Gaugings of four-dimensional $N=3$ supergravity and AdS_4/CFT_3 holography

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We study matter-coupled $N=3$ gauged supergravity in four dimensions with various semisimple gauge groups. When coupled to n vector multiplets, the gauged supergravity contains $3+n$ vector fields and $3n$ complex scalars parametrized by $SU(3,n)/SU(3)\times SU(n)\times U(1)$ coset manifold. Semisimple gauge groups take the form of $G_0\times H\subset SO(3,n)\subset SU(3,n)$ with H being a compact subgroup of $SO(n+3-\dim(G_0))$. The G_0 groups considered in this paper are of the form $SO(3)$, $SO(3,1)$, $SO(2,2)$, $SL(3,\mathbb{R})$ and $SO(2,1)\times SO(2,2)$. We find that $SO(3)\times SO(3)$, $SO(3,1)$ and $SL(3,\mathbb{R})$ gauge groups admit a maximally supersymmetric AdS_4 critical point. The $SO(2,1)\times SO(2,2)$ gauge group admits a supersymmetric Minkowski vacuum while the remaining gauge groups admit both half-supersymmetric domain wall vacua and AdS_4 vacua with completely broken supersymmetry. For the $SO(3)\times SO(3)$ gauge group, there exists another supersymmetric $N=3$ AdS_4 critical point with $SO(3)_{\text{diag}}$ symmetry. We explicitly give a detailed study of various holographic RG flows between AdS_4 critical points, flows to nonconformal theories, and supersymmetric domain walls in each gauge group. The results provide gravity duals of $N=3$ Chern-Simons-matter theories in three dimensions.

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I. INTRODUCTION

The AdS/CFT correspondence has attracted a lot of attention since its original proposal in [1]. The correspondence provides a duality relation between a gravity theory in AdS_{d+1} space and a strongly coupled conformal field theory in d dimensions. The correspondence has also been extended to the case of nonconformal field theories in the form of the domain wall and quantum field theory (DW/QFT) correspondence [2]. These provide a useful tool to understand strongly coupled gauge theories in various spacetime dimensions.

AdS_4/CFT_3 correspondence is particularly interesting in many aspects. In M-theory, $AdS_4\times X^7$ geometries, with X^7 being an internal compact seven-dimensional manifold, arise naturally from a near horizon limit of M2-brane configurations. AdS_4/CFT_3 correspondence is then expected to shed some light on the dynamics of a strongly coupled worldvolume theory on M2-branes [3,4]. And, more recently, the correspondence has also been applied to condensed matter physics systems; see, for example, [5–7].

As in other dimensions, working in lower-dimensional gauged supergravity has proved to be useful and efficient. In the lower-dimensional point of view, the $AdS_4\times X^7$ geometries are identified with the vacua of the scalar potential in the gauged supergravity theory, and the isometries of the internal manifold correspond to the gauge symmetry or its unbroken subgroup at the AdS_4 vacua. For the case of $X^7=S^7$, the resulting $AdS_4\times S^7$ geometry preserves maximal supersymmetry. The effective gauged supergravity in this case is the maximal $N=8$ $SO(8)$ gauged supergravity in four dimensions constructed in [8]. The holographic study within this gauged supergravity has been investigated in many previous works; see, for example, [9–13]. These results give a description of the deformations leading to various types of RG flows in the dual superconformal field theories (SCFTs) in three dimensions.

For $N>2$ supersymmetry, there is a unique nonmaximal AdS_4 solution from a compactification of 11-dimensional supergravity with unbroken $N=3$ supersymmetry in four dimensions [14]. In this case, the internal manifold is a tri-sasakian N^{010} with $SU(2)\times SU(3)$ isometry. The corresponding Kaluza-Klein spectrum has been given in [15], and the structure of $N=3$ multiplets has been further investigated in [16]. The properties of the possible dual SCFT to this background in term of Chern-Simons-matter theory with $SU(3)$ flavor symmetry has been proposed in [17,18]. The gravity dual of this $N=3$ SCFT has been studied in many references; see, for example, [19–24]. In these results, the four-dimensional scalar potentials, encoding various deformations of the dual SCFT, have been obtained from compactifications of 11-dimensional supergravity restricted to particular field configurations.

It has been pointed out in [15] and [16] that $AdS_4\times N^{010}$ compactification can be described by an effective theory in the form of $N=3$, $SO(3)\times SU(3)$ gauged supergravity coupled to eight vector multiplets constructed in [25–27]. Many supersymmetric deformations of the maximally

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supersymmetric AdS_4 critical point including a new AdS_4 critical point with $SO(3)\times U(1)$ symmetry have been identified in a recent work [28]. The eleven-dimensional configurations corresponding to these gravity solutions might be obtained by a consistent reduction ansatz, to be explicitly identified.

Apart from a simple compact gauge group studied in [28], it is natural to consider other types of gauge groups. As in other matter-coupled supergravity, there are many possible gauged groups for $N=3$ gauged supergravity coupled to n vector multiplets, the only existing matter in $N=3$ supersymmetry. These gauge groups are in general subgroups of the global, duality, symmetry group $SU(3,n)$. In this paper, we will consider $N=3$ gauged supergravity coupled to n vector multiplets with compact and noncompact gauge groups $\tilde{G}\subset SO(3,n)\subset SU(3,n)$. In each gauge group, we will study the scalar potential restricted on scalar submanifolds, which are invariant under particular subgroups of the full gauge group under consideration, and identify supersymmetric vacua as well as possible RG flow solutions describing supersymmetric deformations in the dual gauge theories in three dimensions.

The paper is organized as follow. In Sec. II, $N=3$ gauged supergravity coupled to n vector multiplets is reviewed along with possible semisimple gauge groups allowed by supersymmetry. The scalar potential of each gauge group is investigated separately in subsequent sections in which possible supersymmetric vacua in the form of an AdS_4 or a domain wall for different scalar submanifolds are classified. Conclusions and comments on the results are presented in Sec. VIII.

II. $N=3$ GAUGED SUPERGRAVITY WITH COMPACT AND NONCOMPACT GAUGE GROUPS

We begin with a review of $N=3$ gauged supergravity in four spacetime dimensions constructed in [25–27]. We will closely follow most of the notations in [25] but in the mostly plus metric signature $(-+++)$.

$N=3$ supersymmetry in four dimensions contains twelve supercharges. Apart from the supergravity multiplet, the only matter multiplets are in the form of vector multiplets. The supergravity multiplet contains the following field content

$$(e^a_\mu, \psi_{\mu A}, A_{\mu A}, \chi) \tag{1}$$

which are given respectively by a graviton e^a_μ, three gravitini $\psi_{\mu A}$, three vectors $A_{\mu A}$ and one spinor field χ. Indices $A,B,\ldots=1,2,3$ denote the $SU(3)_R$ R-symmetry triplets while $\mu,\nu,\ldots=0,\ldots,3$ and $a,b,\ldots=0,\ldots,3$ are respectively spacetime and tangent space indices. Throughout the paper, spinor indices will not be shown explicitly.

Each of the n vector multiplets has one vector field, four spinor fields which are a triplet and a singlet of $SU(3)_R$, and three complex scalars

$$(A_\mu, \lambda_A, \lambda, z_A)^i \tag{2}$$

with indices $i,j,\ldots=1,\ldots,n$ labeling each of the vector multiplets. All spinors are subject to the chirality projection conditions

$$\psi_{\mu A}=\gamma_5\psi_{\mu A}, \qquad \chi=\gamma_5\chi, \qquad \lambda_A=\gamma_5\lambda_A, \qquad \lambda=-\gamma_5\lambda,$$
$$\psi^A_\mu=-\gamma_5\psi^A_\mu, \qquad \lambda^A=-\gamma_5\lambda^A. \tag{3}$$

When coupled to n vector multiplets, the supergravity theory consists of $3n$ complex or $6n$ real scalar fields ${z_A}^i$ parametrized by the coset space $SU(3,n)/SU(3)\times SU(n)\times U(1)$. The scalars can be parametrized by the coset representative ${L(z)_\Lambda}^\Sigma$ which transforms under the global $G=SU(3,n)$ and the local $H=SU(3)\times SU(n)\times U(1)$ symmetries by left and right multiplications, respectively. Indices $\Lambda,\Sigma,\ldots=(A,i)$ take the values $1,\ldots,n+3$. The indices $i,j,\ldots$ are used to label the fundamental representation of $SU(n)$. The coset representative can be accordingly split as follow ${L_\Lambda}^\Sigma=({L_\Lambda}^A,{L_\Lambda}^i)$. Being an element of $SU(3,n)$, its inverse is related to ${L_\Lambda}^\Sigma$ via the relation

$${(L^{-1})_\Lambda}^\Sigma=J_{\Lambda\Pi}J^{\Sigma\Delta}({L_\Delta}^\Pi)^* \tag{4}$$

where $J_{\Lambda\Sigma}$ is an $SU(3,n)$ invariant tensor given by

$$J_{\Lambda\Sigma}=J^{\Lambda\Sigma}=(\delta_{AB},-\delta_{ij}). \tag{5}$$

There are $n+3$ vector fields, three from the gravity multiplet and n from the vector multiplets, which can be written collectively by a single notation $A_\Lambda=(A_A,A_i)$. Accompanied by their magnetic dual, the $n+3$ vector fields transform in the fundamental representation $\mathbf{n}+\mathbf{3}$ of the global symmetry $SU(3,n)$. The Lagrangian consisting of $n+3$ "electric" vectors is invariant only under the $SO(3,n)$ subgroup of the duality symmetry $SU(3,n)$. It has been argued in [25] that possible gauge groups are subgroups of $SO(3,n)$ which transform the vector fields among themselves. When restricted to $SO(3,n)$, the fundamental, complex, representation of $SU(3,n)$ split into two fundamental, real, representations of $SO(3,n)$

$$(\mathbf{3}+\mathbf{n})_\mathbb{C}\to(\mathbf{3}+\mathbf{n})_\mathbb{R}+(\mathbf{3}+\mathbf{n})_\mathbb{R}. \tag{6}$$

The $(\mathbf{3}+\mathbf{n})_\mathbb{R}$ representation of $SO(3,n)$ in turn will become the adjoint representation of the gauge group.

