Investigations on the charmless decay modes of $Z_c(3900)$ and $Z_c(4025)$

Qi Wu,[*] Gang Li,[†] Fenglan Shao,[‡] and Ruiqin Wang[§]

Department of Physics, Qufu Normal University, Qufu 273165, People's Republic of China

(Received 16 May 2016; published 15 July 2016)

To date, the nature of $Z_c(3900)/Z_c(4025)$ is not clear. In this work, we propose to study the internal structure of $Z_c(3900)/Z_c(4025)$ by exploring the charmless decays. Compared to charmed decays, charmless decay channels may play a unique role since they request the annihilation of the heavy constituents. We calculate the decay width for $Z_c(3900)/Z_c(4025) \to VP$, where P and V denote the pseudoscalar and light vector meson, respectively. Here, the $Z_c(3900)$ and $Z_c(4025)$ are the molecular states of $D^*\bar{D}$ and $D^*\bar{D}^*$. The decay process occurs through intermediate DD^*/D^*D^* meson loops. Based on an effective Lagrangian, we obtain sizable predictions for the branching ratios, which are accessible in experiments. The experimental measurements in the future will not only shed light on the decay mechanism but will also lead to a deeper understanding of the mysterious structure of the $Z_c(3900)/Z_c(4025)$.

DOI: 10.1103/PhysRevD.94.014015

I. INTRODUCTION

In 2013, the charged charmoniumlike structure $Z_c^\pm(3900)$ was first observed by the BESIII Collaboration in the $J/\psi\pi^\pm$ invariant mass spectrum produced in the process $e^+e^- \to J/\psi\pi^+\pi^-$ at 4.26 GeV [1]. Simultaneously, the Belle Collaboration also observed this structure in the same process [2]. The existence of the $Z_c^\pm(3900)$ was later confirmed by the CLEO Collaboration based on their data at the energy of 4.17 GeV [3]. A similar structure $Z_c(3885)$ was observed in the $\pi^\mp$ recoil mass spectrum of $e^+e^- \to (\bar{D}D^*)^\pm\pi^\mp$ at 4.26 GeV and 4.23 GeV [4,5]. The BESIII Collaboration observed the $Z_c(4025)$ near the $D^*\bar{D}^*$ threshold in the $\pi^\mp$ recoil mass spectrum in the process $e^+e^- \to (D^*\bar{D}^*)^\pm\pi^\mp$ [6]. Different from the other observed heavy quarkoniumlike states, such as $X(3872)$, $Y(4260)$, etc., $Z_c(3900)$ and $Z_c(4025)$ are electric charged states and thus cannot be conventional charmonium states. The minimal constituent quark content should be four quarks, so these two states may be ideal candidates for exotic hadrons beyond the conventional $c\bar{c}$ mesons.

Since the observation of $Z_c(3900)$ and $Z_c(4025)$, extensive theoretical discussions have been carried out to explore their structures [7–23]. The measured masses of the $Z_c(3900)$ and $Z_c(4025)$ lie slightly above the $D\bar{D}^*$ and $D^*\bar{D}^*$ mass thresholds, which indicates these two states may be good candidates of the hadronic molecule [7–10]. Besides the molecule explanation, these two states are also identified as tetraquark states [11–14] and kinematical threshold effect [15–17]. Recently, the product and decay behaviors of $Z_c(3900)$ and $Z_c(4025)$ have been extensively studied in the $D\bar{D}^*$ and $D^*\bar{D}^*$ hadronic molecule scenario in Refs. [18–24], and the theoretical calculations were consistent with the corresponding experimental measurements.

