

Gaussian quantum steering and its asymmetry in curved spacetimeJieci Wang,^{1,2,*} Haixin Cao,¹ Jiliang Jing,^{1,†} and Heng Fan^{2,‡}¹*Department of Physics and Collaborative Innovation Center for Quantum Effects and Applications, Hunan Normal University, Changsha, Hunan 410081, China*²*Beijing National Laboratory for Condensed Matter Physics, Institute of Physics, Chinese Academy of Sciences, Beijing 100190, People's Republic of China*

(Received 18 November 2015; published 8 June 2016)

We study Gaussian quantum steering and its asymmetry in the background of a Schwarzschild black hole. We present a Gaussian channel description of quantum state evolution under the influence of Hawking radiation. We find that thermal noise introduced by the Hawking effect will destroy the steerability between an inertial observer Alice and an accelerated observer Bob who hovers outside the event horizon, while it generates steerability between Bob and a hypothetical observer anti-Bob inside the event horizon. Unlike entanglement behaviors in curved spacetime, here the steering from Alice to Bob suffers from a “sudden death” and the steering from anti-Bob to Bob experiences a “sudden birth” with increasing Hawking temperature. We also find that the Gaussian steering is always asymmetric and the maximum steering asymmetry cannot exceed $\ln 2$, which means the state never evolves to an extremal asymmetry state. Furthermore, we obtain the parameter settings that maximize steering asymmetry and find that (i) $s = \operatorname{arccosh} \frac{\cosh^2 r}{1 - \sinh^2 r}$ is the critical point of steering asymmetry and (ii) the attainment of maximal steering asymmetry indicates the transition between one-way steerability and both-way steerability for the two-mode Gaussian state under the influence of Hawking radiation.

DOI: 10.1103/PhysRevD.93.125011

I. INTRODUCTION

Einstein-Podolsky-Rosen steering [1,2], an intermediate type of quantum correlation between entanglement and Bell nonlocality, has recently attracted renewed interest [3–11]. Steering is a quantum phenomenon that allows one to manipulate the state of one subsystem by performing measurements on the other entangled subsystem. After being discovered by Schrödinger [1,2], the concept of quantum steering was studied by Einstein, Podolsky, and Rosen (EPR) in their well-known 1935 paper [12], and was treated as the core concept of the EPR paradox [13]. The experimental detection of quantum steering, i.e., for the demonstration of the EPR paradox, was first proposed by Reid [14], after which several experiments were performed to demonstrate quantum steering and its asymmetry [15–18]. Most recently, Kogias *et al.* [19] proposed an operational measure of quantum steering for bipartite Gaussian states of continuous variable systems. They found that for two-mode Gaussian states, the quantum steering reduces to a form of coherent information and the asymmetry of steering cannot exceed $\ln 2$ [19].

On the other hand, relativistic quantum information [20–33], the study of quantum information processes and concepts in a relativistic setting, has been a blooming

area of research for both conceptual and experimental reasons. Understanding quantum phenomena in a relativistic framework is necessary because the realistic quantum systems are essentially noninertial. It was experimentally demonstrated that the gravitational frequency shift effects have a remarkable influence on the precision of atomic clocks for a variation of 0.33 m in height [34]. In addition, relativistic effects of the Earth notably affect satellite-based quantum information processing tasks [35–37] and quantum clock synchronization [38]. Quantum information also plays a prominent role in the study of the thermodynamics and information loss problem [39,40] of black holes. Therefore, it is of great interest to study how relativistic effects influence the properties of quantum steerability [41] in a curved spacetime.

In this work we present a quantitative investigation of Gaussian quantum steerability for free bosonic modes in the background of an eternal Schwarzschild black hole [27]. We assume that Alice and Bob initially share a two-mode squeezed Gaussian state with squeezing s [22]. Alice is a Kruskal observer who stays stationary at an asymptotically flat region (or freely falls into the black hole), while Bob is a Schwarzschild observer who hovers near the event horizon of the black hole with uniform acceleration. A vacuum state observed by Alice would be detected as a thermal state from Bob's viewpoint. From a general relativity viewpoint, the temperature T of the Hawking thermal bath depends on surface gravity κ of the black hole. In a quantum information scenario, such a process can be described as a bosonic amplification channel acting on

*jcwang@hunnu.edu.cn

†jljing@hunnu.edu.cn

‡hfan@iphy.ac.cn

Bob's quantum state [22,23]. We will calculate the Gaussian quantum steering $\mathcal{G}^{A \rightarrow B}$, which quantifies to what extent Bob's mode can be steered by Alice's measurements, and the steering $\mathcal{G}^{B \rightarrow A}$, to verify the asymmetric property of steerability in the curved space-time. We find that the quantum steerability between Alice and Bob decreases with the increase of the Hawking temperature parameter r , while the steerability between Bob and anti-Bob segregated by the event horizon is generated at the same time. We also find that the attainment of maximal steering asymmetry indicates the transition between one-way steerability and both-way steerability for two-mode Gaussian states under the influence of Hawking radiation.

The outline of the paper is as follows. In Sec. II we briefly introduce the definition and measure of bipartite Gaussian quantum steering. In Sec. III we discuss how the Unruh-Hawking effects of the black hole can be described by a bosonic amplification channel acting on the covariance matrix of a bipartite system. In Sec. IV we study the behavior of Gaussian quantum steering and its asymmetry in the background of a Schwarzschild black hole. The last section is devoted to a brief summary.

