Gravitational waves in $f(R,T)$ and $f(R,T^\phi)$ theories of gravity

M. E. S. Alves*

Instituto de Ciência e Tecnologia, UNESP—Univ Estadual Paulista, São José dos Campos, São Paulo 12247-016, Brazil

P. H. R. S. Moraes†

ITA—Instituto Tecnológico de Aeronáutica—Departamento de Física, São José dos Campos, 12228-900 São Paulo, Brazil

J. C. N. de Araujo‡

INPE—Instituto Nacional de Pesquisas Espaciais—Divisão de Astrofísica, São José dos Campos, 12227-010 São Paulo, Brazil

M. Malheiro§

ITA—Instituto Tecnológico de Aeronáutica—Departamento de Física, São José dos Campos, 12228-900 São Paulo, Brazil

(Received 5 May 2016; published 14 July 2016)

There is a host of alternative theories of gravitation in the literature, among them the $f(R,T)$ and $f(R,T^\phi)$ theories recently elaborated by Harko *et al.* In these theories, R, T and T^ϕ are respectively the Ricci scalar and the traces of the energy-momentum tensors of matter and of a scalar field. There is already in the literature a series of studies of different forms of the $f(R,T)$ and $f(R,T^\phi)$ functions as well as their cosmological consequences. However, there have been no studies so far related to gravitational waves. Here we consider such an issue, in particular, studying the putative extra polarization modes that can appear in the scope of such theories. Different functional forms of $f(R,T^\phi)$ are considered and the gravitational waveforms are found for the extra polarization modes in the cases in which they are present.

DOI: 10.1103/PhysRevD.94.024032

I. INTRODUCTION

Recently, gravitational waves (GWs) were directly detected for the first time [1]. Forthcoming observations will contribute to the study and understanding of a large number of research fields in physics, astrophysics and cosmology, from the absolute ground state of matter [2] to the upper limits on the brane tension values of braneworld cosmologies [3]. In the near future, we may also be able to estimate parameters of compact binary systems [4–7], constrain the equation of state of neutron stars [8–13] and distinguish general relativity (GR) from alternative theories of gravity [14–17].

It is well known that alternative theories of gravity arise as possibilities for evading some standard cosmology shortcomings [18–21]. Recently elaborated by Harko *et al.*, the $f(R,T)$ gravity [22] is one of the promising alternatives.

Although plenty of well-behaved cosmological models have been derived from such a theory (see [23–32] and references therein), no efforts have been made in applying $f(R,T)$ gravity to the study of GWs.

It is the purpose of the present article to explore the physical features of GWs in $f(R,T)$ gravity and in different possible formulations of $f(R,T^\phi)$ gravity. We will show that the physics of GWs is strongly dependent on the functional forms of $f(R,T)$ and $f(R,T^\phi)$, in such a way that different formulations can exhibit different numbers of polarization states. In order to characterize the polarization states of GWs for some formulations of interest, we will evaluate the Newman-Penrose (NP) quantities [33–35] predicted by them. For now, it is worth mentioning that the NP formalism has been applied to different alternative theories of gravity, leading to interesting and testable results [14,36,37].

II. $f(R,T)$ GRAVITY

Proposed by Harko *et al.* [22] as a generalization of the $f(R)$ theories (see [38,39] and references therein), the gravitational part of the $f(R,T)$ action depends not only on a generic function of the Ricci scalar R, as in $f(R)$ gravity theories, but also on a function of T, the trace of the energy-momentum tensor $T_{\mu\nu}$. According to the authors, the dependence on T arises from the consideration of quantum effects (conformal anomaly) which are usually neglected in

*marcio.alves@ict.unesp.br
†moraes.phrs@gmail.com
‡jcarlos.dearaujo@inpe.br
§malheiro@ita.br

$f(R)$ or GR theories, for instance. The full action in $f(R,T)$ gravity reads

$$S = \int d^4x\sqrt{-g}[f(R,T) + L_m], \tag{1}$$

with $f(R,T)$ being an arbitrary function of R and T, g is the determinant of the metric $g_{\mu\nu}$, with greek indices assuming the values 0–3 and $\sqrt{-g}L_m$ is the Lagrangian density of matter. Throughout this work we will use units such that $4\pi G = c = 1$.

By varying Eq. (1) with respect to the metric, one obtains the $f(R,T)$ field equations

$$f_R(R,T)R_{\mu\nu} - \frac{1}{2}f(R,T)g_{\mu\nu} + (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)f_R(R,T)$$
$$= -\frac{1}{2}T_{\mu\nu} - f_T(R,T)T_{\mu\nu} - f_T(R,T)\Theta_{\mu\nu}, \tag{2}$$

with $\Box \equiv \partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu)/\sqrt{-g}$, $\Theta_{\mu\nu} \equiv g^{\alpha\beta}\delta T_{\alpha\beta}/\delta g^{\mu\nu}$, $f_R(R,T) \equiv \partial f(R,T)/\partial R$, $f_T(R,T) \equiv \partial f(R,T)/\partial T$, and where $R_{\mu\nu}$ is the Ricci tensor, ∇_μ is the covariant derivative with respect to the symmetric connection associated to $g_{\mu\nu}$ and the energy-momentum tensor, as usual, reads

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}L_m)}{\delta g^{\mu\nu}}. \tag{3}$$

For reasons that will be presented below, in this article we are also concerned with a different theory for which Harko *et al.* [22] have considered the coupling of gravity with a self-interacting scalar field ϕ, namely $f(R,T^\phi)$ gravity, with T^ϕ standing for the trace of the energy-momentum tensor of the scalar field.