Until now, all these theoretical studies on their decays mainly focus on charmful decay channels. Apart from the charmful decay channels of $Z_c(3900)$ and $Z_c(4025)$, the charmless decay channels can also provides us a good platform to further study $Z_c(3900)$ and $Z_c(4025)$. In this work, we will investigate the charmless decays of $Z_c(3900)$ and $Z_c(4025)$ via intermediate charmed meson loops in an effective Lagrangian approach with quantum numbers $I^G(J^{PC}) = 1^+(1^{+-})$ for the $Z_c(3900)$ and $Z_c(4025)$. Here, we assume $Z_c(3900)$ and $Z_c(4025)$ to be the molecular states of $D^*\bar{D}$ and $D^*\bar{D}^*$. To investigate this process, we calculate the intermediate meson loop (IML) contributions. As is well known, IML transitions have been one of the important nonperturbative transition mechanisms that have been noticed for a long time [25–27]. Recently, this mechanism has been used to study the production and decays of ordinary and exotic states [28–49], and a global agreement with experimental data was obtained. Thus, this approach may be suitable for the process $Z_c(3900)/Z_c(4025) \to VP$, where P and V denote the pseudoscalar and light vector meson, respectively. This paper is organized as follows. After Sec. I, we present the calculation of the decay of $Z_c^\pm$ via the intermediate meson loop and give some relevant formulas in Sec. II. In Sec. III, the numerical results are presented. A summary will be given in Sec. IV.

II. CHARMLESS DECAYS OF $Z_c(3900)$ AND $Z_c(4025)$

In the hadronic molecule scenario, the charmoniumlike states $Z_c(3900)$ and $Z_c(4025)$ couple to their components dominantly via the S wave. The effective Lagrangian

[*] cjtqq@163.com
[†] gli@mail.qfnu.edu.cn
[‡] shaofl@mail.sdu.edu.cn
[§] wangruiqin0211@163.com

describing the couplings of $Z_c(3900)/Z_c(4025)$ and their components are in the form,

$$\mathcal{L}_{Z_c D^* D^*} = i g_{Z_c(4025)D^*D^*} \varepsilon^{\mu\nu\alpha\beta} \partial_\mu Z_{c\nu} D^*_\alpha \bar{D}^*_\beta + g_{Z_c(3900)D^*D} Z^\mu_c (D\bar{D}^*_\mu + D^*_\mu \bar{D}) + \text{H.c.}, \quad (1)$$

where $g_{Z_c(3900)D^*D}$ and $g_{Z_c(4025)D^*D^*}$ are the coupling constants and will be determined later.

Besides the Lagrangian presented in Eq. (1), the effective Lagrangian relevant to the light vector and pseudoscalar mesons is also needed. The effective Lagrangian can be constructed based on the heavy quark limit and chiral symmetry [50,51],

$$\begin{aligned}\mathcal{L} = & -i g_{D^*D\mathcal{P}} (D^i \partial^\mu \mathcal{P}_{ij} D^{*j\dagger}_\mu - D^{*i}_\mu \partial^\mu \mathcal{P}_{ij} D^{j\dagger}) + \frac{1}{2} g^*_D D^* \mathcal{P} \varepsilon_{\mu\nu\alpha\beta} D^{*\mu}_i \partial^\nu \mathcal{P}_{ij} \overleftrightarrow{\partial}^\alpha D^{*\beta\dagger}_j \\ & - i g_{DD\mathcal{V}} D^\dagger_i \overleftrightarrow{\partial}_\mu D^j (\mathcal{V}^\mu)^i_j - 2 f_{D^*D\mathcal{V}} \varepsilon_{\mu\nu\alpha\beta} (\partial^\mu \mathcal{V}^\nu)^i_j (D^\dagger_i \overleftrightarrow{\partial}^\alpha D^{*\beta j} - D^{*\beta\dagger}_i \overleftrightarrow{\partial}^\alpha D^j) \\ & + i g_{D^*D^*\mathcal{V}} D^{*\nu\dagger}_i \overleftrightarrow{\partial}_\mu \mu D^{*j}_\nu (\mathcal{V}^\mu)^i_j + 4 i f_{D^*D^*\mathcal{V}} D^{*\dagger}_i \mu (\partial^\mu \mathcal{V}^\nu - \partial^\nu \mathcal{V}^\mu)^i_j D^{*j}_\nu, \end{aligned} \quad (2)$$

where $\mathcal{D}^{(*)} = (D^{(*+)}, D^{(*)0}, D^{(*)+}_s)$. $\mathcal{P}$ and $\mathcal{V}_\mu$ are 3×3 matrices for the octet pseudoscalar and nonet vector meson, respectively. The coupling constants will be determined in the next section.