II. DEFINITION AND MEASUREMENT OF GAUSSIAN QUANTUM STEERING

Let us first briefly introduce the definition and measurement of Gaussian quantum steering. We consider a continuous variable quantum system [42] represented by $(n+m)$ bosonic modes of a bipartite system ρ_{AB} , composed of a subsystem A of n modes and a subsystem B of m modes. For each mode i , the corresponding phase space variables can be denoted by $\hat{a}_i^A = \frac{\hat{x}_i^A + i\hat{p}_i^A}{\sqrt{2}}$ and $\hat{a}_i^B = \frac{\hat{x}_i^B + i\hat{p}_i^B}{\sqrt{2}}$. The phase-space operators $\hat{x}_i^{A(B)}$, $\hat{p}_i^{A(B)}$ can be grouped together into a vector $\hat{R} = (\hat{x}_1^A, \hat{p}_1^A, \dots, \hat{x}_n^A, \hat{p}_n^A, \hat{x}_1^B, \hat{p}_1^B, \dots, \hat{x}_m^B, \hat{p}_m^B)^\top$, which satisfies the canonical commutation relations $[\hat{R}_i, \hat{R}_j] = i\Omega_{ij}$, with $\Omega = \bigoplus_1^{n+m} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ being the symplectic form. Any Gaussian state ρ_{AB} is completely specified by its first and second statistical moments. The latter is a covariance matrix with elements $\sigma_{ij} = \text{Tr}[\{\hat{R}_i, \hat{R}_j\}_+ \rho_{AB}]$ and can always be put into a block form,

$$\sigma_{AB} = \begin{pmatrix} A & C \\ C^\top & B \end{pmatrix}. \quad (1)$$

The covariance matrix σ_{AB} can describe a physical quantum state if and only if (iff) it satisfies the bona fide uncertainty principle relation

$$\sigma_{AB} + i(\Omega_{AB}) \geq 0. \quad (2)$$

Now let us give the definition of steerability. After Alice performs a set of measurements \mathcal{M}_A , the bipartite state ρ_{AB} is $A \rightarrow B$ steerable (i.e., Alice can steer Bob) iff it is *not* possible for every pair of local observables R_A (on A with outcome r_A) and R_B to express the joint probability as $P(r_A, r_B | R_A, R_B, \rho_{AB}) = \sum_\lambda \varphi_\lambda \varphi(r_A | R_A, \lambda) P(r_B | R_B, \rho_\lambda)$ [3]. In other words, at least one measurement pair (R_A and R_B) is required to violate this expression when φ_λ is fixed across all measurements. Here φ_λ and $\varphi(r_A | R_A, \lambda)$ are probability distributions and $P(r_B | R_B, \rho_\lambda)$ is the conditional probability distribution associated to the extra condition of being evaluated on the state ρ_λ . It has been shown in [3] that a general $(n+m)$ -mode Gaussian state ρ_{AB} is $A \rightarrow B$ steerable by Alice's Gaussian measurements iff the condition

$$\sigma_{AB} + i(0_A \oplus \Omega_B) \geq 0 \quad (3)$$

is violated. Henceforth, a violation of (3) is necessary and sufficient for the Gaussian $A \rightarrow B$ steerability.

One can define the Gaussian $A \rightarrow B$ steering to quantify how much a bipartite Gaussian state σ_{AB} is steerable by the measurements performed by Alice,

$$\mathcal{G}^{A \rightarrow B}(\sigma_{AB}) := \max \left\{ 0, - \sum_{j: \bar{v}_j^B < 1} \ln(\bar{v}_j^B) \right\}, \quad (4)$$

where $\{\bar{v}_j^B\}$ are the symplectic eigenvalues of the Schur complement of A in the covariance matrix σ_{AB} [19].

The $A \rightarrow B$ steering vanishes iff the state described by σ_{AB} is nonsteerable by Alice's measurements, and it generally quantifies the amount by which the condition (3) fails to be fulfilled. The Gaussian steerability $A \rightarrow B$ acquires a particularly simple form when the steered party Bob has one mode only (i.e., $m=1$) [19],

$$\begin{aligned} \mathcal{G}^{A \rightarrow B}(\sigma_{AB}) &= \max \left\{ 0, \frac{1}{2} \ln \frac{\det A}{\det \sigma_{AB}} \right\} \\ &= \max \{ 0, \mathcal{S}(A) - \mathcal{S}(\sigma_{AB}) \}, \end{aligned} \quad (5)$$

where $\mathcal{S}(\sigma) = \frac{1}{2} \ln(\det \sigma)$ is the Rényi-2 entropy [43]. Similarly, a corresponding measure of Gaussian $B \rightarrow A$ steerability can be obtained by swapping the roles of A and B , resulting in an expression like Eq. (5). Unlike quantum entanglement, the quantum steering is an asymmetric property [19]: a quantum state may be steerable from Alice to Bob, but not vice versa. In a quantum information scenario, quantum steering corresponds to the task of entanglement distribution by an untrusted party [3]. If Alice and Bob share a state that is steerable from Alice to Bob, then Alice is able to convince Bob (who does not trust Alice) that their shared state is entangled by performing local measurements and classical communication [3].

III. BOSONIC AMPLIFICATION CHANNEL DESCRIPTION OF THE HAWKING EFFECT

In this section we will show how the Unruh-Hawking radiation of the black hole can be described by a bosonic amplification channel [23], which is a Gaussian channel. The spacetime background near a Schwarzschild black hole is described by the metric

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2), \quad (6)$$

where M represents the mass of the black hole. Throughout this paper we set $G = c = \hbar = \kappa_B = 1$.

In the background of the black hole, a massless bosonic field ϕ obeys the Klein-Gordon equation [44]

$$\frac{1}{\sqrt{-g}} \partial_{x^\mu} \left(\sqrt{-g} g^{\mu\nu} \frac{\partial \phi}{\partial x^\nu} \right) = 0. \quad (7)$$

Solving Eq. (7) near the event horizon, we obtain a set of positive-frequency outgoing modes propagating in the regions inside and outside of the event horizon,

$$\Phi_{\Omega, \text{in}}^+ \sim \phi(r) e^{i\omega u}, \quad (8)$$

$$\Phi_{\Omega, \text{out}}^+ \sim \phi(r) e^{-i\omega u}, \quad (9)$$

where $u = t - r_*$ and $r_* = r + 2M \ln \frac{r-2M}{2M}$ is the tortoise coordinate in Schwarzschild spacetime.

