{3R}, \quad (21)$$

$$\rho \rightarrow -\rho, \quad \eta \rightarrow -i\eta, \quad l_{1L} \rightarrow il_{1L}. \quad (22)$$

The $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \otimes S_3 \otimes Z_2 \otimes Z_4$ invariant quark and lepton Yukawa interactions are

$$\begin{aligned} S_5^{(q)} &= \int d^4x \int_0^{\pi R} dy \sqrt{-g} \left\{ \left[\frac{\lambda_{13}^{(u)}}{M_5^2} \bar{Q}_L \tilde{H} u_{3R} \chi \zeta + \frac{\lambda_{23}^{(u)}}{M_5^2} \bar{Q}_L \tilde{H} u_{3R} \chi \right] \delta(y - \pi R) + \frac{\lambda_{33}^{(u)}}{M_5^2} \bar{q}_{3L} \tilde{H} u_{3R} \right\} \\ &+ \int d^4x \int_0^{\pi R} dy \sqrt{-g} \left[\frac{\lambda_{11}^{(u)}}{M_5^2} \bar{Q}_L \tilde{H} u_{1R} \xi + \frac{\lambda_{22}^{(u)}}{M_5^2} \bar{Q}_L \tilde{H} u_{2R} \xi + \frac{\lambda_{11}^{(d)}}{M_5^2} \bar{Q}_L H d_{1R} \xi \eta \right. \\ &\left. + \frac{\lambda_{22}^{(d)}}{M_5^2} \bar{Q}_L H d_{2R} \xi \rho + \frac{\lambda_{12}^{(d)}}{M_5^2} \bar{Q}_L H d_{2R} \xi \eta^2 + \frac{\lambda_{33}^{(d)}}{M_5^2} \bar{q}_{3L} H d_{3R} \eta \right] \delta(y), \end{aligned} \quad (23)$$

$$\begin{aligned} S_5^{(l)} &= \int d^4x \int_0^{\pi R} dy \sqrt{-g} \left[\frac{\lambda_{11}^{(l)}}{M_5^2} \bar{l}_{1L} H l_{1R} \xi \eta^* + \frac{\lambda_{22}^{(l)}}{M_5^2} \bar{l}_{2L} H l_{1R} \xi + \frac{\lambda_{32}^{(l)}}{M_5^2} \bar{l}_{3L} H l_{1R} \xi \right] \delta(y) \\ &+ \int d^4x \int_0^{\pi R} dy \sqrt{-g} \left[\frac{\lambda_{23}^{(l)}}{M_5^2} \bar{l}_{2L} H l_{3R} + \frac{\lambda_{33}^{(l)}}{M_5^2} \bar{l}_{3L} H l_{3R} \right] \\ &+ \int d^4x \int_0^{\pi R} dy \sqrt{-g} \left[\frac{\lambda_{32}^{(\nu)}}{M_5} \bar{l}_{2L} \tilde{H} \nu_{1R} + \frac{\lambda_{32}^{(\nu)}}{M_5} \bar{l}_{3L} \tilde{H} \nu_{1R} \right] \delta(y) \\ &+ \int d^4x \int_0^{\pi R} dy \sqrt{-g} \left[\frac{\lambda_{11}^{(\nu)}}{M_5^2} \bar{l}_{1L} \tilde{H} \nu_{2R} \eta^* + \frac{\lambda_{21}^{(\nu)}}{M_5} \bar{l}_{2L} \tilde{H} \nu_{2R} + \frac{\lambda_{22}^{(\nu)}}{M_5} \bar{l}_{3L} \tilde{H} \nu_{2R} \right] \delta(y) \\ &+ \int d^4x \int_0^{\pi R} dy \sqrt{-g} [M_{1\nu} \bar{\nu}_{1R} \nu_{1R}^c + M_{2\nu} \bar{\nu}_{2R} \nu_{2R}^c + M_{3\nu} (\bar{\nu}_{1R} \nu_{2R}^c + \bar{\nu}_{2R} \nu_{1R}^c)] \delta(y), \end{aligned} \quad (24)$$

where the dimensionless couplings in Eqs. (23) and (24) are $\mathcal{O}(1)$ parameters. Note that with χ in the TeV brane and η in the Planck brane, terms like $\bar{Q}_L H d_{3R} \chi \eta$ do not contribute and were therefore not included above, even though they are invariant under all the symmetries.

