PHYSICAL REVIEW D **94,** 025021 (2016)

Finite quantum gauge theories

Leonardo Modesto,[1,*] Marco Piva,[2,†] and Lesław Rachwał[1,‡]
[1]*Center for Field Theory and Particle Physics and Department of Physics, Fudan University, 200433 Shanghai, China*
[2]*Dipartimento di Fisica "Enrico Fermi", Università di Pisa, Largo B. Pontecorvo 3, 56127 Pisa, Italy*
(Received 13 August 2015; published 11 July 2016)

We explicitly compute the one-loop exact beta function for a nonlocal extension of the standard gauge theory, in particular, Yang-Mills and QED. The theory, made of a weakly nonlocal kinetic term and a local potential of the gauge field, is unitary (ghost-free) and perturbatively super-renormalizable. Moreover, in the action we can always choose the potential (consisting of one "killer operator") to make zero the beta function of the running gauge coupling constant. The outcome is *a* UV *finite theory for any gauge interaction*. Our calculations are done in $D = 4$, but the results can be generalized to even or odd spacetime dimensions. We compute the contribution to the beta function from two different killer operators by using two independent techniques, namely, the Feynman diagrams and the Barvinsky-Vilkovisky traces. By making the theories finite, we are able to solve also the Landau pole problems, in particular, in QED. Without any potential, the beta function of the one-loop super-renormalizable theory shows a universal Landau pole in the running coupling constant in the ultraviolet regime (UV), regardless of the specific higher-derivative structure. However, the dressed propagator shows neither the Landau pole in the UV nor the singularities in the infrared regime (IR).

DOI: 10.1103/PhysRevD.94.025021

I. INTRODUCTION

We study a class of new actions of fundamental nature for gauge theories that are super-renormalizable or finite at quantum level. In particular, we hereby present four physical objectives to be met in a finite theory of QED and in Yang-Mills gauge interactions: avoiding the Landau pole in QED or for the $U(1)$ sector of the standard model of particle physics (SM), having a better control over divergences in QCD, having more room for unification of the running coupling constants in the super-renormalizable extension of the SM, and stabilizing the Higgs potential. Moreover, whether we want to study gauge theories coupled to super-renormalizable or finite gravity, then the former have to possess the same quantum properties. Furthermore, scale-invariant gauge theories in $D = 4$ can be promoted to conformally invariant ones. We also require the following two guiding principles to be common to all the fundamental interactions: "super-renormalizability or finiteness" and "validity of perturbative expansion" in the quantum field theory framework [1]. The desired theories satisfy the following properties: (i) gauge invariance, (ii) weak nonlocality (or quasipolynomiality), (iii) unitarity, and (iv) quantum super-renormalizability or finiteness. The main difference with quantum perturbative standard Yang-Mills theory (or Abelian quantum electrodynamics) lies in the second requirement, which makes possible to achieve unitarity and renormalizability at the same time in any spacetime dimension D.

Next, by choosing a subclass of theories with a sufficiently high number of derivatives in the UV, we may get even better control over perturbative divergences—we actually may get super-renormalizability. This means that infinities in the perturbative calculus appear only up to some finite loop order. Finally, by adding some operators, which are higher in powers of the gauge field strength, with specially adjusted coefficients, we achieve finiteness, namely, the beta function of gauge coupling can be consistently set to vanish. The outcome is a quantum theory for any gauge interaction free of any divergence at any order in the loop expansion, and the problem of the Landau pole in the UV is solved. Moreover, by shifting the coefficients of the theory, we can easily achieve asymptotic freedom (in the beta function) for all interactions, if this is desired for grand unification.

In a different vein if the theory is one-loop super-renormalizable and with higher-derivatives, then in the beta function we inevitably find a Landau pole at high energy because the beta function is universally negative. However, when looking at the dressed propagator of the theory (or the quantum effective action), we see that the behavior in UV as well as in IR is without additional real poles, and the interactions are suppressed at high energy. Indeed, in the UV it is the nonlocal higher-derivative operator that controls the high energy physics, whereas in IR the theory remains in the perturbative regime because of the universal negative sign of the beta function β_α. To fix the notation, we here define the divergent contribution to

*lmodesto@fudan.edu.cn
†marco.piva@df.unipi.it
‡rachwal@fudan.edu.cn

the effective action in dimension four to be $\Gamma_{\rm div} \equiv \frac{1}{\epsilon}\beta_\alpha \int d^4x {\rm tr}\mathbf{F}^2$, where $\alpha := 1/g^2$ and g is the gauge coupling constant.

II. NONLOCAL GAUGE THEORIES

A consistent gauge-invariant theory for spin one massless particles regardless of the spacetime dimension fits in the following general class of theories [2]:

$$\mathcal{L} = -\frac{1}{4g^2}{\rm tr}[\mathbf{F}e^{H(\mathcal{D}^2_\Lambda)}\mathbf{F} + \mathbf{V}_g]. \tag{1}$$

The theory above consists of a weakly nonlocal kinetic operator and a local curvature potential $\mathbf{V}_g$ crucial to achieve finiteness of the theory as we show later. In (1) the Lorentz indices and tensorial structures have been neglected. The notation on the flat spacetime reads as follows: We use the gauge-covariant box operator defined via $\mathcal{D}^2 = \mathcal{D}_\mu\mathcal{D}^\mu$, where $\mathcal{D}_\mu$ is a gauge-covariant derivative (in the adjoint representation) acting on gauge-covariant field strength $\mathbf{F}_{\rho\sigma} = F^a_{\rho\sigma}T^a$ of the gauge potential A_μ (where T^a are the generators of the gauge group in the adjoint representation.) The metric tensor $g_{\mu\nu}$ has signature $(-+\cdots+)$. We employ the following definition, $\mathcal{D}^2_\Lambda \equiv \mathcal{D}^2/\Lambda^2$, where Λ is an invariant mass scale in our fundamental theory. Finally, the entire function $V^{-1}(z) \equiv \exp H(z)$ $(z \equiv \mathcal{D}^2_\Lambda)$ in (1) satisfies the following general conditions [3], [4]: (i) $V^{-1}(z)$ is real and positive on the real axis, and it has no zeros on the whole complex plane $|z| < +\infty$. This requirement implies, that there are no gauge-invariant poles other than for the transverse and massless gluons. (ii) $|V^{-1}(z)|$ has the same asymptotic behavior along the real axis at $\pm\infty$. (iii) There exists $\Theta \in (0, \pi/2)$ such that asymptotically $|V^{-1}(z)| \to |z|^{\gamma+\frac{D}{2}-2}$, when $|z| \to +\infty$ with $\gamma \geq D/2$ (D is even and γ natural) for complex values of z in the conical regions C defined by $C = \{z| -\Theta < \arg z < +\Theta, \pi - \Theta < \arg z < \pi + \Theta\}$. This condition is necessary to achieve the maximum convergence of the theory in the UV regime. (iv) The difference $V^{-1}(z) - V^{-1}_\infty(z)$ is such that on the real axis

$$\lim_{|z|\to\infty} \frac{V^{-1}(z) - V^{-1}_\infty(z)}{V^{-1}_\infty(z)} z^m = 0, \quad \text{for all } m \in \mathbb{N}, \tag{2}$$

where $V^{-1}_\infty(z)$ is the asymptotic behavior of the form factor $V^{-1}(z)$. Property (iv) is crucial for the locality of counterterms. The entire function $H(z)$ must be chosen in such a way that $\exp H(z)$ tends to a polynomial $p(z)$ in UV hence leading to the same divergences as in higher-derivative theories.

An explicit example of a weakly nonlocal form factor $e^{H(z)}$ that has the properties (i)–(iv) can be easily constructed following [4],

$$e^{H(z)} = e^{\frac{1}{2}[\Gamma(0, e^{-\gamma_E}p(z)^2) + \log(p(z)^2)]} \underset{z\in\mathbb{R}}{=} \sqrt{p(z)^2}\left(1 + \frac{e^{-e^{-\gamma_E}p(z)^2}}{2e^{-\gamma_E}p(z)^2} + \cdots\right), \tag{3}$$

where $\gamma_E \approx 0.577216$ is the Euler-Mascheroni constant and $\Gamma(0, x) = \int_x^{+\infty} dt e^{-t}/t$ is the incomplete gamma function with its first argument vanishing. The polynomial $p(z)$ of degree $\gamma + (D-4)/2$ is such that $p(0) = 0$, which gives the correct low energy limit of our theory coinciding with the standard two-derivative Yang-Mills theory. In this case the Θ-angle defining cones C turns out to be $\pi/(4\gamma + 2(D-4))$.

The theories described by the action in (1) are unitary and perturbatively renormalizable at a quantum level in any dimension as we explicitly show in the following subsections.

Moreover, at the classical level many evidences endorse that we are dealing with "*gauge theories possessing singularity-free exact solutions*." The discussion here is closely analogous to the gravitational case [5–11]. In particular, the static gauge potential for the exponential form factor $\exp(-\Box/\Lambda^2)$ is for weak fields given approximately by

$$\Phi_{\rm gauge}(r) = A_0(r) = g\frac{{\rm Erf}(\frac{\Lambda r}{2})}{r}. \tag{4}$$

We used the form factor $\exp(-\Box/\Lambda^2)$ and $D = 4$ to end up with a simple analytic solution. However, the result is qualitatively the same for the asymptotically polynomial form factor (3), and $\Phi_{\rm gauge}(r) = {\rm const}$ for $r = 0$.