Equations (8) and (9) can be used to expand the scalar field Φ as

$$\Phi = \int d\Omega [\hat{a}_{\Omega}^{\text{out}} \Phi_{\Omega, \text{out}}^+ + \hat{b}_{\Omega}^{\text{out}\dagger} \Phi_{\Omega, \text{out}}^- + \hat{a}_{\Omega}^{\text{in}} \Phi_{\Omega, \text{in}}^+ + \hat{b}_{\Omega}^{\text{in}\dagger} \Phi_{\Omega, \text{in}}^-], \quad (10)$$

where $\hat{a}_{\Omega}^{\text{out}}$ and $\hat{b}_{\Omega}^{\text{out}\dagger}$ are the bosonic particle annihilation and antiboson creation operators acting on the state in the exterior region of the black hole, and $\hat{a}_{\Omega}^{\text{in}}$ and $\hat{b}_{\Omega}^{\text{in}\dagger}$ are the boson annihilation and antiboson creation operators acting on the interior region states. The Schwarzschild vacuum $|0\rangle_S$ can be defined as $\hat{a}_{\Omega}^{\text{out}}|0\rangle_S = \hat{a}_{\Omega}^{\text{in}}|0\rangle_S = 0$; therefore, the modes $\Phi_{\Omega, \text{out}}^\pm$ and $\Phi_{\Omega, \text{in}}^\pm$ are usually called Schwarzschild modes [27,29,45–47].

Making an analytic continuation for Eqs. (8) and (9), we find a complete basis for positive energy modes, i.e., the Kruskal modes, according to the suggestion of Domour and Ruffini [48]. The Kruskal modes can be used to define the Hartle-Hawking vacuum, which corresponds to the Minkowski vacuum in flat spacetime. Then, we can quantize the massless scalar field in the Schwarzschild and Kruskal modes, respectively [24,29], and can obtain the

Bogoliubov transformations [44,49] between the modes and operators in different coordinates. However, as performed in [46], an inertial observer Alice has the freedom to create excitations in any accessible mode Ω_j , $\forall j$. Hence, one cannot map a single-frequency Kruskal mode into a set of single-frequency modes in Schwarzschild coordinates [46]. To avoid this obstacle we employ the Unruh basis, which provides an intermediate step between the Kruskal and Schwarzschild modes. The relations between the Unruh and Schwarzschild operators take the form

$$\begin{aligned} C_{\Omega, \text{R}} &= (\cosh r_{\Omega} \hat{a}_{\Omega, \text{out}} - \sinh r_{\Omega} \hat{b}_{\Omega, \text{in}}^{\dagger}), \\ C_{\Omega, \text{L}} &= (\cosh r_{\Omega} \hat{a}_{\Omega, \text{in}} - \sinh r_{\Omega} \hat{b}_{\Omega, \text{out}}^{\dagger}), \\ D_{\Omega, \text{R}}^{\dagger} &= (-\sinh r_{\Omega} \hat{a}_{\Omega, \text{out}} + \cosh r_{\Omega} \hat{b}_{\Omega, \text{in}}^{\dagger}), \\ D_{\Omega, \text{L}}^{\dagger} &= (-\sinh r_{\Omega} \hat{a}_{\Omega, \text{in}} + \cosh r_{\Omega} \hat{b}_{\Omega, \text{out}}^{\dagger}), \end{aligned} \quad (11)$$

where $\sinh r_{\Omega} = (e^{\frac{2r_{\Omega}}{\kappa}} - 1)^{-\frac{1}{2}}$ and κ is the surface gravity of the black hole, which relates the Hawking temperature T of the black hole by $T = \kappa/2\pi$.

A generic Schwarzschild-Fock state $|nm, pq\rangle_{\Omega}$ describing both particles and antiparticles can be written as

$$|nm, pq\rangle_{\Omega} := \frac{\hat{a}_{\Omega, \text{out}}^{\dagger n} \hat{b}_{\Omega, \text{in}}^{\dagger m} \hat{b}_{\Omega, \text{out}}^{\dagger p} \hat{a}_{\Omega, \text{in}}^{\dagger q}}{\sqrt{n!} \sqrt{m!} \sqrt{p!} \sqrt{q!}} |0\rangle_S, \quad (12)$$

where the \pm sign denotes the particle and antiparticle, respectively. This allows us to write the Unruh vacuum as [45,47]

$$|0_{\Omega}\rangle_{\text{U}} = \frac{1}{\cosh r_{\Omega}^2} \sum_{n, m=0}^{\infty} \tanh r_{\Omega}^{n+m} |nn, mm\rangle_{\Omega}, \quad (13)$$

where $|0_{\Omega}\rangle_{\text{U}}$ is a shortcut notation used to underline that each Unruh mode Ω is mapped into a Schwarzschild mode Ω .

One-particle Unruh states are defined as $|1_j\rangle_{\text{U}}^+ = c_{\Omega, \text{U}}^{\dagger} |0\rangle_{\text{H}}$, $|1_j\rangle_{\text{U}}^- = d_{\Omega, \text{U}}^{\dagger} |0\rangle_{\text{H}}$, where the Unruh particle and antiparticle creation operators are defined as linear combinations of the two Unruh operators $c_{\Omega, \text{U}}^{\dagger} = q_{\text{R}} C_{\Omega, \text{R}}^{\dagger} + q_{\text{L}} C_{\Omega, \text{L}}^{\dagger}$ and $d_{\Omega, \text{U}}^{\dagger} = q_{\text{R}} D_{\Omega, \text{R}}^{\dagger} + q_{\text{L}} D_{\Omega, \text{L}}^{\dagger}$, where q_{R} , q_{L} satisfy $|q_{\text{R}}|^2 + |q_{\text{L}}|^2 = 1$. The operator $c_{\Omega, \text{U}}^{\dagger}$ indicates the creation of a pair of particles [29], i.e., a boson with mode Ω in the exterior region and an antiboson in the interior region of the black hole. Similarly, the creation operator $d_{\Omega, \text{U}}^{\dagger}$ means that an antiboson and a boson are created outside and inside the event horizon, respectively. The particles and antiparticles can radiate randomly toward the inside and outside regions from the event horizon with the total probability $|q_{\text{R}}|^2 + |q_{\text{L}}|^2 = 1$. In this situation, $|q_{\text{R}}| = 1$ means that all the particles move toward the black hole exteriors while all the antiparticles move to the inside

