

W_∞ algebras, Hawking radiation, and information retention by stringy black holes

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We have argued previously, based on the analysis of two-dimensional stringy black holes, that information in stringy versions of four-dimensional Schwarzschild black holes (the singular regions of which are represented by appropriate Wess-Zumino-Witten models) is retained by quantum W symmetries when the horizon area is not preserved due to Hawking radiation. It is key that the exactly marginal conformal world-sheet operator representing a massless stringy particle interacting with the black hole requires a contribution from W_∞ generators in its vertex function. The latter correspond to delocalized, nonpropagating, string excitations that guarantee the transfer of information between the string black hole and external particles. When infalling matter crosses the horizon, these topological states are excited via a process: (stringy black hole) + infalling matter \rightarrow (stringy black hole)*, where the black hole is viewed as a stringy state with a specific configuration of W_∞ charges that are conserved. Hawking radiation is then the reverse process, with conservation of the W_∞ charges retaining information. The Hawking radiation spectrum near the horizon of a Schwarzschild or Kerr black hole is specified by matrix elements of higher-order currents that form a phase-space $W_{1+\infty}$ algebra. We show that an appropriate gauging of this algebra preserves the horizon two-dimensional area classically, as expected because the latter is a conserved Noether charge.

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I. INTRODUCTION AND SUMMARY

The black-hole information problem was posed by the discoveries by Bekenstein [1] and Hawking [2] that four-dimensional black holes have thermodynamical properties such as temperature and nonzero entropy and so must be described by mixed quantum-mechanical states. These discoveries led Hawking [3], in particular, to suggest that information would be lost across the black-hole horizon, giving rise to a transition from a pure to a mixed state.

String theory has provided an explicit theoretical laboratory for probing the black-hole information problem, notably using the two-dimensional black-hole solution found by Witten [4] that has an $SU(1,1)/U(1)$ coset structure [5,6], coupled with dualities [7], and subsequently using four-dimensional stringy black holes constructed using D-branes [8,9]. We argued [10]

that two-dimensional black holes carry an infinite set of quantum numbers associated with a W_∞ symmetry and that these W charges preserve the lost information *in principle*, though this information could not *in practice* be extracted. These observations apply also to spherically symmetric four-dimensional stringy black holes [11], which have horizons of which the geometry is also encoded in an $SU(1,1)/U(1)$ coset structure that possesses a similar W_∞ symmetry and an associated infinite set of W “hair” [12].

D -brane constructions provide examples of four-dimensional black holes of which the microstates could be counted explicitly [8], with results consistent with the Bekenstein-Hawking entropy, suggesting again that the “lost” information was retained in principle. However, there were still questions how this information was transferred from external particles to these microstates, how it was stored, and whether the information retained by the microstates could in practice be extracted from observations of radiated particles.

An interesting, complementary approach to the black-hole information problem has been taken recently by

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Strominger and collaborators [13,14]. They have argued that spherically symmetric four-dimensional black holes carry a previously undiscussed infinite set of soft gravitational hair associated with Bondi-van-de-Burg-Metzner-Sachs (BBMS) supertranslations and super-rotations [15–17] on the retarded null infinity \mathcal{I}^+ . These correspond to vacua that differ by the addition of soft gravitons that could in principle be measured via the gravitational memory effect. Hawking, Perry, and Strominger (HPS) [17] also discuss in detail an analogous infinite set of inequivalent electromagnetic gauge configurations corresponding to soft electromagnetic hair that differ by the addition of soft photons and could also be distinguished in principle by measurements.¹ Assuming additionally that soft hair localized spatially to much less than the Planck length would not be excited in a physical process, HPS found [17] an effective number of soft degrees of freedom proportional to the horizon area, like the Hawking-Bekenstein entropy. At the present stage of this proposal, it is unclear whether supertranslations and the corresponding super-rotations and electromagnetic gauge configurations can encode the information carried by the incoming particles [19], and HPS did not claim to have resolved the information paradox. Moreover, the connection between this approach and one based on stringy black holes is not apparent, and it is the latter approach that we adopt in this paper.

In contrast to the recent work of ‘t Hooft [20], we base our work on string theory at a fundamental level. Our analysis below of supertranslations on the horizon, viewed as the recoil of a D -brane induced by infalling matter, is more similar in spirit to the recent work of Polchinski [21], in which a shock-wave approximation was used to calculate the shift of a generator of the horizon caused by an incoming wave packet. We recall also the work of Refs. [22,23] where the fluctuation of the black-hole horizon induced by infalling matter is argued to play an important role in retaining information. In this connection, we recall that in string theory the interaction of a massless particle with a black hole is represented by a conformal operator on the world sheet of the string, which is exactly marginal *if and only if* contributions from W_∞ generators are included in its vertex function [6,24]. The corresponding renormalization-group (RG) β function would be nonzero without these contributions, leading to a monotonic increase in entropy.

As reviewed recently in Ref. [25], we consider $W_{1+\infty}$ symmetry [26,27] to be essential for “balancing the black-hole information books.” This symmetry is manifest in the effective two-target-space dimensional string theories that describe the excitations in the near-horizon geometry of a spherically symmetric stringy black hole. It is larger than the symmetry group of supertranslations, and we consider

that the latter are insufficient to retain fully the black-hole information. The purpose of this article is to study further the role of these W_∞ symmetries in retaining information and to establish a connection with the work of Refs. [28,29], in which the spectrum of Hawking radiation is related to matrix elements of such an infinite-dimensional W_∞ algebra. In this connection, we recall [10,24] that w_∞ (the classical limit of the quantum W_∞ symmetry) is the algebra of transformations that preserve the two-dimensional phase-space volume of massless (“tachyonic”) stringy matter propagating in the background of a stringy black hole.

The quantum version of this w algebra is a symmetry of the quantum scattering matrix of the corresponding two-dimensional string theory [6,30], in the sense that the operator product expansion between two appropriate vertex operators reproduces the corresponding W algebra. In the flat space-time case (where the string theory is just a two-dimensional Liouville theory), the operators corresponding to the discrete higher-spin operators of the W algebra are discretized tachyon operators. However, as already mentioned, in the presence of a black hole, at the quantum level, the corresponding W_∞ symmetries necessarily mix massless and massive stringy states that are topological and delocalized [6]. The admixture of W_∞ generators in the exactly marginal vertex operator of a massless string excitation makes manifest the transfer of information between a stringy black hole and external particles.

In addition to its spectrum, another important feature of the Hawking radiation is its sparsity at asymptotic infinity. This feature can be explained by viewing the black holes as “particles” and the Hawking radiation process as successive two-body decays [31]. As we shall see, such a picture emerges naturally in string theory, but with essential differences, since the black holes are represented as string states that are completely integrable, due to their infinity of conserved W_∞ charges.

In this article, we synthesize these ideas from a current perspective. In Sec. II, we review the properties and formalism underlying two-dimensional stringy black holes and their embedding in four-dimensional space-times, using them to represent spherically symmetric black-hole configurations in four space-time dimensions. The underlying two-dimensional coset structure of the singularity is essential for the *complete integrability* of these systems and the retention of information via the corresponding w_∞ space-time symmetry current algebras that characterize them. A review of these symmetries and the construction of the corresponding currents in terms of discrete higher-spin string states is also given in this section. We also review in this context our approach to quantifying information loss by representing entropy increase in an evaporating black hole in string theory as world-sheet renormalization-group flow between fixed points in the stringy world-sheet field theory space. We also mention

¹See, however, Ref. [18] for a critical discussion and references to earlier work.

briefly the case of a string-inspired infinitely colored $SU(\infty)$ black hole [32], where the conserved entropy is linked classically to a w_∞ symmetry living on the horizon and preserving its area [25,33,34]. This property is consistent with viewing the classical black-hole area as a conserved Noether charge [35]. In Sec. III, we discuss Hawking radiation in generic four-dimensional spherically symmetric black holes and its link [28] with another form of $W_{1+\infty}$ phase-space symmetry algebra associated with higher-spin currents that correspond to fluxes of Hawking radiation in the effective two-dimensional field theory representation of the Hawking radiation thermal spectrum in the near-horizon geometry. We compare the situation with our stringy case, which involves nonthermal higher-spin discrete delocalised states, also associated with phase-space $W_{1+\infty}$ algebras. In Sec. IV, we discuss the gauging of this algebra and its connection with the preservation of the horizon area at the classical level, interpreted as the conservation of a Noether charge, with some technical details provided in the Appendix. Our conclusions are presented in Sec. V where, in view of the role of W symmetries in preserving information during matter infall or Hawking radiation, the evaporation of the stringy black hole is viewed as successive two-body decays.

II. STRINGY BLACK HOLES, $W_{1+\infty}$ SYMMETRY ALGEBRAS AND INFORMATION RETENTION

A. Two-dimensional stringy black holes as prototypes

The stringy black hole in two dimensions [4] can be formulated as a world-sheet Wess-Zumino-Witten σ -model on the coset space $SL(2, R)_k/U(1)$, where $k = 9/4$ is the Kac-Moody algebra level. The two-dimensional target-space metric induced by the conformal invariance condition for this world-sheet σ -model corresponds to a Euclidean black-hole background

$$ds^2 = dr^2 + \tanh^2 r d\tilde{\theta}^2, \quad (1)$$

where r is a radial coordinate and $\tilde{\theta}$ is a compact ‘‘angular’’ coordinate that plays the role of an external temperature variable. The Euclidean space-time (1) looks like a semi-infinite cigar.

Far away from the singularity, this space-time is asymptotically flat, and the corresponding string theory is a $c = 1$ two-target-dimensional (2D) (X, ρ) Liouville theory with a spacelike Liouville mode ρ and a central charge deficit $Q = 2\sqrt{2}$. In this asymptotic theory, the tachyon field operator is the standard ‘‘massless matter’’ operator of the 2D Liouville theory [30],

$$T^\pm(p) = e^{ipX + (\pm|p| - \sqrt{2})\rho}, \quad (2)$$

which describes a positive-norm physical state. The spectrum also contains discrete higher-spin $J = 0, \frac{1}{2}, 1, 2, \dots$

states with the third component of the internal angular momentum $M = \{-J, -J + 1, \dots, J - 1, J\}$, which also have positive norm, with vertex operators having the asymptotic forms

$$\begin{aligned} \psi_{J,M}^{(\pm)} &\sim (H_-)^{J-M} \psi_{J,J}^{(\pm)} \sim (H_+)^{J+M} \psi_{J,-J}^{(\pm)}, \\ H_\pm &= \int \frac{dz}{2\pi i} T^\pm(\pm\sqrt{2}), \end{aligned} \quad (3)$$

where \sim denotes a normalization factor, H_\pm represents the zero modes of the ladder operator of the $SU(2)$ Kac-Moody currents at the self-dual radius of the $c = 1$ conformal field theory, and $\psi_{J,\pm J}^{(\pm)} = T^\pm(\pm\sqrt{2}J)$.

The operator products (OPs) of these asymptotic discrete states form a classical w_∞ algebra [30],

$$\begin{aligned} &\int \frac{dz}{2\pi i} \psi_{J_1, M_1}^+(z) \psi_{J_2, M_2}^+(0) \\ &= (J_2 M_1 - J_1 M_2) \psi_{J_1+J_2-1, M_1+M_2}^+(0) + \dots, \end{aligned} \quad (4)$$

where the \dots are explained below. This symmetry algebra leads to an infinity of conserved currents and charges and can be used to construct the (nonlocal) effective action in the $c = 1$ 2D target-space Liouville theory [30,36] and its matrix-model extension [37]. The classical w_∞ symmetry is elevated at the quantum level to a $W_{1+\infty}$ algebra with a central extension, where the extra subscript ‘‘1’’ is due to the inclusion of a $U(1)$ spin-1 current.