A. Propagator, unitarity, and divergences

By splitting the gauge field into a background field (with flat gauge connection) plus a fluctuation, fixing the gauge freedom, and computing the quadratic action for the fluctuations, we can invert the kinetic operator to get finally the two-point function. This quantity, also known as the propagator in the Fourier space reads, up to gauge dependent components,

$$\mathcal{O}^{-1}_{\mu\nu}(k) = \frac{-iV(k^2/\Lambda^2)}{k^2 + i\epsilon}\left(\eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right), \tag{5}$$

where we used the Feynman prescription (for dealing with poles). The tensorial structure in (5) is the same as the local Yang-Mills theory, but we see the presence of a new element—multiplicative form factor $V(z)$. If the function $V^{-1}(z)$ does not have any zeros on the whole complex plane, then the structure of poles in the spectrum is the same as in original two-derivative theory. This can be easily proved in the Coulomb gauge, which is manifestly unitary. Therefore, in the spectrum we have exactly the same modes as in two-derivative theories. In this way we have achieved unitarity, but the dynamics is modified from the simple two-derivative to a super-renormalizable one with higher-derivatives. Despite that in the UV regime we recover polynomial higher-derivative theory, the analysis of the tree-level spectrum still gives us a unitary theory without ghosts because the renormalizability is due to the behavior of the theory in the very UV limit, while unitarity is influenced by the behavior at any energy scale.

In the high energy regime (UV), the propagator in momentum space schematically scales as

$$\mathcal{O}^{-1}(k) \sim k^{-(2\gamma+D-2)}. \tag{6}$$

The vertices of the theory can be collected in different sets that may involve or not the entire function $\exp H(z)$. However, to find a bound on quantum divergences, it is sufficient to concentrate on the polynomial operators with the high energy leading behavior in the momenta k [3,4]. These operators scale as the propagator, they cannot have higher power of momentum k in the scaling in order not to break the renormalizability of the theory. The consideration of them gives the following upper bound on the superficial degree of divergence of any graph [4,12–14],

$$\omega(G) \leq DL + (V - I)(2\gamma + D) - E. \tag{7}$$

This bound holds in any spacetime of even or odd dimensionality. In (7) V is the number of vertices, I the number of internal lines, L the number of loops, and E is the number of external legs for the graph G. After plugging the topological relation $I - V = L - 1$ in (7), we get the following simplification:

$$\omega(G) \leq D - 2\gamma(L - 1) - E. \tag{8}$$

We comment on the situation in odd dimensions in the next section. Thus, if in even dimensions $\gamma > (D - E)/2$, in the theory only one-loop divergences survive. Therefore, the theory is one-loop super-renormalizable [4,15–19], and only a finite number of operators of energy dimensions up to M^D has to be included in the action to absorb all perturbative divergences. In a D-dimensional spacetime the renormalizable gauge theory includes all the operators up to energy dimension M^D and schematically reads

$$\mathcal{L}_D = -\frac{1}{4g^2}\mathrm{tr}[\mathbf{F}^2 + \mathbf{F}^3 + \mathbf{F}\mathcal{D}^2\mathbf{F} + \cdots + \mathbf{F}^{D/2}]. \tag{9}$$

In gauge theory the scaling of vertices originating from kinetic terms of the type $\mathbf{F}(\mathcal{D}^2)^{\gamma+(D-4)/2}\mathbf{F}$ is lower than the one seen in the inverse propagator $k^{2\gamma+D-2}$. This is because when computing variational derivatives with respect to the dimensionful gauge potentials (to get higher point functions) we decrease the energy dimension of the result. Hence, the number of remaining partial derivatives, when we put the variational derivative on the flat connection background, must be necessarily smaller. This means that we have a smaller power of momentum when the 3-leg (or higher leg) vertex is written in momentum space. We get the maximal scaling for the gluons' 3-vertex, and it is with the exponent $2\gamma + D - 3$. In this way we can put an upper bound on the degree of divergence for higher-derivative gauge theories even with a little excess. Again, for higher-derivative gauge theories and $\gamma > (D - E)/2$, we have one-loop super-renormalizability. For the minimal choice $E = 2$ (because the tadpole diagram vanishes), we have $\gamma > (D - 2)/2$.

B. Finite gauge theories in odd and even dimensions

In *odd number of dimensions* we can easily show that the theory is finite without need of gauge potential $\mathbf{V}_g$ because in dimensional regularization scheme (DIMREG) *there are no divergences at one-loop and the theory is automatically finite*. The reason is of dimensional nature. In odd dimension the energy dimension of possible one-loop counterterms needed to absorb logarithmic divergences can be only odd. However, at one-loop, such counterterms cannot be constructed in the DIMREG scheme and having at our disposal only Lorentz invariant (and gauge-covariant) building blocks that always have energy dimension two. By elementary building blocks, we mean here field strengths or gauge-covariant box operators or even number of covariant derivatives (an even number is necessary here to be able to contract all indices). For details, we refer the reader to original papers [12].

In *even dimensions* we for simplicity consider the polynomial $p(z)$ to be a monomial, $p_\gamma(z) = \omega z^{\gamma+\frac{D}{2}-2}$ (ω is a positive real parameter). In this minimal setup the monomial in UV gives precisely the highest derivative term of the form $\mathrm{tr}(\mathbf{F}(\mathcal{D}^2_\Lambda)^\gamma\mathbf{F})$ (in $D = 4$). There is only one possible way to take trace over group indices here, and terms with derivatives can be reduced to those with gauge-covariant boxes only by exploiting Bianchi identities in gauge theory. These latter terms take the explicit form $F^a_{\mu\nu}(\mathcal{D}^2_\Lambda)^\gamma F^{\mu\nu}_a$. In four dimensions there is an RG running of only one coupling constant. The contribution to the beta function of the YM coupling constant from this quadratic term is actually a dimensionless constant (independent of the frontal coefficient of the highest derivative term), which has been computed in [20] using Feynman diagrams. This number can be canceled by a contribution coming from a quartic (in field strengths) gauge killer of the form

$$-\frac{s_g}{4g^2\Lambda^4}\mathrm{tr}(\mathbf{F}^2(\mathcal{D}^2_\Lambda)^{\gamma-2}\mathbf{F}^2) \tag{10}$$

(here there are several possibilities of taking traces). The contribution to the beta function is linear in the parameter s_g, and hence, the latter one can be adjusted to make the total beta function vanish.

The action of the finite quantum theory may take the following compact form (for the choice $\gamma = 3$, the general derivative structure is explicit in $D = 4$):

$$\mathcal{L}_{\rm fin} = -\frac{1}{4g^2}\,{\rm tr}\Bigg[\underbrace{\mathbf{F}e^{H(\mathcal{D}^2_\Lambda)}\mathbf{F} + \frac{s_g}{\Lambda^4}\mathbf{F}^2(\mathcal{D}^2_\Lambda)\mathbf{F}^2}_{\text{minimal finite theory}} + \sum_i\sum_{j>2}^{5}\sum_{k=0}^{5-j} c_i^{(j,k)}((\mathcal{D}^2_\Lambda)^k\mathbf{F}^j)_i\Bigg], \qquad (11)$$

where $c_i^{(j,k)}$ are some constant coefficients. The beta function can successfully be killed by the last operator in the first line above. The last terms in the formula (11) have been written in a compact indexless notation, and the index i counts all possible contractions of Lorentz and group indices.

III. THE FINITE THEORY IN $D = 4$

As extensively discussed in the previous section, the minimal nonlocal gauge theory, in $D = 4$, candidate to be finite at the quantum level is

$$\mathcal{L}_{\rm fin} = -\frac{\alpha}{4}\,{\rm tr}\left[\mathbf{F}e^{H(\mathcal{D}^2_\Lambda)}\mathbf{F} + \frac{s_g}{\Lambda^4}\mathbf{F}^2(\mathcal{D}^2_\Lambda)^{\gamma-2}\mathbf{F}^2\right], \qquad (12)$$

where the function $H(z)$ is given in (3). We here evaluate the contribution to the beta function $\beta_\alpha^{(s_g)}$ from the two following independent killer operators quartic in the field strength[1]:

$$1.\quad -\frac{s_g}{4g^2\Lambda^4}F^a_{\mu\nu}F^{\mu\nu}_a\Box_\Lambda^{\gamma-2}(F^b_{\rho\sigma}F^{\rho\sigma}_b), \qquad (15)$$

$$2.\quad -\frac{s_g}{4g^2\Lambda^4}F^a_{\mu\nu}F^{\mu\nu}_b(\mathcal{D}^2_\Lambda)^{\gamma-2}(F^b_{\rho\sigma}F^{\rho\sigma}_a). \qquad (16)$$

All details of the computation are not included in this paper because they are very cumbersome, but the results are

$$1.\quad \beta_\alpha^{(s_g)} = \frac{s_g}{2\pi^2\omega}, \qquad (17)$$

$$2.\quad \beta_\alpha^{(s_g)} = \frac{s_g}{4\pi^2\omega}(1+N_G), \qquad (18)$$

where N_G is the number of generators of the Lie group.

These results have been checked using two different techniques: the method of Feynman diagrams and the Barvinsky-Vilkovisky trace technology [21].

