

Testing Einstein's weak equivalence principle with gravitational wavesXue-Feng Wu,^{1,2,*} He Gao,^{3,†} Jun-Jie Wei,¹ Peter Mészáros,^{4,5,6} Bing Zhang,⁷ Zi-Gao Dai,⁸
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A conservative constraint on Einstein's weak equivalence principle (WEP) can be obtained under the assumption that the observed time delay between correlated particles from astronomical sources is dominated by the gravitational fields through which they move. Current limits on the WEP are mainly based on the observed time delays of photons with different energies. It is highly desirable to develop more accurate tests that include the gravitational wave (GW) sector. The detection by the advanced LIGO/VIRGO systems of gravitational waves will provide attractive candidates for constraining the WEP, extending the tests to gravitational interactions with potentially higher accuracy. Considering the capabilities of the advanced LIGO/VIRGO network and the source direction uncertainty, we show that the joint detection of GWs and electromagnetic signals could probe the WEP to an accuracy down to 10^{-10} , which is one order of magnitude tighter than previous limits, and 7 orders of magnitude tighter than the multimessenger (photons and neutrinos) results by supernova 1987A.

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Einstein's weak equivalence principle (WEP) is one of the main cornerstones of general relativity as well as of many other gravitational theories. One statement of the WEP is that any freely falling, uncharged test body will follow a trajectory independent of its internal composition and structure. It implies that any two different species of massless (or of negligible rest mass) neutral particles, or two particles of the same species with different energies, if emitted simultaneously from the same source and traveling through the same gravitational fields, should reach us at the same time [1,2]. By measuring how closely in time the two different particles arrive, one can test the accuracy of the WEP through the Shapiro (gravitational) time delay effect [3]. In practice, all metric theories of gravity incorporating the WEP predict that all test particles must follow identical trajectories and undergo the same Shapiro time delay. In other words, as long as the WEP is valid, all metric theories

predict $\gamma_1 = \gamma_2 \equiv \gamma$, where γ is the parametrized post-Newtonian (PPN) parameter (γ denotes how much space curvature is provided by unit rest mass of the objects along or near the path of the particles [1,2]) and the subscripts represent two different particles. In this case, the WEP validity can be characterized by limits on the differences of γ value for different test particles (see, e.g., Refs. [4–10]).

Any possible violation of the WEP would have far-reaching consequences for mankind's view of nature, so it is important to extend the tests of its validity by making use of the panoply of new types of astronomical signals being brought to the fore in the multimessenger era. So far, tests of the WEP through the relative differential variations of the γ values have been made using the emissions from supernova 1987A [4,5], gamma-ray bursts (GRBs) [6,7], fast radio bursts (FRBs) [8,9], and TeV blazars [10]. In particular, assuming that the observed time delay between different frequency photons from FRBs are caused mainly by the gravitational potential of the Milky Way, Ref. [8] set the most stringent limits to date on γ differences, yielding $\sim 10^{-8}$. Even more encouragingly, the most recent studies [11] show that the constraints on the WEP accuracy from

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FRBs can be further improved by a few orders of magnitude when taking into account the gravitational potential fluctuations of the large-scale structure, rather than the Milky Way's gravity. In addition, the discovery of a triple system [12], made of a millisecond pulsar PSR J0337 + 1715 and two white dwarves, has recently provided a new interesting test of the equivalence principle. The very large difference in the gravitational binding energies of the pulsar and the white dwarf makes this system very promising on the equivalence principle test.

Although the tests on the WEP have reached high precision, most of the tests rely on the relative arrival time delays of (exclusively) photons with different energies. The first and only WEP test with different species of particles was the measurement of the time delay between the photons and neutrinos from supernova 1987A [4,5]. It was shown that the γ values of photons and neutrinos are equal to an accuracy of approximately 0.34%. New multi-messenger signals exploiting different emission channels are essential for testing the WEP to a higher accuracy.

