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Hybrid metric-Palatini brane system

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It is known that the metric and Palatini formalisms of gravity theories have their own interesting features but also suffer from some different drawbacks. Recently, a novel gravity theory called hybrid metric-Palatini gravity was put forward to cure or improve their individual deficiencies. The action of this gravity theory is a hybrid combination of the usual Einstein-Hilbert action and a $f(\mathcal{R})$ term constructed by the Palatini formalism. Interestingly, it seems that the existence of a light and long-range scalar field in this gravity may modify the cosmological and galactic dynamics without conflicting with the laboratory and Solar System tests. In this paper, we focus on the tensor and scalar perturbations of the thick branes in this novel gravity theory. We consider two models as examples, namely, the thick branes constructed by a background scalar field and by pure gravity. The thick branes in both models have no inner structure. However, affected by the hybrid combination of the metric and Palatini formalisms, the graviton zero mode in the first model has inner structure when the parameter in this model is larger than its critical value, which is different from the cases of general relativity and Palatini $f(\mathcal{R})$ gravity. We find that the effective four-dimensional gravity can be reproduced on the brane for both models and the scalar zero mode in the model without a background scalar field cannot be localized on the brane, which avoids a fifth force. Moreover, the stability of both brane systems against the linear perturbations can also be ensured.

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I. INTRODUCTION

General relativity is a successful gravitational theory at the scale of the Solar System. However, it does not work well at larger scales. Thus, many modified theories of gravity have been put forward to describe cosmological behaviors such as cosmic acceleration and galactic dynamics [1–5]. In general, there are three kinds of formalisms for modified gravity theories, namely, the metric formalism, Palatini formalism (matters do not couple with the priori metric-independent connection), and metric-affine formalism (matters couple with the metric and a priori metric-independent connection) [1]. They all have their own interesting properties and, at the same time, suffer from different drawbacks. Recently, the so-called C theories were proposed in Refs. [6–8] and establish a bridge between the first and second formalisms in order to find ways to cure or improve their individual deficiencies. In C theories, the Levi-Civitá connection $\hat{\Gamma}$ of the metric $\hat{g}_{\mu\nu}$ is conformally related to the spacetime metric $g_{\mu\nu}$, namely, $\hat{g}_{\mu\nu} = \mathcal{C}(\mathcal{R})g_{\mu\nu}$, where $\mathcal{C}$ is an arbitrary function of the Ricci curvature scalar $\mathcal{R} = \mathcal{R}[g,\hat{\Gamma}] = g^{\mu\nu}\mathcal{R}_{\mu\nu}[\hat{\Gamma}]$ only.

Alternatively, another novel modified gravity was presented in Ref. [9], the action of which is a hybrid combination of the usual Einstein-Hilbert action and a $f(\mathcal{R})$ term constructed by the Palatini formalism. It has a dynamically equivalent scalar-tensor representation like the pure metric and pure Palatini cases [9–12]. It also shares the properties of both the metric and Palatini formalisms like C theories. The new feature of such hybrid gravity theory is that it predicts the existence of a light long-range scalar field, which can be used to explain the late-time cosmic acceleration [9].

Considering the characteristics of light and long range, there is a possibility that this scalar field may modify the cosmological and galactic dynamics [9–12] without conflicting with the laboratory and Solar System tests. In Ref. [11], the authors analyzed the criteria for obtaining cosmic acceleration and obtained several cosmological solutions, which describe both accelerating and decelerating universes, depending on the form of the effective scalar potential. The virial theorem was studied in the context of the galaxy cluster, where the mass dispersion relation was modified by a term related to the new scalar field predicted by hybrid metric-Palatini gravity [12]. The stability of the Einstein static Universe was also analyzed in Ref. [13], and a large class of stable solutions was found. In Ref. [14], the authors considered the possibility that wormhole solutions may be supported by hybrid metric-Palatini gravity according to the null energy conditions at the throat and found some specific examples. In Ref. [15], the authors showed that the initial value problem can be well formulated and well posed. Moreover, the dynamics of linear perturbation

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and thermodynamics in hybrid metric-Palatini gravity were also investigated in Refs. [16] and [17], respectively. For a detailed introduction, see the recent review [18].

On the other hand, it has been extensively considered in the past two decades that our four-dimensional world might be just a brane embedded in a higher-dimensional spacetime. This idea provides new insights into solving some long-existing problems, such as the gauge hierarchy and cosmological constant problems [19–24]. Dating back to the original Kaluza-Klein (KK) theory, the extra dimension is compacted into a circle with the Planck scale radius. This makes detecting the extra dimension hopeless, while in brane scenarios, the sizes of the extra dimensions can be the order of submillimeter [21] or infinite [23].

In the Randall-Sundrum-II (RS-II) brane scenario [23], the thickness of the brane is neglected. In more realistic thick brane models, the original singular brane is replaced by a smooth domain wall generated by matter fields. The thick brane models have been extensively studied in the context of higher-dimensional gravity theories [25–40]. The linearization of a brane system is one of the most important issues in the brane models [41–53]. First, it is a key procedure to investigate the stability of the brane solution against the linear perturbations. Second, to reproduce the effective four-dimensional gravity, we need also to study the linear perturbation of the brane system. The localized graviton zero mode produces the Newtonian gravity, and the nonlocalized scalar zero mode avoids a large correction to it. Third, the linear perturbation will result in the interaction of matter fields with the KK gravitons, which can be tested by experiments.

In the previous investigations about a brane system, the metric [49–52] and Palatini formalisms [53,54] were individually considered. Therefore, it is interesting to study the properties of a brane system in a gravity theory containing both formalisms. For example, how does the hybrid of the two formalisms affect the properties of the brane solutions, graviton zero mode, scalar zero modes, and stability of the linear perturbations? This motivates us to investigate the hybrid metric-Palatini brane system. In this paper, inspired by the scalar-tensor representation of hybrid metric-Palatini gravity, we will consider two models: the thick branes constructed by a background scalar field (model A) and by pure gravitational system (model B) in hybrid metric-Palatini gravity. In Refs. [55–62], some brane models have been constructed for pure gravitational systems without matter fields. This scenario is the same as producing an expanding universe from $f(R)$ gravity without introducing an extra scalar field (inflation without the inflaton). To study the issues of the stability of the linear perturbations and the localization of the graviton and scalar zero modes, we will investigate the linearization of these two brane models.

In this work, we focus on the brane model in hybrid metric-Palatini gravity. Therefore, in Sec. II, we briefly introduce the hybrid metric-Palatini model and find the thick brane solutions for model A and model B. The stability of the brane system and localization of the graviton and scalar zero modes are analyzed in Sec. III. Section IV is the conclusion.

II. HYBRID METRIC-PALATINI BRANE MODELS AND SOLUTIONS

Now, let us start with the action of the five-dimensional brane model in hybrid metric-Palatini gravity [9],

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{-g}[R + f(\mathcal{R})] + S_m(g,\chi), \qquad (1)$$

where $\kappa^2 = 8\pi G_5$, with G_5 the five-dimensional Newtonian gravitational constant, and we have set $c = 1$. S_m is the standard matter action, $R = g^{MN}R_{MN}$ is the Einstein-Hilbert Ricci scalar constructed by the metric, and $\mathcal{R} = g^{MN}\mathcal{R}_{MN}$ is the Palatini curvature, where $\mathcal{R}_{MN}$ is defined in terms of a torsionless independent connection, $\hat{\Gamma}$, as

$$\mathcal{R}_{MN} \equiv (\hat{\Gamma}^P_{MN,P} - \hat{\Gamma}^P_{MP,N} + \hat{\Gamma}^P_{PQ}\hat{\Gamma}^Q_{MN} - \hat{\Gamma}^P_{MQ}\hat{\Gamma}^Q_{PN}). \qquad (2)$$

Introducing an auxiliary scalar field ϕ, the action (1) can be deformed as

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{-g}[R + \phi\mathcal{R} - V_1(\phi)] + S_m, \qquad (3)$$

where

$$\phi \equiv F(\mathcal{R}) = \frac{df(\mathcal{R})}{d\mathcal{R}}, \qquad V_1(\phi) \equiv \mathcal{R}F(\mathcal{R}) - f(\mathcal{R}). \qquad (4)$$

The field equations can be obtained by varying the action (3) with respect to the metric g_{MN}, the scalar field ϕ, and the independent connection $\hat{\Gamma}^P_{MN}$,

$$R_{MN} + \phi\mathcal{R}_{MN} - \frac{1}{2}(R + \phi\mathcal{R} - V_1)g_{MN} = \kappa^2 T_{MN}, \qquad (5a)$$

$$\mathcal{R} - V_{1\phi} = 0, \qquad (5b)$$

$$\hat{\nabla}_P(\sqrt{-g}\phi g^{MN}) = 0, \qquad (5c)$$

where $V_{1\phi} \equiv \frac{dV_1(\phi)}{d\phi}$, the matter stress-energy tensor is defined as usual $T_{MN} \equiv -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_m)}{\delta g^{MN}}$, and $\hat{\nabla}_P$ is compatible with the connection $\hat{\Gamma}^P_{MN}$.

