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Electroweak interactions and dark baryons in the sextet BSM model with a composite Higgs particle

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The electroweak interactions of a strongly coupled gauge theory are discussed with an outlook beyond the Standard Model (BSM) under global and gauge anomaly constraints. The theory is built on a minimal massless fermion doublet of the SU(2) BSM flavor group (bsm-flavor) with a confining gauge force at the TeV scale in the two-index symmetric (sextet) representation of the BSM SU(3) color gauge group (bsm-color). The intriguing possibility of near-conformal sextet gauge dynamics could lead to the minimal realization of the composite Higgs mechanism with a light 0^{++} scalar, far separated from strongly coupled resonances of the confining gauge force in the 2–3 TeV range, distinct from Higgsless technicolor. In previous publications we have presented results for the meson spectrum of the theory, including the light composite scalar, which is perhaps the emergent Higgs impostor. Here we discuss the critically important role of the baryon spectrum in the sextet model investigating its compatibility with what we know about thermal evolution of the early Universe including its galactic and terrestrial relics. For an important application, we report the first numerical results on the baryon spectrum of this theory from nonperturbative lattice simulations with baryon correlators in the staggered fermion implementation of the strongly coupled gauge sector. The quantum numbers of composite baryons and their spectroscopy from lattice simulations are required inputs for exploring dark matter contributions of the sextet BSM model, as outlined for future work.

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I. INTRODUCTION

An important strongly coupled near-conformal gauge theory built on the minimally required SU(2) bsm-flavor doublet of two massless fermions, with a confining gauge force at the TeV scale in the sextet representation of the new SU(3) bsm-color, is frequently discussed as an intriguing possibility for the minimal realization of the composite Higgs mechanism. Early discussions of the model as a beyond the Standard Model (BSM) candidate were initiated in systematic explorations of higher fermion representations of color gauge groups [1–3] for extensions of the original Higgsless technicolor paradigm [4,5]. In fact, the first appearance of the particular two-index symmetric SU(3) fermion representation can be traced even further back to quantum chromodynamics (QCD) where a doublet of sextet quarks was proposed as a mechanism for electroweak symmetry breaking (EWSB) without an elementary Higgs field [6]. This idea had to be replaced by a new gauge force at the TeV scale, orders of magnitude stronger than in QCD, to facilitate the dynamics of EWSB just below the lower edge of the conformal window in the new BSM paradigm [1–3]. It should be noted that throughout its early history the important near-conformal behavior

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of the model was not known and definitive results had to wait for recent nonperturbative investigations with lattice gauge theory methods as used in our work.

Near-conformal BSM theories raise the possibility of a light composite scalar, perhaps a Higgs impostor, to emerge from new strong dynamics, far separated from the associated composite resonance spectrum in the few TeV mass range with interesting and testable predictions for the Large Hadron Collider (LHC). This scenario is very different from what was expected from QCD when scaled up to the electroweak scale, as illustrated by the failure of the Higgsless technicolor paradigm. Given the discovery of the 125 GeV Higgs particle at the LHC, any realistic BSM theory must contain a Higgs-like state, perhaps with some hidden composite structure.

Based on our *ab initio* nonperturbative lattice calculations we find accumulating evidence for near-conformal behavior in the sextet theory with the emergent low-mass 0^{++} scalar state far separated from the composite resonance spectrum of bosonic excitations in the 2–3 TeV energy range [7–10]. The identification of the light scalar state is numerically challenging since it requires the evaluation of disconnected fermion loop contributions to correlators with vacuum quantum numbers in the range of light fermion masses we explore. The evidence to date is very promising that the 0^{++} scalar is light in the chiral limit and that the model at this stage remains an important BSM candidate.

In Sec. II critically important features of the strongly coupled sextet gauge sector of the light scalar are briefly reviewed. In Sec. III we discuss the electroweak interactions of sextet fermions and their electroweak multiplet structure. The outlook beyond the Standard Model under global and gauge anomaly constraints is presented in Sec. IV including new BSM physics under the requirement of integer electric charges for dark baryons. In Sec. V dark baryons from the sextet electroweak multiplet structure are constructed with a discussion of model constraints based on galactic and terrestrial relic densities from the early Universe. Section VI describes the construction of lattice baryon operators using staggered lattice fermions in the sextet color representation. Our first nonperturbative lattice results on sextet baryon spectroscopy are presented in Sec. VII. We conclude in Sec. VIII with a brief summary and outlook.

II. THREE CRITICAL FEATURES OF THE SEXTET STRONG FORCE

The foundation of the theory is based on chiral symmetry breaking (χSB) from the sextet gauge force in the massless fermion limit with three Goldstone bosons for the minimal realization of the Higgs mechanism. In our work from lattice simulations, the χSB pattern $\mathrm{SU}(2)_L \otimes SU(2)_R \to SU(2)_V$ in the bsm-flavor group is consistent with the absence of any evidence for a conformal infrared fixed point (IRFP) at scales reached so far with the scale-dependent strong gauge coupling [11]. These two fundamental features are intrinsically interdependent. The existence of an IRFP would make the sextet theory conformal with unbroken chiral symmetry implying the disappearance of the finite-temperature χSB transition of massless fermions in the continuum limit [12]. The phenomenological relevance of the sextet BSM model would be questioned in this case, requiring perhaps speculative changes from previously unexplored new features, like the role of four-fermion operators in strongly coupled gauge dynamics. Since our work on the sextet theory shows no evidence for an IRFP, we do not consider here model modifications, like the role of an added four-fermion operator with large anomalous dimension.

The most important question for future investigations is fermion mass generation in near-conformal BSM models with two-flavor or multiflavor realizations of the Higgs mechanism. Large anomalous dimensions of bilinear fermion operators of the composite theory, when coupled to fermions of the Standard Model, are expected to play an important role in fermion mass generation from effective Yukawa couplings of the light composite scalar to fermions of the Standard Model. Related mechanisms, like top-quark-generated self-energy loop effects reducing the mass of the light scalar, are strategies being investigated outside the scope of this paper [13].

A. The scale-dependent renormalized coupling and its β function

There have been several lattice studies of the renormalized sextet gauge coupling and its β function using different schemes. The first studies using the Schrödinger functional method with tuned massless Wilson fermions were not decisive to rule in or rule out an IRFP in the explored range of the renormalized coupling [14,15]. Our recent study using the gradient flow scheme with exactly massless staggered fermions and with a full investigation of systematic effects in taking the continuum limit shows no evidence that the β function has an infrared fixed point in the gauge coupling range $0 < g^2 < 6.5$ investigated [11]. This finding is consistent with studies of the mass-deformed Goldstone spectrum and the spectrum of the Dirac operator which exhibits the Banks-Casher condensate in the chiral limit [10,16]. Our ongoing investigations include detailed studies of the mass-deformed Goldstone spectrum and the chiral condensate via the Gell-Mann-Oakes-Renner relation in chiral perturbation theory. Additional predictions for χSB from random matrix theory are being tested from the lowest eigenvalues of the Dirac spectrum for high-precision results.

Preliminary results from concurrent studies of the scale-dependent renormalized coupling, using the gradient flow method with Wilson fermions in finite volumes, reported a conformal IRFP in the sextet β function within the $g^2 \sim 5.5$–6.5 range where the three-loop and four-loop perturbative β functions develop zeros in the $\overline{\mathrm{MS}}$ scheme. These preliminary results have been revised by the authors and no IRFP is reported in the sextet β function [17], similar to our findings [11].

It should be noted that the precise determination of the very small β function presents challenges even for the best gradient-flow-based methods which were deployed by both groups under discussion. Nevertheless, the outcome of these difficult scale-dependent gauge coupling studies remains consistent with our expectation that the theory is very close to the lower edge of the conformal window with χSB but without an IRFP.