Assuming that the quark mass and mixing pattern arises from the fermion profiles along the extra dimension, we set the VEVs of the EW scalar singlets with respect to the Wolfenstein parameter $\lambda = 0.225$ and the new physics scale M_5 ,

$$v_\zeta \sim v_\chi \sim v_\rho \sim v_\eta \sim \lambda M_5; \quad v_\xi \sim \lambda^3 M_5. \quad (25)$$

The presence of additional scalars (in the TeV brane) leads to the interesting possibility of extra scalars at colliders. For recent studies on Higgs production and decay in multi-Higgs models (some of which are embedded into 5D warped models), see, for instance, Ref. [98,117–123].

III. QUARK MASSES AND MIXINGS

From the quark Yukawa interactions given by Eq. (23) we get the following mass matrix textures for quarks:

$$\begin{aligned} M_U &= \begin{pmatrix} \epsilon_{11}^{(u)} \frac{v_\xi}{M_5} & 0 & \epsilon_{13}^{(u)} \frac{v_\zeta v_\rho}{M_5^2} \\ 0 & \epsilon_{22}^{(u)} \frac{v_\xi}{M_5} & \epsilon_{23}^{(u)} \frac{v_\chi}{M_5} \\ 0 & 0 & \epsilon_{33}^{(u)} \end{pmatrix} \frac{v}{\sqrt{2}}, \\ M_D &= \begin{pmatrix} \epsilon_{11}^{(d)} \frac{v_\xi v_\eta}{M_5^2} & \epsilon_{12}^{(d)} \frac{v_\xi v_\eta^2}{M_5^3} & 0 \\ 0 & \epsilon_{22}^{(d)} \frac{v_\xi v_\rho}{M_5^2} & 0 \\ 0 & 0 & \epsilon_{33}^{(d)} \frac{v_\eta}{M_5} \end{pmatrix} \frac{v}{\sqrt{2}}, \end{aligned} \quad (26)$$

where

$$\begin{aligned}
\varepsilon_{n3}^{(u)} &= \lambda_{n3}^{(u)} \frac{k}{M_5^{\frac{3}{2}}} \sqrt{\frac{2k(1 \pm \sqrt{4+a})(1-2y_{nL}^{(q)})(1-2y_{3R}^{(u)})}{[e^{2(1 \pm \sqrt{4+a})k\pi R} - 1][e^{(1-2y_{nL}^{(q)})k\pi R} - 1][e^{(1-2y_{3R}^{(u)})k\pi R} - 1]}} e^{(2-y_{nL}^{(q)}-y_{3R}^{(u)} \pm \sqrt{4+a})k\pi R}, \quad n = 1, 2; \\
\varepsilon_{33}^{(u)} &= \lambda_{33}^{(u)} \frac{1}{M_5^{\frac{1}{2}}} \sqrt{\frac{2k(1 \pm \sqrt{4+a})(1-2y_{3L}^{(q)})(1-2y_{3R}^{(u)})}{[e^{2(1 \pm \sqrt{4+a})k\pi R} - 1][e^{(1-2y_{3L}^{(q)})k\pi R} - 1][e^{(1-2y_{3R}^{(u)})k\pi R} - 1]}} \frac{e^{(2-y_{3L}^{(q)}-y_{3R}^{(u)} \pm \sqrt{4+a})k\pi R} - 1}{[2 - y_{3L}^{(q)} - y_{3R}^{(u)} \pm \sqrt{4+a}]}, \\
\varepsilon_{nn}^{(u)} &= \lambda_{nn}^{(u)} \frac{k}{M_5^{\frac{3}{2}}} \sqrt{\frac{2k(1 \pm \sqrt{4+a})(1-2y_{nL}^{(q)})(1-2y_{nR}^{(u)})}{[e^{2(1 \pm \sqrt{4+a})k\pi R} - 1][e^{(1-2y_{nL}^{(q)})k\pi R} - 1][e^{(1-2y_{nR}^{(u)})k\pi R} - 1]}}}, \quad n = 1, 2; \\
\varepsilon_{ij}^{(d)} &= \lambda_{ij}^{(d)} \frac{k}{M_5^{\frac{3}{2}}} \sqrt{\frac{2k(1 \pm \sqrt{4+a})(1-2y_{iL}^{(q)})(1-2y_{jR}^{(u)})}{[e^{2(1 \pm \sqrt{4+a})k\pi R} - 1][e^{(1-2y_{iL}^{(q)})k\pi R} - 1][e^{(1-2y_{jR}^{(u)})k\pi R} - 1]}}}, \quad i, j = 1, 2, 3,
\end{aligned} \tag{27}$$

