

Exploring the simplest purely baryonic decay processes

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Though not considered in general, purely baryonic decays could shed light on the puzzle of the baryon number asymmetry in the universe by means of a better understanding of the baryonic nature of our matter world. As such, they constitute a yet unexplored class of decay processes worth investigating. We propose to search for purely baryonic decay processes at the LHCb experiment. No such type of decay has ever been observed. In particular, we concentrate on the decay $\Lambda_b^0 \rightarrow p\bar{p}n$, which is the simplest purely baryonic decay mode, with solely spin-1/2 baryons involved. We predict its decay branching ratio to be $\mathcal{B}(\Lambda_b^0 \rightarrow p\bar{p}n) = (2.0_{-0.2}^{+0.3}) \times 10^{-6}$, which is sufficiently large to make the decay mode accessible to LHCb. Our study can be extended to other purely baryonic decays such as $\Lambda_b^0 \rightarrow p\bar{p}\Lambda$, $\Lambda_b^0 \rightarrow \Lambda\bar{p}\Lambda$, and $\Lambda_b^0 \rightarrow \Lambda\bar{\Lambda}\Lambda$, as well as to similar decays of antitriplet b baryons such as $\Xi_b^{0,-}$.

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I. INTRODUCTION

It is well known that every (anti)baryon except the (anti) proton decays to a lighter (anti)baryon, such as the beta decay of the neutron, $n \rightarrow pe^-\bar{\nu}_e$, which is the simplest baryonic decay. However, up to now, no *purely* baryonic decay process, with only baryons involved, has yet been observed [1]. By virtue of the baryon number conservation, in order to have a purely baryonic decay, at least three baryons in the final state are needed, e.g., $\mathbf{B}_h \rightarrow \mathbf{B}_{l_1}\bar{\mathbf{B}}_{l_2}\mathbf{B}_{l_3}$, where h and l_i represent heavy and light spin-1/2 baryons, respectively. It is easy to show that the simplest, and lightest, possible purely baryonic decay process is $\Lambda_b^0 \rightarrow p\bar{p}n$ without breaking any known conservation law. Other examples of such decays are $\Lambda_b^0 \rightarrow p\bar{p}\Lambda$, $\Lambda_b^0 \rightarrow \Lambda\bar{p}\Lambda$, and $\Lambda_b^0 \rightarrow \Lambda\bar{\Lambda}\Lambda$ as well as the corresponding $\Xi_b^{0,-}$ decays. Since baryons are the main constituents of our matter world, their production and decay mechanisms should all be explored.

Being one of the three conditions for the baryogenesis to explain the puzzle of the matter and antimatter asymmetry in the universe, CP violation has been a primary topic of study at the B factories and at the LHCb experiment, among others. However, the unique physical CP phase in the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix [2] of the Standard Model (SM) is not sufficient to solve the mystery, leaving room for new physics in the formation of the matter world. On the other hand, for a direct connection to the baryonic contents of the universe, one expects to observe CP violation in purely baryonic processes. Theoretically and experimentally, though, the latter has, to our knowledge, never been studied before. Clearly, it is interesting to discuss CP violating rate asymmetries as well as time-reversal violating spin involved triple correlations due to the rich spin structures

in these purely baryonic decays, to test the SM and search for new physics manifestations.

In the SM, the decay $\Lambda_b^0 \rightarrow p\bar{p}n$, with the tree-level dominated contribution through the $V - A$ quark currents, can be factorized as a color-allowed process, which is insensitive to nonfactorizable effects, where the required matrix elements of the $\Lambda_b^0 \rightarrow p$ transition and the recoiled $\bar{p}n$ pair have been well studied. A reliable prediction of the branching ratio is hence expected, which should be as large as those in the tree-level B decays such as $\bar{B}^0 \rightarrow \pi^+\pi^-$. In addition, the threshold effect around the $\bar{p}n$ invariant mass spectrum, measured as a salient feature in three-body baryonic B decays, could also enhance the contribution.