When a particular subgroup $\tilde{G}\subset SO(3,n)\subset SU(3,n)$ is gauged, the $SO(3,n)$ global symmetry of the Lagrangian is broken to $\tilde{G}$. The gauge field strengths become non-Abelian defined by

$$F_\Lambda = dA_\Lambda + f_\Lambda{}^{\Sigma\Gamma} A_\Sigma \wedge A_\Gamma \tag{7}$$

where $f_{\Lambda\Sigma}{}^\Gamma$ are the structure constants of the gauge group. The gauge generators T_Λ satisfy the $\tilde{G}$ Lie algebra

$$[T_\Lambda, T_\Sigma] = f_{\Lambda\Sigma}{}^\Gamma T_\Gamma. \tag{8}$$

It should be noted that $\tilde{G}$ needs not be simple, and each simple factor can have different coupling constants. Furthermore, in the presence of gaugings, the Mourer-Cartan one-form on the scalar manifold gets modified by the gauge fields appearing in the covariant derivative of $L_\Lambda{}^\Sigma$

$$\Omega_\Lambda{}^\Pi = (L^{-1})_\Lambda{}^\Sigma dL_\Sigma{}^\Pi + (L^{-1})_\Lambda{}^\Sigma f_\Sigma{}^{\Omega\Gamma} A_\Omega L_\Gamma{}^\Pi. \tag{9}$$

In the following, we will omit all of the gauge fields since we are only interested in supersymmetric solutions with only the metric and scalars nonvanishing.

Supersymmetry requires that, for any gauge group consistent with supersymmetry, the tensor

$$f_{\Lambda\Sigma\Gamma} = f_{\Lambda\Sigma}{}^{\Gamma'} J_{\Gamma'\Gamma} = f_{[\Lambda\Sigma\Gamma]} \tag{10}$$

must be totally antisymmetric. The consistency condition can be satisfied by taking $J_{\Lambda\Sigma}$ to be the Killing form of the $(n+3)$-dimensional gauge group $\tilde{G}$. Since $J_{\Lambda\Sigma}$ has indefinite signs of the eigenvalues, the gauge groups can be both compact and noncompact types. Furthermore, since $J_{\Lambda\Sigma}$ has three positive eigenvalues but arbitrarily large number of negative eigenvalues depending on the number of vector fields, the gauge group can have at most three compact or at most three noncompact directions.

Among the possible gauge groups, $SO(3)\times H_n$, $SO(3,1)\times H_{n-3}$ and $SO(2,2)\times H_{n-3}$ groups, with H_n being an n-dimensional compact group, have been pointed out in [25] and [29]. However, the consistency condition and the global symmetry $SO(3,n)$ in which the gauge group can be embedded are very similar to the half-maximal gauged supergravity in seven dimensions constructed in [30], and a number of possible gauge groups have been listed in [31]. We then expect that possible gauge groups of the $N = 3$ gauged supergravity considered here should follow the same structure.

Due to the restriction on the number of compact or noncompact directions of the gauge group mentioned above, all possible semisimple gauge groups accordingly take the form of $G_0 \times H$ with H being a compact group of dimension $n + 3 - \dim(G_0)$. It has been pointed out in [31] that G_0 is a compact or noncompact group taking one of the following forms

$$SO(3), \quad SO(2,2), \quad SO(3,1),$$
$$SO(2,1), \quad SO(2,1)\times SO(2,2), \quad SL(3,\mathbb{R}). \tag{11}$$

All of these G_0 actually give rise to the gauge groups $G_0 \times H$ with $f_{\Lambda\Sigma\Gamma} = f_{[\Lambda\Sigma\Gamma]}$. Therefore, they are admissible gauge groups of the $N = 3$ gauged supergravity coupled to vector multiplets.

The bosonic Lagrangian of the $N = 3$ gauged supergravity, with all but the metric and scalars vanishing, can be written as

$$e^{-1}\mathcal{L} = \frac{1}{4}R - \frac{1}{2}P^{iA}_\mu P^\mu_{Ai} - V. \tag{12}$$

The vielbein $P_i{}^A$ of the $SU(3,n)/SU(3)\times SU(n)\times U(1)$ coset are given by the (A,i)-component of the Mourer-Cartan one-form $\Omega_i{}^A = (\Omega_A{}^i)^*$. The scalar potential is written in terms of the "boosted structure constants"

$$C^\Lambda{}_{\Pi\Gamma} = L_{\Lambda'}{}^\Lambda (L^{-1})_\Pi{}^{\Pi'} (L^{-1})_\Gamma{}^{\Gamma'} f_{\Pi'\Gamma'}{}^{\Lambda'}$$
$$\text{and} \quad C_\Lambda{}^{\Pi\Gamma} = J_{\Lambda\Lambda'} J^{\Pi\Pi'} J^{\Gamma\Gamma'} (C^{\Lambda'}{}_{\Pi'\Gamma'})^* \tag{13}$$

by the following relation

$$\begin{aligned} V &= -2S_{AC}S^{CM} + \frac{2}{3}\mathcal{U}_A\mathcal{U}^A + \frac{1}{6}\mathcal{N}_{iA}\mathcal{N}^{iA} + \frac{1}{6}\mathcal{M}^{iB}{}_A\mathcal{M}_{iB}{}^A \\ &= \frac{1}{8}|C_{iA}{}^B|^2 + \frac{1}{8}|C_i{}^{PQ}|^2 - \frac{1}{4}(|C_A{}^{PQ}|^2 - |C_P|^2) \end{aligned} \tag{14}$$

where $C_P = -C_{PM}{}^M$. Various tensors appearing in the above equation are defined by

$$\begin{aligned} S_{AB} &= \frac{1}{4}(\epsilon_{BPQ}C_A{}^{PQ} + \epsilon_{ABC}C_M{}^{MC}) \\ &= \frac{1}{8}(C_A{}^{PQ}\epsilon_{BPQ} + C_B{}^{PQ}\epsilon_{APQ}), \\ \mathcal{U}^A &= -\frac{1}{4}C_M{}^{MA}, \qquad \mathcal{N}_{iA} = -\frac{1}{2}\epsilon_{APQ}C_i{}^{PQ}, \\ \mathcal{M}_{iA}{}^B &= \frac{1}{2}(\delta^B_A C_{iM}{}^M - 2C_{iA}{}^B). \end{aligned} \tag{15}$$

Other important ingredients for finding supersymmetric solutions are supersymmetry transformations of fermions

$$\delta\psi_{\mu A} = D_\mu\epsilon_A + S_{AB}\gamma_\mu\epsilon^B, \tag{16}$$

$$\delta\chi = \mathcal{U}^A\epsilon_A, \tag{17}$$

$$\delta\lambda_i = -P_{i\mu}{}^A\gamma^\mu\epsilon_A + \mathcal{N}_{iA}\epsilon^A, \tag{18}$$

$$\delta\lambda_{iA} = -P_{i\mu}{}^B\gamma^\mu\epsilon_{ABC}\epsilon^C + \mathcal{M}_{iA}{}^B\epsilon_B. \tag{19}$$

The covariant derivative on the supersymmetry parameter ϵ_A is defined by

$$D\epsilon_A = d\epsilon_A + \frac{1}{4}\omega^{ab}\gamma_{ab}\epsilon_A + Q_A{}^B\epsilon_B + \frac{1}{2}nQ\epsilon_A. \tag{20}$$

${Q_A}^B$ and Q are the $SU(3)\times U(1)$ composite connections. These connections and the corresponding ones for $SU(n)$, ${Q_i}^j$, can be obtained from (A,B) and (i,j) components of the Mourer-Cartan one-form

$$ {\Omega_A}^B = {Q_A}^B - n\delta_A^B Q, \quad {\Omega_i}^j = {Q_i}^j + 3\delta_i^j Q \qquad (21) $$

with the property that ${Q_A}^A = {Q_i}^i = 0$.

We are now in a position to study the scalar potential in each gauge group and classify the corresponding vacua.

III. $SO(3)\times SO(3)$ GAUGE GROUP

We begin with a simple compact gauge group of the form $SO(3)\times SO(3)$ with $G_0 = SO(3)$ and $H_3 = SO(3)$. This gauged supergravity can be obtained from $N=3$ supergravity coupled to three vector multiplets. The structure constants are given by

$$ {f_{\Lambda\Sigma}}^\Gamma = (g_1\epsilon_{ABC}, g_2\epsilon_{i+3,j+3,k+3}), \qquad i,j = 1,2,3. \qquad (22) $$

In this case, there are 18 scalars parametrized by the $SU(3,3)/SU(3)\times SU(3)\times U(1)$ coset manifold. To parametrize this manifold and the other related ones needed in subsequent sections, we introduce the following $6n$ noncompact generators for a general $SU(3,n)/SU(3)\times SU(n)\times U(1)$ coset

$$ \hat{Y}_{iA} = e_{i+3,A} + e_{A,i+3} $$
$$ \text{and} \quad \tilde{Y}_{iA} = -ie_{i+3,A} + ie_{A,i+3} \qquad (23) $$

where $i = 1,\ldots,n$ and $(e_{\Lambda\Sigma})_{\Gamma\Delta} = \delta_{\Lambda\Gamma}\delta_{\Sigma\Delta}$.