where $y_{1L} = y_{2L} = y_L$, as follows from the fact that q_{1L} and q_{2L} are unified into a S_3 doublet as seen from Eq. (15).

We assume that the theory conserves CP and that CP is violated spontaneously only through the complex VEV v_ζ of the ζ scalar. This makes the model more predictive. We fit the parameters in Eq. (26) to reproduce the quark masses and quark mixing parameters.

To reproduce the experimental values of the 10 physical observables of the quark sector, i.e., the 6 quark masses and 4 quark mixing parameters, we fix the right-handed top quark localization profile and the Higgs profile, as follows: $y_{1R}^{(u)} = y_{3R}^{(u)}$, $a = -3$, $\lambda_{ij}^{(u)} = \lambda_{ij}^{(d)} = 10$, whereas we fit the effective 9 free parameters, i.e., the 2 left-handed quark profiles, the remaining 5 right-handed quark profiles, and the magnitude and phase of v_ζ . The values of the quark profiles, the magnitude and phase of v_ζ , corresponding to the results reported in Tables I and II are $y_L^{(q)} = 0.502177$, $y_{3L}^{(u)} = 0.522259$, $y_{1R}^{(u)} = 0.41452$, $y_{2R}^{(u)} = 28.5089$, $y_{1R}^{(d)} = 0.500942$, $y_{2R}^{(d)} = 4.63314$, $y_{3R}^{(d)} = 0.532187$, $|v_\zeta| = 0.4\lambda M_5$, and $\alpha = 67.6164^\circ$ (α being the complex phase of v_ζ). Note that, as all the left handed profiles are $>1/2$, the flavor changing neutral current contributions due to KK modes are suppressed [106,111]. This suppression is due to the fact that the gauge coupling of zero mode fermions to Kaluza-Klein gauge bosons is much smaller than the

electroweak coupling constant when the left-handed fermion profiles are equal to or larger than $1/2$.

The experimental values of the Cabibbo-Kobayashi-Maskawa (CKM) matrix are taken from Ref. [3]. As can be seen, the quark masses and the CKM matrix obtained from these textures are in good agreement with the experimental data. The agreement of our model with the experimental data is as good as in the models of Refs. [15,16,46,69,73,92,94] and better than, for example, those in Refs. [52,114,114,124–129].

IV. LEPTON MASSES AND MIXINGS

This warped S_3 flavor model generates the viable and predictive quark textures proposed in [22] as shown in Sec. III. We now proceed to analyze the lepton sector of the model. From the charged lepton Yukawa terms of Eq. (24) it follows that the charged lepton mass matrix takes the following form:

$$M_l = \begin{pmatrix} \varepsilon_{11}^{(l)} \frac{v_\zeta v_\eta}{M_5^2} & 0 & 0 \\ 0 & \varepsilon_{22}^{(l)} \frac{v_\zeta}{M_5} & \varepsilon_{23}^{(l)} \\ 0 & \varepsilon_{32}^{(l)} \frac{v_\zeta}{M_5} & \varepsilon_{33}^{(l)} \end{pmatrix} \frac{v}{\sqrt{2}} = \begin{pmatrix} w & 0 & 0 \\ 0 & r & s \\ 0 & x & z \end{pmatrix} \frac{v}{\sqrt{2}}, \tag{28}$$

where

TABLE I. Model and experimental values of quark masses.