At the hadronic level, $Z_c(3900)$ and $Z_c(4025)$ are treated as bound states of $D\bar{D}^* + \text{c.c}$ and $D^*\bar{D}^*$, and the decays $Z_c(3900)/Z_c(4025) \to VP$ occur by exchanging charmed mesons as shown in Figs. 1 and 2. Based on the above Lagrangian, we can obtain the explicit transition amplitudes for $Z_c(3900)(p_1) \to [D^{(*)}(q_1)\bar{D}^{(*)}(q_2)]D^{(*)}(q_2) \to V(p_2)P(p_3)$ as shown in Fig. 1,

$$\begin{aligned} M_{DD^*[D]} = & \int \frac{d^4 q_2}{(2\pi)^4} [g_{Z_c(3900)D^*D} \varepsilon_{1\mu}][2 g_{DDV} q_{2\theta} \varepsilon^\theta_2][-g_{D^*DP} p_{3\rho}] \times \frac{i}{q_1^2 - m_1^2} \frac{i}{q_2^2 - m_2^2} \frac{i(-g^{\mu\rho} + q^\mu_3 q^\rho_3/m_3^2)}{q_3^2 - m_3^2} \mathcal{F}(m_2, q_2^2), \\ M_{DD^*[D^*]} = & \int \frac{d^4 q_2}{(2\pi)^4} [g_{Z_c(3900)D^*D} \varepsilon_{1\mu}][-4 f_{D^*DV} \varepsilon_{\theta\phi\kappa\lambda} p^\theta_2 \varepsilon^\phi_2 q^\kappa_2][-g_{D^*D^*P} \varepsilon_{\rho\tau\sigma\xi} p^\tau_3 q^\sigma_3] \\ & \times \frac{i}{q_1^2 - m_1^2} \frac{i(-g^{\lambda\xi} + q^\lambda_2 q^\xi_2/m_2^2)}{q_2^2 - m_2^2} \frac{i(-g^{\mu\rho} + q^\mu_3 q^\rho_3/m_3^2)}{q_3^2 - m_3^2} \mathcal{F}(m_2, q_2^2), \\ M_{D^*D[D^*]} = & \int \frac{d^4 q_2}{(2\pi)^4} [g_{Z_c(3900)D^*D} \varepsilon_{1\mu}][-\varepsilon^\kappa_f [g_{D^*D^*V} g_{\theta\phi} q_{1\kappa} + f_{D^*D^*V} (p_{2\phi} g_{\theta\kappa} - p_{2\theta} g_{\phi\kappa})]][g_{D^*DP} p^{3\rho}] \\ & \times \frac{i(-g^{\mu\theta} + q^\mu_1 q^\theta_1/m_1^2)}{q_1^2 - m_1^2} \frac{i(-g^{\phi\rho} + q^\phi_2 q^\rho_2/m_2^2)}{q_2^2 - m_2^2} \frac{i}{q_3^2 - m_3^2} \mathcal{F}(m_2, q_2^2). \end{aligned} \quad (3)$$