region [29]; i.e., only particles can be detected as Hawking radiation. If we fix $q_R = 1$ and assume that Bob has a detector sensitive only to the particle modes, Eq. (13) reduces to $|0_\Omega\rangle_H = \frac{1}{\cosh r_\Omega} \sum_{n=0}^{\infty} \tanh r_\Omega^n |nn\rangle_\Omega$ and can be described by a bosonic amplification channel [23,46]. Then, the effect of Hawking radiation corresponds to a two-mode squeezing operator acting on the input state $|\psi_0\rangle_{\text{out}}$ for Bob,

$$\rho_{\text{out}} = \text{tr}_{\text{in}} \{ \hat{U}_{\text{out,in}}(r_\Omega) [(|\psi_0\rangle\langle\psi_0|)_{\text{out}} \otimes (|0\rangle\langle 0|)_{\text{in}}] \hat{U}_{\text{out,in}}^\dagger(r_\Omega) \}, \quad (14)$$

where $\hat{U}_{\text{out,in}}(r_\Omega) = e^{r_\Omega(\hat{b}_{\Omega,\text{out}}^\dagger \hat{b}_{\Omega,\text{in}}^\dagger - \hat{a}_{\Omega,\text{out}} \hat{a}_{\Omega,\text{in}})}$ is the two-mode squeezing operator. Hereafter we write r_Ω as r for convenience. It is worth noting that the squeezing transformation $\hat{U}_{\text{out,in}}(r)$ is a Gaussian operation; it will preserve the Gaussianity of the input states. A symplectic phase-space representation of the two-mode squeezing operation $\hat{U}_{\text{out,in}}(r)$ has the form

$$S_{B,\bar{B}}(r) = \begin{pmatrix} \cosh r & 0 & \sinh r & 0 \\ 0 & \cosh r & 0 & -\sinh r \\ \sinh r & 0 & \cosh r & 0 \\ 0 & -\sinh r & 0 & \cosh r \end{pmatrix}. \quad (15)$$

IV. THE EFFECT OF HAWKING RADIATION ON GAUSSIAN QUANTUM STEERABILITY

In this paper we study a massless scalar field ϕ for two Unruh modes A and B whose state, as prepared in an inertial frame, is initialized in a pure, entangled Gaussian two-mode squeezed state with squeezing s [22]. The initial state can be described from an inertial perspective, via its covariance matrix

$$\sigma_{AB}^{(M)}(s) = \begin{pmatrix} \mathcal{C}_2 & 0 & \mathcal{S}_2 & 0 \\ 0 & \mathcal{C}_2 & 0 & -\mathcal{S}_2 \\ \mathcal{S}_2 & 0 & \mathcal{C}_2 & 0 \\ 0 & -\mathcal{S}_2 & 0 & \mathcal{C}_2 \end{pmatrix}, \quad (16)$$

where $\mathcal{C}_2 = \cosh(2s)$ and $\mathcal{S}_2 = \sinh(2s)$, and the state is prepared in Unruh modes A and B . From Eq. (14) one can see that the change from Unruh modes to Schwarzschild modes corresponds to a two-mode squeezing operation associating to the symplectic transformation $S_{B,\bar{B}}(r)$. Under such transformation, mode B is mapped into two sets of Schwarzschild regions for the exterior region (out) and the interior region (in) of the black hole, respectively. From an inertial viewpoint, the system is bipartite, but an extra set of modes \bar{B} becomes relevant from the perspective of a Schwarzschild observer. Therefore, a complete description of the system involves three modes: mode A , described by

Alice; mode B , described by the Schwarzschild observer Bob; and mode \bar{B} , described by a hypothetical observer anti-Bob confined inside the event horizon. The covariance matrix of the Gaussian state describing the complete system is given by [22]

$$\sigma_{AB\bar{B}}^{(a)}(s, r) = [I_A \oplus S_{B,\bar{B}}(r)] [\sigma_{AB}^{(M)}(s) \oplus I_{\bar{B}}] [I_A \oplus S_{B,\bar{B}}(r)], \quad (17)$$

where $S_{B,\bar{B}}(r)$ is the phase-space representation of the two-mode squeezing operation, and we use the fact that the covariance matrix of a vacuum state is an identity matrix.

Because the exterior region is causally disconnected from the interior region of the black hole, Alice and Bob cannot access mode \bar{B} inside the event horizon [24]. Taking the trace over mode \bar{B} , we obtain covariance matrix $\sigma_{AB}(s, r)$ for Alice and Bob,

$$\sigma_{AB}(s, r) = \begin{pmatrix} \mathcal{A}_{AB} & \mathcal{C}_{AB} \\ \mathcal{C}_{AB}^\top & \mathcal{B}_{AB} \end{pmatrix}, \quad (18)$$

with elements $\mathcal{A}_{AB} = \cosh(2s)I_2$, $\mathcal{C}_{AB} = [\cosh(r)\sinh(2s)]Z_2$, and $\mathcal{B}_{AB} = [\cosh(2s)\cosh^2(r) + \sinh^2(r)]I_2$ with $Z_2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. Employing Eq. (5), we obtain an analytic expression of the $A \rightarrow B$ Gaussian steering,

$$\mathcal{G}^{A \rightarrow B}(\sigma_{AB}) = \max \left\{ 0, \ln \frac{\cosh(2s)}{\cosh^2(r) + \cosh(2s)\sinh^2(r)} \right\}. \quad (19)$$

From Eq. (19) we can see that the $A \rightarrow B$ Gaussian steering depends not only the squeezing parameter s , but also the Hawking temperature parameter r ; this means that the Hawking radiation of the black hole will affect the $A \rightarrow B$ steerability because $\sinh r = (e^{\frac{2}{r}} - 1)^{-\frac{1}{2}}$.

