PHYSICAL REVIEW D **93,** 124048 (2016)

Causes of irregular energy density in $f(R,T)$ gravity

Z. Yousaf,[1,*] Kazuharu Bamba,[2,†] and M. Zaeem-ul-Haq Bhatti[1,‡]

[1]*Department of Mathematics, University of the Punjab, Quaid-i-Azam Campus, Lahore-54590, Pakistan*
[2]*Division of Human Support System, Faculty of Symbiotic Systems Science, Fukushima University, Fukushima 960-1296, Japan*

(Received 14 March 2016; published 20 June 2016)

We investigate irregularity factors for a self-gravitating spherical star evolving in the presence of an imperfect fluid. We explore the gravitational field equations and the dynamical equations with the systematic construction in $f(R,T)$ gravity, where T is the trace of the energy-momentum tensor. Furthermore, we analyze two well-known differential equations (which occupy principal importance in the exploration of causes of energy density inhomogeneities) with the help of the Weyl tensor and the conservation laws. The irregularity factors for a spherical star are examined for particular cases of dust and isotropic and anisotropic fluids in dissipative and nondissipative regimes in the framework of $f(R,T)$ gravity. It is found that, as the complexity of the matter with the anisotropic stresses increases, the inhomogeneity factor corresponds more closely to one of the structure scalars.

DOI: 10.1103/PhysRevD.93.124048

I. INTRODUCTION

The influence of modification in gravity theories has attracted significant attention from those in both high energy physics and cosmology. Although there is observational evidence for an accelerating Universe [1–5], certain predictions and compelling theoretical work about the expansion of the Universe are still under consideration. After the successful detection of gravitational waves, it is still possible that the cosmological constant added by Einstein in his field equations can describe the accelerating phase of the Universe. However, the unnatural fine-tuning problem favors the possibility of a dark side in the Universe owing to the dark dynamical effects. The best way to test the viability of any gravitational theory is to compare the predictions with the real object's motion.

A modification in Einstein's theory for discussing dark effects involves generalization in the Lagrangian of the Einstein-Hilbert action. The simplest generalization is to use function $f(R)$ instead of the Ricci scalar in the action. However, in order to include the matter contents, the simplest generalization is to replace R with $f(R,T)$, where T represents the trace of the stress-energy tensor. It is noted that such an addition in the Lagrangian can be observed as the addition of new degrees of freedom. The equation of motion emerging from such a Lagrangian will differ from Einstein's one. In that case, it would be possible to eliminate the cosmological constant to describe the acceleratory phase of the Universe. Such Lagrangians are well suited for studying dark energy (DE) and dark matter problems and little attention has ever been devoted to this direction (for reviews on late-time cosmic acceleration, i.e., the dark energy problem, and modified gravity theories, see, e.g., [6–10]).

Theories involving curvature matter coupling have attracted significant attention for exploring the enigma of cosmic evolution and other cosmological aspects. A geometry matter coupled system results in the existence of extra force due to the nongeodesic motion of test particles, and such systems in the setting of the Lagrangian for $f(R,T)$ were introduced in [11]. It has been observed that the extra force vanishes if one uses the specific form of the Lagrangian for usual matter (e.g., $L_m = p$) for nonminimally coupled $f(R)$ theories [12,13]; however, the extra force does not vanish for a matter geometry coupled system. $f(R,T)$ gravity theory is considered a useful candidate for studying the acceleratory behavior during the cosmic expansion, which is due not only to the scalar-curvature part but also to the matter components. This theory is also considered a useful candidate among the modified gravities which is based on nonminimal curvature matter coupling. The theory holds that the cosmological constant could be considered the trace dependent function, i.e., $\Lambda(T)$ gravity. This is done to make the connection between the usual cosmic matter and DE, which is supported by some modern cosmological data [14].

Faulkner *et al.* [15] explored the $f(R)$ theory and tried to equate it with scalar tensor theories for two classes of models: massive and chameleon $f(R)$ models. In recent years, it has been observed that most of the models proposed in $f(R)$ gravity do not satisfy the weak-field Solar System constraints [16]. Harko *et al.* [11] extended Einstein's standard relativity theory to $f(R,T)$ and found that higher-curvature theories can assist enough to resolve the flatness issue in the rotation curves of galaxies. The field equations for some specific models with explicit

*zeeshan.math@pu.edu.pk
†bamba@sss.fukushima-u.ac.jp
‡mzaeem.math@pu.edu.pk

$f(R,T)$ configurations have also been presented. Reddy *et al.* [17] discussed the Bianchi type III Universe model with perfect matter configuration in the background of $f(R,T)$ gravity while studying the early Universe. Adhav [18] has developed some interesting Bianchi-I Universe models in this theory. It is worthy to stress that the first law of black hole thermodynamics does not hold in $f(R,T)$ gravity [19].

Sharif and Yousaf [20] examined the stability of isotropic compact objects framed within $f(R,T)$ gravity and found relatively more stable and compact objects than those observed in $f(R)$ gravity. Sun and Huang [21] analyzed cosmic evolution in $f(R,T)$ gravity by means of redshift fluctuations against a distance modulus and found good fitting numerical plots consistent with the observational data of astronomy. Baffou *et al.* [22] investigated dynamical evolution along with the stability of power-law and de Sitter cosmic models against linear perturbation. They concluded that such models can be considered potential dark energy candidates. Alves *et al.* [23] explored the physical behavior of gravitational waves in different formalisms of $f(R,T)$ gravity models and showed that the gravitational wave spectrum is strongly dependent upon on the $f(R,T)$ model. Alhamzawi and Alhamzawi [24] explored a considerable contribution of $f(R,T)$ gravity on gravitational lensing and found results comparable to those already present in the literature, thereby suggesting the viability of this theory. Recently, Yousaf and Bhatti [25] observed more restricted unstable Newtonian and post-Newtonian regimes in $f(R,T)$ gravity [as compared to $f(R)$ gravity] for the locally anisotropic collapsing stellar model.

The emergence of curvature singularity for the stellar systems has been discussed in $f(R)$ gravity theory [26,27]. Houndjo [28] performed the cosmological reconstruction of $f(R,T)$ gravity examining the transition from a matter dominated epoch to a late-time accelerated regime. Alvarenga *et al.* [29] formulated energy conditions depending upon the attractive nature of gravity using the Raychaudhuri equation. Also, they investigated the viability of some particular $f(R,T)$ gravity models using these energy conditions. Azizi [30] investigated whether static spherically symmetric traversable wormhole geometries (which are basically exotic cosmic models) exist in $f(R,T)$ gravity.

In the study of accelerated Universe expansion, the viscosity effects resulting from matter configurations are quite vital and appear to be the only nonadiabatic ones in Friedmann-Robertson-Walker (FRW) models. Bulk viscosity disburses negative pressure, thus providing a platform for negative pressure that indicates repulsive gravity. During the particle creation and formation of galaxies and clusters in the early Universe, neutrinos decouple from the cosmic fluid and viscosity arises in the system [31]. Naidu *et al.* [32] studied the cosmological model with the FRW metric in the presence of viscosity in $f(R,T)$ gravity. Reddy *et al.* [33] investigated the Kaluza-Klien Universe model in the presence of viscosity with the background of $f(R,T)$ modified gravity theory. Sharif and his collaborators [34] have explored some physical processes with shear-free as well as expansion and expansion-free self-gravitating collapsing objects. Kiran and Reddy [35] determined the solutions of field equations in $f(R,T)$ gravity theory for the Bianchi type III spatially homogeneous model.

Nojiri and Odintsov [36] claimed that inflationary modified high degree of freedom quantities promote the evolution of Schwarzschild–de Sitter black hole antievaporation in the classical background. Farinelli *et al.* [37] discussed the equilibrium state of hydrostatic celestial objects and concluded that a wide range of compact objects exist in nature in modified gravity. Guo *et al.* [38] investigated the dynamical behavior of spherical relativistic collapse in modified gravity. Albareti *et al.* [39] analyzed homogeneous cosmological models through Raychaudhuri expressions and detailed some viability constraints attributable to the modified gravity theory expansion regimes of the Universe. Hason and Oz [40] observed extended configurations of the Jeans instability condition for the relativistic systems for normal and super fluids.

During the evolution of a star model, a large amount of radiation emits in the form of photons and neutrinos, which gradually increases as the evolution proceeds. The radiating energy can be characterized in two approximations, i.e., diffusion and free-streaming approximation. The diffusion limit is applicable when the typical length of the object is greater than the mean free path of the particles responsible for the motion of energy. In that case, the dissipation is described by a heat flow type vector while, in the other case, it is characterized by an outflow of null fluid. Herrera *et al.* [41] found that the energy density should be inhomogeneous if the system involves a zero expansion condition in a nondissipative fluid background. Herrera [42] explored some factors for a self-gravitating spherical star which are important for describing the irregularities in the matter distribution. Sharif and his collaborators [43] have also explored some factors describing the inhomogeneous density distribution for self-gravitating objects with different matter configurations.