The explicit transition amplitudes for $Z_c(4025)(p_1) \to [D^*(q_1)\bar{D}^*(q_3)]D^{(*)}(q_2) \to V(p_2)P(p_3)$ in Fig. 2 are as follows:

$$\begin{aligned} M_{D^*D^*[D]} = & \int \frac{d^4 q_2}{(2\pi)^4} [g_{Z_c(4025)D^*D^*} \varepsilon_{\mu\nu\alpha\beta} p^\mu_1 \varepsilon^\nu_1][-2 f_{D*DV} \varepsilon_{\theta\phi\kappa\lambda} p^\theta_2 \varepsilon^\phi_2 q^\kappa_2][-g_{D*DP} p_{3\rho}] \\ & \times \frac{i(-g^{\beta\rho} + q^\beta_1 q^\rho_1/m_1^2)}{q_1^2 - m_1^2} \frac{i}{q_2^2 - m_2^2} \frac{i(-g^{\alpha\lambda} + q^\alpha_3 q^\lambda_3/m_3^2)}{q_3^2 - m_3^2} \mathcal{F}(m_2, q_2^2), \\ M_{D^*D^*[D^*]} = & \int \frac{d^4 q_2}{(2\pi)^4} [g_{Z_c(4025)D^*D^*} \varepsilon_{\mu\nu\alpha\beta} p^\mu_1 \varepsilon^\nu_1][-\varepsilon^\kappa_2 [g_{D^*D^*V} g_{\theta\phi} q_{1\kappa} + f_{D^*D^*V} (p_{2\phi} g_{\theta\kappa} - p_{2\theta} g_{\phi\kappa})]] \\ & [-g_{D^*D^*P} \varepsilon_{\rho\tau\sigma\xi} p^\tau_3 q^\sigma_3] \times \frac{i(-g^{\beta\rho} + q^\beta_1 q^\rho_1/m_1^2)}{q_1^2 - m_1^2} \frac{i(-g^{\alpha\theta} + q^\alpha_2 q^\theta_2/m_2^2)}{q_2^2 - m_2^2} \frac{i(-g^{\phi\xi} + q^\phi_3 q^\xi_3/m_3^2)}{q_3^2 - m_3^2} \mathcal{F}(m_2, q_2^2). \end{aligned} \quad (4)$$

In the triangle diagram of Figs. 1 and 2, the exchanged charmed mesons are off shell. To compensate the off-shell effect and to regularize the divergence [52–54], we usually introduce a form factor in the amplitudes. In the present work, we adopt a monopole form factor,

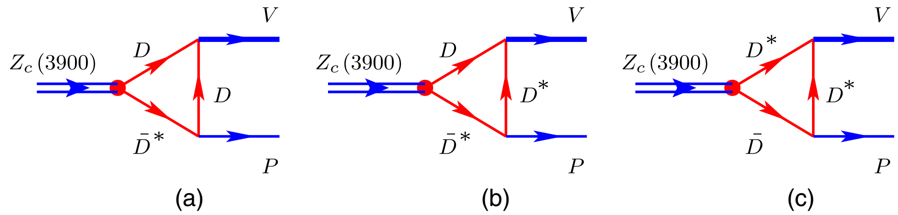

FIG. 1. Diagrams contributing to the decays $Z_c(3900) \to VP$ with $D\bar{D}^* +$ c.c. as the intermediate states.

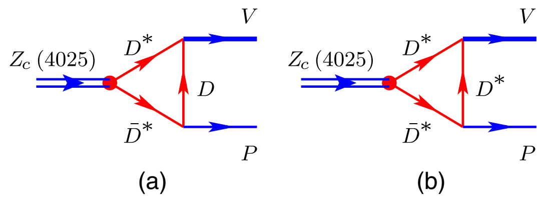

FIG. 2. Diagrams contributing to the decays $Z_c(4025) \to VP$ with $D^*\bar{D}^*$ as the intermediate states.

$$\mathcal{F}(m_2, q_2^2) \equiv \frac{\Lambda^2 - m_2^2}{\Lambda^2 - q_2^2}, \tag{5}$$

where q_2 and m_2 are the momentum and the mass of the exchanged charmed meson, respectively. The parameter $\Lambda \equiv m_2 + \alpha\Lambda_{\rm QCD}$, and the QCD energy scale $\Lambda_{\rm QCD} = 220$ MeV. The dimensionless parameter α is of order 1, which depends on the specific process. This parameter scheme has been applied extensively in other works [41,42,55,56].

