Open/closed string duality and relativistic fluids

Vasilis Niarchos

Crete Center for Theoretical Physics and Crete Center for Quantum Complexity and Nanotechnology,
Department of Physics, University of Crete, Heraklion 71303, Greece

We propose an open/closed string duality in general backgrounds extending previous ideas about open string completeness by Ashoke Sen. Our proposal sets up a general version of holography that works in gravity as a tomographic principle. We argue, in particular, that previous expectations of a supergravity/Dirac-Born-Infeld (DBI) correspondence are naturally embedded in this conjecture and can be tested in a well-defined manner. As an example, we consider the correspondence between open string field theories on extremal D-brane setups in flat space in the large-N, large 't Hooft limit, and asymptotically flat solutions in ten-dimensional type II supergravity. We focus on a convenient long-wavelength regime, where specific effects of higher-spin open string modes can be traced explicitly in the dual supergravity computation. For instance, in this regime we show how the full Abelian DBI action arises from supergravity as a straightforward reformulation of relativistic hydrodynamics. In the example of a $(2 + 1)$-dimensional open string theory this reformulation involves an Abelian Hodge duality. We also point out how different deformations of the DBI action, related to higher-derivative corrections and non-Abelian effects, can arise in this context as deformations in corresponding relativistic hydrodynamics.

I. INTRODUCTION

The primary goal of this paper is to flesh out the possibility of a general holographic connection between open and closed strings in generic backgrounds (including flat space). We will formulate an open/closed string duality based on the conjecture that open string theories are self-consistent quantum mechanical systems without the need to include explicitly couplings to closed strings. The idea of open string completeness has appeared previously in work by Sen in the context of unstable D-branes [1,2], and is closely related to previous observations in studies of open string field theory. We will review Sen's proposal in Sec. II.

In the context of large-N type II open string theories in flat space we postulate a conjecture that opens the road to an extension of standard examples of the AdS/CFT correspondence beyond the low-energy/near-horizon limit. We emphasize that this extension is conceptually distinct from previous attempts to formulate holography in flat space by seeking the rules of a suitable holographic dictionary on an asymptotic boundary (see e.g. attempts [3–6] based on the Bondi-Metzner-Sachs (BMS) group [7,8], or other attempts like [9,11]).

Similar ideas based on open/closed string duality have been proposed by several authors in the past in the context of the AdS/CFT correspondence and extensions of the correspondence beyond anti–de Sitter (AdS). A characteristic (but not exhaustive) sample of previous works that are closely related to our proposal include [10,12–21]. We propose that Sen's completeness conjecture helps streamline and extend certain aspects of previous discussions.

Testing a duality between open and closed strings in critical higher-dimensional spacetimes is admittedly a complicated task. We will attempt to uncover favorable evidence for a precise dictionary in a convenient long-wavelength regime in the large-N, large 't Hooft limit in a special subsector of the full dynamics (related to Abelian singleton dynamics). Unlike the low-energy/near-horizon limit, in the long-wavelength regime of interest it will be possible to keep explicitly effects from the whole open string tower on the open string side. At the same time, the standard large-N, large 't Hooft limit facilitates a tractable description on the closed string side in terms of classical supergravity.

We can summarize the main elements of the evidence we provide in the following way. Taking the traditional path of the 1990s that led to the AdS/CFT correspondence, we consider the properties of extremal (multicharge) p-brane solutions in supergravity. In a specific derivative expansion scheme of the gravitational equations of motion we argue that the study of the long-wavelength perturbations of p-brane solutions leads naturally to an effective $(p + 1)$-dimensional screen outside the near-horizon region where an Abelian effective action can be formulated. Following previous discussions in the context of the blackfold formalism [22,23] we postulate that there is a one-to-one correspondence between the solutions of the equations of motion of this effective action and a certain class of regular solutions of the full-fledged gravitational equations. For extremal solutions in flat space we prove that the action on the gravitational effective screen is identical to the Abelian Dirac-Born-Infeld (DBI) action. We argue that the latter is the Abelian part of the Wilsonian effective action of the holographically dual large-N open string theory making a precise connection between gravity and open strings. We

discuss how non-Abelian effects are incorporated into this picture.

Our general conjecture for a duality between open and closed strings is formulated in Sec. III. In the same section we describe some of the anticipated features of the Wilsonian effective action of open strings, the specifics of the long-wavelength expansions of interest and the implementation of these expansions on the gravitational side. Our main task in this section is to collect and organize the accumulating observations over the years into a coherent story under a single framework. For many of the underlying technical details of the topics that enter this subject we direct the reader to the appropriate references.

One of the main technical tasks of this paper is to verify that the Abelian theory on the gravitational effective screen coincides with the one expected on the open string side. In complete analogy to the fluid-gravity correspondence in AdS/CFT [24], we find that the effective theory that emerges naturally in gravity is formulated in the form of relativistic hydrodynamics. Consequently, open/closed string duality in this context requires a connection between relativistic hydrodynamics and open string effective actions. At zero temperature and finite chemical potential, we show that there is indeed such a direct connection involving the DBI action. Specifically, in Sec. VII we recover the Abelian DBI action from an (anisotropic) hydrodynamic theory of fluids on dynamic elastic hypersurfaces. Sections IV, V and VI prepare the connection between hydrodynamics and gauge theory from a purely hydrodynamic point of view (that as far as we know is novel).

The emergence of the Abelian DBI theory in the context of extremal p-brane solutions in supergravity has a long history and its relation to a putative open/closed string duality has been widely anticipated. Sometimes this relation is referred to in the literature as the supergravity/DBI correspondence. From this perspective two of the main new contributions in this paper are the following.

(i) We propose that the supergravity/DBI correspondence can be made into an algorithmic map within the general formalism of blackfolds in supergravity (extending the proposal of our recent work [25]). In the present paper we provide an important part of this map: the explicit relation between the fluid dynamical variables of the gravitational long-wavelength description and the gauge-theoretic degrees of freedom of the open string description, and the precise relation between the equations of motion that both degrees of freedom obey at extremality. To the best of our knowledge, the details of this relation have not been exhibited before. In fact, the key role that long-wavelength expansions in supergravity play in this connection has not been appreciated. Previous investigations have focused, almost unanimously, on exact (mostly supersymmetric)

supergravity solutions, where the connection with Abelian DBI (see Refs. [26–28] for some examples) and DBI-related structures (e.g. calibrations [29]) has been noticed more on the level of observation and less on the level of a systematic exploration.

As we pointed out in [25] there is a related old approach (first applied to string theory in [30]) that identifies the Abelian part of the brane degrees of freedom in supergravity as collective coordinates associated with large gauge transformations (for a review see [31]). A notable improvement of the blackfold approach is that it encodes rather easily the full nonlinear nature of the DBI action, which is hard to achieve with the techniques of [30].

(ii) The AdS/CFT correspondence is embedded naturally in the big picture that we postulate by taking the standard low-energy/near-horizon limit of Maldacena [32]. Several authors in the past have pointed out the importance of the singleton degrees of freedom for physics outside the near-horizon throat. We reemphasize the key role played by singleton degrees of freedom and point out that the Abelian effective actions that we discuss are singleton effective actions embedded naturally within the full non-Abelian Wilsonian effective action of a dual open string theory. The discussion at this point is closely related to the pre-AdS/CFT considerations of Ref. [33]. We sketch how non-Abelian effects can be incorporated into the singleton descriptions by integrating out interactions between Abelian and non-Abelian degrees of freedom.

Finally, the emergence of hydrodynamics in the above story is interesting for independent reasons. For example, there has been renewed interest in recent investigations, e.g. [34–36], in potential reformulations of fluid dynamics in terms of a Lagrangian variational principle. Our results provide an explicit illustration of an extremal hydrodynamic system where the passage to an action principle is facilitated by a convenient change of variables to a new set of degrees of freedom. The latter are clearly the degrees of freedom favored in the Wilsonian effective description of the underlying (open string) microscopics.

The interplay between hydrodynamics and open string theory holds the promise of interesting lessons about both frameworks. For example, from the gravitationally derived hydrodynamics we learn about various deformations of the Abelian DBI action induced by large-N non-Abelian effects. Moreover, the exact gravitational solutions provide an efficient resummation of all the DBI higher-derivative corrections. Conversely, by studying higher-derivative corrections of the DBI action in open string theory we can make predictions using open/closed string duality about higher-derivative corrections of hydrodynamics that have not yet been computed in supergravity. A preliminary study of this aspect appears in Sec. VIII.

II. SEN'S OPEN STRING COMPLETENESS REVISITED

Before we go into the specifics of our proposal, let us briefly recall a closely related circle of ideas about open/closed string duality put forward by Sen in Ref. [1] in the context of unstable D-branes in string theory.

Unstable D-branes exhibit a rapid time-dependent decay into closed strings [37–39]. During this process closed strings with typical energies of order $1/g_s$ are copiously produced until the end of the process where the D-brane (and the open strings on it) completely disappear. Yet, it was observed at tree level (in the limit where $g_s \rightarrow 0$) that the open string description of this process in terms of a rolling tachyon manages to reproduce correctly many of the features of this process *at all times* without the need to include explicit open/closed string couplings. A review of this evidence can be found in [2]. This led Sen to conjecture in Ref. [1] that there is a quantum open string field theory (OSFT) that describes the full dynamics of the unstable D-brane without an explicit coupling to closed strings.

This statement is consistent with independent studies of open string field theory demonstrating formally that the perturbative expansion of OSFT around the maximum of the tachyon potential is complete [40–42], and that open string amplitudes have the correct poles associated with intermediate closed string states [43–45] (see also [46–48]). The validity of the above conjecture was further tested by Sen in the context of unstable D0-branes in two-dimensional noncritical string theory using the correspondence with double-scaled matrix models.

The assumption that the OSFT on a D-brane setup is a self-consistent quantum mechanical system implies that OSFT contains complete information about the closed strings produced by the D-brane, and therefore suggests that states in closed string theory can be described holographically in a nongravitational language. This does *not* imply that a given OSFT can describe any closed string state. It can only describe those states produced by the decaying D-brane. This introduces an interesting way to think about closed string theory and gravitational dynamics, where we split closed string solutions into separate, quantum mechanically self-consistent, superselection sectors. In this manner, Sen's open/closed string duality works as *tomography*, where different OSFTs slice through different subsectors of the vast configuration space of gravity and closed strings.

There are several apparent differences between this version of open/closed string duality and the more familiar gauge/gravity dualities in the AdS/CFT correspondence. For instance, in the example of Sen there is no large-N limit, and closed strings are produced via a time-dependent process. The correspondence is expected to work even for a single unstable D-brane with an Abelian OSFT. Instead, in the large-N limit of standard AdS/CFT examples closed strings are produced by heavy, typically static and stable, D-brane configurations. Finally, in AdS/CFT there is a

clear holographic screen (boundary) where the dual non-gravitational theory is naturally envisioned. No such screen is visible in Sen's proposal.

In the next section, we conjecture a general framework based on Sen's ideas that attempts to bridge the gap between these apparent disparities.