The computation has been done for the nonlocal theory with general polynomial asymptotic behavior $p_\gamma(z)$ of degree γ. By choosing the monomial $p_\gamma(z) = \omega z^\gamma$, the prototype kinetic term used to evaluated the beta function reads

$$\mathcal{L}_{\rm fin,kin} = -\frac{1}{4g^2}F^a_{\mu\nu}(1+\omega(\mathcal{D}^2_\Lambda)^\gamma)F^{\mu\nu}_a. \qquad (19)$$

As already explained, all the other contributions of the form factor fall off exponentially in the UV and do not contribute to the divergent part of the quantum action. To fix our conventions, we can read the beta function from the counterterm operator, namely,

$$\mathcal{L}_{\rm ct} \coloneqq -\frac{\alpha}{4}(Z_\alpha - 1)F^{\mu\nu}_a F^a_{\mu\nu} = -\mathcal{L}_{\rm div} = -\frac{1}{\epsilon}\beta_\alpha F^{\mu\nu}_a F^a_{\mu\nu}.$$

By using the Batalin-Vilkovisky formalism [22], it is possible to prove that for the theory (12) there is no wave function renormalization for the gauge field A^a_μ. We have only renormalization of the gauge coupling constant. The contribution to the beta function $\beta_\alpha^{(\gamma)}$ due to the nonlocal kinetic term was obtained in [20], namely,

$$\beta_\alpha^{(\gamma)} = -\frac{5+3\gamma+12\gamma^2}{192\pi^2}C_2(G), \quad \gamma \geq 2, \qquad (20)$$

where $C_2(G)$ is the quadratic Casimir of the gauge group G. By imposing the following condition for scale invariance,

$$\beta_\alpha^{(\gamma)} + \beta_\alpha^{(s_g)} = 0, \qquad (21)$$

we can find the special value of the coefficient s_g^* that kills the beta function. Using, for example, the first killer (15), we get

$$s_g^* = -2\pi^2\omega\beta_\alpha^{(\gamma)}, \qquad (22)$$

and the Lagrangian for a finite nonlocal gauge theory in four dimensions can be explicitly written as

[1] It is worth noting that if we choose the gauge group $G = SU(N)$ and in the adjoint representation, it holds

$$\mathrm{tr}(T^aT^bT^cT^d) = \delta^{ab}\delta^{cd} + \delta^{ad}\delta^{bc}. \qquad (13)$$

Therefore, the killers we have considered exhaust all the possible operators we can construct, regarding the structure in the internal indices. On top of this, we have the freedom of using different contractions of Lorentz indices and covariant derivatives in the expressions for quartic killers. Indeed, if we plug the formula above (13) in the following general Lagrangian

$$\mathcal{L}_{\rm killer} = -\frac{s_g}{4g^2\Lambda^4}\,{\rm tr}[\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu}(\mathcal{D}^2_\Lambda)^{\gamma-2}(\mathbf{F}_{\rho\sigma}\mathbf{F}^{\rho\sigma})], \qquad (14)$$

we get the sum of the two killers (15) and (16) with the same front coefficient.

$$\mathcal{L}_{\rm fin} = -\frac{\alpha}{4}\Bigg[F^a_{\mu\nu} e^{H(\mathcal{D}^2_\Lambda)} F^{\mu\nu}_a + \omega \frac{(5+3\gamma+12\gamma^2)}{96\Lambda^4} C_2(G) F^a_{\mu\nu} F^{\mu\nu}_a \Box_\Lambda^{\gamma-2}(F^b_{\rho\sigma} F^{\rho\sigma}_b)\Bigg], \tag{23}$$

where we assumed $\gamma > 2$ (for $\gamma = 2$, we still have running of the vacuum energy, and scale invariance is not properly achieved.)

It is also possible to kill the beta function in nonlocal Abelian gauge theories. For concreteness, we can study the one-loop beta function of QED $\beta_e = e^3/12\pi^2$ for the electric charge e. In terms of the inverse coupling α, this function is expressed as $\beta_\alpha = -1/6\pi^2$, which is a constant and gives logarithmic scaling with the energy for the coupling constant α. Since pure two-derivative QED is a free theory, then the running comes entirely from quantum effects of charged matter. Here, we assume one species of charged fermions coupled minimally to photon field. If we extend QED to the nonlocal version (1) with killer operator (15) and we replace

$$s_g^* = -2\pi^2\omega\beta_\alpha^{(\gamma)} = \frac{\omega}{3} \tag{24}$$

in (12), then the theory is completely finite regardless of the parameter γ. It is important to notice that even in the Abelian case the killer operator has crucial impact on the beta function because it contains photon self-interactions. In this way we solve the problem of the Landau pole for the running of the electric charge in the UV regime of QED. The same can be repeated for any gauge theory coupled to matter, provided that in the matter sector we do not have self-interactions and the coupling to gauge fields is minimal [20,23].

We want to comment on what we can achieve if we stick to one-loop super-renormalizable gauge theories without attempts to make them finite. The final result (20) highlights a universal Landau pole issue in the UV regime for the running coupling constant $g(\mu)$ (where μ is the renormalization scale). This is true for any value of the integer $\gamma \geq 2$, when we do not introduce any potential $\mathbf{V}_g$ with killer operators. The sign of the beta function is negative because the discriminant $\Delta < 0$ of the quadratic polynomial in γ in (20). For the particular choice (22), the theory (12) is one-loop finite, but if the front coefficient s_g has a bigger value than in (22), then we enter the regime in which the UV asymptotic freedom is achieved. We here summarize the three possible scenarios for the value of the s_g:

$$s_g \begin{cases} < \frac{5+3\gamma+12\gamma^2}{96}\omega C_2(G), & \text{Landau pole,} \\ = \frac{5+3\gamma+12\gamma^2}{96}\omega C_2(G) \equiv s_g^*, & \text{finiteness,} \\ > \frac{5+3\gamma+12\gamma^2}{96}\omega C_2(G), & \text{asymptotic freedom.} \end{cases}$$

However, in weakly nonlocal higher-derivative theories we must read out the poles from the quantum effective action and not only from the beta functions of the couplings in the theory. In particular, in the case of theory (1) the one-loop dressed propagator is devoid of any pole because its UV asymptotic behavior is entirely due to the form factor $\exp H(z)$ [4], namely, up to the tensorial structure,

$$-i\frac{e^{-H(k^2)}}{k^2(1+\beta_\alpha e^{-H(k^2)}\log(k^2/\mu_0^2))}. \tag{25}$$

Moreover, as a particular feature of the super-renormalizable theory, when $s_g = 0$ or $s_g < s_g^*$, β_α is negative, signifying that at low energy the theory is weakly coupled. In consequence we do not have any pole in the dressed propagator in the UV nor do we have any problem in the IR as opposite to the local theory.

In local two-derivative theories we usually have a UV Landau pole or an IR singularity of RG flow, so (as, for example, in QED) the theory is weakly coupled in the IR (without confinement), but it becomes nonperturbative in the UV. In QCD we have the reverse. The theory is asymptotically free in the UV where it is perturbative, but a singularity of the RG flow manifests itself in the IR indicating confinement. In the case of two-derivative local theories the singularities of the flow have direct realization as the poles in the effective propagator read from the quantum action. This is not true anymore when higher-derivatives are included. In the theory (12) for $s_g < s_g^*$, the minus sign of the beta function, which usually gives rise to a UV Landau pole, is innocuous because the form factor washes away the $\log(k^2)$ contributions to the dressed propagator in the UV, and there is no possibility for appearance of a new real pole in it. On the other hand, in the IR the analytic form factor does not play any role and there is no pole because the beta function is negative. The outcome is a theory perturbative in both the UV and in the IR regime. Therefore, we are left with two possible options. We can choose completely UV finite (no divergences) nonlocal theories or super-renormalizable nonlocal theories with negative beta functions (β_α) and hence without any singularities in asymptotic behaviors of the couplings. The second option seems to be very appealing in models that attempt to realize a unification of all coupling constants.

IV. REMARKS ON FINITENESS AND RENORMALIZABILITY

The results in this paper are general and can be extended to all local higher derivative gauge theories as well. The construction of our theory is very natural as well as the inclusion of higher derivative operators is natural in the effective field theory framework. As already pointed out we invoked nonlocality only to settle completely the problem of unitarity, but the weakly nonlocal form factor is not crucial to achieve UV finiteness at the quantum level. Moreover, there is also another class of theories compatible with unitarity: the Lee-Wick gauge theories [24–26]. Furthermore, if we restrict our interest to higher derivative local theories, the following Lagrangian is a prototype for a finite four-dimensional gauge theory (with $\gamma = 3$):

$$\mathcal{L}_{\rm fin} = -\frac{\alpha}{4}\Bigg[F^a_{\mu\nu}(1+\omega(\mathcal{D}^2_\Lambda)^3)F^{\mu\nu}_a + \frac{61}{48\Lambda^4}\omega C_2(G)F^a_{\mu\nu}F^{\mu\nu}_a\Box_\Lambda(F^b_{\rho\sigma}F^{\rho\sigma}_b)\Bigg]. \quad (26)$$

We would like to point out that nonlocal field theories commonly arise as quantum effective actions when loop effects are taken into account or heavy modes are integrated out in the domain of effective field theories. In this latter respect the Lagrangian (23) can also naturally arise as a peculiar effective field theory. Therefore, it is inevitable to study nonlocal physics if the effective action is employed as a tool. In all known examples (QED, QCD, etc.) nonlocality appears already at one-loop, and typically, it is characterized by the presence of structures like $\log\Box$ in even dimension or $\sqrt{\Box}$ in odd dimension. The novelty in this paper is that we have studied a quite restrictive operator structure, which is nonlocal (quasipolynomial) already at the classical level (for example, different than $\log\Box$), with the aim to improve the UV behavior of the quantum theory.

