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MSSM from F-theory SU(5) with Klein monodromy

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We revisit a class of $SU(5)$ supersymmetric grand unified theory (SUSY GUT) models which arise in the context of the spectral cover with Klein Group monodromy $V_4 = Z_2 \times Z_2$. We examine the polynomials of the corresponding factorized spectral cover and discuss the constraints imposed on their coefficients for the transitive and nontransitive realization of this monodromy. We show that Z_2 matter parities can be realized via new geometric symmetries respected by the spectral cover.

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I. INTRODUCTION

Over the last decades string theory grand unified theories (GUTs) have aroused considerable interest. Recent progress has been focused in F-theory [1,2] effective models [3–42], which incorporate several constraints attributed to the topological properties of the compactified space. Indeed, in this context the gauge symmetries are associated to the singularities of the elliptically fibred compactification manifold. As such, GUT symmetries are obtained as a subgroup of E_8 and the matter content emerges from the decomposition of the E_8-adjoint representation (for reviews see [8]).

As is well known, GUT symmetries have several interesting features such as the unification of gauge couplings and the accommodation of fermions in simple representations. Yet, they fail to explain the fermion mass hierarchy and more generally, to impose sufficient constraints on the superpotential terms. Hence, depending on the specific model, several rare processes—including proton decay—are not adequately suppressed. We may infer that, a realistic description of the observed low energy physics world, requires the existence of additional symmetry structure of the effective model, beyond the simple GUT group.

Experimental observations on limits regarding exotic processes (such as baryon and lepton number as well as flavor violating cases) and in particular neutrino physics seem to be nicely explained when the standard model or certain GUTs are extended to include Abelian and discrete symmetries. On purely phenomenological grounds, $U(1)$ as well as non-Abelian discrete symmetries such as A_n, S_n, $SLP_2(n)$ and so on, have already been successfully implemented. However, in this context there is no principle to single out the family symmetry group from the enormous number of possible finite groups. Moreover, the choice of the scalar spectrum and the Higgs vacuum expectation value (vev) alignments introduce another source of arbitrariness in the models.

In contrast to the above picture, F-theory constructions offer an interesting framework for restricting both the gauge (GUT and discrete) symmetries as well as the available Higgs sector. In the elliptic fibration we end up with an 8-dimensional theory with a gauge group, which will be of the special unitary (A_n), special orthogonal (D_n), and exceptional (E_n) types in the Cartan classification (ADE in short). In this work we will focus in the simplest unified symmetry which is $SU(5)$ GUT. In the present geometric picture, the $SU(5)$ GUT is supported by 7-branes wrapping an appropriate (del Pezzo) surface S on the internal manifold, while the number of chiral states is given in terms of a topological index formula. Moreover, there is no use of adjoint Higgs representations since the breaking down to the standard model symmetry can occur by turning on a nontrivial $U(1)_Y$ flux along the hypercharge generator [4]. At the same time this mechanism determines exactly the standard model matter content. Further, if the flux parameters are judiciously chosen they may provide a solution to the well-known doublet triplet splitting problem of the Higgs sector. In short, in F-theory one can in principle develop all those necessary tools to determine the GUT group and predict the matter content of the effective theory.

In the present work we will revisit a class of $SU(5)$ supersymmetric (SUSY) GUT models which arise in the context of the spectral cover. The reason is that the recent developments in F-theory provide now a clearer insight and a better perspective of these constructions. For example, developments on computations of the Yukawa couplings [9–20] have shown that a reasonable mass hierarchy and mixing may arise even if more than one of the fermion families reside on the same matter curve. This implies that effective models left over with only a few matter curves after certain monodromy identifications could be viable and it would be worth reconsidering them. More specifically, we will consider the case of the Klein group

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monodromy $V_4 = Z_2 \times Z_2$ [22–25,27]. Interestingly, with this particular spectral cover, there are two main ways to implement its monodromy action, depending on whether V_4 is a transitive or nontransitive subgroup of S_4. A significant part of the present work will be devoted to the viability of the corresponding two kinds of effective models. Another ingredient related to the predictability of the model, is the implementation of R-parity conservation, or equivalently a Z_2 matter parity, which can be realized with the introduction of new geometric symmetries [11] respected from the spectral cover.

The paper is organized as follows. In Sec. II we give a short description of the derivation of $SU(5)$ GUT in the context of F-theory. In Sec. III we describe the action of monodromies and their role in model building. We further focus on the Klein group monodromy and the corresponding spectral cover factorisations which is our main concern in the present work. In Sec. IV we review a few well known mathematical results and theorems which will be used in model building of the subsequent sections. In Sec. V we discuss effective field theory models with Klein group monodromy and implement the idea of matter parity of geometric origin. In Sec. VI we discuss the conclusions.

II. THE ORIGIN OF SU(5) IN F-THEORY

In this section we explain the main setup of these class of models. Focusing on the case under consideration, i.e. the GUT $SU(5)$, the effective four dimensional model can be reached from the maximal E_8 gauge symmetry through the decomposition

$$E_8 \supset SU(5)_{\rm GUT} \times SU(5)_\perp.$$

In the elliptic fibration, we know that an $SU(5)$ singularity is described by the Tate equation

$$y^2 = x^3 + b_0 z^5 + b_2 x z^3 + b_3 y z^2 + b_4 x^2 z + b_5 xy \tag{2.1}$$

where the homologies of the coefficients in the above equation are given by:

$$[b_k] = \eta - k c_1$$
$$\eta = 6c_1 - t$$

where c_1 and t are the Chern classes of the tangent and normal bundles, respectively.