III. OPEN/CLOSED STRING DUALITY AND FLAT SPACE HOLOGRAPHY AS A SPECIAL CASE

A universally expected feature of any type of duality between two theories, **A** and **B** (including holographic dualities), is an equality between their respective effective actions,

$$\mathcal{S}_{\mathbf{A}}[\Phi_{\mathbf{A}}, \mathcal{J}_{\mathbf{A}}] = \mathcal{S}_{\mathbf{B}}[\Phi_{\mathbf{B}}, \mathcal{J}_{\mathbf{B}}], \tag{1}$$

under a specific map between the collection of vacuum expectation values $\Phi_{\mathbf{A}}$ and $\Phi_{\mathbf{B}}$ that label the vacuum state on both sides and the generic sources $\mathcal{J}_{\mathbf{A}}$ and $\mathcal{J}_{\mathbf{B}}$ that represent deformations of the two theories. In quantum mechanical systems with standard Lagrangian formulations the (Wilsonian) effective action is defined formally by a path integral over field configurations ϕ of the form

$$\mathcal{S} = -\log Z = -\log \left(\int [d\phi] e^{-\int \mathcal{L}[\phi]} \right). \tag{2}$$

For example, in the supergravity regime of the standard AdS/CFT correspondence theory **A** is a large-N quantum gauge theory, and theory **B** is a classical supergravity theory in asymptotically AdS spacetimes. The quantities $\Phi_{\mathbf{A}}$ are vacuum expectation values (vev's) of gauge-invariant operators $\mathcal{O}_{\mathbf{A}}$, i.e. $\Phi_{\mathbf{A}} = \langle \mathcal{O}_{\mathbf{A}} \rangle$, and $\mathcal{J}_{\mathbf{A}}$'s are external sources for the same operators. The quantities $\Phi_{\mathbf{B}}$ label the classical profiles of the supergravity fields in gravitational solutions with specified asymptotics $\mathcal{J}_{\mathbf{B}}$. The standard holographic dictionary in AdS/CFT explains how one translates the pairs $(\Phi_{\mathbf{A}}, \mathcal{J}_{\mathbf{A}})$ to the pairs $(\Phi_{\mathbf{B}}, \mathcal{J}_{\mathbf{B}})$.

A. The proposed conjecture

We would like to conjecture a more general holographic duality with the following ingredients. Theory **A** is a (non-Abelian) open string field theory on a stack of D-branes embedded in a specified closed string background Ψ.[1] Theory **B** is a closed string field theory[2] that has Ψ as a

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[1]The specification of a closed string background in the form of a given two-dimensional world sheet conformal field theory is a standard basic ingredient in all known formulations of open string field theory including Witten's cubic open string field theory [49] and subsequent extensions.

[2]A workable formulation of closed string field theory is a notoriously hard technical problem, see [50] for a review of early attempts. Here, we refer to closed string field theory as a putative quantum mechanically consistent framework for closed strings, which we assume to exist beyond the standard first-quantized perturbative formulations.

vacuum state and theory **A** as one of its allowed open string sectors. We add the following requirement: the D-brane configuration should guarantee the existence of an asymptotic region where the D-brane backreaction on closed strings is negligible and the closed string fields asymptote to Ψ. In general, this requirement puts constraints on the codimension of the D-brane configuration, and introduces two standard features of holography: a holographic radial direction, and an asymptotic region that does not change under normalizable deformations.

In this context $\Phi_\mathbf{A}$'s are gauge-invariant vev's of open string fields and $\mathcal{J}_\mathbf{A}$'s are the open/closed string couplings induced by placing the open string theory on the background Ψ. On the other hand, $\Phi_\mathbf{B}$'s label the profiles of closed string field vev's in states with asymptotics defined by Ψ, and $\mathcal{J}_\mathbf{B}$ coincides with the asymptotic closed string state Ψ.

As a concrete example, let us consider the open string theory that resides on a D-brane setup in flat space in perturbative ten-dimensional type II string theory (e.g. open string theory on a stack of D3-branes in flat space). In open/closed string theory we can compute bulk-boundary couplings by evaluating the two-point functions of one open string going to one closed string. From the world sheet point of view, at leading order in g_s, this involves the disk two-point function $\langle \mathcal{V}_{\text{open}} \mathcal{V}_{\text{closed}} \rangle_{\text{disk}}$, where $\mathcal{V}_{\text{open}}$ is a vertex operator inserted at the boundary of the disk and $\mathcal{V}_{\text{closed}}$ is a vertex operator inserted at the center of the disk. Hence, in a small deformation of flat space with nonvanishing profile $\mathcal{J}_\mathbf{B}$ of the closed string fields these couplings dictate uniquely how open string fields are sourced, i.e. they determine the external couplings that we called collectively $\mathcal{J}_\mathbf{A}$.

Consequently, given a collection $\mathcal{J}_\mathbf{B}$ we can, in principle, compute independently two separate quantities: (i) the quantum effective action $\mathcal{S}_\mathbf{A}$ of the open string theory (without explicit coupling to closed strings) in the presence of the sources $\mathcal{J}_\mathbf{A}$ (deduced from the profile of $\mathcal{J}_\mathbf{B}$), in an open string vacuum labeled by $\Phi_\mathbf{A}$, and (ii) the quantum effective action $\mathcal{S}_\mathbf{B}$ of closed string theory (without any coupling to open strings) in a vacuum where the closed string fields asymptote to $\mathcal{J}_\mathbf{B}$. We conjecture that there is a one-to-one map between the open and closed string vacua in points (i) and (ii) such that $\mathcal{S}_\mathbf{A}$ and $\mathcal{S}_\mathbf{B}$ are identical.

In the following subsections we will explore the validity of the above conjecture in a technically convenient large-N, long-wavelength regime, where the description on the closed string side reduces to standard classical supergravity, but the open string side is stringy and we can keep track of explicit open string effects at all orders in α'.

B. Quantum open string effective actions

On the open string side we consider the Wilsonian effective action $\mathcal{S} \equiv \mathcal{S}_\mathbf{A}[\Phi_\mathbf{A}, \mathcal{J}_\mathbf{A}]$ of a non-Abelian open string theory on a stack of D-branes. For concreteness and

simplicity of the presentation we will concentrate on the special case of N coincident Dp-branes ($p < 6$) in flat space in perturbative ten-dimensional type II string theory. The dynamics of open strings in such setups is described by a non-Abelian open string field theory where the open string fields are fields in the adjoint representation of $U(N)$. At low energies this theory reduces to a $U(N)$ supersymmetric gauge theory. Besides the rank of the gauge group N, the other free parameter that characterizes the open string theory in this setup is the string coupling g_s, which is part of the sources $\mathcal{J}_\mathbf{A}$ (determined by the closed string background).

For reasons that are clear already in standard AdS/CFT discussions (and will be repeated in the next subsection) it is convenient to consider the large-N 't Hooft limit where $N \gg 1$, $g_s \ll 1$ and $\lambda = g_s N$ is a fixed tunable parameter.

The $U(N)$ open string field theory on the D-branes is a complicated quantum mechanical system with an infinite tower of interacting open string modes. The Wilsonian effective action in a generic open string vacuum is computed formally from a complicated path integral in string field theory of the general abstract form (2). In a weak 't Hooft coupling expansion (valid when $\lambda \ll 1$) this action, which is a function of the open string field vev's $\Phi_\mathbf{A}$, is expressed by a series of the form

$$
\begin{aligned}
\mathcal{S}[\Phi_\mathbf{A}; \lambda, N] &= \sum_{n=0}^{\infty} \lambda^{-1+n} \mathcal{S}_n[\Phi_\mathbf{A}; N] \\
&= \sum_{g=0,h=1}^{\infty} N^h g_s^{2g-2+h} \mathcal{S}_{g,h}[\Phi_\mathbf{A}] \\
&= \sum_{g=0,h=1}^{\infty} N^{2-2g} \lambda^{2g-2+h} \mathcal{S}_{g,h}[\Phi_\mathbf{A}].
\end{aligned}
\tag{3}
$$

From the world sheet point of view $\mathcal{S}_{g,h}[\Phi_\mathbf{A}; N]$ is an off-shell partition function on a Riemann surface with g handles and h holes [51]. At low energies the corrections in this expansion arise from perturbative loop diagrams in quantum gauge theory. Nonplanar diagrams contribute terms with subleading $1/N$ dependence in the large-N limit.

In the opposite regime, at strong 't Hooft coupling, we expect that the more appropriate description of the effective action is not in terms of the "elementary" open string vev's $\Phi_\mathbf{A}$, but in terms of vev's of gauge-invariant composites, let us call them $\tilde{\Phi}_\mathbf{A}$. Accordingly, we expect a different expansion in inverse powers of λ of the form

$$
\mathcal{S}[\tilde{\Phi}_\mathbf{A}; \lambda, N] = \sum_{n=0}^{\infty} \lambda^{\alpha-n} \tilde{\mathcal{S}}_n[\tilde{\Phi}_\mathbf{A}; N],
\tag{4}
$$

where α is an appropriate constant. Soon we will suggest that the value of α is -1.

In an effort to be more concrete about the evaluation of these expansions, let us concentrate for starters on the first term, $\mathcal{S}_0$, of the weak coupling expansion. This term is essentially classical, and as we mentioned, it can be computed in string perturbation theory from the disk partition function ($g = 0, h = 1$). Although the exact general result of this computation is not known, in the past it has been extremely useful to think about $\mathcal{S}_0$ in a long-wavelength expansion of the vev's $\Phi_\mathbf{A}$ around a suitably symmetric vacuum.

In this approach an open string vacuum is characterized by the vev's of the low-lying massless open string fields, which include among other things the non-Abelian gauge field A_a and transverse scalars $X^\perp$. The indices $a = 0, 1, …, p$ are D-brane worldvolume indices and $\perp$ summarizes collectively the background spacetime indices perpendicular to the brane. The massive open string modes are integrated out and their quantum effects are incorporated in higher-order interactions between the massless fields.

The vacuum around which the long-wavelength expansion is set up must be an exact open string vacuum. In the Abelian case it is known that the vacua with arbitrary constant transverse velocities $\partial_a X^\perp$ and constant gauge field strength F_{ab} are such vacua. The long-wavelength expansion is an expansion in derivatives of these quantities[3]

$$\mathcal{S}_0[A_a, X^\perp; N] = S_0[F_{ab}, \partial_a X^\perp] + S_{\text{higher-order}}[\partial^n F, \partial^m X].$$

(5)

The leading term S_0 on the rhs is exactly on the field strength F and the velocities $\partial X^\perp$ at all orders in α' and $S_{\text{higher-order}}$ is a perturbative expansion in higher derivatives of F and $\partial X^\perp$. A well-defined computation in open string theory determines S_0 as the Abelian DBI action (see [52] for a review and a list of original references)

$$S_{\text{DBI}} = \mathcal{T}_p \int d^{p+1}x \sqrt{-\det(\eta_{\mu\nu}\partial_a X^\mu \partial_b X^\nu + 2\pi\alpha' F_{ab})},$$

(6)

where $\mathcal{T}_p = \frac{1}{g_s\sqrt{\alpha'}}\frac{1}{(2\pi\sqrt{\alpha'})^p}$ is the Dp-brane tension, and $\mu, \nu = 0, 1, …, 9$ are indices for the flat spacetime background with Minkowski metric $\eta_{\mu\nu}$.