In a recent paper, Yousaf *et al.* [44] have formulated some dynamical variables by splitting the Riemann tensor into its constituent trace and trace-free scalar parts in $f(R,T)$ theory. They have also discussed the evolution of shear and expansion using the Raychaudhuri equation. This paper is organized in the following manner. In Sec. II, we will provide some basic equations, including the action of this framework and the equation of motion. In Sec. III, modified field equations, some kinematical and dynamical quantities, and modified Ellis equations are formulated for the construction of our analysis in a systematic way. Section IV explores the irregularity factors involving dissipative and nondissipative matter distribution in certain cases. Finally, we deliver our conclusions in Sec. V.

II. $f(R,T)$ GRAVITY AND SPHERICAL SYSTEMS

The notion of $f(R,T)$ gravity as a possible modification in the gravitational framework of general relativity (GR) received significant attention from researchers. This theory provides numerous interesting results in the field of physics and cosmology, including a plausible explanation for the accelerating cosmic expansion [6,8,9]. The main tenet of this theory is to use an algebraic general function of Ricci as a well trace of the energy-momentum tensor in the standard Einstein-Hilbert action. It can be written [11]

$$S_{f(R,T)} = \int d^4x\sqrt{-g}[f(R,T)+L_M], \tag{1}$$

where g, T are the traces of the metric and standard GR energy-momentum tensors, respectively, while R is the Ricci scalar. In the literature, variety L_M corresponds to particular configurations of relativistic matter distributions. Choosing $L_M=\mu$ (where μ is the system's energy density) and making variation in the above equation with $g_{\alpha\beta}$, the corresponding $f(R,T)$ field equations are given as follows:

$$G_{\alpha\beta} = T_{\alpha\beta}{}^{\mathrm{eff}}, \tag{2}$$

where

$$\begin{aligned} T_{\alpha\beta}{}^{\mathrm{eff}} = \bigg[&(1+f_T(R,T))T^{(m)}_{\alpha\beta} - \mu g_{\alpha\beta} f_T(R,T) \\ &- \left(\frac{f(R,T)}{R} - f_R(R,T)\right)\frac{R}{2} \\ &+(\nabla_\alpha\nabla_\beta + g_{\alpha\beta}\Box)f_R(R,T)\bigg]\frac{1}{f_R(R,T)} \end{aligned}$$

is a nonstandard energy-momentum tensor representing a modified version of a gravitational contribution coming from $f(R,T)$ extra degrees of freedom, while $G_{\alpha\beta}$ is an Einstein tensor. Furthermore, ∇_α represents covariant derivation, while $f_T(R,T)$, $\Box$, and $f_R(R,T)$ represent the $\frac{df(R,T)}{dT}$, $\nabla_\alpha\nabla^\alpha$, and $\frac{df(R,T)}{dR}$ operators, respectively.

We consider a spherical relativistic self-gravitating nonrotating and nonstatic system whose metric can be expressed with the help of the following diagonal form:

$$ds^2 = -A^2(t,r)dt^2 + B^2(t,r)dr^2 + C^2d\theta^2 + C^2\sin^2\theta d\phi^2. \tag{3}$$

It is assumed that this system is filled with shearing viscous, locally anisotropic, and radiating fluid. This fluid can be indicated through the following configurations of the mathematical form:

$$\begin{aligned} T_{\alpha\beta} = {}& P_\perp h_{\alpha\beta} + \mu V_\alpha V_\beta + \Pi\chi_\alpha\chi_\beta + \varepsilon l_\alpha l_\beta \\ &+ q(\chi_\beta V_\alpha + \chi_\alpha V_\beta) - 2\eta\sigma_{\alpha\beta}, \end{aligned} \tag{4}$$

where $P_\perp$ is the tangential pressure, $\Pi \equiv P_r - P_\perp$, and P_r is the fluid pressure along the radial direction. ε is the radiation density, q_β is a vector controlling heat dissipation, $\sigma_{\alpha\beta}$ is a tensor controlling shearing viscosity, and η is its coefficient. Furthermore, $h_{\alpha\beta}$ is the projection tensor, defined as follows:

$$h_{\alpha\beta} = g_{\alpha\beta} + V_\alpha V_\beta.$$

The vectors l^γ, V^γ, and χ^γ represent the null four-vector, the fluid four-velocity, and the radial unit four-vector, respectively. Under comoving coordinates, these four-vectors can be evaluated as $V^\gamma = \frac{1}{A}\delta^\gamma_0$, $\chi^\gamma = \frac{1}{C}\delta^\gamma_1$, $l^\gamma = \frac{1}{A}\delta^\gamma_0 + \frac{1}{B}\delta^\gamma_1$, $q^\gamma = q(t,r)\chi^\gamma$. Moreover, they obey

$$\begin{aligned} &V^\beta V_\beta = -1, \qquad \chi^\beta\chi_\beta = 1, \qquad \chi^\beta V_\beta = 0, \\ &V^\beta q_\beta = 0, \qquad l^\beta V_\beta = -1, \qquad l^\beta l_\beta = 0. \end{aligned}$$

The scalar variable controlling the expansion and contraction of the matter distribution is known as the expansion scalar. This can be obtained through the $\Theta = V^\alpha{}_{;\alpha}$ mathematical expression. For Eq. (3), it is found as follows:

$$\begin{aligned} \Theta &= \frac{1}{A}(\dot{B}B^{-1} + 2\dot{C}C^{-1}), \\ \sigma &= \frac{-1}{A}(\dot{C}C^{-1} - \dot{B}B^{-1}), \end{aligned} \tag{5}$$

where the overdot notation represents the temporal partial derivation.

The $f(R,T)$ field equations (2) for spherical nonstatic interior (3) are found to be

$$G_{00} = \frac{A^2}{f_R}\left[\mu + \varepsilon - \frac{R}{2}\left(\frac{f}{R} - f_R\right) + \frac{\psi_{00}}{A^2}\right], \tag{6}$$

$$G_{01} = \frac{AB}{f_R}\left[-(1+f_T)(q+\varepsilon) + \frac{\psi_{01}}{AB}\right], \tag{7}$$

$$\begin{aligned} G_{11} = \frac{B^2}{f_R}\bigg[&\mu f_T + (1+f_T)\left(P_r + \varepsilon - \frac{4}{3}\eta\sigma\right) \\ &+ \frac{R}{2}\left(\frac{f}{R} - f_R\right) + \frac{\psi_{11}}{B^2}\bigg], \end{aligned} \tag{8}$$

$$\begin{aligned} G_{22} = \frac{C^2}{f_R}\bigg[&(1+f_T)\left(P_\perp + \frac{2}{3}\eta\sigma\right) + \mu f_T \\ &+ \frac{R}{2}\left(\frac{f}{R} - f_R\right) + \frac{\psi_{22}}{C^2}\bigg], \end{aligned} \tag{9}$$

where

$$\psi_{00}=2\partial_{tt}f_R+\left(\frac{\dot{B}}{B}-2\frac{\dot{A}}{A}+2\frac{\dot{C}}{C}\right)\partial_t f_R+\left(A^2\frac{B'}{B}-2AA'-2A^2\frac{C'}{C}\right)\frac{\partial_r f_R}{B^2},$$

$$\psi_{01}=\partial_t\partial_r f_R-\frac{A'}{A}\partial_t f_R-\frac{\dot{B}}{B}\partial_r f_R,$$

$$\psi_{11}=\partial_{rr}f_R-\frac{B^2}{A^2}\partial_{tt}f_R+\left(B^2\frac{\dot{A}}{A}-2B^2\frac{\dot{C}}{C}-2B\dot{B}\right)\frac{\partial_t f_R}{A^2}+\left(\frac{A'}{A}+2\frac{C'}{C}-2\frac{B'}{B}\right)\partial_r f_R,$$

$$\psi_{22}=-C^2\frac{\partial_{tt}f_R}{A^2}+\frac{C^2}{A^2}\left(\frac{\dot{A}}{A}-3\frac{\dot{C}}{C}-\frac{\dot{B}}{B}\right)\partial_t f_R+\frac{C^2}{B^2}\left(\frac{C'}{C}+\frac{A'}{A}-\frac{B'}{B}\right)\partial_r f_R.$$

Here, the prime represents radial partial differentiation.