We proved that the theory (23) is finite because the beta function vanishes. This means that in this theory there is no RG flow. However, here we do not deny the effects, which are very well tested in QCD (like the asymptotic freedom in the deep inelastic scattering) or in QED (the dependence of the scattering amplitude logarithmically with the energy scale) and are typically associated to the presence of running couplings. We only propose a different interpretation of them in the theoretical framework of finite gauge theories. In full generality the RG running of coupling constants is a theoretical feature of (some) quantum field theories, and such an abstract notion is not a subject to experimental verification. What is typically done is that some physical (measurable) effects are traced back to the RG running of the couplings in some theories. However, the latter fact does not mean that the RG flow is experimentally confirmed. Only the physical effects, whose one of the possible explanations is due to RG flow, are being measured. In this paper we provided a different theoretical explanation for such experimental effects. It is important to emphasize that we never found a disagreement with the experiments done in the field of strong or electromagnetic interactions between elementary particles. Moreover, the RG flow is not well tested in QCD but only the measurable physical effects, whose interpretation and explanation is not unique, are verified. Another drawback of RG flows is that beyond one-loop approximation the beta functions are gauge and parametrization dependent; hence, they cannot be physically observable.

Our interpretation of these results in a finite quantum gauge theory is as follows. All the effects, which are typically associated to the RG flow of the couplings (in the standard nonfinite theory) can be mimicked by some special operators (typically nonlocal or with higher derivatives) added to the action of the UV-finite theory. This addition however does not change the finiteness of the theory. One very prominent example of such interpretation naturally comes along with the quantum effective action. If all the quantum (perturbative loop) effects are taken into account in some tree-level action, then there is no need for any further RG effects. Our statement is that in the quantum effective action there is no running of couplings. All the effects are read from it at tree level, and there is no room for the RG flow due to quantum loop effects. In the jargon of RG flow, the quantum effective action stays at a fixed point of the renormalization group. All the physical effects are explained by operators appearing in the quantum effective action, which is anyway a very difficult object to compute. However, when it is given, we do not need to go beyond the tree level. Our situation with finite theories is exactly the same. Our actions for UV-finite theories can be viewed as proposals for the explicit form of quantum effective actions (up to explicit listing of all finite terms there). It is obvious to us that being a proposal for the effective action our finite theories do not have any RG flows but at the same time are able to explain all the experimentally measured effects (because they are all actually explained by effective actions.) As we have already argued there is no any contradiction between our finite theories and effects typically explained by the RG flows in standard nonfinite theories. Furthermore, in our case the quantum effective action will contain only finite contributions.

V. CONCLUSIONS

We have explicitly evaluated the one-loop exact beta function for the weakly nonlocal gauge theory recently proposed in [2]. The higher-derivative structure or quasipolynomiality of the action implies that the theory is super-renormalizable, and in particular, only one-loop divergences survive in any dimension. Once a potential, at least cubic in the field strengths, is switched on, it is always possible to make the theory finite. We evaluated the beta function for the special case of $D = 4$, but the result can be generalized to any dimension where a careful selection of the killer operators should be done.

In short the main achievement of the paper is the following:

We have explicitly shown how to construct a finite theory for gauge bosons in $D = 4$ (23).

In the paper we have considered both cases of Abelian and non-Abelian gauge symmetry groups. The super-renormalizable structure does not change if we add a general extra matter sector that does not exhibit self-interactions.

The minimal nonlocal theory without any killer operator shows a Landau pole for the running coupling constant, regardless of the special asymptotic polynomial structure. This is a universal property shared at least by all the unitary and weakly nonlocal gauge theories with asymptotic polynomial behavior in the UV regime. However, the one-loop dressed propagator does not show any Landau pole in the UV regime because the propagator is dominated by the nonlocal form factor, and it is the nonlocal operator that controls the high energy physics. Moreover, we do not have any pole even in the IR, opposite to the local theory, exactly because of the universal negative sign of the beta function. The outcome is a theory well defined at the perturbative level in both the IR and the UV regime. The same result is achieved in the presence of sufficiently weakly coupled killer operators.

In this paper we mostly considered pure gauge theories, but here we can achieve asymptotic freedom regardless of the number of fermionic fields because it is the interaction between gauge bosons, due to the killer operators, that makes the theory asymptotically free.

The generalization to extra dimensions is straightforward. In particular, the theory is finite in odd dimension without the need to introduce any killer operator, as a mere consequence of dimensional regularization. The results can also be reproduced in cutoff regularization making use of Pauli-Villars operators [27].

We now emphasize the implications of the results in this paper for the high energy physics beyond the standard model of particle physics (SM), namely, a finite theory of all fundamental interactions. Given the gauge symmetry group of the SM coupled to gravity, namely, $\mathcal{G}_{\mathrm{SM+gr}} = GL(4)\times SU(3)_{\mathrm{s}}\times SU(2)_{\mathrm{w}}\times U(1)_Y$, we can easily describe the gravitational [2] and gauge interactions with a quasi-polynomial Lagrangian with a nonlocal form factor (3) having UV monomial behavior $p(z)=z^{\gamma+1}$. It is then sufficient to add up to $3+2$ killer operators to make the gravity-gauge sector of the SM finite. We can use three gauge killers (one for each gauge group) like the one in (10) to make vanish the beta functions for each of the operators F^2 (for each gauge sector). For the gravity sector, two killer operators are enough, namely, $R^2\Box^{\gamma-2}R^2$ and $R^2_{\mu\nu}\Box^{\gamma-2}R^2_{\mu\nu}$ (see [2] for an extensive discussion). The Lagrangian of the matter sector is also weakly nonlocal and free of quantum divergences. We end up with a completely quantum scale-invariant theory for all fundamental interactions.

As an alternative, we notice that the front coefficients of killers can also be chosen to make the SM super-renormalizable, and the gauge coupling constants perfectly meet at the grand unification scale without need of supersymmetry.

For a relatively low energy scale of nonlocality Λ, the future discovery of higher-derivative operators, together with our theoretical guiding principles (unitarity and renormalizability), could confirm or disprove our theory.

Finally, the gauge theories here proposed can have a wide range of applications, not only in the high energy regime but also for the low energy physics. In particular, the nonlocal extension of QED here presented, and other UV and/or infrared nonlocal generalizations, could have applications in condensed matter physics or nuclear physics. Infrared modifications could provide superconductivity without Cooper pairs, while the exact potential (4) without a Coulomb barrier may have implications for research on nuclear fusion.

Finite quantum gauge theories could also play a crucial role in describing critical phenomena. It is known that a theory describing such behaviors is characterized by infinite correlation lengths, where even the discrete atomic systems shows the structure like a continuous medium (described by a continuous field theory). Moreover, such a field theory enjoys scale invariance, which can be promoted to the full conformal symmetry. This can be also naturally explained as the consequence of the fact that the theory of critical phenomena is basically a theory governed by a UV fixed point of RG flow of running coupling constants. In such theory all beta functions must be zero, hence no divergences and UV-finiteness. With our finite gauge theory, we could, in principle, describe critical phenomena with manifest local gauge invariance [28].

ACKNOWLEDGMENTS

We are grateful to D. Anselmi for the very useful discussions in quantum field theory.

APPENDIX: EXPLICIT CALCULATION OF THE ONE-LOOP BETA FUNCTION

We hereby explicitly evaluate the beta function for the nonlocal unitary theory in $D=4$ with polynomial asymptotic behavior $p_n(z)$ of degree n. The minimal theory reads as follows:

$$\mathcal{L}_{\mathrm{fin,YM}} = -\frac{1}{4g^2}\left[F^a_{\mu\nu}e^{H(\mathcal{D}^2_\Lambda)}F^{\mu\nu}_a + \frac{s_g}{\Lambda^4}F^a_{\mu\nu}F^{\mu\nu}_a(\Box_\Lambda)^{n-2}F^b_{\rho\sigma}F^{\rho\sigma}_b\right]. \tag{A1}$$

By choosing $p_n(z)=\omega z^n$, we can focus on the following prototype gauge theory:

$$\mathcal{L}_{\mathrm{fin,YM}} = -\frac{1}{4g^2}\left\{F^a_{\mu\nu}[1+\omega(\mathcal{D}^2_\Lambda)^n]F^{\mu\nu}_a + \frac{s_g}{\Lambda^4}F^a_{\mu\nu}F^{\mu\nu}_a(\Box_\Lambda)^{n-2}F^b_{\rho\sigma}F^{\rho\sigma}_b\right\}. \tag{A2}$$

We can read the beta function from the counterterm operator

$$\mathcal{L}_{\rm ct} \coloneqq -\frac{\alpha}{4}(Z_\alpha - 1)F^{\mu\nu}_a F^a_{\mu\nu} = -\mathcal{L}_{\rm div} = -\frac{1}{\epsilon}\beta_g F^{\mu\nu}_a F^a_{\mu\nu}$$
$$\Rightarrow \frac{\alpha}{4}(Z_\alpha - 1) = \frac{1}{\epsilon}\beta_g, \qquad \alpha = \frac{1}{g^2}. \tag{A3}$$

1. One-loop beta function using Feynman diagrams

We here compute the contribution to the beta function for the killer operator by using Feynman diagrams and with the help of a Mathematica program. We start from the Lagrangian (A2), and we add the following gauge fixing term:

$$\mathcal{L}_{\rm GF} = \bar{C}_a e^{H(\mathcal{D}^2_\Lambda)}\partial^\mu \mathcal{D}^{ab}_\mu C_b - \frac{1}{2\xi}(\partial^\mu A^a_\mu)e^{H(\mathcal{D}^2_\Lambda)}(\partial^\mu A^a_\mu), \tag{A4}$$

while the gluon propagator in momentum space is

$$D^{ab}_{\mu\nu}(k) = \frac{-i\delta_{ab}}{k^2 + i\epsilon}\left(\frac{\eta_{\mu\nu} - (1-\xi)k_\mu k_\nu/k^2}{1+\omega(-k^2)^n}\right), \tag{A5}$$

where we have considered only the asymptotic behavior of the form factor in the gauge fixing.