The solution of Eq. (5c) implies that the independent connection is the Levi-Civitáa connection of a metric $q_{MN} = \phi^{2/3}g_{MN}$. Then, the relation between $\mathcal{R}_{MN}$ and R_{MN} is

$$\mathcal{R}_{MN} = R_{MN} + \frac{4}{3\phi^2}\partial_M\phi\partial_N\phi - \frac{1}{\phi}\left(\nabla_M\nabla_N\phi + \frac{1}{3}g_{MN}\Box\phi\right), \tag{6}$$

where $\Box \equiv g^{KL}\nabla_K\nabla_L$. Using the relation (6), one can obtain a scalar-tensor representation [9–11]:

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{-g}\left[(1+\phi)R + \frac{4}{3\phi}\partial^K\phi\partial_K\phi - V_1(\phi)\right] + S_m. \tag{7}$$

Now, it is clear that the free choice of the form of the $f(\mathcal{R})$ is transformed to the potential $V_1(\phi)$ of a scalar profile ϕ. Inspired by the scalar-tensor representation, we consider two models: model A for the brane constructed by a matter scalar field χ and model B for the brane constructed by the pure gravity without any matter field.

A. Model A: With matter

The action of the matter part with a scalar field is

$$S_m = \int d^5x\sqrt{-g}\left[-\frac{1}{2}g^{MN}\partial_M\chi\partial_N\chi - V_2(\chi)\right]. \tag{8}$$

Substituting Eqs. (6) and (5b) in Eq. (5a), the gravitational field equation can be written as

$$(1+\phi)R_{MN} + \frac{4}{3\phi}\partial_M\phi\partial_N\phi - (\nabla_M\nabla_N\phi - g_{MN}\Box\phi) - \frac{1}{2}g_{MN}\left[(1+\phi)R + \frac{4}{3\phi}\partial^K\phi\partial_K\phi - V_1(\phi)\right] = \kappa^2 T_{MN}. \tag{9}$$

Considering Eq. (5b) and the trace of Eq. (6), one finds that the scalar field ϕ is governed by the second-order evolution equation

$$8\phi\Box\phi - 4\partial_K\phi\partial^K\phi - \phi^2[5V_1(\phi) - 3(1+\phi)V_{1\phi}] + 2\phi^2\kappa^2 T = 0. \tag{10}$$

Our discussion will be limited to the static flat brane scenario, for which the metric is given by

$$ds^2 = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2, \tag{11}$$

with y the extra dimension. Meanwhile, the scalar field, $\phi = \phi(y)$, is independent of the brane coordinates. For the system (9)–(11), the Einstein field equations and equation of motion of the scalar field ϕ are read as

$$3(A'' + 4A'^2)(1+\phi) + 7A'\phi' + \mathrm{V}_1 + 2\kappa^2\mathrm{V}_2 + \phi'' = 0, \tag{12a}$$

$$12(A'' + A'^2)(1+\phi) + 4A'\phi' + \mathrm{V}_1 + 4\phi'' + 2\kappa^2\mathrm{V}_2 + 3\kappa^2\chi'^2 - 4\phi'^2/\phi = 0, \tag{12b}$$

$$32A'\phi'^2\phi - \phi^2(5\phi'\mathrm{V}_1 - 3(1+\phi)\mathrm{V}_1') - 4\phi'^3 - \kappa^2\phi'\phi^2(10\mathrm{V}_2 + 3\chi'^2) + 8\phi''\phi'\phi = 0, \tag{12c}$$

where a prime stands for the derivative with respect to the extradimensional coordinate y.

The equation of motion of the matter field is described by the following equation:

$$4A'\chi' + \chi'' = \frac{dV_2(\chi)}{d\chi}. \tag{13}$$

Equations (12) and (13) describe the whole system. There are five variables, namely, $A(y), \phi(y), \chi(y), V_1(\phi)$, and $V_2(\chi)$. However, there are only three independent equations. So, one needs assume two conditions to solve this system.

In this model, we consider the following configuration for the scalar field $\phi(y)$ and warp factor $A(y)$:

$$\phi(y) = a\tanh^2(ky), \tag{14a}$$

$$A(y) = b\ln[\mathrm{sech}(ky)]. \tag{14b}$$

To avoid the ghost problem, we should ensure the positive definiteness of the coefficient of R in the action (7), so we should take $a > 0$. Now, it can be checked that the system supports the solutions

$$\chi(y) = \tanh(ky)\sqrt{\frac{1}{6}(3(5a+3)\cosh(2ky) + 5a + 9)} + i\sqrt{\frac{10a+9}{3}}\left[E\left(iky, \frac{15a+9}{10a+9}\right) - F\left(iky, \frac{15a+9}{10a+9}\right)\right], \tag{15a}$$

$$V_1(y) = -\frac{1}{2}k^2\mathrm{sech}^4(ky)[(12-8a)\cosh(2ky) + 3(a+1)\cosh(4ky) + 49a + 9], \tag{15b}$$

$$V_2(y) = \frac{5}{2}k^2\mathrm{sech}^2(ky)[-a\mathrm{sech}^2(ky) + 5a + 3], \tag{15c}$$

where we have chosen $b = 1$ and $\kappa = 1$ for simplicity and the functions E and F are two kinds of elliptic integrals. From Fig. 1, it is obvious that the matter field $\chi(y)$ has the shape of a topological soliton and the scalar field ϕ has the shape of a nontopological soliton. The similar solutions