The first $SU(5)$ is defining the GUT group of the effective theory, the second $SU(5)_\perp$ incorporates additional symmetries of the effective theory while it can be described in the context of the spectral cover. Indeed, in this picture, one can depict the non-Abelian Higgs bundle in terms of the adjoint scalar field configuration [6] and work with the Higgs eigenvalues and eigenvectors. For $SU(n)$ these emerge as roots of a characteristic polynomial of nth degree. Thus the $SU(5)$ spectral surface C_5 is represented by the fifth order polynomial

$$C_5 = b_0 s^5 + b_1 s^4 + b_2 s^3 + b_3 s^2 + b_4 s + b_5$$
$$= b_0 \prod_{i=1}^{5} (s - t_i). \tag{2.2}$$

Since the roots are associated to the $SU(5)$ Cartan subalgebra their sum is zero, $\sum_i t_i = 0$, thus we have put $b_1 = 0$.

The $5 + \bar{5}$ and $10 + \overline{10}$ representations are found at certain enhancements of the $SU(5)$ singularity. In particular, for this purpose the relevant quantities are [6]

$$\mathcal{P}_{10} = b_5 = \prod_i t_i \tag{2.3}$$

$$\mathcal{P}_5 = b_3^2 b_4 - b_2 b_3 b_5 + b_0 b_5^2 \propto \prod_{i \neq j} (t_i + t_j). \tag{2.4}$$

At the $\mathcal{P}_{10} = 0$ locus the enhanced singularity is $SO(10)$ and the intersection defines the matter curve accommodating the 10's. Fiveplets are found at a matter curve defined at an $SU(6)$ enhancement associated to the locus $\mathcal{P}_5 = 0$.

In practice, we are interested in phenomenologically viable cases where the spectral cover splits in several pieces. Consider for example the splitting expressed through the breaking chain

$$E_8 \to SU(5) \times SU(5) \to SU(5) \times U(1)^4$$

where we assumed breaking of $SU(5)_\perp$ along the Cartan, $\sum_i t_i = 0$. The presence of four $U(1)$'s in the effective theory leaves no room for a viable superpotential, since many of the required terms, including the top Yukawa coupling, are not allowed. Nevertheless, monodromies imply various kinds of symmetries among the roots t_i of the spectral cover polynomial which can be used to relax these tight constraints. The particular relations among these roots depend on the details of the compactification and the geometrical properties of the internal manifold. All possible ways fall into some Galois group which in the case of $SU(5)_\perp$ is a subgroup of the corresponding Weyl group, i.e., the group S_5 of all possible permutations of the five Cartan weights t_i. It is obvious that there are several options and each of them leads to models with completely different properties and predictions of the effective field theory. Before starting our investigations on the effective models derived in the context of the aforementioned monodromy, we will analyze these issues in the next section.

III. THE IMPORTANCE OF MONODROMY

For the $SU(5)_{\rm GUT}$ model, we have seen that any possible remnant symmetries (embeddable in the E_8 singularity) must be contained in $SU(5)_\perp$. We have already explained that in the spectral cover approach we quotient the theory by the action of a finite group [22] which is expected to

TABLE I. A summary of the permutation cycles of S_4, categorized by cycle size and whether or not those cycles are contained within the transitive subgroups A_4 and V_4. This also shows that V_4 is necessarily a transitive subgroup of A_4, since it contains all the $2+2$-cycles of A_4 and the identity only.

	S_4 cycles	Transitive A_4	Transitive V_4
4-cycles	(1234), (1243), (1324), (1342), (1423), (1432)	No	No
3-cycles	(123), (124), (132), (134), (142), (143), (234), (243)	Yes	No
$2+2$-cycles	(12)(34), (13)(24), (14)(23)	Yes	Yes
2-cycles	(12), (13), (14), (23), (24), (34)	No	No
1-cycles	e	Yes	Yes

descend from a geometrical symmetry of the compactification. Starting from a C_5 spectral cover, the local field theory is determined by the $SU(5)$ GUT group and the Cartan subalgebra of $SU(5)_\perp$ modulo the Weyl group $W(SU(5)_\perp)$. This is the group S_5, the permutation symmetry of five elements which in the present case correspond to the Cartan weights $t_{1,\ldots,5}$.

Depending on the geometry of the manifold, C_5 may split to several factors

$$C_5 = \prod_j C_j.$$

For the present work, we will assume two cases where the compactification geometry implies the splitting of the spectral cover to $C_5 \to C_4 \times C_1$ and $C_5 \to C_2 \times C_2' \times C_1$. Assuming the splitting $C_5 \to C_4 \times C_1$, the permutation takes place between the four roots, say $t_{1,2,3,4}$, and the corresponding Weyl group is S_4. Notwithstanding, under specific geometries to be discussed in the subsequent sections, the monodromy may be described by the Klein group $V_4 \in S_4$. The latter might be either transitive or nontransitive. This second case implies the spectral cover factorization $C_4 \to C_2 \times C_2'$. As a result, there are two nontrivial identifications acting on the pairs (t_1, t_2) and (t_3, t_4) respectively while both are described by the Weyl group $W(SU(2)_\perp) \sim S_2$. Since $S_2 \sim Z_2$, we conclude that in this case the monodromy action is the nontransitive Klein group $Z_2 \times Z_2$. Next, we will analyze the basic features of these two spectral cover factorizations.

A. S_4 subgroups and monodromy actions

The group of all permutations of four elements, S_4, has a total of 24 elements.[1] These include 2,3,4 and $2+2$-cycles, all of which are listed in Table I. These cycles form a total of 30 subgroups of S_4, shown in Fig. 1. Of these there are those subgroups that are transitive subgroups of S_4: the whole group, A_4, D_4, Z_4 and the Klein group.

We focus now on compactification geometries consistent with the Klein group monodromy $V_4 = Z_2 \times Z_2$. We observe that there are three nontransitive V_4 subgroups within S_4 and only one transitive subgroup. This transitive Klein group is the subgroup of the A_4 subgroup. Considering Table I, one can see that A_4 is the group of all even permutations of four elements and the transitive V_4 is that group excluding 3-cycles. The significance of this is that in the case of Galois theory, to be discussed in Sec. IV, the transitive subgroups A_4 and V_4 are necessarily irreducible quartic polynomials, while the nontransitive V_4 subgroups of S_4 should be reducible.

