Comparison of associated Higgs boson-radion and Higgs boson pair production processes

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Many models—in particular, the brane-world models with two branes—predict the existence of the scalar radion, whose mass can be somewhat smaller than those of all the Kaluza-Klein modes of the graviton and Standard Model (SM) particles. Due to its origin the radion interacts with the trace of the energy-momentum tensor of the SM. The fermion part of the radion interaction Lagrangian is different from that for the SM Higgs boson due to the presence of additional terms playing a role for off-shell fermions. It was shown previously [Phys. Rev. D **90**, 095026 (2014)] that for the case of the single radion and single Higgs boson production processes in association with an arbitrary number of SM gauge bosons all the contributions to the perturbative amplitudes appearing due to these additional terms were canceled out, making the processes similar up to a replacement of masses and overall coupling constants. For the case of the associated Higgs boson-radion and the Higgs boson pair-production processes involving the SM gauge bosons, the similarity property also appears. However, a detailed consideration shows that in this case it is not enough to simply replace the masses and the constants ($m_h \to m_r$ and $v \to \Lambda_r$). One should also rescale the triple Higgs coupling by the factor $\xi \equiv 1 + \frac{m_r^2 - m_h^2}{3m_h^2}$.

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I. INTRODUCTION

One of the characteristic features of brane-world models—in particular, of the Randall-Sundrum model [1] with a stabilization of the extra space dimension [2,3]—is the existence of the radion [2,4,5], which is the lowest Kaluza-Klein (KK) mode of the five-dimensional scalar field appearing from the fluctuations of the metric component corresponding to the extra dimension. The radion might be significantly lighter than the other KK modes [6–8], and therefore it is of special interest for collider phenomenology (see, e.g., Refs. [9–23]).

The radion couples to the trace of the energy-momentum tensor of the Standard Model (SM), so the interaction Lagrangian has the following form [2]:

$$L = -\frac{r(x)}{\Lambda_r} T^\mu_\mu, \tag{1}$$

where Λ_r is a dimensional scale parameter, $r(x)$ stands for the radion field, and T^μ_μ is the trace of the SM energy-momentum tensor. In most of the studies the latter is taken at the lowest order in the SM couplings and the fields are supposed to be on the mass shell. Here we consider the additional terms which come into play for the case of off-shell fermions, so the SM energy-momentum tensor has the following form [24]:

$$\begin{aligned} T^\mu_\mu = {} & \frac{\beta(g_s)}{2g_s} G^{ab}_{\rho\sigma} G^{\rho\sigma}_{ab} + \frac{\beta(e)}{2e} F_{\rho\sigma} F^{\rho\sigma} \\ & + \sum_f \left[\frac{3i}{2} ((D_\mu \bar{f}) \gamma^\mu f - \bar{f} \gamma^\mu (D_\mu f)) + 4 m_f \bar{f} f \left(1 + \frac{h}{v}\right) \right] \\ & - (\partial_\mu h)(\partial^\mu h) + 2 m_h^2 h^2 \left(1 + \frac{h}{2v}\right)^2 \\ & - (2 m_W^2 W^+_\mu W^{\mu -} + m_Z^2 Z_\mu Z^\mu) \left(1 + \frac{h}{v}\right)^2, \end{aligned} \tag{2}$$

where the first two terms correspond to the conformal anomaly of massless gluon and photon fields, $\beta(g_s)$ and $\beta(e)$ are the QCD and QED β functions, respectively, h, $W^\pm$, and Z are the SM Higgs, W-, and Z-boson fields, D_μ is the Standard Model covariant derivative, and the summation here is carried out over all the Standard Model fermions.

In the case of on-shell fermions the fermion part of the Lagrangian (1) is the same as for the Higgs boson (with the replacement $\Lambda_r \to v$), but for off-shell fermions additional terms need to be taken into consideration. These terms coming from the covariant derivatives in the Lagrangian give additional momentum-dependent contributions to the three-point fermion-antifermion-radion interaction vertices and new four-point fermion–antifermion–gauge boson–radion vertices. Moreover, another new four-point vertex—the vertex of the fermion-antifermion-Higgs boson-radion interaction—appears from the Yukawa term in the trace of the energy-momentum tensor multiplied by the radion field in the Lagrangian. The three-point vertex of the radion interaction with two Higgs bosons also contains the momenta of the Higgs bosons, which come from the kinetic term of the Higgs field (see Appendixes A and B in Ref. [24]). All this can modify the radion production and decay processes, making them potentially different from the same processes with the Higgs boson.

In Ref. [24] it was shown that all the additional contributions as compared to the Higgs boson case are canceled out in the sum of amplitudes for the single radion production in association with an arbitrary number of any SM vector gauge bosons, and there remain Higgs-like terms only. This property follows from the structure of any massive fermion current emitting the radion and gauge bosons both for the case of real and/or virtual emitted particles as well as for the case of boson and fermion loops [24].

In the present paper we consider the associated Higgs boson-radion production in comparison to the Higgs boson pair-production processes as a continuation of our previous study [24]. We demonstrate that in the case of the associated Higgs boson-radion production the similarity property is more involved. It is not enough to perform the replacement of two constants ($m_r \to m_h$ and $\Lambda_r \to v$) to get the amplitude involving the radion from the corresponding amplitude for the Higgs boson. It is explicitly demonstrated that the amplitude of the associated production of the Higgs boson and the radion and an arbitrary number of gauge bosons can be obtained from the corresponding amplitude involving the Higgs boson pair by the replacement of the Higgs and the radion masses, the constant Λ_r, and the Higgs vacuum expectation value v, and in addition by a rescaling of the triple Higgs coupling by a certain factor. As in our previous study, we do not consider the well-known differences between the Higgs and the radion processes caused by the conformal anomalies.

The investigation of double Higgs boson production is an important task for experimental measurements of the Higgs field potential profile. This problem is rather tricky even in the high-luminosity mode of the LHC, which is one of the key arguments for the construction of the ILC. However, if one of the multidimensional brane-world scenarios occurs in nature, the presence of the radion with a mass close to that of the Higgs boson (which is not excluded by the present experimental data [23]) can further complicate the problem of Higgs potential research due to the similarities between the properties of the Higgs boson and the radion.

II. ASSOCIATED HIGGS BOSON-RADION PRODUCTION IN FERMION-ANTIFERMION ANNIHILATION

Let us first consider the associated Higgs boson-radion production in fermion-antifermion annihilation (Fig. 1).