Observable	Model value	Experimental value
m_u [MeV]	1.48	$1.45_{-0.45}^{+0.56}$
m_c [MeV]	634	635 ± 86
m_t [GeV]	172.1	$172.1 \pm 0.6 \pm 0.9$
m_d [MeV]	3.1	$2.9_{-0.4}^{+0.5}$
m_s [MeV]	56.3	$57.7_{-15.7}^{+16.8}$
m_b [GeV]	2.82	$2.82_{-0.04}^{+0.09}$

TABLE II. Model and experimental values of CKM parameters.

Observable	Model value	Experimental value
$\sin \theta_{12}$	0.22	0.22536 ± 0.00061
$\sin \theta_{23}$	0.0413	0.0414 ± 0.0012
$\sin \theta_{13}$	0.0037	0.00355 ± 0.00015
δ	68°	68°

$$\begin{aligned}
 \varepsilon_{nn}^{(l)} &= \lambda_{nn}^{(l)} \frac{k}{M_5^{\frac{3}{2}}} \sqrt{\frac{2k(1 \pm \sqrt{4+a})(1-2y_{nL}^{(l)})(1-2y_{nR}^{(l)})}{[e^{2(1 \pm \sqrt{4+a})k\pi R} - 1][e^{(1-2y_{nL}^{(l)})k\pi R} - 1][e^{(1-2y_{nR}^{(l)})k\pi R} - 1]}}, \quad n = 1, 2, \\
 \varepsilon_{23}^{(l)} &= \lambda_{23}^{(l)} \frac{k}{M_5^{\frac{3}{2}}} \sqrt{\frac{2k(1 \pm \sqrt{4+a})(1-2y_{2L}^{(l)})(1-2y_{3R}^{(l)})}{[e^{2(1 \pm \sqrt{4+a})k\pi R} - 1][e^{(1-2y_{2L}^{(l)})k\pi R} - 1][e^{(1-2y_{3R}^{(l)})k\pi R} - 1]} \frac{e^{(2-y_{3L}^{(l)}-y_{2R}^{(l)} \pm \sqrt{4+a})k\pi R} - 1}{[2-y_{2L}^{(l)}-y_{3R}^{(l)} \pm \sqrt{4+a}]}, \\
 \varepsilon_{33}^{(l)} &= \lambda_{33}^{(l)} \frac{1}{M_5^{\frac{1}{2}}} \sqrt{\frac{2k(1 \pm \sqrt{4+a})(1-2y_{3L}^{(l)})(1-2y_{3R}^{(l)})}{[e^{2(1 \pm \sqrt{4+a})k\pi R} - 1][e^{(1-2y_{3L}^{(l)})k\pi R} - 1][e^{(1-2y_{3R}^{(l)})k\pi R} - 1]} \frac{e^{(2-y_{3L}^{(l)}-y_{3R}^{(l)} \pm \sqrt{4+a})k\pi R} - 1}{[2-y_{3L}^{(l)}-y_{3R}^{(l)} \pm \sqrt{4+a}]}, \\
 \varepsilon_{32}^{(l)} &= \lambda_{32}^{(l)} \frac{1}{M_5^{\frac{1}{2}}} \sqrt{\frac{2k(1 \pm \sqrt{4+a})(1-2y_{3L}^{(l)})(1-2y_{2R}^{(l)})}{[e^{2(1 \pm \sqrt{4+a})k\pi R} - 1][e^{(1-2y_{3L}^{(l)})k\pi R} - 1][e^{(1-2y_{2R}^{(l)})k\pi R} - 1]}}, \quad (29)
 \end{aligned}$$

where $y_{1R}^{(l)} = y_{2R}^{(l)} = y_R^{(l)}$ as follows from the fact that l_{1R} and l_{2R} are unified into a S_3 doublet as seen from Eq. (15).