The non-Abelian case is richer and comparatively less understood. It is natural to look for a non-Abelian extension of the above expansion as an expansion in small gauge-covariant derivatives, D_a, of the velocities $D_a X^\perp$ and gauge field strength F_{ab}:

$$\mathcal{S}_0[A_a, X^\perp; N] = S_0[F_{ab}, D_a X^\perp] + S_{\text{higher-order}}[D^n F, D^m X].$$

(7)

However, it is already apparent from the identity

$$[D_a, D_b]F_{cd} = [F_{ab}, F_{cd}]$$

(8)

that the leading term S_0 is now ambiguous. Moreover, because of (8) an expansion in small covariant derivatives will be also an expansion around commuting field strengths. Isolating the symmetric covariant derivatives in $S_{\text{higher-order}}$ and using (8) S_0 comprises two pieces—one with F commutators and one without.[4] Reference [53] proposed that the part of S_0 without F commutators is a non-Abelian version of the DBI action (6) with a symmetric trace prescription. For a nice summary of the problems and progress on the non-Abelian DBI action we refer the reader to [54].

Despite the above-mentioned technical difficulties, there are a few robust features expected from the putative non-Abelian S_0, for gauge group $U(N)$, that are useful to highlight for later purposes.

When expanded in a series of powers in F and $DX^\perp$, the leading term in S_0 is quadratic. In the quadratic interactions the vector $U(1)$ and $SU(N)$ parts completely decouple. The $SU(N)$ part includes the $(p + 1)$-dimensional non-Abelian super-Yang-Mills action. At higher orders in this power series the $U(1)$ and $SU(N)$ degrees of freedom are coupled by higher-dimension interactions. From this point of view we can write S_0 schematically as a sum of three terms,

$$S_0 = S_{0,U(1)}[\Phi] + S_{0,SU(N)}[\mathbf{\Phi}] + S_{0,\text{mixed}}[\Phi, \mathbf{\Phi}], \quad (9)$$

where we have denoted compactly the Abelian degrees of freedom by (a normal font) Φ and their non-Abelian counterparts by (a bold font) $\mathbf{\Phi}$. Setting $\mathbf{\Phi} = 0$ [by which we mean explicitly $F = 0$, $DX^\perp = 0$ for the $SU(N)$ vev's] we deduce that $S_{0,U(1)}$ is simply N times the Abelian DBI action (6):

$$S_{0,U(1)} = N S_{\text{abelian DBI}}.$$

(10)

$S_{0,\text{mixed}}$ involves interactions at cubic and higher order between Φ and $\mathbf{\Phi}$.

Varying separately with respect to Φ and $\mathbf{\Phi}$ to determine the Abelian and non-Abelian vacuum expectation values, we find two coupled sets of equations,

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[3]$\mathcal{S}_0$ also includes terms for the action of the superpartner fermions that will be kept implicit in the following discussion.

[4]Terms that involve the covariant velocities $D_a X^\perp$ can be decided by T-duality from the ten-dimensional open string effective action.

$$\frac{\delta S_{0,U(1)}}{\delta \Phi}[\Phi] + \frac{\delta S_{0,\text{mixed}}}{\delta \Phi}[\Phi, \boldsymbol{\Phi}] = 0,$$

$$\frac{\delta S_{0,SU(N)}}{\delta \boldsymbol{\Phi}}[\boldsymbol{\Phi}] + \frac{\delta S_{0,\text{mixed}}}{\delta \boldsymbol{\Phi}}[\Phi, \boldsymbol{\Phi}] = 0, \qquad (11)$$

where δ denotes the standard Euler-Lagrange variation. Since terms in $S_{0,\text{mixed}}$ have to be at least quadratic in the non-Abelian fields, setting $\boldsymbol{\Phi} = 0$ is a consistent *Ansatz* that satisfies automatically the second equation in (11) and leaves

$$\frac{\delta S_{0,U(1)}}{\delta \Phi}[\Phi] = 0 \qquad (12)$$

from the first equation.

Notice the following important property. In a state with a nontrivial Abelian part that solves (12), the higher-dimension interactions in $S_{0,\text{mixed}}$ are *not* infrared irrelevant. In other words, around a nontrivial Abelian vacuum the non-Abelian dynamics does not implicate only terms from $S_{0,SU(N)}$ but also terms from $S_{0,\text{mixed}}$.

We postulate that the structure (9) applies to the full effective action $\mathcal{S}$, not just the classical contribution S_0. For example, in the regime of strong 't Hooft coupling the vev's of gauge-invariant operators are separated naturally into vev's of the Abelian fields $\tilde{\Phi}$, and vev's of the non-Abelian fields $\tilde{\boldsymbol{\Phi}}$. We expect that it is possible to express the equations of motion of the strong coupling effective action (4) in a form analogous to (11), and that the trivial non-Abelian vev's $\boldsymbol{\Phi} = 0$, or $\tilde{\boldsymbol{\Phi}} = 0$, are a consistent *Ansatz* for the full effective action $\mathcal{S}$. We will refer to the vacua with trivial non-Abelian vev's as the origin of the Coulomb branch of the open string field theory.

1. Main aim of the paper

We focus first and foremost on the dynamics of the Abelian part of the effective action $\mathcal{S}$ at the origin of the Coulomb branch in the long-wavelength approximation, and compare descriptions of this sector at weak and strong 't Hooft coupling.

The first thing to notice about the Abelian sector in the long-wavelength approximation is that the Abelian vev's of F and $\partial X^{\perp}$ are automatically gauge invariant. Hence, there is potential for a direct relation between the weak coupling description of these degrees of freedom and their corresponding strong coupling description.

Restricting our attention to the origin of the Coulomb branch ($\boldsymbol{\Phi} = 0$), the small 't Hooft coupling expansion (3) becomes an expansion in terms of the Abelian vev's Φ. Naively the effective action receives corrections from world sheets with an arbitrary number of handles and holes [each one being of the order $\mathcal{O}(N^h g_s^{2g-2+h})$]. Nevertheless, we will soon find from a direct supergravity analysis valid at the strong 't Hooft coupling limit that at leading order in the

long-wavelength derivative expansion the leading term in the $1/N$ expansion of (4) is identical to the leading term in the $1/N$ expansion of the weak coupling series (3), namely

$$\mathcal{S}_0|_{\text{leading } 1/N, \text{leading derivative}, U(1)}$$
$$= \tilde{\mathcal{S}}_0|_{\text{leading } 1/N, \text{leading derivative}, U(1)}. \qquad (13)$$

This observation suggests the possibility of an even stronger relation valid at all orders of the derivative expansion

$$\mathcal{S}_0|_{\text{leading } 1/N, U(1)} = \tilde{\mathcal{S}}_0|_{\text{leading } 1/N, U(1)}. \qquad (14)$$

This equation implies a nonrenormalization theorem of the open string effective action at the origin of the Coulomb branch, where the corrections from world sheets with more than one hole ($h > 1$) are vanishing and the action (expressed in terms of N and g_s) has a trivial linear dependence on N at all values of λ. Moreover, (13) [or the stronger (14)] suggest that the value of the constant α in (4) is -1.

Currently, we are not aware of a conclusive proof of this nonrenormalization theorem. For the purposes of Eq. (14) it would be sufficient to have a proof of the cancellation of the open string corrections coming from world sheets with $h > 1$ at zero genus, $g = 0$. Although such a cancellation may sound plausible in supersymmetric configurations notice that we observe (13) for any extremal configuration irrespective of supersymmetry. Assuming the existence of such a nonrenormalization theorem, the derivation of (14) from supergravity would constitute a direct test of the open/closed string duality formulated in the beginning of this section.

We note that Eq. (13) is an integral part of the long-anticipated correspondence between solutions of the DBI action and extremal supergravity configurations, which was observed experimentally in many examples in the past. In what follows, we describe a framework where such a correspondence can be formulated in a more organized manner.

C. Closed strings: Long-wavelength expansions and effective actions in supergravity

Having discussed some of the features on the open string side of the putative open/closed string duality of Sec. III A we now pass to a corresponding discussion on the closed string side. We continue to focus on the leading order in the large-N, large 't Hooft coupling limit, where standard arguments in the context of the AdS/CFT correspondence show that the low-energy effective field theory description of closed string field theory, $\mathcal{S}_{\mathbf{B}}$, is the ten-dimensional supergravity action.

The trivial vacua of the open string theory on a stack of $N \gg 1$ Dp-branes that we want to consider (as zeroth-order

configurations in subsequent long-wavelength derivative expansions) are captured holographically on the supergravity side by extremal p-brane solutions with translation invariance in the worldvolume directions. To incorporate more features, these solutions may also involve homogeneous fluxes sourced by lower-dimensional branes smeared along the p-brane worldvolume, e.g. the p-brane solution may be an F1-Dp bound state (corresponding to a planar stack of Dp-branes with a constant worldvolume Abelian electric field turned on), or a more complicated multicharge bound state. Let us call this solution $\Phi_\mathbf{B}^{(0)}$. In this case the metric and all other supergravity fields depend nontrivially only on the radial coordinate r. Specific examples will be discussed below. In what follows we want to consider configurations away from the trivial homogeneous vacuum, and to explore if these have a chance to map holographically to solutions of the open string field theory according to the conjecture of Sec. III A.

As we mentioned near the end of the previous subsection, our primary goal is to compare the open string effective action $\mathcal{S}_\mathbf{A}$ in the long-wavelength expansion to the closed string (supergravity) effective action $\mathcal{S}_\mathbf{B}$ in a corresponding expansion. Hence, on the supergravity side it is natural to look for extremal *inhomogeneous* p-brane configurations of the form

$$\Phi_\mathbf{B} = \Phi_\mathbf{B}(x^\mu), \qquad (15)$$

expanded in small derivatives with respect to the p-brane worldvolume coordinates σ^a ($a = 0, 1, \ldots, p$),

$$\Phi_\mathbf{B} = \Phi_\mathbf{B}^{(0)}(r) + \varepsilon \Phi_\mathbf{B}^{(1)}(r, \sigma^a) + \varepsilon^2 \Phi_\mathbf{B}^{(2)}(r, \sigma^a) + \cdots. \quad (16)$$

The dummy variable ε keeps track of the number of worldvolume derivatives ∂_a. This expansion is inserted into the partial differential equations (PDEs) of supergravity which are solved perturbatively order by order.

Our open/closed string conjecture states that there is a unique on-shell map between the open string vev's $\tilde{\Phi}_\mathbf{A}$ and the closed string (supergravity) vev's $\Phi_\mathbf{B}$. Under this map the on-shell value of the open string effective action (4) (at leading order in the $1/N$ and $1/\lambda$ expansion, which is the regime of interest here) should equal the on-shell value of the supergravity action $\mathcal{S}_\mathbf{B}$ at all orders in the long-wavelength derivative expansion.