B. Finite-temperature chiral transition

Based on the existence of χSB at zero temperature, it would be expected that chiral symmetry is restored in the sextet theory in a finite-temperature chiral transition. This has been the focus of recent work with evidence presented for a chiral transition at finite lattice cutoff [12,18,19]. Tracking the cutoff-dependent temperature of the chiral transition as the gauge coupling is varied, the authors conclude that the chiral transition should disappear in the continuum limit and they report new-found indications for a conformal IRFP in the continuum model [12]. This finding is based on the scale-dependent variation of the β function with the bare and renormalized gauge couplings, significantly slower than the expected two-loop perturbative behavior in the renormalized weak coupling range $g_R^2 \sim 1$–3 being tested without the removal of cutoff effects in the β function and without control of other systematics which can qualitatively affect the conclusions of Ref. [12]. In particular, the simple Wilson gauge action and unimproved simple staggered fermion action were used in Ref. [12] with known large cutoff effects and lacking systematic control on the estimate of the renormalized coupling and its continuum limit, restricted to simulations away from the required chiral limit [12]. In contrast, our direct determination of the β function from the gauge field gradient flow method when the cutoff is removed and the continuum limit is taken shows agreement with the two-loop β function in the $g_R^2 \sim 1$–3 range without any sign of a conformal IRFP [11]. To resolve the apparent controversy, definitive and systematic finite-temperature studies of χSB would be needed in the massless fermion limit of this important gauge field theory close to the lower edge of the conformal window.

C. Resonance spectrum

The first direct test of the sextet theory is expected to come from the strongly coupled sector of the new gauge force which predicts resonances in the 2–3 TeV range within the reach of Run 2 at the LHC. As an example, a rho-like vector state has been predicted in the model at approximately 2 TeV which could be observed as a diboson resonance excess above LHC background events [7–10]. In Sec. VII we will briefly comment on the recently reported diboson excess from the ATLAS and CMS collaborations, consistent with our prediction but far from settled. The location of the rho-like resonance at 2 TeV would be less surprising in Higgsless technicolor, but the emergent light 0^{++} scalar at the electroweak scale, far separated from the resonance spectrum in sextet dynamics, is a distinct and unexpected new feature. The recently found diphoton excess from ATLAS and CMS resonance searches [20,21] in the invariant mass range around 750 GeV would require a very specific explanation of a light near-conformal state distinct and separated from the 2–3 TeV composite resonance range in strongly coupled composite gauge theories. An interesting speculative example would be the near-conformal η' state whose light mass with a separation from the resonance spectrum would be associated with the U(1) axial anomaly near the conformal window [22–25]. In composite gauge theories, like the sextet model, another possibility is an emergent second scalar in the 750 GeV mass range as the partner of the light Higgs impostor at 125 GeV.

III. ELECTROWEAK MULTIPLETS AND ANOMALY CONSTRAINTS

Building a BSM theory requires the embedding of the strongly coupled sextet fermion doublet into the $\mathrm{SU}(2)_w \otimes U(1)_Y$ electroweak gauge group with a new outlook beyond the Standard Model under global and gauge anomaly constraints. We will show that the general construction can accommodate new physics at the energy frontier including new heavy leptons and massive neutrinos. The related dark matter content of the theory implies interesting scenarios for future investigations. As a first step, model building requires a consistent electroweak multiplet structure with a simple realization of the composite Higgs mechanism from sextet gauge dynamics.

A. The electroweak multiplet structure of sextet fermions

As in the minimal scheme of Susskind [4] and Weinberg [5], the gauge group of the theory is $\mathrm{SU}(3)_{bsm} \otimes SU(3)_c \otimes SU(2)_w \otimes U(1)_Y$ where $\mathrm{SU}(3)_c$ designates the QCD color gauge group and $\mathrm{SU}(3)_{bsm}$ represents the BSM color gauge group of the new strong gauge force. In addition to quarks and leptons of the Standard Model, we include one SU(2) bsm-flavor doublet (u, d) of fermions which are $\mathrm{SU}(3)_c$ singlets and transform in the six-dimensional sextet representation of bsm-color, distinct from the fundamental color representation of fermions in the original technicolor scheme [4,5]. The formal designation (u, d) for the bsm-flavor doublet of sextet fermions uses a similar notation to the two light quarks of QCD but describes completely different physics. The massless sextet fermions form two chiral doublets $(u,d)_L$ and $(u,d)_R$ under the global symmetry group $\mathrm{SU}(2)_L \otimes SU(2)_R \otimes U(1)_B$. Baryon number is conserved for quarks of the Standard Model separate from baryon number conservation for sextet fermions which carry $1/3$ of the BSM baryon charge associated with the BSM sector of the global $U(1)_B$ symmetry.

It is straightforward to define consistent multiplets for the sextet fermion flavor doublet under the $\mathrm{SU}(2)_w \otimes U(1)_Y$ electroweak gauge group with hypercharge assignments for left- and right-handed fermions transforming under the $\mathrm{SU}(2)_w$ weak isospin group. The two fermion flavors u^{ab} and d^{ab} of the strongly coupled sector carry six colors in two-index symmetric tensor notation, $a,\ b = 1,\ 2,\ 3,$ associated with the gauge force of the $\mathrm{SU}(3)_{bsm}$ group. This is equivalent to a six-dimensional vector notation in the sextet representation.

The fermions transform as left-handed weak isospin doublets and right-handed weak isospin singlets for each color,

$$\psi_L^{ab} = \begin{pmatrix} u_L^{ab} \\ \mathrm{d}_L^{ab} \end{pmatrix}, \qquad \psi_R^{ab} = (u_R^{ab}, d_R^{ab}). \tag{1}$$

With this choice of representations, the normalization for the hypercharge Y of the $U(1)_Y$ gauge group is defined by the relation $Y = 2(Q - T_3)$, with T_3 designating the third component of weak isospin.

Once electroweak gauge interactions are turned on, the chiral-symmetry-breaking pattern $\mathrm{SU}(2)_L \otimes SU(2)_R \to SU(2)_V$ of strong dynamics breaks electroweak symmetry in the expected pattern, $\mathrm{SU}(2)_w \times U(1)_Y \to U(1)_{\mathrm{em}}$, and with the simultaneous dynamical realization of the composite Higgs mechanism. It is important to note that the dynamical Higgs mechanism is facilitated through the electroweak gauge couplings of the sextet fermions and does not depend on the hypercharge assignments of the multiplets [4].

Hypercharges of left-handed doublets and right-handed singlets are determined from anomaly constraints and consistent electric charge assignments for fermions. Apparently, there are two simple solutions to the anomaly constraints with different hypercharge assignments for left-handed doublets. While both solutions represent new physics with sextet baryons, they lead to different consistency conditions on what we know about the relic abundance of dark baryons from their primordial evolution in the early Universe. Before we describe the two different solutions to anomaly constraints in Sec. IV with related cosmological implications, it is useful to briefly review first the merits of the sextet model for the minimal realization of the composite Higgs mechanism which is independent of the two different hypercharge assignments for the left-handed flavor doublets.

B. The minimal composite Higgs in the sextet model

The chiral-symmetry-breaking pattern $\mathrm{SU}(2)_L \otimes SU(2)_R \to SU(2)_V$ of the SU(2) BSM flavor group of sextet fermions generates an isotriplet of three massless Goldstone bosons in the chiral limit. The three Goldstones will become longitudinal modes of the $W^\pm$ and Z^0 weak gauge bosons via the dynamical Higgs mechanism when the electroweak interactions are turned on. This minimal realization of the composite Higgs mechanism comes from the perfect match between the longitudinal electroweak gauge bosons and three massless Goldstone bosons from sextet strong dynamics as one of the most attractive features of this BSM theory. It is near-conformal with just one SU(2) fermion flavor doublet with unexpected spectroscopy, distinct from old technicolor. We already noted that the dynamical Higgs mechanism does not depend on the electric charge assignment of the (u,d) fermion pair with left-handed doublets and right-handed singlets as set by the choice of hypercharges Y under the weak isospin gauge group $\mathrm{SU}(2)_w$ under anomaly constraints. Independent of how Y is set, the three Goldstone bosons always have the correct integer electric charges $(\pm 1, 0)$ to morph into the longitudinal components of the weak bosons as further detailed in Sec. IV. Since the sextet gauge model is naturally located very close to the lower edge of the conformal window without fine-tuning, a light scalar with a far separated resonance spectrum is expected from strong dynamics, making the minimal Higgs mechanism of the model economic and attractive.