It is interesting to note that in this simplest purely baryonic decay $\Lambda_b^0 \rightarrow p\bar{p}n$, a possible intermediate resonant state such as $D_s^-(\rightarrow \bar{p}n)p$ is rather suppressed, which is estimated to have $\mathcal{B}(\Lambda_b \rightarrow p(D_s^- \rightarrow n\bar{p})) \approx \mathcal{B}(\Lambda_b \rightarrow pD_s^-)\mathcal{B}(D_s^- \rightarrow n\bar{p}) \approx 2 \times 10^{-8}$ [3,4]. While the threshold effect, receiving the dominant contribution from the threshold of $m_{B\bar{B}} \approx m_{\mathbf{B}} + m_{\bar{B}}$, has been commonly observed in the baryon pair production [5,6], the meson resonances or the final state interaction due to the multi-particle exchange, which deviates the baryon pair production from the threshold, should be suppressed.

In this article, we concentrate on this simplest purely baryonic decay $\Lambda_b^0 \rightarrow p\bar{p}n$. We will give the theoretical estimation of its decay branching ratio, and stimulate a possible measurement by the LHCb Collaboration. Possible CP and T violating effects in purely baryonic decays are also discussed.

II. THEORETICAL PREDICTIONS

In terms of the effective Hamiltonian at the quark level for the charmless $b \rightarrow u\bar{u}d$ transition in the SM, the

amplitude for $\Lambda_b^0 \rightarrow p\bar{p}n$ in the factorization approach can be written as

$$\mathcal{A}(\Lambda_b^0 \rightarrow p\bar{p}n) \simeq \frac{G_F}{\sqrt{2}} V_{ub} V_{ud}^* a_1 \langle n\bar{p} | (\bar{d}u)_{V-A} | 0 \rangle \times \langle p | (\bar{u}b)_{V-A} | \Lambda_b^0 \rangle, \quad (1)$$

where G_F is the Fermi constant; V_{ij} are the CKM matrix elements; $(\bar{q}_1 q_2)_{V(A)}$ stands for $\bar{q}_1 \gamma_\mu (\gamma_5) q_2$; and $a_i = c_i^{\text{eff}} + c_{i\pm 1}^{\text{eff}}/N_c$ for $i = \text{odd}$ (even) with the effective Wilson coefficients c_i^{eff} defined in Ref. [7] and the color number N_c , which floats from 2 to ∞ to estimate the nonfactorizable effects in the generalized version of the factorization approach. The amplitude in Eq. (1) is dominated by the tree contribution from Fig. 1(a), while that from Fig. 1(c) is primarily the nonfactorizable effect with its size proportional to a_2 . Since the global fittings for a_2 indicate a universal value of $\mathcal{O}(0.2 - 0.3)$ [3,8,9], a_2 is within the uncertainty of a_1 . On the other hand, the penguin contributions in Figs. 1(b) and 1(d) have been neglected due to the suppressed values of $|(V_{tb} V_{td}^*)/(V_{ub} V_{ud}^*) (a_i/a_1)|^2 \leq 0.02$ ($i > 2$).

In Eq. (1), the matrix elements for the baryon pair production are well defined, given by

$$\begin{aligned} \langle n\bar{p} | \bar{d}\gamma_\mu u | 0 \rangle &= \bar{u}_n \left\{ [F_1 + F_2] \gamma_\mu + \frac{F_2}{m_n + m_{\bar{p}}} (p_{\bar{p}} - p_n)_\mu \right\} v_{\bar{p}}, \\ \langle n\bar{p} | \bar{d}\gamma_\mu \gamma_5 u | 0 \rangle &= \bar{u}_n \left\{ g_A \gamma_\mu + \frac{h_A}{m_n + m_{\bar{p}}} q_\mu \right\} \gamma_5 v_{\bar{p}}, \end{aligned} \quad (2)$$

where $q = p_n + p_{\bar{p}}$ is the momentum transfer, $F_{1,2}$, g_A , and h_A are the timelike baryonic form factors, and $u_n(v_{\bar{p}})$ is the neutron (antiproton) spinor. On the other hand, the matrix elements of the $\Lambda_b \rightarrow p$ baryon transition in Eq. (1) have the general forms