We are now interested in evaluating expressions that would be helpful for studying the dynamical phases of spherical anisotropic radiating and shearing viscous interiors in $f(R,T)$ gravity. It is seen that in this gravitational theory, the divergence of the energy-momentum tensor is nonvanishing and is found to be

$$\nabla^\alpha T_{\alpha\beta}=\frac{f_T}{(1-f_T)}\left[(\Theta_{\alpha\beta}+T_{\alpha\beta})\nabla^\alpha\ln f_T-\frac{1}{2}g_{\alpha\beta}\nabla^\alpha T+\nabla^\alpha\Theta_{\alpha\beta}\right].\tag{10}$$

The divergence of the $f(R,T)$ energy-momentum tensor gives the following equations of motion:

$$\begin{aligned}&\dot{\mu}\left(\frac{1+f_T+f_Rf_T}{f_R(1+f_T)}\right)-\frac{\mu}{f_R^2}\partial_t f_R-\left\{\frac{f-Rf_R}{2}\right\}_{,0}+\left(\frac{\psi_{00}}{A^2}\right)_{,0}-\bar{q}'\frac{B}{A}\frac{(1+f_T)}{f_R}-\frac{\bar{q}}{A^2}\left\{\frac{AB(1+f_T)}{f_R}\right\}_{,1}\\&+\frac{1}{A^2}\left(\frac{\psi_{01}}{f_R}\right)_{,1}-\frac{B\dot{B}}{A^2f_R}\left\{(1+f_T)\mu+\varepsilon+(1+f_T)\left(P_r+\varepsilon+\frac{4}{3}\eta\sigma\right)+\frac{\psi_{00}}{A^2}+\frac{\psi_{11}}{B^2}\right\}\\&-\frac{2C\dot{C}}{A^2f_R}\left\{\bar{\mu}+\mu f_T+(1+f_T)\left(P_\perp+\frac{2}{3}\eta\sigma\right)+\frac{\psi_{00}}{A^2}+\frac{\psi_{22}}{C^2}\right\}\\&+\left(\frac{B'}{B}+\frac{A'}{A}-\frac{CC'}{B^2}+2\frac{AA'}{B^2}\right)\frac{1}{f_R}\left\{(1+f_T)AB\bar{q}-\frac{\psi_{01}}{A^2}\right\}\\&+\frac{1}{1+f_T}\left\{(2\mu+\varepsilon)\partial_t f_T+\left(\dot{\mu}+\frac{\dot{T}}{2}\right)f_T\right\}=0,\end{aligned}\tag{11}$$

$$\begin{aligned}&\frac{(1+f_T)}{f_R}\left\{\bar{P}_r'-\frac{4}{3}\eta\sigma'\right\}+\frac{\mu' f_T}{f_R}+\frac{1}{(1+f_T)}\left(\bar{P}_r-\frac{4}{3}\eta\sigma-\mu\right)\partial_r f_T-\frac{A}{Bf_R}\bar{q}'(1+f_T)\\&-\left\{\frac{AB(1+f_T)}{f_R}\right\}_{,0}\frac{1}{B^2}+\frac{1}{B^2}\left(\frac{\psi_{01}}{f_R}\right)_{,0}+\frac{\mu\partial_r f_T}{f_R}-\frac{\mu f_T}{f_R^2}\partial_r f_R\\&+\frac{1}{f_R}\left(\bar{P}_r-\frac{4}{3}\eta\sigma\right)\left\{\partial_r f_T-\frac{(1+f_T)}{f_R}\partial_r f_R\right\}+\left(\frac{f-Rf_R}{2}+\frac{\psi_{11}}{B^2}\right)_{,1}\\&-\frac{AA'}{B^2f_R}\left\{\mu f_T+\bar{\mu}+(1+f_T)\left(\bar{P}_r-\frac{4}{3}\eta\sigma\right)+\frac{\psi_{11}}{B^2}+\frac{\psi_{00}}{A^2}\right\}\\&+\frac{2CC'}{B^2f_R}\left\{(1+f_T)(\bar{P}_r-P_\perp-2\eta\sigma)+\frac{\psi_{11}}{B^2}-\frac{\psi_{22}}{C^2}\right\}-\frac{f_T}{(1+f_T)}\left(\mu'+\frac{\partial_r T}{2}\right)\\&+\left(\frac{\dot{B}}{B}+\frac{\dot{A}}{A}+\frac{2C\dot{C}}{A^2}+2\frac{B\dot{B}}{A^2}\right)\left\{\frac{A}{B}\frac{(1+f_T)}{f_R}\bar{q}-\frac{\psi_{01}}{B^2f_R}\right\}=0.\end{aligned}\tag{12}$$

The matter content within the spherical collapsing stellar geometry can be defined through the general Misner-Sharp formula [45]. This is obtained as

$$m = \left\{\frac{\dot{C}^2}{A^2} - \frac{C'^2}{B^2} + 1\right\}\frac{C}{2}. \quad (13)$$

Before calculating its variations among adjacent surfaces of spherical radiating fluid configurations, we introduce some useful operators. The operators corresponding to proper and radial derivations are found to be

$$D_T = \frac{1}{A}\frac{\partial}{\partial t}, \qquad D_C = \frac{1}{C'}\frac{\partial}{\partial r}. \quad (14)$$

The relativistic velocity associated with the spherical stellar structure can be found by using the above mentioned proper derivative operator. This turns out to be

$$U = D_T C = \frac{\dot{C}}{A}. \quad (15)$$

Now, we define E as a ratio $\frac{C'}{B}$. From Eqs. (13) and (15), one can obtain

$$E = \sqrt{1 + U^2 - \frac{2m(t,r)}{C}}. \quad (16)$$

Using field equations and the above two equations, the radial mass variations is found to be

$$D_C m = \frac{C^2}{2f_R}\left[\bar{\mu} - \frac{R}{2}\left(\frac{f}{R} - f_R\right) + \frac{\psi_{00}}{A^2} + \frac{U}{E}\left\{(1+f_T)\bar{q} - \frac{\psi_{01}}{AB}\right\}\right], \quad (17)$$

whose integration yields

$$m = \frac{1}{2}\int_0^C \frac{C^2}{f_R}\left[\bar{\mu} - \frac{R}{2}\left(\frac{f}{R} - f_R\right) + \frac{\psi_{00}}{A^2} + \frac{U}{E}\left\{(1+f_T)\bar{q} - \frac{\psi_{01}}{AB}\right\}\right] dC, \quad (18)$$

where the overbar indicates the addition of radiation density in the corresponding variable quantity. The particular combinations of radiating matter parameters, $f(R,T)$ higher-curvature terms, and energy density can be achieved via the Misner-Sharp mass formulation. This can be obtained, after using Eq. (18), and is found to be

$$\frac{3m}{C^3} = \frac{3}{2C^3}\int_0^r \left[\bar{\mu} - \frac{R}{2}\left(\frac{f}{R} - f_R\right) + \frac{\psi_{00}}{A^2} + \frac{U}{E}\left\{(1+f_T)\bar{q} - \frac{\psi_{01}}{AB}\right\}C^2 C'\right] dr. \quad (19)$$

This equation can be recast to obtain a scalar related to the tidal forces acting on the anisotropic, radiating, shearing, viscous spherical stellar system

$$\mathcal{E} = \frac{1}{2f_R}\left[\bar{\mu} - (1+f_T)(\bar{\Pi} - 2\eta\sigma) - \frac{R}{2}\left(\frac{f}{R} - f_R\right) + \frac{\psi_{00}}{A^2} - \frac{\psi_{11}}{B^2} + \frac{\psi_{22}}{C^2}\right] - \frac{3m}{C^3}, \quad (20)$$

where $\mathcal{E}$ is the Weyl scalar. It so happened that the Weyl scalar can be decomposed into its magnetic and electric constituents. The magnetic part of the Weyl scalar is zero for the spherical matter distribution. However, an electric part does exist. The scalar, $\mathcal{E}$, is associated with this component of the Weyl tensor. In this way, $\mathcal{E}$ describes the gravity effects coming due to tidal forces in the cosmos. Equation (20) has related tidal forces with the structural properties of the fluid configurations and $f(R,T)$ extra curvature terms. Equation (20) has been evaluated by taking a regular distribution of fluid contents at the central point, i.e., $m(t,0) = 0 = C(t,0)$.

III. EXPANSION-FREE CONDITION AND $f(R,T)$ ELLIS EQUATIONS

In this section, we shall evaluate an expansion-free constraint and then discuss its meaning in the interpretation of the mysterious dark Universe. We then consider the viable and consistent $f(R,T)$ gravity model. We then proceed forward in our analysis by evaluating well-known Ellis equations. The expansion-free equation can be achieved by equating the expansion scalar with zero. Thus, Eq. (5) yields

$$\frac{\dot{B}}{B} = -\frac{2\dot{C}}{C}, \quad (21)$$

which, upon integration, gives

$$B = \frac{h}{C^2}, \quad (22)$$

where h is an arbitrary integration radial function.

Gravitational collapse is the phenomenon that takes place in this accelerating expanding cosmos when the state of hydrostatic equilibrium of a celestial body is destroyed. If a stellar object is massive enough that the gas pressure is insufficient to support it against gravitational forces, then the star undergoes gravitational collapse, giving birth to new stars. It is important to stress that any self-gravitating stellar body would subject to gravitational collapse once it bears through inhomogeneous and irregular surface energy density. Therefore, in the collapse of self-gravitating relativistic fluids, the role of energy density inhomogeneity has gained much significance. If the fluid of the relativistic celestial interiors is expansion free, this study may gain even more attention.