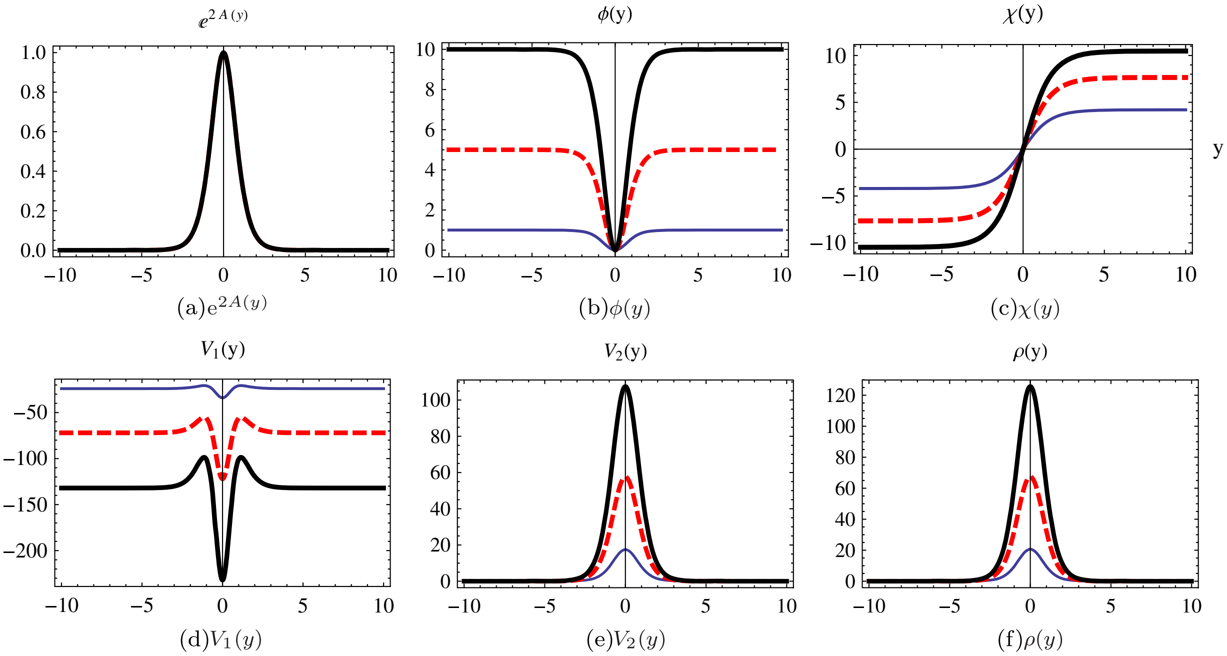


FIG. 1. Plot the shapes of the wrap factor e^{2A}, scalar field $\phi(y)$, matter field $\chi(y)$, scalar potential $V_1(\phi(y))$, scalar potential $V_2(\chi(y))$, and energy density $\rho(y)$ for the model A. The parameters are set to $b=1$, $k=1$, and $a=1$ for thin lines; $a=5$ for red dashed thick lines; and $a=10$ for black thick lines.

have been obtained in Refs. [63–65], which are named “trapping bag” solutions. The shapes of this brane solution and the energy density are shown in Fig. 1. Obviously, the energy density peaks at the origin of the extra dimension, which represents a single brane. It is not difficult to analyze the structure of the five-dimensional spacetime at $y=\pm\infty$, where the curvature $R=-20k^2<0$. This means that the spacetime is asymptotic anti-de Sitter (AdS).

We can also obtain the expression of $f(\mathcal{R})$ from Eq. (4):

$$f(\mathcal{R})=\frac{2a}{3}\mathcal{R}-\frac{a}{120k^2}\mathcal{R}^2+\frac{26ak^2}{3}+12k^2. \quad (16)$$

Then, the complete Lagrangian for gravity can be expressed as

$$\mathcal{L}=R+\frac{2a}{3}\mathcal{R}-\frac{a}{120k^2}\mathcal{R}^2+\frac{26ak^2}{3}+12k^2. \quad (17)$$

B. Model B: Without matter

We can also construct a brane from the scalar profile $\phi(y)$ without introducing the matter field $\chi(y)$. Then, we can obtain the field equations of the whole system just by omitting the terms about the matter field $\chi(y)$ from Eq. (12):

$$3(1+\phi)(A''+4A'^2)+7A'\phi'+\mathrm{V}_1(\phi)+\phi''=0, \quad (18a)$$

$$12\phi(1+\phi)(A''+A'^2)+4A'\phi'\phi+\phi\mathrm{V}_1(\phi)+4\phi\phi''-4\phi'^2=0, \quad (18b)$$

$$8\phi''+32A'\phi'-5\phi\mathrm{V}_1(\phi)+3\phi(1+\phi)\mathrm{V}_{1\phi}=0. \quad (18c)$$

Now, there are only three variables, namely, $A(y),\phi(y)$, and $V_1(\phi)$, but only two independent equations. So, we just need one condition to solve this system.

Subtracting (18a) from (18b) yields

$$9\phi(1+\phi)A''+3\phi\phi''-3\phi A'\phi'-4\phi'^2=0. \quad (19)$$

It is easy to check that this equation yields a thin brane solution, i.e., $\phi(y)=\phi_1$ and $A(y)=-\alpha|y|$, where both ϕ_1 and α are constants. In this paper, we mainly focus on thick brane solution, so we suppose

$$A(y)=b\ln[\mathrm{sech}(ky)], \quad (20)$$

where b is a positive parameter in order to localize the graviton zero mode on the brane [see Sec. III]. To keep the Z_2 symmetry of the extra dimension, we only look for an even function solution for the scalar $\phi(y)$. Thus, the initial condition for $\phi(y)$ can be assumed as

$$\phi(0)=\phi_0, \qquad \phi'(0)=0, \quad (21)$$

and from this and Eq. (19), we can get

$$\phi''(0)=3(1+\phi_0)k^2b. \quad (22)$$

To ensure the positive definiteness of the coefficient of R in the action (7), we require $1+\phi(y)>0$, from which one has $1+\phi_0>0$ and so $\phi''(0)>0$.

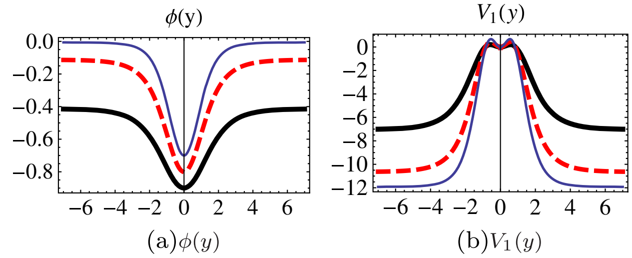


FIG. 2. Figures 2(a) and 2(b) plot the scalar profile $\phi(y)$ and scalar potential $V_1(\phi(y))$ for model B, respectively. The parameters are set to $b = 1$, $k = 1$. The black thick, red dashed thick, and blue thin lines correspond to $\phi_0 = -0.9, -0.8$, and -0.7, respectively.

Considering the asymptotic behavior of the warp factor $A(y \to \pm\infty) \to -bk|y|$, one can obtain the asymptotic behavior of the scalar profile from Eq. (19),

$$\phi(y \to \pm\infty) \to \frac{c_1}{(\mathrm{e}^{c_2} + \mathrm{e}^{-bk|y|})^3} \to c_1, \tag{23}$$

where c_1 and c_2 are integral constants related to $\phi(0)$ and $\phi'(0)$. The numerical solutions of the scalar field and scalar potential $V_1(y)$ are plotted in Fig. 2, from which one can see that c_1 increases with $\phi(0)$.

Next, since the scalar profile ϕ is an even function, it can be expanded around $y = 0$ as

$$\phi(y) = c_0 + c_2 y^2 + c_4 y^4 + \mathcal{O}(y^6), \tag{24}$$

where c_0, c_2, and c_4 are some parameters to be solved. Considering the behavior of the warp factor $A(y \to 0) \to -\frac{by^2}{2} + \frac{by^4}{12} + \mathcal{O}(y^6)$, they can be solved from Eq. (19) as

$$c_0 = \frac{4(8 - 31b)}{45(3b - 2)} + \frac{4\sqrt{2b(8b + 67) + 64}}{45(3b - 2)}, \tag{25}$$

$$c_2 = \frac{(11b - 58)b}{90b - 60} + \frac{2\sqrt{2b(8b + 67) + 64}b}{45b - 30}, \tag{26}$$

$$c_4 = -\frac{157b}{630(3b - 2)} - \frac{(117b + 1108)b^2}{2520(3b - 2)} + \left(\frac{31}{630} - \frac{b}{140}\right)\frac{\sqrt{2b(8b + 67) + 64}b}{3b - 2}. \tag{27}$$

Note that the even parity of the scalar $\phi(y)$ here would have a different influence on the localization of fermions [66] from the case of odd scalar kink solutions in other brane models [26,67].

III. LINEAR PERTURBATIONS AND STABILITY OF THE SOLUTIONS

Since we already have two brane models (model A and model B), we will consider the stability of these two models under the linear perturbations of the spacetime metric and the scalar fields as well as the localization of the graviton and scalar zero modes, which are important issues in the brane model. Generally speaking, the four-dimensional massless graviton should be localized on the brane in order to reproduce the familiar four-dimensional Newtonian potential, and the scalar zero mode should not be localized on the brane to avoid a fifth force. We will analyze these issues in the following context.

