

**Landau levels of cold dense quark matter in a strong magnetic field**Xin-Jian Wen<sup>\*</sup> and Jun-Jun Liang<sup>†</sup>*Institute of Theoretical Physics, Shanxi University, Taiyuan 030006, China*

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The occupied Landau levels of strange quark matter are investigated in the framework of the SU(3) NJL model with a conventional coupling and a magnetic-field dependent coupling respectively. At lower density, the Landau levels are mainly dominated by  $u$  and  $d$  quarks. Threshold values of the chemical potential for the  $s$  quark onset are shown in the  $\mu$ - $B$  plane. The magnetic-field-dependent running coupling can broaden the region of three-flavor matter by decreasing the dynamical masses of  $s$  quarks. Before the onset of  $s$  quarks, the Landau level number of light quarks is directly dependent on the magnetic field strength  $B$  by a simple inverse proportional relation  $k_{i,\max} \approx B_i^0/B$  with  $B_d^0 = 5 \times 10^{19}$  G, which is approximately 2 times  $B_u^0$  of  $u$  quarks at a common chemical potential. When the magnetic field increases up to  $B_d^0$ , almost all three flavors are lying in the lowest Landau level.

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**I. INTRODUCTION**

The study of Quantum Chromodynamics (QCD) matter subject to a strong magnetic field has been a hot topic of intense investigation [1]. The strange quark matter (SQM) is regarded as a ground state composed of deconfined  $u$ ,  $d$ , and  $s$  quarks [2]. The new state is expected to be searched in extreme conditions of high density and/or high temperature. In addition to these environments, the SQM is argued to be subject to strong magnetic fields. The extreme strong magnetic field theoretically seems beyond the scope of the conventional condensed matter, and its origin remains not very clear until now. However, it has been recently proposed to be produced in noncentral collision experiments in the Relativistic Heavy Ion Collider and the Large Hadron Collider on the one hand [3,4], or to be naturally existing in the core of pulsars on the other hand. The large magnetic fields in nature are normally associated with astrophysical objects, where the density is much higher than the nuclear saturation. The typical strength could be the order of  $10^{12}$  G on the surface of pulsars [5]. Some magnetars can have even larger magnetic fields, reaching the surface value as large as  $10^{14}$ – $10^{15}$  G [6]. By comparing the magnetic and gravitational energies, the physical upper limit to the total neutron star is of the order of  $10^{18}$  G. For self-bound quark stars, the limit could go higher [7]. Maximum strengths of  $10^{18}$ – $10^{20}$  G in the interior of stars are proposed by an application of the viral theorem [3,5]. In the LHC/CERN energy, it is possible to produce a field as large as  $5 \times 10^{19}$  G [4].

The special properties of QCD matter are widely affected by strong magnetic fields in many branches, such as the (inverse) magnetic catalysis [8–12], the anisotropies

[13,14], the magnetic oscillations [15], the magnetization [14], the phase diagram with a critical end point [16,17], etc. The magnetic field larger than  $10^{19}$  G can obviously change the spherical symmetry [18]. All of these are essentially resulted due to the Landau level arrangement of charged particles in magnetic fields. In principle, not only quark masses will change in the medium, but also the coupling constant will run in the medium, such as the magnetic-field-dependent coupling and magnetic-temperature-dependent coupling [19]. It is well known that the dressed masses of three flavors are very different, which leads to a flavor-dependent fraction in quark matter and strangelets [20]. Similarly, the threshold condition and the quantum numbers of the Landau level of quarks are also flavor dependent. Generally,  $u$  and  $d$  quarks dominate the bulk matter at low densities. As the density increases, strange flavor begins to take part in its Landau levels. Therefore, the phase was argued to be divided into three regions such as the chirally broken phase (B-phase), the massive phase (C-phase), and the chirally restored phase (A-phase) in previous work [21,22], where the detailed locations of the two-flavor Landau levels were shown in the  $\mu$ - $B$  phase panel. However, at a proper density, the  $s$  quark cannot participate in the previous discussion because its dressed mass is larger than the chemical potential, thus the  $s$  quark could not occupy its lowest Landau level (LLL). In this work, we will show the critical density for the appearance of  $s$  quarks by considering the two kinds of coupling interactions, the conventional coupling constant and the magnetic-field-dependent running coupling respectively. The main aim of this work is to perform a detailed analysis of the Landau levels with and without the  $s$  quark depending on the magnetic field strength.