The expansion-free condition has produced several interesting results at galactic and cosmological scales. Skripkin [46] noticed the captivating process of cavity emergence within nonradiating ideal relativistic matter field. It is seen from the literature that, in null expansion evolution, the innermost boundary surface of the interior fluid configuration slides away from the central point, thus conceiving a vacuum Minkowskian core [47]. The nullity of Θ is sufficient but is not a necessary constraint that guarantees the cavity emergence. The scenario of cavity emergence has been explored in the literature [48] under some kinematical constraints other than $\Theta = 0$. Di Prisco *et al.* [49] and Sharif and Yousaf [50] studied core formation within the relativistic celestial locally anisotropic configurations after its central explosion and demonstrated some expansion-free relativistic solutions.

The possible implementations of the null expansion condition is anticipated for those astronomical settings where a Minkowskian core is probably to be appear. In addition to this, during the process of gravitational collapse, whenever the expansion-free matter moves inside to reach the central point, there will be a strong shear scalar blowup. Joshi *et al.* [51] claimed that the apparent horizon formation could be delayed due to the effects originated from the strong shear of the collapsing system. This suggests the emergence of naked singularity (NS). Therefore, the study of expansion-free relativistic interiors could provide an uncomplicated platform for the analysis of the NS appearance. NS is a spacetime singularity that can be observed directly by a distant observer. It represents the formation of extremely high curvature and strong gravity regions and could provide a source of gravitational waves. Are black holes (BHs) and NS observationally distinguishable from each other? In this perspective, Virbhadra *et al.* [52] gave a very useful mathematical tool for understanding the NS physics.

Virbhadra and Ellis [53] established that one can observationally differentiate NS from BHs by analyzing the corresponding characteristics of gravitational lensing (GL). Claudel *et al.* [54] demonstrated that any photon relativistic spherical body could be around the BH only if it obeys a reasonable energy condition. For the observational study of a cosmic censorship hypothesis, GL could provide a reliable direction. In this context, it is seen from [55] that BHs and NS of the same symmetry and Arnowitt-Deser-Misner mass yield a variety of different images of the same source of light. Furthermore, time of image delays because of GL by a BH is greater than that of NS. This asserts that one can get smaller time delays by choosing extreme values of nakedness variables. NS could also provide images with negative delays of time [56].

Because of zero expansion, matter sources could be effective for the voids explanation. Voids are so-called underdense regions incorporating a substantial amount of information on the cosmological environment [57]. Voids offers a reliable guide to discuss the cosmic structure appearance at large scales. In comparison with GR, they are more rich in modified gravity [58]. Wiltshire [59] claimed that the actual picture of cosmos constitutes spongelike structural bodies in which voids have a dominant role. Furthermore, some cosmological indications assert that about 40%–50% volume of the current cosmos is endowed with cosmological voids with a scale $30\mathrm{h}^{-1}$ Mpc, where h is the nondimensional Hubble parameter, $H_0 = 100\mathrm{h}\,\mathrm{km\,sec}^{-1}\,\mathrm{Mpc}^{-1}$.

The evident relevance of such effects could increase interest in the problem addressed in this paper.

The fascinating phenomenon of accelerating cosmic expansion could be described by taking into account extended gravity models involving curvature matter coupling, like $f(R,T)$ gravity. For theoretically and cosmologically consistent $f(R,T)$ gravity, the choice of $f(R,T)$ function is very crucial. We are considering the following particular $f(R,T)$ model form:

$$f(R,T) = f_1(R) + f_2(T). \tag{23}$$

This model form does involve direct minimal curvature matter coupling. This could be assumed as a possible correction in the well-known $f(R)$ gravity. Here, we take a linear choice of f_2, from which some striking outcomes can be observed on the basis of nontrivial coupling, as compared to $f(R)$ gravity. Thus, we assume $f_2(T) = \nu T$, where ν is a constant. The Lagrangian with this background of f_2 has been broadly examined by many relativistic astrophysicists. Harko *et al.* [60] obtained some cosmic solutions depicting a clear accelerating, expanding picture of the Universe framed within $f_2 = \nu T$.

Now, we are interested in taking a physical feasible generic Ricci invariant function. These may give birth to the existence of some new spherical models. A cosmological viable model needs to obey big-bang nucleosynthesis, radiation as well as matter dominated regimes. Also, they should be expected to allow cosmological perturbations consistent with cosmic restrictions emerging from anisotropies in the cosmic microwave background. In this realm, we consider power-law Ricci scalar corrections, i.e., $f(R) = R + \lambda R^n$, where $\lambda \in \mathbb{R}^+$, with $\mathbb{R}^+$ being the set of positive real numbers and n a constant. Depending upon the selection of n, this model has some physical descriptions. For instance, this model, for $n = 2$, could depict the exponential behavior of the early cosmic expansion proposed by Starobinsky [61]. Such $f(R)$ model could draw dark matter (for $\lambda = \frac{1}{6M^2}$ [62] with $M = 2.7 \times 10^{-12}$ GeV [7]) and DE effects in the gravitational theory. Furthermore, gravity induced under $n = 2$ [63] and $n = 3$ [64] supports the existence of more massive compact objects than GR. The $f(R)$ tanh corrections have also been investigated in the study of stellar collapse [65]. However, the negative n values could helps to explain the dynamics of a stellar object in the presence of late-time accelerating cosmic expansion corrections [6].

In order to delve into the survival of the regular energy density over the dissipative spherical celestial object, we now calculate a couple of well-known equations by following the procedure introduced by Ellis [66]. These expressions in the background of dark source $f(R,T)$ corrections can be found by using Eqs. (6)–(9), (13), (14), (20), and (23) as

$$\left[\mathcal{E}-\frac{1}{2(1+n\lambda R^{n-1})}\left\{\bar{\mu}-(1+\nu)(\bar{\Pi}-2\eta\sigma)-\frac{(1-n)}{2}\lambda R^n-\frac{\nu}{2}T+\frac{\varphi_{00}}{A^2}-\frac{\varphi_{11}}{B^2}+\frac{\varphi_{22}}{C^2}\right\}\right]_{,0}$$
$$=\frac{3\dot{C}}{C}\left[\frac{1}{2(1+n\lambda R^{n-1})}\left\{\bar{\mu}+(1+\nu)\left(P_\perp-\frac{2}{3}\eta\sigma\right)+\mu\nu+\frac{\varphi_{00}}{A^2}+\frac{\varphi_{22}}{C^2}\right\}-\mathcal{E}\right]$$
$$+\frac{3AC'}{2BC(1+n\lambda R^{n-1})}\left\{(1+\nu)\bar{q}-\frac{\varphi_{01}}{AB}\right\}, \tag{24}$$

$$\left[\mathcal{E}-\frac{1}{2(1+n\lambda R^{n-1})}\left\{\bar{\mu}-(1+\nu)(\bar{\Pi}-2\eta\sigma)-\frac{(1-n)}{2}\lambda R^n-\frac{\nu}{2}T+\frac{\varphi_{00}}{A^2}-\frac{\varphi_{11}}{B^2}+\frac{\varphi_{22}}{C^2}\right\}\right]'$$
$$=-\frac{3C'}{C}\left[\mathcal{E}+\frac{1}{2(1+n\lambda R^{n-1})}\left\{(1+\nu)(\bar{\Pi}-2\eta\sigma)+\frac{\varphi_{11}}{B^2}-\frac{\varphi_{22}}{C^2}\right\}\right]$$
$$-\frac{3B\dot{C}}{2AC(1+n\lambda R^{n-1})}\left\{(1+\nu)\bar{q}-\frac{\varphi_{01}}{AB}\right\}, \tag{25}$$

where φ_{ii} encapsulate $f(R,T)$ extra degrees of freedom involved in the evolution of a shearing, viscous, radiating spherical body. These quantities can be evaluated by considering Eqs. (6)–(9) and (23) accordingly.

IV. IRREGULARITIES IN THE DYNAMICAL SYSTEM

In this section, we shall calculate some irregularity factors that cause the appearance of irregularities over the surface of a stellar spherical system with an $f(R,T)$ background. The system enters into the collapsing window once the celestial surface suffers energy density inhomogeneities. Therefore, the understanding of the system's collapsing nature is directly related to the exploration of irregularity factors. For this purpose, we assume that our stellar spherical relativistic system is in a complete homogeneous phase. We shall take some specific choices of matter fields framed within dark source terms coming from the $f(R,T)$ model. As $f(R,T)$ field equations are highly nonlinear, we will confine ourselves to the constant values of the trace of the stress-energy tensor as well as the cosmological Ricci scalar. These are represented by putting a tilde over the respective quantities. We shall also calculate irregularity factors for those spherical relativistic interiors that continue their evolutions by establishing central Minkowskian cavity. This would be achieved by taking an expansion-free condition in the corresponding equations. We shall classify our investigation in two scenarios, i.e., dissipative/radiating and nondissipative/nonradiating systems as follows.

A. Nonradiating matter

Here, we deal with adiabatic noninteracting, ideal, and locally anisotropic forms of relativistic matter distributions coupled and framed within the $f(R,T)$ background.