a. The first killer

The four legs vertex for the killer operator reads as

$$\frac{s_g}{\Lambda^4}F^a_{\mu\nu}F^{\mu\nu}_a(\Box_\Lambda)^{n-2}F^b_{\rho\sigma}F^{\rho\sigma}_b$$
$$= 4\frac{s_g}{\Lambda^4}\partial_\mu A^a_\nu(\partial^\mu A^\nu_a - \partial^\nu A^\mu_a)(\Box_\Lambda)^{n-2}\partial_\rho A^b_\sigma(\partial^\rho A^\sigma_b - \partial^\sigma A^\rho_b). \tag{A6}$$

By switching to the momentum space, we label the four fields in the following way, $A^a_\mu(p), A^b_\nu(k), A^c_\rho(q), A^d_\sigma(l)$, and the integrand of the Fourier transform is

$$4\frac{s_g}{\Lambda^{2n}}(pk\eta^{\mu\nu} - p^\nu k^\mu)[-(q+l)^2]^{n-2}(ql\eta^{\rho\sigma} - q^\sigma l^\rho)$$
$$\times \delta_{ab}\delta_{cd}A^a_\mu(p)A^b_\nu(k)A^c_\rho(q)A^d_\sigma(l), \tag{A7}$$

where $pk = p^\alpha k_\alpha$. To obtain the vertex, we remove the fields A and multiply by i the rest, namely,

$$V^{\mu\nu\rho\sigma}_{abcd}(p,k,q,l) = i4\frac{s_g}{\Lambda^{2n}}(pk\eta^{\mu\nu} - p^\nu k^\mu)[-(q+l)^2]^{n-2}$$
$$\times (ql\eta^{\rho\sigma} - q^\sigma l^\rho)\delta_{ab}\delta_{cd} + \text{perm.} \tag{A8}$$

Choosing the momentum conservation for the incoming momenta, the diagram in dimensional regularization is

$$\Pi^{\mu\nu}_{ab}(p) = \frac{1}{2}\int\frac{d^D q}{(2\pi)^D}V^{\mu\nu\rho\sigma}_{abcd}(p,-p,q,-q)D^{cd}_{\rho\sigma}(q), \tag{A9}$$

where $1/2$ is a symmetry factor. What we found is

$$\Pi^{\mu\nu}_{ab}(p) = \frac{2s_g}{\pi^2\omega\epsilon}i(p^2\eta^{\mu\nu} - p^\mu p^\nu)\delta_{ab}, \tag{A10}$$

where $\epsilon = 4 - D$. Remembering the following relation with the divergent contribution to the one-loop quantum action,

$$i\Gamma^{(1)}_{\rm div} = \frac{1}{2}\int\frac{d^4 p}{(2\pi)^4}A^a_\mu(p)\Pi^{\mu\nu}_{ab}(p)A^b_\nu(-p), \tag{A11}$$

we can write in Fourier transform

$$\Gamma^{(1)}_{\rm div} = \frac{1}{2}\frac{2s_g}{\pi^2\omega\epsilon}\int\frac{d^4p}{(2\pi)^4}d^4xd^4y e^{ip(x-y)}A^a_\mu(-\Box\eta^{\mu\nu} + \partial^\mu\partial^\nu)$$
$$\times\delta_{ab}(x)A^b_\nu(y)$$
$$= \frac{s_g}{\pi^2\omega\epsilon}\int d^4xd^4y\delta^4(x-y)A^a_\mu(x)(-\Box\eta^{\mu\nu} + \partial^\mu\partial^\nu)A^a_\nu(y)$$
$$= \frac{s_g}{\pi^2\omega\epsilon}\int d^4xA^a_\mu(x)(-\Box\eta^{\mu\nu} + \partial^\mu\partial^\nu)A^a_\nu(x)$$
$$= \frac{s_g}{\pi^2\omega\epsilon}\int d^4x(\partial_\mu A^a_\nu\partial^\mu A^\nu_a - \partial_\mu A^a_\nu\partial^\nu A^\mu_a), \tag{A12}$$

where in the last step we integrated by parts. Using the antisymmetry property of $F_{\mu\nu}$, $F^a_{\mu\nu}F^{\mu\nu}_a = 2\partial_\mu A^a_\nu F^{\mu\nu}_a$, we obtain

$$\Gamma^{(1)}_{\rm div} = \frac{s_g}{\pi^2\omega\epsilon}\int d^4x\partial_\mu A^a_\nu(\partial^\mu A^\nu_a - \partial^\nu A^\mu_a)$$
$$= \frac{s_g}{\pi^2\omega\epsilon}\int d^4x\partial_\mu A^a_\nu F^{\mu\nu}_a$$
$$= \frac{s_g}{2\pi^2\omega\epsilon}\int d^4xF^a_{\mu\nu}F^{\mu\nu}_a. \tag{A13}$$

Therefore, the divergent part of the quantum action has the form $\Gamma^{(1)}_{\rm div} = \alpha(Z-1)/4\int d^4xF^a_{\mu\nu}F^{\mu\nu}_a$. By using the Batalin-Vilkovisky formalism [22], it is possible to prove that in the renormalization procedure there are not fields redefinitions and A does not renormalize. It follows that $Z = Z_\alpha$, and the contribution to the beta function is

$$\beta^{(s_g)}_\alpha = \frac{s_g}{2\pi^2\omega}. \tag{A14}$$

b. The second killer

We start from the Lagrangian (A2) but now with the following killer operator:

$$\frac{s_g}{\Lambda^4}\int d^4xF^a_{\mu\nu}F^{\mu\nu}_b(\mathcal{D}^2_\Lambda)^{n-2}F^b_{\rho\sigma}F^{\rho\sigma}_a, \tag{A15}$$

where the color indices are now contracted between fields strength on opposite sides of the d'Alembertian operator.

Using the same procedure, we can compute the contribution from the operator (A15), and the result is

$$\beta_\alpha^{(s_g)} = \frac{s_g}{4\pi^2\omega}(1+N_G), \tag{A16}$$

where N_G is the number of generators of the Lie group.

2. One-loop beta function using the Barvinsky-Vilkovisky technique

The kinetic operator contributing to the beta function is

$$-\frac{\omega}{4}F_{\mu\nu}(\mathcal{D}^2)^n F^{\mu\nu}, \tag{A17}$$

and the variation of field strength on flat gauge space ($A_\mu \to \bar{A}_\mu + A_\mu$ and $\bar{A}_\mu = 0$) reads as follows:

$$\delta F_{\mu\nu} = 2\partial_{[\mu}\delta A_{\nu]}. \tag{A18}$$

The propagator on the flat gauge space is

$$\begin{aligned} -\frac{\omega}{4}\delta^2(F_{\mu\nu}(\mathcal{D}^2)^n F^{\mu\nu}) &= 2\left(-\frac{\omega}{4}\right)(2\partial_{[\mu}\delta A_{\nu]})\Box^n(2\partial^\mu\delta A^\nu) \\ &= 8\left(-\frac{\omega}{4}\right)\partial_{[\mu}\delta A_{\nu]}\Box^n\partial^\mu\delta A^\nu. \end{aligned} \tag{A19}$$

Integrating by parts and under the integral sign, we end up with

$$\begin{aligned} &-\frac{\omega}{4}\delta^2(F_{\mu\nu}(\mathcal{D}^2)^n F^{\mu\nu}) \\ &= 2\omega\delta A_{[\nu}\partial_{\mu]}\Box^n\partial^\mu\delta A^\nu \\ &= -2\omega\delta A_{[\mu}\partial_{\nu]}\Box^n\partial^\mu\delta A^\nu \\ &= -\omega\delta A_\mu\partial_\nu\Box^n\partial^\mu\delta A^\nu + \omega\delta A_\nu\partial_\mu\Box^n\partial^\mu\delta A^\nu \\ &= -\omega\delta A_\mu\partial^\nu\Box^n\partial^\mu\delta A_\nu + \omega\delta A_\nu\Box^{n+1}\delta A^\nu \\ &= \delta A_\mu(-\omega\partial^\nu\Box^n\partial^\mu + \omega\eta^{\mu\nu}\Box^{n+1})\delta A_\nu \\ &\quad -\omega\delta A_\mu(\partial^\mu\partial^\nu\Box^n - \eta^{\mu\nu}\Box^{n+1})\delta A_\nu. \end{aligned} \tag{A20}$$

We now add the higher-derivative gauge fixing, namely,

$$\alpha\chi(\mathcal{D}^2)^n\chi, \qquad \chi = \mathcal{D}^\mu A_\mu, \tag{A21}$$

whose second variation on the flat gauge space is

$$\delta^2(\alpha\chi(\mathcal{D}^2)^n\chi) = 2\alpha\partial^\mu\delta A_\mu\Box^n\partial^\nu\delta A_\nu. \tag{A22}$$

Again under the integral sign we get

$$\begin{aligned} \delta^2(\alpha\chi(\mathcal{D}^2)^n\chi) &= -2\alpha\delta A_\mu\partial^\mu\Box^n\partial^\nu\delta A_\nu \\ &= -2\alpha\delta A_\mu(\partial^\mu\partial^\nu\Box^n)\delta A_\nu. \end{aligned} \tag{A23}$$

Summing together the second variation for the kinetic operator and the gauge fixing, we get the following fully gauge fixed propagator:

$$\delta A_\mu((-\omega-2\alpha)\partial^\mu\partial^\nu\Box^n + \omega\eta^{\mu\nu}\Box^{n+1})\delta A_\nu. \tag{A24}$$

We require the above operator (A24) to be minimal with highest derivative; therefore, the following condition must be imposed:

$$\omega + 2\alpha = 0, \tag{A25}$$

and the kinetic operator turns in

$$\omega\delta A_\mu(\eta^{\mu\nu}\Box^{n+1})\delta A_\nu = \delta A^\mu(\omega\delta_\mu^\nu\Box^{n+1})\delta A_\nu. \tag{A26}$$