This work is organized as follows. In Sec. II, a brief review of the Nambu–Jona-Lasinio (NJL) model description of cold SQM in a strong magnetic field is provided.

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The magnetic-field-dependent running scalar coupling in the SU(3) version is introduced as well as the model parameters in the computation. In Sec. III, the numerical results and discussion are given at common chemical potential and under the  $\beta$  equilibrium respectively. A detailed analysis of the occupied Landau levels with respect to the magnetic field is given. The last section is a short summary.

## II. THERMODYNAMICS OF MAGNETIZED SQM IN THE SU(3) NJL MODEL

The SU(3) NJL Lagrangian density includes both a scalar-pseudoscalar interaction and the t'Hooft six-fermion interaction [23] and can be written as [22]

$$\mathcal{L}_{\text{NJL}} = \bar{\psi}(iD - m)\psi + G \sum_{a=0}^8 [(\bar{\psi}\lambda_a\psi)^2 + (\bar{\psi}\gamma_5\lambda_a\psi)^2] - K \{ \det[\bar{\psi}(1 + \gamma_5)\psi] + \det[\bar{\psi}(1 - \gamma_5)\psi] \}. \quad (1)$$

The field  $\psi = (u, d, s)^T$  represents a quark field with three flavors. Correspondingly,  $m = \text{diag}(m_u, m_d, m_s)$  is the current mass matrix with  $m_u = m_d \neq m_s$ .  $\lambda_0 = \sqrt{2/3}I$  where  $I$  is the unit matrix in the three-flavor space.  $\lambda_a$  with  $0 < a \leq 8$  denotes the Gell-Mann matrix. The gap equations for three-flavor are coupled and should be solved consistently,

$$M_i - m_i + 4G\phi_i - 2K\phi_j\phi_k = 0, \quad (2)$$

where  $(i, j, k)$  is the permutation of  $(u, d, s)$ . The contribution from the quark flavor  $i$  is

$$\phi_i = \phi_i^{\text{vac}} + \phi_i^{\text{mag}} + \phi_i^{\text{med}}. \quad (3)$$

The terms  $\phi_i^{\text{vac}}$ ,  $\phi_i^{\text{mag}}$ , and  $\phi_i^{\text{med}}$  representing the vacuum, magnetic field, and medium contribution to the quark condensation are respectively [24]

$$\phi_i^{\text{vac}} = -\frac{M_i N_c}{2\pi^2} \left[ \Lambda \sqrt{\Lambda^2 + M_i^2} - M_i^2 \ln \left( \frac{\Lambda + \sqrt{\Lambda^2 + M_i^2}}{M_i} \right) \right], \quad (4)$$

$$\phi_i^{\text{mag}} = -\frac{M_i |q_i| B N_c}{2\pi^2} \left\{ \ln[\Gamma(x_i)] - \frac{1}{2} \ln(2\pi) + x_i - \frac{1}{2} (2x_i - 1) \ln(x_i) \right\}, \quad (5)$$

$$\phi_i^{\text{med}} = \sum_{k_i=0}^{k_{i,\text{max}}} a_{k_i} \frac{M_i |q_i| B N_c}{2\pi^2} \ln \left[ \frac{\mu_i + \sqrt{\mu_i^2 - s_i^2}}{s_i} \right]. \quad (6)$$

The effective quantity  $s_i = \sqrt{M_i^2 + 2k_i |q_i| B}$  sensitively depends on the magnetic field. The dimensionless quantity is  $x_i = M_i^2 / (2|q_i| B)$ . The degeneracy label of the Landau energy level is  $a_{k_i} = 2 - \delta_{k_0}$ . The quark condensation is greatly strengthened by the factor  $|q_i B|$  together with the dimension reduction  $D - 2$  [25,26]. The Landau quantum number  $k_i$  and its maximum  $k_{i,\text{max}}$  are defined as

$$k_i \leq k_{i,\text{max}} = \text{Int} \left[ \frac{\mu_i^2 - M_i^2}{2|q_i| B} \right], \quad (7)$$

where ‘‘Int’’ means the number before the decimal point.