In contrast, the condition of near-conformal behavior with fermions in the fundamental representations of SU(3) bsm-color requires a large number of fermion flavors in the BSM construction which leads to an excess of unwanted massless Goldstone bosons. The added complexities can be illustrated with the well-known one-family model from the technicolor era with global flavor symmetry $\mathrm{SU}(8)_L \otimes SU(8)_R$ of eight massless chiral fermions carrying bsm-color in the fundamental representation of the SU(3) gauge group [26]. In the new near-conformal BSM paradigm a minimum of eight flavors is needed in the fundamental color representation to get closer to the conformal window. This motivated recent studies of the eight-flavor fermion model to understand the strong BSM force of the one-family model from several lattice studies [27–33]. The χSB pattern $\mathrm{SU}(8)_L \otimes SU(8)_R \to SU(8)_V$ generates 63 Goldstone bosons with only three needed in the composite Higgs mechanism. Generating masses for the excess unwanted Goldstone bosons presents nontrivial phenomenological challenges, making it more difficult to achieve the desired goal of a near-conformal spectrum with a light 0^{++} scalar far separated from resonance excitations.

IV. CONSTRAINTS FROM GLOBAL AND GAUGE ANOMALIES

Assuming that the strongly coupled gauge sector of the sextet model turns out to be compatible with resonances in the 2–3 TeV region, a more complete BSM outlook of the model would come into focus, guided by anomaly constraints. We will show that the existence of stable baryons in the sextet model when combined with our understanding of the early Universe seems to favor anomalous sextet fermion hypercharge assignment in the electroweak sector. Although the sextet BSM model cannot offer a UV-complete solution for fermion mass generation and the related flavor problem, the outlook for new physics from extending the strongly coupled gauge sector guided by anomaly constraints gives new insights. Anomaly-compensating new physics has several important and immediate aspects without deference to physics on the scale of UV completion. It provides an outlook and framework for new physics the model can predict, or accommodate on several energy scales. Anomalies not only can predict or accommodate plausible new fermion content at the TeV scale but also give insight into electroweak corrections to Standard Model expectations. The example we will provide below for illustration only is the anomaly-canceling pair of two massive lepton doublets (and associated right-handed singlets) to cancel the anomalies in Eqs. (5) and (6) from left-handed sextet fermion doublets. New charged lepton and neutrino masses are partially constrained parameters in this anomaly-matching extension. Even if the new fermion masses are set to very high energy scales in this example, their infrared effects from the associated anomaly content will survive. The best known examples of this footprint from integrating out heavy fermions include the Wess-Zumino effective action and other residual effects at the electroweak scale [34–37].

A. Anomaly conditions in the sextet model

Anomaly constraints have a long history in technicolor-motivated BSM model building with representative examples in Refs. [1,38–41]. The first condition for model construction with left-handed doublets is the global Witten anomaly constraint which requires an even number of left-handed SU(2) multiplets to avoid inconsistency in the theory from a vanishing fermion determinant of the partition function [42].

In addition, gauge anomaly constraints also have to be satisfied [43]. With vector current $V^i_\mu(x) = \bar\psi T^i \gamma_\mu \psi(x)$ and axial current $A^i_\mu(x) = \bar\psi T^i \gamma_\mu \gamma_5 \psi(x)$ constructed from fermion fields and internal symmetry matrices T^i in some group representation R for fermions, the anomaly in the axial vector Ward identity is proportional to $\mathrm{tr}(\{T^i(R), T^j(R)\} T^k(R))$ and must vanish. In the sextet theory fermions are either left-handed doublets or right-handed singlets under the $\mathrm{SU}(2)_w$ gauge group. The matrices T^i will be either the τ^i Pauli matrices or the diagonal U(1) hypercharge Y. Since the SU(2) group is anomaly free, $\mathrm{tr}(\{\tau^i, \tau^j\}\tau^k) = 0$, we only need to consider anomalies where at least one T^i is the hypercharge Y. The nontrivial constraints come from two conditions on hypercharge traces,

$$\mathrm{tr}(Y) = 0, \qquad \mathrm{tr}(Y^3) \propto \mathrm{tr}(Q^2 T_3 - Q T_3^2) = 0, \quad (2)$$

where $Y = 2(Q - T_3)$ with electric charge Q, and T_3 as the third component of weak isospin.

There are two simple solutions for BSM model building with sextet fermions to satisfy the Witten anomaly condition and gauge anomaly constraints on tr(Y) and $\mathrm{tr}(Y^3)$ in Eq. (2). The first solution with the choice $Y(f_L) = 0$ for doublets of left-handed sextet fermions (f_L) leads to half-integer electric charges for composite baryons. The second solution with the choice $Y(f_L) = 1/3$ for doublets of left-handed sextet fermions leads to integer electric charges for composite baryons. The hypercharges of right-handed singlets are automatically set from consistent electric charge assignments in both cases. The two choices are discussed next with their implications.

B. EW content from $Y(f_L) = 0$ with baryons of half-integer electric charge Q

Since left-handed fermion doublets occur with an even number of sextet colors in the strong sector, the global anomaly condition is automatically satisfied for the first solution without the necessity of adding any new left-handed lepton doublets to the theory. Gauge anomaly cancellation in this case requires $Y = 0$ assigned to left-handed doublets of fermions with sextet color (a, b) in the two-index symmetric tensor representation,

$$\begin{aligned} &Y(u_L^{ab}) = 0, \qquad Y(d_L^{ab}) = 0, \\ &Y(u_R^{ab}) = 1, \qquad Y(d_R^{ab}) = -1, \end{aligned} \quad (3)$$

leading to fractional charges $Q(u_L) = 1/2$ and $Q(d_L) = -1/2$ from the $Y = 2(Q - T_3)$ relation. Hypercharges in Eq. (3) are set for right-handed fermions from consistent electric charge assignments $Q(u_R) = 1/2$ and $Q(d_R) = -1/2$.

The minimal electroweak content with the choice $Y(f_L) = 0$ leads to baryon states of three constituents forming flavor isospin doublets with a half unit of positive electric charge for isospin $+1/2$ with (uud) content and a half unit of negative electric charge for isospin $-1/2$ with (udd) content. This is in sharp contrast to electric charges carried by the proton and neutron in QCD where the Witten anomaly of three left-handed color doublets of (u, d) quarks is compensated by the left-handed lepton doublet of the electroweak theory. The first generation of quarks and leptons then allows the well-known choice of electric charges $Q(u) = 2/3$ and $Q(d) = -1/3$, compatible with gauge anomaly constraints and the pattern applied to all three generations. As a consequence, baryons in QCD carry integer electric charges. The unique solution to anomaly constraints in the Standard Model is consistent with direct observations of the full particle content including quark and lepton quantum numbers matching all the anomaly conditions.