The $c = 1$ theory has also *zero-norm (ghostlike) discrete gauge states* (DGSs), which, as explained in Ref. [38], also satisfy the physical-state Virasoro-operator conditions, like the positive-norm discrete states, and represent the same $W_{1+\infty}$ algebra as the positive-norm states. These gauge states have discrete momentum values corresponding exactly to the physical positive-norm discrete states. Detailed formulas for these discrete states are given in Ref. [38] and will not be repeated here. For information, we give the expression for one class of these states, in the ψ^+ sector,

$$\begin{aligned} G_{J,M}^+ &= (J + M + 1)^{-1} \int \frac{dz}{2\pi i} (\psi_{1,-1}^+(z) \psi_{J,M+1}^+(0) \\ &+ \psi_{J,M+1}^+(z) \psi_{1,-1}^+(0)), \end{aligned} \quad (5)$$

where the \dots on the right-hand-side of (4) correspond to ambiguities in the addition of such DGSs.

It can be shown [38] that the right-hand-side of (5) [and all other discrete states in the $(-)$ sector] can be expressed in terms of products of Shur polynomials $S_k(\{-\frac{1}{k}\sqrt{2}\partial_z^k X(0)\})$, with S_k defined through

$$\exp\left(\sum_{k=1}^{\infty} a_k x^k\right) = \sum_{k=0}^{\infty} S_k(\{a_k\}) x^k, \quad (6)$$

$$\{a_k\} \equiv \{a_i : i \in Z_k\},$$

with $S_k = 0$ for $k < 0$. The presence of $\partial_z^k X$ terms implies [26] that the OP of the DGS operators obeys a (classical) w_∞ symmetry algebra of the form (4) [38]:

$$\int \frac{dz}{2\pi i} G_{J_1 M_1}^+(z) G_{J_2 M_2}^+(0) = (J_2 M_1 - J_1 M_2) G_{J_1+J_2-1, M_1+M_2}^+(0). \quad (7)$$

The DGSs are the carriers of the conserved w_∞ charges. They decouple from the correlation functions of the physical states and can be considered as the symmetry parameters of the theory.

In the specific context of strings propagating in target-space black-hole backgrounds, the massless matter particle (tachyon) is associated with the vertex operator,

$$\phi_{-1/2,0,0}^{c,-c} = (g_{++}g_{--})^{-\frac{1}{2}} F\left(\frac{1}{2}; \frac{1}{2}; 1; \frac{g_{+-}g_{-+}}{g_{++}g_{--}}\right), \quad (8)$$

where F denotes a hypergeometric function and g_{ab} , $a, b = +, -$ represent the components of a generic $SL(2, \mathbb{R})$ element. The asymptotic form of this operator gives the massless tachyon vertex operator of the $c=1$ two-dimensional string theory.

This operator is exactly marginal *only* in the *flat-space* two-dimensional string theory. In the presence of a Euclidean black-hole background (or Minkowski, the latter being obtained by analytic continuation of the compact ‘‘temperature’’ variable in the cigar metric of Ref. [4]), the corresponding *exactly marginal* operator is [6]

$$L_0^1 \bar{L}_0^1 = \phi_{-1/2,0,0}^{c,-c} + i(\psi^{++} - \psi^{--}) + \dots, \quad (9)$$

where

$$\psi^{\pm\pm} \equiv :(\bar{J}^\pm)^N (J^\pm)^N (g_{\pm\pm})^{j+m-N}, \quad (10)$$

with $J^\pm \equiv (k-2)(g_{\pm\mp} \partial_z g_{\pm\pm} - g_{\pm\pm} \partial_z g_{\pm\mp})$ and $\bar{J}^\pm \equiv (k-2)(g_{\mp\pm} \partial_z g_{\pm\pm} - g_{\pm\pm} \partial_z g_{\mp\pm})$, where k is the Wess-Zumino level parameter [4]. The combination $\psi^{++} - \psi^{--}$ generates a level-1 massive string mode, and the dots in Eq. (9) represent operators that generate higher-level massive string states.

Another example of an exactly marginal operator is $L_0^2 \bar{L}_0^2 = \psi^{++} + \psi^{--} + \psi^{+-} + \psi^{-+} + \dots$, which also involves in an essential way operators for massive string modes. The coupling corresponding to this world-sheet deformation of the coset model is associated with a global rescaling of the target space-metric [6] and therefore to a global constant shift of the dilaton field. Thus, it produces shifts in the black-hole mass [4], which is of relevance for

the discussion of an evaporating black hole with a time-dependent mass.²

As discussed in Refs. [10,24], these modes are solitonic, with fixed energy and momentum. As such, they are completely *delocalized* in space-time. The vertex operators describing these discrete positive-norm states satisfy a $W_{1+\infty}$ symmetry algebra [with the inclusion of a vector spin-1 $U(1)$ current], which is the gauge symmetry of the string theory in the black-hole background. The conserved W charges are carried by the corresponding discrete gauge states (with zero norm), the asymptotic forms of which coincide with the discrete gauge states of Ref. [38] [cf. Eq. (5)].

B. World-sheet renormalization-group flow, time and the increasing entropy of evaporating black holes

Since the flat-space tachyon vertex operator (8) is not exactly marginal in a black-hole background, the corresponding world-sheet RG β function is nonvanishing. This induces world-sheet renormalization-group flow in the noncritical string theory model of a quantum black hole, which can be identified with the entropy increase rate of an evaporating black hole in string theory, as we now review [24].

The pertinent world-sheet action has deformations of the form

$$S_\sigma = S_\sigma^* + \int_\Sigma d^2\xi \sqrt{-\gamma} g^i V_i, \quad (11)$$

where S_σ^* is a conformal fixed-point σ -model action, summation over repeated indices is implied, γ is a world-sheet metric, and $\{g^i\}$ is an (infinite in general) set of target-space fields associated with the corresponding vertex operators V_i . In a two target-space-dimensional setting, where we start our discussion for instructive purposes [10,24], the only propagating multiplet consists of massless scalar fields (misleadingly called tachyons), whereas the graviton and (the infinity of) higher-spin multiplets are topological ‘‘massive’’ states with discrete momenta. As we discussed in Ref. [11] and review later in this article, such topological states exist also in higher-dimensional target space-times, so their presence is rather generic.

Introducing a global world-sheet RG scale μ , and defining $\mathcal{T} \equiv \ln \mu$, we consider renormalized world-sheet couplings g^i , ‘‘running’’ with \mathcal{T} , according to the following world-sheet RG equation,

²In the two-dimensional case, the horizon has no area, whereas in the four-dimensional case discussed later, the area, and hence the entropy, is proportional to the mass. The fact that mass and hence entropy are changed by $L_0^2 \bar{L}_0^2$, which is not an invariant of $W_{1+\infty}$, is a consequence of the observation that $W_{1+\infty}$ symmetry is required for information retention and hence entropy conservation.

$$\frac{dg^i}{dT} \equiv \beta^i, \quad (12)$$

where β_i is the RG β function for the ‘‘coupling’’ g^i of the two-dimensional world-sheet field theory.³

The presence of relevant deformations in the world-sheet σ model, when only the propagating modes of the string multiplet are taken into account, calls for Liouville dressing [39]. This requires introducing an extra σ -model field, the Liouville mode ρ that, for supercritical deformations such as those in the black-hole case of interest [11], has a target timelike signature, and may be identified with target time, flowing irreversibly [24]. The Liouville mode can be viewed as a local RG scale, as required because the string world sheet is generally curved. For a nonmarginal coupling g^i , e.g., a massless tachyon field in the target-space of a spherically symmetric stringy black hole, the Liouville RG equation replacing (12) is given by [39]

$$\ddot{g}^i + Q(\rho_0)\dot{g}^i = -\beta^i(g) = -\mathcal{G}^{ij} \frac{\delta V[g]}{\delta g^j}, \quad \dot{A} \equiv \frac{d}{d\rho_0} A, \quad (13)$$

where the overdot denotes a derivative with respect the Liouville world-sheet zero mode ρ_0 (with a canonically normalized term in the world-sheet action [39]); β is the world-sheet RG function for the (relevant) coupling g^i , given by (12), expressed as a gradient of an effective target-space potential for the fields/couplings $\{g^i\}$; and \mathcal{G}_{ij} is the Zamolodchikov metric in the space of string theory models [24]. The quantity $Q(\rho_0)$ is the square root of the central charge deficit Q^2 , which is > 0 for supercritical σ models [24,40].

Upon identifying (see Ref. [24]) the flow of the world-sheet zero mode of the Liouville field with the opposite flow of the RG scale \mathcal{T} and of the target time (for the supercritical string case, as explained in detail in Ref. [24]),

$$t = -\mathcal{T} (= -\rho_0), \quad (14)$$

and taking into account [40] that in the case of bosonic target-space background fields g^i there are tachyonic mass shifts in their dispersion relations, i.e., terms in the potential,

$$V[g]_{\text{boson}} \ni -\frac{1}{2} Q^2 g^i g^j \delta_{ij}, \quad (15)$$

one may move half of this term to the left-hand side of the Liouville RG equation (13). One may then write the

³More strictly speaking, in string theory, the target-space dependences of the couplings g^i imply some diffeomorphism variations, which lead to the replacement of the corresponding RG functions β^i by the corresponding Weyl anomaly coefficients, but such complications are not relevant for our main arguments below, so we omit them here. For details, see Ref. [10].

Liouville flow as an equation of motion for the fields g^i obtained from a one-dimensional gauge theory with time (and only) (t) component of the gauge potential $2A_0 = iQ(t)$,

$$\mathcal{L}_{\text{Liouv}} = (D_t g^i)^\dagger \mathcal{G}_{ij} D_t g^j - \tilde{V}[g], \quad D_t = \partial_t - iA_0(t), \quad (16)$$

where \dagger denotes Hermitian conjugation and \tilde{V} contains the remaining part of the tachyonic mass shift terms of the form $-\frac{1}{4} Q^2 g^i g^j \delta_{ij}$ along with the rest of the interaction terms for the fields g^i . We shall make use of (16) when we discuss scalar modes of Hawking radiation in the horizon of a spherically symmetric black hole.

We also remark that, upon the identification (14) in a noncritical string theory [24], the effective central charge $C[g] = Q^2$ obeys an irreversible flow equation,

$$\frac{d}{d\rho_0} Q^2 = -\frac{d}{dt} Q^2 \sim -\beta^i \mathcal{G}_{ij} \beta^j < 0, \quad (17)$$

provided the Zamolodchikov metric is positive definite. This happens in string theories with constant dilaton backgrounds and in Euclidean target space-times, as used to describe a finite-temperature black-hole space-time.

Given that the central charge counts the degrees of freedom of a system, the relation (17) implies that the system flows toward an increase of its degrees of freedom (and thus its entropy) as time t progresses, or equivalently during the evolution from infrared to ultraviolet on the world sheet, whenever a string propagates in a nonconformal background. Hence, the entropy associated with massless ‘‘matter’’ increases inexorably, i.e, information is lost, *if the higher-level string modes in (9) are neglected*. Conversely, if the discrete solitonic string modes (9), (10) are taken into account, the corresponding RG β function vanishes, and entropy does not increase with the world-sheet RG flow, which we identify with the target-space temporal time flow in our approach. Thus, there is no information loss; it is stored by the higher-level string modes.