The corresponding contributions to the amplitude simplified using the Dirac equation and the identity $(\not{k} - m_f)\frac{\not{k}+m_f}{k^2-m_f^2} = 1$ have the following form:

$$M_1^{(rh)} = \bar{v}^r(p_1)\frac{-im_f}{v}u^s(p_2)\frac{i}{k_h^2 - m_h^2}\frac{-i}{\Lambda_r}(-2k_h p_h + 4m_h^2)r(p_r)h(p_h), \tag{3}$$

$$\begin{aligned} M_2^{(rh)} &= \bar{v}^r(p_1)\frac{-i}{\Lambda_r}\left\{\frac{3}{2}[(\not{p}_1 + m_f) - (\not{k} - m_f)] + m_f\right\}r(p_r)i\frac{\not{k} + m_f}{k^2 - m_f^2}i\frac{-m_f}{v}u^s(p_2)h(p_h) \\ &= -i\bar{v}^r(p_1)\frac{1}{\Lambda_r}\frac{m_f}{v}\left\{-\frac{3}{2} + m_f\frac{\not{k} + m_f}{k^2 - m_f^2}\right\}r(p_r)u^s(p_2)h(p_h), \end{aligned} \tag{4}$$

$$\begin{aligned} M_3^{(rh)} &= \bar{v}^r(p_1)i\frac{-m_f}{v}h(p_h)i\frac{\not{k}' + m_f}{k'^2 - m_f^2}\frac{-i}{\Lambda_r}\left\{\frac{3}{2}[(-\not{k}' + m_f) - (\not{p}_2 - m_f)] + m_f\right\} \\ &\quad r(p_r)u^s(p_2) \\ &= -i\bar{v}^r(p_1)\frac{1}{\Lambda_r}\frac{m_f}{v}\left\{-\frac{3}{2} + m_f\frac{\not{k}' + m_f}{k'^2 - m_f^2}\right\}r(p_r)u^s(p_2)h(p_h), \end{aligned} \tag{5}$$

$$M_4^{(rh)} = \bar{v}^r(p_1)\frac{-i}{\Lambda_r}\frac{4m_f}{v}h(p_r)r(p_r)u^s(p_2) = -i\bar{v}^r(p_1)\frac{1}{\Lambda_r}\frac{m_f}{v}\{4\}r(p_r)u^s(p_2)h(p_h). \tag{6}$$

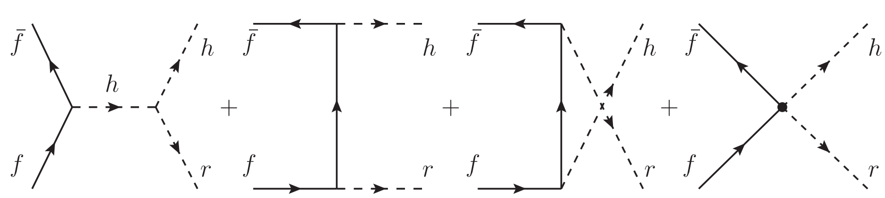

FIG. 1. Feynman diagrams contributing to the associated Higgs boson-radion production in fermion-antifermion annihilation.

For clarity one can modify Eq. (3) using the simple kinematics relation

$$k_h = p_h + p_r \Rightarrow p_h = k_h - p_r, \tag{7}$$

$$\begin{aligned} 2k_h p_h &= 2(p_h + p_r)p_h = (p_h + p_r)^2 + p_h^2 - p_r^2 \\ &= k_h^2 + p_h^2 - p_r^2 = k_h^2 + m_h^2 - m_r^2, \end{aligned} \tag{8}$$

and therefore

$$\begin{aligned} \frac{-(2k_h p_h) + 4m_h^2}{k_h^2 - m_h^2} &= \frac{-k_h^2 - m_h^2 + m_r^2 + 4m_h^2}{k_h^2 - m_h^2} \\ &= -1 + \frac{m_r^2 + 2m_h^2}{k_h^2 - m_h^2}. \end{aligned} \tag{9}$$

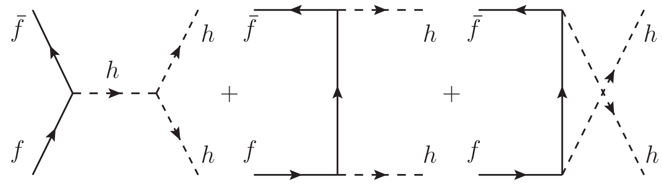

FIG. 2. Feynman diagrams contributing to the double Higgs boson production in fermion-antifermion annihilation.

So Eq. (3) can be rewritten as follows:

$$M_1^{(rh)} = -i\frac{m_f}{\Lambda_r v}\bar{v}^r(p_1)r(p_r)h(p_h)\left\{-1+\frac{m_r^2+2m_h^2}{k_h^2-m_h^2}\right\}u^s(p_2). \tag{10}$$

It is easy to put Eqs. (4), (5), (6), and (10) together and write down the total amplitude for the $f\bar{f} \to rh$ process,

$$\begin{aligned} M_{\text{tot}}^{(rh)} = -i\frac{m_f}{\Lambda_r v}r(p_r)h(p_h)\bar{v}^r(p_1)\Bigg\{ & m_f\frac{\not{k} + m_f}{k^2 - m_f^2} + m_f\frac{\not{k}' + m_f}{k'^2 - m_f^2} - \frac{3}{2} - \frac{3}{2} + 4 - 1 \\ & + \frac{m_r^2 + 2m_h^2}{k_h^2 - m_h^2}\Bigg\}u^s(p_2). \end{aligned} \tag{11}$$

Now let us write down the contributions to the amplitude of the double Higgs boson production process $f\bar{f} \to hh$ (Fig. 2) and compare the results:

$$\begin{aligned} M_1^{(hh)} &= \bar{v}^r(p_1)i\frac{-m_f}{v}u^s(p_2)\frac{i}{k_h^2 - m_h^2}\frac{-i}{v}3m_h^2 h(p_{h_1})h(p_{h_2}) \\ &= -i\frac{m_f}{v^2}h(p_{h_1})h(p_{h_2})\bar{v}^r(p_1)\left\{\frac{3m_h^2}{k_h^2 - m_h^2}\right\}u^s(p_2), \end{aligned} \tag{12}$$

$$\begin{aligned} M_2^{(hh)} &= \bar{v}^r(p_1)i\frac{-m_f}{v}h(p_{h_1})i\frac{\not{k} + m_f}{k^2 - m_f^2}i\frac{-m_f}{v}h(p_{h_2})u^s(p_2) \\ &= -i\frac{m_f^2}{v^2}h(p_{h_1})h(p_{h_2})\bar{v}^r(p_1)\left\{\frac{\not{k} + m_f}{k^2 - m_f^2}\right\}u^s(p_2), \end{aligned} \tag{13}$$

$$\begin{aligned} M_3^{(hh)} &= \bar{v}^r(p_1)i\frac{-m_f}{v}h(p_{h_2})i\frac{\not{k}' + m_f}{k'^2 - m_f^2}i\frac{-m_f}{v}h(p_{h_1})u^s(p_2) \\ &= -i\frac{m_f^2}{v^2}h(p_{h_1})h(p_{h_2})\bar{v}^r(p_1)\left\{\frac{\not{k}' + m_f}{k'^2 - m_f^2}\right\}u^s(p_2). \end{aligned} \tag{14}$$

Notice that in this case there is no contribution like Eq. (6).