Such a formulation was developed in a cosmological perspective in [23] and gave rise to a complete scenario of the Universe's evolution, able to describe the inflationary, radiation, matter and dark energy eras.

For this case, the full action is given by

$$S = \int d^4x\sqrt{-g}[f(R,T^\phi) + L(\phi,\nabla_\mu\phi) + L_m], \tag{4}$$

where $L(\phi,\nabla_\mu\phi)$ is the usual Lagrangian for the scalar field, namely

$$L(\phi,\nabla_\mu\phi) = \frac{1}{2}\nabla_\alpha\phi\nabla^\alpha\phi - V(\phi), \tag{5}$$

and $V(\phi)$ is a self-interacting potential. Notice that in this theory, the matter fields have only a minimal coupling with gravity and they do not couple with ϕ.

From (3) and (5), the energy-momentum tensor for the scalar field reads

$$T^\phi_{\mu\nu} = \nabla_\mu\phi\nabla_\nu\phi - \frac{1}{2}g_{\mu\nu}\nabla_\alpha\phi\nabla^\alpha\phi + g_{\mu\nu}V(\phi) \tag{6}$$

and its trace is given by

$$T^\phi = -\nabla_\alpha\phi\nabla^\alpha\phi + 4V(\phi). \tag{7}$$

III. GWS IN THE $f(R,T)$ GRAVITY

In order to find the number of polarization modes of GWs of a theory we need to examine the far-field, linearized, vacuum field equations of the theory. For vacuum, the $f(R,T)$ and $f(R)$ field equations [38,39] are the same, namely,

$$f_R R_{\mu\nu} - \frac{1}{2}fg_{\mu\nu} - \nabla_\mu\nabla_\nu f_R + g_{\mu\nu}\Box f_R = 0. \tag{8}$$

The calculation of the NP parameters for such a theory was carried out in Ref. [14]. The authors considered $f(R) = R - \alpha R^{-\beta}$, with α and β being constants, and found out that if $\alpha \neq 0$ and $\beta \neq 0$, one has for the NP quantities

$$\Psi_2 \neq 0, \qquad \Psi_3 = 0, \qquad \Psi_4 \neq 0, \quad \text{and} \quad \Phi_{22} \neq 0, \tag{9}$$

showing that this theory presents the scalar longitudinal mode (Ψ_2) and the "breathing" scalar transversal mode (Φ_{22}) in addition to the usual tensor modes represented by Ψ_4. But it is worth emphasizing that since $\Psi_2 \neq 0$, this is the only observer-independent mode. The presence or absence of all other modes depends on the observer (see, e.g., [14,33]).

Therefore, the standard $f(R,T)$ formalism does not give new information about GW polarization states, because in vacuum the $f(R)$ formalism is retrieved.

IV. GWS IN THE $f(R,T^\phi) = -R/4 + f(T^\phi)$ THEORY

Because one expects scalar field terms to appear in the $f(R,T^\phi)$ field equations for vacuum, new polarization states of GWs can be present in such theories. The present and the following sections will deal with this issue.

In what follows we consider GWs in the absence of matter and therefore we take $L_m = 0$. The field equations of the $f(R,T^\phi) = -R/4 + f(T^\phi)$ theory read [23]

$$G_{\mu\nu} = 2[T^\phi_{\mu\nu} - g_{\mu\nu}f(T^\phi) - 2f_T(T^\phi)\nabla_\mu\phi\nabla_\nu\phi], \tag{10}$$

where $G_{\mu\nu}$ is the usual Einstein tensor. It is also useful to know explicitly the Ricci scalar, namely

$$R = -2[T^\phi - 4f(T^\phi) - 2f_T(T^\phi)\nabla_\alpha\phi\nabla^\alpha\phi]. \tag{11}$$

The equation of motion for the scalar field can be found by applying the covariant divergence of the field equations (10). One then obtains

$$(1-2f_T)\Box\phi+(1-4f_T)\frac{\partial V}{\partial\phi}-2f_{TT}\nabla^\nu\phi\nabla_\nu T=0, \quad (12)$$

with $f_{TT}\equiv\partial^2 f(R,T)/\partial T^2$.

The properties of GWs depend upon the particular choice for $f(T^\phi)$, for which we analyze some well-motivated possibilities below.

A. The $f(T^\phi)=2\lambda T^\phi$ case

One interesting particular case is to consider that f depends linearly on the trace of the energy-momentum tensor of the scalar field, i.e., $f(T^\phi)=2\lambda T^\phi$, where λ is a constant. Such a functional form has already been used in $f(R,T^\phi)$ models, yielding a description of a complete cosmological scenario, as can be seen in [23]. For this case, Eq. (12) reduces to

$$\Box\phi+\left(\frac{1-8\lambda}{1-4\lambda}\right)\frac{\partial V(\phi)}{\partial\phi}=0, \quad (13)$$

for $\lambda\neq 1/4$.