These topological modes are not detectable in a local scattering experiment, leading to an apparent ‘‘loss’’ of quantum coherence, which is an artifact of the phenomenological truncation of the scattering process within a local effective field theory framework. Associated with this apparent loss of quantum coherence, there is an apparent ‘‘increase’’ in entropy at a rate quantified by the right-hand side of (17), since the *truncated* RG β^i functions of the nonmarginal propagating modes do not vanish. Nevertheless, the conserved W-hair charges are in principle measurable, and ways for doing so in principle have been outlined in Ref. [11].

C. Embedding of the two-dimensional black hole in four space-time dimensions

The coset singularity structure of the two-dimensional stringy black hole and generic properties of its associated

discrete states have counterparts for spherically symmetric black-hole configurations in four space-time dimensions. This can be seen by embedding the two-dimensional coset describing the singularity in a four-dimensional space-time [11] with the structure $SU(1, 1)/U(1) \otimes S^2$, where S^2 is a two-dimensional manifold with the topology of the sphere that is to be identified with the horizon of the four-dimensional black hole. This enables us to place our arguments on the importance of W_∞ symmetries in a more generic perspective.

To see this, consider a spherically symmetric gravitational background of black-hole type, which is a solution of the generalized Einstein equations in the effective field theory derived from string theory. The metric tensor takes the form

$$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta + e^{W(r,t)} d\Omega^2, \quad (18)$$

where $W(r, t)$ is a nonsingular function, the r, t coordinates are denoted by $x^{\alpha,\beta}$, and the line element on a fixed spherical surface is denoted by $d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2$. The standard Schwarzschild solution describing a spherically symmetric four-dimensional black hole can be cast in the form (18) with an appropriate transformation of variables.

When written in Kruskal-Szekeres coordinates, the Schwarzschild solution takes the form [41]

$$ds^2 = -\frac{32M^3}{r} e^{-\frac{r}{2M}} dudv + r^2 d\Omega^2, \quad (19)$$

where r is a function of u, v , given by

$$\left(\frac{r}{2M} - 1\right) e^{\frac{r}{2M}} = -uv. \quad (20)$$

Although the two-dimensional metric components depend on the variables u, v , the black-hole solution is nevertheless static.⁴ Changing variables to

$$\begin{aligned} e^{-\frac{r}{4M}} u &= u', \\ e^{-\frac{r}{4M}} v &= v', \end{aligned} \quad (21)$$

we can write the two-dimensional metric in the form

$$g_{bh}(u', v') = \frac{e^{D(u', v')}}{1 - u'v'} du' dv', \quad (22)$$

with the scale factor being given by $16M^2 e^{-\frac{r'(u', v')}{2M}} J(u', v')$, where r' is the coordinate r reexpressed in terms of the coordinates u', v' , and J is the Jacobian of the

⁴Moreover, in pure gravity, all the *classical* spherically symmetric solutions to the equations of motion obtained from higher-derivative gravitational actions with an arbitrary number of curvature tensors are *static* [42], and a similar result holds for stringy black holes at tree level.

transformation of the area element $dudv$. The metric (22) is a conformally rescaled form of Witten's two-dimensional black-hole solution [4]. The latter is described by an exact conformal field theory, so the same is true after this conformal rescaling, which simply represents a change of renormalization scheme according to the σ -model point of view. The function $D(u, v)$ in (22) can also be regarded as part of the two-dimensional dilaton in the given renormalization scheme.

The global properties, such as singularities, are the same as in the two-dimensional string case. In particular, the infinite-dimensional W symmetry associated with the $SU(1, 1)/U(1)$ coset structure of the dilaton-graviton sector in the two-dimensional model is also a model-independent feature of spherically symmetric four-dimensional string configurations. Such structures are associated with topological solitonic nonpropagating states, which are spherically symmetric solutions of the low-energy equations of motion obtained from string theory in a manifold with topology $SU(1, 1)/U(1) \times \mathcal{M}^2$, where \mathcal{M}^2 is a two-dimensional manifold of constant curvature. They correspond to jumps in the number of degrees of freedom at discrete energy-momentum values, resulting from the relaxation of certain gauge theory constraints, as shown below. The simplest example of such a manifold is where $\mathcal{M}^2 = S^2$, which describes a spherically symmetric four-dimensional black hole. The infinite number of associated discrete topological (nonpropagating) states, with discrete energy-momentum values, couple to the massless propagating tachyon string matter and render the associated σ -model action conformally invariant, as in the two-dimensional stringy black hole [4], described above.

A w_∞ symmetry also arises in the phase space of matter coupled to another example of a two-dimensional string theory embedding in four dimensions [9], namely a four-dimensional extremal solitonic black-hole background in the context of $N = 2, D = 4$ supergravity. This is a BPS (Bogomolnyi-Prasad-Sommerfeld) solution that interpolates between an $AdS_2 \times H^2$ geometry (where AdS_2 refers to the radial-coordinate/time part of the space-time and H^2 is a hyperbolic two-dimensional manifold of constant curvature describing the angular part of the space-time) that characterizes the space-time near the horizon of the black hole and a maximally supersymmetric AdS_4 space-time at large radial distances. It was shown in Ref. [9] that a quantum-mechanical massive particle with nontrivial magnetic charge in the near-horizon geometry has dynamics described by a one-spatial-dimensional Hamiltonian H , with a w_∞ symmetry that preserves the two-dimensional phase-space area symplectic form $\Omega = dp \wedge dq - dH \wedge dt$, where q is the spatial coordinate, p is the canonical momentum and t is the time.

The energy spectrum of this particle is continuous and bounded from below, $E > 0$, but the ground state is non-normalizable, with an IR divergence, which was regularized

in Ref. [9] by putting the system in a box. The IR-regularized system is also invariant under a w_∞ that contains a Virasoro symmetry (26), which can be associated with the asymptotic symmetries of the AdS_2 space-time, i.e., the diffeomorphisms that leave invariant the AdS_2 metric, the quantum version of which includes a central extension. Such asymptotic symmetries are symmetries of the quantum-gravity scattering matrix for the full four-dimensional $\text{AdS}_2 \times H^2$ extremal black hole of Ref. [9].⁵

The particle system in this example is characterized by an infinite set of conserved charges corresponding to diffeomorphisms preserving a two-dimensional symplectic area two-form Ω defined for the “coordinates” x, y :

$$\Omega = dy \wedge dx. \quad (23)$$

These area-preserving diffeomorphisms are generated by the quantities

$$v_m^\ell = y^{\ell+1} x^{\ell+m+1}, \quad (24)$$

where ℓ and m are integers. The Poisson brackets of these generators satisfy the classical w_∞ algebra

$$\{v_m^\ell, v_{m'}^{\ell'}\} = [m(\ell' + 1) - m'(\ell + 1)] v_{m+m'}^{\ell+\ell'}. \quad (25)$$

This includes a Virasoro symmetry generated by the operators $L_n = v_n^0$, the Poisson brackets of which obey the algebra

$$\{L_n, L_m\} = (m - n)L_{m+n}, \quad (26)$$

which is a subalgebra of the w_∞ algebra (25).

In the case of the effective two-dimensional mechanics of particles in the near-horizon geometry of the four-dimensional black hole, the roles of the symplectic coordinates x, y are played by appropriate combinations of the *phase-space* coordinates of the particle [9], and hence the system is completely integrable. The role of the w_∞ algebra for the particle near the horizon of the four-dimensional black hole of Ref. [9] is exactly analogous to that preserving the phase-space area for massless tachyonic string matter in the two-dimensional stringy black hole [10]—or its four-dimensional extension with topology $\text{SU}(1, 1)/\text{U}(1) \times S^2$ —as discussed previously.

It was suggested in Ref. [10] that the infinite set of W charges in the quantum version of the classical w_∞ symmetry provides an *infinite set of discrete gauge hair* (called W-hair), which maintains the quantum coherence for the two-dimensional stringy black hole. This was based on the fact that the quantum-gravity scattering matrix obtained from correlation functions of marginal world-

sheet vertex operators is invariant under the quantum W symmetry. The existence of an infinite set of conservation laws for a particle in the near-horizon geometry of the black hole discussed in Ref. [9], and hence an infinite set of conserved charges v_m^ℓ , also guarantees quantum coherence by retaining information during the black-hole evaporation. The elevation of the classical phase-space area-preserving w_∞ symmetry algebra to a quantum algebra capable of preserving coherence necessarily involves the discrete massive topological states of the string, as discussed above. It is their mixing with the propagating massless matter states that guarantees the conformal symmetry of the corresponding vertex operators in a stringy black-hole background [10] and thereby preserves quantum coherence.

There are infinitely many discrete topological gauge states in a string theory in a D -dimensional target space, which have a similar nature to those in the two-dimensional case [30,36,38] that appear in (5). The existence of these states can be understood by examining the gauge conditions for a rank- n tensor multiplet,

$$D^{\mu_1} A_{\mu_1 \mu_2 \dots \mu_n} = 0, \quad (27)$$

where D_μ is a (curved-space) covariant derivative. To see this, consider for example weak gravitational perturbations around flat space with a linear dilaton field of the form $\Phi(X) = Q_\mu X^\mu$. In this case, one may Fourier transform (27) to find

$$(p + Q)^{\mu_1} \tilde{A}(k)_{\mu_1 \mu_2 \dots \mu_n} = 0. \quad (28)$$

This shows that the number of degrees of freedom increases at the discrete momentum $p = -Q$. Since this momentum is fixed, it corresponds to a complete delocalized state, which should be regarded as a quasitopological, non-propagating solitonlike state. Such states carry a small statistical weight in ordinary string theories, relative to the continuous spectrum of the continuum string modes. However, these discrete states assume particular importance when strings propagate in spherically symmetric four-dimensional background space-times. These backgrounds are effectively two-dimensional, and Ward identities of the form (27) may be used to gauge away all the transverse modes of higher-rank tensors, except for these topological modes. These s -wave topological modes constitute the final stages of the evaporation of four-dimensional spherically symmetric black holes [10] and play key roles in maintaining quantum coherence [10,11].

These discrete solitonic states can be regarded as singular gauge configurations [12], similar to the discrete gauge states in two-dimensional strings (5), and their conserved W charges could in principle be measured via generalized Aharonov-Bohm phase effects. These higher-spin topological string states also leave their imprints via

⁵An asymptotic symmetry of the quantum-gravity scattering matrix under supertranslations of generic black-hole backgrounds has been examined in Refs. [13,14].

selection rules in the scattering matrix, where they appear as (resonance) poles at discrete energies and momenta. There is an infinite set of such black-hole soliton states in the stringy black-hole case, which can be classified by the quadratic Casimir and “magnetic” quantum numbers of an internal symmetry group [12]. These resonances appear at calculable energies and decay into distinctive combinations of light final-state particles.

The stringy scattering matrix in such a black-hole background is well defined in general, since the correlation functions among the appropriate exactly marginal vertex operators on the world sheet are unitary, since these operators contain an infinity of nonpropagating topological states as well as the parts corresponding to propagating string states. In practice, these delocalized states cannot be detected in laboratory scattering experiments, since they involve a finite number of localized (in space-time) particle states. Hence, there would apparently be decoherence from the point of view of a local low-energy observer, even though there are no pathologies in the full stringy theory of quantum gravity.