The minimal operator $H_\mu{}^\nu$ on flat gauge space reads as follows:

$$H_\mu{}^\nu = \omega\delta_\mu^\nu\Box^{n+1}, \tag{A27}$$

with the highest derivative term (the DeWitt metric is here the Minkowski metric).

a. The first killer

The first killer we consider is given by one of the options in contracting the color indices. We here consider the following product of two traces:

$$s_g F^a_{\mu\nu}F^{a\mu\nu}(\mathcal{D}^2)^{n-2}F^b_{\rho\sigma}F^{b\rho\sigma}. \tag{A28}$$

The second variation of the above operator on a general gauge background (up to square field strength order) reads

$$\begin{aligned} &\delta^2(s_g F^a_{\mu\nu}F^{a\mu\nu}(\mathcal{D}^2)^{n-2}F^b_{\rho\sigma}F^{b\rho\sigma}) \\ &= 2s_g\delta(F^a_{\mu\nu}F^{a\mu\nu})(\mathcal{D}^2)^{n-2}\delta(F^b_{\rho\sigma}F^{b\rho\sigma}) \\ &= 8s_g F^{a\mu\nu}(\delta F^a_{\mu\nu})\Box^{n-2}F^b_{\rho\sigma}(\delta F^{b\rho\sigma}) \\ &= 8s_g F^{a\mu\nu}F^b_{\rho\sigma}(\delta F^a_{\mu\nu})\Box^{n-2}(\delta F^{b\rho\sigma}) \\ &= 8s_g F^{a\mu\nu}F^b_{\rho\sigma}(2\partial_{[\mu}\delta A^a_{\nu]})\Box^{n-2}(2\partial^{[\rho}\delta A^{b\sigma]}) \\ &= 32s_g F^{a\mu\nu}F^b_{\rho\sigma}\partial_\mu\delta A^a_\nu\Box^{n-2}\partial^\rho\delta A^{b\sigma}. \end{aligned} \tag{A29}$$

Integrating by parts, we get from this expression (neglecting derivatives on background fields and commutation of derivatives)

$$\begin{aligned} &-32s_g F^{a\mu\nu}F^b_{\rho\sigma}\delta A^a_\nu\partial_\mu\Box^{n-2}\partial^\rho\delta A^{b\sigma} \\ &\quad = -32s_g F^{a\mu\nu}F^b_{\rho\sigma}\delta A^a_\nu\partial_\mu\partial^\rho\Box^{n-2}\delta A^{b\sigma} \\ &\quad = 32s_g F^{a\mu\rho}F^{b\sigma\nu}\delta A^a_\mu(\partial_\rho\partial_\sigma\Box^{n-2})\delta A^b_\nu. \end{aligned} \tag{A30}$$

We now check if it is the self-adjoint part of the operator (to this level):

$$
\begin{aligned}
&32s_g F^{a\mu\rho}F^{b\sigma\nu}\delta A^a_\mu(\partial_\rho\partial_\sigma\Box^{n-2})\delta A^b_\nu \\
&\quad = 32s_g F^{a\mu\rho}F^{b\sigma\nu}\delta A^b_\nu(\partial_\rho\partial_\sigma\Box^{n-2})\delta A^a_\mu \\
&\quad = 32s_g F^{a\mu\rho}F^{b\sigma\nu}\delta A^a_\mu(\partial_\rho\partial_\sigma\Box^{n-2})\delta A^b_\nu. \qquad (A31)
\end{aligned}
$$

For our purpose, the relevant part of the operator H is

$$
H_\mu{}^{\nu ab} = \omega\delta^\nu_\mu\delta^{ab}\Box^{n+1} + \cdots + 32s(F^a{}_\mu{}^\rho F^{b\sigma\nu})\partial_\rho\partial_\sigma\Box^{n-2} + \ldots \qquad (A32)
$$

Now we have to take the trace of the logarithm of the operator (A32),

$$
\begin{aligned}
\mathrm{Tr}\ln H_\mu{}^{\nu ab} &= \mathrm{Tr}\ln(\omega\delta^\nu_\mu\delta^{ab}\Box^{n+1} + \cdots + 32s_g(F^a{}_\mu{}^\rho F^{b\sigma\nu})\partial_\rho\partial_\sigma\Box^{n-2} + \cdots) \\
&= \mathrm{Tr}\ln(\omega\delta^\kappa_\mu\delta^{ac}\Box^{n+1}(\delta^\nu_\kappa\delta^{cb} + \cdots 32s_g\omega^{-1}(F^c{}_\kappa{}^\rho F^{b\sigma\nu})\partial_\rho\partial_\sigma\Box^{-3} + \cdots)) \\
&= (n+1)\mathrm{Tr}\ln(\delta^\kappa_\mu\delta^{ac}\Box) + \mathrm{Tr}\ln(\delta^\nu_\kappa\delta^{cb} + \cdots + 32s_g\omega^{-1}(F^c{}_\kappa{}^\rho F^{b\sigma\nu})\partial_\rho\partial_\sigma\Box^{-3} + \cdots). \qquad (A33)
\end{aligned}
$$

We concentrate on the second contribution, and we expand it in Taylor series,

$$
\begin{aligned}
\mathrm{Tr}\ln(\delta^\nu_\kappa\delta^{cb} + \cdots + 32s_g\omega^{-1}(F^c{}_\kappa{}^\rho F^{b\sigma\nu})\partial_\rho\partial_\sigma\Box^{-3} + \cdots) &= \mathrm{Tr}(32s_g\omega^{-1}(F^c{}_\kappa{}^\rho F^{b\sigma\nu})\partial_\rho\partial_\sigma\Box^{-3}) \\
&= \frac{32s_g}{\omega}\mathrm{Tr}((F^c{}_\kappa{}^\rho F^{b\sigma\nu})\partial_\rho\partial_\sigma\Box^{-3}) \\
&= \mathrm{Tr}(U_\kappa{}^{\nu cb,\rho\sigma}\partial_\rho\partial_\sigma\Box^{-3}), \qquad (A34)
\end{aligned}
$$

where we introduced the following definition

$$
U_\kappa{}^{\nu cb,\rho\sigma} = \frac{32s_g}{\omega}F^c{}_\kappa{}^\rho F^{b\sigma\nu}. \qquad (A35)
$$

Using formula (4.60) from the Barvinsky-Vilvovisky physics report for the particular case $n = 3$,

$$
\nabla_{\mu_1}\nabla_{\mu_2}\frac{\hat{1}}{\Box^3}\delta(x,y)|^{\mathrm{div}}_{y=x} = \frac{i\ln L^2}{16\pi^2}\frac{g^{1/2}g^{(1)}_{\mu_1\mu_2}}{2^1 2!}\hat{1}. \qquad (A36)
$$

The last can be rewritten in our case (flat spacetime) as

$$
\partial_\rho\partial_\sigma\frac{\hat{1}}{\Box^3}\delta(x,y)|^{\mathrm{div}}_{y=x} = \frac{i\ln L^2}{64\pi^2}\eta_{\rho\sigma}\hat{1}. \qquad (A37)
$$

Therefore,

$$
\begin{aligned}
\mathrm{Tr}(U_\kappa{}^{\nu cb,\rho\sigma}\partial_\rho\partial_\sigma\Box^{-3}) &= \mathrm{tr}\left(U_\kappa{}^{\nu cb,\rho\sigma}\frac{i\ln L^2}{64\pi^2}\eta_{\rho\sigma}\right) \\
&= \frac{i\ln L^2}{64\pi^2}\mathrm{tr}(U_\kappa{}^{\nu cb,\rho\sigma}\eta_{\rho\sigma}) \\
&= \frac{32s_g}{\omega}\frac{i\ln L^2}{64\pi^2}\mathrm{tr}(F^c{}_\kappa{}^\rho F^{b\sigma\nu}\eta_{\rho\sigma}) \\
&= \frac{s_g}{\omega}\frac{i\ln L^2}{2\pi^2}\mathrm{tr}(F^c{}_{\kappa\sigma}F^{b\sigma\nu}) \\
&= \frac{s_g}{\omega}\frac{i\ln L^2}{2\pi^2}\mathrm{tr}(F^c{}_{\nu\sigma}F^{b\sigma\nu}) \\
&= \frac{-s_g}{\omega}\frac{i\ln L^2}{2\pi^2}\mathrm{tr}(F^c{}_{\mu\nu}F^{b\mu\nu}) \\
&= \frac{-s_g}{\omega}\frac{i\ln L^2}{2\pi^2}F^a{}_{\mu\nu}F^{a\mu\nu}.
\end{aligned}
$$

The relation between the cutoff scale L and the infinitesimal parameter epsilon in dimensional regularization can be read in [21] [Formula (4.38)],

$$
\frac{1}{2-\frac{D}{2}} = \ln L^2, \qquad \frac{D}{2} = 2 - 0^+ = 2 - \frac{\epsilon}{2} \Rightarrow \ln L = \frac{1}{\epsilon}. \qquad (A38)
$$