$$\begin{aligned} \langle p | \bar{u}\gamma_\mu b | \Lambda_b \rangle &= \bar{u}_p \left[f_1 \gamma_\mu + \frac{f_2}{m_{\Lambda_b}} i\sigma_{\mu\nu} q^\nu + \frac{f_3}{m_{\Lambda_b}} q_\mu \right] u_{\Lambda_b}, \\ \langle p | \bar{u}\gamma_\mu \gamma_5 b | \Lambda_b \rangle &= \bar{u}_p \left[g_1 \gamma_\mu + \frac{g_2}{m_{\Lambda_b}} i\sigma_{\mu\nu} q^\nu + \frac{g_3}{m_{\Lambda_b}} q_\mu \right] \gamma_5 u_{\Lambda_b}, \end{aligned} \quad (3)$$

where f_j (g_j) ($j = 1, 2, 3$) are the form factors, with $f_1 = g_1$ and $f_{2,3} = g_{2,3} = 0$ resulting from the SU(3) flavor and SU(2) spin symmetries [3], which agree with the results based on the heavy-quark and large-energy symmetries in Ref. [10].

For the numerical analysis, the theoretical inputs of the CKM matrix elements in the Wolfenstein parametrization are given by [1]

$$V_{ub} = A\lambda^3(\rho - i\eta), \quad V_{ud} = 1 - \lambda^2/2, \quad (4)$$

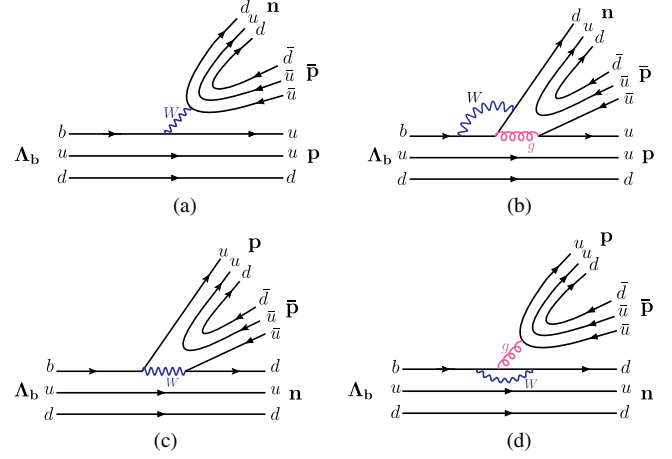


FIG. 1. (a), (c) Tree-level and (b), (d) penguin-level Feynman diagrams contributing to the $\Lambda_b^0 \rightarrow p\bar{p}n$ decay.

with $(\lambda, A, \rho, \eta) = (0.225, 0.814, 0.120 \pm 0.022, 0.362 \pm 0.013)$. We adopt $(c_1^{\text{eff}}, c_2^{\text{eff}}) = (1.168, -0.365)$ in Ref. [7], and obtain $a_1 = 1.05_{-0.06}^{+0.12}$. For the timelike baryonic form factors, it is adopted that $F_1(g_A) = C_{F_1(g_A)}/t^2 [\ln(t/\Lambda_0^2)]^{-\gamma}$, $h_A = C_{h_A}/t^2$, and $F_2 = F_1/(t \ln[t/\Lambda_0^2])$ with $\gamma = 2.148$, $\Lambda_0 = 0.3$ GeV, and $t \equiv q^2$ in perturbative QCD (pQCD) counting rules [11–13]. The form factors at the timelike region may possess the strong phase with the analytical continuation to the spacelike region [14], since it can be derived as an overall factor to all subprocesses in Fig. 1. This strong phase, in fact, has no effect and is neglected in our paper. The form factors with the asymptotic behaviors in pQCD counting rules have been justified to agree with the $e^+e^- \rightarrow p\bar{p}(n\bar{n})$ data at $t = (4 - 10)$ GeV² [5,6]. Besides, they have been used to explain the branching ratios, and the so-called threshold effect in the baryon pair invariant mass spectra, which presents the peak around the threshold of $t \simeq 4$ GeV² and gradually turns out to be flat around $t = 10 - 16$ GeV² [9,15,16], being observed as the common feature in the three-body baryonic B decays. For the $\Lambda_b \rightarrow p$ transition, the $f_1(g_1)$ is presented as the double-pole momentum dependences: $f_1(g_1) = C_{f_1(g_1)}/(1 - t/m_{\Lambda_b}^2)^2$ [17]. Since the form factors are associated with the studies of baryonic B decays and b -baryon decays, we use the numerical results in Refs. [3,10,13,16–18] to give $(C_{F_1}, C_{g_A}, C_{h_A}) = (196.1 \pm 37.6, 101.0 \pm 37.6, -4.5 \pm 2.2)$ GeV⁴, and $C_{f_1} = C_{g_1} = 0.136 \pm 0.009$.