We discussed why Abelian deformations at the origin of the Coulomb branch is a computationally opportune context. What is the corresponding description of this sector on the supergravity side?

The zeroth-order supergravity profile $\Phi_\mathbf{B}^{(0)}$, which corresponds to an open string vacuum at the origin of the Coulomb branch, is labeled by a set of constants that parametrize the asymptotic charges, e.g. mass, angular momentum, brane charges. These parameters, and other parameters associated with the breaking of the global

symmetries of the asymptotic background by the p-brane solution, can be viewed as collective coordinates of the supergravity solution. A restricted class of supergravity solutions can be constructed perturbatively around $\Phi_\mathbf{B}^{(0)}$ with a supergravity *Ansatz* that promotes the collective coordinates into slowly varying functions of the worldvolume coordinates. This is a special case of the general expansion (16). It was argued long ago [33] (and also more recently in [25]) that the above collective modes are the supergravity manifestation of the massless $U(1)_\text{vector}$ degrees of freedom of the dual open string theory. Consequently, we propose that the supergravity deformations within this sector are the holographic duals of the open string configurations with vanishing non-Abelian vev's.

A systematic development of long-wavelength expansions of the type we have just described in general (super) gravity theories has been initiated in recent years in the context of the blackfold formalism starting with Refs. [22,23]. We refer the reader to the existing literature for a more detailed technical exposition of current results in this (still developing) framework. Here it will be useful to highlight some of the key conceptual features of the formalism that play a role in our general discussion.

(a) *(Super)gravity expansions.*—Perturbative supergravity solutions in the blackfold approach are constructed using the method of matched asymptotic expansions (MAEs) (see [55] for an instructive application of MAEs to caged black holes). An exact zeroth-order p-brane solution is perturbed in a *near-zone* region ($r \ll \mathcal{R}$, where $\mathcal{R}$ is the typical scale of the long-wavelength perturbation) by promoting the collective modes to slowly varying functions of the worldvolume coordinates. At the same time the supergravity fields are corrected appropriately to achieve a perturbative solution of the full set of supergravity equations of motion. The *Ansatz* in this region is performed in a very similar way conceptually to analogous constructions for AdS black branes in the fluid-gravity correspondence in AdS/CFT [24]. Simultaneously, an independent computation is performed in a Newtonian approximation far from the horizon in the *far-zone* region ($r \gg r_H$, where r_H is the typical near-horizon radius of the p-brane solution; for extremal p-brane solutions this is the charge radius that appears explicitly in Sec. VII). A matched asymptotic expansion is performed order by order by matching the near-zone and far-zone solutions in the large overlap region $r_H \ll r \ll \mathcal{R}$, whose existence is the basis of the long-wavelength expansion.

(b) *Collective mode equations.*—During this process one discovers that a part of the supergravity equations (constraint equations in the near-zone analysis) reduces to a system of $(p + 1)$-dimensional equations for the collective coordinates. These lower-dimensional equations, which we call *blackfold*, or

collective mode, equations, are expressed naturally as conservation equations for a set of currents; these include the energy-momentum tensor T_{ab}, and q-form currents $J_{a_1 \cdots a_q}$ related to the charges of the p-brane solution. In general bound state solutions there are several different values of $q \leq p$. These equations are naturally formulated as hydrodynamic equations for fluids propagating on dynamical hypersurfaces.

One of the main technical computations below will be to exhibit the precise relation between these supergravity collective mode equations and the Abelian DBI equations of motion making a part of the relation (14) manifest within the postulated open/closed string duality.

Finally, it has been conjectured [23] that solutions of the blackfold equations are in one-to-one correspondence with solutions of the full supergravity equations order by order in the long-wavelength expansion. A general proof of this conjecture (referred to as the *blackfold conjecture* below) is not available at the moment. However, a proof in special cases at first order in the derivative expansion has appeared in [56–58].

(c) *Dynamical holographic screen.*—Similar to the AdS/CFT correspondence, where the dual theory is naturally thought of as a theory residing on the asymptotic boundary of AdS, in the present case the dual open string theory is naturally thought of as a theory residing on a D-brane stack embedded in the given asymptotic background. In the main examples of this paper the asymptotic background is flat space. This feature is emerging almost automatically from gravity in the long-wavelength blackfold expansions. The currents whose conservation defines a set of lower-dimensional dynamical equations in supergravity (collective mode equations) are computed in the overlap region, which can be viewed as the asymptote of the near-zone region deep inside the asymptotically flat far-zone region. Hence, in the long-wavelength approximation we discover naturally within supergravity the emergence of a dynamical lower-dimensional holographic screen embedded in the asymptotic background.

(d) *Higher-derivative corrections.*—The order-by-order solution of the gravitational PDEs results in a perturbative higher-derivative modification of the blackfold equations in a fixed background. In extremal setups we postulate that such derivative corrections are in direct correspondence with the derivative corrections to the Abelian DBI action which can be computed from standard calculations on the disk world sheet of the dual open string theory. This postulate assumes the validity of the nonrenormalization relation (14). A preliminary discussion of this correspondence appears in Sec. VIII below.

It is also interesting to note in this context that we can interpret an exact inhomogeneous p-brane solution in gravity (with flat space asymptotics) as the dual of a nonperturbative resummation of the derivative expansion of the open string effective action including all stringy effects in the leading order in the $1/N$ and $1/\lambda$ expansions.

(e) *Low-energy/near-horizon limits.*—The low-energy (small field-strength) limit on the open string side corresponds to the near-horizon limit of the (deformed) supergravity solutions. As we discussed in Sec. III B, around the trivial undeformed vacuum the low-energy limit on the open string side results in a decoupling of the $SU(N)$ and $U(1)$ sectors. The $SU(N)$ part is strongly interacting and has a dual AdS/CFT description in terms of gravity in the near-horizon region of the homogeneous zeroth-order supergravity solution. The $U(1)$ part also has a well-known description in the near-horizon limit. It corresponds to singleton degrees of freedom that are topological in the bulk and are fully supported on the boundary. From this point of view, it is natural to think of the blackfold effective field theory that we formulate as a singleton effective field theory [25,33].

In accordance with the discussion in Sec. III B, the near-horizon limit around a *deformed solution* does not have to lead to a complete infrared decoupling of the singleton degrees of freedom. In fact, in some cases a deformed brane solution may not even have a single near-horizon limit as BIon-type solutions in supergravity exhibit [27,59,60].

(f) *Non-Abelian effects.*—The holographic encoding of the strongly interacting non-Abelian degrees of freedom of the open string theory is admittedly one of the most interesting aspects of the proposed open/closed string duality. One expects that the full non-Abelian physics is captured in the bulk by the most general asymptotically flat p-brane solution. In this context the virtues of the near-horizon limit are well known and much studied in the context of the AdS/CFT correspondence. In this paper we work outside the near-horizon limit and we have chosen to focus on the Abelian part around a trivial non-Abelian vacuum, which provides a comparatively more tractable situation. Yet, we can easily imagine more complicated cases where non-Abelian effects play a pronounced role.

From the point of view of the long-wavelength expansions, we can imagine a zeroth-order supergravity solution that captures holographically a vacuum with nontrivial non-Abelian properties. It is possible to extract interesting information about non-Abelian physics by studying the Abelian sector around this more general vacuum with a suitable

modification of the above-mentioned long-wavelength expansions in supergravity.

One example would be to consider an extremal zeroth-order solution with a nonvanishing non-Abelian condensate. The analysis of Sec. III B suggests that the supergravity *Ansatz* is now more complicated with additional degrees of freedom and additional interactions. Indeed, as an illustration, deformations of multicenter solutions do exhibit these features. It would be very interesting to analyze such examples in more detail, and to attempt to probe further our open/closed string conjecture in this direction. Classifying zeroth-order solutions by their near-horizon AdS/CFT description might prove to be a useful approach.

Another example involves p-brane solutions at finite temperature. Around such vacua the Abelian open string effective theory incorporates thermal non-Abelian effects and is no longer the standard DBI action. Typically, it is hard to compute these effects explicitly on the open string side. On the gravitational side, however, we are instructed to repeat the blackfold derivative expansion around a finite-temperature zeroth-order solution. The blackfold equations provide a relatively easy way to compute the corresponding modifications of the DBI equations of motion. For instance, finite-temperature modifications of the DBI action have been considered in this way in [61–64]. In the case of finite-temperature configurations at the trivial Abelian vacuum the low-energy/near-horizon limit of the resulting effective theory is expected to reduce to the fluid dynamical effective description of non-Abelian dynamics that is familiar from the fluid-gravity correspondence. A related illustration of this statement appeared in [65].

D. Open/closed string duality in a long-wavelength regime

Closing this section it is useful to summarize some of the most prominent features of the above discussion highlighting some obvious parallels with the discussion of open string completeness by Sen. We continue to focus on the long-wavelength regime of the proposed open/closed string duality and the Abelian sector at the origin of the Coulomb branch.

In the long-wavelength regime we are postulating a picture where the blackfold equations act as a useful mediator between gravity and open string theory. This occurs in the way schematically summarized in Fig. 1. Within the blackfold expansion scheme a part of the supergravity equations reduces to a set of lower-dimensional equations of motion for the collective modes of the supergravity solution, which are naturally formulated as hydrodynamic equations. We will show explicitly in the ensuing sections (for extremal configurations) that these equations can be reformulated as equations of motion of a recognizable
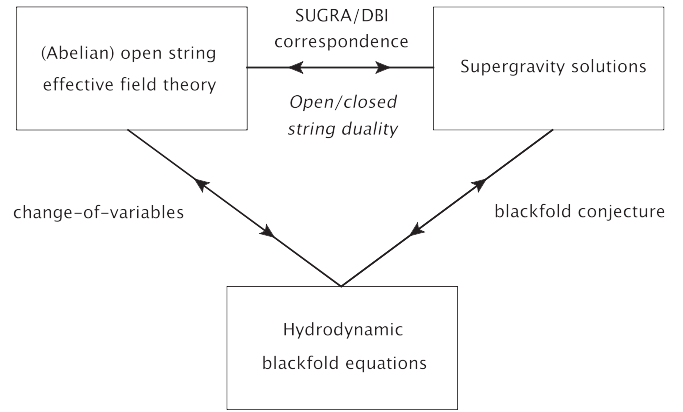

FIG. 1. A diagrammatic depiction of the main components of our open/closed string conjecture in a long-wavelength derivative expansion.

dual open string effective action. This direct link between gravity and open string theory is represented by the left arrow, labeled *change of variables*, in Fig. 1.

The second ingredient of the construction (the right arrow in Fig. 1) is the conjecture mentioned above, which we dub the blackfold conjecture, stating that solutions of the blackfold equations are in one-to-one correspondence with a class of regular p-brane solutions.

Combining these two ingredients we obtain a manifest long-wavelength realization of the proposed open/closed string duality: solutions of the Abelian action are in one-to-one correspondence with brane supergravity solutions. In this form, we set the stage for an explicit algorithmic formulation of the long-anticipated supergravity/DBI correspondence, which has been implicit in many previous investigations of brane solutions in supergravity.