In terms of group elements, the Klein group that is transitive in S_4 has the elements:

$$\{(1),(12)(34),(13)(24),(14)(23)\} \tag{3.1}$$

which are the $2+2$-cycles shown in Table I along with the identity. On the other hand, the nontransitive Klein groups within S_4 are isomorphic to the subgroup containing the elements:

$$V_4 = \{(1),(12),(34),(12)(34)\}. \tag{3.2}$$

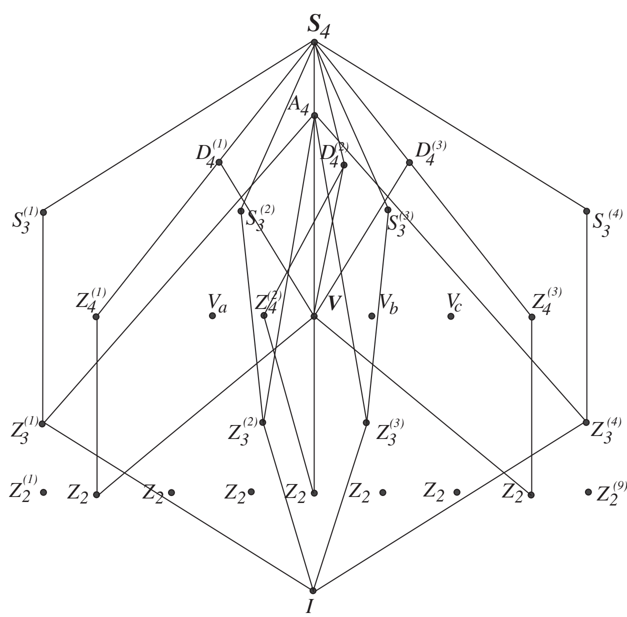

FIG. 1. Pictorial summary of the subgroups of S_4, the group of all permutations of four elements—representative of the symmetries of a cube.

[1]The order of an S_N group is given by $N!$

The distinction here is that the group elements are not all within one cycle, since we have two 2-cycles and one $2+2$-cycle. These types of subgroup must lead to a factorization of the quartic polynomial, as we shall discuss in Sec. IV. Referring to Fig. 1, these Klein groups are the nodes disconnected from the web, while the central V_4 is the transitive group.

B. Spectral cover factorization

In this section we will discuss the two possible factorizations of the spectral surface compatible with a Klein group monodromy, in accordance with the previous analysis. In particular, we shall be examining the implications of a monodromy action that is a subgroup of S_4—the most general monodromy action relating four weights. In particular we shall be interested in the chain of subgroups $S_4 \to A_4 \to V_4$, which we shall treat as a problem in Galois theory.

1. C_4 spectral cover

This set of monodromy actions requires the spectral cover of Eq. (2.2) to split into a linear part and a quartic part:

$$C_5 \to C_4 \times C_1 \tag{3.3}$$

$$C_5 \to (a_5 s^4 + a_4 s^3 + a_3 s^2 + a_2 s + a_1)(a_6 + a_7 s). \tag{3.4}$$

The $b_1 = 0$ condition must be enforced for $SU(5)$ tracelessness. This can be solved by consistency in Eq. (3.4),

$$b_1 = a_5 a_6 + a_4 a_7 = 0. \tag{3.5}$$

Let us introduce a new section a_0, enabling one to write a general solution of the form:

$$\begin{aligned} a_4 &= \pm a_0 a_6 \\ a_5 &= \mp a_0 a_7. \end{aligned}$$

Upon making this substitution, the defining equations for the matter curves are

$$C_5 \colon = a_1 a_6 \tag{3.6}$$

$$\begin{aligned} C_{10} \colon = &(a_2^2 a_7 + a_2 a_3 a_6 \mp a_0 a_1 a_6^2) \\ &\times (a_3 a_6^2 + (a_2 a_6 + a_1 a_7) a_7), \end{aligned} \tag{3.7}$$

which is the most general, pertaining to an S_4 monodromy action on the roots. By consistency between Eq. (3.4) and Eq. (2.2), we can calculate that the homologies of the coefficients are

$$\begin{aligned} [a_i] &= \eta - (i-6)c_1 - \chi \\ &\quad i = 1,2,3,4,5 \\ [a_6] &= \chi \\ [a_7] &= c_1 + \chi \\ [a_0] &= \eta - 2(c_1 + \chi) \end{aligned}$$

2. The $C_2 \times C_2' \times C_1$ case

If the V_4 actions are not derived as transitive subgroups of S_4, then the Klein group is isomorphic to:

$$A_4 \not\supset V_4 \colon \{(1), (12), (12)(34), (34)\}. \tag{3.8}$$

This is not contained in A_4, but is admissible from the spectral cover in the form of a monodromy $C_5 \to C_2 \times C_2' \times C_1$.

Then, the $\mathbf{10} \in SU(5)$ GUT ($\in SU(5)_\perp$) spectral cover reads

$$C_5 \colon (a_1 + a_2 s + a_3 s^2)(a_4 + a_5 s + a_6 s^2)(a_7 + a_8 s). \tag{3.9}$$

We may now match the coefficients of this polynomial in each order in s to the ones of the spectral cover with the b_k coefficients:

$$\begin{aligned} b_0 &= a_{368} \\ b_1 &= a_{367} + a_{358} + a_{268} \\ b_2 &= a_{357} + a_{267} + a_{348} + a_{258} + a_{168} \\ b_3 &= a_{347} + a_{257} + a_{167} + a_{248} + a_{158} \\ b_4 &= a_{247} + a_{157} + a_{148} \\ b_5 &= a_{147} \end{aligned} \tag{3.10}$$

following the notation $a_{ijk} = a_i a_j a_k$ in [23]. In order to find the homology classes of the new coefficients a_i, we match the coefficients of the above polynomial in each order in s to the ones of Eq. (2.2) such that we get relations of the form $b_k = b_k(a_i)$.