A. Tensor perturbation

Since the scalar, vector, and tensor fluctuations are decoupled from each other, we can write the spacetime metric under the tensor fluctuation as

$$ds^2 = e^{2A(y)}(\eta_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu + dy^2, \tag{28}$$

where $h_{\mu\nu}$ represents the tensor fluctuation and it is transverse traceless, i.e., $\eta^{\mu\beta}\partial_\beta h_{\mu\nu} = 0$ and $h \equiv \eta^{\mu\nu}h_{\mu\nu} = 0$. The field equation of the tensor perturbation reads

$$h''_{\mu\nu} + \left(4A' + \frac{\phi'}{1 + \phi}\right)h'_{\mu\nu} + \mathrm{e}^{-2A}\Box^{(4)}h_{\mu\nu} = 0, \tag{29}$$

where $\Box^{(4)} = \eta^{\mu\nu}\partial_\mu\partial_\nu$ stands for the four-dimensional d'Alembertian operator. By making a coordinate transformation $dy = \mathrm{e}^A dz$, Eq. (29) can be rewritten as

$$\partial_z^2 h_{\mu\nu} + \left(3\partial_z A + \frac{\partial_z\phi}{1 + \phi}\right)\partial_z h_{\mu\nu} + \Box^{(4)}h_{\mu\nu} = 0. \tag{30}$$

After making the KK decomposition $h_{\mu\nu} = \varepsilon_{\mu\nu}(x)f(z)H(z)$, we can get the two equations

$$\Box^{(4)}\varepsilon_{\mu\nu}(x) = m^2\varepsilon_{\mu\nu}(x), \tag{31a}$$

$$-\partial_z^2 H(z) - (3\partial_z A + \partial_z \ln(f^2(1 + \phi)))\partial_z H(z) - \left(\frac{\partial_z^2 f}{f} + 3\frac{\partial_z A\partial_z f}{f} + \frac{\partial_z\phi}{1 + \phi}\frac{\partial_z f}{f}\right)H(z) = m^2 H(z), \tag{31b}$$

where Eq. (31a) is the Klein-Gordon equation for the four-dimensional massless ($m = 0$) or massive ($m \neq 0$) graviton. To get a Schrödinger-like equation of the KK mode $H(z)$, its first-order derivation should be vanishing. Thus, the function $f(z)$ can be solved from $3\partial_z A + \partial_z \ln(f^2(1 + \phi)) = 0$ as

$$f(z) = \frac{e^{-3A/2}}{\sqrt{1+\phi}}. \quad (32)$$

Then, Eq. (31b) can be rewritten as

$$(-\partial_z^2 + U(z))H(z) = m^2 H(z), \quad (33)$$

where the effective potential for the KK mode is given by

$$\begin{aligned} U(z) &= 2\frac{(\partial_z f)^2}{f^2} - \frac{\partial_z^2 f}{f} \\ &= \frac{3}{2}\partial_z^2 A + \frac{9}{4}(\partial_z A)^2 - \frac{(\partial_z \phi)^2}{4(1+\phi)^2} \\ &\quad + \frac{3\partial_z A \partial_z \phi + \partial_z^2 \phi}{2(1+\phi)}, \end{aligned} \quad (34)$$

which can be rewritten in the y coordinate as

$$U(z(y)) = e^{2A}\left(\frac{3}{2}A'' + \frac{15}{4}A'^2 - \frac{\phi'^2}{4(\phi+1)^2} + \frac{4A'\phi' + \phi''}{2(\phi+1)}\right). \quad (35)$$

Equation (33) is the equation of motion for the KK mode $H(z)$, and it can be factorized as the supersymmetric form $L^\dagger L H(z) = m^2 H(z)$ with $L = (\frac{d}{dz} + \frac{\partial_z f}{f})$ and $L^\dagger = (-\frac{d}{dz} + \frac{\partial_z f}{f})$. The Hermitian and positive definite of the operator $L^\dagger L$ ensure that $m^2 \geq 0$. Thus, there is no tachyonic KK mode.

By setting $m = 0$ in Eq. (33), the graviton zero mode can be solved as

$$H_0(z) = N_0 f^{-1}(z) = N_0\sqrt{1+\phi}e^{3A/2}, \quad (36)$$

where N_0 is a normalization constant. The normalization of the zero mode is expressed as

$$\int H_0^2 dz = \int H_0^2 e^{-A} dy = N_0^2 \int (1+\phi)e^{2A} dy = 1. \quad (37)$$

For model A with the other parameters set to $b = 1$, $k = 1$, and $\kappa = 1$, N_0 can be calculated as $N_0 = \sqrt{\frac{3}{6+2a}}$. For arbitrary positive parameters, it can be shown that the integral in Eq. (37) is finite. So, the graviton zero mode in model A can be localized on the brane. Figure 3 shows the shapes of the effective potential of the gravitational fluctuation and non-normalized graviton zero mode. It can be seen that the shape of the effective potential changes from a volcanolike well to a double well with increasing a. From Eq. (35), we can obtain $U''(0) = -4a^2 - 18a + 27/2$. It is obvious that the shape of the effective potential is volcanolike for

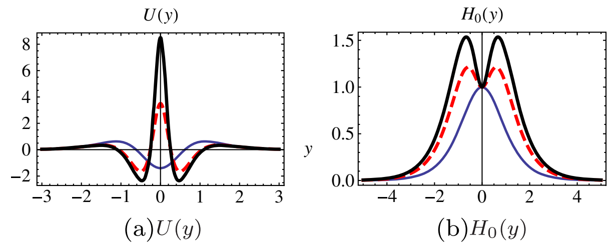


FIG. 3. The left and right figures plot the effective potential $U(y)$ of the gravitational fluctuation and the graviton zero mode $H(y)$ for model A, respectively. The parameters are set to $b = 1$, $k = 1$, $\kappa = 1$, and $a = 0.1$ for thin lines; $a = 5$ for red dashed thick lines; and $a = 10$ for black thick lines.

$U''(0) > 0$ and double well for $U''(0) < 0$. The critical value of the parameter a is $a_c = \frac{3}{4}(\sqrt{15} - 3)$ since we only need positive a. Thus, the effective potential has a single well and a double well for $0 < a < a_c$ and $a > a_c$, respectively.

It can also be seen that the graviton zero mode is localized gradually far away from the origin of the extra dimension [see Fig. 3(b)] because the shape of the effective potential changes from volcanolike to double well [see Fig. 3(a)]. This character does not mean a double brane but a single brane because the energy density still peaks at the origin of the extra dimension [see Fig. 1f]. Therefore, even though the brane has no inner structure, the effective potential has a double well, and the graviton zero mode has a split for large a. This is a new result of this model that is different from the previous ones in the literature.

Figure 4 shows the shapes of the effective potential and graviton zero mode in model B. From Eqs. (20), (21), (22), and (35), one can get

$$U(0) = \frac{\phi''(0)}{2(\phi_0+1)} - \frac{3}{2}bk^2 = \frac{3}{2}bk^2 - \frac{3}{2}bk^2 = 0.$$

Thus, the shape of the effective potential of the gravitational fluctuation is always a double well for any positive parameters b and k. From the asymptotic behavior of the warp factor $A(y \to \pm\infty) \to -bk|y|$ and scalar profile $\phi(y \to \pm\infty) \to c_1$, it is easy to check that the corresponding

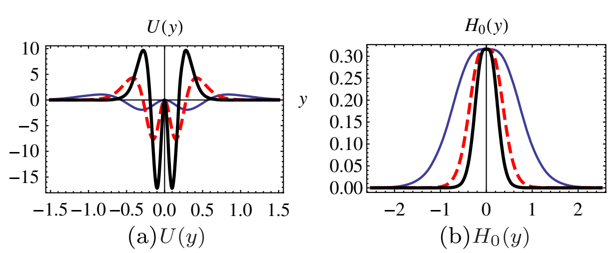


FIG. 4. The left and right figures plot the effective potential $U(y)$ of the gravitational fluctuation and the graviton zero mode fluctuation and the graviton zero mode $H(y)$ for model B, respectively. The parameters are set to $b = 3$, $\phi_0 = -0.9$, and $k = 1$ for thin lines; $k = 2$ for red dashed thick lines; and $k = 3$ for black thick lines.

graviton zero mode for model B can also be normalizable: $\int H_0^2(z)dz < \infty$. Therefore, the graviton zero mode can be localized on the brane.