Therefore, $M_l M_l^T$ can be approximately diagonalized by a rotation matrix R_l according to

$$R_l^T M_l M_l^T R_l = \begin{pmatrix} m_e^2 & 0 & 0 \\ 0 & m_\mu^2 & 0 \\ 0 & 0 & m_\tau^2 \end{pmatrix}, \quad R_l = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \gamma & -\sin \gamma \\ 0 & \sin \gamma & \cos \gamma \end{pmatrix}, \quad \tan 2\gamma = \frac{2(rx + sz)}{(r^2 + s^2) - (x^2 + z^2)}. \quad (30)$$

The masses for charged leptons take the form

$$\begin{aligned}
 m_e &= \varepsilon_{11}^{(l)} \frac{v_\xi v}{\sqrt{2}M_5}, \\
 m_\mu &= \sqrt{\frac{1}{2}(r^2 + s^2 + x^2 + z^2) - \frac{1}{2}\sqrt{(r^2 + 2rz + s^2 - 2sx + x^2 + z^2)(r^2 - 2rz + s^2 + 2sx + x^2 + z^2)}} \frac{v}{\sqrt{2}}, \\
 m_\tau &= \sqrt{\frac{1}{2}(r^2 + s^2 + x^2 + z^2) + \frac{1}{2}\sqrt{(r^2 + 2rz + s^2 - 2sx + x^2 + z^2)(r^2 - 2rz + s^2 + 2sx + x^2 + z^2)}} \frac{v}{\sqrt{2}}. \quad (31)
 \end{aligned}$$

To show that the charged lepton texture can fit the experimental data and to simplify our analysis, we adopt a benchmark where we set

$$y_{1L}^{(l)} = y_{2L}^{(l)} = y_{3L}^{(l)} = y_L^{(l)}, \quad \lambda_{23}^{(l)} = \lambda_{33}^{(l)} = \lambda^{(l)}, \quad \lambda_{32}^{(l)} = 1. \quad (32)$$

We assume equality of left-handed leptonic profiles to avoid a huge hierarchy in the entries of the neutrino mass matrix, as required by the neutrino oscillation experimental data which favor a moderate hierarchy among these entries. Our two assumptions concerning the charged lepton Yukawa couplings are motivated by the partial universality of these couplings and by naturalness arguments. Let us note that the charged lepton mass hierarchy is caused by the different localizations of the right-handed charged leptonic fields l_R and l_{3R} in the extra dimension as well as by the different power dependences in terms of the Wolfenstein parameter $\lambda = 0.225$ presented in the different columns of the charged lepton mass matrix. Consequently it is natural to assume partial universality in the charged lepton Yukawa couplings.

In addition, the S_3 discrete symmetry leads to the following constraint:

$$y_{1R}^{(l)} = y_{2R}^{(l)} = y_R^{(l)}. \quad (33)$$

Then, we proceed to adjust the parameters $y_R^{(l)}$, $y_L^{(l)}$, $y_{3R}^{(l)}$, $\lambda^{(l)}$, $\lambda_{11}^{(l)}$, and $\lambda_{22}^{(l)}$ to reproduce the experimental values of the charged lepton masses finding the following best fit result:

$$\begin{aligned}
 \lambda_{11}^{(l)} &= 3.147, \quad \lambda_{22}^{(l)} = 2.218, \quad \lambda^{(l)} = 0.484, \\
 y_L^{(l)} &= 7.323713, \quad y_R^{(l)} = 0.5207487, \quad y_{3R}^{(l)} = 0.501. \quad (34)
 \end{aligned}$$

The full 5×5 neutrino mass matrix arises from the neutrino Yukawa interactions given in Eq. (24) and is given by

$$M_\nu = \begin{pmatrix} \mathbf{0}_{3 \times 3} & M_\nu^D \\ (M_\nu^D)^T & M_R \end{pmatrix}, \quad (35)$$

where

$$M_\nu^D = \begin{pmatrix} 0 & \epsilon_{12}^{(\nu)} \frac{v_n}{M_5} \\ \epsilon_{21}^{(\nu)} & \epsilon_{22}^{(\nu)} \\ \epsilon_{31}^{(\nu)} & \epsilon_{32}^{(\nu)} \end{pmatrix} \frac{v}{\sqrt{2}} = \begin{pmatrix} 0 & f \\ b & c \\ h & d \end{pmatrix} \frac{v}{\sqrt{2}}, \quad M_R = \begin{pmatrix} M_{1\nu} & M_{3\nu} \\ M_{3\nu} & M_{2\nu} \end{pmatrix},$$