In the beginning we argued that it is natural to view this construction as a large-N manifestation of Sen's open string completeness. As an obvious similarity to the examples analyzed by Sen [2], we note that the Abelian open string quantum effective action that we consider in the context of blackfolds is, as in [2], a natural description of configurations at energies of the order of the D-brane tension, namely energies of the order $\mathcal{O}(1/g_s)$. The production of closed strings is large and the gravitational solution is deformed, but we postulate that the open string description is in itself self-consistent and there is no need to consider a coupled system of open and closed strings. In the blackfold derivative expansion this statement is related to the claim that constraint equations can be phrased as equations for the collective modes in a fixed supergravity background at all orders in the perturbative expansion.

More generally, in complete analogy with Sen's proposal, we do not attempt to set up a holographic relation in terms of a universal boundary theory that captures all possible gravity (closed string theory) configurations in the bulk, but rather we set up holography as a tomographic principle that works in superselection sectors. Different

open string theories capture holographically different sub-sectors in closed string configuration space. An open string theory can only capture those closed string configurations that are sourced by the D-brane setup on which the open string theory resides.

This picture has interesting implications for closed string theory and gravity as quantum mechanical theories. A viewpoint that pronounces the role of open strings suggests that we should separate closed string solutions into different quantum mechanically consistent subsectors, each one with its own nonperturbative microscopic definition in terms of a dual open string theory. On the other hand, a viewpoint centered around a putative single quantum mechanically consistent formulation of closed strings and gravity would suggest that we should view closed string theory as an overarching theory of theories for diverse quantum open string theories (and related quantum field theories) in different subsectors.

IV. RELATIVISTIC FLUIDS AS A LINK BETWEEN OPEN AND CLOSED STRINGS

Our next task is to exhibit the direct relation between the blackfold equations (derived from gravity as collective mode equations) and the Abelian Dirac-Born-Infeld action, making explicit the change-of-variables link depicted in Fig. 1. We focus on extremal configurations.

We noted already that the blackfold equations are naturally formulated as conservation equations of a set of currents which are functionals of the collective modes. With specific constitutive relations provided by the thermodynamics of the zeroth-order solution, these equations are automatically formulated within gravity as hydrodynamic equations for a fluid that propagates on an elastic medium [23]. In generic situations of multicharge p-branes this is an anisotropic fluid parametrized by several conserved charges, equivalently chemical potentials [66]. As we will verify soon in explicit examples, the resulting hydrodynamic description is nontrivial even at zero temperature if there are nonvanishing chemical potentials.

The passage from hydrodynamics to the DBI theory, described by the change-of-variables arrow in Fig. 1, is also interesting for another reason. The long-standing problem of Lagrangian reformulations of hydrodynamic systems has been revisited recently in several works with promising results, e.g. [34–36]. Our analysis provides a different example of such a reformulation. At leading order in the derivative expansion, where we encounter ideal relativistic fluids, we discover a reformulation of hydrodynamics in terms of a gauge theory with a generalized BF-type interaction. At higher orders in the derivative expansion the expected connection to open string theory predicts hydrodynamics with specific higher-derivative corrections. In the examples that we consider, we find (see Sec. VIII below) that these corrections are different in superstring theory compared to the bosonic string, and do not have any

second-order dissipative terms (as one might have anticipated from extremal systems).

V. STANDARD IDEAL RELATIVISTIC FLUIDS

Before delving into the details of the hydrodynamic systems that arise in (super)gravity expansions it will be useful first to set useful notation and quickly remind the reader of some pertinent facts from the theory of ideal relativistic fluids. This topic is very familiar (for a review we refer the reader to [67–69]), and does not warrant a special introduction. We slightly generalize the typical setup and consider $(p + 1)$-dimensional ideal fluids on a *dynamical* hypersurface propagating in an ambient $(d + 1)$-dimensional spacetime. We will denote the metric of the ambient spacetime by $g_{\mu\nu}$ $(\mu, \nu = 0, 1..., d)$, and the induced metric on the fluid hypersurface by $\gamma_{ab} = g_{\mu\nu}\partial_a X^\mu \partial_b X^\nu$ $(a, b = 0, 1, ..., p)$.[5] X^μ are the embedding scalars of the $(p + 1)$-dimensional hypersurface inside the $(d + 1)$-dimensional spacetime. To recover the fluid on a fixed background geometry we can simply freeze the dynamics of these scalars.

It is well known that the equations of motion of an *irrotational*[6] relativistic ideal fluid can be formulated as the Euler-Lagrange equations of the action

$$S = \int d^{p+1}x \sqrt{-\gamma}[J^a\partial_a\theta + f(\sqrt{-J^aJ_a}) + b_a(J^a - \rho u^a)$$
$$+ \lambda(u^a u_a + 1)]. \quad (17)$$

The fields b_a, λ are Lagrange multipliers enforcing the standard relation between the fluid current J^a, the fluid density ρ and the unit timelike fluid velocity vector u^a. The equations of motion of ρ and u^a set the on-shell values of the Lagrange multipliers $\lambda = 0$, $b_a = 0$, and the function f is an arbitrary function that controls the precise equation of state of the fluid (in a manner to be specified momentarily).

There are three remaining equations of motion. The first one comes from the variation of the current J^a,

$$\partial_a\theta = \frac{J_a}{\sqrt{-J^2}}f'(\sqrt{-J^2}), \quad (18)$$

where f' denotes the derivative of f with respect to its argument and $J^2 = J^a J_a$. Indices are lowered and raised with the use of the induced metric γ_{ab}. In differential form language this equation implies

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[5]Throughout the paper we will use small greek letters $\mu, \nu, ...$ for the ambient spacetime indices, and small Latin letters $a, b, ...$ for indices of the fluid hypersurface. The determinant of the induced metric $\det(\gamma_{ab})$ will be denoted as γ.

[6]The extention beyond irrotational fluids is also known and requires replacing $\partial_a\theta$ with $\partial_a\theta + \alpha\partial_a\beta$ in the action given here, where α and β are extra fields. We will focus on the irrotational case that is most relevant for our purposes below.

$$d\left(\frac{f'(\sqrt{-J^2})}{\sqrt{-J^2}}J\right) = 0. \tag{19}$$

d is the exterior derivative and J the current 1-form.

The second equation of motion that follows from the variation of θ provides the conservation equation of the current

$$D_a J^a = 0, \qquad \text{equivalently} \quad d*J = 0. \tag{20}$$

D_a is the covariant derivative with respect to γ_{ab}. $*$ denotes the $(p + 1)$-dimensional Hodge dual of a differential form with respect to the same metric.

Finally, we can vary the embedding scalars X^μ that express the induced metric. The resulting equations [23] can be massaged into the form (from Carter [70])

$$K_{ab}{}^{\hat{i}}T^{ab} = 0, \tag{21}$$

where T^{ab} is the energy-momentum tensor of the fluid and $K_{ab}{}^{\hat{i}}$ is the extrinsic curvature tensor. The latter is expressed in terms of the second derivatives of the embedding scalars (for explicit formulas see for example [23]), and $\hat{i}$ is a spacetime index along directions perpendicular to the fluid hypersurface.

The energy-momentum tensor of the fluid is [after the use of the $(\lambda, \rho, u^a, b_a)$ equations of motion]

$$T^{ab} = \frac{2}{\sqrt{-\gamma}}\frac{\delta S}{\delta \gamma_{ab}} = (\varepsilon + P)u^a u^b + P\gamma^{ab} \tag{22}$$

with energy density

$$\varepsilon = f(\rho) \tag{23}$$

and pressure

$$P = \rho f'(\rho) - f(\rho). \tag{24}$$

We can see that the arbitrary, but given from the start, function f controls the equation of state of the fluid. Equations (23) and (24) constitute the standard equation of state of relativistic fluids that guarantees constant specific entropy [71]. It is straightforward to verify that the equations of motion (19) and (20) imply the conservation of the energy-momentum tensor T^{ab}.

Equivalently, it is common to summarize the full set of fluid equations as the conservation equations,[7]

$$D_a T^{ab} = 0, \qquad K_{ab}{}^{\hat{i}}T^{ab} = 0. \tag{25}$$

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[7]The elastic equation $K_{ab}{}^{\hat{i}}T^{ab} = 0$ can also be formulated as energy-momentum conservation with an index in directions transverse to the fluid hypersurface [70].

Since we are considering irrotational flow we need to supplement these equations with the irrotational flow condition (19)

$$d\left(\frac{f'(\rho)}{\rho}J\right) = 0, \tag{26}$$

which is only partially encoded by the energy-momentum conservation condition.

VI. DUALITY IN $2 + 1$ DIMENSIONS AS A MAP FROM FLUID DYNAMICS TO GAUGE THEORY

Actions of the form (17) have a close relation to classical actions of Abelian gauge theories. This relation is most straightforward in $2 + 1$ dimensions. In that case we can dualize the current J^a into a 2-form F_{ab}

$$F_{ab} = \sqrt{-\gamma}\varepsilon_{abc}J^c, \qquad \text{or equivalently} \quad F = *J, \tag{27}$$

where ε_{abc} denotes the Levi-Civita antisymmetric symbol ($\varepsilon_{012} = 1$). Then, we notice that the current conservation equation (20) translates into the Bianchi identity

$$dF = 0 \tag{28}$$

and for that reason we can reinterpret F as the field strength of an Abelian $(2 + 1)$-dimensional gauge field A, namely $F = dA$.

With these specifications we can reformulate the ideal relativistic fluid (17) in terms of a modified Abelian Yang-Mills action on a dynamical membrane

$$S = \int d^{2+1}x\sqrt{-\gamma}\left[f\left(\sqrt{\frac{1}{2}F^{ab}F_{ab}}\right)\right.$$
$$\left. + B^{ab}(F_{ab} - \rho\varepsilon_{abc}u^c) + \lambda(u^a u_a + 1)\right]. \tag{29}$$

Having assumed (27) and (28), the term $J^a\partial_a\theta$ is now a total derivative and up to surface terms it does not contribute. B^{ab} is the Hodge dual of the Lagrange multiplier b_a, i.e. $b = *B$.

The Euler-Lagrange equations of the action (29) provide an alternative derivation of the ideal fluid equations of the previous section (in $2 + 1$ dimensions). In particular, the gauge field equations reproduce Eq. (19), which was closely tied to the irrotational nature of the fluid.

The reader should appreciate that because of the Lagrange multiplier B^{ab} our system is not simply an Abelian gauge theory; it is an Abelian gauge theory with a specific magnetic *Ansatz* for the gauge field

$$F_{ab} = \rho\varepsilon_{abc}u^c. \tag{30}$$

Because of the BF coupling we can also view (29) as a generalized BF-type gauge theory.

It is probable that this simple gauge theory reformulation of ideal relativistic hydrodynamics in $2 + 1$ dimensions has been noticed before, but I am not aware of an explicit presentation of this observation in the literature. Clearly, the duality with a gauge field is specific to $2 + 1$ dimensions and would not work in exactly the same manner in other dimensions. Nevertheless, we will soon see that there are other types of fluids that have a close connection with Abelian gauge theory in arbitrary spacetime dimensions.