III. NUMERICAL RESULTS

In this section, we will first determine the coupling constants in the above section and then present our numerical results. Here, we assume that the total widths of $Z_c(3900)$ and $Z_c(4025)$ are saturated by the decays $Z_c(3900) \to D^+\bar{D}^{*0} + \bar{D}^0 D^{*+}$ and $Z_c(4025) \to D^{*+}\bar{D}^{*0}$ and the relevant coupling constants in Eq. (1) can be determined as follows:

$$g_{Z_c(3900)DD^*} = 1.74^{+0.24}_{-0.25}\ \text{GeV}, \qquad g_{Z_c(4025)D^*D^*} = 0.35^{+0.05}_{-0.06}. \tag{6}$$

In the heavy quark and chiral limits, the charmed meson couplings to light vector and pseudoscalar mesons have the relationship [50]

$$g_{D^*D^*P} = \frac{g_{D^*DP}}{\sqrt{m_D m_{D^*}}} = \frac{2}{f_\pi} g, \qquad g_{DDV} = g_{D^*D^*V} = \frac{\beta g_V}{\sqrt{2}},$$
$$f_{D^*DV} = \frac{f_{D^*D^*V}}{m_{D^*}} = \frac{\lambda g_V}{\sqrt{2}}, \tag{7}$$

where $f_\pi = 132$ MeV is the pion decay constant and the parameter g_V respects the relation $g_V = m_\rho/f_\pi$ [50]. We take $\lambda = 0.56\ \text{GeV}^{-1}$ and $g = 0.59$ [57].

TABLE I. The predicted branching ratios of $Z_c^+(3900) \to VP$ decays with different α values.

Final states	$\alpha = 0.4$	$\alpha = 0.6$	$\alpha = 0.8$
$\omega\pi^+$	1.92×10^{-3}	4.31×10^{-3}	7.61×10^{-3}
$\rho^+\eta$	1.21×10^{-3}	2.71×10^{-3}	4.79×10^{-3}
$\rho^+\eta'$	5.55×10^{-4}	1.24×10^{-3}	2.19×10^{-3}

TABLE II. The predicted branching ratios of $Z_c^+(4025) \to VP$ decays with different α values.

Final states	$\alpha = 0.4$	$\alpha = 0.6$	$\alpha = 0.8$
$\omega\pi^+$	2.96×10^{-5}	6.58×10^{-5}	1.16×10^{-4}
$\rho^+\eta$	1.95×10^{-5}	4.23×10^{-5}	7.43×10^{-5}
$\rho^+\eta'$	8.50×10^{-6}	1.88×10^{-5}	3.29×10^{-5}

In Table I, we present the predicted branching ratios of $Z_c(3900)$ for each decay channel with cutoff parameters $\alpha = 0.4$, 0.6, and 0.8, respectively. As can be seen in Table I, the predicted branching ratios of $Z_c(3900) \to VP$ are less sensitive to the cutoff parameter α. The predicted branching ratios of each channel $Z_c(3900) \to VP$ are of order of 10^{-3} at $\alpha = 0.4 \sim 0.8$. In Table II, we list the branching ratios of $Z_c(4025)$ for each decay channel with the same cutoff parameters. The behavior is similar to that of $Z_c(3900)$ in Table I. The predicted branching ratios are of order 10^{-5} at $\alpha = 0.4 \sim 0.8$, which is about 2 orders of magnitude smaller than that of $Z_c(3900)$. As shown in Figs. 1 and 2, there are three kinds of diagrams for $Z_c(3900) \to VP$, while there are only two kinds of diagrams for $Z_c(4025) \to VP$. The uncertainties of our numerical results mainly come from the coupling constants and off-shell effects arising from the exchanged particles of the loops, and the cutoff parameter can also be different in decay channels.