With all theoretical inputs, we find

$$\mathcal{B}(\Lambda_b^0 \rightarrow p\bar{p}n) = (2.0_{-0.2}^{+0.3} \pm 0.1 \pm 0.1) \times 10^{-6}, \quad (5)$$

where the errors come from the form factors, the nonfactorizable effects, and the CKM matrix elements, respectively. By combining the uncertainties, we obtain the first prediction, $\mathcal{B}(\Lambda_b^0 \rightarrow p\bar{p}n) = (2.0_{-0.2}^{+0.3}) \times 10^{-6}$, on the branching ratio of this purely baryonic decay, which is

sizable and comparable to the branching ratios of other baryonic B decays observed at the B factories and LHCb.

Figure 2 displays the invariant mass spectra of $m_{n\bar{p}}$ and $m_{p\bar{p}}$. The neutron-antiproton invariant mass spectrum in $\Lambda_b^0 \rightarrow p\bar{p}n$ presents the threshold effect due to the form of $1/t^n$ for the $n\bar{p}$ -pair production, which is similar to the peak around the threshold area of $m_{\mathbf{B}\bar{\mathbf{B}}'} \approx m_{\mathbf{B}} + m_{\bar{\mathbf{B}}'}$ that enhances the branching ratios in the three-body baryonic $B \rightarrow \mathbf{B}\bar{\mathbf{B}}'M$ decays. On the other hand, the $m_{p\bar{p}}$ distribution is in accordance with the fact that the proton and antiproton are not pair produced. Note that the spectra in Fig. 2 are partly a consequence of ignoring the contributions from the diagrams in Figs. 1(c) and 1(d). Future measurements of dibaryon spectra should be able to test our assumptions, in particular the factorization approach.

Since purely baryonic decays are directly connected to the baryonic contents of the universe, it is worthwhile to have systematic investigations of their decay branching ratios and direct CP violating rate asymmetries as well as the possible T -odd triple correlations. Besides the simplest mode $\Lambda_b^0 \rightarrow p\bar{p}n$, example decays are $\Lambda_b^0 \rightarrow p\bar{p}\Lambda$, $\Lambda_b^0 \rightarrow \Lambda\bar{p}\Lambda$, and $\Lambda_b^0 \rightarrow \Lambda\bar{\Lambda}\Lambda$ decays, and decays of other antitriplet b baryons such as $\Xi_b^{0,-}$. The direct CP violating rate asymmetry can be defined by

$$\mathcal{A}_{CP} = \frac{\Gamma(\mathbf{B}_h \rightarrow \mathbf{B}_{l_1} \bar{\mathbf{B}}_{l_2} \mathbf{B}_{l_3}) - \Gamma(\bar{\mathbf{B}}_h \rightarrow \bar{\mathbf{B}}_{l_1} \mathbf{B}_{l_2} \bar{\mathbf{B}}_{l_3})}{\Gamma(\mathbf{B}_h \rightarrow \mathbf{B}_{l_1} \bar{\mathbf{B}}_{l_2} \mathbf{B}_{l_3}) + \Gamma(\bar{\mathbf{B}}_h \rightarrow \bar{\mathbf{B}}_{l_1} \mathbf{B}_{l_2} \bar{\mathbf{B}}_{l_3})}. \quad (6)$$

If both weak (γ) and strong (δ) phases are nonvanishing, one has that

$$\mathcal{A}_{CP} \propto \sin \gamma \sin \delta. \quad (7)$$

For T violation in purely baryonic decays, we can examine the triple product correlations of the T -odd form $\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3)$ where \vec{v}_i are spins or momenta of the baryons. Explicitly, for this class of decays

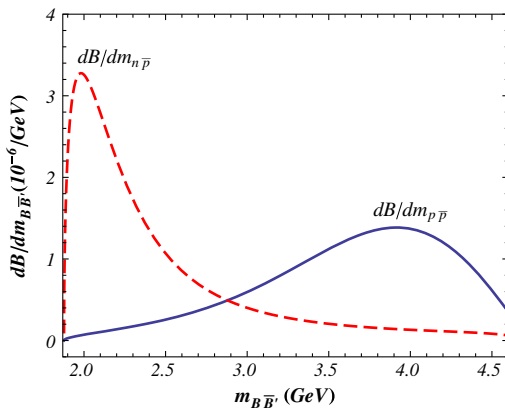


FIG. 2. The dibaryon invariant mass spectra for $\Lambda_b^0 \rightarrow p\bar{p}n$.