D. Horizon-area-preserving w_∞ classical symmetries

The area of a black hole can be regarded classically as a conserved Noether charge [35]. The diffeomorphisms on the black-hole horizon that preserve the horizon area belong to the classical w_∞ symmetry algebra of transformations of the horizon coordinates. These preserve the area of the horizon of an isolated spherically symmetric four-dimensional black hole by construction, thereby conserving its entropy in agreement with the results of Ref. [35].

A classical w_∞ symmetry may manifest itself in a number of ways, as we now discuss. The horizon of a stringy black hole may be regarded as a thick brane, which is known to be describable as an $SU(\infty)$ gauge theory [33,34], leading to the infinitely colored $SU(\infty)$ black-hole model of Ref. [32]. In this approach, open-string states terminating on the horizon brane carry the $SU(\infty)$ charges. Such an $SU(\infty)$ symmetry is classically isomorphic to the classical w_∞ algebra preserving a two-dimensional area, which can be identified in this case with the area of the horizon of the infinitely colored, spherically symmetric black hole.

As we have discussed, there is a classical w_∞ algebra that preserves the two-dimensional area of an “internal space” with the topology of a sphere [26,27]. The question is then whether this internal sphere can be identified with the horizon of the four-dimensional spherically symmetric Schwarzschild black hole. To analyze this issue, we considered in Ref. [25] examples of four-dimensional spherically symmetric black holes with infinitely colored hair, which realize explicitly a classical w_∞ symmetry as discussed in the previous paragraph. This black-hole solution of $SU(N \rightarrow \infty)$ gauge theory is formulated in a four-dimensional anti-de Sitter (AdS) space-time with negative cosmological constant, which plays the role of a regulator

making the black-hole solution well defined [32]. This AdS regulator was given physical significance via the AdS/CFT bulk/boundary correspondence and turned out to be physically important, as we argued in Ref. [25] and discuss briefly below.

Our interest in these black holes is motivated by the classical isomorphism between $SU(N \rightarrow \infty)$ and w_∞ [26,33].⁶ To see this correspondence, one replaces the $SU(2)$ generators S_i by rescaled versions, $T_i \equiv \frac{2}{N} S_i$, and finds [25] that in the limit $N \rightarrow \infty$ the $N^2 - 1$ matrices $T_{\ell m}^{(N)}$ obey

$$\frac{N}{2i} [T_{\ell,m}^{(N)}, T_{\ell',m'}^{(N)}] \rightarrow \{Y_{\ell,m}, Y_{\ell',m'}\}, \quad N \rightarrow \infty, \quad (29)$$

where we denote by $Y_{\ell m}(\theta, \phi)$ the spherical harmonics on the sphere S^2 . It is well known that the (classical) Poisson algebra of the spherical harmonics is that of $SDiff(S^2)$, the infinite set of area-preserving diffeomorphisms on the sphere,

$$\begin{aligned} \{Y_{\ell,m}, Y_{\ell',m'}\} &= \frac{M(\ell + \ell' - 1, m + m')}{M(\ell, m)M(\ell', m')} \\ &\times (\ell' m - \ell m') Y_{\ell+\ell'-1, m+m'} \\ &+ \sum_{n=1} g_{2n}(\ell, \ell') C_{\ell, m, \ell', m'}^{\ell+\ell'-1-2n, m+m'} \\ &\times Y_{\ell+\ell'-1-2n, m+m'}, \end{aligned} \quad (30)$$

where the structure constants C are given in the fourth paper in Ref. [26] and M and g_{2n} are normalization factors. This algebra is isomorphic to the classical area-preserving w_∞ algebra.

Writing the gauge fields of the $SU(N \rightarrow \infty)$ gauge theory using the matrices $T_{\ell, m}^{(N)}$ as a basis, obeying (29), we see that this area-preserving diffeomorphism symmetry preserves the horizon area in this example of an infinitely colored gauge black hole, upon identification of the internal sphere S^2 with the horizon sphere of the spherically symmetric $SU(\infty)$ black hole. In this case, the entropy of the black hole can be preserved classically by the w_∞ symmetry and its associated infinite set of W hair.

Viewing this $SU(\infty)$ gauge model as a low-energy limit of some string-theory black hole indicates, according to our world-sheet RG interpretation of the target time, that the classical conservation of the horizon area reflects the conformal invariance of the corresponding world-sheet field theory, which guarantees that the right-hand side of (17) vanishes because the β^i functions of relevant combinations of the couplings g^i are zero. Here, the set $\{g^i\}$ comprises the graviton $G_{\mu\nu}$ and the $SU(\infty)$ gauge field modes: $A_\mu^a, a = 1 \dots \infty$. In Ref. [25], the AdS regulator of the

⁶The geometry of $SU(N)$ gauge theories with finite N is noncommutative [34], commutativity appearing only in the limit $N \rightarrow \infty$.

$SU(\infty)$ model was given physical significance in guaranteeing the vanishing of these β functions, upon mixing the graviton with the gauge-field contributions, analogously to our previous stringy black-hole case, where higher-spin states mix with the massless string matter contributions to ensure conformal invariance on the world sheet.

Proceeding further by describing the horizon of the four-dimensional black hole as a 2-brane [25], one can treat the recoil when an open string, representing a matter state, meets the horizon surface. When one of the ends of the open string attaches to the horizon, the latter recoils so as to conserve momentum. This horizon recoil causes it to fluctuate in a way characterized by a logarithmic conformal field theory on the world sheet [43,44] that describes the transfer of information [45]. Such a recoiling black-hole horizon may be represented as a “thick” stack of $N \rightarrow \infty$ concentric D -branes. In the case of a macroscopic black hole, the horizon of which is large compared to the wavelength of the infalling matter, these concentric branes may be regarded approximately as a stack of parallel and flat $N \rightarrow \infty$ -branes, which are equivalent to an $SU(N \rightarrow \infty)$ gauge theory [46]. This can be seen intuitively by considering the different ways [$N^2 - 1$ in the case of an $SU(N)$ gauge theory] in which an open string can be attached to a stack of N D -branes. Therefore, when matter reaches the thick brane model of the black-hole horizon, the recoil is described by excitations that carry the $SU(\infty)$ charges. These correspond to the infinite W hair of the black hole and the classical horizon area-preserving w_∞ symmetry discussed above. In this example, the $SU(\infty)$ symmetry plays a role as a coherence-preserving symmetry of the scattering matrix involving the $SU(\infty)$ black hole.

However, the presence of classical infinite-dimensional horizon-preserving symmetries is more generic than the above example and is in fact associated with the observation of Ref. [28] that the entire spectrum of Hawking radiation of black holes (either Schwarzschild as we consider here or rotating Kerr type) may be represented in terms of higher-spin currents of the associated conformal field theory near the horizon surface. We proceed to discuss this case next and then place it in the context of our string theory considerations.

III. HAWKING RADIATION FROM GENERIC SCHWARZSCHILD BLACK HOLES AND $W_{1+\infty}$ ALGEBRAS

In order to discuss explicitly the connection of Hawking radiation to our $W_{1+\infty}$ symmetries, we first review briefly some interesting results [28,29] connecting Hawking radiation in rather generic, nonstringy, spherically symmetric black holes to a $W_{1+\infty}$ algebra realized by higher-spin states associated with the moments of the Hawking radiation. These $W_{1+\infty}$ currents are sourced by background fields of these higher-spin states, which can be identified with the discrete gauge states in our stringy approach.

The effective two-dimensional conformal field theory representation [47] of the dynamics of matter fields in the near-horizon geometry of a spherically symmetric black hole [10] is crucial for the connection of Hawking radiation to W_∞ algebras. It is known that the outgoing Hawking quanta radiated from the horizon of a spherically symmetric black hole break general covariance. As shown in Ref. [47], this symmetry is restored through the cancellation of the corresponding gravitational anomalies in the quantum-gravity path integral for a $(1 + 1)$ -dimensional black body at the Hawking temperature of a black hole [2]. One can represent the effective two-dimensional field theory of the Hawking radiation at the black-hole horizon as a two-dimensional field theory with an infinity of two-dimensional conformal quantum fields obeying a thermal spectrum, with the left movers corresponding to infalling matter and the right movers to outgoing matter.

We restrict ourselves here to Schwarzschild black holes, which emit Hawking radiation with a Planck distribution

$$N^\pm(\omega) = \frac{1}{e^{\beta\omega} \pm 1}, \quad (31)$$

where ω is the frequency (energy) of the radiated quantum, β is the inverse of the Hawking temperature [2], and $+$ ($-$) for fermions (bosons). The full spectrum of the radiation is encoded in the infinite set of moments of the Hawking radiation spectrum [29],

$$F_{2n}^+ = \int_0^\infty \frac{d\omega}{2\pi} \omega^{n-1} N^+(\omega) = (1 - 2^{1-2n}) \frac{B_{2n}}{8\pi n} \kappa^{2n}, \quad (32)$$

or

$$F_{2n}^- = \int_0^\infty \frac{d\omega}{2\pi} \omega^{n-1} N^-(\omega) = \frac{B_{2n}}{8\pi n} \kappa^{2n}, \quad (33)$$

where the B_{2n} are the Bernoulli numbers and $\kappa = 2\pi/\beta$ is the surface gravity of the black hole. For example, the energy flux is given by the second moment of $N^\pm(\omega)$, $F_2(\omega) = \int_0^\infty \frac{d\omega}{2\pi} \omega N^\pm(\omega)$.

It was proposed in Ref. [29] that, generalizing the connection of the energy flux of the black hole to a spin-2 current with matrix element $F_2(\omega)$, the higher moments F_{2n} , $n > 1$ could be identified as matrix elements of phenomenological higher-spin currents that could be regarded as generalizations of the energy-momentum tensor.

These higher-spin currents can be expressed [29] as (appropriately normal-ordered) products of two-dimensional boson and fermion fields and their space-time derivatives. In terms of the light-cone variables

$$u = t + r_*, \quad v = t - r_*, \quad r_* \cdot \frac{\partial r_*}{\partial r} = f(r)^{-1}, \quad (34)$$

where r_* is the so-called tortoise coordinate, a metric of the form, $ds^2 = f(r)dt^2 - \frac{1}{f(r)}dr^2$, which may be used to represent the effective two-dimensional space-time in the near-horizon geometry of the spherically symmetric black hole, becomes that of a conformally flat space-time:

$$ds^2 = e^{2\varphi(t,r^*)}(dt^2 - d(r^*)^2) = e^{2\varphi(u,v)}dudv, \\ e^{2\varphi(u,v)} = f(r). \quad (35)$$

Outgoing scalar radiation is then described by holomorphic (v -independent) currents of the form [29]

$$\text{Outgoing radiation scalar currents: } J_{uu\dots u}^B \\ = \text{linear combinations of: } (-1)^{n+m} \partial_u^m \phi \partial_u^{2n-m} \phi :, \quad (36)$$

and for fermions $\psi(u)$, one has

$$\text{Outgoing radiation fermion currents: } J_{uu\dots u}^F \\ = \text{linear combinations of: } \bar{\psi} \partial_u^n \psi :, \quad (37)$$

where $:\dots:$ denotes the appropriate normal ordering, as defined in Ref. [29].