In the sextet BSM theory we do not have direct observations on new heavy baryons to set unique hypercharge assignments for left-handed doublets and right-handed singlets of sextet fermions from one of two alternate solutions to the anomaly conditions. The viability of the choices $Y(f_L) = 0$, or $Y(f_L) = 1/3$, is affected by the different electric charge assignments they imply. With heavy baryon masses in the 3 TeV range, as determined from our lattice simulations in Secs. VI and VII outside the reach of immediate accelerator searches, our understanding of the early Universe provides important input concerning the two simple anomaly solutions. The seemingly minimal solution with $Y = 0$ for left-handed doublets would lead to intriguing predictions of baryon states with half-integer electric charges for future accelerator searches and relics with fractional electric charges from the early Universe with observable consequences. Problems with half-integer electric charges, from the choice $Y(f_L) = 0$ in our case, were anticipated earlier from strong observational limits on stable fractional charges in the early Universe and their terrestrial relics [44,45]. Specifically, the sextet model inheriting this problem (with a related discussion deferred to Sec. V) favors the noncontroversial $Y(f_L) = 1/3$ anomaly solution with a new outlook and new BSM implications.

C. EW content from $Y(f_L) = 1/3$ with baryons of integer electric charge Q

Motivated by problems of the anomaly-free selection with half-integer electric charges, we are now led to consider the electroweak content with sextet model baryons carrying integer electric charges which requires nonzero hypercharge for left-handed fermions with sextet color, in close analogy with the Standard Model pattern of fractional electric charges carried by three colors of quarks and integer charges carried by baryons in QCD. Hypercharge assignment $Y = 1/3$ is set for the left-handed sextet fermion doublets with consistent choices required for right-handed singlets,

$$Y(u_L^{ab}) = 1/3, \qquad Y(d_L^{ab}) = 1/3,$$
$$Y(u_R^{ab}) = 4/3, \qquad Y(d_R^{ab}) = -2/3. \tag{4}$$

Equation (4) leads to QCD-like electric charge assignments for sextet fermions with $Q(u) = 2/3$ and $Q(d) = -1/3$. The $Y = 1/3$ hypercharge assignment for left-handed doublets implies integer electric charges for composite baryons, built from three fermions of sextet color. The fermion content of the baryon doublet is given by (uud)isospin$=+1/2$, $Q=+1$ and (udd)isospin$=-1/2$, $Q = 0$, in contrast to the hypercharge selection $Y = 0$ leading to baryons of half-integer electric charges.

With six left-handed doublets there is no Witten anomaly, but $Y = 1/3$ for the left-handed doublets, which is necessary to get integer electric charges for composite baryons, leads to gauge anomalies,

$$\mathrm{tr}(Y) = 6\left\{\frac{1}{3} \times 2 + \frac{4}{3} - \frac{2}{3}\right\} = 8, \tag{5}$$

$$\begin{aligned}\mathrm{tr}(Y^3) &\propto \mathrm{tr}(Q^2 T_3 - Q T_3^2)\\ &= 6\left\{\left(\frac{2}{3}\right)^2 \times \frac{1}{2} - \left(\frac{1}{3}\right)^2 \times \frac{1}{2} - \frac{2}{3} \times \frac{1}{4} + \frac{1}{3} \times \frac{1}{4}\right\}\\ &= \frac{1}{2}.\end{aligned} \tag{6}$$

The anomalies in Eqs. (5) and (6) have to be compensated with new physics from some unknown scale. Once a commitment is made to the $Y = 1/3$ choice with integer electric charges for sextet model baryons, infrared effects from anomaly-compensating new physics cannot be ignored and will affect electroweak precision tests and other predictions on the electroweak scale. As we will argue in Sec. V, the noncontroversial second type of hypercharge choice, $Y(f_L) = 1/3$, leads to relic stable baryons from the early Universe. These baryons are neutral without direct conflict from limits on galactic and terrestrial charged relics but they require new physics to compensate the anomalies in Eqs. (5) and (6).

D. Example for $Y(f_L) = 1/3$ anomaly cancellation with new left-handed lepton doublets

The $Y(f_L) = 1/3$ choice for fermions in the strongly coupled sextet gauge sector is illustrated by the anomaly cancellation mechanism of new left-handed fermion doublets. The absence of a global SU(2) anomaly in the strongly coupled gauge sector requires the addition of a pair of left-handed fermion doublets. They are introduced as singlets under SU(3) QCD color and SU(3) bsm-color. Gauge anomaly constraints require the hypercharge assignment $Y \neq 0$ for the left-handed fermion doublets to compensate the anomalies from sextet fermions as counted in Eqs. (5) and (6). Consistent hypercharge assignments for right-handed singlets completes the solution for anomaly cancellation. For simplicity, we will use the notation of lepton families with a new family label $\alpha = 1, 2$,

$$\begin{pmatrix} N_L^{(\alpha)} \\ E_L^{(\alpha)} \end{pmatrix}, \qquad (N_R^{(\alpha)}, E_R^{(\alpha)}), \qquad \alpha = 1, 2. \tag{7}$$

It should be noted that the addition of the two lepton families is different from adding complete generations of quarks and leptons in the Standard Model. Here lepton families are added only to the sextet fermion doublet of the strongly coupled gauge sector.

As described before, the hypercharge assignment $Y = 1/3$ is set for the left-handed sextet fermion doublet with $Y = 4/3$ for the right-handed singlet u_R and $Y = -2/3$ for d_R, leading to QCD-like charge assignments for sextet fermions with $Q(u) = 2/3$ and $Q(d) = -1/3$. The added pair of lepton doublets, without introducing global anomalies, allows QCD-like charge assignment for the sextet fermion doublet by canceling the gauge anomalies when hypercharge $Y = -1$ is set for the new left-handed leptons $N_L^{(\alpha)}$, $E_L^{(\alpha)}$. The right-handed singlets $N_R^{(\alpha)}$ are assigned $Y = 0$, and $Y = -2$ is set for the right-handed singlets $E_R^{(\alpha)}$. It is easy to check that both gauge anomaly constraints of Eq. (2) are satisfied. The new leptons $E^{(\alpha)}$ carry electric charge $Q = -1$ and electric charge $Q = 0$ is set for the massive neutrinos $N^{(\alpha)}$.

The Lagrangian of the two lepton flavors includes gauge-invariant mass terms for charged leptons and massive neutrinos [41,46]. The most general mass matrix with mixing describes Dirac masses for the charged leptons $E_-^{(\alpha)}$ with two additional terms representing Majorana masses for $N_L^{(\alpha)}$ and $N_R^{(\alpha)}$. After diagonalization of the 2×2 mass matrix, each lepton family will have two neutrino mass eigenstates M_1 and M_2 in addition to the charged lepton mass M_E and a tunable mixing angle Θ from the Lagrangian mass parameters [46]. The two families can be allowed to mix which leads to more options in the full mass spectrum.

A more comprehensive analysis of the extended lepton sector in the sextet BSM model is beyond the scope of this work and will be reported in a separate publication [47]. Here the lepton sector serves to illustrate the most plausible framework for a BSM extension of the sextet model, compatible with anomaly conditions and integer electric charges for baryons. The lepton sector with its spectrum and quantum numbers also provides useful general guidance for expected new effects at the electroweak scale from a broad range of mass parameters, or equivalently the low-energy effective action with a Wess-Zumino term and other residual anomaly effects if we seek to integrate out the leptons asymptotically in the heavy-mass limit. Clearly, the BSM outlook of the sextet model remains quite flexible and interesting. Here we only briefly summarize our main findings so far:

(i) There exists a range of charged lepton masses and heavy neutrino masses which are not in conflict with electroweak precision tests. In addition, in that range the charged leptons can decay and the lowest stable particle is a Majorana neutrino.

(ii) Stable and heavy Majorana neutrinos are interesting dark matter candidates in the mass range where their relic densities are not in conflict with direct dark matter experiments, like XENON100 [48] and LUX2013 [49]. Generally heavy neutrinos could mix with the active ones and this mixing is constrained by the invisible width of the Z boson as discussed in an earlier reference [41].