A. Special case I: Maxwell theory

In the special case where $f(\rho) = \frac{1}{8}\rho^2$, we obtain the ordinary Maxwell theory with a magnetic *Ansatz*. In the Maxwell case the energy density and pressure are both positive and equal, $\varepsilon = P = \frac{1}{8}\rho^2$.

B. Special case II: Dirac-Born-Infeld theory

Another interesting case with a different equation of state,

$$\varepsilon = -\frac{c^2}{P}, \tag{31}$$

where c is an arbitrary constant and the pressure P is negative, can be obtained by setting

$$f(\rho) = \sqrt{c^2 + \rho^2}. \tag{32}$$

Because of the identity

$$\frac{1}{c}\det\left(c\delta^a_b + F^a{}_b\right) = c^2 + \frac{1}{2}F^{ab}F_{ab} \tag{33}$$

in three dimensions, one can easily show that the fluid equations of motion in the case at hand can be written equivalently as the Euler-Lagrange equations of the Dirac-Born-Infeld action

$$S = \int d^{2+1}x\sqrt{-\gamma}\left[c\sqrt{\det\left(\delta^a{}_b + \frac{1}{c}F^a{}_b\right)}\right.$$

$$\left. + B^{ab}(F_{ab} - \rho\varepsilon_{abc}u^c) + \lambda(u^a u_a + 1)\right]. \tag{34}$$

In type IIB string theory this action (with the appropriate overall rescaling) has a familiar connotation: it describes the Abelian dynamics of *extremal* D2-branes with dissolved D0 charge flowing along the velocity vector u^a inside the D2-brane worldvolume. In the next section we will reencounter this fluid from a rather different point of

view, that of the supergravity (blackfold) analysis of the extremal D0-D2 solution.

VII. DBI RECONSTRUCTION FROM SUPERGRAVITY

After this short detour on relativistic hydrodynamics we return to the long-wavelength expansions of interest in supergravity. To exhibit the relation with the DBI action we will consider in detail two representative examples. The first is based on the D0-D2 bound state that we encountered already in the previous section. The second example is based on the analysis of the F1-Dp bound state in flat space.

We will see that different bound states reconstruct the DBI action in different subclasses of configurations. In this sense the DBI action arises as a master action for whole families of effective hydrodynamic descriptions that arise from supergravity.

A. D0-D2 deformations

1. Supergravity analysis

The homogeneous planar D0-D2 bound state in ten-dimensional flat space that forms the zeroth-order solution in the long-wavelength expansions of interest is [72,73]

$$ds^2 = (-H^{-\frac{1}{2}}fu_a u_b + DH^{-\frac{1}{2}}(\gamma_{ab} + u_a u_b))d\sigma^a d\sigma^b$$

$$+ H^{\frac{1}{2}}(dr^2 + r^2 d\Omega_6^2),$$

$$e^{2\phi} = H^{\frac{1}{2}},$$

$$B_2 = \tan\vartheta(H^{-1}D - 1) * u,$$

$$C_1 = \sin\vartheta\coth\alpha(H^{-1} - 1)u,$$

$$C_3 = \sec\vartheta\coth\alpha(H^{-1}D - 1) * \mathbf{1}, \tag{35}$$

where

$$f(r) = 1 - \left(\frac{r_0}{r}\right)^5, \qquad H(r) = 1 + \left(\frac{r_0}{r}\right)^5\sinh^2\alpha,$$

$$D^{-1}(r) = \cos^2\vartheta + \sin^2\vartheta H^{-1}. \tag{36}$$

The solution, which has been boosted with a general $SO(1,2)$ transformation along the 2-brane worldvolume coordinates $(\sigma^0, \sigma^1, \sigma^2)$, is parametrized by the scalars

$$\vartheta \in [0, 2\pi), \qquad \alpha \in \mathbb{R}, \qquad r_0 \in \mathbb{R}_+, \tag{37}$$

and the unit velocity vector

$$u^a, \qquad u_a u^a = -1. \tag{38}$$

These parameters constitute part of the collective modes of the solution. In the metric and B_2, C_1, C_3 potentials we can also see collective coordinates associated with the

breaking of the transverse $SO(7)$ symmetry, which appear through the induced metric γ_{ab} [to which the Hodge star ($*$) refers]. These collective modes comprise seven transverse scalars $X^{\hat{i}}$. As usual, it is more convenient to formulate these modes more covariantly using ten scalars X^μ, in terms of which the induced worldvolume metric takes the form

$$\gamma_{ab} = g_{\mu\nu}\partial_a X^\mu \partial_b X^\nu. \tag{39}$$

In our problem $g_{\mu\nu}$ is the background Minkowski metric $\eta_{\mu\nu}$. In the planar solution (35) the transverse scalars have a fixed constant value and the induced worldvolume metric is the three-dimensional flat space metric η_{ab}. Writing explicitly γ_{ab} instead of η_{ab} in (35) is useful, because it prepares us for the types of supergravity *Ansätze* that lead to deformed brane solutions [25].

In the extremal limit, which will be the main case of interest in this paper,

$$r_0 \to 0, \quad \alpha \to \infty, \quad \text{with} \quad r_H^n := r_0^n \sinh^2\alpha \quad \text{held fixed.} \tag{40}$$

In this limit the temperature vanishes and the solution is 1/2-BPS.

As explained in previous sections, we want to set up a derivative expansion in supergravity where inhomogeneous extremal (but not necessarily supersymmetric) solutions are constructed order by order around (35). The supergravity *Ansatz* is based on the promotion of the above-mentioned collective coordinates to slowly varying functions of the worldvolume coordinates σ^a [22,23]. Analyzing the constraint equations of supergravity within this *Ansatz* at leading order in the derivative expansion, one arrives at the extremal blackfold equations [74]

$$D_a T^{ab} = 0, \qquad K_{ab}{}^{\hat{i}} T^{ab} = 0, \tag{41}$$

$$d * \tilde{J} = 0, \qquad d * J = 0, \qquad d * J_3 = 0, \tag{42}$$

with currents

$$T_{ab} = -\mathcal{C} r_H^5 (-\sin^2\vartheta u_a u_b + \cos^2\vartheta \gamma_{ab}), \qquad \mathcal{C} := \frac{5\Omega_{(6)}}{16\pi G}, \tag{43}$$

$$J = \mathcal{C}\sin\vartheta r_H^5 u, \qquad \tilde{J} = \cos\vartheta * J, \tag{44}$$

$$J_3 = \mathcal{C}\cos\vartheta r_H^5 * \mathbf{1}. \tag{45}$$

$\Omega_{(d)} = \frac{2\pi^{\frac{d-1}{2}}}{\Gamma(\frac{d-1}{2})}$ is the volume of the unit round d-sphere. The presence of the 1-, 2-, 3-form currents $J, \tilde{J}, J_3$ is closely related to the fact that in (35) the solution sources the 1-, 2-, 3-form potentials C_1, B_2, C_3.

We notice that the last equation, $d * J_3 = 0$, in (42) is expressing trivially the fact that the 2-brane charge, which is a quantized quantity, is a worldvolume constant

$$\partial_a(r_H^5 \cos\vartheta) = 0 \Leftrightarrow r_H^5 = \frac{c}{\mathcal{C}\cos\vartheta}. \tag{46}$$

Here c is an integration constant whose precise value will not play an important role in the ensuing. Consequently, r_H is not a true collective mode, and should be substituted into Eqs. (41) and (42) in terms of ϑ. Equation (46) is consistent with the proposal that open/closed string duality works within superselection sectors. The superselection sectors in this case are labeled by the value of the integration constant c.

To summarize, after the substitution (46) the leading order collective mode (blackfold) equations for D0-D2 solutions deduced from supergravity are comprised of the energy-momentum conservation equations

$$D_a T^{ab} = 0, \qquad K_{ab}{}^{\hat{i}} T^{ab} = 0,$$
$$T_{ab} = (\varepsilon + P)u^a u^b + P\gamma_{ab}, \tag{47}$$

with energy ε and pressure P,

$$\varepsilon(\vartheta) = c(\cos\vartheta)^{-1}, \qquad P(\vartheta) = -c\cos\vartheta, \tag{48}$$

and the current equations

$$d(\cos\vartheta J) = 0, \qquad d * J = 0, \qquad J = \rho u, \tag{49}$$

with charge density

$$\rho = c\tan\vartheta. \tag{50}$$

This is a complete set of dynamical equations for the unknown functions ϑ, u^a, X^μ. The blackfold conjecture states that solutions of this system are in one-to-one correspondence with regular first-order corrected inhomogeneous D0-D2 solutions within the blackfold *Ansatz*.[8] In higher orders of the derivative expansion the conserved currents are corrected with higher-derivative terms, but the background asymptotic geometry $g_{\mu\nu} = \eta_{\mu\nu}$ is not modified. We will discuss higher-order corrections in Sec. VIII.

2. Equivalence with DBI

It is evident that the collective mode equations of the supergravity analysis reduce at leading order to the hydrodynamic equations of a standard charged ideal fluid on a dynamical surface similar to the one reviewed in Sec. V. The constitutive relations are those of an extremal irrotational fluid with

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[8]This is a natural generalization of the corresponding statement in the fluid-gravity correspondence for large AdS black holes.

$$\varepsilon = f(\rho) = \sqrt{c^2 + \rho^2}. \qquad (51)$$

The irrotational flow condition (26) coincides with the supergravity equation $d(\cos \vartheta J) = 0$ in (49). As we noted in Sec. V not all equations in (47) and (49) are independent. It is enough to consider Eq. (47) combined with the irrotational flow condition.

In this case the relation to DBI is explained at the end of Sec. VI as special case II. The Abelian duality of the current J to a gauge field strength (27) converts the hydrodynamic equations (which are part of the supergravity equations here) to the equations of motion of the DBI action with Lagrange multipliers (29). The basic gauge/gravity map is expressed by Eq. (30), which translates an open string degree of freedom—the $U(1)$ gauge field with a magnetic *Ansatz*—to the quantities ρ, u^a that express components of the gravitational fields.

B. F1-Dp deformations

We now move to a different example in a $(p + 1)$-dimensional open string theory $(p > 1)$ with an electric gauge field explicitly turned on.