Recently, the Laser Interferometer Gravitational-wave Observatory (LIGO) team report their discovery of the first gravitational wave (GW) source, GW 150914 [13], opening a brand new channel for studying the Universe, which could lead to breakthroughs in both fundamental physics and astrophysics. In fact, the next generation of gravitational detectors, including advanced LIGO, advanced VIRGO, and KAGRA, appear poised to detect a plethora of increasingly sophisticated GW signals in the very near future [14–18]. Phenomenologically, one may treat the GWs with different frequencies as different gravitons to test the WEP. In extending the constraints on $\Delta\gamma$, tests of the WEP using GW measurements become more robust against various assumptions, because, e.g., GWs do not suffer absorption or scattering along their path, in contrast to photons. In the following, we illustrate the progress that can be expected in testing the WEP with the reported/future GW observations.

II. DESCRIPTION OF THE METHOD

The Shapiro time delay effect [3] causes the time interval for particles to pass through a given distance to be longer in the presence of a gravitational potential $U(r)$ by

$$\Delta t_{\text{gra}} = -\frac{1+\gamma}{c^3} \int_{r_e}^{r_o} U(r) dr, \quad (1)$$

where γ is a PPN parameter, and r_o and r_e correspond to locations of observation and the source of particle emission.

Assuming that the observed time delays (Δt_{obs}) between correlated particles from the same astronomical source are mainly caused by the gravitational potential of the Milky Way, and adopting the Keplerian potential for the Milky Way, we have [5,19]

$$\Delta t_{\text{obs}} > \Delta t_{\text{gra}} = \Delta\gamma \frac{GM_{\text{MW}}}{c^3} \times \ln \left\{ \frac{[d + (d^2 - b^2)^{1/2}][r_G + s_n(r_G^2 - b^2)^{1/2}]}{b^2} \right\}, \quad (2)$$

where $\Delta\gamma$ is the difference between the γ values for different test particles, $M_{\text{MW}} \simeq 6 \times 10^{11} M_\odot$ is the Milky Way mass [20,21], d represents the distance from the source to the center of the Milky Way (if the source is of extra-Galactic or cosmological origin, d is approximated as the distance from the source to Earth), $r_G \simeq 8$ kpc is the distance from the Sun to the center of the Milky Way, b denotes the impact parameter of the particle paths relative to the Milky Way center, and $s_n = \pm 1$ is the sign of the correction of the source direction. If the source is located along the direction of the Galactic center, $s_n = +1$; $s_n = -1$ corresponds to the source located along the direction pointing away from the Galactic center. Note that the impact parameter b is on the order of the distance of the Sun from the Galactic center, i.e., $b \leq r_G$. With Eq. (2), one can constrain the WEP by putting a strict limit on the differences of γ value [4–10].

We notice that although the method adopted in this work can provide severe constraints on the accuracy of the WEP, which is one of the important postulates of GR, it cannot be directly used to distinguish between specific gravity theories, such as GR and its alternatives. Many precise methods have been devised to test the accuracy of GR through the measurement of the absolute value of γ based on the fact that GR predicts $\gamma = 1$ (see Ref. [2] for a recent review). However, it is worth pointing out that $\gamma = 1$ is not a sufficient condition to identify general relativity, because it is not the only theory that predicts $\gamma = 1$ [2]. Thus, further investigations would be essential for developing more accurate tests of the WEP and for distinguishing between GR and other alternative gravity theories.

III. WEP TEST USING GW SIGNALS

The process of compact binary coalescence (CBC)—either neutron-star (NS) binary, black-hole (BH) binary, or NS-BH binary—provides the primary targets for the second generation of GW detectors, such as advanced LIGO/VIRGO [14–18]. The first reported GW detection, GW 150914, is a BH-BH merger with two BH masses $36_{-4}^{+5} M_\odot$ and $29_{-4}^{+4} M_\odot$, respectively [13]. Of significant interest for CBC GW detections is the fact that some relevant fundamental physics postulates, including the WEP, may be constrained using gravitational radiation alone [2,22]. This could be done exploiting the fact that the frequency of the gravitational radiation sweeps from low frequencies at the initial moment of observation (in-spiral phase) to a higher frequency at the final moment (coalescence phase). Note that the GW frequency eventually saturates to a constant