When representing the higher moments of the Hawking radiation in terms of conformal fields on the horizon, there are ambiguities in the relative coefficients of the various terms appearing in the holomorphic currents (36) and (37), and the currents are not normalizable in general. These issues were resolved in Ref. [28] by requiring that the coefficients be fixed by a symmetry principle, specifically by postulating that there is a W_∞ algebra on the horizon of the black hole, generalizing the Virasoro algebra.⁷

In the case of flat two-dimensional space-times, upon Euclideanization and replacing the coordinates u, v by the complex variables z, \bar{z} , respectively, the bosonic w_∞ currents with conformal spin s can be written as follows [28],⁸

$$j_{z\dots z}^{(s)B} = q^{s-2} \frac{2^{s-3} s!}{(2s-3)!!} \sum_{k=1}^{s-1} (-1)^k \left[\frac{1}{s-1} \binom{s-1}{k} \binom{s-1}{s-k} \right] \\ : \partial_z^k \phi(z) \partial_{\bar{z}}^{s-k} \bar{\phi}(\bar{z}) :, \quad (38)$$

where q is a complex deformation parameter [26]. The holomorphic free fields $\phi(z)$ are assumed to have the

⁷Their higher-derivative holomorphic structure, $\partial_z^n \chi$ ($\chi = \phi, \psi$) [26], led one to expect that the currents could realize such an infinite-dimensional algebra.

⁸From now on, we use complex scalar fields to represent the Hawking spectrum, so as to make a direct correspondence with the $W_{1+\infty}$ -algebra formalism.

following nonvanishing two-point function $\langle \phi(z) \bar{\phi}(z') \rangle = -\ln(z - z')$. The spin $s = 2$ current is independent of the deformation parameter q , as expected because this current can be unambiguously identified with the holomorphic stress tensor

$$j_{uu}^{(2)} = -2\pi T_{uu}^{\text{hol}},$$

but the higher-spin currents depend on parameter q . It can be fixed [28] by demanding that the currents (38), when covariantized as appropriate in the curved space-time of the spherical symmetric black hole, reproduce the higher moments of the Hawking flux, leading to $q = -i/4$ [28].

The work of Ref. [28] also discussed appropriately normalized higher-spin currents for the fermion fields, working in a Kerr metric with Kerr parameter $a \neq 0$, corresponding to a nontrivial angular momentum of the rotating black hole. In this case, the corresponding gravitational covariant derivative for Dirac fermions in the effective two-dimensional space-time (35) has an extra gauge U(1) potential, proportional to the parameter a . The fermionic currents then satisfy a $W_{1+\infty}$ algebra, as a result of the inclusion of the conformal spin-1, U(1) gauge vector current in the construction [28]. One may take the limit $a \rightarrow 0$ to consider the Schwarzschild black-hole case, and although the background gauge potential in such a case vanishes, nevertheless the corresponding current is nonzero, which implies that the U(1) spin-1 current has to be included in the construction in the case of fermion matter. For completeness, we note that in a Euclideanized flat space-time with coordinates z, \bar{z} these currents are given by [26]

$$j_{z\dots z}^{(s)F} = q^{s-2} \frac{2^{s-3} s!}{(2s-3)!!} \sum_{k=1}^{s-1} (-1)^k \frac{1}{s} \binom{s-1}{s-k}^2 \\ : \partial_{\bar{z}}^{s-k} \Psi^\dagger(z) \partial_z^{k-1} \Psi(z) :, \quad (39)$$

where $\Psi(z)$ denotes the two-dimensional conformal fermion field and $:\dots:$ again denotes the appropriate normal ordering.

The fact that the currents $j^{(s)}$ are quadratic in the fields is an important feature that we explore in the next section, when we discuss the gauging of the W algebra and its role in providing a mechanism for the classical conservation of the horizon area of a black hole. This feature is carried over in the covariantized currents, which are at most linear functions of the currents $j^{(s)}$ and their gravitationally covariant derivatives $\nabla j^{(s)}$, as can be explicitly seen by the expressions of the first few of them, that we list below for concreteness [28]. The bosonic currents with conformal spin ≤ 6 are [28]

$$\begin{aligned}
 J_{uu}^{(2)B} &= j_{uu}^{(2)B} - \frac{\hbar}{6} T, \\
 J_{uuu}^{(3)B} &= j_{uuu}^{(3)B}, \\
 J_{uuuu}^{(4)B} &= j_{uuuu}^{(4)B} + \frac{\hbar}{30} T^2 + \frac{2}{5} T J_{uu}^{(2)B}, \\
 J_{uuuuu}^{(5)B} &= j_{uuuuu}^{(5)B} + \frac{10}{7} T J_{uuu}^{(3)B}, \\
 J_{uuuuuu}^{(6)B} &= \left(-\frac{2\hbar}{63} T^3 + \frac{5\hbar}{504} (\partial_u T)^2 - \frac{\hbar}{126} T \partial_u^2 T - \frac{2}{3} T^2 J_{uu}^{(2)B} - \frac{1}{21} T \nabla_u^2 J_{uu}^{(2)B} - \frac{1}{21} (\partial_u^2 T) J_{uu}^{(2)B} + \frac{5}{42} (\partial_u T) \nabla_u J_{uu}^{(2)B} \right. \\
 &\quad \left. - \frac{5}{21} \Gamma T \nabla_u J_{uu}^{(2)B} - \frac{5}{21} \Gamma^2 T J_{uu}^{(2)B} + \frac{5}{21} \Gamma (\partial_u T) J_{uu}^{(2)B} \right) - \frac{5}{24} T J_{uuuu}^{(4)B} + j_{uuuuuu}^{(6)B}, \tag{40}
 \end{aligned}$$

where

$$\Gamma = \partial_u \varphi, \quad T = \partial_u^2 \varphi - \frac{1}{2} (\partial_u \varphi)^2 = \partial_u^2 \varphi - \frac{1}{2} \Gamma^2, \tag{41}$$

where $\varphi(u, v)$ was defined in (35) and is associated with the effective two-dimensional curved metric near the horizon written in a conformally flat form.

The corresponding fermionic currents with spin ≤ 5 are

$$\begin{aligned}
 J_u^{(1)F} &= j_u^{(1)F} + \frac{i\hbar}{2q} A_u, \\
 J_{uu}^{(2)F} &= \left(-\frac{T}{12} \right) \hbar - 2A_u J_u^{(1)F} + j_{uu}^{(2)F}, \\
 J_{uuu}^{(3)F} &= -4J_u^{(1)F} A_u^2 - 4J_{uu}^{(2)F} A_u + \left(\frac{8A_u^3}{3} - \frac{A_u T}{3} \right) \hbar + \frac{T J_u^{(1)F}}{6} + j_{uuu}^{(3)F}, \\
 J_{uuuu}^{(4)F} &= +\hbar \left(4A_u^4 - \frac{7TA_u^2}{5} - \frac{2}{5} (\nabla_u^2 A_u) A_u + \frac{7T^2}{240} + \frac{3}{5} (\nabla_u A_u)^2 \right) - 8J_u^{(1)F} A_u^3 - 12J_{uu}^{(2)F} A_u^2 \\
 &\quad + \left(\frac{1}{5} \nabla_u^2 J_u^{(1)F} + \frac{7T J_u^{(1)F}}{5} - 6J_{uu}^{(3)F} \right) A_u - \frac{3}{5} (\nabla_u A_u) (\nabla_u J_u^{(1)F}) + \frac{1}{5} (\nabla_u^2 A_u) J_u^{(1)F} + \frac{7T J_{uu}^{(2)F}}{10} + j_{uuuu}^{(4)F}, \\
 J_{uuuuu}^{(5)F} &= \hbar \left(\frac{32A_u^5}{5} - \frac{104TA_u^3}{21} - \frac{16}{7} (\nabla_u^2 A_u) A_u^2 + \frac{27T^2 A_u}{70} + \frac{24}{7} (\nabla_u A_u)^2 A_u + \frac{1}{35} (\nabla_u^2 T) A_u - \frac{1}{7} (\nabla_u A_u) (\nabla_u T) + \frac{2}{21} T (\nabla_u^2 A_u) \right) \\
 &\quad - 16J_u^{(1)F} A_u^4 - 32J_{uu}^{(2)F} A_u^3 + \frac{8}{7} (\nabla_u^2 J_u^{(1)F}) A_u^2 + \frac{52}{7} T J_u^{(1)F} A_u^2 - 24J_{uuu}^{(3)F} A_u^2 - \frac{24}{7} (\nabla_u A_u) (\nabla_u J_u^{(1)F}) A_u + \frac{12}{35} (\nabla_u^2 J_{uu}^{(2)F}) A_u \\
 &\quad + \frac{16}{7} (\nabla_u^2 A_u) J_u^{(1)F} A_u + \frac{52}{7} T J_{uu}^{(2)F} A_u - 8J_{uuuu}^{(4)F} A_u + \frac{1}{14} (\nabla_u T) (\nabla_u J_u^{(1)F}) - \frac{12}{7} (\nabla_u A_u) (\nabla_u J_{uu}^{(2)F}) \\
 &\quad - \frac{1}{21} T (\nabla_u^2 J_u^{(1)F}) - \frac{27T^2 J_u^{(1)F}}{140} - \frac{12}{7} (\nabla_u A_u)^2 J_u^{(1)F} - \frac{1}{70} (\nabla_u^2 T) J_u^{(1)F} + \frac{8}{7} (\nabla_u^2 A_u) J_{uu}^{(2)F} + \frac{13T J_{uuu}^{(3)F}}{7} + j_{uuuuuu}^{(5)F}, \tag{42}
 \end{aligned}$$

where A_u is a component of the background gauge potential that characterizes the motion of fermionic matter in the Kerr black hole, which is proportional to the angular momentum quantum number a . We note that the presence of \hbar in the above formulas is necessary for dimensional reasons and that the propagator of the fermionic field is given by $\langle \Psi^\dagger(z) \Psi(w) \rangle = \hbar(z-w)^{-1}$.

In the limit of the Schwarzschild black hole of interest to us here, the gauge field $a \rightarrow 0$, and so $A_u \rightarrow 0$. Thus, the fermionic current expressions become

$$\begin{aligned}
 J_u^{(1)F} &= j_u^{(1)F}, \\
 J_{uu}^{(2)F} &= \left(-\frac{T}{12}\right)\hbar + j_{uu}^{(2)F}, \\
 J_{uuu}^{(3)F} &= \frac{TJ_u^{(1)F}}{6} + j_{uuu}^{(3)F}, \\
 J_{uuuu}^{(4)F} &= +\hbar\frac{7T^2}{240} + \frac{7TJ_{uu}^{(2)F}}{10} + j_{uuuu}^{(4)F}, \\
 J_{uuuuu}^{(5)F} &= \frac{1}{14}(\nabla_u T)(\nabla_u J_u^{(1)F}) - \frac{1}{21}T(\nabla_u^2 J_u^{(1)F}) \\
 &\quad - \frac{27T^2 J_u^{(1)F}}{140} - \frac{1}{70}(\nabla_u^2 T)J_u^{(1)F} + \frac{13TJ_{uuu}^{(3)F}}{7} + j_{uuuuu}^{(5)F}.
 \end{aligned} \tag{43}$$

We now remark that, as shown in Ref. [28], the covariantized versions of the currents (38), (39) with spins higher than 2 are free of, or at most have trivial, conformal, or diffeomorphism anomalies. This is consistent with the fact that the higher moments of the Hawking radiation are expected to describe a gravitational anomaly-free theory, since only the spin-2 current (stress tensor) of the theory has diffeomorphism or conformal anomalies, and it is the requirement of their cancellation that requires the appearance of the Hawking radiation spectrum [47]. If these currents had conformal anomalies, then they would correspond to new (nongauge) quantum numbers for black holes, which would violate the no-hair theorem.