A. AdS_4 vacua and RG flows with $SO(3)$ symmetry

We first consider scalars which are singlets of $SO(3)_{\text{diag}} \subset SO(3)\times SO(3)$. The 18 scalars transform in representations $(\mathbf{3},\bar{\mathbf{3}})_{-2} + (\bar{\mathbf{3}},\mathbf{3})_2$ of the local $SU(3)\times SU(3)\times U(1)$. From now on, we will neglect all the $U(1)$ charges for simplicity since they will not play any important role. With the embedding of $SO(3)$ in $SU(3)$ such that $\mathbf{3}\to\mathbf{3}$ and $\bar{\mathbf{3}}\to\mathbf{3}$, there are two $SO(3)_{\text{diag}}$ singlets among the 18 scalars according to the decomposition

$$ \mathbf{3}\times\mathbf{3} + \mathbf{3}\times\mathbf{3} = (\mathbf{1}+\mathbf{3}+\mathbf{5}) + (\mathbf{1}+\mathbf{3}+\mathbf{5}). \qquad (24) $$

These singlets correspond to the following $SU(3,3)$ noncompact generators:

$$ Y_1 = \hat{Y}_{11} + \hat{Y}_{22} + \hat{Y}_{33}, \qquad Y_2 = \tilde{Y}_{11} + \tilde{Y}_{22} + \tilde{Y}_{33}. \qquad (25) $$

The coset representative can be parametrized by

$$ L = e^{\Phi_1 Y_1} e^{\Phi_2 Y_2}. \qquad (26) $$

The scalar potential is computed to be

$$ \begin{aligned} V = &-\frac{3}{32}\cosh(2\Phi_2) \\ &\times [4\cosh(2\Phi_1)[1+\cosh(2\Phi_1)\cosh(2\Phi_2)]^2 g_1^2 \\ &+ 2\sinh(2\Phi_1) \\ &\times [\cosh(4\Phi_1) - 3 + 2\cosh^2(2\Phi_1)\cosh(4\Phi_2)]g_1 g_2 \\ &+ 4\cosh(2\Phi_1)[\cosh(2\Phi_1)\cosh(2\Phi_2) - 1]^2 g_2^2]. \qquad (27) \end{aligned} $$

We find that this potential admits two supersymmetric AdS_4 critical points. The first one occurs at $\Phi_1 = \Phi_2 = 0$ with the cosmological constant and the AdS_4 radius given by

$$ V_0 = -\frac{3}{2}g_1^2, \qquad L^2 = -\frac{3}{2V_0} = \frac{1}{g_1^2}. \qquad (28) $$

Another critical point is given by

$$ \Phi_1 = \frac{1}{2}\ln\left[\frac{g_2 - g_1}{g_2 + g_1}\right], \qquad \Phi_2 = 0, $$
$$ V_0 = -\frac{3g_1^2 g_2^2}{2(g_2^2 - g_1^2)}, \qquad L^2 = \frac{g_2^2 - g_1^2}{g_1^2 g_2^2}. \qquad (29) $$

It should be noted that reality of Φ_1 requires that $g_2^2 - g_1^2 > 0$, so the critical point is AdS_4 with $V_0 < 0$.

At the trivial critical point with all scalars vanishing and $SO(3)\times SO(3)$ symmetry unbroken, all scalars have the same mass $m^2L^2 = -2$ corresponding to the dual operators of dimensions $\Delta = 1, 2$ in the dual $N=3$ SCFT. At the $SO(3)_{\text{diag}}$ critical point, we can compute the scalar masses as shown in Table I. All masses satisfy the BF bound as expected for a supersymmetric critical point. Furthermore, there are three massless Goldstone bosons from the $SO(3)\times SO(3)\to SO(3)$ symmetry breaking.

To check for the unbroken supersymmetry and set up BPS equations for studying supersymmetric domain wall solutions, we consider supersymmetry transformations of χ, λ_i, λ_{iA} and $\psi_{\mu A}$. The four-dimensional metric is taken to be

$$ ds^2 = e^{2A(r)}dx_{1,2}^2 + dr^2, \qquad (30) $$

TABLE I. Scalar masses at the $N=3$ supersymmetric AdS_4 critical point with $SO(3)_{\text{diag}}$ symmetry and the corresponding dimensions of the dual operators in $SO(3)\times SO(3)$ gauge group.

$SO(3)_{\text{diag}}$ representations	m^2L^2	Δ
1	$4, -2$	4, (1,2)
3	$0_{(\times 3)}, -2_{(\times 3)}$	3, (1,2)
5	$-2_{(\times 10)}$	(1,2)

and the two scalars $\Phi_{1,2}$ only depend on r. $\delta\chi = 0$ equations are identically satisfied since $C_M{}^{MA} = 0$ in the present case. We will use Majorana representation for gamma matrices in which all of the gamma matrices γ^a are real. The chirality matrix $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ is then purely imaginary. This implies that ϵ_A and ϵ^A are related by a complex conjugation, $\epsilon_A = (\epsilon^A)^*$.

In the following analysis, we will use the same procedure as in [28]. With the projection condition

$$\gamma^{\hat{r}}\epsilon_A = e^{i\Lambda}\epsilon^A, \tag{31}$$

where $e^{i\Lambda}$ is a phase factor, the equations for $\delta\lambda_i = 0$ and $\delta\lambda_{iA} = 0$ reduce to two equations,

$$\begin{aligned} e^{i\Lambda}[\cosh(2\Phi_2)\Phi_1' \pm i\Phi_2'] &= -\frac{1}{2}[\sinh(2\Phi_1) + i\cosh(2\Phi_1\sinh(2\Phi_2))] \\ &\quad\times[\cosh\Phi_2(g_1\cosh\Phi_1 + g_2\cosh\Phi_1) \\ &\quad - i\sinh\Phi_2(g_1\sinh\Phi_1 + g_2\cosh\Phi_1)], \end{aligned} \tag{32}$$

where $' \equiv \frac{d}{dr}$.

For this particular coset representative consisting of only $SO(3)_{\mathrm{diag}}$ singlets, S_{AB} is diagonal with

$$S_{AB} = \frac{1}{2}\mathcal{W}\delta_{AB} \tag{33}$$

where the "superpotential" $\mathcal{W}$ is given by

$$\begin{aligned} \mathcal{W} = &-[\cosh\Phi_1\cosh\Phi_2 - i\sinh\Phi_1\sinh\Phi_2] \\ &\times[\cosh\Phi_1\cosh\Phi_2 + i\sinh\Phi_1\sinh\Phi_2]^2 g_1 \\ &+[\sinh\Phi_1\cosh\Phi_2 - i\cosh\Phi_1\sinh\Phi_2] \\ &\times[\sinh\Phi_1\cosh\Phi_2 + i\cosh\Phi_1\sinh\Phi_2]^2 g_2. \end{aligned} \tag{34}$$

With this, $\delta\psi_{\mu A} = 0$ equations for $\mu = 0, 1, 2$ become

$$A' e^{i\Lambda} + \mathcal{W} = 0. \tag{35}$$

By writing $\mathcal{W} = |\mathcal{W}|e^{i\omega}$ and separating the real and imaginary parts of (35), we find

$$A' + \frac{1}{2}|\mathcal{W}|(e^{i\omega - i\Lambda} + e^{-i\omega + i\Lambda}) = 0, \tag{36}$$

$$\frac{1}{2}|\mathcal{W}|(e^{i\omega - i\Lambda} - e^{-i\omega + i\Lambda}) = 0 \tag{37}$$

where $W = |\mathcal{W}|$ will play the role of the "real superpotential". The second equation gives $e^{i\Lambda} = \pm e^{i\omega}$.

Equation (32) implies $\Phi_2' = 0$. Consistency with the field equations requires that $\Phi_2 = 0$. We then set $\Phi_2 = 0$ in the remaining analysis. Furthermore, setting $\Phi_2 = 0$ gives a real $\mathcal{W}$ since $\omega = 0$. In this case, we simply have $e^{i\Lambda} = \pm 1$, and the BPS equations (32) and (35) become

$$\Phi_1' = \mp\sinh\Phi_1\cosh\Phi_1(g_1\cosh\Phi_1 + g_2\sinh\Phi_1), \tag{38}$$

$$A' = \pm(g_1\cosh^3\Phi_1 + g_2\sinh^3\Phi_1). \tag{39}$$

These equations admit precisely two AdS_4 solutions with $N = 3$ supersymmetry identified previously. The corresponding Killing spinors could be obtained from $\delta\psi_{rA} = 0$ which eventually gives, as in many other cases, $\epsilon_A = e^{\frac{A}{2}}\epsilon_A^{(0)}$ for constant spinors $\epsilon_A^{(0)}$ satisfying $\gamma^r\epsilon_A^{(0)} = \pm\epsilon^{(0)A}$.

It should also be noted that equations (38) and (39) are similar to those studied in [28] within the $N = 3$ gauged supergravity with $SO(3) \times SU(3)$ gauge group. The solution interpolating between the two supersymmetric AdS_4 critical points can be found similarly. The upper signs will be chosen in order to identify the UV critical point at $\Phi_1 = 0$ with $r \to \infty$. The resulting solution is given by

$$\begin{aligned} g_1 g_2 r = {} & 2g_1\tan^{-1}e^{\Phi_1} + g_2\ln\left[\frac{e^{\Phi_1}+1}{e^{\Phi_1}-1}\right] \\ & - 2\sqrt{g_2^2 - g_1^2}\tanh^{-1}\left[e^{\Phi_1}\sqrt{\frac{g_2 + g_1}{g_2 - g_1}}\right], \end{aligned} \tag{40}$$

$$\begin{aligned} A = {} & \Phi_1 - \ln(1 - e^{4\Phi_1}) \\ & + \ln[(e^{2\Phi_1} + 1)g_1 + (e^{\Phi_1} - 1)g_2] \end{aligned} \tag{41}$$

where we have omitted all irrelevant additive integration constants.

As $r \to \infty$, we find

$$\Phi \sim e^{-g_1 r} \sim e^{-\frac{r}{L_{\mathrm{UV}}}}, \qquad A \sim g_1 r \sim \frac{r}{L_{\mathrm{UV}}}. \tag{42}$$

This implies that the flow is driven by a relevant operator of dimension $\Delta = 1, 2$ in the UV. In the IR as $r \to -\infty$, we find

$$\Phi_1 \sim e^{\frac{g_1 g_2 r}{\sqrt{g_2^2 - g_1^2}}} \sim e^{\frac{r}{L_{\mathrm{IR}}}}, \qquad A \sim \frac{g_1 g_2 r}{\sqrt{g_2^2 - g_1^2}} \sim \frac{r}{L_{\mathrm{IR}}} \tag{43}$$

which shows that the operator dual to Φ_1 becomes irrelevant with dimension $\Delta = 4$. This precisely agrees with the scalar masses given previously.

Other interesting IR behaviors of the above solution are flows to large values of $|\Phi_1|$. These correspond to flows from conformal field theories, identified with the AdS_4 critical points, to nonconformal gauge theories in the IR. As $\Phi_1 \to \infty$, the above solution gives

$$\Phi_1 \sim -\frac{1}{3}\ln[r(g_1+g_2)+C], \qquad A \sim -\Phi_1,$$
$$ds^2 = [r(g_1+g_2)+C]^{\frac{2}{3}}dx_{1,2}^2 + dr^2. \tag{44}$$

where C is a constant that can be removed by shifting the coordinate r.