So, we can conclude that the brane can be constructed by the background scalar field or by pure gravity, and the Newtonian potential on the brane can be reproduced on both models since the graviton zero mode can be localized on the brane.

B. Scalar perturbation

In this subsection, we will first analyze the scalar perturbation for model B in detail. Before analyzing the more complex model A, we will give a brief analysis about the scalar perturbations for a general multifield system by using a covariant approach. It is convenient to analyze the scalar perturbation in the Einstein frame. Thus, we will first rewrite the action (7) in the Einstein frame.

After making a coordinate transformation $dy = e^{A(z)}dz$, the metric can be written as

$$g_{MN} = e^{2A(z)}\eta_{MN}. \tag{38}$$

Then, we introduce a conformal transformation

$$\tilde{g}_{MN} = e^{2\omega}g_{MN}. \tag{39}$$

Under this conformal transformation, the Ricci scalar transforms as [68]

$$R = e^{2\omega}\tilde{R} + 8\tilde{g}^{MN}e^{\omega}\tilde{\nabla}_M\tilde{\nabla}_N e^{\omega} - 20\tilde{g}^{MN}\tilde{\nabla}_M e^{\omega}\tilde{\nabla}_N e^{\omega}, \tag{40}$$

where $\tilde{\nabla}_M$ is the covariant derivative compatible with the metric $\tilde{g}_{MN}$. Then, from Eq. (7), the action of model B in the Einstein frame reads

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{-\tilde{g}}\left[\tilde{R} + \frac{4\tilde{g}^{MN}\partial_M\phi\partial_N\phi}{3\phi(\phi+1)^2} - \frac{V_1(\phi)}{(\phi+1)^{5/3}}\right]. \tag{41}$$

By the way, the second term of the action (41) can also be rewritten in canonical form. Here, we will not do this because we can make use of the solutions of the scalar field ϕ and the potential $V_1(\phi)$ obtained in the previous section directly.

The field equations can be obtained by varying the action (41) with respect to the metric g_{MN} and the scalar field ϕ,

$$\tilde{R}_{MN} - \frac{1}{2}\tilde{g}_{MN}\tilde{R} + \frac{4}{3\phi(\phi+1)^2}\partial_M\phi\partial_N\phi - \frac{1}{2}\tilde{g}_{MN}\left[\frac{4\tilde{g}^{KL}\partial_K\phi\partial_L\phi}{3\phi(\phi+1)^2} - \frac{V_1}{(\phi+1)^{5/3}}\right] = 0, \tag{42}$$

$$(\phi+1)\phi'' + 3\tilde{A}'\phi' + 3\phi\tilde{A}'\phi' - \frac{3}{2}\phi'^2 - \frac{\phi'^2}{2\phi} + \left(\frac{1}{8} + \frac{\phi}{4} + \frac{\phi^2}{8}\right)\frac{3e^{2\tilde{A}}\phi}{(\phi+1)^{2/3}}V_{1\phi} - \frac{5}{8}e^{2\tilde{A}}\phi(\phi+1)^{1/3}V_1 = 0, \tag{43}$$

where the prime in this subsection denotes the derivative with respect to the coordinate z, and $\tilde{A} = A + \frac{1}{3}\ln(\phi+1)$.

In the longitudinal gauge, the perturbed metric can be written in the familiar form

$$ds^2 = e^{2\tilde{A}}((1+2\alpha)\eta_{\mu\nu}dx^\mu dx^\nu + (1+2\beta)dz^2). \tag{44}$$

By the way, instead of choosing a gauge, we can also construct gauge-invariant variables [42,43,69].

Then, we get the equations for the scalar perturbations:

$$(\mu,\nu):\ \left[-3\alpha'' - 9\tilde{A}'\alpha' + 3\tilde{A}'\beta' - \frac{e^{2\tilde{A}}V_1}{(\phi+1)^{5/3}}\alpha + \left(6\tilde{A}'^2 - \frac{4\phi'^2}{3\phi(\phi+1)^2} + 6\tilde{A}''\right)(\beta-\alpha) - \eta^{\rho\sigma}\partial_\rho\partial_\sigma(\beta+2\alpha)\right]\delta^\mu_\nu + \eta^{\mu\rho}\partial_\rho\partial_\nu(\beta+2\alpha) = \left[\frac{e^{2\tilde{A}}(3(\phi+1)V_{1\phi} - 5V_1)}{6(\phi+1)^{8/3}}\delta\phi + \frac{2(3\phi+1)\phi'^2}{3\phi^2(\phi+1)^3}\delta\phi - \frac{4\phi'}{3\phi(\phi+1)^2}\delta\phi'\right]\delta^\mu_\nu, \tag{45}$$

$$(\mu,z):\ -3\tilde{A}'\partial_\mu\beta + 3\partial_\mu\alpha' = \frac{4\phi'}{3\phi(\phi+1)^2}\partial_\mu\delta\phi, \tag{46}$$

$$(z,z):\ 12\tilde{A}'\alpha' + 3\eta^{\mu\nu}\partial_\mu\partial_\nu\alpha + \frac{e^{2\tilde{A}}V_1}{(\phi+1)^{5/3}}\beta = \left[\frac{e^{2\tilde{A}}(5V_1 - 3(\phi+1)V_{1\phi})}{6(\phi+1)^{8/3}} + \frac{2(3\phi+1)\phi'^2}{3\phi^2(\phi+1)^3}\right]\delta\phi - \frac{4\phi'}{3\phi(\phi+1)^2}\delta\phi', \tag{47}$$

$$\text{matter}:\ \eta^{\mu\nu}\partial_\mu\partial_\nu\delta\phi + \delta\phi'' + \left[3\tilde{A}' - \frac{(3\phi+1)\phi'}{\phi(\phi+1)}\right]\delta\phi' + \left[\frac{5e^{2\tilde{A}}\phi}{3(\phi+1)^{5/3}}V_1 - \frac{5e^{2\tilde{A}}\phi}{4(\phi+1)^{3/2}}V_{1\phi} + \frac{3}{8}e^{2\tilde{A}}\phi(\phi+1)^{1/3}\frac{d^2V_1}{d\phi^2} + \frac{(6\phi^2+4\phi+1)\phi'^2}{\phi^2(\phi+1)^2} - \frac{(3\phi+1)(3\tilde{A}'\phi' + \phi'')}{\phi(\phi+1)}\right]\delta\phi = -6\phi'\alpha' - \left[12\tilde{A}'\phi' + 4\phi'' - \frac{2(3\phi+1)\phi'^2}{\phi(\phi+1)}\right]\alpha. \tag{48}$$

From Eq. (46), we get

$$\delta\phi = \frac{9\phi(\phi+1)^2}{4\phi'}(\alpha' - \tilde{A}'\beta), \tag{49}$$

and from the off-diagonal part of Eq. (45), we get

$$2\alpha + \beta = 0. \tag{50}$$

Substituting Eqs. (43), (49), and (50) into Eq. (45), we obtain the scalar perturbation equation of the system,

$$\eta^{\mu\nu}\partial_\mu\partial_\nu\alpha + \alpha'' + H(z)\alpha' + F(z)\alpha = 0, \tag{51}$$

where

$$F(z) = -2\tilde{A}'^2 + 6\tilde{A}'' - \frac{4\tilde{A}'\phi''}{\phi'} + \frac{2(3\phi+1)\tilde{A}'\phi'}{\phi(\phi+1)} - \frac{8\phi'^2}{9\phi(\phi+1)^2}, \tag{52}$$

$$H(z) = 3\tilde{A}' - \frac{2\phi''}{\phi'} + \frac{(3\phi+1)\phi'}{\phi(\phi+1)}. \tag{53}$$