(iii) The Wess-Zumino action and its effects at the electroweak scale can be identified from the footprints of heavy leptons and neutrinos of the model at very high energy scales. Weak isospin violation will constrain the mass splittings to be small on the 100 GeV scale between the upper and lower components of the new doublets so that the footprints do not violate electroweak precision constraints in the T-parameter. This will make the mass spectrum of the new leptons very different in comparison to the Standard Model.

(iv) Restrictions on heavy fermions from vacuum instability of the effective potential with the composite Higgs remain a difficult and unresolved problem.

Leaving further analysis of the lepton sector for future reporting, we will turn in Secs. VI and VII to the baryon spectrum from nonperturbative simulations since it is not affected by the intriguing properties of the lepton sector. In Sec. V we will explain first in some detail why fractional electric charge assignment for baryons is problematic and might be excluded by what we understand from the early history of the Universe. The well-motivated other solution $Y(f_L) = 1/3$ to the anomaly constraints with two lepton flavors leads to stable relic baryons from the early Universe. These baryons are neutral and contribute to the missing dark matter content of the Universe. Their relic abundance and direct detection limits in dark matter experiments require quantitative analysis decoupled from future developments in the lepton sector.

V. SEXTET MODEL BARYONS AND THE EARLY UNIVERSE

There is an exactly conserved U(1) symmetry which, when combined with exact electric charge conservation, implies a lightest stable baryon state in the spectrum which will be the primary focus of the forthcoming discussion. There are several questions to consider:

(i) the electric charge of the lightest and stable sextet model baryon,

(ii) their galactic and terrestrial relic abundance, and

(iii) limits from direct detection in dark matter experiments.

We will discuss these questions with two different choices of hypercharges for the left-handed doublets of sextet fermions.

A. Sextet model baryons from $Y(f_L) = 0$ with half unit of electric charge

The choice $Y(f_L) = 0$ for left-handed bsm-flavor doublets of fermions in the sextet bsm-color representation leads to composite baryon states in the 3 TeV mass range with spin one-half and electric charge $Q = \pm 1/2$. The two lightest baryon states form a degenerate pair and transform as an isospin doublet (uud, udd) of bsm-flavor carrying half-integer electric charges of opposite sign. Since the lightest sextet baryon carries half-integer charge it remains stable after its formation in the early Universe below the electroweak transition temperature. Additional speculations on some charge-conservation-violating mechanism to make fractionally charged baryons unstable are outside the scope of the model and our discussion. The lowest stable baryon state of the sextet model with spin one-half and electric charge one-half under this anomaly-free scenario belongs to the class of fractionally charged massive particles which have been discussed in several aspects before [45,50].

Arguments were presented against fractionally charged leptons with detailed estimates on their relic terrestrial density from the early Universe, strongly violating observational limits [45,51]. Estimates of the relic terrestrial density of sextet model baryons with half-integer charge proceed along similar lines with some uncertainties from nonperturbative strong gauge dynamics binding the fermions into baryons. We will briefly review the charge-symmetric evolution of these baryons in the early Universe. Some assumptions we will make on annihilation cross sections from strong gauge dynamics are not likely to affect the qualitative conclusions on this problematic anomaly-free choice. Charge-asymmetric evolution would make the scenario even less likely.

In the symmetric thermal evolution under discussion, baryons and antibaryons will remain in thermal equilibrium with decreasing charge-symmetric densities well below the electroweak transition temperature. At some freeze-out temperature T_* the annihilation rate of baryons and antibaryons cannot keep up any longer with the expansion rate of the Universe. The total number of baryons and antibaryons remains approximately constant after freeze-out for $T \ll T_*$ and the relic abundance level is set from the solution of the Boltzmann equation [52]. The freeze-out temperature and the relic sextet baryon number density n_{B_6} relative to the ordinary baryon number density n_B will depend on the sextet baryon mass M_{B_6} and the thermally averaged annihilation rate $\langle \sigma v \rangle_{\rm ann}$ of sextet model baryons and antibaryons,

$$\frac{n_{B_6}}{n_B} \approx \frac{10^{-25}}{M_{B_6} \langle \sigma v \rangle_{\rm ann}}. \tag{8}$$

The velocity-dependent annihilation cross section σ is known for ordinary nucleons and will be estimated for sextet model baryons whose mass is approximately $M_{B_6} = 3$ TeV, as reported in Sec. VII from our nonperturbative lattice simulations. For a qualitative estimate, the thermally averaged annihilation rate of sextet model baryons is scaled down from the nucleon-antinucleon annihilation cross sections of QCD according to the generally accepted approximation,

$$\langle v\sigma \rangle_{\rm ann} \approx \langle v\sigma \rangle_{\rm ann}^{\rm nuc} \times M_{\rm nuc}^2 / M_{B_6}^2. \tag{9}$$

Based on the value of M_{B_6} we determined and using the rough estimate of the annihilation cross section, the freeze-out temperature and the ratio $x = M_{B_6}/T_*$ can be approximately determined, with the ratio logarithmically dependent on the thermally averaged annihilation cross section. This leads to an approximate relic sextet baryon number density as a fraction of nucleon number density,

$$\frac{n_{B_6}}{n_B} \approx 3 \times 10^{-7}, \tag{10}$$

far exceeding terrestrial limits of stable fractional charges. The factor 3 in Eq. (10) is associated with the particular assumption about the annihilation rate and the details of the freeze-out calculation. Only the order of magnitude estimate is relevant for the argument in what will follow. As pointed out in Ref. [45], fractionally charged baryons and antibaryons will get rethermalized at ≈300 K on Earth and continue annihilating over the 4.5 Gyr life of Earth. This, at first thought, perhaps would bring their terrestrial density below acceptable observational limits, many orders of magnitude less than the 3×10^{-7} freeze-out relic abundance. Unfortunately the terrestrial annihilation mechanism is blocked by some overlooked new mechanism in the early Universe where negatively charged sextet model baryons will capture alpha particles with calculable estimates of the capture rate [45] and significant relic density of compound particles $(\alpha^{++}B^-)$. This will block the terrestrial annihilation for a large fraction of the positively charged free baryons with negatively charged bound baryons which are screened by alpha particles of the compound, hiding behind a repulsive Gamow barrier. As noted in Ref. [45] the terrestrial annihilation is unlikely to continue at the necessary rate for fractionally charged leptons because negative charges will bind to alpha particles with calculable rate estimates with the Gamow barrier blocking annihilation. Similarly, relic positively charged baryons cannot continue terrestrial annihilation at the needed rate, blocked by the repulsive Coulomb barrier between unbound positively charged relic baryons and the compound $(\alpha^{++}B^-)$ objects, so that the terrestrial bounds most likely remain in violation. Other difficulties were also noted, like the symmetric distribution of opposite-sign fractional charges in the interstellar medium of the Galaxy which is also problematic given detection limits [45].

Unless unforeseen considerations bring new arguments for the viability of stable sextet model baryons with half-unit electric charges, their existence from the early Universe makes the $Y(f_L) = 0$ anomaly-free choice unlikely. We will discuss next the more realistic solution to the anomaly-free construction.

B. Sextet model baryons from $Y(f_L) = 1/3$ with integer units of electric charge

The lightest baryons in the strongly coupled sextet gauge sector are expected to form isospin-flavor doublets (uud, udd), similar to the pattern in QCD. As we noted earlier, baryons in the sextet model should carry integer multiples of electric charges if $Y(f_L) \neq 0$ to avoid problems with the relics of the early Universe. This leads to the simplest choice $Y(f_L) = 1/3$ with gauge anomalies to be compensated. A new pair of left-handed lepton doublets emerged from this choice in Sec. IV as the simplest manifestation of the anomalies and the electroweak extension of the strongly coupled sextet gauge sector.