For $\Phi_1 \to -\infty$, we find

$$\Phi_1 \sim \frac{1}{3}\ln[r(g_1-g_2)+C], \qquad A \sim \Phi_1,$$
$$ds^2 = [r(g_1-g_2)+C]^{\frac{2}{3}}dx_{1,2}^2 + dr^2. \tag{45}$$

In the above solutions, there is a singularity at $r \sim -\frac{C}{g_1 \pm g_2}$. However, the singularity is physically acceptable according to the criterion of [32] since the potential is bounded above as can be checked from (27) that

$$V(\Phi_1 \to \pm\infty, \Phi_2 = 0) \to -(g_1 \pm g_2)^2\infty. \tag{46}$$

B. RG flows with $SO(2) \times SO(2)$ symmetry

We now move to a scalar submanifold invariant under $SO(2)_{\text{diag}} \subset SO(2) \times SO(2) \subset SO(3) \times SO(3)$ symmetry. There are six singlets corresponding to $SU(3,3)$ noncompact generators

$$Y_1 = \hat{Y}_{33}, \quad Y_2 = \tilde{Y}_{33}, \quad Y_3 = \hat{Y}_{11} + \hat{Y}_{22},$$
$$Y_4 = \tilde{Y}_{11} + \tilde{Y}_{22}, \quad Y_5 = \hat{Y}_{21} - \hat{Y}_{12}, \quad Y_6 = \tilde{Y}_{21} - \tilde{Y}_{12}. \tag{47}$$

The coset representative can be parametrized by

$$L = e^{\Phi_1 Y_1} e^{\Phi_2 Y_2} e^{\Phi_3 Y_3} e^{\Phi_4 Y_4} e^{\Phi_5 Y_5} e^{\Phi_6 Y_6}. \tag{48}$$

The scalar potential turns out to be far more complicated than the $SO(3)$ singlet scalars. We will present the results for some consistent truncations of the full potential.

We first give the result for $SO(2) \times SO(2)$ singlet scalars. These scalars correspond to Φ_1 and Φ_2. The scalar potential is given by

$$V = -\frac{1}{2}g_1^2 e^{-2\Phi_1}[e^{2\Phi_1} + (1 + e^{4\Phi_1})\cosh(2\Phi_2)]. \tag{49}$$

It is clearly seen that this potential admits only a critical point at $\Phi_1 = \Phi_2 = 0$ which is the $SO(3) \times SO(3)$ critical point.

By using the same projector as in the previous case, we can set up the relevant BPS equations as follow. In this case, the matrix S_{AB} is given by

$$S_{AB} = \frac{1}{2}\text{diag}(\mathcal{W}_1, \mathcal{W}_1, \mathcal{W}_2) \tag{50}$$

where

$$\mathcal{W}_1 = -g_1 \cosh\Phi_1 \cosh\Phi_2,$$
$$\mathcal{W}_2 = -g_1(\cosh\Phi_1 \cosh\Phi_2 + i\sinh\Phi_1 \sinh\Phi_2). \tag{51}$$

It should be noted that, when $\Phi_1 = 0$ or $\Phi_2 = 0$, $\mathcal{W}_1$ and $\mathcal{W}_2$ coincide. For $\Phi_1 \neq 0$ and $\Phi_2 \neq 0$, it turns out that $\mathcal{W}_2$ provides the true superpotential in term of which the scalar potential (49) can be written as

$$V = \frac{1}{2}G^{\alpha\beta}\frac{\partial|\mathcal{W}_2|}{\partial\Phi_\alpha}\frac{\partial|\mathcal{W}_2|}{\partial\Phi_\beta} - \frac{3}{2}|\mathcal{W}_2|^2. \tag{52}$$

With the scalar kinetic terms

$$-\frac{1}{2}P_\mu^{Ai}P_{iA}^\mu = -\frac{1}{2}[\cosh^2(2\Phi_2)\Phi_1'^2 + \Phi_2'^2], \tag{53}$$

we find $G_{\alpha\beta} = \text{diag}(-\cosh^2(2\Phi_2), -1)$, and $G^{\alpha\beta}$ is the inverse of $G_{\alpha\beta}$ with $\Phi_\alpha = (\Phi_1, \Phi_2)$.

The BPS equations coming from $\delta\psi_{\mu A} = 0$, $\mu = 0, 1, 2$, become

$$A' = \mp|\mathcal{W}_2| = \pm\frac{1}{2}g_1\sqrt{2 + 2\cosh(2\Phi_1)\cosh(2\Phi_2)} \tag{54}$$

and $e^{i\Lambda} = \pm e^{i\omega}$ with $\mathcal{W}_2 = |\mathcal{W}_2|e^{i\omega}$. It should also be noted that for $\Phi_1 \neq 0$ and $\Phi_2 \neq 0$, only the supersymmetry corresponding to ϵ_3 can be preserved since we need to set $\epsilon_{1,2} = 0$ in the $\delta\psi_{\mu A}$ equations. Therefore, together with the γ^r projection, the solution will preserve only two supercharges or $N = 1$ Poincare supersymmetry in three dimensions.

The conditions $\delta\lambda_{iA} = 0$ are identically satisfied for $\epsilon_{1,2} = 0$ while $\delta\lambda_i = 0$ equations give

$$[e^{i\Lambda}[\cosh(2\Phi_2)\Phi_1' + i\Phi_2']$$
$$+ g_1(\sinh\Phi_1\cosh\Phi_2 - i\cosh\Phi_1\sinh\Phi_2)]\epsilon^3 = 0. \tag{55}$$

This will give the flow equations for Φ_1 and Φ_2. Using the above result for $e^{i\Lambda} = \pm e^{i\omega}$, it can be verified that the flow equations can be written as

$$\Phi_\alpha' = \pm G^{\alpha\beta}\frac{\partial|\mathcal{W}_2|}{\partial\Phi_\beta} \tag{56}$$

or explicitly

$$\Phi_1' = \mp\frac{\sinh(2\Phi_1)\text{sech}(2\Phi_2)g_1}{\sqrt{2 + \cosh(2\Phi_1)\cosh(2\Phi_2)}},$$
$$\Phi_2' = \mp\frac{\cosh(2\Phi_1)\sinh(2\Phi_2)g_1}{\sqrt{2 + \cosh(2\Phi_1)\cosh(2\Phi_2)}}. \tag{57}$$

We are not able to solve the above equations completely, but by combining the two equations, we find a relation between Φ_1 and Φ_2

$$\coth(2\Phi_2) = \frac{e^{2\Phi_1}}{2 - 2e^{4\Phi_1}}. \quad (58)$$

The full flow solution would require some numerical analysis. In the following, we will simply give the asymptotic behaviors at $\Phi_{1,2} \sim 0$ and large $|\Phi_\alpha|$.

Identifying $r \to \infty$ as the UV fixed point, we find

$$\Phi_1 \sim \Phi_2 \sim e^{-g_1 r} \quad (59)$$

As $\Phi_2 \to \pm\infty$, we find

$$\Phi_1 \sim \Phi_0, \qquad \Phi_2 \sim \mp \ln(g_1 r)$$
$$ds^2 = r^2 dx_{1,2}^2 + dr^2 \quad (60)$$

where Φ_0 is a constant. For convenience, we have put the singularity at $r = 0$ by choosing an integration constant.

For $\Phi_1 \to \pm\infty$, the solution becomes

$$\Phi_1 \sim \mp \ln(g_1 r), \qquad \Phi_2 \sim \Phi_0,$$
$$ds^2 = r^2 dx_{1,2}^2 + dr^2. \quad (61)$$

All of these flows give $V \to -\infty$ and are physical.

As noted before for Φ_1 or Φ_2 vanishing, the eigenvalues of S_{AB} degenerate $\mathcal{W}_1 = \mathcal{W}_2$, and the BPS equations coming from $\delta\lambda_i = 0$ and $\delta\lambda_{iA} = 0$ are identical. The resulting equations for $\Phi_1 = 0$ and $\Phi_2 = 0$ cases turn out to be symmetric. In the following, we will set $\Phi_2 = 0$ for definiteness. The flow equations reduce to

$$\Phi_1' = -g_1 \sinh \Phi_1,$$
$$A' = g_1 \cosh \Phi_1 \quad (62)$$

with a simple solutions

$$\Phi_1 = \pm \ln\left[\frac{e^{g_1 r - C} + 1}{e^{g_1 r - C} - 1}\right],$$
$$A = -g_1 r + \ln(e^{2g_1 r - 2C} - 1). \quad (63)$$

At large r, we find $\Phi_1 \sim e^{-g_1 r}$ and $A \sim g_1 r$ which is the UV AdS_4. For $g_1 r \sim C$, the solution becomes

$$\Phi_1 \sim \pm \ln(g_1 r - C), \qquad A \sim \ln(g_1 r - C),$$
$$ds^2 = (g_1 r - C)^2 dx_{1,2}^2 + dr^2. \quad (64)$$

This solution is also physical and preserves $N = 3$ Poincare supersymmetry in three dimensions. We then find two classes of deformations that break conformal symmetry. One of them with Φ_1 and Φ_2 nonvanishing breaks $N = 3$ supersymmetry to $N = 1$ while the other with Φ_1 or Φ_2 vanishing preserves $N = 3$ supersymmetry. On the other hand, both of them preserve $SO(2) \times SO(2)$ symmetry.

C. RG flows with $SO(2)$ symmetry

The scalar potential and BPS equations for $SO(2)_{\text{diag}}$ singlet scalars are far more complicated than the $SO(2) \times SO(2)$ case. We will only give the result for a truncation with $\Phi_2 = \Phi_4 = \Phi_6 = 0$. We have verified that this is a consistent truncation both for the BPS equations and the corresponding field equations.