Equation (51) can be written as a Schrödinger-type form in terms of the variable [42,43,47]

$$\mathcal{G} = \theta\delta\phi - \gamma\alpha, \tag{54}$$

where $\theta \equiv \sqrt{\frac{-4e^{3\tilde{A}}}{3\kappa\phi(\phi+1)^2}}$ and $\gamma \equiv \theta\frac{\phi'}{\tilde{A}'}$. The reason why one must discuss this variable is that it is $\mathcal{G}$ rather than α, which can diagonalize the quadratic action. Thus, one should use $\mathcal{G}$ as the normal mode in the quantization (for more details, see Refs. [42,43,47]). Then, from Eqs. (48) and (51), the equation for the canonical normal mode $\mathcal{G}$ can be obtained as [42,43,47]

$$\mathcal{G}'' + \eta^{\mu\nu}\partial_\mu\partial_\nu\mathcal{G} - \frac{\gamma''}{\gamma}\mathcal{G} = 0. \tag{55}$$

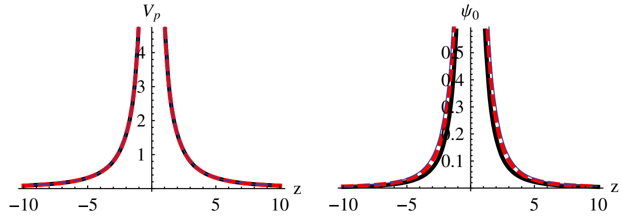
With the expansion

$$\mathcal{G} = \psi(z)e^{ipx}, \tag{56}$$

Eq. (55) becomes

$$(-\partial_z^2 + V_p)\psi(z) = m^2\psi(z), \tag{57}$$

where $p^2 = -m^2$ and the effective potential $V_p = \frac{\gamma''}{\gamma}$. Equation (57) can also be factorized as a supersymmetric form $\mathcal{A}\mathcal{A}^\dagger\psi = m^2\psi$ with $\mathcal{A} = (\frac{d}{dz} + \frac{\gamma'}{\gamma})$ and $\mathcal{A}^\dagger = (-\frac{d}{dz} + \frac{\gamma'}{\gamma})$. The Hermitian and positive definite of the operator $\mathcal{A}\mathcal{A}^\dagger$ ensure that $m^2 \geq 0$. Thus, there is no tachyonic KK mode.



(a) V_p (b) $\psi_0(y)$

FIG. 5. The left and right figures plot the effective potential V_p and zero mode ψ_0 of scalar perturbation of model B, respectively. The parameters are set to $b = 1$ and $k = 1$. The black thick, red dashed thick, and blue thin lines correspond to $\phi_0 = -0.9, -0.8$, and -0.7, respectively.

By the way, Eq. (51) can also be factorized as a supersymmetric form $\mathcal{A}^\dagger\mathcal{A}\alpha_m = m^2\alpha_m$, which indicates the spectra of $\mathcal{G}$ and α are related [42].

From Eq. (57), the scalar zero mode can be solved as

$$\psi_0 = n_0\gamma, \tag{58}$$

where n_0 is a normalization constant. The normalization condition is given by

$$\int|\psi_0(z)|^2dz = n_0^2\int|\gamma|^2dz = 2n_0^2\int_0^{+\infty}|\gamma|^2e^{-\tilde{A}}dz = 2n_0^2\left(\int_0^\epsilon + \int_\epsilon^{+\infty}\right)\frac{-4e^{2\tilde{A}}\phi'^2}{3\kappa\tilde{A}'^2\phi(\phi+1)^2}dy, \tag{59}$$

where ϵ is an infinitesimal positive number. Considering the behavior of the scalar profile $\phi(y)$ [see Eq. (24)] and warp factor $A(y)$ around $y = 0$, the first part of Eq. (59) can be calculated as $2n_0^2\int_0^\epsilon\frac{-4e^{2\tilde{A}}\phi'^2}{3\kappa\tilde{A}'^2\phi(\phi+1)^2}dy \sim 2n_0^2\int_0^\epsilon(\frac{d_1}{y^4} + \frac{d_2}{y^2})dy$, which is divergent. The second part of Eq. (59) is finite because the convergent of the integrand referring to Eq. (23). The divergence of Eq. (59) implies that the zero mode cannot be localized on the brane.

Besides, from Fig. 5(a), we can see that the effective potential is positive everywhere and diverges at the origin of extra dimension $z = 0$, which also implies that the zero mode of the scalar perturbation cannot be localized on the brane in model B.

Next, we will give some comment on the scalar perturbation in model A. There are two scalar fields, which are coupled together. Thus, it is more complex than model B. The scalar perturbations of multiple scalar fields were considered in Ref. [65] by Giovannini, who investigated the perturbations of the gravitating multidefects constructed by two scalar fields in five dimensions. The two scalar perturbation equations are coupled together and can be written in matrix notation. Giovannini showed that the system is stable under the scalar perturbations and the zero modes of them cannot be localized on the multidefects.

For more examples, see Refs. [70,71]. However, the kinetic term of each scalar field in Refs. [65,70,71] is canonical, and there is no coupling between them. In general, the kinetic term of a scalar field may not be canonical, and the different scalar fields may be coupled together. It is convenient to deal with these general scalar fields in the field space formalism, where the kinetic terms of the scalar fields can be written in a compact form,

$$\mathcal{L}_{\rm kin} = -\frac{1}{2} g^{MN} G_{IJ} \partial_M \Phi^I \partial_N \Phi^J, \tag{60}$$

where $G_{IJ} = G_{IJ}(\Phi^K)$ is a metric in the field space. Here, some Latin letters label spacetime indices, $M, N, P, Q, \ldots = 0, 1, 2, 3, 5$, and some other Latin letters, $I, J, K, L, \ldots = 1, 2, 3, \ldots$, label the field space indices. Thus, the scalar fields in Refs. [65,70,71] correspond a flat metric in the field space, $G_{IJ} = \delta_{IJ}$.

In this paper, the action (7) of model A in the Einstein frame is

$$S = \int d^5x \sqrt{-\tilde{g}} \bigg[\frac{\tilde{R}}{2\kappa^2} + \frac{2\tilde{g}^{MN} \partial_M \phi \partial_N \phi}{3\kappa^2 \phi(\phi+1)^2} - \frac{\tilde{g}^{MN} \partial_M \chi \partial_N \chi}{2(\phi+1)} - \frac{V_1(\phi) + 2\kappa^2 V_2(\chi)}{2\kappa^2 (\phi+1)^{5/3}} \bigg]. \tag{61}$$

In the field space formalism, the above action (61) can be rewritten as

$$S = \int d^5x \sqrt{-\tilde{g}} \bigg[\frac{\tilde{R}}{2\kappa^2} - \frac{1}{2} \tilde{g}^{MN} G_{IJ} \partial_M \Phi^I \partial_N \Phi^J - V \bigg], \tag{62}$$

where

$$\Phi^1 = \phi(y), \qquad \Phi^2 = \chi(y),$$
$$V = \frac{V_1(\phi) + 2\kappa^2 V_2(\chi)}{2\kappa^2 (\phi+1)^{5/3}}, \tag{63}$$

and the nonvanishing components of the field space metric G_{IJ} are $G_{11} = -\frac{1}{\kappa^2} \frac{4}{3\phi(\phi+1)^2}$ and $G_{22} = \frac{1}{\phi+1}$.

In Ref. [72], the authors investigated the scalar perturbations of multifield inflationary models with an arbitrary field space metric in cosmology. They calculated the second-order action in the linear perturbations and divided the scalar perturbations into an adiabatic mode and entropy modes and analyzed their behavior individually. To obtain the higher-order action of cosmological perturbations easily and systematically, the authors of Ref. [73] introduced a covariant approach for general multiple scalar field system. They obtained the quadratic order action and cubic order action in the covariant form.

Although the field space formalism is very powerful in analyzing the multiple scalar perturbations in cosmology, we do not know how to deal with it in the brane world. In the brane world, the most interesting things are the stability of the system and the localization of the scalar zero modes, which have not been solved. We leave this for the next work.