In Figs. 3 and 4, we present the α dependence of the total branching ratios of $Z_c^+(3900) \to VP$ and $Z_c(4025) \to VP$ listed in Tables I and II, respectively. For the commonly accepted α range, the branching ratios obtained are not drastically sensitive to the cutoff parameter α to some extent. As shown in this figure, at the same α, the intermediate charmed meson loop effects turn out to be more important in $Z_c(3900) \to VP$ than in $Z_c(4025) \to VP$.

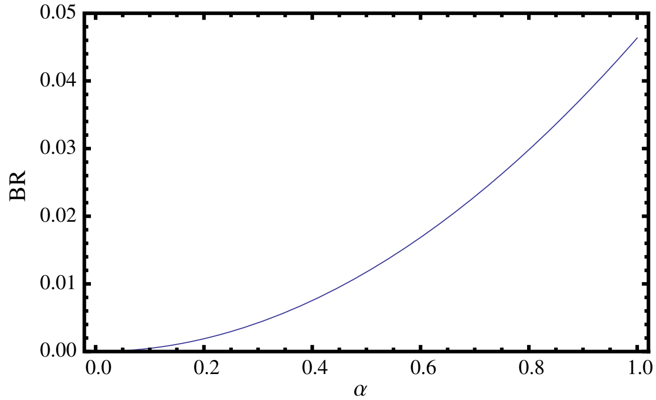

FIG. 3. The α dependence of the total branching ratios of $Z_c^+(3900) \to VP$.

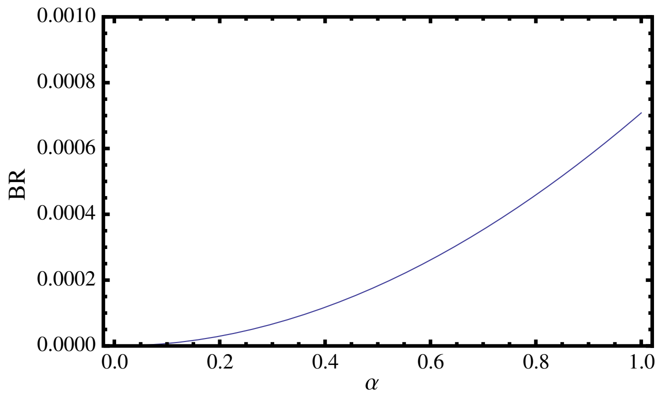

FIG. 4. The α dependence of the total branching ratios of $Z_c^+(4025) \to VP$.

IV. SUMMARY

It is likely that the $Z_c(3900)$ and $Z_c(4025)$ are the hadronic molecules with the $D\bar{D}^*$ and $D^*\bar{D}^*$ constituents, respectively. Their neutral partners of these two resonances have the quantum numbers $I^G(J^{PC}) = 1^+(1^{+-})$. In this work, we have proposed that charmless decays are valuable toward a better understanding of their nature.

We have investigated the charmless decays of $Z_c(3900)$ and $Z_c(4025)$ via intermediate charmed meson loops in an effective Lagrangian approach. Our results show that the obtained branching ratios are not drastically sensitive to the cutoff parameter α to some extent. The masses of $Z_c(3900)$ and $Z_c(4025)$ are close to the two particle $D\bar{D}^*$ and $D^*\bar{D}^*$ thresholds, which indicates a relatively small cutoff range. The predicted branching ratios of the $Z_c(3900) \to VP$ channels are typically at the order of 10^{-3}, while for $Z_c(4025) \to VP$, the predicted branching ratios are of order of 10^{-5} in the same cutoff range. In 2015, BESIII reported a search for $Z_c^\pm(3900) \to \omega\pi^+$ at 4.23 and 4.26 GeV, but no $Z_c^\pm(3900)$ signal has been found [58]. The experimental upper limit for branching fractions is still above our predictions, and thus there is no contradiction. With more data to be accumulated by BESIII and other experimental facilities in the future, the $Z_c(3900) \to VP$ may be observed on the experimental side, and then the mechanism for charmless decays can be explored in detail. We believe a deeper understanding of the nature of $Z_c(3900)$ and $Z_c(4025)$ will be gained accordingly.