Comparing to the homologies of the unsplit spectral cover, a solution for the above can be found for the homologies of a_i. Notice, though, that we have 6 well-defined homology classes for b_j with only 8 a_i coefficients, therefore the homologies of a_i are defined up to two homology classes:

$$\begin{aligned} [a_{n=1,2,3}] &= \chi_1 + (n-3)c_1 \\ [a_{n=4,5,6}] &= \chi_2 + (n-6)c_1 \\ [a_{n=7,8}] &= \eta + (n-8)c_1 - \chi_1 - \chi_2. \end{aligned} \tag{3.11}$$

We have to enforce the $SU(5)$ tracelessness condition, $b_1 = 0$. An Ansatz for the solution was put forward in [23],

$$a_2 = -c(a_6 a_7 + a_5 a_8)$$
$$a_3 = c a_6 a_8 \tag{3.12}$$

which introduces a new section, c, whose homology class is completely defined by

$$[c] = -\eta + 2\chi_1. \tag{3.13}$$

With this anstaz for the solution of the splitting of spectral cover, P_{10} reads

$$P_{10} = a_1 a_4 a_7 \tag{3.14}$$

while the P_5 splits into

$$P_5 = a_5(a_6 a_7 + a_5 a_8)(a_6 a_7^2 + a_8(a_5 a_7 + a_4 a_8)) \times (a_1 - a_5 a_7 c) \tag{3.15}$$

$$(a_1^2 - a_1(a_5 a_7 + 2a_4 a_8)c + a_4(a_6 a_7^2 + a_8(a_5 a_7 + a_4 a_8))c^2). \tag{3.16}$$

An extended analysis of this interesting case will be presented in the subsequent sections.

IV. A LITTLE BIT OF GALOIS THEORY

So far, we have outlined the properties of the most general spectral cover with a monodromy action acting on four of the roots of the perpendicular $SU(5)$ group. This monodromy action is the Weyl group S_4, however a subgroup is equally admissible as the action. Transitive subgroups are subject to the theorems of Galois theory, which will allow us to determine what properties the coefficients of the quartic factor of Eq. (3.4) must have in order to have roots with a particular symmetry [37–40]. In this paper we shall focus on the Klein group, $V_4 \cong Z_2 \times Z_2$. As already mentioned, the transitive V_4 subgroup of S_4 is contained within the A_4 subgroup of S_4, and so shall share some of the same requirements on the coefficients.

While Galois theory is a field with an extensive literature to appreciate, in the current work we need only reference a handful of key theorems. We shall omit proofs for these theorems as they are readily available in the literature and are not required for the purpose at hand.

Theorem 1.—Let K be a field with characteristic different than 2, and let $f(X)$ be a separable, polynomial in $K(X)$ of degree n.

(i) If $f(X)$ is irreducible in $K(X)$ then its Galois group over K has order divisible by n.

(ii) The polynomial $f(X)$ is irreducible in $K(X)$ if and only if its Galois group over K is a transitive subgroup of S_n.

This first theorem offers the key point that any polynomial of degree n, that has nondegenerate roots, but cannot be factorized into polynomials of lower order with coefficients remaining in the same field must necessarily have a Galois group relating the roots that is S_n or a transitive subgroup thereof.

Theorem 2.—Let K be a field with characteristic different than 2, and let $f(X)$ be a separable, polynomial in $K(X)$ of degree n. Then the Galois group of $f(X)$ over K is a subgroup of A_n if and only if the discriminant of f is a square in K.

As already stated, we are interested specifically in transitive V_4 subgroups. Theorem 2 gives us the requirement for a Galois group that is A_4 or its transitive subgroup V_4—both of which are transitive in S_4. Note that no condition imposed on the coefficients of the spectral cover should split the polynomial ($C_4 \to C_2 \times C_2$), due to Theorem 1. We also know by Theorem 2 that both V_4 and A_4 occur when the discriminant of the polynomial is a square, so we necessarily require another mechanism to distinguish the two.

A. The cubic resolvent

The so-called cubic resolvent, is an expression for a cubic polynomial in terms of the roots of the original quartic polynomial we are attempting to classify. The roots of the cubic resolvent are defined to be,

$$x_1 = (t_1 t_2 + t_3 t_4), \qquad x_2 = (t_1 t_3 + t_2 t_4),$$
$$x_3 = (t_1 t_4 + t_2 t_3) \tag{4.1}$$

and one can see that under any permutation of S_4 these roots transform between one another. However, in the event that the polynomial has roots with a Galois group relation that is a subgroup of S_4, the roots need not all lie within the same orbit. The resolvent itself is defined trivially as:

$$(x - (t_1 t_2 + t_3 t_4))(x - (t_1 t_3 + t_1 t_4))(x - (t_1 t_4 + t_3 t_2)) = g_3 x^3 + g_2 x^2 + g_1 x + g_0. \tag{4.2}$$

The coefficients of this equation can be determined by relating of the roots to the original C_4 coefficients. This resulting polynomial is

$$g(x) = a_5^3 x^3 - a_3 a_5^2 x^2 + (a_2 a_4 - 4a_1 a_5) a_5 x - a_2^2 a_5 + 4a_1 a_3 a_5 - a_1 a_4^2. \tag{4.3}$$

Note that this may be further simplified by making the identification $y = a_5 x$.

$$g(y) = y^3 - a_3 y^2 + (a_2 a_4 - 4a_1 a_5) y - a_2^2 a_5 + 4a_1 a_3 a_5 - a_1 a_4^2. \tag{4.4}$$

TABLE II. A summary of the conditions on the partially symmetric polynomials of the roots and their corresponding Galois group.