In this truncation, S_{AB} is diagonal

$$S_{AB} = \frac{1}{2}\mathcal{W}\delta_{AB} \quad (65)$$

where $\mathcal{W}$ is real and given by

$$W = \mathcal{W} = -\frac{1}{2} g_1 \cosh\Phi_1[1 + \cosh(2\Phi_3)]\cosh(2\Phi_5)$$
$$+ g_2[1 - \cosh(2\Phi_3)\cosh(2\Phi_5)]\sinh\Phi_1. \quad (66)$$

With the scalar kinetic terms

$$-\frac{1}{2}P_\mu^{iA}P_{Ai}^\mu = -\frac{1}{2}\Phi_1'^2 - \frac{1}{4}e^{-4\Phi_5}(1 + e^{4\Phi_5})^2\Phi_3'^2 - \Phi_5'^2, \quad (67)$$

the scalar potential can be written as

$$V = -\frac{1}{2}\frac{\partial W}{\partial\Phi_1}\frac{\partial W}{\partial\Phi_1} - \frac{e^{4\Phi_5}}{(1 + e^{4\Phi_5})^2}\frac{\partial W}{\partial\Phi_3}\frac{\partial W}{\partial\Phi_3} - \frac{1}{4}\frac{\partial W}{\partial\Phi_5}\frac{\partial W}{\partial\Phi_5} - \frac{3}{2}W^2$$
$$= \frac{1}{32}[-4[1 + \cosh(2\Phi_3)\cosh(2\Phi_5)][2\cosh(2\Phi_3)\cosh(2\Phi_5)$$
$$+ \cosh(2\Phi_1)[1 + 3\cosh(2\Phi_3)]\cosh(2\Phi_5)]g_1^2$$
$$- 6[\cosh(4\Phi_3) + 2\cosh^2(2\Phi_3)\cosh(4\Phi_5) - 3]\sinh(2\Phi_1)g_1 g_2,$$
$$+ 2[2\cosh(2\Phi_3)\cosh(2\Phi_5) - 2][2\cosh(2\Phi_3)\cosh(2\Phi_5)$$
$$+ 2\cosh(2\Phi_1)[1 - 3\cosh(2\Phi_3)\cosh(2\Phi_5)]]g_2^2]. \quad (68)$$

All of the BPS equations coming from $\delta\lambda_i = 0$ and $\delta\lambda_{iA} = 0$ are solved by the following flow equations,

$$\Phi_1' = \pm\frac{\partial W}{\partial \Phi_1} = \mp\frac{1}{2}[g_1[1+\cosh(2\Phi_3)\cosh(2\Phi_5)]\sinh\Phi_1 + g_2\cosh\Phi_1[\cosh(2\Phi_3)\cosh(2\Phi_5)-1]], \tag{69}$$

$$\Phi_3' = \pm\frac{2e^{4\Phi_5}}{(1+e^{4\Phi_5})^2}\frac{\partial W}{\partial \Phi_3} = \mp\frac{e^{2\Phi_5}}{1+e^{4\Phi_5}}\sinh(2\Phi_3)[g_1\cosh\Phi_1 + g_2\sinh\Phi_1], \tag{70}$$

$$\Phi_5' = \pm\frac{1}{2}\frac{\partial W}{\partial \Phi_5} = \mp\frac{1}{2}\cosh(2\Phi_3)\sinh(2\Phi_5)[g_1\cosh\Phi_1 + g_2\sinh\Phi_1], \tag{71}$$

$$A' = \mp W, \tag{72}$$

after using the projector $\gamma^r\epsilon_A = \pm\epsilon^A$. The solution to these equations then preserves $N = 3$ supersymmetry in three dimensions. Apart from the trivial critical point with all $\Phi_i = 0$ and the $SO(3)_{\text{diag}}$ with $\Phi_5 = 0$ and $\Phi_1 = \pm\Phi_3 = \frac{1}{2}\ln[\frac{g_2+g_1}{g_2-g_1}]$, the above equations admit no new critical points.

We are not able to solve the above equations analytically for general values of g_1 and g_2. However, for $g_2 = g_1$ and $\Phi_5 = 0$, an analytic solution can be found:

$$A = \Phi_1 - \frac{1}{2}\ln(e^{4\Phi_1}-1),$$
$$\Phi_3 = \cosh^{-1}\left[e^{\frac{\Phi_1}{2}}\sqrt{\cosh\Phi_1}\right],$$
$$g_1 r = \tan^{-1}e^{\Phi_1} + \frac{1}{2}\ln\left[\frac{e^{\Phi_1}+1}{e^{\Phi_1}-1}\right]. \tag{73}$$

This solution describes an RG flow from the trivial AdS_4 critical point to an $N = 3$ nonconformal gauge theory in the IR. At $\Phi_1 \sim \Phi_3 \sim 0$, the above solution approaches the UV AdS_4:

$$\Phi_1 \sim e^{-2g_1 r}, \qquad \Phi_3 \sim e^{-g_1 r}, \qquad A \sim g_1 r. \tag{74}$$

Near the IR singularity $r \sim 0$, the solution behaves as

$$\Phi_1 \sim -\ln(g_1 r), \qquad \Phi_3 \sim \Phi_1, \qquad A \sim -\Phi_1 \sim \ln(g_1 r),$$
$$ds^2 = (g_1 r)^2 dx_{1,2}^2 + dr^2 \tag{75}$$

for $\Phi_1 > 0$ and

$$\Phi_1 \sim \ln(g_1 r), \qquad \Phi_3 \sim \text{constant}, \qquad A \sim \Phi_1 \sim \ln(g_1 r),$$
$$ds^2 = (g_1 r)^2 dx_{1,2}^2 + dr^2 \tag{76}$$

for $\Phi_1 < 0$. Both of these singularities give $V \sim -\infty$ and, hence, are physical. Therefore, the solution gives a gravity dual of an RG flow from $N = 3$ SCFT with $SO(3)\times SO(3)$ symmetry to $N = 3$ gauge theory with $SO(2)$ symmetry in three dimensions.

IV. $SO(3,1)$ GAUGE GROUP

We still work with the $n = 3$ case but with $SO(3,1)$ gauge group. The structure constants in this case are given by $f_{\Lambda\Sigma}{}^{\Gamma} = f_{\Lambda\Sigma\Gamma'}J^{\Gamma'\Gamma}$, where

$$f_{\Lambda\Sigma\Gamma} = g(\epsilon_{ABC}, \epsilon_{i+3,j+3,A}), \tag{77}$$

and $\epsilon_{i+3,j+3,A}$ are totally antisymmetric with $\epsilon_{345} = \epsilon_{156} = \epsilon_{264} = 1$.

A. RG flows with $SO(3)$ symmetry

We now proceed as in the previous section by considering the $SO(3) \subset SO(3,1)$ singlet scalars. Under this $SO(3)$, the decomposition of the representation for all 18 scalars is similar to (24) since the $SO(3)$ maximal compact subgroup of $SO(3,1)$ is embedded in $SO(3,1)$ as a diagonal subgroup of $SO(3)\times SO(3) \subset SO(3,3)$. Accordingly, there are two singlets given by the $SU(3,3)$ noncompact generators:

$$Y_1 = \hat{Y}_{11} - \hat{Y}_{22} + \hat{Y}_3, \qquad Y_2 = \tilde{Y}_{11} - \tilde{Y}_{22} + \tilde{Y}_{33}. \tag{78}$$

We then parametrize the coset representative by

$$L = e^{\Phi_1 Y_1}e^{\Phi_2 Y_2}, \tag{79}$$

which gives the potential

$$V = -\frac{3}{64}g^2 e^{-6\Phi_1}[2e^{6\Phi_1}[13\cosh(2\Phi_1)+3\cosh(6\Phi_1)] \times\cosh(2\Phi_2) + (e^{4\Phi_1}-1)^2[(1+e^{4\Phi_1})\cosh(6\Phi_2) - 16e^{2\Phi_1}\cosh^2(2\Phi_2)]]. \tag{80}$$

This potential admits a trivial critical point at $\Phi_1 = \Phi_2 = 0$ at which the $SO(3,1)$ gauge symmetry is broken to its maximal compact subgroup $SO(3)$. The values of the cosmological constant and AdS_4 radius are given by

$$V_0 = -\frac{3}{2}g^2, \qquad L^2 = \frac{1}{g^2}. \tag{81}$$

Scalar masses are given in Table II. We again see that there are three Goldstone bosons.

TABLE II. Scalar masses at the $N = 3$ supersymmetric AdS_4 critical point with $SO(3)$ symmetry and the corresponding dimensions of the dual operators in $SO(3,1)$ gauge group.

$SO(3)$ representations	m^2L^2	Δ
1	$4, -2$	4, (1,2)
3	$0_{(\times 3)}, -2_{(\times 3)}$	3, (1,2)
5	$-2_{(\times 10)}$	(1,2)

We also find a nonsupersymmetric critical point given by

$$\Phi_1 = \frac{1}{2}\ln\left[\frac{4 \pm \sqrt{7}}{3}\right], \qquad \Phi_2 = 0,$$
$$V_0 = -\frac{11}{9}g^2, \qquad L^2 = \frac{27}{22g^2}. \tag{82}$$

This critical point is however unstable since some of the scalar masses violate the BF bound. All scalar masses are given in Table III.

We now consider possible supersymmetric RG flow solutions within the $N = 3$ $SO(3,1)$ gauged supergravity. Since we have not found any nontrivial supersymmetric AdS_4 critical points in this gauge group, we will consider only supersymmetric RG flows to nonconformal theories. Similar to the $SO(3) \times SO(3)$ gauge group, we find that the BPS equations coming from $\delta\lambda_i = 0$ and $\delta\lambda_{iA} = 0$ give rise to the following equations

$$e^{i\Lambda}[\cosh(2\Phi_2)\Phi_1' \pm i\Phi_2']$$
$$= g\sinh^3\Phi_1 \cosh\Phi_2 + \frac{1}{2}g\cosh\Phi_1[\sinh(2\Phi_1)\cosh(3\Phi_2)$$
$$- 2i[1 - 2\sinh^2\Phi_1\cosh(2\Phi_2)]\sinh\Phi_2] \tag{83}$$

which again imply $\Phi_2' = 0$. Consistency with the second order field equations requires that $\Phi_2 = 0$. This gives rise to real superpotential.

Follow the same procedure as in the previous section with an appropriate sign choice, we find the relevant BPS equations

$$\Phi_1' = \frac{1}{4}e^{-3\Phi_1}g(e^{2\Phi_1} + e^{6\Phi_1} - e^{4\Phi_1} - 1), \tag{84}$$

$$A' = -\frac{1}{4}e^{-3\Phi_1}g(1 + e^{6\Phi_1} - 3e^{2\Phi_1} - 3e^{4\Phi_1}). \tag{85}$$

TABLE III. Scalar masses at the nonsupersymmetric AdS_4 critical point with $SO(3)$ symmetry in $SO(3,1)$ gauge group.