$\mathbf{B}_h \rightarrow \mathbf{B}_{l_1} \bar{\mathbf{B}}_{l_2} \mathbf{B}_{l_3}$, there are many T -odd correlations of $\vec{s}_{\mathbf{B}_j} \cdot (\vec{p}_{\mathbf{B}_k} \times \vec{p}_{\mathbf{B}_l})$ and $\vec{p}_{\mathbf{B}_j} \cdot (\vec{s}_{\mathbf{B}_k} \times \vec{s}_{\mathbf{B}_l})$, corresponding to one and two spins, respectively, due to the rich spin structures of the baryons. The asymmetry depending on these correlations is defined by

$$A_T \equiv \frac{\Gamma(\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3) > 0) - \Gamma(\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3) < 0)}{\Gamma(\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3) > 0) + \Gamma(\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3) < 0)}, \quad (8)$$

which results in

$$A_T \propto \sin(\gamma + \delta). \quad (9)$$

Note that A_T may not indicate the real CP violating effect in the gauge theory. To obtain the true effect, one can construct the asymmetry by using

$$\mathcal{A}_T \equiv \frac{1}{2}(A_T - \bar{A}_T), \quad (10)$$

where \bar{A}_T is measured in the CP -conjugate decay process, and consequently one finds that

$$\mathcal{A}_T \propto \sin \gamma \cos \delta, \quad (11)$$

which is in general nonzero as long as $\gamma \neq 0$, no matter whether the strong phase δ exists. We remark that, for the baryonic decays with mesons, such as $\Lambda_b \rightarrow pK^{(*)-}$, $\bar{B}^0 \rightarrow \Lambda\bar{p}\pi^+$, and $B^- \rightarrow p\bar{p}K^{(*)-}$, their branching ratios, and the CP and T violating asymmetries have been well studied [17,19,20]. In contrast, the purely baryonic decays are conceptually new species and have not been well explored yet. In the SM, the direct CP violating asymmetry in Eq. (6) and the T violating asymmetry in Eq. (10) are estimated to be both around -4% . As some new CP violating mechanism is anticipated, these asymmetries could be sensitive to it. For example, the asymmetries can be enhanced in the CP violating models with charged Higgs and gauge bosons.

III. EXPERIMENTAL CONSIDERATIONS

Experimentally speaking, the presence of a neutron in the final state gives rise to a signature resembling that of a semileptonic decay. The LHCb experiment has already published studies of such topologies with two charged tracks and a particle escaping detection, for example, $\Lambda_b^0 \rightarrow p\mu\bar{\nu}_\mu$ [21]. The analysis can be seen as a proof of concept for the study of $\Lambda_b^0 \rightarrow p\bar{p}n$.

The observation of the $\Lambda_b^0 \rightarrow p\bar{p}n$ signal is nevertheless rather challenging given a branching ratio significantly lower than that of a typical semileptonic decay. Decays with a $p\bar{p}$ pair in the final state and extra invisible or nonreconstructed particles are potentially dangerous