Group	Discriminant	Cubic resolvent
S_4	$\Delta \neq \delta^2$	Irreducible
A_4	$\Delta = \delta^2$	Irreducible
D_4/Z_4	$\Delta \neq \delta^2$	Reducible
V_4	$\Delta = \delta^2$	Reducible

If the cubic resolvent is factorizable in the field K, then the Galois group does not contain any three cycles. For example, if the Galois group is V_4, then the roots will transform only under the $2+2$-cycles:

$$V_4 \subset A_4 = \{(1), (12)(34), (13)(24), (14)(23)\}. \quad (4.5)$$

Each of these actions leaves the first of the roots in Eq. (4.1) invariant, thus implying that the cubic resolvent is reducible in this case. If the Galois group were A_4, the 3-cycles present in the group would interchange all three roots, so the cubic resolvent is necessarily irreducible. This leads us to a third theorem, which classifies all the Galois groups of an irreducible quartic polynomial (see also Table II).

Theorem 3.—The Galois group of a quartic polynomial $f(x) \in K$, can be described in terms of whether or not the discriminant of f is a square in K and whether or not the cubic resolvent of f is reducible in K.

V. KLEIN MONODROMY AND THE ORIGIN OF MATTER PARITY

In this section we will analyze a class of four-dimensional effective models obtained under the assumption that the compactification geometry induces a $Z_2 \times Z_2$ monodromy. As we have seen in the previous section, there are two distinct ways to realize this scenario, which depends on whether the corresponding Klein group is transitive or nontransitive.

There are significant differences in the phenomenological implications of these models since in a factorized spectral surface matter and Higgs are associated with different irreducible components.[2]

In the present work we will choose to explore the rather promising case where the monodromy Klein group is nontransitive. In other words, this essentially means that the spectral cover admits a $C_2 \times C_2' \times C_1$ factorization. The case of a transitive Klein group is more involved and it is not easy to obtain a viable effective model, and we will consider this issue in a future work.

[2]Further phenomenological issues concerning proton decay and unbroken $U(1)$ factors beyond a local spectral cover can be found in [25,26].

Hence, turning our attention to the nontransitive case, the basic structure of the model obtained in this case corresponds to one of those initially presented in [22] and subsequently elaborated by other authors [23–27]. This model possesses several phenomenologically interesting features and we consider it is worth elaborating it further.

A. Analysis of the $Z_2 \times Z_2$ model

To set the stage, we first present a short review of the basic characteristics of the model following mainly the notation of [23]. The $Z_2 \times Z_2$ monodromy case implies a $2+2+1$ splitting of the spectral fifth-degree polynomial which has already been given in (3.9). Under the action (3.8), for each element, either x_2 and x_3 roots defined in (4.1) are exchanged or the roots are unchanged.

The effective model is characterized by three distinct 10 matter curves, and five 5 matter curves. The matter curves, along with their charges under the perpendicular surviving $U(1)$ and their homology classes are presented in Table III.

Knowing the homology classes associated to each curve allows us to determine the spectrum of the theory through the units of Abelian fluxes that pierce the matter curves. Namely, by turning on a flux in the $U(1)_X$ directions, we can endow our spectrum with chirality and break the perpendicular group. In order to retain an anomaly free spectrum we need to allow for

$$\sum M_5 + \sum M_{10} = 0, \quad (5.1)$$

where M_5 (M_{10}) denote $U(1)_X$ flux units piercing a certain 5 (10) matter curve.

A nontrivial flux can also be turned on along the hypercharge. This will allow us to split GUT irreps, which will provide a solution for the doublet-triplet splitting problem. In order for the hypercharge to remain unbroken, the flux configuration should not allow for a Green-Schwarz mass, which is accomplished by

$$F_Y \cdot c_1 = 0, \qquad F_Y \cdot \eta = 0. \quad (5.2)$$

For the new, unspecified, homology classes, χ_1 and χ_2 we let the flux units piercing them to be

$$F_Y \cdot \chi_1 = N_1, \qquad F_Y \cdot \chi_2 = N_2, \quad (5.3)$$

where N_1 and N_2 are flux units, and are free parameters of the theory.

For a fiveplet, 5 one can use the above construction as a *doublet-triplet splitting solution* as

$$n(3,1)_{-1/3} - n(\bar{3},1)_{1/3} = M_5, \quad (5.4)$$

$$n(1,2)_{1/2} - n(1,2)_{-1/2} = M_5 + N, \quad (5.5)$$

where the states are presented in the SM basis. For a 10 we have

TABLE III. Matter curves and their charges and homology classes.

Curve	$U(1)$ Charge	Defining equation	Homology class
10_1	t_1	a_1	$-2c_1+\chi_1$
10_3	t_3	a_4	$-2c_1+\chi_2$
10_5	t_5	a_7	$\eta-c_1-\chi_1-\chi_2$
5_1	$-2t_1$	$a_6a_7+a_5a_8$	$\eta-c_1-\chi_1$
5_{13}	$-t_1-t_3$	$a_1^2-a_1(a_5a_7+2a_4a_8)c+a_4(a_6a_7^2+a_8(a_5a_7+a_4a_8))c^2$	$-4c_1+2\chi_1$
5_{15}	$-t_1-t_5$	$a_1-a_5a_7c$	$-2c_1+\chi_1$
5_{35}	$-t_3-t_5$	$a_6a_7^2+a_8(a_5a_7+a_4a_8)$	$2\eta-2c_1-2\chi_1-\chi_2$
5_3	$-2t_3$	a_5	$-c_1+\chi_2$

$$n(3,2)_{1/6}-n(\bar{3},2)_{-1/6}=M_{10}, \quad (5.6)$$

$$n(\bar{3},1)_{-2/3}-n(3,1)_{2/3}=M_{10}-N, \quad (5.7)$$

$$n(1,1)_1-n(1,1)_{-1}=M_{10}+N. \quad (5.8)$$

In the end, given a value for each M_5, M_{10}, N_1, N_2 the spectrum of the theory is fully defined as can be seen in Table IV.