Thus, the total amplitude of the double Higgs production process yields

$$M_{\text{tot}}^{(hh)} = -i\frac{m_f}{v^2}h(p_{h1})h(p_{h2})\bar{v}^r(p_1)\left\{m_f\frac{\not{k} + m_f}{k^2 - m_f^2} + m_f\frac{\not{k}' + m_f}{k'^2 - m_f^2} + \frac{3m_h^2}{k_h^2 - m_h^2}\right\}u^s(p_2). \tag{15}$$

Finally, one can compare Eqs. (11) and (15) and see the explicit cancellation of all the contributions that make up the difference between the associated Higgs boson-radion and the double Higgs boson production.

In other words, Eq. (11) can be written in terms of Eq. (15) in the following way:

$$M_{\rm tot}^{(rh)} \sim \bar{v}^r(p_1)\left\{m_f \frac{\not{k}+m_f}{k^2-m_f^2} + m_f \frac{\not{k}'+m_f}{k'^2-m_f^2} + \xi \frac{3m_h^2}{k_h^2-m_h^2}\right\} u^s(p_2), \quad (16)$$

i.e., the expressions for the total amplitudes (11) and (15) coincide up to the replacements of the masses $m_r \to m_h$ and the denominators of the coupling constants $\Lambda_r \to v$ and to the renormalization of the triple Higgs coupling by the factor ξ, where

$$\xi \equiv 1 + \frac{m_r^2 - m_h^2}{3m_h^2}.$$

III. ASSOCIATED HIGGS BOSON-RADION PRODUCTION IN gg FUSION

As another example let us compare two processes involving gluons: the associated Higgs boson-radion production ($gg \to rh$) and the double Higgs boson production ($gg \to hh$); the corresponding diagrams are shown below.

One can see that the first four diagrams for the process of radion production (Fig. 3) are similar to those which appear in the SM (Fig. 4). But the other three diagrams contain the Higgs boson–fermion–fermion–radion vertex which does not exist in the SM.

One can notice that all the contributions to the amplitudes of these processes have the following similar structure:

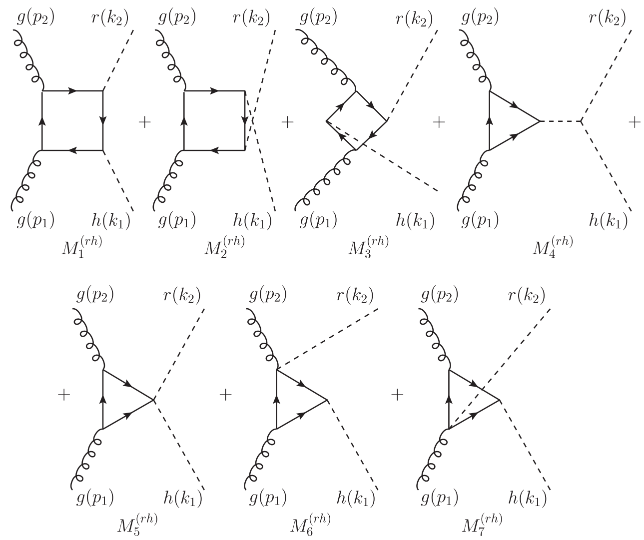

FIG. 3. Feynman diagrams contributing to the associated Higgs boson-radion production ($gg \to rh$).

$$M_i^{(rh)} = \frac{g_c^2}{v\Lambda_r}\epsilon(p_1)_\mu \epsilon(p_2)_\nu h(k_1) r(k_2) \times \int \frac{d^d l}{(2\pi)^d} X_i^{\mu\nu}(p_1, p_2, k_1, k_2), \quad (17)$$

for $gg \to rh$, where $i = 1, 2, \ldots, 7$;

$$M_i^{(hh)} = \frac{g_c^2}{v^2}\epsilon(p_1)_\mu \epsilon(p_2)_\nu h(k_1) h(k_2) \times \int \frac{d^d l}{(2\pi)^d} Y_i^{\mu\nu}(p_1, p_2, k_1, k_2), \quad (18)$$

for $gg \to hh$, where $i = 1, 2, 3, 4$, and where

$$X_1^{\mu\nu} \equiv Sp[\gamma^\mu S_1 \gamma^\nu S_2 \Gamma_{2,3} S_3(-m) S_4], \qquad Y_1^{\mu\nu} \equiv Sp[\gamma^\mu S_1 \gamma^\nu S_2(-m) S_3(-m) S_4],$$
$$X_2^{\mu\nu} \equiv Sp[\gamma^\mu S_1 \gamma^\nu S_2(-m) S_5 \Gamma_{5,4} S_4], \qquad Y_2^{\mu\nu} \equiv Sp[\gamma^\mu S_1 \gamma^\nu S_2(-m) S_5(-m) S_4],$$
$$X_3^{\mu\nu} \equiv Sp[\gamma^\mu S_1(-m) S_6 \gamma^\nu S_5 \Gamma_{5,4} S_4], \qquad Y_3^{\mu\nu} \equiv Sp[\gamma^\mu S_1(-m) S_6 \gamma^\nu S_5(-m) S_4],$$
$$X_4^{\mu\nu} \equiv Sp[\gamma^\mu S_1 \gamma^\nu S_2(-m) S_4] D\Gamma', \qquad Y_4^{\mu\nu} \equiv Sp[\gamma^\mu S_1 \gamma^\nu S_2(-m) S_4] D\Gamma',$$
$$X_5^{\mu\nu} \equiv Sp[\gamma^\mu S_1 \gamma^\nu S_2(+4m) S_4],$$
$$X_6^{\mu\nu} \equiv Sp[\gamma^\mu S_1(-3\gamma^\nu) S_3(-m) S_4],$$
$$X_7^{\mu\nu} \equiv Sp[(-3\gamma^\mu) S_1 \gamma^\nu S_2(-m) S_5],$$

and $\Gamma_{2,3}$, $\Gamma_{5,4}$, Γ' have the form

$$\Gamma_{2,3} = \frac{3}{2} S_2^{-1} + \frac{3}{2} S_3^{-1} - m, \quad (19)$$

$$\Gamma_{5,4} = \frac{3}{2} S_5^{-1} + \frac{3}{2} S_4^{-1} - m, \quad (20)$$

$$\Gamma' = \begin{cases} -3m_h^2, \\ 2\{(k_1+k_2)_\mu k_1^\mu - 2m_h^2\}. \end{cases} \quad (21)$$