$SO(3)$ representations	m^2L^2
1	$-\frac{168}{11}, -\frac{36}{11}$
3	$0_{(\times 3)}, -\frac{36}{11}\vert_{(\times 3)}$
5	$-\frac{24}{11}\vert_{(\times 5)}, -\frac{36}{11}\vert_{(\times 5)}$

Since the operator dual to Φ_1 has dimension $\Delta = 4$ corresponding to an irrelevant operator, we then expect the AdS_4 to appear in the IR of the RG flow driven by Φ_1. The solution to the above equations can be readily found

$$gr = \ln\left[\frac{e^{\Phi_1} - 1}{e^{\Phi_1} + 1}\right] + \frac{1}{\sqrt{2}}\ln\left[\frac{1 + \sqrt{2}e^{\Phi_1} + e^{2\Phi_1}}{\sqrt{2}e^{\Phi_1} - 1 - e^{2\Phi_1}}\right], \tag{86}$$

$$A = \Phi_1 + \ln(e^{2\Phi_1} - 1) - \ln(1 + e^{4\Phi_1}). \tag{87}$$

As $\Phi_1 \sim 0$, the solution gives

$$\Phi_1 \sim e^{gr} \sim e^{\frac{r}{L}}, \qquad A \sim gr \sim \frac{r}{L} \tag{88}$$

which is the AdS_4 critical point.

At large $|\Phi_1|$, we find that for $\Phi_1 > 0$ the solution behaves as

$$\Phi_1 \sim -\frac{1}{3}\ln(gr + C), \qquad A \sim -\Phi_1,$$
$$ds^2 = (gr + C)^{\frac{2}{3}}dx_{1,2}^2 + dr^2 \tag{89}$$

while for $\Phi_1 < 0$, the solution becomes

$$\Phi_1 \sim \frac{1}{3}\ln(C - gr), \qquad A \sim \Phi_1,$$
$$ds^2 = (C - gr)^{\frac{2}{3}}dx_{1,2}^2 + dr^2. \tag{90}$$

Both of these singularities are physical since

$$V(\Phi_1 \to \pm\infty, \Phi_2 = 0) \to -\infty. \tag{91}$$

B. RG flows with $SO(2)$ symmetry

For $SO(2)$ singlet scalars, the coset representative can be parametrized by

$$L = e^{\Phi_1 Y_1}e^{\Phi_2 Y_2}e^{\Phi_3 Y_3}e^{\Phi_4 Y_4}e^{\Phi_5 Y_5}e^{\Phi_6 Y_6} \tag{92}$$

where the $SU(3,3)$ noncompact generators are defined by

$$Y_1 = \hat{Y}_{33}, \quad Y_2 = \tilde{Y}_{33}, \quad Y_3 = \hat{Y}_{11} - \hat{Y}_{22},$$
$$Y_4 = \tilde{Y}_{11} - \tilde{Y}_{22}, \quad Y_5 = \hat{Y}_{12} + \hat{Y}_{21}, \quad Y_6 = \tilde{Y}_{12} + \tilde{Y}_{21}. \tag{93}$$

The resulting scalar potential is very complicated. After making a truncation by setting $\Phi_2 = \Phi_4 = \Phi_6 = 0$, we find a much simpler potential

$$V = \frac{1}{8}g^2[16\cosh(2\Phi_5)\sinh(2\Phi_1)\sinh(2\Phi_3)$$
$$- 3\cosh(2\Phi_1)[3 + \cosh(4\Phi_3)]$$
$$+ 2[2 + (2 - 3\cosh(2\Phi_1))\cosh(4\Phi_5)]\sinh^2(2\Phi_3)]. \tag{94}$$

Apart from the trivial critical point, there are no other supersymmetric critical points from this potential.

We now move to the BPS equations. The S_{AB} matrix in this truncation is diagonal and proportional to the identity matrix with the superpotential

$$W = -g\cosh\Phi_1 + g\cosh(2\Phi_5)\sinh\Phi_1\sin(2\Phi_3). \tag{95}$$

As usual, the scalar potential can be written in term of W as

$$V = -\frac{1}{2}\left(\frac{\partial W}{\partial\Phi_1}\right)^2 - e^{4\Phi_5}(1+e^{4\Phi_5})^{-2}\left(\frac{\partial W}{\partial\Phi_3}\right)^2 - \frac{1}{4}\left(\frac{\partial W}{\partial\Phi_5}\right)^2 - \frac{3}{2}W^2. \tag{96}$$

The flow equations are then given by

$$\Phi_1' = \pm\frac{\partial W}{\partial\Phi_1} = \pm[-g\sinh\Phi_1 + g\cosh\Phi_1\cosh(2\Phi_5)\sinh(2\Phi_3)], \tag{97}$$

$$\Phi_3' = \pm 2e^{4\Phi_5}(1+e^{4\Phi_5})^{-2}\frac{\partial W}{\partial\Phi_3} = \pm\frac{4e^{4\Phi_5}}{(1+e^{4\Phi_5})^2}g\cosh(2\Phi_3)\cosh(2\Phi_5)\sinh\Phi_1, \tag{98}$$

$$\Phi_5' = \pm\frac{1}{2}\frac{\partial W}{\partial\Phi_5} = \pm g\sinh\Phi_1\sinh(2\Phi_3)\sinh(2\Phi_5), \tag{99}$$

$$A = \mp W. \tag{100}$$

We are not able to solve these equations analytically. We will therefore only discuss the asymptotic behaviors of the flow solution and leave the full solution for a numerical analysis. Near the AdS_4 critical point, we find

$$\Phi_1 \sim \Phi_3 \sim e^{gr} \sim e^{\frac{r}{L}}, \qquad \Phi_5 \sim \text{constant}, \qquad A \sim gr \sim \frac{r}{L}. \tag{101}$$

We see that Φ_1 and Φ_3 are dual to irrelevant operators of dimension four while Φ_5 is dual to a marginal operator. Actually, Φ_5 is one of the Goldstone bosons.

Near the singularity at large $|\Phi_3|$, we find $\Phi_5' = 0$. In what follows, we will choose $\Phi_5 = 0$ for simplicity. The asymptotic behaviors of the flow solution are given by

$$\Phi_1 \sim \pm\Phi_3 \sim \pm\frac{1}{3}\ln\left|C \pm \frac{3}{4}gr\right|, \qquad A \sim \frac{1}{3}\ln\left|C \pm \frac{3}{4}gr\right|, \qquad ds^2 = \left(C \pm \frac{3}{4}gr\right)^{\frac{2}{3}}dx_{1,2}^2 + dr^2. \tag{102}$$

It can also be checked that both of these singularities are physical.

V. $SO(2,2)$ GAUGE GROUP

For $n = 3$ vector multiplets, there is another possible gauge group namely $SO(2,2) \sim SO(2,1)\times SO(2,1)$. The structure constants are given by

$$f_{\Lambda\Sigma}{}^{\Gamma} = (g_1\epsilon_{\bar{A}\bar{B}\bar{D}}\eta^{\bar{D}\bar{C}}, g_2\epsilon_{\bar{i}\bar{j}\bar{l}}\eta^{\bar{l}\bar{k}}), \qquad \bar{A},\bar{B},\ldots = 1,2,6, \qquad \bar{i},\bar{j},\ldots = 3,4,5 \tag{103}$$

with $\eta^{\bar{A}\bar{B}} = \mathrm{diag}(1,1,-1)$ and $\eta^{\bar{i}\bar{j}} = \mathrm{diag}(1,-1,-1)$.

We will consider the scalar potential for $SO(2)_{\mathrm{diag}}$ invariant scalars. There are six singlets parametrized by the coset representative

$$L = e^{\Phi_1(\hat{Y}_{11}+\hat{Y}_{22})}e^{\Phi_2(\tilde{Y}_{11}+\tilde{Y}_{22})}e^{\Phi_3\hat{Y}_{33}}e^{\Phi_4\tilde{Y}_{33}} \times e^{\Phi_5(\hat{Y}_{21}-\hat{Y}_{12})}e^{\Phi_6(\tilde{Y}_{21}-\tilde{Y}_{12})}. \tag{104}$$

The scalar potential turns out be much involved. We will only give the potential for a truncation $\Phi_2 = \Phi_4 = \Phi_6 = 0$ for brevity

$$\begin{aligned} V = \frac{1}{16}[&4\cosh(2\Phi_1)\cosh(2\Phi_5)[\cosh(2\Phi_1)\cosh(2\Phi_5)(g_1^2-g_2^2)+g_1^2+g_2^2] \\ &- 2\cosh(2\Phi_3)[g_1^2+g_2^2+\cosh(2\Phi_1)\cosh(2\Phi_5)[3\cosh(2\Phi_1)\cosh(2\Phi_5)(g_1^2+g_2^2)+4(g_1^2-g_2^2)]] \\ &+ 3g_1g_2\sinh(2\Phi_3)[2\cosh(4\Phi_5)\cosh^2(2\Phi_1)+\cosh(4\Phi_1)-3]] \end{aligned} \tag{105}$$

This potential admits an AdS_4 critical point at $\Phi_i = 0$, $i = 1,\ldots,6$ with $V_0 = -\frac{1}{2}g_1^2$ and $L^2 = \frac{3}{g_1^2}$. This critical point is however nonsupersymmetric. This can be seen by considering the supersymmetry transformations

$$\delta\lambda_i = \delta_{i3}g_1\epsilon^3 \quad \text{and} \quad \delta\lambda_{iA} = \delta_{i3}g_1(\delta_{A2}\epsilon^1 - \delta_{A1}\epsilon^2). \tag{106}$$

We see that the only way these variations will vanish is to set $\epsilon^A = 0$, so this critical point breaks all supersymmetries.

This critical point is also unstable as can be seen from the scalar masses in Table IV.

On the other hand, a half-supersymmetric vacuum in the form of a domain wall is possible. Use the domain wall ansatz for the metric and proceed as in the previous cases, we find a set of very complicated BPS equations for $SO(2)_{\text{diag}}$ singlet scalars. To give an example of this solution, we will consider a simpler case of $SO(2) \times SO(2)$ symmetry. Setting all scalars but Φ_3 and Φ_4 to zero results in a simple scalar potential

$$V = -\frac{1}{2} g_1^2 e^{-2\Phi_3}[(1 + e^{4\Phi_3}) \cosh(2\Phi_4) - e^{2\Phi_3}]. \quad (107)$$

The gravitini variations give

$$S_{AB} = \frac{1}{2} \text{diag}(\mathcal{W}_1, \mathcal{W}_1, \mathcal{W}_2) \quad (108)$$

where

$$\mathcal{W}_1 = g_1 \sin \Phi_3 \cosh \Phi_4, \quad (109)$$

$$\mathcal{W}_2 = g_1 \cosh \Phi_4 \sinh \Phi_3 + i g_1 \cosh \Phi_3 \sinh \Phi_4. \quad (110)$$

As in the $SO(3) \times SO(3)$ case, only supersymmetry generated by ϵ_3 is preserved. Carrying out a similar analysis gives the following BPS equations

$$\Phi_3' = \pm \cosh^{-2}(2\Phi_4) \frac{\partial W}{\partial \Phi_3} = \pm \frac{g_1 \text{sech}(2\Phi_4) \sinh(2\Phi_3)}{\sqrt{2}\sqrt{\cosh(2\Phi_3)\cosh(2\Phi_4) - 1}}, \quad (111)$$

$$\Phi_4' = \mp \frac{\partial W}{\partial \Phi_4} = \mp \frac{g_1 \cosh(2\Phi_3) \sinh(2\Phi_4)}{\sqrt{2}\sqrt{\cosh(2\Phi_3)\cosh(2\Phi_4) - 1}}, \quad (112)$$

$$A' = \mp W \quad (113)$$

where

$$W = |\mathcal{W}_2| = \sqrt{2} g_1 \sqrt{\cosh(2\Phi_3)\cosh(2\Phi_4) - 1}. \quad (114)$$

TABLE IV. Scalar masses at the nonsupersymmetric AdS_4 critical point with $SO(2) \times SO(2)$ symmetry in $SO(2,2)$ gauge group.