$$M_R^{-1} = \begin{pmatrix} \frac{M_{2\nu}}{M_\nu M_{2\nu} - M_{3\nu}^2} & -\frac{M_{3\nu}}{M_\nu M_{2\nu} - M_{3\nu}^2} \\ -\frac{M_{3\nu}}{M_\nu M_{2\nu} - M_{3\nu}^2} & \frac{M_\nu}{M_\nu M_{2\nu} - M_{3\nu}^2} \end{pmatrix} = \begin{pmatrix} p & r \\ r & q \end{pmatrix}, \quad (36)$$

$$\epsilon_{n1}^{(\nu)} = \lambda_{n1}^{(\nu)} \frac{k}{M_5} \sqrt{\frac{2(1 \pm \sqrt{4+a})(1 - 2y_{nL}^{(\nu)})}{[e^{2(1 \pm \sqrt{4+a})k\pi R} - 1][e^{(1-2y_{nL}^{(\nu)})k\pi R} - 1]}}, \quad n = 2, 3,$$

$$\epsilon_{j2}^{(\nu)} = \lambda_{j2}^{(\nu)} \frac{k}{M_5} \sqrt{\frac{2(1 \pm \sqrt{4+a})(1 - 2y_{jL}^{(\nu)})}{[e^{2(1 \pm \sqrt{4+a})k\pi R} - 1][e^{(1-2y_{jL}^{(\nu)})k\pi R} - 1]}}, \quad j = 1, 2, 3. \quad (37)$$

As the Majorana neutrino masses are much larger than the electroweak symmetry breaking scale $v = 256$ GeV, i.e., $(M_R)_{ii} \gg v$, active neutrinos acquire small masses via a type-I seesaw mechanism mediated by the Majorana neutrinos. Then, the light neutrino mass matrix reads

$$M_L = M_\nu^D M_R^{-1} (M_\nu^D)^T = \begin{pmatrix} 0 & f \\ b & c \\ h & d \end{pmatrix} \begin{pmatrix} p & r \\ r & q \end{pmatrix} \begin{pmatrix} 0 & b & h \\ f & c & d \end{pmatrix} \frac{v^2}{2}$$

$$= \begin{pmatrix} f^2 q & f(br + cq) & f(dq + hr) \\ f(br + cq) & pb^2 + 2rbc + qc^2 & b(dr + hp) + c(dq + hr) \\ f(dq + hr) & b(dr + hp) + c(dq + hr) & qd^2 + 2rdh + ph^2 \end{pmatrix} \frac{v^2}{2}. \quad (38)$$

To show that the light neutrino mass matrix given above is consistent with the experimental data on neutrino oscillations and to simplify our analysis, we adopt a benchmark where we set

$$p = q, \quad h = -d, \quad b = c, \quad (39)$$

so that the light active neutrino mass matrix takes the form

$$M_L = \begin{pmatrix} f^2 p & bf(p+r) & df(p-r) \\ bf(p+r) & 2b^2(p+r) & 0 \\ df(p-r) & 0 & 2d(dp-dr) \end{pmatrix} \frac{v^2}{2}, \quad (40)$$

which implies that the light active neutrino mass matrix does not contribute to $\sin^2 \theta_{23}$. The leptonic mixing parameter $\sin^2 \theta_{23}$ solely arises from the charged lepton sector and is found to be equal to 0.507, which is consistent with its corresponding experimental value within the 1σ experimentally allowed range. Varying the parameters p , r , b , d , and f we fitted Δm_{21}^2 , Δm_{31}^2 (note that we define $\Delta m_{ij}^2 = m_i^2 - m_j^2$), $\sin^2 \theta_{12}$, and $\sin^2 \theta_{13}$ to the experimental values [130] in Table III for the normal hierarchy neutrino mass spectrum. The best fit result is