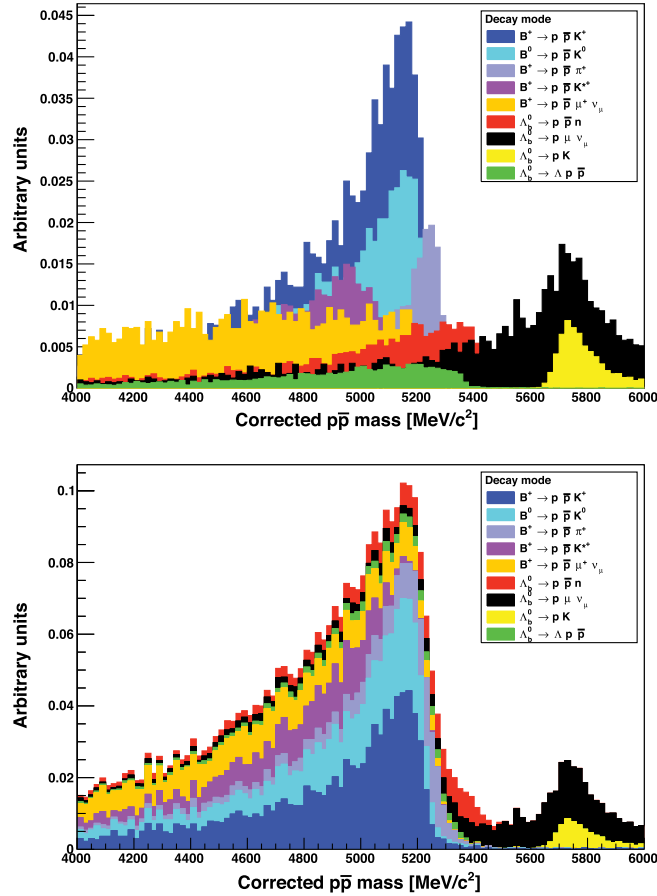


FIG. 3. Distributions of the $p\bar{p}$ pair corrected mass for the signal $\Lambda_b^0 \rightarrow p\bar{p}n$ and various sources of background resulting from a toy simulation study. The top part presents all distributions normalized to the unit area, whereas the bottom part stacks all contributions so as to give a more realistic picture of the kind of spectrum at hand. Refer to the text for further details.

sources of background, with branching ratios in the same range— 10^{-6} —as the signal.

The Λ_b^0 candidate can be reconstructed using the so-called *corrected mass* [22] defined by

$$m_{\text{corr}} = \sqrt{m_{p\bar{p}}^2 + p_\perp^2} + p_\perp,$$

where $m_{p\bar{p}}$ is the invariant mass of the $p\bar{p}$ pair and p_\perp its momentum transverse to the Λ_b^0 direction of flight. Figure 3 shows the distributions of the corrected $p\bar{p}$ mass for the signal and a few typical background decay modes resulting from a toy simulation study. Care has been taken to smear

the momentum resolution of tracks according to the average resolution published by the LHCb experiment. Also the relative contributions have been scaled taking into account the experimental branching ratios (reasonable assumptions on the branching ratio were made when the decay mode has not yet been observed) and typical misidentification rates in LHCb [23].

The bottom figure gives in particular the sum over all contributions. The signal appears as a shoulder around the region 5300–5500 MeV/c². It is evident that isolation requirements similar to those implemented, e.g., in Ref. [21], need to be exploited in order to control the decays to $p\bar{p}X$ final states, where X represents one or several charged and/or neutral particles. An interesting alternative is the identification of the signal neutron with the calorimeter of the LHCb experiment. The authors are aware that no publication from LHCb has ever studied neutrons, which makes the route challenging. These preliminary studies do indicate, though, that the observation of $\Lambda_b^0 \rightarrow p\bar{p}n$ at LHCb is promising.

For CP and/or T violation, although the current sensitivity of LHCb is unable to reach the level predicted in the SM [24], it is still worthwhile to explore the CP/T violating asymmetries of fully reconstructed baryonic decays— $\Lambda_b^0 \rightarrow p\bar{p}\Lambda$ is a prominent example—as they could be large in the new CP violating models.

IV. CONCLUSIONS

Since CP violation in fully baryonic decay processes is directly related to the matter and antimatter asymmetry of the universe, we have studied the simplest case of $\Lambda_b^0 \rightarrow p\bar{p}n$ to investigate its accessibility at the LHCb experiment. With the predicted $\mathcal{B}(\Lambda_b^0 \rightarrow p\bar{p}n) = (2.0_{-0.2}^{+0.3}) \times 10^{-6}$, this decay can be the new frontier to test the SM and search for new physics. One can, and should, study other purely baryonic decay modes: $\Lambda_b^0 \rightarrow p\bar{p}\Lambda$, $\Lambda_b^0 \rightarrow \Lambda\bar{p}\Lambda$, and $\Lambda_b^0 \rightarrow \Lambda\bar{\Lambda}\Lambda$, and other similar decays of antitriplet b baryon such as $\Xi_b^{0,-}$.

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