It is well known that the symmetric properties of quantum steering is a crucial issue. For example, it was recently found that the quantum steerability from A to B is asymmetric [19] to the $B \rightarrow A$ steerability in a Gaussian setting, which has been experimentally demonstrated in [16] in a flat spacetime. To obtain understanding of this issue, we here calculate the steerability $\mathcal{G}^{B \rightarrow A}$ and check if the relation $\mathcal{G}^{A \rightarrow B} = \mathcal{G}^{B \rightarrow A}$ is satisfied in the Schwarzschild spacetime. After some calculations, the $B \rightarrow A$ steering is found to be $\mathcal{G}^{B \rightarrow A}(\sigma_{AB}) = \max \left\{ 0, \ln \frac{\cosh^2(r)\cosh(2s) + \sinh^2(r)}{\cosh^2(r) + \cosh(2s)\sinh^2(r)} \right\}$. To check the degree of steerability asymmetric in the curved spacetime, we define the Gaussian steering asymmetry as

$$\mathcal{G}_{AB}^\Delta = |\mathcal{G}^{B \rightarrow A} - \mathcal{G}^{A \rightarrow B}|. \quad (20)$$

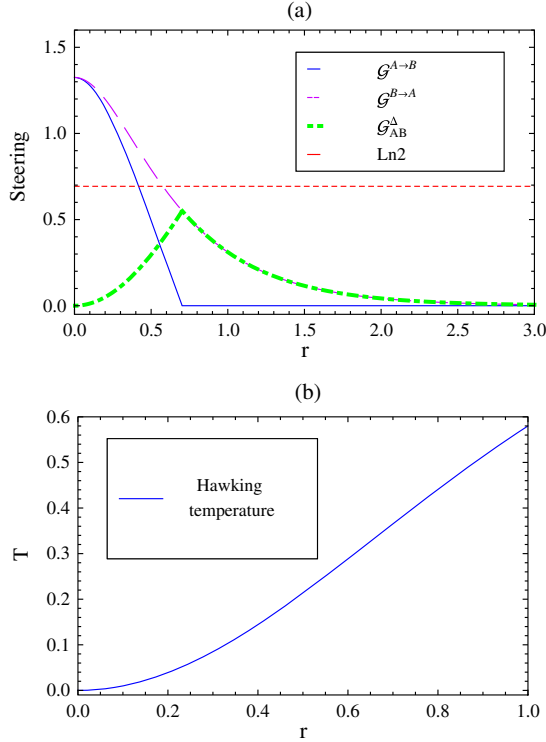


FIG. 1. (a) The Gaussian quantum steering $\mathcal{G}^{A \rightarrow B}$ (solid line), $\mathcal{G}^{B \rightarrow A}$ (dashed line), and steering asymmetry $\mathcal{G}_{AB}^{\Delta}$ (dot-dashed line) between Alice and Bob as a function of the Hawking temperature parameter r of the black hole. The squeezing parameter s of the initial state is fixed as $s = 1$. (b) The relation between the parameter r and Hawking temperature T by fixing $\Omega = 1$.

In Fig. 1(a) we plot the steerabilities $\mathcal{G}^{B \rightarrow A}$, $\mathcal{G}^{A \rightarrow B}$ as well as the Gaussian steering asymmetry $\mathcal{G}_{AB}^{\Delta}$ as a function of the black hole's Hawking temperature parameter r for a fixed squeezing $s = 1$. The relation between the parameter r and Hawking temperature T is given in Fig. 1(b) which shows that T is a monotonically increasing function of r . From Fig. 1(a) we can see that both the $A \rightarrow B$ and $B \rightarrow A$ steering decrease with the increase of Hawking temperature parameter r , which means that the thermal noise introduced by Hawking effect will destroy the steerability between an inertial and an accelerated observer. It is shown that the $A \rightarrow B$ steering decreases quickly and suffers from a “sudden death” with increasing r ; this is quite different from the behavior of quantum entanglement in a relativistic setting [21,22,24], where entanglement reduces to zero only at the limit of $r \rightarrow \infty$. It is shown that the $B \rightarrow A$ steering is always bigger than the $A \rightarrow B$ steering and avoids sudden death with the increase of r , which indicates that the inertial part steering the noninertial part is easier than the noninertial part steering the inertial part. From Fig. 1(a) we can see that $\mathcal{G}^{A \rightarrow B} \neq \mathcal{G}^{B \rightarrow A}$ for any finite-valued r , which means that the steering is always asymmetric between Alice and Bob in the curved spacetime. The

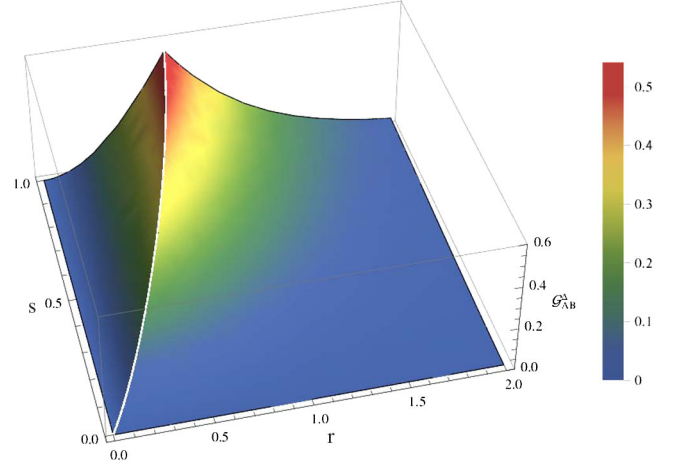


FIG. 2. The Gaussian steering asymmetry $\mathcal{G}_{AB}^{\Delta}$ as functions of the Hawking temperature parameter r and the squeezing parameter s .