1. Noninteracting relativistic particles

This subsection addresses geodesically moving noninteracting fluid configurations. So, we take all pressure gradients effects to be zero, $\hat{P}_r=0=P_\perp=\hat{q}$, along with $A=1$. Then the $f(R,T)$ Ellis equations (24) and (25) reduce to

$$\left[\mathcal{E}-\frac{1}{2(1+n\lambda\tilde{R}^{n-1})}\left\{\mu-\frac{(1-n)}{2}\lambda\tilde{R}^n-\frac{\nu}{2}\tilde{T}\right\}\right]_{,0}=\frac{3\dot{C}}{C}\left[\frac{1}{2(1+n\lambda\tilde{R}^{n-1})}\{\mu(1+\nu)\}-\mathcal{E}\right],$$
$$\left[\mathcal{E}-\frac{1}{2(1+n\lambda\tilde{R}^{n-1})}\left\{\mu-\frac{(1-n)}{2}\lambda\tilde{R}^n-\frac{\nu}{2}\tilde{T}\right\}\right]'=-3\frac{C'}{C}\mathcal{E}.$$

Using Eqs. (5), (11), and (23) in the above equations, we have

$$\dot{\mathcal{E}}+\frac{3\dot{C}}{C}\mathcal{E}=\frac{\mu(1+\lambda)}{2(1+n\lambda\tilde{R}^{n-1})}\left[\frac{3\dot{C}}{C}+\frac{(1+\nu)B^2C^2}{\{1+(1+n\lambda\tilde{R}^{n-1})(1+\nu)\}}\Theta\right], \tag{26}$$

$$\mathcal{E}'+\frac{3C'}{C}\mathcal{E}=\frac{\mu'}{2(1+n\lambda\tilde{R}^{n-1})}. \tag{27}$$

It can seen from Eq. (27) that, if $\mu'=\mu(t)$, then

$$\mathcal{E}=0,$$

thereby indicating that the existence of the Weyl scalar is directly proportional to the existence of the regular energy density of noninteracting self-gravitating particles. This is the very result found in GR by many relativistic astrophysicists. Thus, we conclude that $f(R,T)$ extra curvature terms have not altered or disturbed the Weyl curvature role in the conformally flat solutions of a relativistic dust cloud. Now, we solve Eq. (26) to investigate which quantities are in fact making an impact over the contribution of the Weyl scalar in $f(R,T)$ gravity. The solution of Eq. (26) yields

$$\mathcal{E}=\frac{(1+\nu)}{2(1+n\lambda\tilde{R}^{n-1})C^3}\int_0^t\left[3C^2\dot{C}+\frac{(1+\nu)B^2C^5}{\{1+(1+n\lambda\tilde{R}^{n-1})(1+\nu)\}}\Theta\right]\mu dt. \tag{28}$$

This shows that the Weyl scalar for dust particles in the $f(R,T)$ model is directly related to the temporal integrals of the energy density and expansion scalar. If we take the null expansion scenario, the above equation gives

$$\mathcal{E}=\frac{3(1+\nu)}{2(1+n\lambda\tilde{R}^{n-1})C^3}\int_0^t\mu C^2\dot{C}dt. \tag{29}$$

The relativistic systems that evolve by encapsulating the Minkowskian core in the Universe should satisfy the above constraint in order to enter into the inhomogeneous phase. In other words, for the regular distribution of dust expansion-free particles in $f(R,T)$ gravity, one needs to take Eq. (29) to be zero.

2. Isotropic fluid

Here, we consider the case of an ideal self-gravitating fluid in the environment of $f(R,T)$ gravity. The extended Ellis equations (24) and (25) give rise to the following set of differential equations:

$$\left[\mathcal{E}-\frac{1}{2(1+n\lambda\tilde{R}^{n-1})}\left\{\mu-\frac{(1-n)}{2}\lambda\tilde{R}^n-\frac{\nu}{2}\tilde{T}\right\}\right]_{,0}=\frac{3\dot{C}}{C}\left[\frac{1}{2(1+n\lambda\tilde{R}^{n-1})}\{(\mu+P)(1+\nu)\}-\mathcal{E}\right],$$

$$\left[\mathcal{E}-\frac{1}{2(1+n\lambda\tilde{R}^{n-1})}\left\{\mu-\frac{(1-n)}{2}\lambda\tilde{R}^n-\frac{\nu}{2}\tilde{T}\right\}\right]'=-3\frac{C'}{C}\mathcal{E}.$$

Equations (5) and (12) provide

$$\dot{\mathcal{E}}+\frac{3\dot{C}}{C}\mathcal{E}=\frac{(1+\lambda)(\mu+P)}{2(1+n\lambda\tilde{R}^{n-1})}\left[\frac{3\dot{C}}{C}+\frac{(1+\nu)B^2C^2}{\{1+(1+n\lambda\tilde{R}^{n-1})(1+\nu)\}}\Theta\right], \tag{30}$$

$$\mathcal{E}'+\frac{3C'}{C}\mathcal{E}=\frac{\mu'}{2(1+n\lambda\tilde{R}^{n-1})}. \tag{31}$$

It is seen from the second of the above equations that energy density will be regular as long as $\mathcal{E}=0$. However, the solution of Eq. (30) yields

$$\mathcal{E}=\frac{(1+\nu)}{2(1+n\lambda\tilde{R}^{n-1})C^3}\int_0^t\left[3C^2\dot{C}+\frac{(1+\nu)B^2C^5}{\{1+(1+n\lambda\tilde{R}^{n-1})(1+\nu)\}}\Theta\right](\mu+P)dt. \tag{32}$$

This indicates that the influence of tidal forces is controlled by the linear combination of the system energy density and a locally isotropic pressure gradient. This also highlights the importance of an expansion scalar in the modeling of a homogeneous spherical geometry coupled with isotropic matter configurations in the presence of $f(R,T)$ corrections. However, if we eliminate this scalar with the help of Eq. (21), we have

$$\mathcal{E}=\frac{3(1+\nu)}{2(1+n\lambda\tilde{R}^{n-1})C^3}\int_0^t(\mu+P)C^2\dot{C}dt. \tag{33}$$

This suggests that a pressure gradient has increased the impact of tidal forces over the isotropic spherical stellar interior. Furthermore, $f(R,T)$ corrections tend to reduce the influence of the Weyl scalar due to its nonattractive nature.

3. Anisotropic fluid

This subsection is aimed at extending our previous work. Here, we introduce effects of anisotropic stresses; thus, $\Pi\neq 0$ in our analysis. In this realm, the $f(R,T)$ Ellis equations (24) and (25) take the following forms:

$$\left[\mathcal{E}-\frac{1}{2(1+n\lambda\tilde{R}^{n-1})}\left\{\mu-(1+\nu)\Pi-\frac{(1-n)}{2}\lambda\tilde{R}^n-\frac{\nu}{2}\tilde{T}\right\}\right]_{,0}=\frac{3\dot{C}}{C}\times\left[\frac{1}{2(1+n\lambda\tilde{R}^{n-1})}\{(\mu+P_\perp)(1+\nu)\}-\mathcal{E}\right],$$

$$\left[\mathcal{E}-\frac{1}{2(1+n\lambda\tilde{R}^{n-1})}\left\{\mu-(1+\nu)\Pi-\frac{(1-n)}{2}\lambda\tilde{R}^n-\frac{\nu}{2}\tilde{T}\right\}\right]'=-3\frac{C'}{C}\times\left[\mathcal{E}+\frac{(1+\nu)\Pi}{2(1+n\lambda\tilde{R}^{n-1})}\right],$$

which can be manipulated, after using Eq. (12), in the following manner:

$$\begin{aligned}&\left[\mathcal{E}+\frac{(1+\lambda)\Pi}{2(1+n\lambda\tilde{R}^{n-1})}\right]_{,0}+3\left[\mathcal{E}+\frac{(1+\lambda)\Pi}{2(1+n\lambda\tilde{R}^{n-1})}\right]\frac{\dot{C}}{C}\\&\quad=\frac{3[\mu+(1+\nu)P_r]\dot{C}}{2C(1+n\lambda\tilde{R}^{n-1})}+\frac{(1+\nu)^2(1+n\lambda\tilde{R}^{n-1})^{-1}}{2A\{1+(1+n\lambda\tilde{R}^{n-1})(1+\nu)\}}\left[(\mu+P_r)B^2C^5\Theta+\frac{2C\Pi\dot{C}}{A}\right],\end{aligned} \tag{34}$$

$$\left[\mathcal{E}+\frac{(1+\lambda)\Pi}{2(1+n\lambda\tilde{R}^{n-1})}\right]_{,1}+3\left[\mathcal{E}+\frac{(1+\lambda)\Pi}{2(1+n\lambda\tilde{R}^{n-1})}\right]\frac{C'}{C}=\frac{\mu'}{2(1+n\lambda\tilde{R}^{n-1})}. \tag{35}$$

It is well known from the work of several relativistic astrophysicists that, in GR [67] as well as in $f(R,T)$ [44], one can break the Riemann tensor into a couple of tensors, namely, $X_{\alpha\beta}$ and $Y_{\alpha\beta}$. The traceless part of $X_{\alpha\beta}$ yields the following (for details please see [44]):

$$X_{TF}=-\mathcal{E}-\frac{(1+\lambda)\Pi}{2(1+n\lambda\tilde{R}^{n-1})}. \tag{36}$$