The first line in Eq. (21) corresponds to the Higgs boson–fermion–fermion vertex in the $gg \to hh$ process, and the

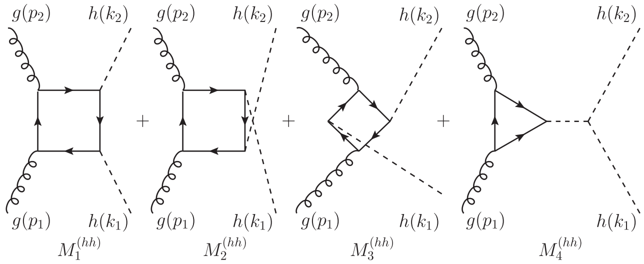

FIG. 4. Feynman diagrams contributing to the double Higgs boson production ($gg \to hh$).

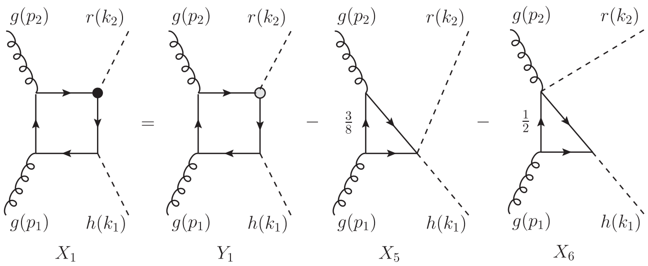

FIG. 5. Reduction of a box diagram with the radion to a linear combination of one box diagram with a vertex such as that of the Higgs boson (the empty point) and two triangle diagrams.

second line corresponds to the Higgs boson–radion–fermion–fermion vertex in the $gg \to rh$ process:

$$S_j^{-1} = (\not{l} - \not{q}_j) - m, \qquad D^{-1} = (k_1 + k_2)^2 - m^2, \qquad (22)$$

$$q_1 = 0, \qquad q_2 = -p_2, \qquad q_3 = -p_2 + k_2,$$
$$q_4 = -p_2 + k_2 + k_1,$$
$$q_5 = -p_2 + k_1, \qquad q_6 = k_1.$$

Let us notice that the radion-fermion-fermion vertex $\Gamma_{i,j}$ contains the inverse propagators S_i^{-1} and S_j^{-1}. In the expressions $X_1^{\mu\nu}$, $X_2^{\mu\nu}$, and $X_3^{\mu\nu}$ for the box diagrams this vertex is surrounded by the propagators S_i and S_j. This gives a reduction of a box diagram with the radion to a linear combination of two triangle diagrams and one box diagram with a vertex such as that of the Higgs boson, as demonstrated in Fig. 5.

This can be easily understood with the help of the tree-level illustration of a fermion current with the emission of the radion, a gauge boson, and the Higgs boson (Fig. 6). The product of the radion-fermion-fermion vertex (the black point in Fig. 6) and two propagators leads to three terms, respectively,

$$S_i \Gamma_{i,j} S_j = S_i \left\{ -m + \frac{3}{2} S_j^{-1} + \frac{3}{2} S_i^{-1} \right\} S_j$$
$$= -S_i m S_j + \frac{3}{2} S_i + \frac{3}{2} S_j,$$

that is, a Higgs-like term with the vertex proportional to the fermion mass (the empty point in Fig. 6), a term with the propagator S_j omitted, i.e., with the radion and the Higgs boson emission from the same point, and a term with the propagator S_i omitted, i.e., with the radion and the gauge boson emission from the same point. In this way we get a reduction of each box diagram with the radion to a sum of other contributions.

One can substitute Eqs. (19)–(21) explicitly into the expressions $X_i^{\mu\nu}$ and open the brackets in order to get a representation of $X_1^{\mu\nu}$, $X_2^{\mu\nu}$, and $X_3^{\mu\nu}$ as sums of other $X_i^{\mu\nu}$ and $Y_i^{\mu\nu}$ contributions with the corresponding numerical factors

$$X_1^{\mu\nu} = Y_1^{\mu\nu} - \frac{3}{8} X_5^{\mu\nu} - \frac{1}{2} X_6^{\mu\nu}, \qquad (23)$$

$$X_2^{\mu\nu} = Y_2^{\mu\nu} - \frac{3}{8} X_5^{\mu\nu} - \frac{1}{2} X_7^{\mu\nu}, \qquad (24)$$

$$X_3^{\mu\nu} = Y_3^{\mu\nu} - \frac{1}{2} X_6^{\mu\nu} - \frac{1}{2} X_7^{\mu\nu}. \qquad (25)$$

The term $X_4^{\mu\nu}$ can be represented as a combination of $X_i^{\mu\nu}$ and $Y_i^{\mu\nu}$ contributions as

$$X_4^{\mu\nu} = -\frac{1}{4} X_5^{\mu\nu} + \frac{m_r^2 + 2m_h^2}{(k_1 + k_2)^2 - m_h^2} Y_4^{\mu\nu} \qquad (26)$$

by transforming the Higgs boson-radion vertex as

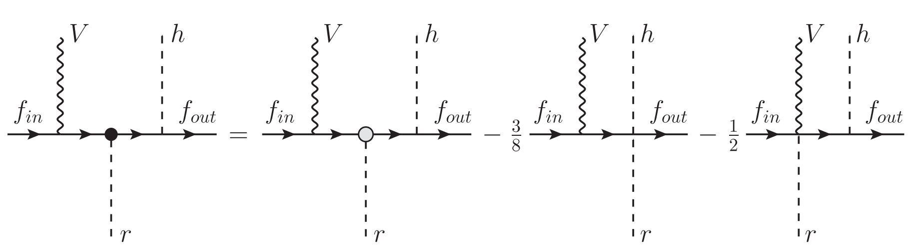

FIG. 6. Fermion current with emission of the radion, a gauge boson, and the Higgs boson expressed through the term with the Higgs-like vertex (the empty point) and two terms with four-point radion-boson vertices with corresponding numerical factors.