$$\Delta m_{21}^2 = 7.55 \times 10^{-5} \text{ eV}^2, \quad \Delta m_{31}^2 = 2.51 \times 10^{-3} \text{ eV}^2,$$

$$m_{\nu_1} = 0, \quad m_{\nu_2} \simeq 9 \text{ meV}, \quad m_{\nu_3} \simeq 50 \text{ meV},$$

$$\sin^2 \theta_{12} = 0.311, \quad \sin^2 \theta_{13} = 0.024, \quad \sin^2 \theta_{23} = 0.507,$$

$$b \simeq 0.38, \quad f \simeq 0.52, \quad d \simeq 1.57,$$

$$q \simeq -1.72 \times 10^{-16} \text{ GeV}^{-1},$$

$$r \simeq -4.98 \times 10^{-16} \text{ GeV}^{-1}. \quad (41)$$

In Table IV we show the model and experimental values for the physical observables of the lepton sector for the normal hierarchy neutrino mass spectrum. Comparing Eq. (41) with Table III we see that the mass squared splittings Δm_{21}^2 and Δm_{31}^2 and mixing parameters $\sin^2 \theta_{12}$, $\sin^2 \theta_{13}$, and $\sin^2 \theta_{23}$ are in excellent agreement with the experimental data as they are inside the 1σ experimentally allowed range. Note that here we considered all leptonic parameters to be real for simplicity, but a nonvanishing CP phase in the PMNS mixing matrix can be generated by making complex any of the off diagonal charged lepton Yukawa couplings, e.g., $\lambda_{23}^{(l)}$ or $\lambda_{32}^{(l)}$. It is noteworthy that, in view of the parametric freedom of our model, it is always possible to fit any physical observable

TABLE III. Experimental ranges of neutrino squared mass differences and leptonic mixing angles, from Ref. [130], for the normal hierarchy neutrino mass spectrum.

Parameter	Δm_{21}^2 (10^{-5} eV ²)	Δm_{31}^2 (10^{-3} eV ²)	$(\sin^2 \theta_{12})_{\text{exp}}$	$(\sin^2 \theta_{23})_{\text{exp}}$	$(\sin^2 \theta_{13})_{\text{exp}}$
Best fit	7.60	2.48	0.323	0.567	0.0234
1 σ range	7.42–7.79	2.41–2.53	0.307–0.339	0.439–0.599	0.0214–0.0254
2 σ range	7.26–7.99	2.35–2.59	0.292–0.357	0.413–0.623	0.0195–0.0274
3 σ range	7.11–8.11	2.30–2.65	0.278–0.375	0.392–0.643	0.0183–0.0297

of both quark and lepton sectors independently of its degree of precision. Thus, for example, an increase in the precision of the experimental value of the leptonic mixing parameter $\sin^2 \theta_{13}$ will not rule out our model.

Now we can predict the amplitude for neutrinoless double beta ($0\nu\beta\beta$) decay, which is proportional to the effective Majorana neutrino mass

$$m_{\beta\beta} = \left| \sum_k U_{ek}^2 m_{\nu_k} \right|, \quad (42)$$

where U_{ek}^2 and m_{ν_k} are the PMNS mixing matrix elements and the Majorana neutrino masses, respectively.

Then, we predict the following effective neutrino mass:

$$m_{\beta\beta} \approx 1.4 \text{ meV}. \quad (43)$$

Our value for the effective neutrino mass is beyond the reach of the present and forthcoming neutrinoless double beta decay experiments. The current best upper bound for the effective neutrino mass, i.e., $m_{\beta\beta} \leq 160$ meV arises from the EXO-200 experiment [131] $T_{1/2}^{0\nu\beta\beta}({}^{136}\text{Xe}) \geq 1.6 \times 10^{25}$ yr at the 90% C.L. An improvement of this upper bound is expected within the not too far future. The GERDA experiment [132,133] is upgrading to “phase-II,” and is expected to reach $T_{1/2}^{0\nu\beta\beta}({}^{76}\text{Ge}) \geq 2 \times 10^{26}$ yr, corresponding to $m_{\beta\beta} \leq 100$ MeV. A bolometric CUORE