B. Matter parity

Some major issues in supersymmetric GUT model building, including operators leading to fast proton decay and other flavor processes at unacceptable rates, are usually solved by introducing the concept of R-parity. In early F-theory model building [23,41], such matter parity symmetries where introduced by hand. Here, in the present approach, the conjecture is that as in the case of the GUT symmetries which are associated with the manifold singularities, R-parity can also be attributed to the geometric properties of the manifold.

In this work we concentrate on models with matter being distributed on different matter curves in contrast to the models where all three families reside on a single curve. In such models the Higgs field, H_u, is accommodated on a suitable curve so that a tree-level coupling for the up-quark fermion mass matrix is ensured. Similarly, we may require at most one tree level coupling for down-type quarks. Because of $U(1)$ symmetries left over under some chosen monodromy action, all other mass entries are generated at higher orders. However, despite the existence of $U(1)$ symmetries, it is possible that other trilinear (tree-level) couplings among the fermion fields themselves are still allowed in the effective superpotential. In the present $2+2+1$ spectral cover splitting for example, we can see that more than one down-quark type trilinear coupling exists, since any of the following $10_1(\bar{5}_{13}\bar{5}_{35}+\bar{5}_{15}\bar{5}_3)$, $10_3(\bar{5}_{13}\bar{5}_{15}+\bar{5}_1\bar{5}_{35})$ and $10_5(\bar{5}_1\bar{5}_3+\bar{5}_{13}\bar{5}_{13})$ are invariant under all symmetries. A similar picture emerges for the up-quark sector. Such terms are also present in $2+1+1+1$ as well as other splittings as can be easily checked. Assigning the Higgs in the appropriate fiveplet, one of the above terms may account for the quark mass of the third generation. Of course, we might seek appropriate flux parameters to eliminate chiral states on the unwanted fiveplets involved in the remaining terms, but this is not always possible. In such cases additional restrictions are required and a possible solution to this drawback is the concept of R-parity.

In an F-theory framework, we can think of three different ways to introduce R-parity in the model: As a first approach, we may impose *ad hoc* a Z_2 symmetry on the grounds of the desired low energy phenomenology. As has already been said, this has been suggested in early F-theory constructions. However, inasmuch as F-theory gauge symmetries are intimately connected to geometric properties, it would be desirable that R-parity has also a geometric origin. A second possibility, then, is to seek such a symmetry within the properties of the spectral cover. Finally, a third way to deal with the annihilation of the

TABLE IV. Matter curve spectrum. Note that $N=N_1+N_2$ has been used as short hand.

Curve	Weight	Homology	N_Y	N_X	Spectrum
10_1	t_1	$-2c_1+\chi_1$	N_1	M_{10_1}	$M_{10_1}Q+(M_{10_1}-N_1)u^c+(M_{10_1}+N_1)e^c$
10_3	t_3	$-2c_1+\chi_2$	N_2	M_{10_3}	$M_{10_3}Q+(M_{10_3}-N_2)u^c+(M_{10_3}+N_2)e^c$
10_5	t_5	$\eta-c_1-\chi_1-\chi_2$	$-N_1-N_2$	M_{10_5}	$M_{10_5}Q+(M_{10_5}+N)u^c+(M_{10_5}-N)e^c$
5_1	$-2t_1$	$\eta-c_1-\chi_1$	$-N_1$	M_{5_1}	$M_{5_1}\overline{d^c}+(M_{5_1}-N_1)\bar{L}$
5_{13}	$-t_1-t_3$	$-4c_1+2\chi_1$	$2N_1$	$M_{5_{13}}$	$M_{5_{13}}\overline{d^c}+(M_{5_{13}}+2N_1)\bar{L}$
5_{15}	$-t_1-t_5$	$-2c_1+\chi_1$	N_1	$M_{5_{15}}$	$M_{5_{15}}\overline{d^c}+(M_{5_{15}}+N_1)\bar{L}$
5_{35}	$-t_3-t_5$	$2\eta-2c_1-2\chi_1-\chi_2$	$-2N_1-N_2$	$M_{5_{35}}$	$M_{5_{35}}\overline{d^c}+(M_{5_{35}}-2N_1-N_2)\bar{L}$
5_3	$-2t_3$	$-c_1+\chi_2$	N_2	M_{5_3}	$M_{5_3}\overline{d^c}+(M_{5_3}+N_2)\bar{L}$

perilous Yukawa couplings is to introduce new symmetries emerging from specific elliptic fibrations possessing rational sections. Indeed, these imply the existence of new $U(1)$ symmetries [43] of the Mordell-Weil type, beyond those embedded in the non-Abelian part. Such symmetries may prevent undesirable terms.

Given the fact that the GUT symmetries in F-theory are associated with geometric singularities, in the present work we think it is also worth exploring the possibility that R-parity may be of a similar nature. Of course, imposing R-parity in a bottom up approach is always possible, however, we will follow the second path and attempt to describe R-parity from geometric symmetries associated with the spectral cover. Such a conjecture might also look *ad hoc* but in the following we will try to give a kind of "evidence" of this correlation.

It was proposed [11], that in local F-theory constructions there are geometric discrete symmetries of the spectral cover that manifest on the final field theory. In F-theory the relevant data originate from the geometric properties of the Calabi-Yau four-fold and the G_4-flux. For example, for a surface of the type $S = \mathbb{P}^2$, it was shown in [11] that a Z_2 transformation acting on S generates also a Z_2 transformation on spinors. If this transformation is a symmetry of the specific geometric configuration, it should also be a symmetry of the spectral surface and this is indeed the case.