ACKNOWLEDGMENTS

This work is supported in part by the National Natural Science Foundation of China (Grants No. 11275113, No. 11575100, and No. 11505104) and the Natural Science Foundation of Shandong Province (Grant No. ZR2015JL001). We would like to acknowledge Wei Wang for carefully reading the manuscript and useful suggestions.

[1] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. Lett. **110**, 252001 (2013).
[2] Z. Q. Liu *et al.* (Belle Collaboration), Phys. Rev. Lett. **110**, 252002 (2013).
[3] T. Xiao, S. Dobbs, A. Tomaradze, and K. K. Seth, Phys. Lett. B **727**, 366 (2013).
[4] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. Lett. **112**, 022001 (2014).
[5] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. D **92**, 092006 (2015).
[6] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. Lett. **112**, 132001 (2014).
[7] F. K. Guo, C. Hidalgo-Duque, J. Nieves, and M. P. Valderrama, Phys. Rev. D **88**, 054007 (2013).
[8] Q. Wang, C. Hanhart, and Q. Zhao, Phys. Rev. Lett. **111**, 132003 (2013).
[9] C. Y. Cui, Y. L. Liu, W. B. Chen, and M. Q. Huang, J. Phys. G **41**, 075003 (2014).
[10] J. R. Zhang, Phys. Rev. D **87**, 116004 (2013).
[11] E. Braaten, Phys. Rev. Lett. **111**, 162003 (2013).
[12] L. Maiani, V. Riquer, R. Faccini, F. Piccinini, A. Pilloni, and A. D. Polosa, Phys. Rev. D **87**, 111102 (2013).
[13] Z. G. Wang, Commun. Theor. Phys. **63**, 466 (2015).
[14] C. F. Qiao and L. Tang, Eur. Phys. J. C **74**, 2810 (2014).
[15] D. Y. Chen, X. Liu, and T. Matsuki, Phys. Rev. D **88**, 036008 (2013).
[16] E. S. Swanson, Phys. Rev. D **91**, 034009 (2015).
[17] A. P. Szczepaniak, Phys. Lett. B **747**, 410 (2015).