The total thermodynamic potential density in the mean field approximation reads

$$\Omega = \sum_{i=u,d,s} (\Omega_i^{\text{vac}} + \Omega_i^{\text{mag}} + \Omega_i^{\text{med}} + 2G\phi_i^2) - 4K\phi_u\phi_d\phi_s, \quad (8)$$

where the first term in the summation is the vacuum contribution to the thermodynamic potential, i.e.,

$$\Omega_i^{\text{vac}} = \frac{N_c}{8\pi^2} \left[ M_i^4 \ln \left( \frac{\Lambda + \epsilon_\Lambda}{M_i} \right) - \epsilon_\Lambda \Lambda (\Lambda^2 + \epsilon_\Lambda^2) \right], \quad (9)$$

where the quantity  $\epsilon_\Lambda$  is defined as  $\epsilon_\Lambda = \sqrt{\Lambda^2 + M_i^2}$ . The ultraviolet divergence in the vacuum part of the thermodynamic potential  $\Omega$  is removed by the momentum cutoff. In the literature, a form factor is introduced in the diverging zero energy as a smooth regularization procedure [27]. The magnetic field and medium contributions are respectively

$$\Omega_i^{\text{mag}} = -\frac{N_c(|q_i|B)^2}{2\pi^2} \left[ \zeta'(-1, x_i) - \frac{1}{2}(x_i^2 - x_i) \ln(x_i) + \frac{x_i^2}{4} \right], \quad (10)$$

$$\Omega_i^{\text{med}} = -\frac{|q_i|BN_c}{4\pi^2} \sum_{k=0}^{k_{\text{max}}} a_k \left\{ \mu_i \sqrt{\mu_i^2 - (M_i^2 + 2k_i|q_i|B)} - (M_i^2 + 2k_i|q_i|B) \ln \left[ \frac{\mu_i + \sqrt{\mu_i^2 - (M_i^2 + 2k_i|q_i|B)}}{\sqrt{M_i^2 + 2k_i|q_i|B}} \right] \right\}, \quad (11)$$

where  $\zeta(a, x) = \sum_{n=0}^{\infty} \frac{1}{(a+n)^x}$  is the Hurwitz zeta function. From the thermodynamic potential (8), one can easily obtain the quark density as

$$n_i(\mu, B) = \sum_{k=0}^{k_{\text{max}}} a_k \frac{|q_i|BN_c}{2\pi^2} \sqrt{\mu_i^2 - (M_i^2 + 2k_i|q_i|B)}. \quad (12)$$

The corresponding pressure from the flavor  $i$  contribution is

$$P_i(\mu_i, B) = -\Omega_i = -(\Omega_i^{\text{vac}} + \Omega_i^{\text{mag}} + \Omega_i^{\text{med}}). \quad (13)$$

Under strong magnetic fields, the system total pressure should be a sum of the matter pressure and the field pressure contribution [24,28]. So we have

$$P_i(\mu_i, B) = -\Omega_i + \frac{B^2}{2}, \quad (14)$$

where the magnetic field term  $B^2/2$  is due to the electromagnetic Maxwell contribution. It is well known to us that the energy density and pressure should vanish in vacuum. So the pressure and the thermodynamic potential should be normalized by requiring the zero pressure at the zero density as [24]

$$P_i^{\text{eff}}(\mu_i, B) = P_i(\mu_i, B) - P_i(0, B). \quad (15)$$

In the normalization result, the field term is automatically absent. According to the fundamental thermodynamic relation, the free energy density at zero temperature is

$$\varepsilon_i = -P_i^{\text{eff}} + \mu_i n_i. \quad (16)$$

The system pressure and energy density are written as

$$P = \sum_i P_i^{\text{eff}}, \quad \varepsilon = \sum_i \varepsilon_i, \quad (17)$$

where the summation goes over  $u, d$  quarks, and electrons.