To be more precise, we analyze this in some detail for the $SU(5)$ group where the spectral surface is described by the equation $\sum_{k=0}^{5} b_k s^{5-k} = 0$. We consider the GUT divisor S_{GUT} and three open patches S, T, U covering S_{GUT}; we define a phase $\phi_N = \frac{2\pi}{N}$ and a map σ_N such that

$$\sigma_N\colon\ [S\colon\ T\colon\ U] \to [e^{i\phi_N} S\colon\ e^{i\phi_N} T\colon\ U].$$

For a Z_2 symmetry discussed in [11] one requires a Z_2 background configuration, with a Z_2 action so that the mapping is

$$\sigma_2\colon\ [S\colon\ T\colon\ U] \to [-S\colon\ -T\colon\ U] \quad \text{or} \quad [S\colon\ T\colon\ -U].$$

To see if this is a symmetry of the local geometry for a given divisor, we take local coordinates for the three trivialization patches. These can be defined as $(t_1, u_1) = (T/S, U/S)$, $(s_2, u_2) = (S/T, U/T)$ and $(s_3, t_3) = (S/U, T/U)$. Assuming that $\sigma_2(p)$, is the map of a point p under σ_2 transformation the corresponding local coordinates are mapped according to

$$\begin{aligned} (t_1, u_1, \xi_s)|_{\sigma_2(p)} &= (t_1, -u_1, -\xi_s)|_p \\ (s_2, u_2, \xi_t)|_{\sigma_2(p)} &= (s_2, -u_2, -\xi_t)|_p \\ (s_3, t_3, \xi_u)|_{\sigma_2(p)} &= (-s_3, -t_3, \xi_u)|_p \end{aligned} \tag{5.9}$$

This is an $SU(3)$ rotation on the three complex coordinates, which acts on the spinors in the same way. Hence, starting from a Z_2 symmetry of the three-fold we conclude that a Z_2 transformation is also induced on the spinors. The required discrete symmetry must be a symmetry of the local geometry. This can happen if the defining equation of the spectral surface is left invariant under the corresponding discrete transformation. Consequently we expect nontrivial constraints on the polynomial coefficients b_k, which carry the information of local geometry. In order to extract these constraints we focus on a single trivialization patch and take s to be the coordinate along the fiber. Under the mapping of points $p \to \sigma(p)$ we consider the phase transformation

$$s(\sigma(p)) = s(p) e^{i\phi}, \qquad b_k(\sigma(p)) = b_k(p) e^{i(\chi - (6-k)\phi)}.$$

Under this action, each term in the spectral cover equation transforms the same way

$$b_k s^{5-k} \to e^{i(\chi - \phi)} b_k s^{5-k}.$$

It can be readily observed that a nontrivial solution accommodates a Z_N symmetry for $\phi = \frac{2\pi}{N}$. Thus, for $N = 2$, we have $\phi = \pi$ and the transformation reduces to

$$s \to -s, \qquad b_k \to (-1)^k e^{i\chi} b_k. \tag{5.10}$$

Further, we may assume that this symmetry is communicated from the $\mathcal{C}_5$ theory to the split spectral cover geometry. On matter curves GUT symmetry is enhanced while their geometric description is given by the defining equations. Clearly, the properties of their coefficients are determined from b_k's. Our conjecture is that the R-parity is determined in analogy with the bulk surface fields. In this respect, for a Z_2 choice, to all fields residing on a specific matter curve, we assign either even or odd parity in accordance with the property of its corresponding defining equation.

Returning to the present construction, for curves accommodating minimal supersymmetric standard model (MSSM) chiral matter we will assume that R-parity is defined by the corresponding "parity" of its defining equation, which is fixed through its relation with the $\mathcal{C}_5$ coefficients. Thus the chiral matter fields on the same matter curve must necessarily have the same parity, since it is a symmetry arising from the matter curve itself. For the specific models of this work, we can use [27] the equations relating

$$b_k \propto a_l a_m a_n, \quad \text{with} \quad l + m + n = 17 \tag{5.11}$$

to find the transformation rules of the a_k such that the spectral cover equation respects the symmetry of Eq. (5.11). Consistency with Eq. (5.11) implies that the coefficients a_n should transform as

$$a_n \to e^{i\psi_n} e^{i(11/3 - n)\phi} a_n. \tag{5.12}$$

We now note that the above transformations can be achieved by a Z_N symmetry if $\phi = 3\frac{2\pi}{N}$. In that case one can

TABLE V. All possible matter parity assignments.

Curve	Charge	Parity	All possible assignments							
10_1	t_1	i	+	−	+	−	+	−	+	−
10_3	t_3	j	+	+	−	−	+	+	−	−
10_5	t_5	k	+	+	+	+	−	−	−	−
5_1	$-2t_1$	jk	+	+	−	−	−	−	+	+
5_{13}	$-t_1-t_3$	+	+	+	+	+	+	+	+	+
5_{15}	$-t_1-t_5$	i	+	−	+	−	+	−	+	−
5_{35}	$-t_3-t_5$	j	+	+	−	−	+	+	−	−
5_3	$-2t_3$	$-j$	−	−	+	+	−	−	+	+

find, by looking at the equations (3.10) for $b_k \propto a_l a_m a_n$ that we have

$$\psi_1 = \psi_2 = \psi_3 \tag{5.13}$$

$$\psi_4 = \psi_5 = \psi_6 \tag{5.14}$$

$$\psi_7 = \psi_8 \tag{5.15}$$

meaning that there are three distinct cycles, and

$$\chi = \psi_1 + \psi_4 + \psi_7. \tag{5.16}$$

Furthermore, the section c introduced to split the matter conditions (3.12) has to transform as

$$c \to e^{i\phi_c} c, \tag{5.17}$$

with

$$\begin{aligned} \phi_c &= \psi_3 - \psi_6 - \psi_7 + \left(-\frac{11}{3} + 11\right)\phi, \\ \phi_c &= \psi_2 - \psi_5 - \psi_8 + \left(-\frac{11}{3} + 11\right)\phi. \end{aligned} \tag{5.18}$$

We can now deduce what would be the matter parity assignments for Z_2 with $\phi = 3(2\pi/2)$. Let $p(x)$ be the parity of a section (or products of sections), x. We notice that there are relations between the parities of different coefficients, for example one can easily find

$$\frac{p(a_1)}{p(a_2)} = -1 \tag{5.19}$$

amongst others, which allow us to find that all parity assignments depend only on three independent parities

$$p(a_1) = i \tag{5.20}$$

$$p(a_4) = j \tag{5.21}$$

$$p(a_7) = k \tag{5.22}$$

$$p(c) = ijk, \tag{5.23}$$

where we notice that $i^2 = j^2 = k^2 = +$. The parities for each matter curve—both in form of a function of i, j, k and all possible assignments—are presented in Table V.