$$\Gamma' = 2\{(k_1+k_2)_\mu k_1^\mu - 2m_h^2\} = (k_1+k_2)^2 + k_1^2 - k_2^2 - 4m_h^2$$
$$= [(k_1+k_2)^2 - m_h^2] - m_r^2 - 2m_h^2 \quad (27)$$

and multiplying it by the propagator D [Eq. (22)],

$$D\Gamma' = 1 - \frac{m_r^2 + 2m_h^2}{(k_1+k_2)^2 - m_h^2}. \quad (28)$$

Finally, the sum of all $X_i^{\mu\nu}$ for the $gg \to rh$ process can be written in terms of $Y_i^{\mu\nu}$ and the parameter ξ, as was done in the previous example [see Eq. (16)],

$$\sum_{i=1}^{7} X_i^{\mu\nu} = Y_1^{\mu\nu} + Y_2^{\mu\nu} + Y_3^{\mu\nu} + \xi Y_4^{\mu\nu}, \quad (29)$$

which looks very similar to the expression for the $gg \to hh$ process,

$$\sum_{i=1}^{4} Y_i^{\mu\nu} = Y_1^{\mu\nu} + Y_2^{\mu\nu} + Y_3^{\mu\nu} + Y_4^{\mu\nu}. \quad (30)$$

Thus, once again we see that the amplitudes for these two processes coincide up to the parameter ξ and to the replacements of the masses and the denominators of the coupling constants.

In other words, the explicit amplitude of the associated Higgs boson-radion production process—which contains additional nonstandard Feynman graphs and a complicated vertex structure—can be obtained form the well-studied result for the Higgs boson pair production by a simple rescaling of the parameters.

IV. CANCELLATIONS OF ADDITIONAL TO THE HIGGS-LIKE CONTRIBUTIONS IN ASSOCIATED HIGGS BOSON-RADION PRODUCTION

In Ref. [24] it was shown that all the additional contributions as compared to the Higgs boson case are canceled out in the amplitudes of the single radion production processes. This property follows from the structure of any massive fermion current emitting the radion and an arbitrary number of any SM gauge bosons. Now we show that a similar general property appears in the case of the associated Higgs boson-radion production. We have already demonstrated the explicit cancellation using the example of the associated Higgs boson-radion production in fermion-antifermion annihilation and in gluon fusion. These were the processes with only two bosons (rh or hh) in the final state. For the general proof let us consider a fermion current (or a fermion loop) with the emission of an arbitrary number, say N, of SM bosons (vector gauge V or Higgs h) with all possible permutations. N_V stands for the number of gauge bosons and N_h stands for the number of Higgs bosons, $N_V + N_h = N$. Now we add another Higgs boson (for $f\bar{f} \to h_1, \ldots, h_{N_h+1}, V_1, \ldots, V_{N_V}$) or another radion (for $f\bar{f} \to r, h_1, \ldots, h_{N_h}, V_1, \ldots, V_{N_V}$) to this current in all possible ways.

There are two possibilities of adding a Higgs boson: a) the one emitted from the fermion line, and b) the one emitted from the boson (V or h) line. For the radion there are the same options plus another one: c) the radion is emitted directly from the four-point vertex with a V or h boson (Fig. 7).

Let us first set the notations for all the vertices. For each vertex here all the lines are considered to be incoming. We will use latin indices (a, b, c) for a parallel consideration of cases with gauge and Higgs bosons. In the case of gauge bosons the latin indices take values of vector indices (α, β, γ, μ) and in the case of Higgs bosons they just reduce to the sign (h) for Higgs boson vertices.

So, Γ_a stands for the Lorentz part of the fermion–fermion–gauge boson vertex and for the fermion–fermion–Higgs boson vertex, respectively,

$$\Gamma_a = \begin{cases} \Gamma_\mu \\ \Gamma_{(h)} \end{cases} = i\begin{cases} \gamma_\mu(a_f + b_f\gamma_5), \\ -\frac{m_f}{v}. \end{cases} \quad (31)$$

In the same manner, we denote the vertex with two gauge bosons and the Higgs boson and the triple Higgs vertex

$$[\Gamma'_{(h)}]^a_b = \begin{cases} [\Gamma'_{(h)}]^\alpha_\beta \\ \Gamma'_{(h)} \end{cases} = i\begin{cases} \frac{2m_V^2}{v}\delta^\alpha_\beta, \\ -\frac{3m_h^2}{v}. \end{cases} \quad (32)$$

Now for the radion, the fermion-fermion-radion vertex $\Gamma_{(r)}$ is a function of the momenta of the incoming fermions, say p and l. It can be rewritten in terms of the inverse propagators S^{-1} [24] and takes the following form:

$$\Gamma_{(r)}(p,l) = -\frac{i}{\Lambda_r}\left\{\frac{3}{2}[(\not{p} + m_f) - (\not{l} - m_f)] + m_f\right\}$$
$$= \frac{i}{\Lambda_r}\left\{\frac{3}{2}[S^{-1}(l) + S^{-1}(-p)] - m_f\right\}$$
$$= \frac{v}{\Lambda_r}\Gamma_{(h)} + \frac{i3}{2\Lambda_r}[S^{-1}(l) + S^{-1}(-p)]. \quad (33)$$

The notation $[\Gamma'_{(r)}]^a_b$ unifies the vertices for two gauge bosons and the radion interaction (excluding the anomalies) and for two Higgs bosons and the radion interaction, respectively,

$$[\Gamma'_{(r)}]^a_b = \begin{cases} [\Gamma'_{(r)}]^\alpha_\beta \\ \Gamma'_{(r)} \end{cases} = i\begin{cases} \frac{2m_V^2}{\Lambda_r}\delta^\alpha_\beta, \\ -\frac{2}{\Lambda_r}\{(p\cdot k) + 2m_h^2\}. \end{cases} \quad (34)$$

The term $\Gamma''_{(r),a}$ stands for gauge boson–fermion–fermion–radion and Higgs boson–fermion–fermion–radion four-point vertices:

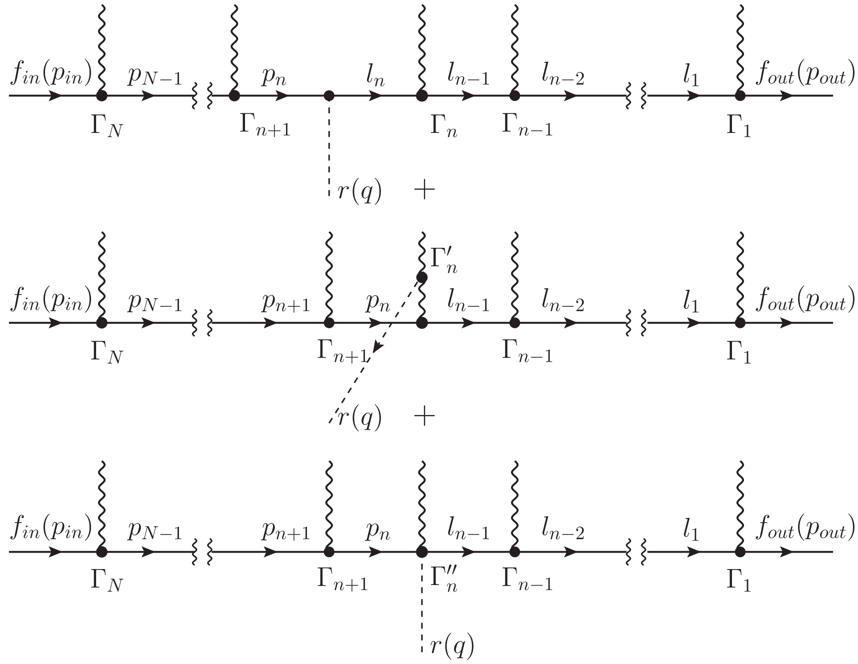

FIG. 7. Fermion current radiating the radion and N SM Higgs or vector gauge bosons, $l_j \equiv p_j - q$.

$$\Gamma''_{(r),a} = \begin{cases} [\Gamma''_{(r)}]_\mu \\ \Gamma''_{(r)} \end{cases} = \frac{i}{\Lambda_r} \begin{cases} 3\gamma_\mu (a_f + b_f \gamma_5) \\ -\frac{4m_f}{v} \end{cases} = \frac{1}{\Lambda_r} \begin{cases} 3\Gamma_\mu \\ 4\Gamma_{(h)} \end{cases} = \frac{3}{\Lambda_r}\Gamma_a + \frac{1}{\Lambda_r} \begin{cases} 0, \\ \Gamma_{(h)}. \end{cases} \tag{35}$$

Now one can write down the contributions to the amplitudes in the following form:

$$M[\varphi]_n = i^N \bar{f}(p_{\rm out}) \left[\prod_{j=1}^{n} \epsilon(k_j)^{a_j} \Gamma_{a_j} S(p_j - q) \right] \varphi(q) \Gamma_{(\varphi)} \left[\prod_{j=n+1}^{N} S(p_{j-1}) \epsilon(k_j)^{a_j} \Gamma_{a_j} \right] f(p_{\rm in}), \tag{36}$$

$$M'[\varphi]_n = i^N \bar{f}(p_{\rm out}) \left[\prod_{j=1}^{n-1} \epsilon(k_j)^{a_j} \Gamma_{a_j} S(p_j - q) \right] (\epsilon(k_n)^{a_n} \varphi(q) [\Gamma'_{(\varphi)}]^c_{a_n} G(k_n + q)^b_c \Gamma_b) \left[\prod_{j=n+1}^{N} S(p_{j-1}) \epsilon(k_j)^{a_j} \Gamma_{a_j} \right] f(p_{\rm in}), \tag{37}$$

$$M''[\varphi]_n = i^{N-1} \bar{f}(p_{\rm out}) \left[\prod_{j=1}^{n-1} \epsilon(k_j)^{a_j} \Gamma_{a_j} S(p_j - q) \right] (\epsilon(k_n)^{a_n} r(q) [\Gamma''_{(r)}]_{a_n}) \left[\prod_{j=n+1}^{N} S(p_{j-1}) \epsilon(k_j)^{a_j} \Gamma_{a_j} \right] f(p_{\rm in}), \tag{38}$$

where φ is either h or r in Eqs. (36) and (37) and only r in Eq. (38), $\epsilon(k_j)^{a_j}$ is either $\epsilon(k_j)^{\mu_j}$ (for the gauge boson) or $h(k_j)$ (for the Higgs boson), $G(k_n+q)^b_c$ is either $G(k_n+q)^\beta_\gamma$ (the gauge boson propagator) or $G(k_n+q)$ (the Higgs boson propagator), and

$$p_{n-1} = p_n - k_n, \qquad p_{\rm in} = p_N, \qquad p_{\rm out} = p_0 - q.$$

The number n runs from 1 to N ($n = 1, 2, \ldots, N$). In the case of real initial and final fermions one should take into account another amplitude,

$$M[\varphi]_0 = i^N \left[\prod_{j=1}^{N} \epsilon(k_j)^{a_j} \right] \varphi(q) \bar{f}(p_{\rm out}) \Gamma_{(\varphi)} \left[\prod_{j=1}^{N} S(p_{j-1}) \Gamma_{a_j} \right] f(p_{\rm in}). \tag{39}$$

If the added particle is the Higgs boson, the amplitude takes the form

$$M[h]_0 = i^N \bar{f}(p_{\rm out}) h(q) \Gamma_{(h)} \left[\prod_{j=1}^{N} S(p_{j-1}) \epsilon(k_j)^{a_j} \Gamma_{a_j} \right] f(p_{\rm in}). \tag{40}$$

If the added particle is the radion, one can write the following amplitude:

$$\begin{aligned} M[r]_0 &= \frac{v}{\Lambda_r} i^N \left[\prod_{j=1}^{N} \epsilon(k_j)^{a_j} \right] r(q) \bar{f}(p_{\rm out}) \Gamma_{(h)} \left[\prod_{j=1}^{N} S(p_{j-1}) \Gamma_{a_j} \right] f(p_{\rm in}) + \frac{i3}{2\Lambda_r} i^N \left[\prod_{j=1}^{N} \epsilon(k_j)^{a_j} \right] r(q) \bar{f}(p_{\rm out}) S^{-1}(p_0) \\ &\quad \times \left[\prod_{j=1}^{N} S(p_{j-1}) \Gamma_{a_j} \right] f(p_{\rm in}) \\ &= \frac{v}{\Lambda_r} M[h]_0 + \frac{i3}{2\Lambda_r} i^N \left[\prod_{j=1}^{N} \epsilon(k_j)^{a_j} \right] r(q) \bar{f}(p_{\rm out}) \left[\prod_{j=1}^{N} \Gamma_{a_j} S(p_j) \right] \Gamma_{a_N} f(p_{\rm in}), \end{aligned} \tag{41}$$

where we used Eq. (33) and the equation of motion $\bar{f}(p_{\rm out}) S^{-1}(p_{\rm out}) = 0$ ($p_{\rm out}$ is the outgoing momentum).