In principle, the interaction coupling constant between quarks should be solved by the RG equation, or can be phenomenological expressed in an effective potential [29–31]. In the infrared region at low energy, the dynamical gluon mass represents the confinement feature of QCD [32]. Furthermore, in the presence of a strong magnetic field, the gluon mass becomes large together with a decreasing of the interaction constant, which leads to a

damping of the chiral condensation. For sufficiently strong magnetic fields  $eB \gg \Lambda_{\text{QCD}}^2$ , the coupling constant  $\alpha_s$  is proposed to be related to the magnetic field [11,25]. Motivated by the work of Miransky and Shovkovy [25], the similar ansatz of the magnetic-field-dependent coupling constant is introduced in the SU(3) NJL models [11]. The simple ansatz of the running coupling is probably suitable for the SU(3) NJL model if we include the  $s$  quarks [11],

$$G'(eB) = \frac{G}{\ln(e + |eB|/\Lambda_{\text{QCD}}^2)}, \quad (18)$$

where the parameter  $\Lambda_{\text{QCD}} = 300$  MeV. We can find the running coupling constant versus the field  $B$  approaches gradually to the constant value  $G'(B \rightarrow 0) \sim G$ . In the computation of the SU(3) NJL model, we adopt the parameters  $\Lambda = 602.3$  MeV,  $m_u = m_d = 5.5$  MeV,  $m_s = 140.7$  MeV,  $G = 1.835/\Lambda^2$ , and  $K = 12.36/\Lambda^5$  [33].

### III. NUMERICAL RESULTS AND DISCUSSION

In a strong magnetic field with a certain direction, quarks wrap around the magnetic field and the orbital motion will be ruled by the Landau energy level. Because dressed masses of quarks are different, the occupations of the discretized Landau level are flavor dependent. At a proper density, the dynamical masses of the  $u$  and  $d$  quarks are smaller than their chemical potential and have a real distribution in the Landau levels. But the dynamical mass of the  $s$  quark is much heavier than that of the  $u/d$  quark, and thus it cannot occur until the critical chemical potential above its dynamical mass is reached. In Fig. 1, the critical chemical potential is shown by the solid line for the conventional constant coupling  $G$ , and by the dashed line for the running coupling  $G'(eB)$  respectively. When the chemical potential is above the line, the  $s$  quarks have real distributions in the Landau levels. In the region below the line, there are only two-flavor quarks in their Landau levels, and the  $s$  quark is excluded because of its dressed mass larger than  $\mu$ . It can be found that at much higher field strengths, the regions of three quark matter in the  $\mu$ - $B$  plane become wider. Furthermore, the region is much wider with the running coupling  $G'(eB)$  than the constant coupling  $G$ . Therefore, it is concluded that the running coupling interaction could broaden the region of three-flavor quark matter. In other words, the strange quark can exist at lower density with the running coupling than the constant coupling case.

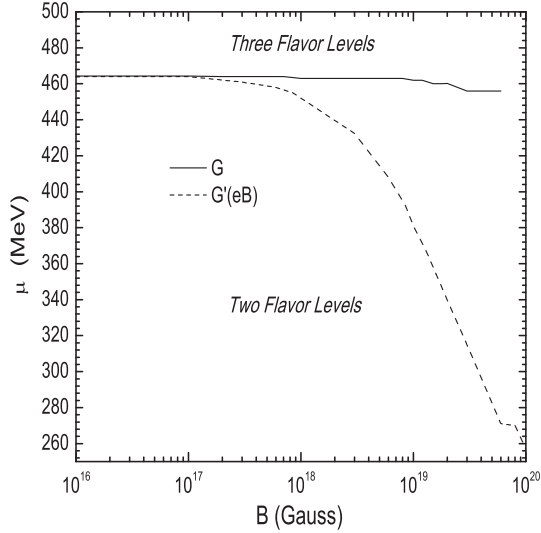


FIG. 1. The lines [solid and dashed lines for couplings  $G$  and  $G'(eB)$  respectively] for the critical chemical potential  $\mu$  separate the panel into two regions. In the region above the line, there are strange quark distributions in Landau levels. In the region below the line, only two-flavor light quarks fill in their Landau levels, where the  $s$  quark is excluded because of its dressed mass larger than  $\mu$ .

It is well known that the chemical potential dominates the energy spectrum of the particle without the magnetic field. Now we study how large the magnetic field effect is on the distribution of Landau levels at a fixed chemical potential. In Table I, we adopt  $\mu = 350$  MeV, and give the maximum number of the Landau levels of the  $u$ ,  $d$ , and  $s$  quarks for several magnetic fields. It can be found that the much higher magnetic field can accommodate quarks in lower levels. Furthermore, the onset of the  $s$  quark can be seen in its LLL until the field reaches  $2 \times 10^{19}$  G. While at lower magnetic field, the quantum number of filled Landau levels is larger and the quantization effects are washed out.