It is seen that some terms involved in Eqs. (34) and (35) have the same configurations as that of the traceless part of the second dual of the Riemann curvature tensor mentioned in Eq. (33). In this context, Eqs. (34) and (35) can be recast as

$$\dot{X}_{TF}+\frac{3X_{TF}\dot{C}}{C}=-\frac{3[\mu+(1+\nu)P_r]\dot{C}}{2C(1+n\lambda\tilde{R}^{n-1})}-\frac{(1+\nu)^2(1+n\lambda\tilde{R}^{n-1})^{-1}}{2A\{1+(1+n\lambda\tilde{R}^{n-1})(1+\nu)\}}\left[(\mu+P_r)B^2C^5\Theta+\frac{2C\Pi\dot{C}}{A}\right], \tag{37}$$

$$X'_{TF}+\frac{3X_{TF}C'}{C}=\frac{-\mu'}{2(1+n\lambda\tilde{R}^{n-1})}. \tag{38}$$

The second of the above equations points out that if $\mu'=0$, then $X_{TF}=0$, and vice versa. This indicates X_{TF} as an entity supervising inhomogeneities in the energy density of the anisotropic spherical fluids. This result supports the conclusions of [44]. Now, we are interested in finding out on which factors this X_{TF} depends, in the presence of dark source terms resulting from $f(R,T)$ gravity. The solution of Eq. (37) yields

$$X_{TF}=\frac{-3}{2(1+n\lambda\tilde{R}^{n-1})C^3}\int_0^t[\mu+(1+\nu)P_r]C^2\dot{C}dt-\frac{(1+\nu)^2(1+n\lambda\tilde{R}^{n-1})^{-1}}{2A\{1+(1+n\lambda\tilde{R}^{n-1})(1+\nu)\}}\int_0^t\left[(\mu+P_r)B^2C^5\Theta+\frac{2C\Pi\dot{C}}{A}\right]dt. \tag{39}$$

This illustrates the importance of pressure anisotropy and an expansion scalar in the modeling of regular energy density of the celestial spherical geometry in $f(R,T)$ gravity. Now, using Eqs. (21) and (22), we get

$$X_{TF}=\frac{-3}{2(1+n\lambda\tilde{R}^{n-1})C^3}\int_0^t[\mu+(1+\nu)P_r]C^2\dot{C}dt-\frac{(1+\nu)^2(1+n\lambda\tilde{R}^{n-1})^{-1}}{A^2\{1+(1+n\lambda\tilde{R}^{n-1})(1+\nu)\}}\int_0^t C\Pi\dot{C}dt. \tag{40}$$

This provides that inhomogeneity factor; i.e., X_{TF} depends upon the anisotropic pressure gradients in the scenario of $f(R,T)$ gravity. Since we know that, in the null expansion stellar body, the central point is covered by another metric appropriately joined with the rest of the matter distributions.

B. Radiating shearing viscous noninteracting particles

This subsection discusses the irregularity factors in the realm of dissipation with both free-streaming and diffusion limits, but with a special case of viscous particles. Therefore, we consider $P_r=0=P_\perp$ in the matter field, and the evolution is characterized by geodesics. This assumption is well established in the background of some theoretical developments. Then, Eqs. (24) and (25) give

$$\left[\mathcal{E}-\frac{1}{2(1+n\lambda\tilde{R}^{n-1})}\left\{\mu-2(1+\nu)\eta\sigma-\frac{(1-n)}{2}\lambda\tilde{R}^n-\frac{\nu}{2}\tilde{T}\right\}\right]_{,0}=\frac{3\dot{C}}{C}\times\left[\frac{1}{2(1+n\lambda\tilde{R}^{n-1})}\left\{\bar{\mu}-\frac{2}{3}(1+\nu)\eta\sigma+\mu\nu\right\}-\mathcal{E}\right]+\frac{3A(1+\nu)\bar{q}C'}{2BC(1+n\lambda\tilde{R}^{n-1})}, \tag{41}$$

$$\left[\mathcal{E}-\frac{1}{2(1+n\lambda\tilde{R}^{n-1})}\left\{\bar{\mu}+2(1+\nu)\eta\sigma-\frac{(1-n)}{2}\lambda\tilde{R}^n-\frac{\nu}{2}\tilde{T}\right\}\right]'=-3\frac{C'}{C}\times\left\{\mathcal{E}+\frac{(1+\nu)\eta\sigma}{(1+n\lambda\tilde{R}^{n-1})}\right\}-\frac{3B(1+\nu)\bar{q}\,\dot{C}}{2AC(1+n\lambda\tilde{R}^{n-1})}. \tag{42}$$

It has been investigated with the above equation that the quantity which is controlling irregularities is Ψ, defined as follows:

$$\Psi\equiv\mathcal{E}+\frac{(1+\nu)}{C^3(1+n\lambda\tilde{R}^{n-1})}\left[\eta\int_0^r\left(\sigma'-\frac{3C'}{C}\sigma\right)C^3dr-\frac{3}{2}\int_0^r B\dot{C}\,\bar{q}\,C^2dr\right]. \tag{43}$$

Thus, if there is a regular configuration of energy density, i.e., $\mu'=\mu(t)$, then $\Psi=0$, and vice versa. Thus, in order to enter into the homogeneous window by the radiating dust cloud, it should vanish the above quantity Ψ. It can be seen that Ψ controls shearing viscosity and heat flux. Making use of Eqs. (5) and (12) in Eq. (41), the Ψ evolution equation is found to be as follows:

$$\begin{aligned}\dot{\Psi}-\frac{\dot{\Omega}}{C^3}=&\frac{(1+\nu)^2(1+n\lambda\tilde{R}^{n-1})^{-1}}{2\{1+(1+n\lambda\tilde{R}^{n-1})(1+\nu)\}}\left[\bar{q}'B+B^2C^2\left(\bar{\mu}+\frac{2}{3}\eta\sigma\right)\Theta+B\left(\varepsilon+\frac{2}{3}\eta\sigma\right)\dot{B}\right]\\&+\frac{\dot{\varepsilon}}{2(1+n\lambda\tilde{R}^{n-1})}\left\{1-\frac{(1+\nu)}{\{1+(1+n\lambda\tilde{R}^{n-1})(1+\nu)\}}\right\}\\&+\frac{\eta(1+\nu)}{(1+n\lambda\tilde{R}^{n-1})}\left(\dot{\sigma}+\frac{\dot{C}}{C}\sigma\right)+\frac{3\dot{C}}{2C(1+n\lambda\tilde{R}^{n-1})}\{\varepsilon+\mu(1+\nu)\}-\frac{3\dot{C}}{C}\Psi\\&+\frac{(1+\nu)\bar{q}}{2(1+n\lambda\tilde{R}^{n-1})}\left\{\frac{3C'}{C}-\frac{(1+\nu)}{\{1+(1+n\lambda\tilde{R}^{n-1})(1+\nu)\}}\left(\frac{B'}{B}-\frac{CC'}{B^2}\right)\right\},\end{aligned} \tag{44}$$

whose solution leads to

$$\Psi=\frac{1}{C^3}\int_0^t\left[\dot{\Omega}+\left\{\frac{(1+\nu)^2(1+n\lambda\tilde{R}^{n-1})^{-1}C^3}{2\{1+(1+n\lambda\tilde{R}^{n-1})(1+\nu)\}}\left\{B^2C^2\left(\bar{\mu}+\frac{2}{3}\eta\sigma\right)\Theta+\bar{q}'B+\left(\varepsilon+\frac{2}{3}\eta\sigma\right)B\dot{B}\right\}\right\}\right.$$
$$+\frac{\dot{\varepsilon}C^3}{2(1+n\lambda\tilde{R}^{n-1})}\left\{1-\frac{(1+\nu)}{\{1+(1+n\lambda\tilde{R}^{n-1})(1+\nu)\}}\right\}$$
$$+\frac{\eta(1+\nu)}{(1+n\lambda\tilde{R}^{n-1})}\left(\dot{\sigma}+\frac{\dot{C}}{C}\sigma\right)C^3+\frac{3\dot{C}C^2}{2(1+n\lambda\tilde{R}^{n-1})}\{\varepsilon+\mu(1+\nu)\}$$
$$\left.+\frac{(1+\nu)\bar{q}}{2C^3(1+n\lambda\tilde{R}^{n-1})}\left\{\frac{3C'}{C}-\frac{(1+\nu)}{\{1+(1+n\lambda\tilde{R}^{n-1})(1+\nu)\}}\left(\frac{B'}{B}-\frac{CC'}{B^2}\right)\right\}\right]. \tag{45}$$