Now let us take any number n, $n = 1, \ldots, N$. In the case of adding a Higgs boson the sum of all amplitudes for the chosen n is

$$\begin{aligned} M[h]_n + M'[h]_n &= i^N \left[\prod_{j=1}^{N} \epsilon(k_j)^{a_j} \right] h(q) \bar{f}(p_{\rm out}) \left[\prod_{j=1}^{n-1} \Gamma_{a_j} S(p_j - q) \right] \\ &\quad \times \{ \Gamma_{a_n} S(p_n - q) \Gamma_{(h)} + [\Gamma'_{(h)}]^c_{a_n} G(k+q)^b_c \Gamma_b \} \left[\prod_{j=n+1}^{N} S(p_{j-1}) \Gamma_{a_j} \right] f(p_{\rm in}). \end{aligned} \tag{42}$$

For the case of adding a radion the sum has the following form:

$$\begin{aligned} M[r]_n + M'[r]_n + M''[r]_n &= i^N \left[\prod_{j=1}^{N} \epsilon(k_j)^{a_j} \right] r(q) \bar{f}(p_{\rm out}) \left[\prod_{j=1}^{n-1} \Gamma_{a_j} S(p_j - q) \right] \\ &\quad \times \{ \Gamma_{a_n} S(p_n - q) \Gamma_{(r)} + [\Gamma'_{(r)}]^c_{a_n} G(k+q)^b_c \Gamma_b - i[\Gamma''_{(r)}]_{a_n} \} \left[\prod_{j=n+1}^{N} S(p_{j-1}) \Gamma_{a_j} \right] f(p_{\rm in}). \end{aligned} \tag{43}$$

One can calculate the part in curly brackets in Eq. (43) and compare it with that in Eq. (42),

$$\begin{aligned} \Gamma_{a_n} S(p_n - q) \Gamma_{(r)} + [\Gamma'_{(r)}]^c_{a_n} G(k+q)^b_c \Gamma_b - i[\Gamma''_{(r)}]_{a_n} &= \frac{v}{\Lambda_r} \left(\Gamma_{a_n} S(p_n - q) \Gamma_{(h)} + \left\{ \begin{matrix} 1 \\ \xi \end{matrix} \right\} [\Gamma'_{(h)}]^c_{a_n} G(k+q)^b_c \Gamma_b \right) \\ &\quad + \frac{i3}{2\Lambda_r} \Gamma_{a_n} (S(p_n - q) S^{-1}(p_n) - 1). \end{aligned} \tag{44}$$

Substituting the result (44) into Eq. (43) and opening the brackets, one gets

$$\begin{aligned} M[r]_n + M'[r]_n + M''[r]_n &= i^N \left[\prod_{j=1}^{N} \epsilon(k_j)^{a_j} \right] r(q) \bar{f}(p_{\rm out}) \left[\prod_{j=1}^{n-1} \Gamma_{a_j} S(p_j - q) \right] \frac{v}{\Lambda_r} \left(\Gamma_{a_n} S(p_n - q) \Gamma_{(h)} \right. \\ &\quad \left. + \left\{ \begin{matrix} 1 \\ \xi \end{matrix} \right\} [\Gamma'_{(h)}]^c_{a_n} G(k+q)^b_c \Gamma_b \right) \left[\prod_{j=n+1}^{N} S(p_{j-1}) \Gamma_{a_j} \right] f(p_{\rm in}) + i^N \left[\prod_{j=1}^{N} \epsilon(k_j)^{a_j} \right] r(q) \bar{f}(p_{\rm out}) \\ &\quad \times \left[\prod_{j=1}^{n-1} \Gamma_{a_j} S(p_j - q) \right] \frac{i3}{2\Lambda_r} \Gamma_{a_n} (S(p_n - q) S^{-1}(p_n) - 1) \left[\prod_{j=n+1}^{N} S(p_{j-1}) \Gamma_{a_j} \right] f(p_{\rm in}). \end{aligned} \tag{45}$$

Here the first term has almost the same form as in the case of the Higgs boson (the absolute similarity would take place if $\xi = 1$ and $\Lambda_r = v$). In the second term one can open the brackets and get

$$
\begin{aligned}
&i^N\left[\prod_{j=1}^{N}\epsilon(k_j)^{a_j}\right]r(q)\bar{f}(p_{\rm out})\left[\prod_{j=1}^{n-1}\Gamma_{a_j}S(p_j-q)\right]\frac{i3}{2\Lambda_r}\Gamma_{a_n}(S(p_n-q)S^{-1}(p_n)-1)\left[\prod_{j=n+1}^{N}S(p_{j-1})\Gamma_{a_j}\right]f(p_{\rm in})\\
&=\frac{i3}{2\Lambda_r}i^N\left[\prod_{j=1}^{N}\epsilon(k_j)^{a_j}\right]r(q)\bar{f}(p_{\rm out})\left[\prod_{j=1}^{n}\Gamma_{a_j}S(p_j-q)\right]\Gamma_{a_{n+1}}\left[\prod_{j=n+2}^{N}S(p_{j-1})\Gamma_{a_j}\right]f(p_{\rm in})\\
&-\frac{i3}{2\Lambda_r}i^N\left[\prod_{j=1}^{N}\epsilon(k_j)^{a_j}\right]r(q)\bar{f}(p_{\rm out})\left[\prod_{j=1}^{n-1}\Gamma_{a_j}S(p_j-q)\right]\Gamma_{a_n}\left[\prod_{j=n+1}^{N}S(p_{j-1})\Gamma_{a_j}\right]f(p_{\rm in})=U_{n+1}-U_n, \qquad (46)
\end{aligned}
$$

where

$$
U_n=\frac{i3}{2\Lambda_r}i^N\left[\prod_{j=1}^{N}\epsilon(k_j)^{a_j}\right]r(q)\bar{f}(p_{\rm out})\left[\prod_{j=1}^{n-1}\Gamma_{a_j}S(p_j-q)\right]\Gamma_{a_n}\left[\prod_{j=n+1}^{N}S(p_{j-1})\Gamma_{a_j}\right]f(p_{\rm in}) \qquad (47)
$$

for $n < N$.