value in the vicinity of the light ring. The amplitude, however, decreases monotonically after reaching its peak at the light ring. Any WEP violation will cause a distortion of the observed phasing of the waves, and would result in a shorter (or longer) than expected overall time of passage of a given number of cycles. It is worth pointing out that there are many effects that can change the lifespan of a waveform: spin corrections, eccentricity, spin precession, etc. However, it is difficult to disentangle these effects from a WEP violation. Our upper limits on the WEP accuracy are based on very conservative estimates of the observed time delay (i.e., the whole time delay is assumed to be caused by the WEP violation). In fact, the inclusion of contributions from the neglected effects in the observed waveform could improve the limits on WEP to some degree. In this case, since no electromagnetic (EM) counterparts are required, all CBC GW detections would be relevant. For instance, the signal of GW 150914 increases in frequency and amplitude in about 8 cycles (over 0.2 s) from 35 to 150 Hz, where the amplitude reaches a maximum [13]. Considering the localization information of GW 150914, we could tighten the limit on the WEP to $\Delta\gamma \sim 10^{-9}$.

More recently, the Fermi Gamma-Ray Burst Monitor (GBM) team reported that GBM observations at the time of GW150914 reveal the presence of a weak transient source above 50 keV, 0.4 s after the GW event was detected, with a false alarm probability of 0.0022 [23]. If this is indeed the EM counterpart of GW150914 (see possible interpretations in [24]), with the aforementioned method, we could further extend the WEP test with GWs and photons, setting a severe limit on WEP to an accuracy of 10^{-8} , which is 5 orders of magnitude tighter than the results set by the photons and neutrinos from supernova 1987A.

Besides BH-BH mergers, GW signals from binary NSs and NS-BH mergers are also expected to be detected in the near future [25], for which a variety of detectable EM counterparts have been widely discussed [26–28], including the following representative cases: the prompt short GRB emission, the afterglow emission of the on-beam ultrarelativistic outflows, and the macronova/kilonova emission of the subrelativistic r-process material ejected during the merger. For NS-NS mergers, if the merger product is a massive millisecond pulsar instead of a BH, the detectable EM signatures from the system become much richer and brighter (see Ref. [29] for details). Joint detections of GW/EM signals, once achieved, could be used to give important constraints on the WEP.

Consider the case of a joint detection of GW/EM signals from a NS-NS or NS-BH coalescence event in the advanced LIGO/VIRGO era. Since the sky- and binary-orientation-averaged sensitivity of the advanced LIGO/VIRGO network for CBC is of the order of ~ 100 Mpc [14–18], here we assume the distance from the GW source to the Earth to be $d = 200$ Mpc. It is worth pointing out that the constraints on the WEP are not greatly affected by the source

distance uncertainty (see Ref. [8] for more explanations). To account for the source direction uncertainty, and based on the fact that the impact parameter $b \leq r_G$, here we present four extreme cases by assuming $b = 0.001r_G$ and $s_n = +1$, $b = 0.001r_G$ and $s_n = -1$, $b = 0.999r_G$ and $s_n = +1$, and $b = 0.999r_G$ and $s_n = -1$, respectively. The real results should lie within the range circumscribed by these extreme cases.

Regarding the EM counterpart of the GW detection, suppose we are lucky to detect all the promising emission types, i.e., the short GRB prompt emission, the on-beam GRB afterglow emission, and the macronova emission. Recently, Ref. [30] discussed the time lags between the GW signal and all these EM counterparts in some detail, and suggested that the time delay Δt_{obs} is expected to be of the order of ~ 0.01 –1 s (short GRB), 0.01–1 day (on-beam afterglow), or 1–10 days (macronova), respectively. With these expected time delays and with the location information in hand, we would be able to set bounds on the WEP from Eq. (2). The expected constraints on the differences of the γ values are shown in Fig. 1. It has been suggested that the macronova emission may be the most frequently detectable EM signal of the coalescence events [27,28]. If the macronova emission is detected at $\Delta t_{\text{obs}} \sim 1$ day after the merger, a strict limit on the WEP will be $\Delta\gamma < 10^{-3}$. One can see from this plot that much more severe constraints would be achieved ($\sim 10^{-3}$ – 10^{-5} or 10^{-8} – 10^{-10}) if the EM counterpart is an on-beam afterglow or a short GRB. Note that the compact binary coalescence and the EM counterpart do not occur at the same time, since Δt_{obs} has a contribution from the intrinsic emission time lag (Δt_{lag}) between the photons and the GW signals. Here we take $\Delta t_{\text{lag}} = 0$ to give a conservative estimate of the WEP. More severe constraints could be achieved with a better