Finally, the killer's contribution to the divergent part of the above functional trace is given by

$$
-\frac{1}{\epsilon}\frac{i}{\pi^2}\frac{s_g}{\omega}F^2. \qquad (A39)
$$

Hence, the divergent part of the one-loop effective action is

$$
\begin{aligned}
\Gamma^{(1)}_{\mathrm{div}} &= \frac{i}{2}\mathrm{Tr}\ln H_\mu{}^\nu = -\frac{1}{\epsilon}\frac{i}{2}\frac{i}{\pi^2}\frac{s_g}{\omega}\int d^4xF^2 \\
&= \frac{1}{\epsilon}\left(\frac{1}{2\pi^2}\frac{s_g}{\omega}\right)\int d^4xF^2 \coloneqq \frac{1}{\epsilon}\beta^{s_g}_\alpha\int d^4xF^2. \qquad (A40)
\end{aligned}
$$

b. The second killer

The second killer consists in taking the trace in a different way, namely,

$$
s_gF^a_{\mu\nu}F^{b\mu\nu}(\mathcal{D}^2)^{n-2}F^a_{\rho\sigma}F^{b\rho\sigma} = s_gF^a_{\mu\nu}F^{b\mu\nu}(\mathcal{D}^2)^{n-2}F^b_{\rho\sigma}F^{a\rho\sigma}. \qquad (A41)
$$

Variation of the above expression in respect to the gauge background (again up to square field strength order) is

$$\begin{aligned}\delta^2(s_g F^a_{\mu\nu} F^{b\mu\nu} (\mathcal{D}^2)^{n-2} F^a_{\rho\sigma} F^{b\rho\sigma}) &= 2s_g \delta(F^a_{\mu\nu} F^{b\mu\nu})(\mathcal{D}^2)^{n-2} \delta(F^a_{\rho\sigma} F^{b\rho\sigma}) \\ &= 2s_g (F^{a\mu\nu} \delta F^b_{\mu\nu} + \delta F^a_{\mu\nu} F^{b\mu\nu}) \Box^{n-2} (F^a_{\rho\sigma} \delta F^{b\rho\sigma} + \delta F^a_{\rho\sigma} F^{b\rho\sigma}) \\ &= 2s_g [F^{a\mu\nu} F^{b\rho\sigma} (\delta F^b_{\mu\nu}) \Box^{n-2} (\delta F^a_{\rho\sigma}) + F^a_{\rho\sigma} F^{b\mu\nu} (\delta F^a_{\mu\nu}) \Box^{n-2} (\delta F^{b\rho\sigma}) \\ &\quad + 2F^{a\mu\nu} F^a_{\rho\sigma} (\delta F^b_{\mu\nu}) \Box^{n-2} (\delta F^{b\rho\sigma})] \\ &= 4s_g F^{a\mu\nu} F^{b\rho\sigma} (\delta F^b_{\mu\nu}) \Box^{n-2} (\delta F^a_{\rho\sigma}) + 4s_g F^{a\mu\nu} F^a_{\rho\sigma} (\delta F^b_{\mu\nu}) \Box^{n-2} (\delta F^{b\rho\sigma}) \\ &= 4s_g F^{a\mu\nu} F^{b\rho\sigma} (2\partial_{[\mu} \delta A^b_{\nu]}) \Box^{n-2} (2\partial^{[\rho} \delta A^{a\sigma]}) + 4s_g F^{a\mu\nu} F^a_{\rho\sigma} (2\partial_{[\mu} \delta A^b_{\nu]}) \Box^{n-2} (2\partial^{[\rho} \delta A^{b\sigma]}) \\ &= 16 s F^{a\mu\nu} F^{b\rho\sigma} \partial_\mu \delta A^b_\nu \Box^{n-2} \partial^\rho \delta A^{a\sigma} + 16 s F^{a\mu\nu} F^a_{\rho\sigma} \partial_\mu \delta A^b_\nu \Box^{n-2} \partial^\rho \delta A^{b\sigma}. \end{aligned} \tag{A42}$$

Integrating by parts, we get from this expression (neglecting derivatives of the background fields and commutation of derivatives)

$$\begin{aligned} &- 16 s_g F^{a\mu\nu} F^{b\rho\sigma} \delta A^b_\nu \partial_\mu \Box^{n-2} \partial^\rho \delta A^{a\sigma} - 16 s_g F^{a\mu\nu} F^a_{\rho\sigma} \delta A^b_\nu \partial_\mu \Box^{n-2} \partial^\rho \delta A^{b\sigma} \\ &\quad = -16 s_g F^{a\mu\nu} F^{b\rho\sigma} \delta A^b_\nu \partial_\mu \partial^\rho \Box^{n-2} \delta A^{a\sigma} - 16 s_g F^{a\mu\nu} F^a_{\rho\sigma} \delta A^b_\nu \partial_\mu \partial^\rho \Box^{n-2} \delta A^{b\sigma} \\ &\quad = 16 s_g F^{b\mu\sigma} F^{a\rho\nu} \delta A^a_\mu (\partial_\rho \partial_\sigma \Box^{n-2}) \delta A^b_\nu + 16 s_g F^{c\mu\sigma} F^{c\rho\nu} \delta A^a_\mu (\delta^{ab} \partial_\rho \partial_\sigma \Box^{n-2}) \delta A^b_\nu \\ &\quad = 16 s_g \delta A^a_\mu (F^{b\mu\sigma} F^{a\rho\nu} \partial_\rho \partial_\sigma \Box^{n-2} + F^{c\mu\sigma} F^{c\rho\nu} \delta^{ab} \partial_\rho \partial_\sigma \Box^{n-2}) \delta A^b_\nu. \end{aligned} \tag{A43}$$

We now check whether it is the self-adjoint part of the operator (to this level), namely,

$$\begin{aligned} 16 s_g \delta A^a_\mu (F^{b\mu\sigma} F^{a\rho\nu} \partial_\rho \partial_\sigma \Box^{n-2} + F^{c\mu\sigma} F^{c\rho\nu} \delta^{ab} \partial_\rho \partial_\sigma \Box^{n-2}) \delta A^b_\nu &= 16 s_g \delta A^b_\nu (F^{b\mu\sigma} F^{a\rho\nu} \partial_\rho \partial_\sigma \Box^{n-2} + F^{c\mu\sigma} F^{c\rho\nu} \delta^{ab} \partial_\rho \partial_\sigma \Box^{n-2}) \delta A^a_\mu \\ &= 16 s_g \delta A^a_\mu F^{b\mu\sigma} F^{a\rho\nu} \partial_\rho \partial_\sigma \Box^{n-2} \delta A^b_\nu \\ &\quad + 16 s_g \delta A^a_\mu F^{c\mu\sigma} F^{c\rho\nu} \delta^{ab} \partial_\rho \partial_\sigma \Box^{n-2} \delta A^b_\nu. \end{aligned} \tag{A44}$$

The relevant part of the operator H contributing to the beta function is

$$H_\mu{}^{\nu ab} = \omega \delta^\nu_\mu \delta^{ab} \Box^{n+1} + \cdots + 16 s_g (F^{b\mu\sigma} F^{a\rho\nu}) \partial_\rho \partial_\sigma \Box^{n-2} + 16 s_g (F^{c\mu\sigma} F^{c\rho\nu}) \delta^{ab} \partial_\rho \partial_\sigma \Box^{n-2} + \cdots. \tag{A45}$$

Now, we have to take the trace of the logarithm of the above operator H, namely,

$$\begin{aligned} \mathrm{Tr} \ln H_\mu{}^{\nu ab} &= \mathrm{Tr} \ln (\omega \delta^\nu_\mu \delta^{ab} \Box^{n+1} + \cdots + 16 s_g (F^{b\mu\sigma} F^{a\rho\nu}) \partial_\rho \partial_\sigma \Box^{n-2} + 16 s (F^{c\mu\sigma} F^{c\rho\nu}) \delta^{ab} \partial_\rho \partial_\sigma \Box^{n-2} + \cdots) \\ &= \mathrm{Tr} \ln \left[\omega \delta^\kappa_\mu \delta^{ac} \Box^{n+1} \left(\delta^\nu_\kappa \delta^{cb} + \cdots + 16 s_g \omega^{-1} (F^b{}_\kappa{}^\sigma F^{c\rho\nu}) \partial_\rho \partial_\sigma \frac{1}{\Box^3} + 16 s_g \omega^{-1} (F^d{}_\kappa{}^\sigma F^{d\rho\nu}) \delta^{cb} \partial_\rho \partial_\sigma \frac{1}{\Box^3} + \cdots \right) \right] \\ &= (n+1) \mathrm{Tr} \ln (\delta^\kappa_\mu \delta^{ac} \Box) \\ &\quad + \mathrm{Tr} \ln (\delta^\nu_\kappa \delta^{cb} + \cdots + 16 s_g \omega^{-1} (F^b{}_\kappa{}^\sigma F^{c\rho\nu}) \partial_\rho \partial_\sigma \Box^{-3} + 16 s_g \omega^{-1} (F^d{}_\kappa{}^\sigma F^{d\rho\nu}) \delta^{cb} \partial_\rho \partial_\sigma \Box^{-3} + \cdots). \end{aligned} \tag{A46}$$