The covariant higher-spin- s currents $J_{\mu_1\mu_2\dots\mu_n}^{(s)B,F}$ are sourced by appropriate background fields $\mathcal{B}_{\mu_1\mu_2\dots\mu_n}^{(s)B,F}$,

$$J_{\mu_1\dots\mu_n}^{(s)B,F} = \frac{1}{\sqrt{g}} \frac{\delta}{\delta \mathcal{B}_{\mu_1\mu_2\dots\mu_n}^{(s)B,F}} S, \tag{44}$$

where S is the two-dimensional effective action of the Hawking radiation in the near-horizon geometry of the spherically symmetric black hole. The relevant interactions in this effective geometry are then given simply by

$$S_{\text{int}} = \int_{\text{near horizon 2D space-time}} d^2x \sqrt{g} \sum_s \sum_{\alpha=B,F} \mathcal{B}^{(s)\alpha\mu_1\dots\mu_n} J_{\mu_1\dots\mu_n}^{(s)\alpha}, \tag{45}$$

with x denoting two-dimensional space-time coordinates (e.g., in one frame $x = \{u, v\}$). The background fields $\mathcal{B}^{(s)\alpha\mu_1\dots\mu_n}$ may be taken to vanish at asymptotic spatial infinity, far away from the horizon. Due to the quadratic nature of the current, after appropriate partial integrations, the action (45) can be written schematically in the form

$$\begin{aligned}
 S_{\text{int}} &= \int_{\text{near horizon 2D space-time}} d^2x \sqrt{g} \left[\mathcal{V}(x) \right. \\
 &\quad \left. + \sum_s \sum_{\alpha=B,F} \chi^{\dagger\alpha}(x) \mathcal{F}^{(s)\alpha}(\partial_\mu) \chi^\alpha(x) \right],
 \end{aligned} \tag{46}$$

where χ^α is a scalar [$\phi(u, v)$ for $\alpha = B$] or fermionic [$\psi(u, v)$ for $\alpha = F$] field and the $\mathcal{F}(\partial_\mu)$ are appropriate functions containing multiple derivatives ∂_μ , $\mu = u, v$ with respect to the two-dimensional horizon space-time. The quantity $\mathcal{V}[\phi(x)]$, which is a χ -independent function of the scalar field $\phi(x)$ and its derivatives, plays the role of a vacuum energy term in the two-dimensional horizon effective field theory. It arises from the χ -independent terms of the covariant currents (40), (43), which are generically functions of Γ and T (i.e., of $\partial_u \phi$, $\partial_u^2 \phi$) and their covariant derivatives.

In the spin-2 case, the corresponding spin-2 current (the stress tensor) couples to the graviton field, $\int d^2x \sqrt{g} T^{\mu\nu} g_{\mu\nu}$, which is characterized by diffeomorphism invariance $\delta g_{\mu\nu} = \partial_{(\mu} \xi_{\nu)}$ for an infinitesimal diffeomorphism $\xi_\mu \rightarrow x_\mu + \xi_\mu$, provided the stress tensor is conserved.⁹ Generalizing this, the higher-spin currents, which are free from conformal and diffeomorphism anomalies [28], are conserved exactly, and their conservation is associated with an infinity of Abelian gauge symmetries of the form

$$\mathcal{B}_{\mu_1\dots\mu_n}^{(s)} \rightarrow \mathcal{B}_{\mu_1\dots\mu_n}^{(s)} + \partial_{(\mu_1} \Xi_{\mu_2\dots\mu_n)}, \tag{47}$$

where the (...) among indices indicates appropriate symmetrization. The presence of this infinite set of gauge symmetries is consistent with the no-hair theorem, as the spatial integrals of the currents correspond to conserved charges.

The existence of a W_∞ symmetry of matter in the near-horizon geometry, larger than the Virasoro algebra, results in the complete integrability of the matter system and is analogous to the cases of matter in the near-horizon geometries of black-hole structures in the context of string theory [9,10]. These W_∞ algebras are phase-space-preserving algebras, like the W_∞ algebras discussed in the stringy cases earlier. To see this, one may rewrite the (traceless) energy-momentum tensor of the two-dimensional effective theory using a point-splitting method [28], as follows (we consider scalar fields ϕ for concreteness):

$$\begin{aligned}
 T_{\mu\nu} &= \lim_{y \rightarrow 0} \partial_\mu \phi(x-y) \partial_\nu \phi(x+y) - g_{\mu\nu} (\text{stress-tensor trace}) \\
 &= \sum_{i=0} \sum_{j=0} \frac{(-1)^i}{i!j!} : y^{\mu_1} \dots y^{\mu_i} y^{\nu_1} \dots y^{\nu_j} \partial_\mu \partial_{\mu_1} \dots \partial_{\mu_i} \phi(x) \\
 &\quad \times \partial_\nu \partial_{\nu_1} \dots \partial_{\nu_j} \phi(x) :.
 \end{aligned} \tag{48}$$

This expression can be covariantized by replacing the partial derivatives by covariant derivatives, making the right-hand side of (48) a complicated expression in terms of products of the higher-spin currents discussed above with

⁹In the black-hole case, as we have discussed above, the diffeomorphism invariance is broken by the outgoing flux, but the form of the transformation is included in (47).

the y -dependent factors (48), that correspond to higher-level background tensors $\mathcal{B}_{\mu_1 \dots \mu_n}^{(s)}$ that source the higher-spin currents. For our purposes, the most important feature of (48) is the fact that the right-hand side depends not only on the coordinate x^μ but also on the coordinate $y^\mu = dx^\mu$ of the cotangent bundle, and thus on a symplectic phase-space manifold, showing that the corresponding $W_{1+\infty}$ algebra generated by the higher-spin currents of the Hawking radiation spectrum is indeed a phase-space algebra.

IV. FIELD THEORY OF HAWKING RADIATION AND HORIZON-AREA-PRESERVING CLASSICAL SYMMETRIES

In this section, we demonstrate that one may associate W symmetry with classical horizon-area-preserving diffeomorphisms, following the discussion of Ref. [25] for the $SU(\infty)$ -colored black-hole case. To this end, we first consider the completely integrable field theory system of bosonic (scalar) field currents (38) in a flat space-time and then generalize it to the curved space-time case (40).

We consider a holomorphic, i.e., one-dimensional, Euclideanized scalar field theory given by the appropriate flat space-time limit of (45), which involves only the bosonic higher-spin currents $J_{\mu_1 \dots \mu_n}^{(s)}$ that are quadratic in the fields $\phi(z)$,

$$S_{\text{int}} = \int_{\text{near-horizon 2D space-time}} dz \sum_s \mathcal{B}^{B(s)z \dots z} j_z^{B(s)}, \quad (49)$$

and we take $\mathcal{B}^{B(s)z \dots z}$ to be asymptotically constant. Note that these background fields are *not* functions of the $\phi(z)$ fields but may be functions of the holomorphic coordinate z . Introducing the Fourier transforms of the fields

$$\phi(z) = \int dp e^{ipz} \tilde{\phi}(p) \quad (50)$$

and defining

$$\mathcal{U}_{pz} \equiv \tilde{\phi}(p) e^{ipz} \neq \mathcal{U}_{zp}, \quad (51)$$

we observe that the quantity

$$\int_{-\infty}^{\infty} dz \int_{-\infty}^{\infty} dp U_{pz} = 2\pi \tilde{\phi}(0) \quad (52)$$

is proportional to \mathcal{U}_{0z} . Thus, if we view the indices p, z as spanning a set of *discrete values*: $0, 1, \dots, N-1$ with $N \rightarrow \infty$ (the set becoming continuous in the limit only), we observe that, on account of the constraint (52), the (complex) field variables \mathcal{U}_{pz} have only $N^2 - 1$ independent degrees of freedom, as $N \rightarrow \infty$. If we label these degrees of freedom as

$$U_{pz} \rightarrow f_\alpha, \alpha = 1, \dots, N^2 - 1, \quad (53)$$

the interacting action (49) becomes

$$S_{\text{near horizon scalars}} = \sum_{\alpha, \beta} f_\alpha \mathcal{C}^{\alpha\beta} f_\beta, \quad \alpha, \beta = 1, \dots, N^2 - 1, \quad (54)$$

and the (matrix) coefficients $\mathcal{C}^{\alpha\beta}$ contain terms $z^m p^n$, with m, n positive integers, where (in operator form) $p = -\partial_z$.

There is a local (gauge) symmetry characterizing the action (54) since, in the spirit of Ref. [27], one may redefine the field variables by unitary matrices \mathcal{V}_α^β ,

$$f_\alpha \rightarrow \mathcal{V}_\alpha^\beta f_\beta, \quad (55)$$

and integrate over \mathcal{V} in a path integral. The original action (54) may then be viewed as a ‘‘gauge fixed’’ version of the theory, where \mathcal{V} is fixed in an appropriate form.

The ‘‘time’’ in this Euclideanized near-horizon geometry is not the target time included in the light-cone variable z , but it is the Liouville RG time (14), which characterizes the evolution of the scalar modes $\phi(z)$. From a stringy black-hole viewpoint, the latter correspond to propagating modes, and as such they correspond to nonmarginal deformations in a σ -model that describes string propagation in the neighborhood of the black hole [6,10,11]. This Liouville time t leads to a one-dimensional adjoint Higgs model for the Hawking-radiation scalar matter, as a consequence of the Lagrangian (16), namely

$$S_H = \int dt \text{tr} \left[\frac{1}{2} (\partial_0 \hat{M}(t) - [\hat{\mathcal{A}}_0, \hat{M}](t)) (\partial^0 \hat{M}(t) - [\hat{\mathcal{A}}^0, \hat{M}](t)) - v(\hat{M}) \right], \quad (56)$$

where the index 0 denotes the time variable, the trace is over group indices, and the matrix-valued field $M(t) = f_\alpha T^\alpha$, with $\alpha = 1, \dots, N^2 - 1$, and the $(N^2 - 1) \times (N^2 - 1)$ matrices T^α form an adjoint representation of the $SU(N \rightarrow \infty)$ gauge group. The properties of such a theory connected with the W_∞ gauge symmetries are discussed in the Appendix, following the analysis of Ref. [27]. The fact that we are using an adjoint matrix representation of the $SU(\infty)$ algebra is important, because—rigorously speaking [48]—it is the large- N limit of the $(N^2 - 1) \times (N^2 - 1)$ matrix generators in the adjoint representation of the $SU(N)$ gauge group that become those of the Poisson algebra in the classical limit. In contrast, the $N \times N$ matrices in the fundamental representation of $SU(N)$ diverge and are not well defined in the $N \rightarrow \infty$ limit. The gauge potential $\hat{\mathcal{A}}_0$ is identified appropriately with the square root of the central charge deficit of the world-sheet theory $Q(t)$, according to the discussion leading to (16). The potential in our case contains at most quadratic

terms in the adjoint Higgs field \hat{M} , given the form of the action (54).