Neutron-like udd sextet model baryons (n_6) will carry no electric charge and proton-like uud sextet model baryons (p_6) have one unit of positive electric charge from the choice $Y(f_L) = 1/3$. The two baryon masses are split by electromagnetic interactions. The ordering of the two baryon masses in the chiral limit of massless sextet fermions will require nonperturbative *ab initio* lattice calculations of the electromagnetic mass shifts to confirm intuitive expectations that the neutron-like n_6 baryon has lower mass than the proton-like p_6 baryon. In QCD this pattern was confirmed by recent lattice calculations [53]. We expect the same ordering in the sextet model so that the proton-like p_6 baryon will decay very fast, $p_6 \to n_6 + \ldots$, with a lifetime $\tau \ll 1$ second. It is unlikely for rapidly decaying p_6 baryons to leave any relic footprints from dark nucleosynthesis before they decay.

With BSM baryon number conservation the neutral n_6 baryon is stable and observational limits on its direct detection from experiments like XENON100 [48] and LUX2013 [49] have to be estimated. In charge-symmetric thermal evolution sextet model baryons are produced with relic number density ratio $n_{B_6}/n_B \approx 3 \times 10^{-7}$ [Eq. (10)]. For 3 TeV sextet model baryon masses we can estimate the detectable dark matter ratio of respective mass densities ρ_{B_6} and ρ_B as $\rho_{B_6}/\rho_B \approx 10^{-4}$, about 5×10^4 times less than the full amount of unaccounted dark mass, $\rho_{\rm dark} \approx 5 \cdot \rho_B$. We will use this mass density estimate to guide observational limits on relic sextet model baryons emerging from charge-symmetric thermal evolution where tests of dark baryon detection come from elastic collisions with nuclei in dark matter detectors. The neutral and stable n_6 baryon can interact in several different ways with heavy nuclei in direct detection experiments including (a) magnetic dipole interaction, (b) Z-boson exchange, (c) Higgs boson exchange, and (d) electric polarizability. The goals of our calculations are similar to earlier important work setting dark matter limits in some BSM models [54].

A brief review of our estimates of these interactions will lead us to important observations from what follows. It turns out that cross sections from (a) and (b) can be parametrized and well estimated without lattice simulations.

(a) The magnetic moment $\mu_6 = g \cdot e/2M_{n_6}$ of the neutral n_6 sextet baryon can be calculated from first principles on the lattice but the only unknown quantity, g, is not needed in our estimate. The magnetic moment μ_6 of n_6 controls the coherent scattering cross section from magnetic moments of protons and neutrons in heavy nuclei with slow elastic recoil in direct detection experiments. If the mass density of relic n_6 baryons would be large enough to match all the missing dark matter, a limit on the magnetic dipole g-factor from LUX2013 would be set to $g^2 \leq (M_{n_6}/5.1\ \text{TeV})^3$, otherwise they would have been detected [49]. With $M_{n_6} = 3$ TeV the limiting value $g = 0.45$ can be far exceeded due to the much lower relic mass density of n_6 in the symmetric thermal evolution of the Universe. The precise value of g would require a straightforward lattice calculation which is less important with low magnetic dipole cross sections well below observational limits in comparison with cross section estimates from Z exchange.

(b) The sextet model n_6 baryon carries isospin $1/2$ and hypercharge $Y = 1$ which is the source of the Z-boson field. Coherent Z exchange between the n_6 baryon and heavy nuclei in detectors of direct searches leads to a larger cross section than magnetic dipole scattering and detectability has to be carefully calculated and compared with XENON100 data. Is the sextet model with its relic baryon density still safe against the most sensitive detection limits in charge-symmetric thermal evolution? This turns out to be the most sensitive test of the model. For dark matter candidates at $M_{n_6} = 3$ TeV XENON100 sets a cross section bound of approximately 10^{-43} cm^2 per nucleon under the assumption of missing dark matter density of $\rho_{\rm dark} \approx 5\ \rho_B$. Now our estimate of the cross section from Z exchange per nucleon for n_6 is approximately 10^{-39} cm^2, seemingly 4 orders of magnitude above detection threshold. This is not the case however, because the n_6 relic charge-symmetric mass density is about 5×10^4 times less than the full amount of unaccounted dark mass. This leads to the interesting observation that the n_6 sextet model baryons might be detectable in the next generation of direct searches. There are several caveats to this including uncertainties in estimating the relic density from hypothesized annihilation cross sections and the potential complications of asymmetric thermal evolution.

(c) The Higgs exchange effect is expected to be small but it will require lattice calculations to determine the coupling of the composite light scalar to the n_6 sextet model baryon. We will return to this problem in a future report.

(d) Similarly, the estimate of the scattering cross section from the electric polarizability of n_6 baryons requires lattice calculations which are left for future work. This effect is expected to be much smaller than cross sections for (a) and (b).

Based on these estimates we conclude that the sextet BSM model is consistent with observational limits and will contribute only a small fraction to the missing dark matter content. As a last and important step of our analysis, we turn now to the nonperturbative lattice determination of the baryon masses.

VI. CONSTRUCTION OF THE SEXTET NUCLEON OPERATOR

We next discuss how to build a sextet baryon operator that can be used in lattice simulations to isolate the baryon state and measure its mass. In the first two parts of this section we discuss the color, spin and flavor structure of the sextet baryon state in the continuum. We will see that a symmetric color contraction is needed in order to construct a color-singlet three-fermion state when fermions are in the sextet representation of SU(3). This is opposite to the behavior in QCD, where the baryon color wave function is antisymmetric. Consequently, the construction of the sextet baryon operator is nontrivial using staggered lattice fermions, which we describe in the third part of this section.

A. Color structure

Three SU(3) sextet fermions can give rise to a color singlet. The tensor product $6 \otimes 6 \otimes 6$ can be decomposed into irreducible representations of SU(3) as [55]

$$\begin{aligned} &6\otimes 6\otimes 6 \\ &\quad = 1\oplus 2\times 8\oplus 10\oplus \overline{10}\oplus 3\times 27\oplus 28\oplus 2\times 35, \end{aligned} \tag{11}$$

where irreps are denoted by their dimensions and $\overline{10}$ is the complex conjugate of 10. The color-singlet state corresponds to the unique singlet above. Fermions in the 6-representation ψ_{ab} are symmetric in the two indices and transform as

$$\psi_{aa'} \to U_{ab}U_{a'b'}\psi_{bb'} \tag{12}$$

and the color-singlet combination is given by

$$\epsilon_{abc}\epsilon_{a'b'c'}\psi_{aa'}\psi_{bb'}\psi_{cc'}. \tag{13}$$

(We earlier used superscripts for clarity.) Let us introduce the index $A = 1, \ldots, 6$ for the six components of the symmetric ψ_{ab}, i.e. we switch notation to $\psi_{ab} = \Psi_A$. Then the above color-singlet operator may be written as

$$\epsilon_{abc}\epsilon_{a'b'c'}\psi_{aa'}\psi_{bb'}\psi_{cc'} = T_{ABC}\Psi_A\Psi_B\Psi_C, \tag{14}$$

with a completely symmetric 3-index tensor T_{ABC}. The contrast with QCD where the baryon color contraction is antisymmetric is here explicit.

B. Spin-flavor structure

As we have seen the color contraction is symmetric for the sextet representation and hence the overall antisymmetry of the baryon wave function with respect to the interchange of any two fermions must come from the spin-flavor structure. Our operator construction is fully relativistic; we look here at the nonrelativistic limit for illustration, omitting color indices. We label the two flavors u and d as in QCD and the nonrelativistic spin will be either $\uparrow$ or $\downarrow$. We start with $|\uparrow u, \uparrow d, \downarrow u\rangle$ and build the desired state by requiring it to be antisymmetric under all possible interchanges, leading to

$$\begin{aligned} |\uparrow\psi\rangle &= |\uparrow u,\uparrow d,\downarrow u\rangle + |\downarrow u,\uparrow u,\uparrow d\rangle + |\uparrow d,\downarrow u,\uparrow u\rangle \\ &\quad - |\downarrow u,\uparrow d,\uparrow u\rangle - |\uparrow d,\uparrow u,\downarrow u\rangle - |\uparrow u,\downarrow u,\uparrow d\rangle, \end{aligned} \tag{15}$$

which is similar to the wave function of triton [56].