As such, models from $Z_2 \times Z_2$ are completely specified by the information present in Table VI.

C. Application of geometric matter parity

We study now the implementation of the explicit $Z_2 \times Z_2$ monodromy model presented in [23] alongside the matter parity proposed above. The model under consideration is defined by the flux data

$$N_1 = M_{5_{15}} = M_{5_{35}} = 0 \tag{5.24}$$

$$N_2 = M_{10_3} = M_{5_1} = 1 = -M_{10_5} = -M_{5_3} \tag{5.25}$$

TABLE VI. All the relevant information for model building with $Z_2 \times Z_2$ monodromy.

Curve	Charge	Matter parity	Spectrum
10_1	t_1	i	$M_{10_1}Q + (M_{10_1} - N_1)u^c + (M_{10_1} + N_1)e^c$
10_3	t_3	j	$M_{10_3}Q + (M_{10_3} - N_2)u^c + (M_{10_3} + N_2)e^c$
10_5	t_5	k	$M_{10_5}Q + (M_{10_5} + N_1 + N_2)u^c + (M_{10_5} - N_1 - N_2)e^c$
5_1	$-2t_1$	jk	$M_{5_1}\bar{d}^c + (M_{5_1} - N_1)\bar{L}$
5_{13}	$-t_1 - t_3$	+	$M_{5_{13}}\bar{d}^c + (M_{5_{13}} + 2N_1)\bar{L}$
5_{15}	$-t_1 - t_5$	i	$M_{5_{15}}\bar{d}^c + (M_{5_{15}} + N_1)\bar{L}$
5_{35}	$-t_3 - t_5$	j	$M_{5_{35}}\bar{d}^c + (M_{5_{35}} - 2N_1 - N_2)\bar{L}$
5_3	$-2t_3$	$-j$	$M_{5_3}\bar{d}^c + (M_{5_3} + N_2)\bar{L}$

TABLE VII. Spectrum and allowed geometric parities for the $Z_2 \times Z_2$ monodromy model.

Curve	Charge	Spectrum	All possible assignments							
10_1	t_1	$3Q + 3u^c + 3e^c$	+	−	+	−	+	−	+	−
10_3	t_3	$Q + 2e^c$	+	+	−	−	+	+	−	−
10_5	t_5	$-Q - 2e^c$	+	+	+	+	−	−	−	−
5_1	$-2t_1$	$D_u + H_u$	+	+	−	−	−	−	+	+
5_{13}	$-t_1 - t_3$	$-3\bar{d}^c - 3\bar{L}$	+	+	+	+	+	+	+	+
5_{15}	$-t_1 - t_5$	0	+	−	+	−	+	−	+	−
5_{35}	$-t_3 - t_5$	$-\bar{H}_d$	+	+	−	−	+	+	−	−
5_3	$-2t_3$	$-\bar{D}_d$	−	−	+	+	−	−	+	+

$$M_{10_1} = 3 = -M_{5_{13}} \tag{5.26}$$

which leads to the spectrum presented in Table VII alongside all possible geometric parities.

Inspecting Table VII one can arrive at some conclusions. For example, looking at the spectrum from each curve it is immediate that all matter is contained in 10_1 and 5_{13}, while the Higgses come from 5_1 and 5_{35}, and the rest of the states are exotics that come in vectorlike pairs. Immediately we see that there will be R-parity violating terms since 5_{13} has positive parity.

Of the possible combinations $\{i, j, k\}$ for the geometric parity assignments, the only choices that allow for a tree-level top quark mass are

$$\begin{aligned} \{i,j,k\} &= \{+,+,+\} \\ \{i,j,k\} &= \{-,+,+\} \\ \{i,j,k\} &= \{+,-,-\} \\ \{i,j,k\} &= \{-,-,-\}. \end{aligned} \tag{5.27}$$

The option that most closely resembles the R-parity imposed in the model [23] corresponds to the choice $i = -$, $j = k = +$. However, if R-parity has a geometric origin the parity assignments of matter curves cannot be arbitrarily chosen. Using the *Mathematica* package presented in [44], it is straightforward to produce the spectrum of operators up to an arbitrary mass dimension. One can readily observe that its implementation allows a number of operators that could cause bilinear R-parity violation (BRPV) at unacceptably high rates.

It transpires that in a similar way, all the models with this flux assignment must be ruled out when we apply this geometric parity. This is due to the tension between BRPV terms and exotic masses, which seem to always be at odds in models with this novel parity. This motivates one to search for models without any exotics, as these models will not have any constraining features coming from exotic masses.

VI. CONCLUSIONS

We have revisited a class of $SU(5)$ SUSY GUT models which arise in the context of the spectral cover with Klein group monodromy $V_4 = Z_2 \times Z_2$. By investigating the symmetry structures of the spectral cover equation and the defining equations of the mater curves it is possible to understand the F-theory geometric origin of matter parity, which has hitherto been just assumed in an *ad hoc* way. In particular, we have shown how the simplest Z_2 matter parities can be realized via the new geometric symmetries respected by the spectral cover. By exploiting the various ways that these symmetries can be assigned, there are a large number of possible variants. The results of our analysis are presented in the Tables of Sec. V C. where various examples can be easily worked out.

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