Since we need to analyze the field equations in the linearized regime, we expand the potential around its minimum, obtaining [33]

$$\Box\phi+\left(\frac{1-8\lambda}{1-4\lambda}\right)m^2(\phi-\phi_0)=0, \quad (14)$$

where now $\Box=\eta^{\mu\nu}\partial_\mu\partial_\nu$ with $\eta^{\mu\nu}$ being the Minkowski metric, $m^2=(\partial^2V/\partial\phi^2)_{\phi=\phi_0}$ and ϕ_0 locates the minimum of the potential, which could be obtained from cosmological boundary conditions. A solution of this equation reads

$$\phi(x)=\phi_0+\phi_1 e^{iq_\alpha x^\alpha}, \quad (15)$$

where ϕ_1 is a small amplitude and q_α is the wave vector which obeys the following equation,

$$q_\alpha q^\alpha=\left(\frac{1-8\lambda}{1-4\lambda}\right)m^2, \quad (16)$$

with $\lambda\leq 1/8$ or $\lambda>1/4$.

The Ricci scalar (11) for $f(T^\phi)=2\lambda T^\phi$ reads

$$R=2[(1-4\lambda)\nabla_\alpha\phi\nabla^\alpha\phi-4(1-8\lambda)V(\phi)], \quad (17)$$

and by considering terms up to first order in ϕ we find a constant curvature scalar, namely

$$R=-8(1-8\lambda)V_0+\mathcal{O}(\phi_1^2), \quad (18)$$

where V_0 is the minimum value of the potential.

Therefore, from the point of view of the propagation of GWs, the overall effect of the inclusion of a minimally coupled scalar field to first order is equivalent to considering an effective cosmological constant (CC)

$$\Lambda=2(1-8\lambda)V_0, \quad (19)$$

which is positive for $\lambda<1/8$, null for $\lambda=1/8$ and negative otherwise. As it is well known, Λ does not introduce any additional polarization states for GWs [40]. Therefore, we can conclude that GWs in the $f(R,T^\phi)=-R/4+2\lambda T^\phi$ theory have only the two usual polarizations of GR, i.e., $+$ and $\times$.

The above result shows that the linearized field equations of this theory, in the absence of matter, have the Minkowski metric as background only if $\lambda=1/8$ or $V_0=0$. In this case, the GW equations are exactly the same as those of GR theory.

Otherwise, in order to obtain the first order equations for the GWs, we need to expand the metric around the de Sitter metric. Here we do not consider such an issue; instead we refer the reader to Ref. [41] for a study of linear fields on de Sitter space-time.

B. The $f(T^\phi)=2\lambda(T^\phi)^n$ case with $V_0\neq 0$

If the minimum value of the potential $V_0\neq 0$, and by assuming $m^2\phi_1\ll V_0$, the case $f(T^\phi)=2\lambda(T^\phi)^n$, with n being a constant, does not provide a first order term in ϕ in the right-hand side of Eq. (10) or (11). The first two non-null terms of the expansion are the zero and second order terms. Therefore, as in the previous case, there are GWs just with the $+$ and $\times$ polarizations. The overall effect is just a redefinition of Λ in the de Sitter background metric as

$$\Lambda=2[V_0-2\lambda(4V_0)^n]. \quad (20)$$

C. The $f(T^\phi)=2\lambda\sqrt{T^\phi}$ case

Such an $f(T^\phi)$ functionality has already been studied as an $f(R,T)$ gravity case in [42]. This particular $f(T^\phi)$ is the only one in which the conservation law is respected in a minimal coupling of matter and geometry. This $f(T^\phi)$ exhibits the following equation for ϕ,

$$(\sqrt{T^\phi}-2\lambda)\Box\phi+(\sqrt{T^\phi}-4\lambda)\frac{\partial V}{\partial\phi}+\lambda\nabla^\mu\phi\nabla_\mu(\ln T^\phi)=0. \quad (21)$$

Following the same procedure as before, expanding $V(\phi)$ around a non-null minimum value V_0 and keeping terms up to first order in ϕ in the above equation we find

$$\Box\phi+\left(\frac{\sqrt{V_0}-2\lambda}{\sqrt{V_0}-\lambda}\right)m^2(\phi-\phi_0)=0. \quad (22)$$

The solution of this equation is of the form (15) with

$$q_\alpha q^\alpha = \left(\frac{\sqrt{V_0} - 2\lambda}{\sqrt{V_0} - \lambda}\right) m^2, \tag{23}$$

and $\lambda \leq \sqrt{V_0}/2$ or $\lambda > \sqrt{V_0}$. However, this solution does not imply any additional polarization states for GWs since this is a particular case of the precedent section with $n = 1/2$ in Eq. (20).