$SO(2) \times SO(2)$ representations	m^2L^2
(1, 1)	$-6, -6$
(2, 1)	$0_{(\times 2)}, -\frac{15}{2}\|_{(\times 2)}$
(1, 2)	$0_{(\times 2)}, -\frac{3g_2^2}{2g_1^2}\|_{(\times 2)}$
(2, 2)	$-\frac{3}{2}\frac{g_1^2+g_2^2}{g_1^2}\|_{(\times 8)}$

From these equations, we immediately see that there is no supersymmetric AdS_4 critical point. We can also solve for A and Φ_3 as a function of Φ_4 as follow

$$\Phi_3 = \frac{1}{2} \ln \left[\frac{1}{4}[\text{csch}(2\Phi_4)\sqrt{10\cosh(4\Phi_4) - 6} - 2\coth(2\Phi_4)]\right], \quad (115)$$

$$A = -\frac{1}{2} \ln \sinh(2\Phi_4) - iF(2i\Phi_4, 5) \quad (116)$$

where F is the elliptic function of the first kind defined by

$$iF(i\Phi_3, 5) = \int_0^{\Phi_3} \frac{d\chi}{\sqrt{1 - 25\sinh^3\chi}}. \quad (117)$$

However, we are not able to solve for $\Phi_4(r)$ in a closed form.

For $\Phi_4 = 0$, $|\mathcal{W}_1| = |\mathcal{W}_2|$, we find much simpler BPS equations

$$\Phi_3' = \pm g_1 \cosh \Phi_3, \quad (118)$$

$$A' = \pm g_1 \sinh \Phi_3. \quad (119)$$

It should be noted that in this case the supersymmetry is enhanced to $N = 3$ as in the $SO(3) \times SO(3)$ case. An analytic solution to these equations can be completely obtained

$$\Phi_3 = \ln \tan \left[\frac{g_1 r + C}{2}\right], \quad A = -\ln \sin(g_1 r + C), \quad (120)$$

$$ds^2 = \sin^{-2}(g_1 r + C) dx_{1,2}^2 + dr^2. \quad (121)$$

The solution preserves $N = 3$ Poincare supersymmetry in three dimensions due to the projection $\gamma^r \epsilon_A = \pm \epsilon^A$. According to the DW/QFT correspondence, this solution should be dual to a three-dimensional $N = 3$ gauge theory.

We end this section by giving a remark on $SO(2,1)$ gauge group. This gauge group can be obtained by coupling one vector multiplet to the $N = 3$ supergravity and gauging the theory by using the structure constant

$$f_{\Lambda\Sigma}{}^{\Gamma} = g \epsilon_{\bar{A}\bar{B}\bar{D}} \eta^{\bar{D}\bar{C}}, \qquad \bar{A}, \bar{B}, \ldots = 1, 2, 4, \quad \eta_{\bar{A}\bar{B}} = \text{diag}(1, 1, -1). \quad (122)$$

The resulting potential and BPS equations for $SO(2) \subset SO(2,1)$ invariant scalars are the same as the above results for $SO(2,2)$ gauge group with $g_2 = 0$. Therefore, $SO(2,1)$ gauge group also admits a nonsupersymmetric AdS_4 critical point with all scalars vanishing and a half-supersymmetric domain wall. In particular, the domain wall with $SO(2)$ symmetry has the same form as the solution given in (121).

VI. $SL(3,\mathbb{R})$ GAUGE GROUP

This gauge group can be gauged by coupling five vector multiplets to $N=3$ supergravity. To identify the structure constants $f_{\Lambda\Sigma}{}^{\Gamma} = g\tilde{f}_{\Lambda\Sigma}{}^{\Gamma}$, we define the following $SL(3,\mathbb{R})$ generators

$$T_\Lambda = (i\lambda_2, i\lambda_7, i\lambda_5, \lambda_1, \lambda_3, \lambda_4, \lambda_6, \lambda_8) \tag{123}$$

where λ_i are Gell-mann matrices. The structure constants can be extracted from the $SL(3,\mathbb{R})$ algebra

$$[T_\Lambda, T_\Sigma] = \tilde{f}_{\Lambda\Sigma}{}^{\Gamma} T_\Gamma. \tag{124}$$

There are 30 scalars transforming as $(\mathbf{3},\bar{\mathbf{5}}) + (\bar{\mathbf{3}},\mathbf{5})$ under the $SU(3)\times SU(5)$ local symmetry. The $SO(3)$ maximal compact subgroup of $SL(3,\mathbb{R})$ is embedded by $\mathbf{3}\to\mathbf{3}$ and $\mathbf{8}\to\mathbf{3}+\mathbf{5}$. The 30 scalars transform under this $SO(3)$ as

$$(\mathbf{3}\times\mathbf{5}) + (\mathbf{3}\times\mathbf{5}) = (\mathbf{3}+\mathbf{5}+\mathbf{7}) + (\mathbf{3}+\mathbf{5}+\mathbf{7}). \tag{125}$$

There are accordingly no singlets under $SO(3)$ symmetry. We then consider scalars which are singlets under $SO(2)\subset SO(3)$. Further decomposing the above representations give six singlets, each of these representations giving one singlet, corresponding to the following non-compact generators of $SU(3,5)$

$$Y_1 = \hat{Y}_{24} + \hat{Y}_{33}, \quad Y_2 = \hat{Y}_{23} - \hat{Y}_{34}, \quad Y_3 = \hat{Y}_{15},$$
$$Y_4 = \tilde{Y}_{24} + \tilde{Y}_{33}, \quad Y_5 = \tilde{Y}_{23} - \tilde{Y}_{34}, \quad Y_6 = \tilde{Y}_{15}. \tag{126}$$

With the coset representative

$$L = e^{\Phi_1 Y_1} e^{\Phi_2 Y_2} e^{\Phi_3 Y_3} e^{\Phi_4 Y_4} e^{\Phi_5 Y_5} e^{\Phi_6 Y_6}, \tag{127}$$

we find the following potential

$$\begin{aligned} V = -\frac{1}{32} e^{-4\Phi_2-4\Phi_3} g^2 [&16\sqrt{3} e^{2\Phi_2}(e^{4\Phi_2}-1)(e^{4\Phi_3}-1)\cosh(2\Phi_4)\cosh(2\Phi_5)\cosh(2\Phi_6) \\ &+ \cosh^2(2\Phi_5)[3e^{2\Phi_3}(2e^{4\Phi_2} - 3e^{8\Phi_2} - 3) - 12 e^{2\Phi_3}(e^{4\Phi_2})^2 \cosh(4\Phi_4) \\ &+ (1+e^{4\Phi_3})[2(3+e^{4\Phi_2}+3e^{8\Phi_2}) + (9-2e^{4\Phi_2}+9e^{8\Phi_2})\cosh(4\Phi_4)]\cosh(2\Phi_6)] \\ &+ (1+e^{4\Phi_3})\cosh(2\Phi_6)[3+4e^{4\Phi_2}+3e^{8\Phi_2} + (3-4e^{4\Phi_2}+3e^{8\Phi_2})\sinh^2(2\Phi_5)] \\ &- e^{2\Phi_3}[3+14e^{4\Phi_2}+3e^{8\Phi_2}+3(1-6e^{4\Phi_2}+e^{8\Phi_2})\sinh^2(2\Phi_5) \\ &- 8\sqrt{3}e^{2\Phi_2}(1+e^{4\Phi_2})\cosh(2\Phi_4)\sinh(4\Phi_5)\sinh(2\Phi_6)]]. \end{aligned} \tag{128}$$

Apart from the trivial critical point at all $\Phi_i = 0$, we have not found any other critical points. At the trivial AdS_4 point, we find

$$V_0 = -\frac{3}{2}g^2, \qquad L^2 = \frac{1}{g^2} \tag{129}$$

and the scalar masses given in Table V. Apart from the Goldstone bosons, there are marginal deformations corresponding to the scalar fields in the **7** representation of the unbroken $SO(3)$ symmetry.

We will not give the full BPS equations here due to their complexity. To find some supersymmetric deformations of the $N=3$ SCFT dual to the AdS_4 critical point, we will consider a truncation to Φ_1, Φ_2 and Φ_3. Within this truncation, we find that $S_{AB} = \frac{1}{2}W\delta_{AB}$ and the system of BPS equations

$$\Phi_1' = 0, \tag{130}$$

$$\Phi_2' = \pm\frac{1}{2}\frac{\partial W}{\partial \Phi_2} = \mp\sqrt{3}g\cosh(2\Phi_2)\sinh(\Phi_3), \tag{131}$$

$$\begin{aligned}\Phi_3' &= \pm\frac{\partial W}{\partial\Phi_3} \\ &= \mp g[\sqrt{3}\cosh\Phi_3\sinh(2\Phi_2) + \sinh\Phi_3],\end{aligned} \tag{132}$$

$$A' = \mp W \tag{133}$$

where the superpotential is given by

$$W = -g[\cosh\Phi_3 + \sqrt{3}\sinh(2\Phi_2)\sinh\Phi_3]. \tag{134}$$

TABLE V. Scalar masses at the $N=3$ supersymmetric AdS_4 critical point with $SO(3)$ symmetry and the corresponding dimensions of the dual operators in $SL(3,\mathbb{R})$ gauge group.