It is easy to write U_1 in the following form:

$$
U_1=\frac{i3}{2\Lambda_r}i^N\left[\prod_{j=1}^{N}\epsilon(k_j)^{a_j}\right]r(q)\bar{f}(p_{\rm out})\left[\prod_{j=1}^{N-1}\Gamma_{a_j}S(p_j)\right]\Gamma_{a_N}f(p_{\rm in}). \qquad (48)
$$

Getting back to the expression for $M[r]_0$ [Eq. (41)], one finds

$$
M[r]_0=\frac{v}{\Lambda_r}M[h]_0+U_1. \qquad (49)
$$

For $n = N$ one must separately consider the case of real initial and final fermions and the case of a fermion loop. For the first case the equation of motion $S^{-1}(p_N)f(p_{\rm in}) = 0$ is valid, and thus

$$
\begin{aligned}
&i^N\left[\prod_{j=1}^{N}\epsilon(k_j)^{a_j}\right]r(q)\bar{f}(p_{\rm out})\left[\prod_{j=1}^{N-1}\Gamma_{a_j}S(p_j-q)\right]\frac{i3}{2\Lambda_r}\Gamma_{a_N}(S(p_N-q)S^{-1}(p_N)-1)\\
&=-\frac{i3}{2\Lambda_r}i^N\left[\prod_{j=1}^{N}\epsilon(k_j)^{a_j}\right]r(q)\bar{f}(p_{\rm out})\left[\prod_{j=1}^{N-1}\Gamma_{a_j}S(p_j-q)\right]\Gamma_{a_N}f(p_{\rm in})=-U_N. \qquad (50)
\end{aligned}
$$

Finally, for the case of real initial and final fermions we have

$$
\begin{aligned}
M[r]_0+\sum_{n=1}^{N}(M[r]_n+M'[r]_n+M''[r]_n)&=\frac{v}{\Lambda_r}M[h]_0+\frac{v}{\Lambda_r}\sum_{n=1}^{N}\left(M[h]_n+\begin{Bmatrix}1\\ \xi\end{Bmatrix}M'[h]_n\right)+U_1+(U_2-U_1)\\
&\quad+(U_3-U_2)+\cdots+(U_N-U_{N-1})-U_N\\
&=\frac{v}{\Lambda_r}M[h]_0+\frac{v}{\Lambda_r}\sum_{n=1}^{N}\left(M[h]_n+\begin{Bmatrix}1\\ \xi\end{Bmatrix}M'[h]_n\right). \qquad (51)
\end{aligned}
$$

In the case of the fermion loop ($p_{\rm in} = p_{\rm out}$) one can move $\bar{f}(p_{\rm out})$ by cyclic permutations to the end of the matrix product, which leaves the trace invariant, so $f(p_{\rm in})\bar{f}(p_{\rm out}) = S(p_{\rm in}) = S(p_{\rm out})$ and therefore $S^{-1}(p_N)f(p_{\rm in})\bar{f}(p_{\rm out}) = 1$:

$$i^N\left[\prod_{j=1}^{N}\epsilon(k_j)^{a_j}\right]r(q)\bar{f}(p_{\rm out})\left[\prod_{j=1}^{N-1}\Gamma_{a_j}S(p_j-q)\right]\frac{i3}{2\Lambda_r}\Gamma_{a_N}(S(p_N-q)S^{-1}(p_N)-1)f(p_{\rm in})$$
$$=\frac{i3}{2\Lambda_r}i^N\left[\prod_{j=1}^{N}\epsilon(k_j)^{a_j}\right]r(q)\left[\prod_{j=1}^{N}\Gamma_{a_j}S(p_j-q)\right]-U_N. \tag{52}$$

It is easy to check that the second to last term in Eq. (52) is equal to U_1. Indeed, it can be shown by means of the same trick: moving $f(p_{\rm in})$ to the beginning and shifting the loop momentum by the value q. Thus, just as in the case of real fermions we get Eq. (51).

V. CONCLUSIONS

In the current work we have discussed the Higgs boson-radion similarity in their associated production processes. First, the associated Higgs boson-radion production was considered in two examples: fermion-antifermion annihilation and gluon fusion. In both cases we have shown explicitly that the analytical expressions for the full amplitudes of the associated Higgs boson-radion and the Higgs boson pair-production processes coincide up to the replacement of the masses and the denominators of the coupling constants and a rescaling of the triple Higgs coupling. Next, the general proof of this property was provided for the case of the radion production in association with an arbitrary number of SM gauge or Higgs bosons at the tree and loop levels.

This property of the amplitudes can significantly simplify the explicit symbolic computations of the full perturbative amplitudes of associated Higgs boson-radion production processes, because we can use the well-studied and less-complicated Higgs pair production amplitudes instead.

Due to this property the radion contribution can mimic the deviation in the triple Higgs coupling in the case of close radion and Higgs boson masses. This fact should be taken into account in the investigation of the triple Higgs coupling. Namely, in the case where the Higgs and the radion masses are close to each other and the Higgs boson-radion mixing is zero or small, a detailed analysis is needed to understand how accurately one can measure the triple Higgs coupling, keeping in mind that even in the SM it is a difficult task to measure the coupling at future collider experiments with an accuracy better than 30%. As it was shown in Ref. [23], this case is still not excluded by the experimental data.

Moreover, if there are other Higgs bosons (as it occurs in various SM extensions) and the mass of the heavy Higgs boson is close to the radion mass, we can face the same problem in measuring the triple couplings and clarifying to which particles the coupling corresponds. One can use other properties (other production channels and decay modes) to try to answer this question, but surely an additional investigation would be needed in each specific case.

Of course, there exists the well-known difference between the radion and the Higgs boson because of the presence of the radion anomalous interaction. In addition to the enhancement of the radion decay modes to two gluons and to two photons, the anomalous radion-gluon-gluon interaction contributes differently to the associated Higgs boson-radion and to the Higgs pair production. In the latter case the Higgs boson pair production can occur via the radion decay. The corresponding diagram does not participate in the cancellations and turns into a diagram of the same Λ_r^{-1} order in the case of the resonant Higgs boson production ($m_r > 2m_h$), while in the case of the nonresonant Higgs boson production ($m_r < 2m_h$) this diagram is of the next order (Λ_r^{-2}). In fact, the anomalous radion-gluon-gluon interaction gives the leading contribution to the Higgs pair production via the radion decay. A phenomenological analysis of the differences in the production and decay properties of the radion and the Higgs boson taking into account the mentioned rescaling of the parameters and the anomalies in the radion interactions with the massless gauge bosons is out of the scope of the current work, and could be an interesting subject for further investigations.

The radion pair production was not considered in the current work, as it is a model-dependent and complicated study where the next orders ought to be taken into account.

It is important to mention that the considered property is valid not only for the radion in the brane-world models with two branes, but it can also take place in scalar-tensor gravity theories (for example, Brans-Dicke theory) or theories involving a dilaton where the scalar field interacts with the trace of the energy-momentum tensor of matter.

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