steering asymmetry increases with decreasing steerability either way; this means that the Hawking radiation destroys the symmetry of steerability. We find that the parameter setting maximizing the steering asymmetry of the state σ_{AB} is $s = \text{arccosh}(\frac{\cosh^2 r}{1 - \sinh^2 r})$. This condition is the same as that when the $A \rightarrow B$ steering experiences sudden death in Fig. 1(a). That is, the steering asymmetry is maximal when the state is nonsteerable in the $A \rightarrow B$ way. Therefore, the parameter r attaining the peak of steering asymmetry is that which indicates the system is currently experiencing a transformation from both-way steerability to one-way steerability. In other words, attainment of maximal steering asymmetry indicates the transition between one-way steerability and both-way steerability for two-mode Gaussian states under the influence of Hawking radiation.

To better understand the interplay between squeezing and the Hawking effect in the generation of Gaussian quantum steering, we plot the Gaussian steering asymmetry $\mathcal{G}_{AB}^{\Delta}$ as functions of the Hawking temperature parameter r and the squeezing s in Fig. 2. It is shown that $\mathcal{G}_{AB}^{\Delta}$ equals zero; i.e., the steerability is asymmetric when $s = 0$ and $r \rightarrow 0$ because $\mathcal{G}^{A \rightarrow B} = \mathcal{G}^{B \rightarrow A} = 0$ in these two cases. The steering asymmetry monotonically increases with increasing squeezing parameter s , which means that the quantum resources shared in the initial state play a dominant role in quantum steering. In addition, the maximal steerability point enlarges its value with increasing s .

We then study the steering between mode B and mode \bar{B} , which propagate, respectively, outside and inside the event horizon. Tracing over the modes in A , we obtain the covariance matrix $\sigma_{B\bar{B}}(s, r)$ for Bob and anti-Bob,

$$\sigma_{B\bar{B}}(s, r) = \begin{pmatrix} \mathcal{A}_{B\bar{B}} & \mathcal{C}_{B\bar{B}} \\ \mathcal{C}_{B\bar{B}}^{\top} & \mathcal{B}_{B\bar{B}} \end{pmatrix}, \quad (21)$$

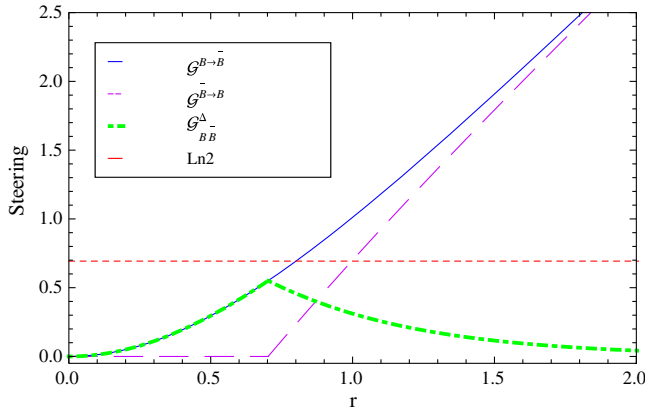


FIG. 3. The Gaussian quantum steering $\mathcal{G}^{B \rightarrow \bar{B}}$ (solid line), $\mathcal{G}^{\bar{B} \rightarrow B}$ (dashed line), and steering asymmetry $\mathcal{G}_{BB}^{\Delta}$ (dot-dashed line) between Bob and anti-Bob as a function of the Hawking temperature parameter r of the black hole. The squeezing parameter s of the initial state is fixed as $s = 1$.

where $\mathcal{A}_{B\bar{B}} = [\cosh(2s)\cosh^2(r) + \sinh^2(r)]I_2$, $\mathcal{C}_{B\bar{B}} = [\cosh^2(s) \sinh(2r)]Z_2$, and $\mathcal{B}_{B\bar{B}} = [\cosh^2(r) + \cosh(2s) \sinh^2(r)]I_2$. Using Eqs. (5) and (20), we obtain analytic expressions of the $B \rightarrow \bar{B}$ and $\bar{B} \rightarrow B$ steering, which are $\mathcal{G}^{B \rightarrow \bar{B}}(\sigma_{B\bar{B}}) = \max\{0, \ln[\cosh^2(r) + \frac{\sinh^2(r)}{\cosh(2s)}]\}$ and $\mathcal{G}^{\bar{B} \rightarrow B}(\sigma_{B\bar{B}}) = \max\{0, \ln[\sinh^2(r) + \frac{\cosh^2(r)}{\cosh(2s)}]\}$, respectively. The Gaussian steering asymmetry $\mathcal{G}_{BB}^{\Delta}$ between Bob and anti-Bob can be computed in a similar way.

In Fig. 3 we plot the Gaussian quantum steering and steering asymmetry between Bob and anti-Bob as a function of the Hawking temperature parameter r with fixed squeezing $s = 1$. It is shown that quantum steerability is generated between Bob and anti-Bob with the increase of the Hawking temperature parameter r . The steerability $\mathcal{G}^{B \rightarrow \bar{B}}$ is nonzero for any r , while the steerability from anti-Bob to Bob appears as “sudden birth” behavior with the increase of the Hawking temperature parameter r . It is interesting to find that the maximizing condition for the $\sigma_{B\bar{B}}$ steering asymmetry is also $s = \text{arccosh}(\frac{\cosh^2 r}{1 - \sinh^2 r})$. Therefore, we arrive at the conclusion that this value of s is a critical point of steering asymmetry in the curved spacetime. Again, the maximal steering asymmetry for the state $\sigma_{B\bar{B}}$ is obtained when the $\bar{B} \rightarrow B$ steering appears as sudden birth. That is, the parameter r attaining the peak of $\bar{B} \rightarrow B$ steering asymmetry is the one that indicates the system is experiencing a transformation from one-way steerability to both-way steerability. In addition, we find that Bob and anti-Bob can steer each other when the parameter r is bigger than a critical point, even though they are separated by the event horizon; this verifies that quantum steering is a nonlocal quantum correlation. We again find that the quantum steering between Bob and anti-Bob is always asymmetric for any Hawking temperature and the maximum steering asymmetry cannot exceed $\ln 2$,

which means that the state never evolves to an extremal state under the effects of Hawking radiation.