We concentrate on the second contribution, and we expand it in Taylor series, namely,

$$\begin{aligned} &\mathrm{Tr} \ln (\delta^\nu_\kappa \delta^{cb} + \cdots + 16 s_g \omega^{-1} (F^b{}_\kappa{}^\sigma F^{c\rho\nu}) \partial_\rho \partial_\sigma \Box^{-3} + 16 s_g \omega^{-1} (F^d{}_\kappa{}^\sigma F^{d\rho\nu}) \delta^{cb} \partial_\rho \partial_\sigma \Box^{-3} + \ldots) \\ &\quad = \mathrm{Tr} (16 s_g \omega^{-1} (F^b{}_\kappa{}^\sigma F^{c\rho\nu}) \partial_\rho \partial_\sigma \Box^{-3} + 16 s_g \omega^{-1} (F^d{}_\kappa{}^\sigma F^{d\rho\nu}) \delta^{cb} \partial_\rho \partial_\sigma \Box^{-3}) \\ &\quad = 16 \frac{s_g}{\omega} \mathrm{Tr} ((F^b{}_\kappa{}^\sigma F^{c\rho\nu}) \partial_\rho \partial_\sigma \Box^{-3} + (F^d{}_\kappa{}^\sigma F^{d\rho\nu}) \delta^{cb} \partial_\rho \partial_\sigma \Box^{-3}) \\ &\quad = 16 \frac{s_g}{\omega} \mathrm{Tr} ((F^b{}_\kappa{}^\sigma F^{c\rho\nu} + F^d{}_\kappa{}^\sigma F^{d\rho\nu} \delta^{cb}) \partial_\rho \partial_\sigma \Box^{-3}) = \mathrm{Tr} (U_\kappa{}^{\nu cb, \rho\sigma} \partial_\rho \partial_\sigma \Box^{-3}), \end{aligned} \tag{A47}$$

where we defined

$$U_\kappa{}^{\nu cb, \rho\sigma} = 16 \frac{s_g}{\omega} (F^b{}_\kappa{}^\sigma F^{c\rho\nu} + F^d{}_\kappa{}^\sigma F^{d\rho\nu} \delta^{cb}). \tag{A48}$$

Using again (A36) and (A37), the trace (A47) is

$$\begin{aligned}
\mathrm{Tr}(U_\kappa{}^{\nu cb,\rho\sigma}\partial_\rho\partial_\sigma\Box^{-3}) &= \mathrm{tr}\left(U_\kappa{}^{\nu cb,\rho\sigma}\frac{i\ln L^2}{64\pi^2}\eta_{\rho\sigma}\right) = \frac{i\ln L^2}{64\pi^2}\mathrm{tr}(U_\kappa{}^{\nu cb,\rho\sigma}\eta_{\rho\sigma}) \\
&= 16\frac{s_g}{\omega}\frac{i\ln L^2}{64\pi^2}\mathrm{tr}((F^b{}_\kappa{}^\sigma F^{c\rho\nu} + F^d{}_\kappa{}^\sigma F^{d\rho\nu}\delta^{cb})\eta_{\rho\sigma}) = \frac{s_g}{\omega}\frac{i\ln L^2}{4\pi^2}\mathrm{tr}(F^b{}_{\kappa\rho}F^{c\rho\nu} + F^d{}_{\kappa\rho}F^{d\rho\nu}\delta^{cb}) \\
&= \frac{s_g}{\omega}\frac{i\ln L^2}{4\pi^2}\mathrm{tr}(F^b{}_{\nu\rho}F^{c\rho\nu} + F^d{}_{\nu\rho}F^{d\rho\nu}\delta^{cb}) = -\frac{s_g}{\omega}\frac{i\ln L^2}{4\pi^2}\mathrm{tr}(F^b{}_{\nu\rho}F^{c\nu\rho} + F^d{}_{\nu\rho}F^{d\nu\rho}\delta^{cb}) \\
&= -\frac{s_g}{\omega}\frac{i\ln L^2}{4\pi^2}[\mathrm{tr}(F^b{}_{\mu\nu}F^{c\mu\nu}) + F^d{}_{\mu\nu}F^{d\mu\nu}\mathrm{tr}(\delta^{cb})] = -\frac{s_g}{\omega}\frac{i\ln L^2}{4\pi^2}[F^b{}_{\mu\nu}F^{b\mu\nu} + F^d{}_{\mu\nu}F^{d\mu\nu}N_G] \\
&= -\frac{s_g}{\omega}\frac{i\ln L^2}{4\pi^2}F^b{}_{\mu\nu}F^{b\mu\nu}(N_G+1) = -\frac{s_g}{\omega}\frac{i\ln L^2}{4\pi^2}F^2(N_G+1), \qquad (A49)
\end{aligned}$$

where $N_G = \delta^{cc}$ is the number of generators of the Lie group. Using again $\ln L = 1/\epsilon$, we end up with the following contribution to the beta function:

$$-\frac{1}{\epsilon}\frac{i}{2\pi^2}\frac{s_g}{\omega}F^2(N_G+1). \qquad (A50)$$

Hence, the divergent part of one-loop effective action equals

$$\Gamma^{(1)}_{\mathrm{div}} = \frac{i}{2}\mathrm{Tr}\ln H_\mu{}^{\nu ab} = -\frac{1}{\epsilon}\frac{i}{2}\frac{i}{2\pi^2}\frac{s_g}{\omega}(N_G+1)\int d^4xF^2 = \frac{1}{\epsilon}\frac{N_G+1}{4\pi^2}\frac{s_g}{\omega}\int d^4xF^2. \qquad (A51)$$

All the results in the text are obtained making the replacement $n \to \gamma$.

[1] D. Anselmi, http://renormalization.com.
[2] L. Modesto and L. Rachwal, Nucl. Phys. **B900**, 147 (2015); **B889**, 228 (2014); P. Don, S. Giaccari, L. Modesto, L. Rachwal, and Y. Zhu, J. High Energy Phys. 08 (2015) 038; Y. D. Li, L. Modesto, and L. Rachwal, J. High Energy Phys. 12 (2015) 173.
[3] Yu. V. Kuz'min, Yad. Fiz. **50**, 1630 (1989) [Sov. J. Nucl. Phys. **50**, 1011 (1989)].
[4] E. T. Tomboulis, arXiv:hep-th/9702146v1.
[5] V. P. Frolov, Phys. Rev. Lett. **115**, 051102 (2015); V. P. Frolov, A. Zelnikov, and T. de Paula Netto, J. High Energy Phys. 06 (2015) 107; V. P. Frolov, arXiv:1411.6981; J. High Energy Phys. 05 (2014) 049; V. P. Frolov and G. A. Vilkovisky, Phys. Lett. **106B**, 307 (1981); Report No. IC-79-69.
[6] L. Modesto, J. W. Moffat, and P. Nicolini, Phys. Lett. B **695**, 397 (2011); P. Nicolini, A. Smailagic, and E. Spallucci, Phys. Lett. B **632**, 547 (2006); E. Spallucci, A. Smailagic, and P. Nicolini, Phys. Rev. D **73**, 084004 (2006); P. Nicolini, Int. J. Mod. Phys. A **24**, 1229 (2009).
[7] C. Bambi, D. Malafarina, and L. Modesto, Phys. Rev. D **88**, 044009 (2013).
[8] C. Bambi, D. Malafarina, and L. Modesto, Eur. Phys. J. C **74**, 2767 (2014).
[9] G. Calcagni, L. Modesto, and P. Nicolini, Eur. Phys. J. C **74**, 2999 (2014).
[10] A. S. Koshelev, Classical Quantum Gravity **30**, 155001 (2013); A. S. Koshelev and S. Y. Vernov, Phys. Part. Nucl. **43**, 666 (2012); A. S. Koshelev, Romanian Journal of Physics **57**, 894 (2012); S. Y. Vernov, Phys. Part. Nucl. **43**, 694 (2012); A. S. Koshelev and S. Y. Vernov, Phys. Part. Nucl. Lett. **11**, 960 (2014); A. S. Koshelev, L. Modesto, L. Rachwal, and A. A. Starobinsky, arXiv:1604.03127.
[11] L. Modesto, T. d. P. Netto, and I. L. Shapiro, J. High Energy Phys. 04 (2015) 098.
[12] L. Modesto, Phys. Rev. D **86**, 044005 (2012); Astron. Rev. **8**, 4 (2013); arXiv:1202.0008; arXiv:1302.6348.
[13] D. Anselmi, Phys. Rev. D **89**, 125024 (2014).
[14] D. Anselmi, Phys. Rev. D **89**, 045004 (2014).
[15] J. W. Moffat, Eur. Phys. J. Plus **126**, 43 (2011).
[16] N. J. Cornish, Mod. Phys. Lett. A **07**, 631 (1992).
[17] N. V. Krasnikov, Teor. Mat. Fiz. **73**, 235 (1987) [Theor. Math. Phys. **73**, 1184 (1987)].
[18] G. V. Efimov, *Nonlocal Interactions* in Russian] (Nauka, Moscow, 1977); V. A. Alebastrov and G. V. Efimov, Commun. Math. Phys. **31**, 1 (1973); **38**, 11 (1974); G. V. Efimov, Teor. Mat. Fiz. **128**, 395 (2001) [Theor. Math. Phys. **128**, 1169 (2001)].
[19] O. Babelon and M. A. Namazie, J. Phys. A **13**, L27 (1980).
[20] M. Piva, Report No. etd-11192014-163737, 2014.

[21] A. O. Barvinsky and Vilkovisky, Phys. Rep. **119**, 1 (1985).
[22] I. A. Batalin and G. A. Vilkovisky, Phys. Lett. **102B**, 27 (1981); Phys. Rev. D **28**, 2567 (1983); **30**, 508(E) (1984); S. Weinberg, *The Quantum Theory of Fields*, Vol. II (Cambridge University Press, Cambridge, England, 1995).
[23] M. W. Kalinowski, Fortschr. Phys. **64**, 190 (2016).
[24] T. D. Lee and G. C. Wick, Phys. Rev. D **2**, 1033 (1970); Nucl. Phys. **B9**, 209 (1969).
[25] I. L. Shapiro, Phys. Lett. B **744**, 67 (2015); L. Modesto and I. L. Shapiro, Phys. Lett. B **755**, 279 (2016).
[26] L. Modesto, Nucl. Phys. **B909**, 584 (2016).
[27] D. Anselmi, Phys. Rev. D **45**, 4473 (1992); **48**, 680 (1993); **48**, 5751 (1993).
[28] M. Jaccard, M. Maggiore, and E. Mitsou, Phys. Rev. D **88**, 044033 (2013).