C. From continuum Dirac to lattice staggered basis

We next convert from continuum to staggered lattice fermion operators. The lattice operators that create the state (15) belong to a suitable multiplet of SU(4) taste symmetry. Our staggered fermion operator construction follows Refs. [57–60]. We first convert from continuum operators to lattice operators in the Dirac basis, then we switch to lattice staggered fields. For simplicity we want to have operators as local as possible, thus in the Dirac basis, our sextet baryon operator takes the form

$$N^{\alpha i}(2y) = T_{ABC}u_A^{\alpha i}(2y)\cdot[u_B^{\beta j}(2y)(C\gamma_5)_{\beta\gamma}(C^*\gamma_5^*)_{jk}d_C^{\gamma k}(2y)] \tag{16}$$

where greek letters and lower case latin letters denote spin and taste indices respectively. C is the charge-conjugation matrix satisfying

$$\begin{aligned} &C\gamma_\mu C^{-1} = -\gamma_\mu^T, \\ &-C = C^T = C^\dagger = C^{-1}. \end{aligned} \tag{17}$$

The coordinate y labels elementary staggered hypercubes. Staggered fields are defined as

$$u^{\alpha i}(2y) = \frac{1}{8}\sum_\eta \Gamma_\eta^{\alpha i}\chi_u(2y+\eta),$$

where $\Gamma(\eta)$ is an element of the Euclidean Clifford algebra labeled by the four-vector η whose elements are defined

TABLE I. The set of staggered lattice baryon operators we used to determine the baryon mass.

Label	Operators (set a)	Operators (set b)
IV_{xy}	$\chi_u(1,1,0,0)\chi_u(0,0,0,0)\chi_d(0,0,0,0)$	$\chi_u(0,0,0,0)\chi_u(1,1,0,0)\chi_d(1,1,0,0)$
IV_{yz}	$\chi_u(0,1,1,0)\chi_u(0,0,0,0)\chi_d(0,0,0,0)$	$\chi_u(0,0,0,0)\chi_u(0,1,1,0)\chi_d(0,1,1,0)$
IV_{zx}	$\chi_u(1,0,1,0)\chi_u(0,0,0,0)\chi_d(0,0,0,0)$	$\chi_u(0,0,0,0)\chi_u(1,0,1,0)\chi_d(1,0,1,0)$
VIII	$\chi_u(1,1,1,0)\chi_u(0,0,0,0)\chi_d(0,0,0,0)$	$\chi_u(0,0,0,0)\chi_u(1,1,1,0)\chi_d(1,1,1,0)$

mod 2 as usual. More precisely $\Gamma(\eta) = \gamma_1^{\eta_1}\gamma_2^{\eta_2}\gamma_3^{\eta_3}\gamma_4^{\eta_4}$ where $\eta \equiv (\eta_1, \eta_2, \eta_3, \eta_4)$. Written in terms of the staggered fields,

$$N^{\alpha i}(2y) = -T_{ABC}\frac{1}{8^3}\sum_{\eta'}\Gamma^{\alpha i}_{\eta'}\chi^A_u(2y+\eta') \cdot \sum_{\eta} S(\eta)\chi^B_u(2y+\eta)\chi^C_d(2y+\eta), \quad (18)$$

where $S(\eta)$ is a sign factor. To obtain a single time-slice operator an extra term has to be either added to or subtracted from the diquark operator to cancel the spread over two time slices of the unit hypercube. This is similar to what is done to construct the single time-slice meson operators in QCD. This extra term corresponds to the parity partner of the nucleon. The single time-slice nucleon operator reads

$$N^{\alpha i}(2y) = -T_{ABC}\frac{1}{8^3}\sum_{\vec{\eta}'}\Gamma^{\alpha i}_{\vec{\eta}'}\chi^A_u(2y+\vec{\eta}') \cdot \sum_{\vec{\eta}} S(\vec{\eta})\chi^B_u(2y+\vec{\eta})\chi^C_d(2y+\vec{\eta}). \quad (19)$$

This operator is a sum of $8 \times 8 = 64$ terms over the elementary cube in a given time slice. The local terms vanish individually after the symmetric color contraction. The nonvanishing terms are those where a diquark resides on a corner of the cube at a fixed time slice and the third fermion resides on any of the other corners. The nucleon operator is thus the sum of 56 such terms with appropriate sign factors. In order to find the mass of the lowest-lying state any one of these 56 terms can in principle be used. We list in Table I the operators that we have implemented.

VII. LATTICE SIMULATIONS

We use the same lattice action as in our other studies of the sextet model, namely the tree-level Symanzik-improved gauge action and the staggered fermion matrix with two stout steps of exponential smearing of the gauge link variables [61,62]. We implement the Rational Hybrid Monte Carlo algorithm with the rooting procedure in all simulations to study the model with two fermion flavors. To accelerate the molecular dynamics time evolution we use multiple time scales [63] and the Omelyan integrator [64]. The results we show here are at one lattice spacing corresponding to the bare gauge coupling $\beta = 6/g^2 = 3.2$, which is defined as the overall prefactor of the Symanzik lattice action. We have continuing studies of the spectrum on finer lattice spacings to allow us to quantify lattice artifacts and determine the continuum limit of the spectrum, which we will report on in future publications. We examine the time histories of the correlators, the fermion condensate, the topological charge, and the gauge field energy on the gradient flow to estimate autocorrelation times. For the estimate of the statistical errors of hadron masses we used correlated fitting of the effective masses with a double jackknife procedure applied to the covariance matrices [65].

A. Nucleon operator comparison

We investigate the quality of the signal for the operators listed in Table I on ensembles of approximately 1000 trajectories, with each measurement separated by 50 trajectories during the molecular dynamics evolution, on a lattice volume $V = 32^3 \times 64$ and at fermion mass $m = 0.007$. For each operator the nucleon mass, M_N, is determined by correlated fitting of the effective mass with a double jackknife procedure applied to the covariance matrices from time separation $t_{\min}$ to $t_{\max}$. Figure 1 compares the corresponding fits for various values of

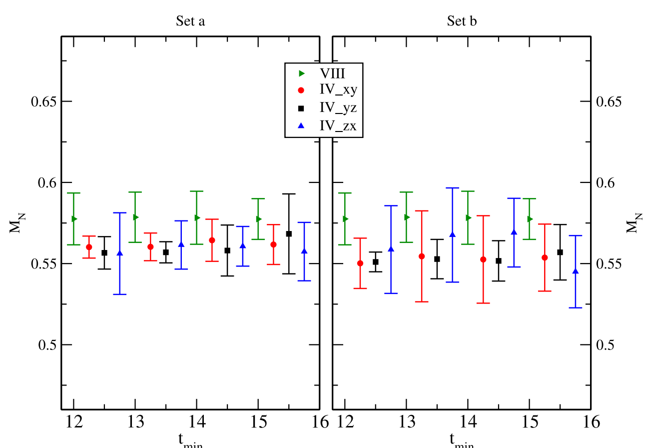

FIG. 1. Comparison of M_N from different operators varying $t_{\min}$ with fixed $t_{\max} = 20$. The calculation is performed on lattices with $\beta = 3.20$, $V = 32^3 \times 64$ and $m = 0.007$ over approximately 1000 trajectories. The $t_{\min}$ values for the operators IV_{xy}, IV_{yz} and IV_{zx} are shifted by 0.25, 0.5 and 0.75 respectively for clarity. Sets a and b correspond to the location of the diquark operator as in Table I.

t_{min} at $t_{max} = 20$. It is observed that, for all operators, the fits are not very sensitive to the choice of the fit range. Moreover, all operators give consistent results for the nucleon mass within errors. The noise-to-signal ratio is around ∼5% for all operators, none of which is significantly less noisy than the others. Therefore the quality of the resulting spectroscopy is independent of the choice of operator, and in the following analysis we use the operator IV_{xy} from set a.