Finally, let us present a physical interpretation of the generation of quantum steering across the event horizon. In this paper, anti-Bob is a hypothetical observer inside the event horizon of the black hole. It is well known that Hawking radiation can be explained as the spontaneous creation of particles and antiparticles by quantum fluctuations near the event horizon. The particles and antiparticles will randomly radiate ingoing to and outgoing from the event horizon. If a particle is observed and measured by the observer Bob outside the event horizon, the state of the other antiparticle is steered and might observed by anti-Bob. Therefore, the mode \bar{B} can be steered by measuring mode B because they are pair generated and shared initial entanglement from the event horizon.

V. CONCLUSIONS

The effect of the Hawking effect on Gaussian quantum steering and its asymmetry in Schwarzschild spacetime are investigated. We consider three subsystems: subsystem A , observed by an inertial observer Alice; subsystem B , observed by accelerated Bob hovers near the event horizon; and subsystem \bar{B} , observed by an imaginary observer anti-Bob inside the event horizon. We obtain a phase-space description of quantum state evolution under the influence of the thermal bath induced by Hawking radiation. It is shown that quantum steerability between Alice and Bob decreases as the Hawking temperature parameter r increases. That is to say, thermal noise introduced by the Hawking effect will destroy the steerability between an inertial and an accelerated observer. However, the steerability between two observers segregated by the event horizon of the black hole is generated due to the effect of Hawking radiation. It is found that the steering from Alice to Bob suffers from a sudden death and the steering from anti-Bob to Bob experiences a sudden birth with increasing Hawking temperature, which is quite different from the behavior of quantum entanglement in accelerated setting [21,22] and curved spacetime [24,29]. It is intriguing to find that the steering is always asymmetric and is endowed with a maximum steering asymmetry for a fixed r , and that the maximum steering asymmetry cannot exceed $\ln 2$ in the curved spacetime. It has been shown that $s = \text{arccosh}(\frac{\cosh^2 r}{1 - \sinh^2 r})$ is a critical point of steering asymmetry under the influence of Hawking radiation. In addition, the parameters attaining the peaks of steering asymmetry are obtained when the $A \rightarrow B$ steering experiences sudden death or the $\bar{B} \rightarrow B$ steering experiences sudden birth. That is, the attainment of maximal steering asymmetry indicates a transition point of the two-mode Gaussian state in the Schwarzschild spacetime. These results should be significant both for giving us more information from a black hole by measuring the

Hawking radiation and for our general understanding of quantum steering in a relativistic quantum system.

ACKNOWLEDGMENTS

We acknowledge the reviewer for helpful comments, and Ioannis Kogias and Gerardo Adesso for helpful discussions. J. Wang is supported the National Natural Science Foundation of China under Grant No. 11305058, the

Doctoral Scientific Fund Project of the Ministry of Education of China under Grant No. 20134306120003, and the Postdoctoral Science Foundation of China under Grants No. 2014M560129 and No. 2015T80146. J. Jing is supported the National Natural Science Foundation of China under Grant No. 11475061. H. Fan is supported the National Natural Science Foundation of China under Grant No. 91536108.