For an expansion-free condition, the inhomogeneity factor for the viscous dissipative system is found to be as follows:

$$\Psi=\frac{1}{C^3}\int_0^t\left[\dot{\Omega}+\left\{\frac{(1+\nu)^2(1+n\lambda\tilde{R}^{n-1})^{-1}C^3}{2\{1+(1+n\lambda\tilde{R}^{n-1})(1+\nu)\}}\left\{\frac{h\bar{q}'}{C^2}-\frac{2h^2\dot{C}}{C^5}\left(\varepsilon+\frac{2}{3}\eta\sigma\right)\right\}\right\}\right.$$
$$+\frac{\dot{\varepsilon}C^3}{2(1+n\lambda\tilde{R}^{n-1})}\left\{1-\frac{(1+\nu)}{\{1+(1+n\lambda\tilde{R}^{n-1})(1+\nu)\}}\right\}+\frac{\eta(1+\nu)}{(1+n\lambda\tilde{R}^{n-1})}\left(\dot{\sigma}+\frac{\dot{C}}{C}\sigma\right)C^3$$
$$+\frac{3\dot{C}C^2}{2(1+n\lambda\tilde{R}^{n-1})}\{\varepsilon+\mu(1+\nu)\}$$
$$\left.+\frac{(1+\nu)\bar{q}}{2C^3(1+n\lambda\tilde{R}^{n-1})}\left\{\frac{3C'}{C}-\frac{(1+\nu)}{\{1+(1+n\lambda\tilde{R}^{n-1})(1+\nu)\}}\left(\frac{h'}{h}-\frac{2C'}{C}-\frac{C^5C'}{h^2}\right)\right\}\right]. \tag{46}$$

This asserts the importance of matter parameters as the irregularity factor has some correspondence with the fluid variables, especially the shearing viscosity, heat flux, and structural properties of the system.

V. CONCLUSIONS

In this paper, we have studied the impact of modified gravity on the distribution of matter configuration for a self-gravitating spherical star. The disturbance in the hydrostatic equilibrium of a celestial object leads to homogeneous or inhomogeneous matter state. We have taken into consideration the spherically symmetric source in the gravitational field of $f(R,T)$ gravity. The geometry is filled with an imperfect fluid due to anisotropic stresses and dissipation which are designed in both limits, i.e., diffusion and the free-streaming limit. We have constructed the modified field equations and the corresponding dynamical equations using conservation laws. Some kinematical and dynamical quantities are formulated to explain the evolutionary development of such objects. The mass function using the Misner-Sharp [45] approach is calculated for our spherical object and the curvature tensor, as well as the Weyl tensor, is explored in this framework. It has been found that the Weyl tensor has its constituent tensor, like its electric and magnetic parts. Its magnetic part vanishes due to the symmetry of the underconsidered problem, while only its electric part exists.

It is a well-established fact that the Weyl tensor is responsible for the emergence of tidal forces which makes the object more inhomogeneous over the evolution of time. In our case, the spherical system suffers the inhomogeneous states due to the presence of its electric part only. We have established an explicit expression between the Weyl tensor and the matter variables—heat flux, anisotropic stresses, etc.—which is significantly important in light of Penrose's proposal [68]. Penrose provided the idea of a relationship between the Weyl tensor with inhomogeneities in energy density and isotropic pressure; however, such a link is no longer valid in the presence of anisotropies. In this manuscript, we have established such a relation between the Weyl tensor and fluid parameters in the background of higher order curvature terms emerging due to the $f(R,T)$ gravitational field.

We have also disintegrated the curvature tensor into its constituent parts using the comoving coordinates. These are found to be structure scalars, as was already obtained in the framework of $f(R)$ gravity. These scalars have gained significant importance in light of the Newtonian and general relativistic star models. It is also observed that these scalars are used to find the solutions of the Einstein field equations [69]. Moreover, these scalars are also used to discuss the irregular distribution of matter density. It is still unclear how different physical factors emerging in fluid configuration can affect the production of inhomogeneities in energy density. Here, we have found some factors creating the irregularities in the matter distribution with $f(R,T)$ extra curvature ingredients. Our analysis will strictly depend upon two differential equations emerging from the explicit expression of Weyl tensor with matter variables and the mass function. These equations are carried out by using Ellis's procedure as adopted in his paper. We have constructed our analysis to demonstrate the inhomogeneities in two regimes, i.e., with dissipative and nondissipative fluids. The results obtained in the particular cases of dust, isotropic, and anisotropic matter are given as follows.

(i) In the absence of dissipative effects and with dust cloud and $f(R,T)$ dark source terms, we have found that the evolutionary motion of the celestial bodies will be homogeneous if the Weyl scalar is zero, with extra curvature terms of the theory. In other words, the Weyl tensor attributed to its electric part and the impact of modified gravity makes the system more inhomogeneous in the gravitational arrow of time. This result can be seen from Eqs. (28) and (29); i.e., if we have a homogeneous matter distribution, then the Weyl tensor and the dark source term should vanish.

(ii) By increasing the complexity in the previous case with isotropic pressure, we have observed the same factors creating the irregularities in the density distribution, but in the presence of isotropic pressure.

(iii) For the nondissipative anisotropic fluid model, it has been found that a linear combination of matter profile is now responsible for the emergence of density inhomogeneity. Furthermore, we have examined that such a linear combination corresponds to one of the structure scalars obtained in Eq. (40).

(iv) For the dissipative dust cloud case, we have factored in controlling the density distribution as a combination of geometrical and physical variables in the background of $f(R,T)$ gravity theory, as obtained in Eqs. (45) and (46).

We mention that this study can be generalized to study the density inhomogeneity to $f(R,T,R_{\mu\nu}T^{\mu\nu})$ gravity. We would like to draw attention to a recent paper [70], in which Ayuso *et al.* discussed the consistency of a recently proposed class of theories described by an arbitrary function of the Ricci scalar, the trace of the energy-momentum tensor, and the contraction of the Ricci tensor with the energy-momentum tensor. Therein they briefly discussed the limitations of including the energy-momentum tensor in the action, as it is a non-fundamental quantity but a quantity that should be derived from the action (in agreement with Koivisto and Padmanabhan arguments). All of our results reduce to general relativity if we take $f(R,T) = R$.

ACKNOWLEDGMENTS

We sincerely appreciate the kind encouragement for this work from Professor Muhammad Sharif. This work was partially supported by JSPS Grant-in-Aid for Young Scientists (B) No. 25800136 and the research funds presented by Fukushima University (K. B.).

[1] S. Perlmutter *et al.* (Supernova Cosmology Project Collaboration), Astrophys. J. **517**, 565 (1999); A. G. Riess *et al.* (Supernova Search Team Collaboration), Astron. J. **116**, 1009 (1998).
[2] E. Komatsu *et al.* (WMAP Collaboration), Astrophys. J. Suppl. Ser. **192**, 18 (2011); G. Hinshaw *et al.* (WMAP Collaboration), *ibid.* **208**, 19 (2013); P. A. R. Ade *et al.* (Planck Collaboration), arXiv:1502.01589; arXiv:1502.02114; , Phys. Rev. Lett. **112**, 241101 (2014); P. A. R. Ade *et al.* (BICEP2 and Planck Collaborations), *ibid.* **114**, 101301 (2015); P. A. R. Ade *et al.* (BICEP2 and Keck Array Collaborations), *ibid.* **116**, 031302 (2016).
[3] M. Tegmark *et al.* (SDSS Collaboration), Phys. Rev. D **69**, 103501 (2004); U. Seljak *et al.* (SDSS Collaboration), *ibid.* **71**, 103515 (2005).
[4] D. J. Eisenstein *et al.* (SDSS Collaboration), Astrophys. J. **633**, 560 (2005).
[5] B. Jain and A. Taylor, Phys. Rev. Lett. **91**, 141302 (2003).
[6] S. Nojiri and S. D. Odintsov, Int. J. Geom. Methods Mod. Phys. **04**, 115 (2007).
[7] T. P. Sotirou and V. Faraoni, Rev. Mod. Phys. **82**, 451 (2010).
[8] S. Nojiri and S. D. Odintsov, Phys. Rep. **505**, 59 (2011).
[9] S. Capozziello and M. D. Laurentis, Phys. Rep. **509**, 167 (2011).
[10] S. Capozziello and V. Faraoni, *Beyond Einstein Gravity* (Springer, Dordrecht, 2010); K. Bamba, S. Capozziello, S. Nojiri, and S. D. Odintsov, Astrophys. Space Sci. **342**, 155 (2012); A. de la Cruz-Dombriz and D. Sáez-Gómez, Entropy **14**, 1717 (2012); A. Joyce, B. Jain, J. Khoury, and M. Trodden, Phys. Rep. **568**, 1 (2015); K. Koyama, Rep. Prog. Phys. **79**, 046902 (2016); K. Bamba, S. Nojiri, and S. D. Odintsov, arXiv:1302.4831; K. Bamba and S. D. Odintsov, *Proc. Sci.*, KMI2013 (2014) 023 [arXiv:1402.7114]; Symmetry **7**, 220 (2015).
[11] T. Harko, F. S. N. Lobo, S. Nojiri, and S. D. Odintsov, Phys. Rev. D **84**, 024020 (2011).
[12] O. Bertolami, C. G. Boehmer, T. Harko, and F. S. N. Lobo, Phys. Rev. D **75**, 104016 (2007).