B. First results

In this section we present our first results for the nucleon spectroscopy. Simulations and data analysis of the sextet model are continuously ongoing; what we show here is a snapshot of the results at one lattice spacing. The nucleon correlator of operator IV_{xy} in set a is measured on ensembles with lattice volume $V = 32^3 \times 64$ and fermion masses ranging from $m = 0.003$ to $m = 0.008$, each with 200 to 300 configurations, with each configuration separated by five Monte Carlo trajectories. In Fig. 2 we show the chiral extrapolation of M_N, as well as extrapolations of the Goldstone boson, and the a_1 and ρ mesons, denoted by M_π, M_{a_1} and M_ρ respectively [7,10]. For all except the Goldstone boson state we assume linear dependence on the fermion mass towards the chiral limit. We see that the baryon remains significantly split from the meson sector of the spectrum in the chiral limit.

To convert to physical units, one can use the scale set by the chiral limit of the Goldstone boson decay constant F_π, denoted by F. The left plot of Fig. 3 shows the chiral extrapolation of F_π, measured on lattices of size $48^3 \times 96$ at gauge coupling $\beta = 3.20$ for $m = 0.003$ and $32^3 \times 64$ for the heavier fermion masses. A detailed description of the analysis is given in Ref. [10]. The result we use here is $F = 0.0253(4)$ with a systematic uncertainty larger than the small statistical error would indicate. The value in the chiral limit is likely to get considerable corrections, due to additional lattice simulations pushing to even lighter fermion masses, as well as modifications of the analysis taking into account lattice artifacts in chiral perturbation theory. The BSM implementation of the theory identifies the chiral limit of the Goldstone boson decay constant with the value of the scalar vacuum expectation value in the Standard Model, namely $F = v_R = 246$ GeV. This requirement follows from the generation of the electroweak gauge boson mass $m_W = (gF)/2$ due to Goldstone boson contributions to the gauge boson vacuum polarization. With this conversion, the baryon in the chiral limit is at approximately 3 TeV with systematic uncertainties in the 10–20 percent range. The nucleon, vector and axial-vector meson masses are shown in physical units in the right panel of Fig. 3. Our initial studies indicate a composite scalar in the sextet model which is light, with M_H/F in the range 1 to 3 in the chiral limit [10]. The large uncertainty is due to the difficulty of extracting a state with vacuum quantum numbers, which requires disconnected fermion diagrams to be calculated, which is a notoriously challenging computational problem. In Sec. II we commented on top-quark-generated self-energy loop effects reducing the mass of the light scalar as being investigated outside the scope of this paper [13]. The lightness of the scalar, far separated from the remainder of the spectrum, means in the BSM context that experimental studies would need to explore the few TeV range for this model to be critically tested [66].

A vector resonance, which in our context would be the ρ state, is a particularly pronounced signature for searches at the LHC. As indicated in Fig. 3, the simulation results predict that the vector state is at roughly 2 TeV in the sextet model. This prediction will be refined in our ongoing study, including the dependence on the lattice cutoff. Searches for

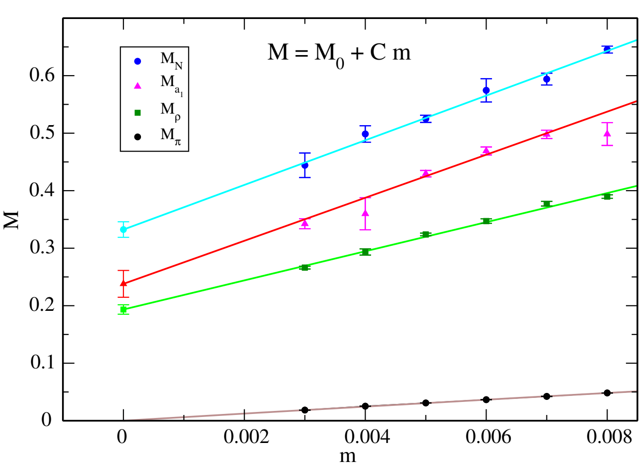

FIG. 2. Chiral extrapolation of M_N (blue) in comparison with M_π, M_{a_1} and M_ρ. The calculation is performed on lattices with $\beta = 3.20$, $V = 48^3 \times 96$ for $m = 0.003$ (except for M_N on $V = 32^3 \times 64$) and $V = 32^3 \times 64$ for the heavier fermion masses, using 200 to 300 configurations at each mass.

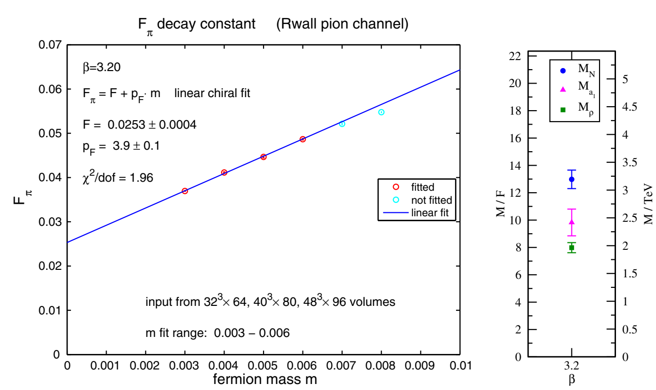

FIG. 3. Chiral extrapolation of F_π. Left: The simulations are performed on lattices with $\beta = 3.20$, $V = 48^3 \times 96$ for fermion mass $m = 0.003$ and $V = 32^3 \times 64$ for the heavier fermion masses, using 200 to 300 configurations; see Ref. [10] for more details. Right: Hadron spectroscopy at $\beta = 3.20$ converted to physical units.

resonances in the 2–3 TeV range in Run 2 of the LHC will provide an important test on the viability of the sextet BSM theory. As a curiosity, it is noted that the ATLAS and CMS collaborations have reported an observed excess in WW, WZ and ZZ diboson pairs at around 2 TeV [67,68] with large statistical uncertainty and without knowing the ultimate fate of the reported excess. In Sec. II we already commented briefly on the recently found diphoton excess in ATLAS and CMS resonance searches [20,21].

VIII. SUMMARY AND OUTLOOK

We have developed the operator technology to extract baryon states using staggered lattice fermions in the sextet model. The first results are encouraging that the lightest baryon mass can be nailed down with good precision from the current generation of lattice simulations. The emerging picture is of a spectrum with the baryon significantly above the vector and axial-vector mesons in the chiral limit. The next step will be to extend the analysis, allowing more control over the chiral extrapolation and removing the distortion of the spectrum due to lattice artifacts and possible finite-volume contamination.

The study of the baryon is a natural extension of our ongoing work to explore the meson spectrum of the sextet model and shore up the case that the theory is near-conformal, with a massive spectrum in the chiral limit. Whether or not the model is ultimately viable will largely hinge on the fate of the composite scalar, which should be light for the minimal model to dynamically generate a Higgs impostor. The embedding of the sextet theory into the Standard Model brings other facets into play, such as constraints on the relic abundance of sextet model baryons in the early Universe, and experimental limits on stable particles with fractional charge [51]. The scenario of a neutral sextet baryon with additional lepton doublets for anomaly cancellation shows the possibility for rich BSM physics beyond simply a composite Higgs state, and will be brought into sharp relief if the model continues to warrant future study.

After completing this work, it was called to our attention by the authors of an earlier paper that the second solution to the anomaly constraints with two lepton doublets was also discussed in their work [69].

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