experiment, using ${}^{130}\text{Te}$ [134], is under construction. Its sensitivity is estimated at about $T_{1/2}^{0\nu\beta\beta}({}^{130}\text{Te}) \sim 10^{26}$ yr, which corresponds to $m_{\beta\beta} \leq 50$ meV. There are several proposals for ton-scale future $0\nu\beta\beta$ experiments with ${}^{136}\text{Xe}$ [135,136] and ${}^{76}\text{Ge}$ [132,137] estimating sensitivities over $T_{1/2}^{0\nu\beta\beta} \sim 10^{27}$ yr, corresponding to $m_{\beta\beta} \sim 12\text{--}30$ meV. Recent experimental reviews can be found in Ref. [138] and references therein. Consequently, as indicated by Eq. (43) we get a prediction of $T_{1/2}^{0\nu\beta\beta}$, which is at the level of sensitivities of the next generation or next-to-next generation $0\nu\beta\beta$ experiments.

V. CONCLUSIONS

In this paper we proposed an extension of the Standard Model where a warped extra dimension is supplemented by the discrete flavor symmetry $S_3 \times Z_2 \times Z_4$ and the particle content is extended by including two heavy right-handed Majorana neutrinos and scalar fields responsible for the breaking of the discrete symmetry. To the best of our knowledge, our model is the first implementation of the S_3 flavor symmetry in a five-dimensional warped framework. We examined a particular choice of S_3 assignments in the quark sector, which leads to a mixing inspired texture where the down-type quark sector contributes to the Cabibbo mixing, whereas the up-type quark sector contributes to the remaining mixing angles. In the lepton sector, the effective neutrino masses are obtained through a type I seesaw with two right-handed neutrinos, and the charged lepton mass texture contributes to the mixing in the 2–3 plane. The flavor symmetry is responsible for several texture zeros in the mass matrices of each sector. The model is capable of a very good fit to the fermion data, and the mass hierarchy and mixing angles are explained by a combination of the discrete symmetries and the exponential suppressions arising from the warped spacetime.

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TABLE IV. Model and experimental values of the charged lepton masses, neutrino mass squared splittings, and leptonic mixing parameters for the normal hierarchy neutrino mass spectrum.

Observable	Model value	Experimental value
m_e [MeV]	0.487	0.487
m_μ [MeV]	103	102.8 ± 0.0003
m_τ [GeV]	1.75	1.75 ± 0.0003
Δm_{21}^2 (10^{-5} eV ²)	7.60	$7.60_{-0.18}^{+0.19}$
Δm_{31}^2 (10^{-3} eV ²)	2.48	$2.48_{-0.07}^{+0.05}$
$\sin^2 \theta_{12}$	0.311	0.323 ± 0.016
$\sin^2 \theta_{23}$	0.507	$0.567_{-0.128}^{+0.032}$
$\sin^2 \theta_{13}$	0.024	0.0234 ± 0.0020

where part of this work was done. A. E. C. H. was supported by Fondecyt (Chile) Grant No. 11130115 and by DGIP internal Grant No. 111458.

APPENDIX: THE PRODUCT RULES FOR S_3

The S_3 discrete group contains 3 irreducible representations: **1**, **1'**, and **2**. Considering $(x_1, x_2)^T$ and $(y_1, y_2)^T$ as the basis vectors for two S_3 doublets and y' an S_3 nontrivial singlet, the multiplication rules of the S_3 group for the case of real representations take the form [30]

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_2 \otimes \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}_2 = (x_1 y_1 + x_2 y_2)_1 + (x_1 y_2 - x_2 y_1)_{1'} + \begin{pmatrix} x_2 y_2 - x_1 y_1 \\ x_1 y_2 + x_2 y_1 \end{pmatrix}_2, \quad (\text{A1})$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_2 \otimes (y')_{1'} = \begin{pmatrix} -x_2 y' \\ x_1 y' \end{pmatrix}_2, \quad (\text{A2})$$

$$(x')_{1'} \otimes (y')_{1'} = (x' y')_{1'}.$$

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