On the other hand, if we now adopt $V_0 = 0$ from the beginning we find that $\sqrt{T^\phi}$ is of first order in ϕ. Then, keeping terms up to first order in ϕ in Eq. (21) (which is now equivalent to saying that $\sqrt{T^\phi} \ll \lambda$) we find

$$\Box\phi + 2m^2(\phi - \phi_0) - \frac{1}{2}\partial^\mu\phi\partial_\mu \ln(T^\phi) = 0. \tag{24}$$

If we assume a propagating solution like (15) for this equation we find that it is identically satisfied only if $m = 0$. However, it does not imply $q_\mu q^\mu = 0$. Accordingly, the curvature scalar is given by

$$R = 12\lambda\sqrt{q_\mu q^\mu}\phi_1 e^{iq_\alpha x^\alpha} + \mathcal{O}(\phi^2) \tag{25}$$

and, considering the scalar wave ϕ and the GW propagating in the z direction, we choose $q^\mu = (\omega, 0, 0, k)$, where ω is the frequency of the scalar field and k is the z component of the wave vector. Thus, the non-null components of the Ricci tensor are

$$R_{00} = 2\lambda\left(\frac{3\omega^2 - k^2}{\sqrt{\omega^2 - k^2}}\right)\phi_1 e^{iq_\alpha x^\alpha}, \tag{26}$$

$$R_{33} = 2\lambda\left(\frac{3k^2 - \omega^2}{\sqrt{\omega^2 - k^2}}\right)\phi_1 e^{iq_\alpha x^\alpha}, \tag{27}$$

$$R_{03} = R_{30} = -4\lambda\left(\frac{\omega k}{\sqrt{\omega^2 - k^2}}\right)\phi_1 e^{iq_\alpha x^\alpha}, \tag{28}$$

$$R_{11} = R_{22} = -2\lambda\sqrt{\omega^2 - k^2}\phi_1 e^{iq_\alpha x^\alpha}. \tag{29}$$

Now, by using the above results together with the definitions (A5)–(A8) and with the help of Eqs. (A9)–(A13), we evaluate the NP parameters, namely

$$\Psi_2 = \lambda\sqrt{\omega^2 - k^2}\phi_1 e^{iq_\alpha x^\alpha}, \tag{30}$$

$$\Psi_3 = 0, \tag{31}$$

$$\Phi_{22} = -\lambda\left[\frac{(\omega + k)^2}{\sqrt{\omega^2 - k^2}}\right]\phi_1 e^{iq_\alpha x^\alpha}. \tag{32}$$

Note also that since there are no further constraints on the components of the Riemann tensor, $\Psi_4 \neq 0$. Thus, similarly to the $f(R)$ gravity case mentioned above, we are led to conclude that Eq. (9) holds once again, but now the presence or absence of the scalar longitudinal mode and of the scalar transversal mode depends on λ. If $\lambda = 0$ these extra polarization modes disappear and we recover GR theory with the only non-null parameter Ψ_4.

It is interesting to notice that in the usual scalar-tensor theories of gravity, the presence of a propagating Ψ_2 mode is related to the mass of the scalar field, in such a way that if the mass is zero, this mode does not exist (see [33,36]). On the other hand, we showed that the equations of the $f(R, T^\phi) = -R/4 + 2\lambda\sqrt{T^\phi}$ theory in the linearized regime implied a null mass for the scalar field, but the Ψ_2 mode is still present.

Although there are no initial constraints on $q_\mu q^\mu$, the result we have obtained is not valid for $q_\mu q^\mu = 0$ $(\omega = k)$ since the invariant Φ_{22} diverges. Furthermore, we should have $q_\mu q^\mu = \omega^2 - k^2 > 0$ in order to not violate the causality. Therefore, in the weak field regime of this theory, we have a massless scalar field with a speed smaller than the speed of light. As a consequence, since the GW modes associated with Ψ_2 and Φ_{22} have the same speed as that of the scalar field, they also have a speed $v_{\mathrm{GW}} < c$.

V. GWS IN THE $f(R,T^\phi) = f_1(R) + f_2(T^\phi)$ THEORY

In this section we follow closely the method used in Ref. [14]. Considering the case for which $f(R, T^\phi) = f_1(R) + f_2(T^\phi)$, the field equations (2) read

$$f_{1R}R_{\mu\nu} - \frac{1}{2}f_1 g_{\mu\nu} + (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)f_{1R} = -\frac{1}{2}[T^\phi_{\mu\nu} - g_{\mu\nu}f_2(T^\phi) - 2f_{2T}\nabla_\mu\phi\nabla_\nu\phi] \tag{33}$$

whose corresponding trace is given by

$$f_{1R}R - 2f_1 + 3\Box f_{1R} = -\frac{1}{2}(T^\phi - 4f_2 - 2f_{2T}\nabla_\alpha\phi\nabla^\alpha\phi). \tag{34}$$

By restricting $f(R, T)$ to the case $f_1(R) = -\frac{1}{4}(R - \alpha R^{-\beta})$ and $f_2(T^\phi) = 2\lambda T^\phi$, and ignoring terms of order two or higher in ϕ, we find a dynamical equation for the Ricci scalar, namely

$$\Box R^{-(1+\beta)} + \frac{\beta + 2}{3\beta}R^{-\beta} - \frac{1}{3\alpha\beta}R + \frac{8(8\lambda - 1)V_0}{3\alpha\beta} = 0, \tag{35}$$

which must be solved in order to verify if there is a propagating GW polarization mode associated with R. Considering $\beta \geq 1$, we have $R^{-\beta} \gg R$ in the weak field regime and the above equation now reads (similar calculations can be carried out by assuming other range of values for β)

$$\Box\psi + \frac{\beta+2}{3\beta}\psi^{\frac{\beta}{1+\beta}} + \frac{8(8\lambda-1)V_0}{3\alpha\beta} = 0, \tag{36}$$

where, for convenience, we have renamed $\psi = R^{-(1+\beta)}$.