[13] T. Harko and F. S. N. Lobo, Eur. Phys. J. C **70**, 373 (2010).
[14] N. J. Poplawski, arXiv:gr-qc/0608031.
[15] T. Faulkner, M. Tegmark, E. F. Bunn, and Y. Mao, Phys. Rev. D **76**, 063505 (2007).
[16] T. Chiba, Phys. Lett. B **575**, 1 (2003); A. L. Erickcek, T. L. Smith, and M. Kamionkowski, Phys. Rev. D **74**, 121501 (2006).
[17] D. R. K. Reddy, R. Santikumar, and R. L. Naidu, Astrophys. Space Sci. **342**, 249 (2012).
[18] K. S. Adhav, Astrophys. Space Sci. **339**, 365 (2012).
[19] M. Jamil, D. Momeni, and R. Myrzakulov, Chin. Phys. Lett. **29**, 109801 (2012).
[20] M. Sharif and Z. Yousaf, Astrophys. Space Sci. **354**, 471 (2014).
[21] G. Sun and Y.-C. Huang, Int. J. Mod. Phys. D **25**, 1650038 (2016).
[22] E. H. Baffou, A. V. Kpadonou, M. E. Rodrigues, M. J. S. Houndjo, and J. Tossa, Astrophys. Space Sci. **356**, 173 (2015).
[23] M. E. S. Alves, P. H. R. S. Moraes, J. C. N. de Araujo, and M. Malheiro, arXiv:1604.03874.
[24] A. Alhamzawi and R. Alhamzawi, Int. J. Mod. Phys. D **25**, 1650020 (2016).
[25] Z. Yousaf and M. Z. Bhatti, Eur. Phys. J. C **76**, 267 (2016).
[26] E. V. Arbuzova and A. D. Dolgov, Phys. Lett. B **700**, 289 (2011).
[27] K. Bamba, S. Nojiri, and S. D. Odintsov, Phys. Lett. B **698**, 451 (2011).
[28] M. J. S. Houndjo, Int. J. Mod. Phys. D **21**, 1250003 (2012).
[29] F. G. Alvarenga, M. J. S. Houndjo, A. V. Monwanou, and J. B. C. Orou, J. Mod. Phys. **4**, 130 (2013).
[30] T. Azizi, Int. J. Theor. Phys. **52**, 3486 (2013).
[31] B. L. Hu, L. J. Fang, and R. Ruffini, *Advances in Astrophysics* (World Scientific, Singapore, 1983).
[32] R. L. Naidu *et al.*, Glob. J. Sci. Front. Res. A **13** (2013).
[33] D. R. K. Reddy, R. L. Naidu, K. D. Naidu, and T. R. Prasad, Astrophys. Space Sci. **346**, 261 (2013).
[34] M. Sharif and Z. Yousaf, Mon. Not. R. Astron. Soc. **440**, 3479 (2014); J. Cosmol. Astropart. Phys. 06 (2014) 019; Astropart. Phys. **56**, 19 (2014); Astrophys. Space Sci. **355**, 317 (2015); Eur. Phys. J. C **75**, 194 (2015); M. Sharif and M. Z. Bhatti, J. Cosmol. Astropart. Phys. 10 (2013) 056; Astrophys. Space Sci. **347**, 337 (2013); **352**, 883 (2014); **355**, 389 (2015).
[35] M. Kiran and D. R. K. Reddy, Astrophys. Space Sci. **346**, 521 (2013).
[36] S. Nojiri and S. D. Odintsov, Classical Quantum Gravity **30**, 125003 (2013).
[37] R. Farinelli, M. De Laurentis, S. Capozziello, and S. D. Odintsov, Mon. Not. R. Astron. Soc. **440**, 2894 (2014).
[38] J. Guo, D. Wang, and A. V. Frolov, Phys. Rev. D **90**, 024017 (2014).
[39] F. D. Albareti, J. A. R. Cembranos, A. de la Cruz-Dombriz, and A. Dobadob, J. Cosmol. Astropart. Phys. 03 (2014) 012.
[40] I. Hason and Y. Oz, Eur. Phys. J. C **74**, 3183 (2014).
[41] L. Herrera, G. Le Denmat, and N. O. Santos, Phys. Rev. D **79**, 087505 (2009).
[42] L. Herrera, Int. J. Mod. Phys. D **20**, 1689 (2011).
[43] M. Sharif and M. Z. Bhatti, Mod. Phys. Lett. A **27**, 1250141 (2012); **29**, 1450094 (2014); **29**, 1450129 (2014); M. Sharif and Z. Yousaf, Astrophys. Space Sci. **352**, 321 (2014); **354**, 431 (2014); **357**, 49 (2015); Eur. Phys. J. C **75**, 58 (2015); Gen. Relativ. Gravit. **47**, 48 (2015); Can. J. Phys. **93**, 905 (2015).
[44] Z. Yousaf, K. Bamba, and M. Z. ul Haq Bhatti, Phys. Rev. D **93**, 064059 (1960).
[45] C. W. Misner and D. H. Sharp, Phys. Rev. **136**, B571 (1964).
[46] V. A. Skripkin, Sov. Phys. Dokl. **135**, 1183 (1960).
[47] L. Herrera, N. O. Santos, and A. Wang, Phys. Rev. D **78**, 084026 (2008).
[48] L. Herrera, N. O. Santos, and G. Le Denmat, Classical Quantum Gravity **27**, 135017 (2010).
[49] A. Di Prisco, L. Herrera, J. Ospino, N. O. Santos, and V. M. Vinã-Cervantes, Int. J. Mod. Phys. D **20**, 2351 (2011).
[50] M. Sharif and Z. Yousaf, Can. J. Phys. **90**, 865 (2012); Int. J. Mod. Phys. D **21**, 1250095 (2012).
[51] P. S. Joshi, N. Dadhich, and R. Maartens, Phys. Rev. D **65**, 101501 (2002).
[52] K. S. Virbhadra, D. Narasimha, and S. M. Chitre, Astron. Astrophys. **337**, 1 (1998).
[53] K. S. Virbhadra and G. F. R. Ellis, Phys. Rev. D **65**, 103004 (2002).
[54] C. M. Claudel, K. S. Virbhadra, and G. F. R. Ellis, J. Math. Phys. (N.Y.) **42**, 818 (2001).
[55] K. S. Virbhadra and G. F. R. Ellis, Phys. Rev. D **62**, 084003 (2000); K. S. Virbhadra, Phys. Rev. D **79**, 083004 (2009).
[56] K. S. Virbhadra and C. R. Keeton, Phys. Rev. D **77**, 124014 (2008).
[57] R. Van de Weygaert and E. Platen, Int. J. Mod. Phys. Conf. Ser. **01**, 41 (2011).
[58] B. Li, G. B. Zhao, and K. Koyama, Mon. Not. R. Astron. Soc. **421**, 3481 (2012); M. Sharif and Z. Yousaf, Phys. Rev. D **88**, 024020 (2013).
[59] D. L. Wiltshire, in *Dark Matter in Astroparticle and Particle Physics: Proceedings of the 6th International Heidelberg Conference* edited by H. V. Klapdor-Kleingrothaus and G. F. Lewis (World Scientific, Singapore, 2008), pp. 565–596.
[60] M. W. Hossain, R. Myrzakulov, M. Sami, and E. N. Saridakis, Phys. Rev. D **89**, 123513 (2014).
[61] A. A. Starobinsky, Phys. Lett. **91B**, 99 (1980).
[62] J. A. R. Cembranos, Phys. Rev. Lett. **102**, 141301 (2009); J. Phys. Conf. Ser. **315**, 012004 (2011).
[63] Z. Yousaf and M. Z. Bhatti, Mon. Not. R. Astron. Soc. **458**, 1785 (2016).
[64] A. V. Astashenok, S. Capozziello, and S. D. Odintsov, J. Cosmol. Astropart. Phys. 12 (2013) 040.
[65] M. Z. Bhatti and Z. Yousaf, Eur. Phys. J. C **76**, 219 (2016).
[66] G. F. R. Ellis, Gen. Relativ. Gravit. **41**, 581 (2009).
[67] L. Herrera, J. Ospino, and A. Di Prisco, Phys. Rev. D **84**, 107502 (2011).
[68] R. Penrose, in *General Relativity, An Einstein Centenary Survey*, edited by S. W. Hawking and W. Israel (Cambridge University Press, Cambridge, England, 1979).
[69] L. Herrera, A. Di Prisco, and J. Ospino, Gen. Relativ. Gravit. **44**, 2645 (2012); M. Sharif and M. Z. Bhatti, Gen. Relativ. Gravit. **44**, 2811 (2012); M. Sharif and Z. Yousaf, Astrophys. Space Sci. **357**, 49 (2015).
[70] I. Ayuso, J. B. Jimenez, and A. D. L. Cruz-Dombriz, Phys. Rev. D **91**, 104003 (2015).