Before the onset of the  $s$  quark, the  $u$  and  $d$  quarks dominate the quark matter. Because of the identity  $q_u = -2q_d$ , the level number of the  $d$  quarks is exactly 2 times the level number of the  $u$  quark in order to meet the global charge neutrality. In Fig. 2, the Landau levels of the  $u$  and  $d$  quarks with the same chemical potential  $\mu$  are shown in the range of the magnetic field ( $10^{16}$ – $10^{19}$  G). We

TABLE I. The quantum number of Landau levels occupied by quarks for the fixed chemical potential  $\mu = 350$  MeV at several magnetic fields. The number “0” means the LLL.

Magnetic field (G)	$k_{u,\max}$	$k_{d,\max}$	$k_{s,\max}$
$1.0 \times 10^{17}$	115	230	No
$1.0 \times 10^{18}$	15	30	No
$1.0 \times 10^{19}$	1	3	No
$2.0 \times 10^{19}$	0	1	0

use the logarithm to label both the vertical axis and the horizontal axis. Then the Landau level number as functions of the magnetic field vary linearly, which is very near the red dotted line ( $\log k = 19.7 - \log B$ ) for  $d$  quarks. Consequently, we can easily find that the  $u/d$  quark Landau level number  $k_{i,\max}$  and the magnetic field  $B$  nearly satisfy a simple inverse proportional relation,

$$k_{i,\max} \approx B_i^0/B, \quad (19)$$

at the coupling  $G$ , where the scale is  $B_d^0 = 5 \times 10^{19}$  G for  $d$  quarks, which is 2 times  $B_u^0$  of  $u$  quarks. The exponential formula indicates the constraint on the strong magnetic field. In strong magnetic fields, charged fermions acquire infrared phase space proportional to  $|eB|$ . As the magnetic field strength increases, quarks are suppressed to the lower levels. At the same time the degeneracy factor of each energy level is enlarged to  $eB$ . When the magnetic field increases up to the order of  $B_0$  or so, almost all three flavors are concentrated on the LLL. The LLL would make the QCD matter more interesting, where the quarks are independent on the magnetic field with zero transverse energy [26]. After taking into account the running coupling  $G'(eB)$  (marked by dashed lines) in Fig. 2, the line deviates from the straight line at the field strength larger than  $10^{18}$  G. Therefore the running coupling can move the location of the LLL to a field strength slightly lower than  $B_0$ . It can be expected that at the same magnetic field, the SQM can be more easily realized at the running coupling than the conventional coupling.

The strong magnetic field drastically affects the structural properties and the thermodynamics. As far as we know, the chemical potential increases together with the

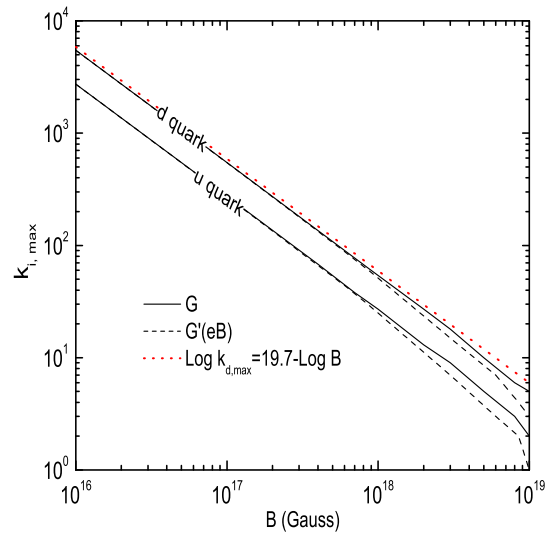


FIG. 2. The maximum Landau levels for  $u$  and  $d$  quarks change with the strong magnetic field. The solid line and dashed line are, respectively, for the constant coupling  $G$  and the running coupling  $G'(eB)$ .