Nevertheless, Eq. (36) has the following form,

$$\Box\psi + \frac{\partial U}{\partial\psi} = 0, \tag{37}$$

with the potential given by

$$U(\psi) = \left[\frac{(\beta+1)(\beta+2)}{3\beta(2\beta+1)}\right]\psi^{\frac{2\beta+1}{\beta+1}} + \frac{8(8\lambda-1)V_0}{3\alpha\beta}\psi. \tag{38}$$

Therefore, since it is Lorentz invariant, it can be solved by the following method used in Ref. [43]. Let us first consider the static solution of (37),

$$\frac{d^2\psi}{dz^2} = \frac{\partial U}{\partial\psi}, \tag{39}$$

which can be written as

$$\frac{1}{2}\left(\frac{d\psi}{dz}\right)^2 = U(\psi). \tag{40}$$

Substituting the potential (38) in the above equation and noticing that the last term of the potential is much smaller than the first, we find that

$$\psi^{\frac{1}{2(\beta+1)}} + \frac{4(8\lambda-1)(2\beta+1)V_0}{\alpha(\beta+1)(\beta+2)(2\beta-1)}\psi^{-\frac{2\beta-1}{2(\beta+1)}} = \left[\frac{(\beta+2)}{6\beta(\beta+1)(2\beta+1)}\right]^{\frac{1}{2}}(z+C), \tag{41}$$

with C being an integration constant.

The most simple solution of the above equation can be found for $\beta = 1$, namely

$$R(z) = \psi^{-\frac{1}{2}}(z) = \xi^2\left[(z+C) \pm \sqrt{(z+C)^2 - 4\sqrt{3}/\xi}\right]^2, \tag{42}$$

where

$$\xi = \frac{\alpha}{8\sqrt{3}(8\lambda-1)V_0}. \tag{43}$$

Now, since R is Lorentz invariant, we can obtain a time-dependent solution from the static solution (42) by considering a Lorentz transformation

$$R(t,z) = \xi^2\{\gamma(z-vt) + C \pm \sqrt{[\gamma(z-vt)+C]^2 - 4\sqrt{3}/\xi}\}^2, \tag{44}$$

where $\gamma = (1-v^2)^{-\frac{1}{2}}$ is the Lorentz factor.

Now, with this result in the field equations (33) we are able to find the following relevant components of the Ricci tensor,

$$R_{00} = F(t,z)G(t,z) - \frac{1}{2}R(t,z), \tag{45}$$

$$R_{33} = v^2F(t,z)G(t,z) + \frac{1}{2}R(t,z), \tag{46}$$

$$R_{03} = -vF(t,z)G(t,z), \tag{47}$$

where the functions $F(t,z)$ and $G(t,z)$ are given respectively by

$$F(t,z) = \frac{16\gamma^2}{[\gamma(z-vt)+C]^2 - 4\sqrt{3}/\xi} \tag{48}$$

and

$$G(t,z) = 1 \pm \frac{1}{4}\frac{[\gamma(z-vt)+C]}{\sqrt{[\gamma(z-vt)+C]^2 - 4\sqrt{3}/\xi}}. \tag{49}$$

The components R_{11} and R_{22} are also non-null but since they do not enter in the calculation of the NP parameters, we do not quote them here. Finally, from Eqs. (A5)–(A8) and with the help of Eqs. (A9)–(A13), we are able to find the NP parameters

$$\Psi_2 = \frac{1}{12}R(t,z), \tag{50}$$

$$\Psi_3 = 0, \tag{51}$$

$$\Phi_{22} = -\frac{1}{4}(1+v)^2F(t,z)G(t,z). \tag{52}$$

Additionally, since the theory does not exhibit further constraints in the spacetime geometry, we conclude that $\Psi_4 \neq 0$ although it is not possible to obtain its behavior from the curvature scalar or from the Ricci tensor (because Ψ_4 is the NP invariant associated with the Weyl tensor). Therefore, again we find that Eq. (9) holds.

Now, the presence or absence of the scalar polarization states does not depend on λ. This is because they can be generated simply by the particular choice of the function $f_1(R)$ we considered. On the other hand, by taking $\alpha = 0$ we have $R = F = 0$ and then $\Psi_2 = \Phi_{22} = 0$, which is in accordance with our previous assertion that the

$f(R,T^{\phi}) = -R/4 + 2\lambda T^{\phi}$ theory exhibits only the two usual tensor polarizations of GR.