[18] E. Wilbring, H.-W. Hammer, and U.-G. Meißner, Phys. Lett. B **726**, 326 (2013).
[19] Y. Dong, A. Faessler, T. Gutsche, and V. E. Lyubovitskij, Phys. Rev. D **88**, 014030 (2013).
[20] Y. Dong, A. Faessler, T. Gutsche, and V. E. Lyubovitskij, Phys. Rev. D **89**, 034018 (2014).
[21] G. Li, X. H. Liu, and Z. Zhou, Phys. Rev. D **90**, 054006 (2014).
[22] T. Gutsche, M. Kesenheimer, and V. E. Lyubovitskij, Phys. Rev. D **90**, 094013 (2014).
[23] A. Esposito, A. L. Guerrieri, and A. Pilloni, Phys. Lett. B **746**, 194 (2015).
[24] G. Li, Eur. Phys. J. C **73**, 2621 (2013).
[25] H. J. Lipkin, Nucl. Phys. **B291**, 720 (1987).
[26] H. J. Lipkin and S. F. Tuan, Phys. Lett. B **206**, 349 (1988).
[27] P. Moxhay, Phys. Rev. D **39**, 3497 (1989).
[28] X.-H. Liu and G. Li, Phys. Rev. D **88**, 014013 (2013).
[29] F.-K. Guo, C. Hanhart, U.-G. Meißner, Q. Wang, and Q. Zhao, Phys. Lett. B **725**, 127 (2013).
[30] Q. Wang, C. Hanhart, and Q. Zhao, Phys. Lett. B **725**, 106 (2013).
[31] M. Cleven, Q. Wang, F.-K. Guo, C. Hanhart, U.-G. Meißner, and Q. Zhao, Phys. Rev. D **87**, 074006 (2013).
[32] D.-Y. Chen and X. Liu, Phys. Rev. D **84**, 094003 (2011).
[33] G. Li and X.-H. Liu, Phys. Rev. D **88**, 094008 (2013).
[34] M. B. Voloshin, Phys. Rev. D **87**, 074011 (2013).
[35] M. B. Voloshin, Phys. Rev. D **84**, 031502 (2011).
[36] A. E. Bondar, A. Garmash, A. I. Milstein, R. Mizuk, and M. B. Voloshin, Phys. Rev. D **84**, 054010 (2011).
[37] F.-K. Guo, C. Hanhart, G. Li, U.-G. Meißner, and Q. Zhao, Phys. Rev. D **82**, 034025 (2010).
[38] D.-Y. Chen, X. Liu, and T. Matsuki, Phys. Rev. D **84**, 074032 (2011).
[39] D.-Y. Chen, X. Liu, and T. Matsuki, Chin. Phys. C **38**, 053102 (2014).
[40] D.-Y. Chen, X. Liu, and T. Matsuki, Phys. Rev. D **88**, 014034 (2013).
[41] F. K. Guo, C. Hanhart, G. Li, U. G. Meissner, and Q. Zhao, Phys. Rev. D **83**, 034013 (2011).
[42] G. Li, F. l. Shao, C.-W. Zhao, and Q. Zhao, Phys. Rev. D **87**, 034020 (2013).
[43] F.-K. Guo, C. Hanhart, and U.-G. Meißner, Phys. Rev. Lett. **103**, 082003 (2009); F.-K. Guo, C. Hanhart, and U.-G. Meißner, Phys. Rev. Lett. **104**, 109901(E) (2010).
[44] G. Li and Z. Zhou, Phys. Rev. D **91**, 034020 (2015).
[45] G. Li, C. S. An, P. Y. Li, D. Liu, X. Zhang, and Z. Zhou, Chin. Phys. C **39**, 063102 (2015).
[46] G. Li and W. Wang, Phys. Lett. B **733**, 100 (2014).
[47] G. Li, X. H. Liu, and Q. Zhao, Eur. Phys. J. C **73**, 2576 (2013).
[48] G. Li, X. h. Liu, Q. Wang, and Q. Zhao, Phys. Rev. D **88**, 014010 (2013).
[49] G. Li and Q. Zhao, Phys. Rev. D **84**, 074005 (2011).
[50] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio, and G. Nardulli, Phys. Rep. **281**, 145 (1997).
[51] P. Colangelo, F. De Fazio, and T. N. Pham, Phys. Rev. D **69**, 054023 (2004).
[52] M. P. Locher, Y. Lu, and B. S. Zou, Z. Phys. A **347**, 281 (1994).
[53] X. Q. Li and B. S. Zou, Phys. Lett. B **399**, 297 (1997).
[54] X.-Q. Li, D. V. Bugg, and B.-S. Zou, Phys. Rev. D **55**, 1421 (1997).
[55] X. H. Liu and Q. Zhao, Phys. Rev. D **81**, 014017 (2010).
[56] X. H. Liu and Q. Zhao, J. Phys. G **38**, 035007 (2011).
[57] C. Isola, M. Ladisa, G. Nardulli, and P. Santorelli, Phys. Rev. D **68**, 114001 (2003).
[58] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. D **92**, 032009 (2015).