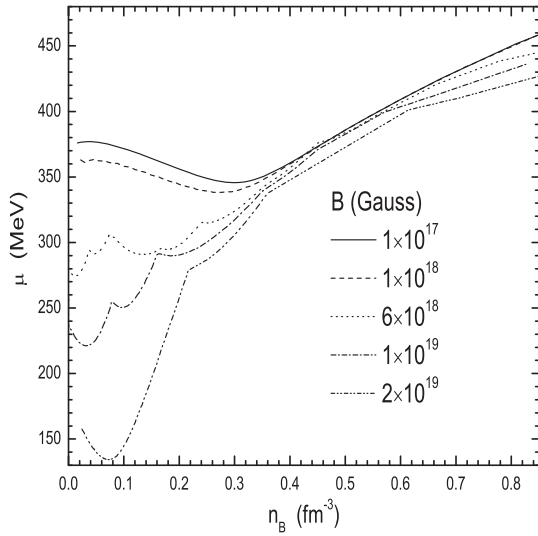


FIG. 3. The chemical potential versus the baryon number density at several values of the magnetic field.

number density. But for the SQM under a strong magnetic field, the variation relation between the chemical potential with the number density is not always monotonous. In Fig. 3, the quark chemical potential changes with the baryon number density at several values of the magnetic fields. The curves from top to down denote the increasing of the magnetic field. At a low density less than  $0.35 \text{ fm}^{-3}$ , the effect of the magnetic field strength is very important, where the degeneracy contribution from the magnetic field is much larger than the fermion momentum. So at the same density, the chemical potentials are very different for different magnetic fields. Furthermore, a small number of quarks under the influence of the strong fields can easily produce the oscillation behavior of the chemical potential. While in the high density region, the fermion momentum increases and the oscillation behavior cannot be easily found anymore.

In realistic situations for neutron stars, the chemical potentials for different flavors will be different and related by the physical constraints in a neutron star. So we can do the calculation by assuming the three-flavor quark matter is in  $\beta$  equilibrium. Now there are three dynamical masses and two independent chemical potentials, which can be determined by the three gap equations (2), the baryon number conservation, and the neutral charge condition,

$$2n_u - n_d - n_s - 3n_e = 0. \quad (20)$$

Under the  $\beta$  equilibrium condition  $\mu_d = \mu_s = \mu_u + \mu_e$  in Fig. 4, we can get the similar result as in Fig. 1, namely, the critical potential  $\mu_s$  for the onset of  $s$  quark is about 465 MeV with the coupling  $G$ . But at much higher magnetic field, the value of  $\mu_s$  has an apparent drop due to the contribution of electrons.

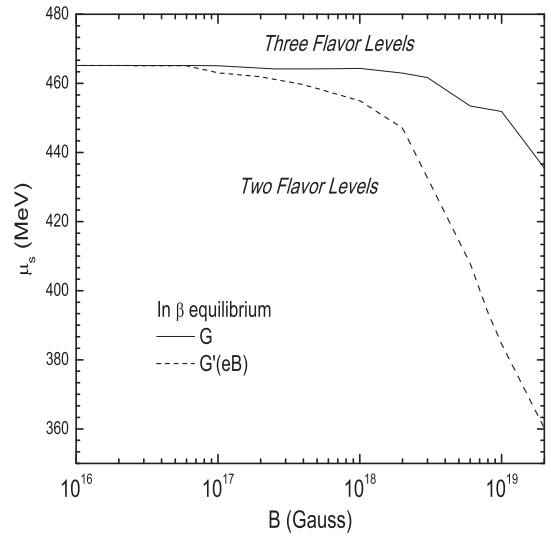


FIG. 4. The critical chemical potential  $\mu_s$  as the Fig. 1 is shown under the  $\beta$  equilibrium condition.

Before the onset of the  $s$  quarks, the Landau level number is approximately inverse proportional to the magnetic field. In Fig. 5, we can also find the similar inverse proportional relation between the maximum Landau quantum number and the magnetic field. Under the  $\beta$  equilibrium, it should be changed as

$$k_{i,\max}^\beta \approx B_i^\beta / B. \quad (21)$$

We can see that the contribution of electrons can hardly change the relation of  $d$  quarks. But the value of  $B_u^\beta$  is decreased to  $1.7 \times 10^{19} \text{ G}$  because its chemical potential is reduced to  $\mu_s - \mu_e$ . In fact, the charge neutral condition can still be reached due to the reduction of the  $d$  quark density