It is worth stressing that the value $\beta = 1$ was chosen in Eq. (41) only for simplicity of the subsequent calculations, but it should be remembered that the theory $f(R) = -\frac{1}{4}(R - \alpha R^{-1})$ suffers the well-known Dolgov-Kawasaki instability [44]. Although the waveforms for Ψ_2 and Φ_{22} depend on the choice of β, we do not expect qualitative changes in the final result.

VI. CONCLUSIONS

With the recent detection of GWs by the Advanced LIGO team [1], a new window to observe the Universe has finally been opened. The high detection rate expected for some events, as the one detected (black hole–black hole merger), allied to some electromagnetic counterparts, may lead us to understand physics at extreme regimes of gravitational fields, densities etc.

The GW spectrum and its polarization modes are theory dependent. Previously motivated by shortcomings of the standard cosmological scenario, the alternative theories of gravity can also contribute to the study of GWs, being able to generate observables to be corroborated by experiment.

In this article, we have presented a study of GWs in the $f(R,T)$ and $f(R,T^{\phi})$ theories of gravity. The $f(R,T)$ theories consider the gravitational part of the action to be dependent not only on a generic function of R, but also on a function of T. The dependence on T comes from the consideration of exotic fluids or quantum effects. The $f(R,T)$ models depend on a source term, which represents the variation of the energy-momentum tensor with respect to the metric. On the other hand, in the $f(R,T^{\phi})$ theories, it is considered a function of R and of the trace of the energy-momentum tensor of a self-interacting scalar field ϕ, while the energy-momentum tensor of matter fields enters the field equations in the usual way. In both cases, the field equations of the $f(R)$ gravity are recovered if $T = 0$ or $T^{\phi} = 0$.

It is the first time that GWs are considered in $f(R,T)$ and $f(R,T^{\phi})$ theories. The first steps of this investigation have shown us that, in terms of the polarization modes, it is not possible to distinguish $f(R,T)$ gravity from $f(R)$ gravity. This is because in order to find the number of polarization modes of GWs in a given theory one has to examine the theory in a region far from the source of GWs where $T = 0$. In this regime, $f(R,T)$ gravity retrieves $f(R)$ theory. However, it is expected that it would be possible to distinguish the two theories by analyzing the waveforms produced by a given source, a binary system for instance, since the energy-momentum tensor of the source enters in a different manner in the $f(R,T)$ gravity.

In Ref. [23], through the introduction of a scalar field, the $f(R,T^{\phi})$ theory was considered. In that paper, there is a contribution coming from the T^{ϕ} term, and, consequently, the theory is distinguishable from the $f(R)$ gravity even for $T = 0$ regimes, namely radiation era and vacuum. Starting from such a formulation, we have shown in the present article that indeed it is possible to obtain $f(R,T^{\phi})$ gravity information in vacuum regime without necessarily recovering $f(R)$ gravity. By using the field equations of the theory, we have obtained the NP quantities and we have found out extra polarization states of GWs.

As expected, the properties of GWs depend upon the functional form of $f(R,T^{\phi})$. In Sec. IV we took $f(R,T^{\phi}) = -R/4 + f(T^{\phi})$ and analyzed different forms for $f(T^{\phi})$ along with different assumptions for the scalar field potential. For $f(T^{\phi}) = 2\lambda T^{\phi}$ we showed that the effects of the inclusion of the scalar field up to first order terms are equivalent to considering that the usual CC $\Lambda \to 2(1 - 8\lambda)V_0$. However, it has already been shown that a CC does not introduce any additional polarization states for GWs (see Ref. [40]).

It is well known that in a $\Lambda = 0$ case, in order to study isolated systems in the weak field regime, one investigates the linearized gravitational fields in Minkowski space-time. Such GW equations are recovered in $f(R,T^{\phi}) = -R/4 + 2\lambda T^{\phi}$ gravity when $\lambda = 1/8$ or $V_0 = 0$. On the other hand, for the $\Lambda > 0$ case, it seems natural to replace the Minkowski metric with the de Sitter one, as quoted in Sec. IV A.

A particular form for $f(T^{\phi})$, namely $f(T^{\phi}) = 2\lambda\sqrt{T^{\phi}}$, exhibits a quite different scenario. In this theory we have shown that GWs can have two scalar polarization modes (longitudinal and transversal) beyond the usual Einstein polarizations. Nevertheless, it is worth remembering that since $\Psi_2 \neq 0$, the $E(2)$ classification of the theory is II_6; i.e., Ψ_2 is the only observer-independent mode. The presence or absence of all other modes depends on the observer. Additionally, we found out that these scalar polarization modes have a speed $v_{\mathrm{GW}} < c$, as in the massive gravity case [37].

Similar results were obtained for the theory $f(R,T^{\phi}) = -\frac{1}{4}(R - \alpha R^{-\beta}) + 2\lambda T^{\phi}$, but now the presence or absence of the extra scalar polarization modes does not depend on the presence of the term $2\lambda T^{\phi}$ since the $f(R)$ gravity also presents these modes. However, the waveforms of the NP parameters Ψ_2 and Φ_{22} we have obtained are quite different from those of the $f(R)$ gravity (as one can compare with the expressions obtained in Ref. [14]), which could be a way to distinguish between the two theories.