IV. CONCLUSIONS AND DISCUSSION

In this paper, we investigated two thick brane models (model A and model B) in hybrid metric-Palatini gravity. The brane in model A was constructed by a background scalar field χ. This brane system can be solved analytically. On the other hand, inspired by the scalar-tensor representation of this gravity, we considered the possibility of the thick brane constructed by pure gravity, which is called model B. We obtained a set of numerical solutions for the brane system in this model. Then, we derived the field equation of the tensor perturbation (29). After the KK decomposition, we obtained a Schrödinger-like equation of the KK modes $H(z)$, which is the equation of motion of the graviton along the extra dimension. This equation can be factorized as a supersymmetric form, which ensures the stability of the brane system. Furthermore, we also gave the condition that avoids the ghost gravitons. Then, we analyzed the scalar perturbation of model B in detail and gave a brief introduction about the multiple scalar field perturbations.

To produce the four-dimensional Newtonian potential, we analyzed the graviton zero modes in both models. The graviton zero mode in model A splits from one peak to two peaks with the increase of the parameter a; however, the brane does not split. This means that the graviton zero mode is localized gradually far away from the origin of the extra dimension with the parameter a increasing. The reason is that the shape of the effective potential of the gravitational fluctuation changes from volcanolike ($0 < a < a_c$) to double well ($a > a_c$). This is a new feature compared with the former literature, where the splitting graviton zero mode only appears with the splitting brane [33,41,42,44–46,48–53]. This may be caused by the hybrid of the metric and Palatini formalisms. The graviton zero mode in model B is localized around the origin of the extra dimension and becomes thinner with the parameter k increasing. The shape of the effective potential of the gravitational fluctuation is always a double well. The graviton zero modes in both models are localized on the branes. So, we can obtain the familiar four-dimensional Newtonian potential for both models.

On the other hand, we analyzed the scalar perturbation of model B in detail. We found that the effective potential of the scalar perturbation is positive definite. Thus, the zero mode cannot be localized on the brane, avoiding a fifth force. In model A, the two scalar fields are coupled together, which is hard to deal with. Perturbations in the multiple scalar fields with nontrivial metric in the field space have been investigated in the cosmology, whereas, we do not know how to deal with it in the corresponding brane world models. This problem will be left for our next work.

Next, we will give a brief analysis about the localization of the bulk matter fields. We first consider the localization of a massless real free bulk scalar field. The effective potential of the scalar KK modes can be obtained as [74–78]

$$V_0(z) = \frac{3}{2}A'' + \frac{9}{4}A'^2 = \frac{3k^2(5k^2z^2 - 2)}{4(k^2z^2+1)^2}. \tag{64}$$

The zero mode of the bulk scalar field can be solved as $\varphi_0(z) = c_0 e^{3A(z)/2} = \frac{c_0}{(k^2z^2+1)^{3/4}}$. It can be easily shown that this zero mode can be localized on the branes in models A and B.

In general, the zero mode of a massless free bulk vector field cannot be localized on the RS-type thick brane with codimension 1 [75–79]. To obtain a confined zero mode of a bulk vector field, one should introduce a dynamical mass term [80,81] or consider other brane models [82,83]. After a mass term has been added on the Lagrangian density of a bulk vector field, the effective potential of the vector KK modes can be obtained as [80,81]

$$V_1(z) = A'' + A'^2 = \frac{k^2(2k^2z^2 - 1)}{(k^2z^2+1)^2}. \tag{65}$$

The vector zero mode is $\rho_0 = c_1 e^A = \frac{c_1}{\sqrt{k^2z^2+1}}$ and is localized on the branes in both models.

To obtain a confined four-dimensional massless fermion field, one needs to introduce the Yukawa coupling between the bulk fermion and background scalar fields, i.e., $\eta\bar{\Psi}\Pi(\chi)\Psi$, in which the background scalar field χ is an odd function of the extra dimension [74,76–78,84–87]. In model A, this background scalar field can be chosen as the matter field χ because it is an odd function. Then, the effective potential of the fermion KK modes with $\Pi(\chi) = \chi$ can be obtained as [76–78]

$$V_{\rm L,R}(z) = (\eta e^A \chi)^2 \mp \partial_z(\eta e^A \chi). \tag{66}$$

The fermion zero modes read $f_{\rm L0,R0}(z) = c_{1/2}\exp(\mp\eta\int e^A\chi dz)$. It can be easily shown that the left-chiral fermion can be localized on the brane for positive coupling constant η, which is similar to the results given in Refs. [76–78].

However, there is only one even background scalar field ϕ in model B. Therefore, the Yukawa coupling does not work. Reference [66] introduced a new mechanism with a coupling term $\eta\bar{\Psi}\Gamma^M\partial_M\mathcal{F}(\phi)\Psi$ to localize fermions. With the new coupling term, the effective potentials of the fermion KK modes with $\mathcal{F}(\phi) = \frac{1}{\phi_0^2-\phi^2}$ are given by

$$\begin{aligned} V_{\rm L,R}(z) &= (\eta\partial_z\mathcal{F})^2 \mp \partial_z(\eta\partial_z\mathcal{F}) \\ &= \frac{1}{(\phi_0^2-\phi^2)^4}[\mp 2\eta(\partial_z\phi)^2(\mp 2(\eta \mp \phi_0^2)\phi^2 \\ &\quad - 3\phi^4 + \phi_0^4) \mp 2\eta\phi(\phi_0^2-\phi^2)^2(\partial_z^2\phi)]. \end{aligned} \tag{67}$$

The zero modes of a bulk fermion field can be solved as $f_{\rm L0,R0}(z) = c_{1/2}\exp(\mp\eta\mathcal{F}) = c_{1/2}\exp(\frac{\mp\eta}{\phi_0^2-\phi^2})$. Then, a confined left-chiral fermion can be obtained for positive coupling constant η.

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[1] T. P. Sotiriou and V. Faraoni, Rev. Mod. Phys. **82**, 451 (2010).
[2] A. De Felice and S. Tsujikawa, Living Rev. Relativ. **13**, 3 (2010).
[3] S. Capozziello and M. De Laurentis, Phys. Rep. **509**, 167 (2011).
[4] T. Clifton, P. G. Ferreira, A. Padilla, and C. Skordis, Phys. Rep. **513**, 1 (2012).
[5] S. Nojiri and S. D. Odintsov, Phys. Rep. **505**, 59 (2011).
[6] L. Amendola, K. Enqvist, and T. S. Koivisto, Phys. Rev. D **83**, 044016 (2011).
[7] T. S. Koivisto, Phys. Rev. D **83**, 101501 (2011); **84**, 121502 (2011).
[8] T. S. Koivisto, D. F. Mota, and M. Sandstad, arXiv:1305.4754.
[9] T. Harko, T. S. Koivisto, F. S. N. Lobo, and G. J. Olmo, Phys. Rev. D **85**, 084016 (2012).
[10] S. Capozziello, T. Harko, F. S. N. Lobo, and G. J. Olmo, Int. J. Mod. Phys. D **22**, 1342006 (2013).