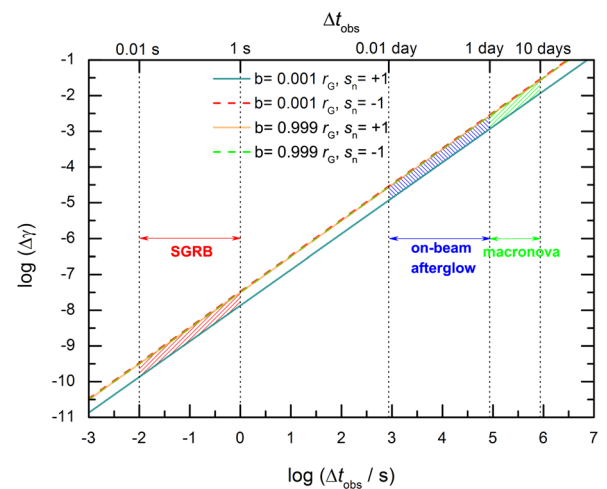


FIG. 1. Expected limits on the differences of the γ values between the GW signals and the photons for various types of EM counterparts. The vertical lines correspond to different characteristic times.

understanding of the nature of Δt_{lag} , allowing one to remove its contribution from Δt_{obs} . On the other hand, it should be underlined that these upper limits are based on very conservative estimates of the gravitational potential of the Milky Way. If the gravitational potential fluctuations from the intervening large-scale structures are taken into consideration, our constraint results would be further improved by orders of magnitude [11].

IV. SUMMARY AND DISCUSSION

In conclusion, we show that new WEP tests can be carried out with potentially much higher accuracy in the GW era. For all kinds of CBC GW detections, regardless of whether EM counterparts are detected or not, we can always use GWs with different frequencies to give stringent constraints on the accuracy of the WEP. Taking GW 150914 as an example, it takes less than one second for the GW signals emitted from lower frequency to higher frequency where the signal amplitude reaches a maximum (e.g., 35 Hz to 150 Hz), resulting in a tightening of the limit on the WEP to approximately 10^{-9} , which is as good as the current most stringent results from FRBs [8,9].

Once EM counterparts of the GW signals are firmly detected, an interesting WEP test could be performed by using the measured time delay between the GWs and any associated photons. Also, taking GW 150914 as an example, if the claimed short GRB, GW150914-GBM, is indeed the EM counterpart of GW150914, a severe limit on WEP could be set to an accuracy of 10^{-8} , 5 orders of magnitude tighter than the results set by the photons and neutrinos from supernova 1987A [4,5].

Finally, considering the capabilities of the advanced LIGO/VIRGO network and the source direction uncertainty, we found that for the expected GW detection from NS-NS/BH mergers, if the prompt short GRB emission and/or its afterglow emission is detected, a stringent limit on the WEP could be set at the level of $\Delta\gamma < (10^{-8} - 10^{-10})$ (prompt) or $\sim 10^{-3} - 10^{-5}$ (afterglow). Because of the low detection rates of GRB-accompanied GW signals,

the first positively identified electromagnetic counterpart of a GW signal is very likely to be a macronova. If the macronova emission is detected at $\Delta t_{\text{obs}} \sim 1$ day after the merger, a strict limit on the WEP will be $\Delta\gamma < 10^{-3}$.

In sum, the main result of this paper is to propose a method to test the WEP, which can be applied when future robust GW/EM associations become available. For GW 150914, we have applied our method to the available data (the GW data and the putative EM signal following the GW signal) and derived some stringent limits on the WEP that were not achievable by previous analyses. There are astrophysical uncertainties in applying our method. Examples of such astrophysical uncertainties are, e.g., the distance of a purely GW-detected sources such as GW 150914, the astrophysical time lags between EM and GW emission mentioned in Ref. [30], the detection of more than one EM emission component of a short GRB, etc. Such astrophysical uncertainties, however, will certainly diminish in time, with improvements in EM instruments and observations, the addition of further GW detectors at different Earth locations, and so forth.

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