The recent detection of GWs by the LIGO team is consistent with a binary black hole system in general relativity [1]. However, because of the similar orientations of the Hanford and Livingston LIGO instruments, the data cannot exclude the presence of non-Einsteinian polarization modes. To determine the polarization content of a signal requires a network of detectors with different orientations, such as Virgo [45]. Also, with only two detectors and in the absence of an electromagnetic or neutrino counterpart, there is a large uncertainty in the sky location of the source. As a consequence, there is an uncertainty in the speed of the GWs estimated from the difference of the time of arrival of the signal in each detector; thus $v_{\mathrm{GW}} < c$ cannot be ruled out. Therefore, the $f(R,T)$ formalism discussed here, as well as several other alternative theories of gravitation, are not excluded from the point of view of the polarization modes or the speed of GWs. We hope that with the future detection of GWs stronger bounds could be established for such parameters.

ACKNOWLEDGMENTS

The authors would like to thank the Brazilian agency São Paulo Research Foundation (FAPESP) for financial support (Grant No. 13/26258-4). P. H. R. S. M. thanks FAPESP, Grant No. 2015/08476-0, for financial support and O. D. Miranda for suggesting the study of extra polarization states of gravitational waves in $f(R,T)$ gravity. J. C. N. A. thanks the Brazilian agency CNPq (308983/2013-0) for partial support.

APPENDIX: AN OVERVIEW OF THE NEWMAN-PENROSE FORMALISM

A powerful tool to study the properties of GWs was developed in [33]. The basic idea is to analyze all the physically relevant components of the Riemann tensor which cause relative acceleration among test particles. In [33], the authors used a null tetrad basis, which is specially suitable to treat approximately null waves, to calculate the NP quantities [34,35], which are directly related to the GW polarization states in a given theory. Those quantities are given in terms of the irreducible parts of the Riemann tensor, i.e., the Weyl tensor, the traceless Ricci tensor and the Ricci scalar.

The analysis in [33] showed that there are up to six possible modes of polarization for GWs, depending on the theory, which can be corroborated by experiments. Therefore it is possible to categorize a given theory from its non-null NP quantities.

At a given point, the complex tetrad $(\mathbf{k},\mathbf{l},\mathbf{m},\bar{\mathbf{m}})$ is related to the usual Cartesian tetrad $(\mathbf{e}_t,\mathbf{e}_x,\mathbf{e}_y,\mathbf{e}_z)$ as

$$\mathbf{k} = \frac{1}{\sqrt{2}}(\mathbf{e}_t + \mathbf{e}_z), \tag{A1}$$

$$\mathbf{l} = \frac{1}{\sqrt{2}}(\mathbf{e}_t - \mathbf{e}_z), \tag{A2}$$

$$\mathbf{m} = \frac{1}{\sqrt{2}}(\mathbf{e}_x + i\mathbf{e}_y), \tag{A3}$$

$$\bar{\mathbf{m}} = \frac{1}{\sqrt{2}}(\mathbf{e}_x - i\mathbf{e}_y). \tag{A4}$$

In general, the NP quantities are independent. In the study of approximately plane waves, there are some differential and symmetrical properties of the Riemann tensor which reduce the number of non-null independent components from 20 (ten Ψ's, nine Φ's and Λ) to six. Therefore we can choose the set $\{\Psi_2,\Psi_3,\Psi_4,\Phi_{22}\}$ to describe, in a given coordinate system, the six independent components of a wave in a given theory. Consequently, in the tetrad basis and in the case of plane waves, the NP quantities are given by

$$\Psi_2 = -\frac{1}{6}R_{lklk}, \tag{A5}$$

$$\Psi_3 = -\frac{1}{2}R_{lkl\bar{m}}, \tag{A6}$$

$$\Psi_4 = -R_{l\bar{m}l\bar{m}}, \tag{A7}$$

$$\Phi_{22} = -R_{lml\bar{m}}, \tag{A8}$$

with $R_{\alpha\beta\mu\nu}$ being the Riemann tensor. Note that Ψ_3 and Ψ_4 are complex quantities, so that each of them represents two independent polarization states, one represented by the real part and the other by the imaginary part of Ψ_3 and Ψ_4.

Other useful expressions for the NP formalism are the following,

$$R_{lk} = R_{lklk}, \tag{A9}$$

$$R_{ll} = 2R_{lml\bar{m}}, \tag{A10}$$

$$R_{lm} = R_{lklm}, \tag{A11}$$

$$R_{l\bar{m}} = R_{lkl\bar{m}}, \tag{A12}$$

$$R = -2R_{lk} = -2R_{lklk}, \tag{A13}$$

with $R_{\mu\nu}$ being the Ricci tensor.