$SO(3)$ representations	m^2L^2	Δ
3	$10_{(\times 3)}$, $-2_{(\times 3)}$	5, (1,2)
5	$0_{(\times 5)}$, $-2_{(\times 5)}$	3, (1,2)
7	$0_{(\times 7)}$, $-2_{(\times 7)}$	3, (1,2)

With the scalar kinetic terms

$$-\frac{1}{4}e^{-4\Phi_2}(1+e^{4\Phi_2})^2\Phi_1'^2 - \Phi_2'^2 - \frac{1}{2}\Phi_3'^2, \quad (135)$$

the scalar potential can be written as

$$V = -\frac{1}{4}\frac{\partial W}{\partial \Phi_2} - \frac{1}{2}\frac{\partial W}{\partial \Phi_3} - \frac{3}{2}W^2$$
$$= -\frac{1}{4}g^2[2+\cosh(2\Phi_3)+\cosh(4\Phi_2-)[9\cosh(2\Phi_3)-6] + 8\sqrt{3}\sinh(2\Phi_2)\sinh(2\Phi_3)]. \quad (136)$$

We now analyze asymptotic behaviors of the solution near the UV and IR of the flow. Near the AdS_4 critical point, we find

$$\frac{1}{\sqrt{3}}\Phi_2 + \Phi_3 \sim e^{-3g_1 r} \sim e^{-\frac{3r}{L}},$$
$$\Phi_3 - \frac{\sqrt{3}}{2}\Phi_2 \sim e^{2g_1 r} \sim e^{\frac{2r}{L}}, \qquad A \sim g_1 r \sim \frac{r}{L}. \quad (137)$$

We see that $\frac{1}{\sqrt{3}}\Phi_2 + \Phi_3$ is dual to a vacuum expectation value of a marginal operator while $\Phi_3 - \frac{\sqrt{3}}{2}\Phi_2$ is dual to an irrelevant operator of dimension $\Delta = 5$. Since a marginal operator does not break conformal symmetry, we expect that the flow involves the operator dual to $\Phi_3 - \frac{\sqrt{3}}{2}\Phi_2$. In this case, the UV SCFT dual to the supersymmetric AdS_4 critical point should appear in the IR since the operator driving the flow is irrelevant at the fixed point.

Near the singularity, we find for large $|\Phi_2|$,

$$\Phi_3 \sim \Phi_2 \sim \mp\frac{1}{3}\ln\left[\frac{3\sqrt{3}gr}{4}\right], \qquad A \sim \frac{1}{3}\ln r,$$
$$ds^2 = r^{\frac{2}{3}}dx_{1,2}^2 + dr^2. \quad (138)$$

This leads to a physical singularity and describes an RG flow in the dual $N = 3$ supersymmetric field theory to a conformal fixed point in the IR.

VII. $SO(2,1)\times SO(2,2)$ GAUGE GROUP

The last gauge group to be considered in this paper is $SO(2,1)\times SO(2,2)\sim SO(2,1)\times SO(2,1)\times SO(2,1)$. This gauge group can be obtained by coupling six vector multiplets to $N = 3$ supergravity with the following structure constants

$$f_{\Lambda\Sigma}{}^{\Gamma} = (g_1\epsilon_{\bar{A}\bar{B}\bar{D}}\eta^{\bar{D}\bar{C}}, g_2\epsilon_{\bar{i}\bar{j}\bar{l}}\eta^{\bar{l}\bar{k}}, g_3\epsilon_{\tilde{i}\tilde{j}\tilde{l}}\eta^{\tilde{l}\tilde{k}}) \quad (139)$$

where $\bar{A},\bar{B},\ldots = 1,4,5$, $\bar{i},\bar{j},\ldots = 2,6,7$, $\tilde{i},\tilde{j},\ldots = 3,8,9$ and

$$\eta_{\bar{A}\bar{B}} = \mathrm{diag}(1,-1,-1), \qquad \eta_{\bar{i}\bar{j}} = \mathrm{diag}(1,-1,-1),$$
$$\eta_{\tilde{i}\tilde{j}} = \mathrm{diag}(1,-1,-1). \quad (140)$$

At the vacua, the full gauge group $SO(2,1)\times SO(2,2)$ will be broken to its maximal compact subgroup $SO(2)\times SO(2)\times SO(2)$. We will consider scalars which are invariant under the $SO(2)\times SO(2)$ residual symmetry chosen to be the first two $SO(2)$'s. In this case, there are twelve singlets given by

$$Y_1 = \hat{Y}_{15}, \quad Y_2 = \hat{Y}_{16}, \quad Y_3 = \hat{Y}_{25}, \quad Y_4 = \hat{Y}_{26},$$
$$Y_5 = \hat{Y}_{35}, \quad Y_6 = \hat{Y}_{36}, \quad Y_7 = \tilde{Y}_{15}, \quad Y_8 = \tilde{Y}_{16},$$
$$Y_9 = \tilde{Y}_{25}, \quad Y_{10} = \tilde{Y}_{26}, \quad Y_{11} = \tilde{Y}_{35}, \quad Y_{12} = \tilde{Y}_{36}. \quad (141)$$

The coset representative can be parametrized by

$$L = \prod_{i=1}^{12} e^{\Phi_i Y_i}. \quad (142)$$

The potential is highly complicated. We refrain from giving its explicit form here but only note that the resulting potential admits a Minkowski vacuum at $\Phi_i = 0$, for $i = 1,\ldots,12$ preserving $N = 3$ supersymmetry and $SO(2)\times SO(2)\times SO(2)$ symmetry. It can also be checked that there are precisely six massless Goldstone bosons of the symmetry breaking $SO(2,1)\times SO(2,2)\to SO(2)\times SO(2)\times SO(2)$.

VIII. CONCLUSIONS

In this paper, we have studied $N = 3$ gauged supergravity in four dimensions with various types of semisimple gauge groups and classified their vacua. We now summarize the main results found in this paper. For $SO(3)\times SO(3)$, $SO(3,1)$ and $SL(3,\mathbb{R})$ gauge groups, there exists a maximally supersymmetric AdS_4 critical point at which all scalars vanishing. The critical point has $SO(3)$ symmetry in $SO(3,1)$ and $SL(3,\mathbb{R})$ gauge groups and $SO(3)\times SO(3)$ symmetry for $SO(3)\times SO(3)$ gauge group. In the latter case, we have also found a nontrivial AdS_4 critical point with $SO(3)_{\mathrm{diag}}$ symmetry and unbroken $N = 3$ supersymmetry. A holographic RG flow interpolating between the $SO(3)\times SO(3)$ and $SO(3)_{\mathrm{diag}}$ critical points including a number of RG flows to nonconformal gauge theories have also been given. The nonconformal RG flows break conformal symmetry but preserve $N = 3$ or $N = 1$ supersymmetries. A similar study has also been carried out in the case of $SO(3,1)$ and $SL(3,\mathbb{R})$. These results might be useful in the holographic study of $N = 3$ Chern-Simons-matter theories in three dimensions.

For $SO(2,1)\times SO(2,2)$ gauge group, the gauged supergravity admits $N = 3$ Minkowki vacuum when all scalars vanish. In the case of $SO(2,1)$ and $SO(2,2) \sim SO(2,1) \times SO(2,1)$ gauge groups, the resulting gauged supergravities admit a half-maximal supersymmetric domain wall as a supersymmetric vacuum. This solution should be useful in the context of the DW/QFT correspondence for studying strongly coupled gauge theories in three dimensions. When all scalars vanish, there exists a nonsupersymmetric AdS_4 critical point with $SO(2)$ and $SO(2) \times SO(2)$ symmetries, respectively. This critical point and all of the nonsupersymmetric critical points identified in this paper are unstable.

The results have some similarity to the gaugings of $N = 2$ gauged supergravity in seven dimensions studied in [33], but in the present case, nonconformal flows with partially broken supersymmetry are possible. It should also be remarked that although we have considered only the G_0 part of the full semisimple gauge group $G_0 \times H$, all of the vacua and solutions we have found are valid in the full theory with $G_0 \times H$ gauge group. This is a consequence of the fact that all scalars in $SU(3,n)/SU(3) \times SU(n) \times U(1)$ we have considered are H singlets. By the argument given in [34], solutions identified within the scalar submanifold parametized by $H' \times H$, with $H' \subset G_0$, singlets are solutions of the full $G_0 \times H$ theory.

It would be interesting to identify the SCFTs or nonconformal gauge theories that are dual to the gravity solutions obtained here. Looking for more general domain walls with the truncated scalars restored could be useful since these scalars correspond to relevant operators. From the analysis of this paper, we expect these scalars to break some supersymmetry. Investigating their role in the dual SCFT should give some insight to relevant deformations of the dual three-dimensional SCFT. Among the interesting supergravity solutions, a domain wall interpolating between the $N = 3$ AdS_4 critical point and the AdS_4 critical point with $N = 1$ unbroken supersymmetry discovered in [35] deserves some investigation. This $N = 1$ critical point is not accesible from our simple scalar parametrization. Looking for this solution requires a more complicated scalar submanifold of $SU(3,n)/SU(3) \times SU(n) \times U(1)$. Finally, it would also be interesting to look for supersymmetric Janus solutions which are dual to some conformal interface in the $N = 3$ SCFTs. A number of these solutions have been obtained within the maximal $N = 8$ gauged supergravity in [36].

It should be noted that all gaugings considered here are of "electric" type in which only electric gauge fields are involved. Similar to the maximal $N = 8$ and half-maximal $N = 4$ gauged supergravities [37,38], it could be interesting to apply the embedding tensor formalism to the $N = 3$ gauged supergravity and look for more general gaugings such as the magnetic or dyonic gaugings in which magnetic gauge fields also participate in the process of gauging. We then expect many other possible gauge groups will arise from the embedding tensor formalism similar to the $N = 4$ gauged supergravity with $SU(1,1)/U(1) \times SO(6,n)/SO(6) \times SO(n)$ scalar manifold studied in [38].

The $N = 3$ AdS_4 critical point with $SO(3) \times SO(3)$ symmetry within the dyonic $ISO(7)$ gauged $N = 8$ supergravity is known [39–41], and the holographic study of the possible dual SCFT has been given in [42,43]. Furthermore, this $N = 3$ AdS_4 solution has known massive type IIA origin [42,44]. Similarly, investigating the embedding of the results presented in this paper in higher dimensions could be of interest and will give rise to new $N = 3$ AdS_4 backgrounds within the context of string/M-theory. We will leave these interesting issues for future works.

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