The area-preserving nature of the associated classical w_∞ symmetries is seen by viewing the action (56) as a gauge theory over an extended $(2+1)$ -dimensional space-time, where the internal space is viewed as a stereographic projection of a spherical surface (z, \bar{z}) representing the black-hole horizon of a macroscopic (semiclassical) black hole. In such a large-area black hole, the horizon can be approximated by an almost flat surface, and hence the approach [27] of constructing W_∞ gauge theories and their classical limit outlined in the Appendix applies. The corresponding adjoint Higgs action is given by (A9) with $d = 1$, and the local limit (A15) yields an invariance of the corresponding limiting action (A16) under the horizon-area-preserving diffeomorphisms (A17).

We turn next to the fermionic currents in the near-horizon geometry, which (like the bosons) can also be represented as a one-dimensional quantum-mechanical system, by labelling the holomorphic fermions as

$$\Psi(z, t) = \psi_z(t), \quad (57)$$

with t denoting the Liouville RG time. As in the bosonic case, we view the (continuous) suffix z as an internal fermion index, a limiting case of a discrete index: $\bar{z} = 1, \dots, N$, $N \rightarrow \infty$. In this case, any integration over z becomes a sum over internal fermion indices: $\int dz \sum_{\bar{z}}$. Likewise, any determinant of the two-dimensional black-hole metric and other functions of the original holomorphic variables z become functions of the (discretized) \bar{z} .

Using this representation and including the RG time dependence of the fermion fields, the fermionic conformal field theory near the black-hole horizon, with interaction terms (45), becomes a quantum-mechanical theory (field theory in $d = 1$ space-time dimension) of the fermions $\psi_z(t)$ (57) of the generic form,

$$\mathcal{S}_\psi = \int dt \sum_{\bar{z}} \psi_{\bar{z}}^\dagger(t) \left(i \frac{d}{dt} + h^{(1)}[\partial_{\bar{z}}] \right) \psi_{\bar{z}}, \quad (58)$$

where the structure $h^{(1)}[\partial_{\bar{z}}]$ represents the complicated operators in the interaction terms (45) after appropriate partial integration. [We assume that the background source fields $\mathcal{B}_{\mu_1 \dots \mu_n}$ are static, and the upper index (1) in $h^{(1)}$ indicates that it pertains to fermion bilinear terms only.] The gauge nature of the theory can be seen by observing that such constructions hide an infinite-dimensional gauge theory [27,37] (a gauged w_∞ algebra). of the fermion transformations

$$\psi_{\bar{z}} \rightarrow U_{\bar{z}}^{\bar{z}'} \psi_{\bar{z}'}, \quad (59)$$

where $U_{\bar{z}}^{\bar{z}'}$ is an $N \times N$ unitary matrix, in the fundamental representation of the $SU(N \rightarrow \infty)$ group, as opposed to the adjoint representation in the bosonic case.

The proper gauging procedure for the associated W_∞ algebra pertaining to the fermion case is also presented in the Appendix, where the fermion action (58) is represented as a $(1+2)$ -dimensional action over the extended (horizon) surface z, \bar{z} . However, because the fermions are in the fundamental representation, the appropriate Poisson large- N limit cannot be defined [48]. Indeed, according to the discussion in the Appendix, upon taking the local limit $\ell \rightarrow 0$ in (A15), by means of which one defines a classical contraction w_∞ of the quantum W_∞ algebra, one obtains a trivial fermion action $\mathcal{S}_\psi \rightarrow 0$, as $\ell \rightarrow 0$. Thus, the area-preserving nature of the classical w_∞ symmetries that characterize the black-hole horizon is realized nontrivially via the bosonic Hawking radiation fields.

The above results have been obtained in the flat space-time limit. Nevertheless, covariantizing the flat-space current theory and going to a curved space-time metric using (40) does not change qualitatively the above features of the flat space-time theory, as can readily be seen from the form of the covariant effective action (46). In a phase-space representation, the function $\mathcal{F}^{(\alpha)}(\partial_\mu)$ will still play the role of a Hamiltonian operator as in (54) and (58), and the only remnant of the curved metric would be the vacuum energy term $\int d^2x \sqrt{-g} \mathcal{V}[\varphi]$, which is invariant under the gauge W symmetries, being field ϕ, ψ independent.¹⁰

The area-preserving classical w_∞ symmetries are consistent with the view of the horizon area of a classical black hole as a conserved Noether charge [35]. On the other hand, at a quantum level, the W_∞ quantum symmetries, although they are phase-space area-preserving symmetries for matter in the near-horizon geometry that maintain the complete integrability of the matter system (and hence preserve quantum coherence [10]), do not preserve the horizon area. This feature is in agreement with the shrinking of the latter quantity with increasing time, due to the Hawking evaporation process.

V. CONCLUSIONS AND OUTLOOK

We conclude by reiterating the main points underlying the microscopic mechanism for maintaining quantum coherence and retaining information in an evaporating spherically symmetric stringy black hole. The corresponding effective theory is a two-dimensional string theory, with a singularity structure of which the dynamics is described by an integrable physical system characterized by an infinity of mutually commuting $W_{1+\infty}$ conserved charges, carried by nonpropagating delocalized discrete gauge states, corresponding to an infinity of higher-spin states. These discrete states have zero norm and discrete momenta, which, however, take on the same values as those

¹⁰The metric field $\varphi(u, v)$, corresponding to the conformal factor of the near-horizon metric in the particular conformal-frame representation (35) of the two-dimensional horizon geometry, does not transform under the gauge symmetries in question.

corresponding to the infinity of physical (positive norm) propagating string states of the effective two-dimensional string. The two-dimensional substructure is essential to this argument and can always be embedded in four dimensions by considering near-horizon geometries of the form $SU(1,1)/U(1) \times H^{(2)}$, where $H^{(2)}$ a two-dimensional compact or noncompact manifold.

In this picture, the infall of matter into the black-hole horizon and the Hawking radiation process are viewed as “particle interactions” in the following sense. Consider first the case of matter falling into this black hole, specifically massless matter (represented as a tachyon propagating mode in the above effective-two-dimensional string theory context), which starts from spatial infinity. Initially, world-sheet conformal invariance of this tachyon background is guaranteed without mixing with the higher-spin states. However, upon reaching the horizon, discrete delocalized string modes of higher spin are excited, in order to dress the tachyon background and make it conformal on the world sheet. In this sense, given that the W_∞ charges are conserved, the backreaction of the black hole leads to an excited state, so that the whole process can be represented as (stringy black hole) + (massless matter) \Rightarrow (stringy black hole)*, where the star denotes an excited state and a rearrangement of the W_∞ charges. The black hole is viewed as a string state in the background of discrete gauge states and other topological states in this picture, as per our description above and in the previous literature.

The Hawking evaporation process can be thought of similarly as successive steps of the time-reversed process. This is reminiscent of the arguments of Ref. [31] for viewing black holes as particles and their Hawking radiation as consisting of successive “two-body” decays, in accordance with the sparsity of the Hawking radiation at infinity. However, our picture is very different in essence, as the black holes are stringy states characterized by an infinity of charges, thus integrable systems. The emitted massless matter reaches spatial infinity “decoupled” (in the sense that its world-sheet β -function vanishes) from the topological states, but the latter (due to conservation of the W_∞ charges) are omnipresent as a nonthermal discrete environment, carrying information. In string theory, the thermal Hawking radiation spectrum is only “part of” the whole picture, associated with propagating modes, whereas the discrete states provide a specific “Ariadne’s thread” for external measurements capable [12] of reconstructing the information “mislaid” within the black-hole labyrinth.

Comparing finally with the supertranslation approach [13,14,17], we recall that the latter also lead to an infinity of conserved charges on the two-dimensional horizon, which correspond to currents excited during the interaction of infalling matter. This can be seen straightforwardly in the representation of the black-hole horizon as a recoiling D -brane [25]. However, the supertranslation charges are not responsible for balancing the information books, for the

reasons stated above. This role is played by the W_∞ -charge-carrying topological discrete states, as can be seen in the simple example of the two-dimensional black hole [4], where these states are responsible for retaining information even in the absence of an horizon, which is only a spatial point in two dimensions.

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APPENDIX: W_∞ SYMMETRIES IN d SPACE-TIME DIMENSIONS AS $d + 2$ -DIMENSIONAL GAUGE SYMMETRIES

We discuss in this Appendix the connection between the Hawking radiation fields on the horizon of the spherically symmetric black hole with classical $SU(\infty)$ area-preserving gauge symmetries.

We consider the construction [27] of quantum W_∞ (and classical w_∞) gauge theories in $(d + 2)$ dimensions, where d is the dimension of the space-time where the algebras live: $d = 2$ in the black-hole case of interest to us. The W_∞ quantum algebra may be defined as the algebra of commutators of Hermitian operators $\xi(a, a^\dagger)$, where a, a^\dagger are harmonic-oscillator annihilation and creation operators. The operators $\xi(a, a^\dagger)$ may be parametrized using coherent states on a Euclidean space [27], spanned by complex coordinates z, \bar{z} , which can be identified with the coordinates of a stereographic projection of the horizon sphere S^2 [25],

$$:\xi(\hat{a}, \hat{a}^\dagger): = \int d^2z e^{-|z|^2} |z\rangle \xi(z, \bar{z}) \langle z|,$$

where $|z\rangle = e^{\hat{a}^\dagger z} |0\rangle$, $\langle z| = \langle 0| e^{\hat{a} \bar{z}}$, $\langle z'|z\rangle = e^{\bar{z}'z}$, $\hat{a}|z\rangle = z|z\rangle$, $\langle z|\hat{a}^\dagger = \langle z|\bar{z}$, we use the normalization condition $\int d^2z e^{-|z|^2} |z\rangle \langle \bar{z}| = 1$ with $d^2z \equiv \frac{1}{\pi} \text{Re}z \text{Im}z$, and $:\xi(\hat{a}, \hat{a}^\dagger):$ is a(n anti-)normal-ordered operator, in which the annihilation operators are always placed to the left of the creation operators.

One may regard [27] the coordinates z, \bar{z} as a group-theoretical (“color”) space and introduce a gauge potential $A_\mu(x, \hat{a}, \hat{a}^\dagger)$, where $\mu = 1, \dots, d$ is a d -dimensional space-time $\{x\}$ index:

$$\hat{A}_\mu(x) \equiv A_\mu(x, \hat{a}, \hat{a}^\dagger) = \int d^2z e^{|z|^2} |z\rangle A_\mu(x, z, \bar{z}) \langle z|. \tag{A1}$$

One may then introduce an infinite-dimensional set of infinitesimal W_∞ gauge transformations,

$$\begin{aligned} \delta \hat{A}_\mu(x) &= \partial_\mu \hat{\xi}(x) + i[\hat{\xi}(x), \hat{A}_\mu(x)], \\ \delta A_\mu(x, z, \bar{z}) &= \partial_\mu \xi(x, z, \bar{z}) - \{ \{ \xi, A_\mu \} \}_{\text{Moyal}}(x, z, \bar{z}), \end{aligned} \quad (\text{A2})$$

where the symbol $\{ \{ \cdot, \cdot \} \}_{\text{Moyal}}$ denotes a Moyal bracket, defined by

$$\begin{aligned} \{ \{ \xi_1, \xi_2 \} \}_{\text{Moyal}}(z, \bar{z}) &\equiv i \sum_{n=1}^{\infty} \frac{(-1)^n}{n} (\partial_z^n \xi_1(z, \bar{z}) \partial_{\bar{z}}^n \xi_2(z, \bar{z}) \\ &\quad - \partial_{\bar{z}}^n \xi_1(z, \bar{z}) \partial_z^n \xi_2(z, \bar{z})). \end{aligned} \quad (\text{A3})$$

The generators of W_∞ , $\rho[\xi]$, are linear functionals of $\xi(z, \bar{z})$ in this construction, satisfying

$$[\rho[\xi_1], \rho[\xi_2]] = i\rho[\{ \{ \xi_1, \xi_2 \} \}_{\text{Moyal}}] \quad (\text{A4})$$

at the quantum level [27]. The classical area-preserving w_∞ Lie algebra, as obtained from W_∞ by the appropriate contraction discussed in Ref. [27], is then

$$[\rho[\xi_1], \rho[\xi_2]] = i\rho[\{ \{ \xi_1, \xi_2 \} \}_{\text{Poisson}}], \quad (\text{A5})$$

where $\{ \cdot, \cdot \}_{\text{Poisson}}$ denotes the (classical) Poisson bracket.