[1] B. P. Abbott *et al.*, Phys. Rev. Lett. **116**, 061102 (2016).
[2] P. H. R. S. Moraes and O. D. Miranda, Mon. Not. R. Astron. Soc. Lett. **445**, L11 (2014).
[3] P. H. R. S. Moraes and O. D. Miranda, Astrophys. Space Sci. **354**, 2121 (2014).
[4] C. P. L. Berry *et al.*, Astrophys. J. **804**, 114 (2015).
[5] J. Veitch *et al.*, Phys. Rev. D **91**, 042003 (2015).
[6] C. Cutler and É. E. Flanagan, Phys. Rev. D **49**, 2658 (1994).
[7] E. Poisson and C. M. Will, Phys. Rev. D **52**, 848 (1995).
[8] K. Takami, L. Rezzolla, and L. Baiotti, Phys. Rev. Lett. **113**, 091104 (2014).
[9] B. D. Lackey and L. Wade, Phys. Rev. D **91**, 043002 (2015).
[10] W. Del Pozzo, T. G. F. Li, M. Agathos, C. Van Den Broeck, and S. Vitale, Phys. Rev. Lett. **111**, 071101 (2013).
[11] A. Bauswein, H.-T. Janka, K. Hebeler, and A. Schwenk, Phys. Rev. D **86**, 063001 (2012).
[12] M. Agathos, J. Meidam, W. Del Pozzo, T. G. F. Li, M. Tompitak, J. Veitch, S. Vitale, and C. Van Den Broeck, Phys. Rev. D **92**, 023012 (2015).
[13] J. S. Read, C. Markakis, M. Shibata, K. Uryū, J. D. E. Creighton, and J. L. Friedman, Phys. Rev. D **79**, 124033 (2009).
[14] M. E. S. Alves, O. D. Miranda, and J. C. N. de Araujo, Phys. Lett. B **679**, 401 (2009).
[15] W. Del Pozzo, J. Veitch, and A. Vecchio, Phys. Rev. D **83**, 082002 (2011).
[16] R. Konoplya and A. Zhidenko, Phys. Lett. B **756**, 350 (2016).
[17] K. Yagi and T. Tanaka, Phys. Rev. D **81**, 064008 (2010).
[18] T. Clifton, P. G. Ferreira, A. Padilla, and C. Skordis, Phys. Rep. **513**, 1 (2012).
[19] T. Padmanabhan *et al.*, Phys. Rep. **380**, 235 (2003).
[20] A. Kehagias, Phys. Lett. B **600**, 133 (2004).
[21] N. Arkani-Hamed, S. Dimopoulos, N. Kaloper, and R. Sundrum, Phys. Lett. B **480**, 193 (2000).
[22] T. Harko, F. S. N. Lobo, S. Nojiri, and S. D. Odintsov, Phys. Rev. D **84**, 024020 (2011).
[23] P. H. R. S. Moraes and J. R. L. Santos, Eur. Phys. J. C **76**, 60 (2016).
[24] P. H. R. S. Moraes and R. A. C. Correa, Astrophys. Space Sci. **361**, 91 (2016).
[25] P. H. R. S. Moraes, Int. J. Theor. Phys. **55**, 1307 (2016).
[26] P. H. R. S. Moraes, Eur. Phys. J. C **75**, 168 (2015).
[27] P. H. R. S. Moraes, Astrophys. Space Sci. **352**, 273 (2014).
[28] M. F. Shamir, Eur. Phys. J. C **75**, 354 (2015).
[29] E. H. Baffou, A. V. Kpadonou, M. E. Rodrigues, M. J. S. Houndjo, and J. Tossa, Astrophys. Space Sci. **356**, 173 (2015).
[30] C. P. Singh and P. Kumar, Eur. Phys. J. C **74**, 3070 (2014).
[31] H. Shabani and M. Farhoudi, Phys. Rev. D **90**, 044031 (2014).
[32] T. Harko, Phys. Rev. D **90**, 044067 (2014).
[33] D. M. Eardley, D. L. Lee, and A. P. Lightman, Phys. Rev. D **8**, 3308 (1973).
[34] E. Newman and R. Penrose, J. Math. Phys. (N.Y.) **3**, 566 (1962).
[35] E. Newman and R. Penrose, J. Math. Phys. (N.Y.) **4**, 998 (1963).
[36] M. E. S. Alves, O. D. Miranda, and J. C. N. de Araujo, Classical Quantum Gravity **27**, 145010 (2010).
[37] W. L. S. de Paula, O. D. Miranda, and R. M. Marinho, Classical Quantum Gravity **21**, 4595 (2004).
[38] S. Nojiri, S. D. Odintsov, and D. Sáez-Gómez, Phys. Lett. B **681**, 74 (2009).
[39] S. Nojiri and S. D. Odintsov, Phys. Lett. B **657**, 238 (2007).
[40] J. Näf, P. Jetzer, and M. Sereno, Phys. Rev. D **79**, 024014 (2009).
[41] A. Ashtekar, B. Bonga, and A. Kesavan, Phys. Rev. D **92**, 044011 (2015).
[42] H. Shabani and M. Farhoudi, Phys. Rev. D **88**, 044048 (2013).
[43] R. Rajaraman, *Solitons and Instantons: An Introduction to Solitons and Instantons in Quantum Field Theory* (Elsevier Science Publishers, Amsterdam, The Netherlands, 1982).
[44] A. D. Dolgov and M. Kawasaki, Phys. Lett. B **573**, 1 (2003).
[45] F. Acernese *et al.* (Virgo Collaboration), Classical Quantum Gravity **32**, 024001 (2015).