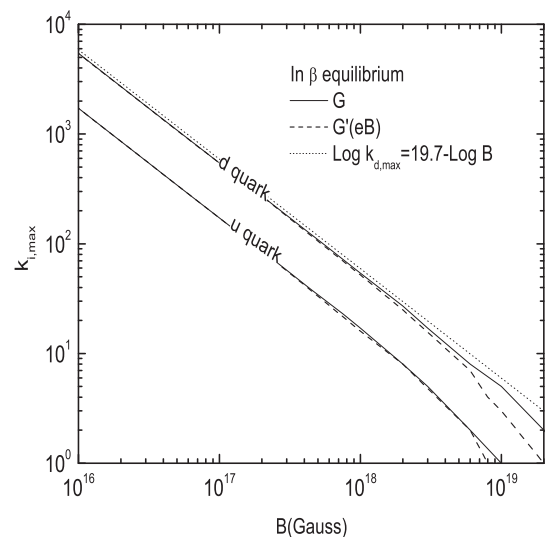


FIG. 5. The maximum Landau levels for  $u$  and  $d$  quarks versus the magnetic field as in Fig. 2 are shown under the  $\beta$  equilibrium.

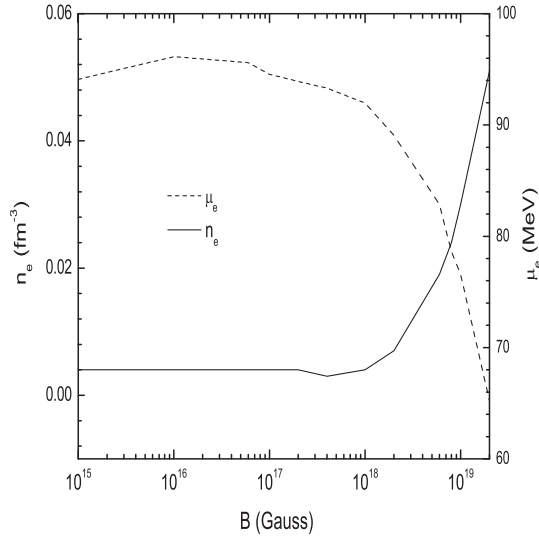


FIG. 6. The density (on the left axis) and the chemical potential (on the right axis) of electrons versus the magnetic field.

in addition to the contribution of electrons. In Fig. 6, the corresponding contribution of the electron density is shown on the left axis. The chemical potential  $\mu_e$  is shown on the right axis. We can see that the  $\mu_e$  will decrease as the magnetic field increases. On the contrary, due to the degeneracy factor  $eB$ , the density will keep increasing monotonously at the higher magnetic field.

#### IV. SUMMARY

In this paper we have studied the energy level of the SQM in a strong magnetic field within the SU(3) NJL model. The critical chemical potential was shown for the onset of  $s$  quarks. We found that the running coupling scheme can broaden the existing region of the three-flavor quarks in the  $\mu$ - $B$  plane. As the density increases, the quarks in the high level participate in the system. Just before the onset of  $s$  quarks, we found that the maximum

quantum number of the  $u/d$  Landau level is directly dependent on the field strength through a simple inverse proportional relation  $k_{i,\max} \approx B_i^0/B$ . The value of  $B_0$  is certainly flavor dependent. In particular, the value  $B_d^0$  of  $d$  quarks is approximately 2 times  $B_u^0$  of  $u$  quarks. It should be pointed out that the chemical potential does not monotonously vary as the density increases. At high densities, the chemical potential keeps increasing with the density. But at low density, the chemical potential could be a decreasing function of the baryon number density, otherwise, the oscillation behavior becomes clear. The corresponding work can be done under the  $\beta$  equilibrium. The running coupling can still broaden the window of three-flavor matter. Because the common chemical potential relation is changed to  $\mu_d = \mu_s = \mu_u + \mu_e$ , the inverse proportional relation should be written as  $k_{i,\max}^\beta \approx B_i^\beta/B$ , where  $B_u^\beta$  is no longer half of  $B_d^\beta$ . These results will be helpful in realistic situations for neutron stars. The magnitude of magnetic field in compact stars decreases from the core to the surface of stars. The study of the relation of magnetic field and Landau level is helpful to know the components of stars for a given radius.

The quarks in the LLL make the QCD more interesting and illustrate the nonperturbative effects, which is further enhanced by the strong magnetic field. We give the condition of the magnetic field larger than  $B_i^0$ , under which all three-flavor quarks are lying in the LLLs. We expect that all of these considerations would be helpful to the theoretical investigation and future experiments searching for the SQM under an extremely strong magnetic field.

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