We observe that, in this representation, the W_∞ gauge fields $A_\mu(x, z, \bar{z})$ are defined with coordinates in a $d+2$ -dimensional space-time $\{x, z, \bar{z}\}$ with a two-dimensional internal color space spanned by the $\{z, \bar{z}\}$ coordinates. We consider the following Yang-Mills-type action \mathcal{S} , which is invariant under the W_∞ gauge transformations (A2),

$$\begin{aligned} \mathcal{S} &= -\frac{1}{4g^2} \int d^d x \frac{1}{4} \text{Tr}(\hat{\mathcal{F}}_{\mu\nu} \hat{\mathcal{F}}^{\mu\nu}): \\ \hat{\mathcal{F}}_{\mu\nu} &= \partial_\mu \hat{A}_\nu(x) - \partial_\nu \hat{A}_\mu(x) - i[\hat{A}_\mu, \hat{A}_\nu], \end{aligned} \quad (\text{A6})$$

where g is a coupling constant. This can be rewritten using the coherent-state representation as [27]

$$\begin{aligned} \mathcal{S} &= -\frac{1}{4g^2} \int d^d x d^2 z \sum_{n=0}^{\infty} \frac{(-1)^n}{n} \partial_z^n \mathcal{F}_{\mu\nu}(x, z, \bar{z}) \partial_{\bar{z}}^n \mathcal{F}^{\mu\nu}(x, z, \bar{z}): \\ \mathcal{F}_{\mu\nu} &= \partial_\mu A_\nu(x, z, \bar{z}) - \partial_\nu A_\mu(x, z, \bar{z}) + \{ \{ A_\mu, A_\nu \} \}_{\text{Moyal}}(x, z, \bar{z}). \end{aligned} \quad (\text{A7})$$

We note that the action is nonlocal in terms of the z, \bar{z} variables. Indeed, as stressed in Ref. [27], it is this nonlocal nature of the action that differentiates the quantum W_∞ from the classical w_∞ symmetry, as concerns the association with the $\text{SU}(\infty)$ gauge theory. It is the W_∞ that can be viewed as the $N \rightarrow \infty$ limit of $\text{SU}(N)$, not the classical w_∞ .

Next, we consider a scalar field in an adjoint representation, which we call a Higgs field:

$$\hat{M}(x) \equiv M(x, \hat{a}, \hat{a}^\dagger) = \int d^2 z e^{-|z|^2} |z\rangle M(x, z, \bar{z}) \langle z|. \quad (\text{A8})$$

The Yang-Mills-Higgs action is then given by

$$\begin{aligned} S_H &= \int d^d x \text{tr} \left[\frac{1}{2} (\partial_\mu \hat{M}(x) - [\hat{A}_\mu, \hat{M}](x)) (\partial^\mu \hat{M}(x) - [\hat{A}^\mu, \hat{M}](x)) - v(\hat{M}) \right] \\ &= \int d^d x \left[\int d^2 z \frac{1}{2} \sum_{m=0}^{\infty} \frac{(-1)^m}{m!} \partial_z^m (\partial_\mu M(x, z, \bar{z}) - \{ \{ A_\mu, M \} \}(x, z, \bar{z})) \times \partial_{\bar{z}}^m (\partial^\mu M(x, z, \bar{z}) - \{ \{ A^\mu, M \} \}(x, z, \bar{z})) \right] - \text{tr} v(\hat{M}), \\ v(\hat{M}) &= \sum_n g_n \hat{M}^n, \quad \dim(g_n) = d \left(\frac{n}{2} - 1 \right) - n. \end{aligned} \quad (\text{A9})$$

We require that the fields and their z, \bar{z} derivatives should fall to zero at $z = \infty$. It is easy to check that this action is invariant under the W_∞ gauge transformation (A2) and

$$\delta M(x, z, \bar{z}) = \{ \{ \xi, M \} \}(x, z, \bar{z}), \quad \delta \hat{M}(x) = -i[\hat{\xi}(x), \hat{M}(x)]. \quad (\text{A10})$$

Notice that here again the interactions are nonlocal in the internal (z, \bar{z}) space.

As a last example of a W_∞ gauge theory, let us introduce a fermion field in a fundamental representation of W_∞ , namely a field that transforms as a bra or ket vector in the Hilbert space of a harmonic oscillator:

$$|\psi(x)\rangle = \int |z\rangle d^2 z e^{-|z|^2} \langle z|\psi(x)\rangle \equiv \int |z\rangle d^2 z e^{-|z|^2} \psi(x, \bar{z}). \quad (\text{A11})$$

We can write the action as

$$\begin{aligned} S_F &= \int d^n x \langle \psi(x) | \gamma^\mu (i\partial_\mu - \hat{\mathcal{A}}_\mu(x)) | \psi(x) \rangle \\ &= \iint d^n x d^2 z e^{-|z|^2} \bar{\psi}(x, z) \gamma^\mu (i\partial_\mu - \mathcal{A}_\mu(x, z, \bar{z})) \psi(x, \bar{z}), \end{aligned} \quad (\text{A12})$$

which is invariant under the W_∞ gauge transformation and

$$\begin{aligned} \delta |\psi(x)\rangle &= -i\hat{\xi}(x) |\psi(x)\rangle, \\ \delta \psi(x, \bar{z}) &= -i\ddagger \xi(\partial_{\bar{z}}, \bar{z}) \ddagger \psi(x, \bar{z}), \end{aligned} \quad (\text{A13})$$

where $\ddagger \dots \ddagger$ indicates that the derivatives are placed on the left of \bar{z} .

To consider the classical limit of the gauge algebra, as appropriate for the horizon-area-preserving symmetry of

the classical black hole, where the horizon area is viewed as a Noether charge [35], one defines the variables σ_i , $i = x, y$,

$$z = \frac{1}{\sqrt{2\ell}} (\sigma_x + i\sigma_y), \quad \bar{z} = \frac{1}{\sqrt{2\ell}} (\sigma_x - i\sigma_y), \quad (\text{A14})$$

and takes the limit of the length $\ell \rightarrow 0$. As is well known [27], this limiting procedure will yield the contracted classical area-preserving [in the internal (z, \bar{z}) space] w_∞ symmetry from the quantum W_∞ symmetry. For fields in the adjoint representation of the $SU(N)$ group, such as the gauge field \mathcal{A}_μ and the adjoint Higgs M field, this procedure is straightforward. To see this, we first pass from the z, \bar{z} variables to the σ_i , $i = x, y$ variables in the effective actions above and then consider the limit $\ell \rightarrow 0$, while performing the simultaneous rescaling of the various fields:

$$\begin{aligned} \mathcal{A}_\mu(x, z, \bar{z}) &= \ell^{-2} \mathcal{A}_\mu(x, \sigma_x, \sigma_y), & M(x, z, \bar{z}) &= \sqrt{2\pi} \ell^{-2} M(x, \sigma_x, \sigma_y), \\ g^2 &= \tilde{g}^2 \ell^{-6}, & g_n &= \tilde{g} \sqrt{2\pi} \ell^{2-n}. \end{aligned} \quad (\text{A15})$$

The actions (A6) and (A9) then become

$$\begin{aligned} S_{YM} &= -\frac{1}{4\tilde{g}^2} \int d^d x d^2 \bar{\sigma} F_{\mu\nu}(x, \bar{\sigma}) F^{\mu\nu}(x, \bar{\sigma}): \\ F_{\mu\nu}(x, \bar{\sigma}) &= \partial_\mu \mathcal{A}_\nu(x, \bar{\sigma}) - \partial_\nu \mathcal{A}_\mu(x, \bar{\sigma}) + \epsilon^{ij} \partial_i \mathcal{A}_\mu(x, \bar{\sigma}) \partial_j \mathcal{A}_\nu(x, \bar{\sigma}), \\ S_H &= \int d^d x d^2 \bar{\sigma} \left[\frac{1}{2} (\partial_\mu M(x, \bar{\sigma}) - \epsilon^{ij} \partial_i \mathcal{A}_\mu(x, \bar{\sigma}) \partial_j M(x, \bar{\sigma})) \times (\partial^\mu M(x, \bar{\sigma}) - \epsilon^{ij} \partial_i \mathcal{A}^\mu(x, \bar{\sigma}) \partial_j M(x, \bar{\sigma})) - \tilde{v}(M) \right]: \\ \tilde{v}(M) &= \sum_n \tilde{g}_n M^n(x, \bar{\sigma}). \end{aligned} \quad (\text{A16})$$

Here again, we require that the fields vanish at $\bar{\sigma} = \infty$.

Setting $\xi(x, z, \bar{z}) = l^{-2} \xi(x, \bar{\sigma})$, we find the w_∞ gauge transformations:

$$\begin{aligned} \delta \mathcal{A}^\mu(x, \bar{\sigma}) &= \partial^\mu \xi(x, \bar{\sigma}) - \epsilon^{ij} \partial_i \xi(x, \bar{\sigma}) \partial_j \mathcal{A}^\mu(x, \bar{\sigma}), \\ \delta M(x, \bar{\sigma}) &= \epsilon^{ij} \partial_i \xi(x, \bar{\sigma}) \partial_j M(x, \bar{\sigma}). \end{aligned} \quad (\text{A17})$$

One can check that the actions (A16) are indeed invariant under the (classical) w_∞ gauge transformation (A17). The reader should notice that the second equation of (A17) can be written as

$$\begin{aligned} \delta M(x, \bar{\sigma}) &= M(x, \bar{\sigma} + \delta \bar{\sigma}(x, \bar{\sigma})) - M(x, \bar{\sigma}), \\ \delta \sigma^i(x, \bar{\sigma}) &= -\epsilon^{ij} \partial_j \xi(x, \bar{\sigma}), \end{aligned} \quad (\text{A18})$$

which is a local area-preserving coordinate transformation in the internal two-dimensional space.

It is important to stress that the damping factor $e^{-|z|^2}$ cancels out in Lagrangians for the fields in the adjoint representation such as \mathcal{A}_μ and M , due to the property of the trace in the coherent-state representation. This allows the limit $\ell \rightarrow 0$ in the change of variables (A15) to be well behaved, yielding nontrivial actions (A16) in that limit.

This is a feature of fields in the adjoint representation of $SU(\infty)$. The same cannot be said for the fermion fields in (A12), which belong to the fundamental representation of $SU(\infty)$. For the latter action, there are no damping $e^{-|z|^2}$ factors in the $d + 2$ extended space-time, which implies that, formally, the fermion action vanishes in the classical $\ell \rightarrow 0$ limit (A15), $S_F \rightarrow 0$.

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