1. Supergravity analysis

Our starting point, as a zeroth-order solution in our long-wavelength expansion, is the homogeneous, planar F1-Dp bound state in ten-dimensional flat space. This involves the following supergravity profiles [73,75]:

$$ds^2 = (D^{-\frac{1}{2}}H^{-\frac{1}{2}}((1-f)u_a u_b + \hat{h}_{ab})$$
$$+ D^{\frac{1}{2}}H^{-\frac{1}{2}}\hat{\perp}_{ab})d\sigma^a d\sigma^b + D^{-\frac{1}{2}}H^{\frac{1}{2}}(dr^2 + r^2 d\Omega_{8-p}^2),$$

$$e^{2\phi} = D^{\frac{p-5}{2}}H^{\frac{3-p}{2}},$$

$$B_2 = \sin \vartheta \coth \alpha (H^{-1} - 1)u \wedge v,$$

$$C_{p-1} = (-)^p \tan \vartheta (H^{-1}D - 1) * (u \wedge v),$$

$$C_{p+1} = (-)^p \cos \vartheta \coth \alpha D(H^{-1} - 1) * \mathbf{1}, \qquad (52)$$

where

$$f(r) = 1 - \left(\frac{r_0}{r}\right)^n, \qquad H(r) = 1 + \left(\frac{r_0}{r}\right)^n \sinh^2 \alpha,$$

$$D^{-1} = \cos^2 \vartheta + \sin^2 \vartheta H^{-1}, \qquad n = 7 - p. \qquad (53)$$

As in the previous section, the solution has been boosted with a general $SO(1, p)$ transformation along the p-brane worldvolume coordinates $(\sigma^0, \ldots, \sigma^p)$. Accordingly, it is parametrized by the scalars

$$\vartheta \in [0, 2\pi), \qquad \alpha \in \mathbb{R}, \qquad r_0 \in \mathbb{R}_+, \qquad (54)$$

and the orthogonal vectors

$$u^a, \qquad v^a, \qquad (u_a u^a = -1, v_a v^a = 1, v_a u^a = 0) \quad (55)$$

that define the projectors

$$\hat{h}_{ab} = -u_a u_b + v_a v_b, \qquad \hat{\perp}_{ab} = \gamma_{ab} - \hat{h}_{ab}. \quad (56)$$

The induced metric γ_{ab}, which is flat in the zeroth-order solution, incorporates the dependence on the $9 - p$ transverse scalars $X^{\hat{i}}$. The Hodge star $(*)$ is defined with respect to this metric.

Once again, we will focus on the extremal limit

$$r_0 \to 0, \quad \alpha \to \infty, \quad \text{with} \quad r_H^n := r_0^n \sinh^2 \alpha \quad \text{held fixed,}$$
$$(57)$$

where the temperature vanishes and the zeroth-order solution is 1/2 BPS.

Reanalyzing the leading constraint equations in supergravity one arrives at the extremal blackfold equations [74]

$$D_a T^{ab} = 0, \qquad K_{ab}{}^{\hat{i}} T^{ab} = 0, \qquad (58)$$

$$d * J_{p-1} = 0, \qquad d * J_2 = 0, \qquad d * J_{p+1} = 0, \qquad (59)$$

with currents

$$T_{ab} = -\mathcal{C}_n r_H^n (\sin^2 \vartheta \hat{h}_{ab} + \cos^2 \vartheta \gamma_{ab}),$$

$$\mathcal{C}_n := \frac{n\Omega_{(n+1)}}{16\pi G}, \qquad n = 7 - p, \qquad (60)$$

$$J_2 = \mathcal{C}_n \sin \vartheta r_H^n u \wedge v, \qquad J_{p-1} = \cos \vartheta * J_2, \quad (61)$$

$$J_{p+1} = \mathcal{C}_n \cos \vartheta r_H^n * \mathbf{1}. \qquad (62)$$

There are three currents expressed as 2-, $(p - 1)$- and $(p + 1)$-forms corresponding to three spacetime potentials of the corresponding degree.

The last equation, $d * J_{p+1} = 0$, in (59) is expressing the fact that the p-brane charge is a worldvolume constant

$$\partial_a(r_H^n \cos \vartheta) = 0 \Leftrightarrow r_H^n = \frac{c}{\mathcal{C}_n \cos \vartheta}. \qquad (63)$$

We solve it in terms of the integration constant c that captures the quantized Dp-brane charge. Then, r_H is substituted into Eqs. (58) and (59) in terms of ϑ.

After this substitution the leading order F1-Dp blackfold equations are

$$D_a T^{ab} = 0, \qquad K_{ab}{}^{\hat{i}} T^{ab} = 0,$$
$$T_{ab} = (\varepsilon + P_T)u^a u^b - (P_T - P_L)v^a v^b + P_T \gamma_{ab}, \quad (64)$$

with energy, transverse and longitudinal pressures

$$\varepsilon(\vartheta) = c\frac{1}{\cos\vartheta}, \qquad P_T(\vartheta) = -c\cos\vartheta,$$

$$P_L(\vartheta) = -c\frac{1}{\cos\vartheta}, \tag{65}$$

and the current conservation equations

$$d(\cos\vartheta J_2) = 0, \qquad d * J_2 = 0,$$
$$J_2 = c\tan\vartheta u \wedge v. \tag{66}$$

The blackfold conjecture states that solutions of this system are in one-to-one correspondence with regular first-order corrected inhomogeneous F1-Dp solutions in the blackfold *Ansatz*.

2. Equivalence with DBI

In this case we obtain a set of dynamical equations for an augmented set of unknown functions $\vartheta, u^a, v^a, X^\mu$. They are hydrodynamic equations for an anisotropic fluid propagating on a dynamical $(p+1)$-dimensional hypersurface. The general fluid of this type obeys the relations [66]

$$\varepsilon + P_T = \mathcal{T}s + \mu q, \qquad P_T - P_L = \mu q, \tag{67}$$

where $\mathcal{T}$ is the local temperature, s the entropy density, μ the string chemical potential and q the string charge density. As is evident in (67) the relation $\varepsilon = -P_L$ is a consequence of extremality ($\mathcal{T} = 0$).

Generalizing the arguments of Sec. VI in a slightly different direction, which is not merely an Abelian Hodge duality, we will now show that the dynamical system (64), (65), (66) is classically equivalent to the equations of motion of the DBI action with an electric field constraint imposed by a 2-form Lagrange multiplier

$$S = \int d^{p+1}x\sqrt{-\gamma}[\sqrt{\det(\delta^a{}_b + F^a{}_b)}$$
$$+ B^{ab}(F_{ab} - \sin\vartheta(u_a v_b - v_a u_b))$$
$$+ \lambda_1(u^a u_a + 1) + \lambda_2(v^a v_a - 1) + \lambda_3 u^a v_a]. \tag{68}$$

The electric field *Ansatz*

$$F = \sin\vartheta u \wedge v \tag{69}$$

is the expected DBI description of the F1-Dp system. The specific form of the field strength (69) arises if we do an arbitrary pointwise spacetime-dependent Lorentz transformation of the constant electric field $F_{01} = \sin\vartheta$ that describes the planar, homogeneous F1-Dp solution. $\sin\vartheta$ is being used to express the familiar fact that F (being electric) cannot grow larger than the critical value $F_{01} = 1$ where the determinant inside the square root vanishes.[9] The

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[9]We restrict $\vartheta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$.

orthonormality of the vectors u, v, which is part of the definition of (69), is enforced by the variation of the Lagrange multipliers λ_1, λ_2, λ_3.

The remaining equations of motion of the action (68) are

$$T^{ab}K_{ab}{}^{\hat{i}} = 0, \tag{70}$$

$$D_a\left(\frac{1}{\cos\vartheta}F^{ab}\right) = 0. \tag{71}$$

In addition, we have the Bianchi identity

$$dF = 0. \tag{72}$$

With the *Ansatz* (69) these equations are obviously the same as the hydrodynamic equations (64), (65), (66).

To verify this, first we notice that the energy-momentum tensor of this system (after the use of the $\lambda_{1,2,3}$, u_a, v_a, B^{ab}, ϑ equations) is

$$T^{ab} = \frac{\sin^2\vartheta}{\cos\vartheta}(u^a u^b - v^a v^b) - \cos\vartheta\gamma^{ab}, \tag{73}$$

the same as that encountered in the blackfold equations.

Second, since

$$J_2 = \frac{c}{\cos\vartheta}F \tag{74}$$

we observe that the gauge field equation (71) is identical to the string current conservation equation $d * J_2 = 0$. The Bianchi equation (72), which is identical to the first equation in (66), $d(\cos\vartheta J_2) = 0$, can be viewed as a property closely related to the irrotational condition (19) of standard relativistic fluids in Sec. V.

C. Extensions

More general configurations of p-brane bound states with dissolved lower-dimensional charges in flat space admit a similar analysis. The constraint equations of the perturbative supergravity analysis always reduce to a hydrodynamic system, which admits a direct reformulation at extremality as a DBI action along the lines described above.

Let us summarize several interesting extensions of the exercises of the previous two subsections. An important extension concerns the higher-derivative corrections. There are such corrections both on the blackfold supergravity analysis and on the open string side as corrections to the Abelian DBI effective action. We will discuss the latter in the next section.

Another interesting direction, which was highlighted already in Sec. III, concerns the incorporation of non-Abelian effects. The imprint of such effects in the Abelian sector and its effective description can be captured in a

conceptually straightforward manner in the blackfold supergravity approach by redoing the perturbative analysis around other zeroth-order p-brane solutions.

For instance, we can consider flat p-brane bound states at finite temperature. In the above-mentioned D0-D2, F1-Dp examples, this introduces an additional degree of freedom [both r_0 and α participate without the scaling (57)]. It is interesting to ask whether there is a finite-temperature deformation of the DBI action that reproduces the on-shell finite-temperature hydrodynamic equations. This question was considered for stationary configurations in Refs. [61–63]. However, the question of a general deformation of the DBI action independent of stationarity remains open. Recent advances, e.g. in Ref. [35], could prove a useful avenue for this problem. We should point out that thermal small temperature corrections to the DBI action have been computed at weak coupling in Ref. [63], yet the exact form of finite-temperature corrections in open string theory is hard to obtain.[10]

Deformations of the Abelian DBI action can also be obtained from supergravity at extremality by considering other exact p-brane solutions at zeroth order. For instance, deformations of the zeroth-order solution will appear necessarily under external forcing, i.e. when a p-brane solution is embedded in a nonflat asymptotic background with fluxes. As a concrete example consider brane solutions in AdS.[11] In this case the hydrodynamic blackfold equations are modified in two different ways compared to the equations in flat space. First, there is a background-induced deformation of the conserved currents. Second, there are extra force terms in the equations, which are analogs of the Wess-Zumino couplings in the DBI action. A general formulation of such couplings in the blackfold expansion will appear in [74]. Further studies of such effects should contribute significantly to the understanding of the open/closed string duality proposed in Sec. III.

VIII. HIGHER-ORDER HYDRODYNAMICS FROM HIGHER-DERIVATIVE CORRECTIONS TO DBI

The DBI action is the leading term in a long-wavelength derivative expansion of open string field theory. Open string theory dictates very specific higher-derivative corrections to the DBI action. Such corrections were determined in flat space in [79–82] using the S-matrix or σ-model approach (for a review see [52]). Since we make

a connection with hydrodynamics it is interesting to ask how such corrections translate into the hydrodynamic language.

In what follows we will assume the validity of the nonrenormalization relation (14). Combined with open/closed string duality this relation allows us to translate information from a weak coupling open string analysis into a set of predictions for appropriate supergravity solutions in a long-wavelength derivative expansion.

Before we go into the details of the connection between open string and fluid dynamical higher-derivative connections, it is useful to recall that the subject of higher-derivative (dissipative) corrections in relativistic hydrodynamics has a long history. The mere addition of first-order derivative corrections to the energy-momentum tensor and the current [83,84] is well known to be inadequate and leads to unacceptable problems with causality and stability. These problems are amended in the Israel-Stewart approach [85–87], where higher-order corrections are added, or in other formalisms like Carter's canonical formalism (for a review see e.g. [67,68]).