-
- [1] E. Schrödinger, *Proc. Cambridge Philos. Soc.* **31**, 555 (1935).
- [2] E. Schrödinger, *Proc. Cambridge Philos. Soc.* **32**, 446 (1936).
- [3] H. M. Wiseman, S. J. Jones, and A. C. Doherty, *Phys. Rev. Lett.* **98**, 140402 (2007).
- [4] P. Skrzypczyk, M. Navascués, and D. Cavalcanti, *Phys. Rev. Lett.* **112**, 180404 (2014).
- [5] S. P. Walborn, A. Salles, R. M. Gomes, F. Toscano, and P. H. Souto Ribeiro, *Phys. Rev. Lett.* **106**, 130402 (2011).
- [6] J. Bowles, T. Vértesi, M. T. Quintino, and N. Brunner, *Phys. Rev. Lett.* **112**, 200402 (2014).
- [7] Q. Y. He, Q. H. Gong, and M. D. Reid, *Phys. Rev. Lett.* **114**, 060402 (2015).
- [8] C.-M. Li, K. Chen, Y.-N. Chen, Q. Zhang, Y.-A. Chen, and J.-W. Pan, *Phys. Rev. Lett.* **115**, 010402 (2015).
- [9] M. Marciniak, A. Rutkowski, Z. Yin, M. Horodecki, and R. Horodecki, *Phys. Rev. Lett.* **115**, 170401 (2015).
- [10] Q. Y. He, L. Rosales-Zárate, G. Adesso, and M. D. Reid, *Phys. Rev. Lett.* **115**, 180502 (2015).
- [11] A. B. Sainz, N. Brunner, D. Cavalcanti, P. Skrzypczyk, and T. Vertesi, *Phys. Rev. Lett.* **115**, 190403 (2015).
- [12] A. Einstein, B. Podolsky, and N. Rosen, *Phys. Rev.* **47**, 777 (1935).
- [13] M. D. Reid, P. D. Drummond, W. P. Bowen, E. G. Cavalcanti, P. K. Lam, H. A. Bachor, U. L. Andersen, and G. Leuchs, *Rev. Mod. Phys.* **81**, 1727 (2009).
- [14] M. D. Reid, *Phys. Rev. A* **40**, 913 (1989).
- [15] D. J. Saunders, S. J. Jones, H. M. Wiseman, and G. J. Pryde, *Nat. Phys.* **6**, 845 (2010).
- [16] V. Handchen, T. Eberle, S. Steinlechner, A. Samblowski, T. Franz, R. F. Werner, and R. Schnabel, *Nat. Photonics* **6**, 598 (2012); S. Wollmann, Nathan Walk, Adam J. Bennet, Howard M. Wiseman, and Geoff J. Pryde, *Phys. Rev. Lett.* **116**, 160403 (2016); K. Sun, Xiang-Jun Ye, Jin-Shi Xu, Xiao-Ye Xu, Jian-Shun Tang, Yu-Chun Wu, Jing-Ling Chen, Chuan-Feng Li, and Guang-Can Guo, *Phys. Rev. Lett.* **116**, 160404 (2016).
- [17] K. Sun, J.-S. Xu, X.-J. Ye, Y.-C. Wu, J.-L. Chen, C.-F. Li, and G.-C. Guo, *Phys. Rev. Lett.* **113**, 140402 (2014).
- [18] S. Kocsis, M. J. W. Hall, A. J. Bennet, and G. J. Pryde, *Nat. Commun.* **6**, 5886 (2015).
- [19] I. Kogias, A. R. Lee, S. Ragy, and G. Adesso, *Phys. Rev. Lett.* **114**, 060403 (2015).
- [20] A. Peres and D. R. Terno, *Rev. Mod. Phys.* **76**, 93 (2004).
- [21] I. Fuentes-Schuller and R. B. Mann, *Phys. Rev. Lett.* **95**, 120404 (2005).
- [22] G. Adesso, I. Fuentes-Schuller, and M. Ericsson, *Phys. Rev. A* **76**, 062112 (2007).
- [23] M. Aspachs, G. Adesso, and I. Fuentes, *Phys. Rev. Lett.* **105**, 151301 (2010).
- [24] J. Wang, Q. Pan, and J. Jing, *Phys. Lett. B* **692**, 202 (2010).
- [25] D. J. Hosler, C. van de Bruck, and P. Kok, *Phys. Rev. A* **85**, 042312 (2012).
- [26] N. Friis, A. R. Lee, K. Truong, C. Sabín, E. Solano, G. Johansson, and I. Fuentes, *Phys. Rev. Lett.* **110**, 113602 (2013).
- [27] J. Doukas, E. G. Brown, A. Dragan, and R. B. Mann, *Phys. Rev. A* **87**, 012306 (2013).
- [28] D. Su, and T. C. Ralph, *Phys. Rev. D* **90**, 084022 (2014).
- [29] J. Wang, J. Jing, and H. Fan, *Phys. Rev. D* **90**, 025032 (2014).
- [30] M. Ahmadi, D. E. Bruschi, and I. Fuentes, *Phys. Rev. D* **89**, 065028 (2014); M. Ahmadi, A. R. H. Smith, and A. Dragan, *Phys. Rev. A* **92**, 062319 (2015).
- [31] A. Chęcińska and A. Dragan, *Phys. Rev. A* **92**, 012321 (2015).
- [32] A. Blasco, L. J. Garay, M. Martín-Benito, and E. Martín-Martínez, *Phys. Rev. Lett.* **114**, 141103 (2015).
- [33] B. Richter and Y. Omar, *Phys. Rev. A* **92**, 022334 (2015).
- [34] C. W. Chou, D. B. Hume, T. Rosenband, and D. J. Wineland, *Science* **329**, 1630 (2010).
- [35] J. Y. Wang *et al.*, *Nat. Photonics* **7**, 387 (2013).
- [36] D. E. Bruschi, T. C. Ralph, I. Fuentes, T. Jennewein, and M. Razavi, *Phys. Rev. D* **90**, 045041 (2014).
- [37] D. E. Bruschi, A. Datta, R. Ursin, T. C. Ralph, and I. Fuentes, *Phys. Rev. D* **90**, 124001 (2014).
- [38] J. Wang, Z. Tian, J. Jing, and H. Fan, *Phys. Rev. D* **93**, 065008 (2016).
- [39] L. Bombelli, R. K. Koul, J. Lee, and R. D. Sorkin, *Phys. Rev. D* **34**, 373 (1986).
- [40] S. W. Hawking, *Commun. Math. Phys.* **43**, 199 (1975); *Phys. Rev. D* **14**, 2460 (1976); H. Terashima, *Phys. Rev. D* **61**, 104016 (2000).
- [41] C. Sabín and G. Adesso, *Phys. Rev. A* **92**, 042107 (2015); D. Mondal and C. Mukhopadhyay, [arXiv:1510.07556](https://arxiv.org/abs/1510.07556).
- [42] C. Weedbrook, S. Pirandola, R. García-Patrón, N. J. Cerf, T. C. Ralph, J. H. Shapiro, and S. Lloyd, *Rev. Mod. Phys.* **84**, 621 (2012).

- [43] G. Adesso, D. Girolami, and A. Serafini, *Phys. Rev. Lett.* **109**, 190502 (2012).
- [44] N.D. Birrell and P.C.W. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, United Kingdom, 1982).
- [45] A. Fabbri and J. Navarro-Salas, *Modeling Black Hole Evaporation* (Imperial College Press, London, United Kingdom, 2005).
- [46] D. E. Bruschi, J. Louko, E. Martín-Martínez, A. Dragan, and I. Fuentes, *Phys. Rev. A* **82**, 042332 (2010).
- [47] D. E. Bruschi, A. Dragan, I. Fuentes, and J. Louko, *Phys. Rev. D* **86**, 025026 (2012).
- [48] T. Damour and R. Ruffini, *Phys. Rev. D* **14**, 332 (1976).
- [49] S. M. Barnett and P. M. Radmore, *Methods in Theoretical Quantum Optics* (Oxford University Press, New York, 1997), pp. 67–80.