[11] S. Capozziello, T. Harko, T. S. Koivisto, F. S. N. Lobo, and G. J. Olmo, J. Cosmol. Astropart. Phys. 04 (2013) 011.
[12] S. Capozziello, T. Harko, T. S. Koivisto, F. S. N. Lobo, and G. J. Olmo, J. Cosmol. Astropart. Phys. 07 (2013) 024.
[13] C. G. Bohmer, F. S. N. Lobo, and N. Tamanini, Phys. Rev. D **88**, 104019 (2013).
[14] S. Capozziello, T. Harko, T. S. Koivisto, F. S. N. Lobo, and G. J. Olmo, Phys. Rev. D **86**, 127504 (2012).
[15] S. Capozziello, T. Harko, F. S. N. Lobo, G. J. Olmo, and S. Vignolo, Int. J. Geom. Methods Mod. Phys. **11**, 1450042 (2014).
[16] N. A. Lima, Phys. Rev. D **89**, 083527 (2014).
[17] T. Azizi and N. Borhani, Astrophys. Space Sci. **357**, 146 (2015).
[18] S. Capozziello, T. Harko, T. S. Koivisto, F. S. N. Lobo, and G. J. Olmo, Universe **1**, 199 (2015).
[19] I. Antoniadis, Phys. Lett. B **246**, 377 (1990).
[20] N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali, Phys. Lett. B **429**, 263 (1998).
[21] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali, Phys. Lett. B **436**, 257 (1998).
[22] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999).
[23] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999).
[24] R. Maartens and K. Koyama, Living Rev. Relativ. **13**, 5 (2010).
[25] W. D. Goldberger and M. B. Wise, Phys. Rev. Lett. **83**, 4922 (1999).
[26] M. Gremm, Phys. Lett. B **478**, 434 (2000).
[27] M. Gremm, Phys. Rev. D **62**, 044017 (2000).
[28] O. DeWolfe, D. Z. Freedman, S. S. Gubser, and A. Karch, Phys. Rev. D **62**, 046008 (2000).
[29] C. Csaki, J. Erlich, T. J. Hollowood, and Y. Shirman, Nucl. Phys. **B581**, 309 (2000).
[30] T. Gherghetta and A. Pomarol, Nucl. Phys. **B586**, 141 (2000).
[31] N. Arkani-Hamed, M. Porrati, and L. Randall, J. High Energy Phys. 08 (2001) 017.
[32] A. Campos, Phys. Rev. Lett. **88**, 141602 (2002).
[33] S. Kobayashi, K. Koyama, and J. Soda, Phys. Rev. D **65**, 064014 (2002).
[34] A. Z. Wang, Phys. Rev. D **66**, 024024 (2002).
[35] C. Charmousis, S.C. Davis, and J.F. Dufaux, J. High Energy Phys. 12 (2003) 029.
[36] D. Bazeia and A. R. Gomes, J. High Energy Phys. 05 (2004) 012.
[37] Y. X. Liu, L. Zhao, X. H. Zhang, and Y. S. Duan, Nucl. Phys. **B785**, 234 (2007).
[38] V. Dzhunushaliev, V. Folomeev, and M. Minamitsuji, Phys. Rev. D **79**, 024001 (2009).
[39] Y. X. Liu, Y. Zhong, Z. H. Zhao, and H. T. Li, J. High Energy Phys. 06 (2011) 135.
[40] W. J. Geng and H. Lu, Phys. Rev. D **93**, 044035 (2016).
[41] J. Garriga and T. Tanaka, Phys. Rev. Lett. **84**, 2778 (2000).
[42] M. Giovannini, Phys. Rev. D **64**, 064023 (2001).
[43] M. Giovannini, Classical Quantum Gravity **20**, 1063 (2003).
[44] D. Bazeia, C. Furtado, and A. R. Gomes, J. Cosmol. Astropart. Phys. 02 (2004) 002.
[45] A. Herrera-Aguilar, D. Malagon-Morejon, and R. R. Mora-Luna, J. High Energy Phys. 11 (2010) 015.
[46] K. Yang, Y. X. Liu, Y. Zhong, X. L. Du, and S. W. Wei, Phys. Rev. D **86**, 127502 (2012).
[47] Y. Zhong and Y. X. Liu, Phys. Rev. D **88**, 024017 (2013).
[48] A. Ahmed and B. Grzadkowski, J. High Energy Phys. 01 (2013) 177.
[49] W. T. Cruz, L. J. S. Sousa, R. V. Maluf, and C. A. S. Almeida, Phys. Lett. B **730**, 314 (2014).
[50] N. Barbosa-Cendejas, A. Herrera-Aguilar, U. Nucamendi, I. Quiros, and K. Kanakoglou, Gen. Relativ. Gravit. **46**, 1631 (2014).
[51] M. Higuchi and S. Nojiri, Gen. Relativ. Gravit. **46**, 1822 (2014).
[52] D. Bazeia, A. S. Lobao, L. Losano, R. Menezes, and G. J. Olmo, Phys. Rev. D **91**, 124006 (2015).
[53] B. M. Gu, B. Guo, H. Yu, and Y. X. Liu, Phys. Rev. D **92**, 024011 (2015).
[54] D. Bazeia, L. Losano, R. Menezes, G. J. Olmo, and D. Rubiera-Garcia, Eur. Phys. J. C **75**, 569 (2015).
[55] V. Dzhunushaliev, V. Folomeev, B. Kleihaus, and J. Kunz, J. High Energy Phys. 04 (2010) 130.
[56] H. Liu, H. Lu, and Z. L. Wang, J. High Energy Phys. 02 (2012) 083.
[57] Y. Zhong and Y. X. Liu, Eur. Phys. J. C **76**, 321 (2016).
[58] M. Parry, S. Pichler, and D. Deeg, J. Cosmol. Astropart. Phys. 04 (2005) 014.
[59] K. A. Bronnikov and S. G. Rubin, Gravitation Cosmol. **13**, 191 (2007).
[60] N. Deruelle, M. Sasaki, and Y. Sendouda, Prog. Theor. Phys. **119**, 237 (2008).
[61] A. Balcerzak and M. P. Dabrowski, Phys. Rev. D **77**, 023524 (2008).
[62] A. Herrera-Aguilar, D. Malagon-Morejon, and R. R. Mora-Luna, J. High Energy Phys. 11 (2010) 015.
[63] M. Giovannini, Phys. Rev. D **74**, 087505 (2006).
[64] M. Giovannini, Classical Quantum Gravity **23**, L73 (2006).
[65] M. Giovannini, Phys. Rev. D **75**, 064023 (2007).
[66] Y. X. Liu, Z. G. Xu, F. W. Chen, and S. W. Wei, Phys. Rev. D **89**, 086001 (2014).
[67] M. Cvetic and H. H. Soleng, Phys. Rep. **282**, 159 (1997).
[68] S. Carroll, *Spacetime and Geometry: An Introduction to General Relativity* (Addison-Wesley, New York, 2004).
[69] J. M. Bardeen, Phys. Rev. D **22**, 1882 (1980).
[70] S. M. Aybat and D. P. George, J. High Energy Phys. 09 (2010) 010.
[71] D. P. George, Phys. Rev. D **83**, 104025 (2011).
[72] D. Langlois and S. Renaux-Petel, J. Cosmol. Astropart. Phys. 04 (2008) 017.
[73] J. O. Gong and T. Tanaka, J. Cosmol. Astropart. Phys. 03 (2011) 015; 02 (2012) E01.
[74] V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B **125**, 136 (1983).
[75] B. Bajc and G. Gabadadze, Phys. Lett. B **474**, 282 (2000).
[76] Y. X. Liu, H. Guo, Chun-E. Fu, and H. T. Li, Phys. Rev. D **84**, 044033 (2011).
[77] M. Gogberashvili, J. High Energy Phys. 09 (2012) 056.
[78] Q. Y. Xie, Z. H. Zhao, Y. Zhong, J. Yang, and X. N. Zhou, J. Cosmol. Astropart. Phys. 03 (2015) 014.
[79] G. Alencar, R. R. Landim, M. O. Tahim, and R. N. Costa Filho, Phys. Lett. B **739**, 125 (2014).

[80] Z. H. Zhao, Q. Y. Xie, and Y. Zhong, Classical Quantum Gravity **32**, 035020 (2015).
[81] C. A. Vaquera-Araujo and O. Corradini, Eur. Phys. J. C **75**, 48 (2015).
[82] Y. X. Liu, Z. H. Zhao, S. W. Wei, and Y. S. Duan, J. Cosmol. Astropart. Phys. 02 (2009) 003.
[83] Y. X. Liu, C.-E. Fu, H. Guo, and H. T. Li, Phys. Rev. D **85**, 084023 (2012).
[84] A. E. R. Chumbes, A. E. O. Vasquez, and M. B. Hott, Phys. Rev. D **83**, 105010 (2011).
[85] N. Barbosa-Cendejas, D. Malagn-Morejn, and R. Rigel Mora-Luna, Gen. Relativ. Gravit. **47**, 77 (2015).
[86] S. Randjbar-Daemi and M. Shaposhnikov, Phys. Lett. B **492**, 361 (2000).
[87] C. Ringeval, P. Peter, and J. P. Uzan, Phys. Rev. D **65**, 044016 (2002).