The embedding of the DBI action in open string theory and its map to gravity suggests a class of hydrodynamic systems with higher-derivative corrections derived from string theory. The consistency of the latter implies that the usual issues with causality observed in generic (low order) hydrodynamic constructions should be absent here. Keep in mind that compared to the generic case discussed in the hydrodynamics literature, in this paper we have focused mainly on zero-temperature, finite-density fluids. This necessarily entails some obvious differences compared to the standard discussion of finite-temperature relativistic fluids that will become apparent soon.

Following [80] one finds from an open string theory computation that the general form of the leading higher-derivative corrections to the DBI action in superstring theory in flat space is[12]

$$S_{\text{super}} = \int d^{p+1}x \sqrt{-\det(\eta_{ab} + F_{ab})}[1 + \mathcal{F}^{k\ell mnabcd}$$
$$\times (F)\partial_k \partial_\ell F_{mn} \partial_a \partial_b F_{cd} + \mathcal{O}(\partial^6)]. \tag{75}$$

For simplicity, in (75) we have frozen the background geometry and the transverse scalar dynamics. The transverse scalar dynamics can be derived from this action in ten dimensions by T-duality.[13] We notice that the leading higher-derivative correction comes at the order of four derivatives. The function $\mathcal{F}(F) \sim F^2 + F^4 + \cdots$ has an in principle computable expansion in powers of the field strength F. For example, a four-vector superstring amplitude calculation on the disk gives up to order $\mathcal{O}(\partial^4 F^6)$ [80]

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[10]For a discussion of thermal corrections to gauge theory from a D-brane probe analysis in the context of the AdS/CFT correspondence see [76].

[11]AdS black holes in the blackfold approximation were considered in [77,78].

[12]We drop factors of π and α' that can be easily reinstated.

[13]It would be interesting to work out these corrections explicitly and compare them with the general theory of relativistic elasticity discussed in [88–92].

$$\mathcal{F}^{k\ell mnabcd}\partial_k\partial_\ell F_{mn}\partial_a\partial_b F_{cd}$$

$$= -\frac{1}{96}\left(\partial_a\partial_b F_{mn}\partial^a\partial^b F^{n\ell}F_{\ell r}F^{rm}\right.$$

$$+\frac{1}{2}\partial_a\partial_b F_{mn}F^{n\ell}\partial^a\partial^b F_{\ell r}F^{rm}$$

$$-\frac{1}{4}\partial_a\partial_b F_{mn}F^{mn}\partial^a\partial^b F^{\ell r}F_{\ell r}$$

$$\left.-\frac{1}{8}\partial_a\partial_b F_{mn}\partial^a\partial^b F^{mn}F_{\ell r}F^{\ell r}\right) + \mathcal{O}(\partial^4 F^6). \quad (76)$$

Interestingly, the corrections are different in the bosonic string.[14] Quoting [79,80]

$$S_{\text{bosonic}} = \int d^{p+1}x\sqrt{-\det(\eta_{ab}+F_{ab})}$$

$$\times [1+\mathcal{F}^{kmnacd}(F)\partial_k F_{mn}\partial_a F_{cd}+\mathcal{O}(\partial^4)]. \quad (77)$$

From a four-vector amplitude on the disk one finds

$$\mathcal{F}^{kmnacd}(F)\partial_k F_{mn}\partial_a F_{cd}$$

$$= -\frac{1}{48\pi}[F_{k\ell}F^{k\ell}\partial_a F_{mn}\partial^a F^{mn}$$

$$+ 8F_{k\ell}F^{\ell m}\partial_a F_{mn}\partial^a F^{nk}$$

$$- 4F_{\ell a}F^{\ell b}\partial^a F^{mn}\partial_b F_{mn} + \mathcal{O}(\partial^2 F^6)]. \quad (78)$$

In this case the derivatives start at a lower order, $\mathcal{O}(\partial^2)$.

The strategy developed in the previous sections suggests a natural connection of these actions (supplemented with specific *Ansätze* for the gauge field strength) with higher-derivative hydrodynamic systems. For example, a D0-D2–type higher-derivative fluid in three dimensions arises from the action

$$S = S_{\text{OS}} - \int d^{2+1}x\sqrt{-\gamma}[B^{ab}(F_{ab}-\rho\varepsilon_{abc}u^c)$$

$$+ \lambda(u^a u_a + 1))], \quad (79)$$

where S_{OS} is the open string–derived DBI action with higher-derivative corrections (S_{super} or S_{bosonic} above). As we noted previously the Lagrange multiplier B^{ab} enforces the *Ansatz* $F_{ab} = \rho\varepsilon_{abc}u^c$ and then the Bianchi identity $dF = 0$ guarantees the current conservation $d*J = 0$, where $J^a = \rho u^a$. This particular identification of the current (unchanged by the presence of the derivative corrections) means that we have chosen to work in the Eckart frame.

A significant part of the gauge field equations can be reexpressed as the energy-momentum conservation conditions $D_a T^{ab} = 0$. As an illustration let us consider the higher-derivative corrections to the energy-momentum tensor that arise in the case of the bosonic string. After the implementation of the B^{ab},λ,ρ,u^a equations the energy-momentum tensor of the resulting fluid takes the following form up to $\mathcal{O}(\partial^2)$:

$$T^{ab} = T^{ab}_{\text{ideal}} + T^{ab}_{\text{higher-derivative}} \quad (80)$$

with

$$T^{ab}_{\text{ideal}} = \frac{1}{\sqrt{1+\rho^2}}(\rho^2 u^a u^b - \gamma^{ab}) \quad (81)$$

and

$$T^{ab}_{\text{higher-derivative}}$$

$$= \left(T^{ab}_{\text{ideal}}\tilde{F}^{k\ell}_{mn} + \sqrt{1+\rho^2}[\tilde{F}^{k\ell}_{mn}]^{ab}\right)\partial_k(\rho u^m)\partial_\ell(\rho u^n). \quad (82)$$

The tensor structures

$$\tilde{F}^{k\ell}_{mn} = \varepsilon_{dem}\varepsilon_{fgn}\mathcal{F}^{kde\ell fg},$$

$$\times [\tilde{F}^{k\ell}_{mn}]^{ab}\partial_k(\rho u^m)\partial_\ell(\rho u^n)$$

$$= \varepsilon_{dem}\varepsilon_{fgn}\frac{\partial}{\partial\gamma_{ab}}[\mathcal{F}^{kdefg\ell}D_k(\rho u^m)D_\ell(\rho u^n)] \quad (83)$$

are functions of ρ and u without derivatives. At the order of Eq. (78) we obtain the more specific expressions

$$\tilde{F}^{k\ell}_{mn}\partial_k(\rho u^m)\partial_\ell(\rho u^n)$$

$$= \frac{\rho^2}{12\pi}(-3\partial_a\rho\partial^a\rho + \rho^2\partial_a u_b\partial^a u^b + u_a\partial^a\rho u_b\partial^b\rho$$

$$- 2\rho^2 u_a\partial^a u_c u_b\partial^b u^c), \quad (84)$$

$$[\tilde{F}^{k\ell}_{mn}]^{ab}\partial_k(\rho u^m)\partial_\ell(\rho u^n)$$

$$= \frac{\rho^2}{6\pi}(\mathcal{A}^{ab}((\partial\rho)^2) + \rho\mathcal{B}^{ab}(\partial\rho\partial u) + \rho^2\mathcal{C}^{ab}((\partial u)^2)), \quad (85)$$

where

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[14]I would like to thank E. Kiritsis for emphasizing this point.

$$\mathcal{A}^{ab} = \partial^a \rho \partial^b \rho - 2 \hat{\mathbb{1}}^{ab} u^c \partial_c \rho u^d \partial_d \rho - 4 u^{(a} \partial^{b)} \rho u^c \partial_c \rho$$
$$+ 2 \varepsilon^{act} \varepsilon^{bds} u_t u_s \partial_c \rho \partial_d \rho,$$
$$\mathcal{B}^{ab} = 4 \partial_c \rho \partial^c u^{(a} u^{b)} - 4 u^c \partial_c \rho u^d \partial_d u^{(a} u^{b)},$$
$$\mathcal{C}^{ab} = \partial_c u^a \partial^c u^b + \partial^a u^c \partial^b u_c - 3 u^a u^b \partial_c u_d \partial^c u^d$$
$$- 2 u^c \partial_c u^a u^d \partial_d u^b$$
$$+ 4 u^{(a} \partial^{b)} u^d u^c \partial_c u_d + 2 \gamma^{ab} u^c \partial_c u_e u^d \partial_d u^e$$
$$+ 4 \varepsilon^{aks} \varepsilon^{bmt} u_s u_t \partial_c u_k \partial^c u^m$$
$$- 2 \varepsilon^{act} \varepsilon^{bds} u_t u_s \partial_c u_e \partial_d u^e. \tag{86}$$

To derive these expressions we promoted the worldvolume metric η_{ab} to γ_{ab} covariantizing all couplings in (77) and (78), then took a derivative with respect to γ_{ab} and finally set $\gamma_{ab} = \eta_{ab}$. Potential higher-derivative couplings of the worldvolume metric do not affect this computation.

Several comments are in order at this point.

  (i) We observe that the usual dissipative corrections at order $\mathcal{O}(\partial)$ associated with the shear and bulk viscosity are absent. This is due to the extremal nature of the configurations that we are considering. At nonzero temperature such corrections are expected to appear. In fact we would expect that the ratio of the shear viscocity over the entropy density, $\frac{\eta}{s}$, is nonvanishing at nonzero temperature. The connection with gravity in previous sections suggests that this ratio is the constant $1/4\pi$. Then, for the system at hand we expect that as we take the zero-temperature limit η/s remains nonvanishing, while both η and s go simultaneously to zero.

 (ii) The leading corrections occur at $\mathcal{O}(\partial^2)$. Qualitatively these are corrections of the same general form as the corrections in the Israel-Stewart formalism [85–87].

(iii) It is interesting to ask how field redefinitions affect the above formulas. For instance, in a different frame, e.g. the Landau frame, where the current J is $\rho u^a + \mathcal{O}(\partial)$ corrections, the action (77) will also receive derivative corrections from the expansion of

the DBI square root. Notice that the leading corrections remain $\mathcal{O}(\partial^2)$.

Repeating the same exercise with the superstring action (75) we find a different set of higher-derivative corrections. For example, in the case of the D0-D2 configurations the above analysis would yield a fluid whose energy-momentum tensor receives its leading higher-derivative contributions at $\mathcal{O}(\partial^4)$. It would be interesting to know if this feature is related to the improved convergence properties of DBI solutions in superstring theory that have been observed throughout the literature over the years, e.g. the BIon solutions [93].

A similar analysis of open string–derived derivative corrections can be performed for the F1-Dp configurations (which require the *Ansatz* $F = \sin \vartheta u \wedge v$), or other more general configurations of the gauge field that lead to anisotropic fluids. We will not spell out the details here.

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