Post-Newtonian parameters and cosmological constant of screened modified gravity

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Screened modified gravity (SMG) is a kind of scalar-tensor theory with screening mechanisms, which can generate a screening effect to suppress the fifth force in high density environments and pass the solar system tests. Meanwhile, the potential of the scalar field in the theories can drive the acceleration of the late Universe. In this paper, we calculate the parametrized post-Newtonian (PPN) parameters γ and β, the effective gravitational constant $G_{\rm eff}$, and the effective cosmological constant Λ for SMG with a general potential V and coupling function A. The dependence of these parameters on the model parameters of SMG and/or the physical properties of the source object are clearly presented. As an application of these results, we focus on three specific theories of SMG (chameleon, symmetron, and dilaton models). Using the formulas to calculate their PPN parameters and cosmological constant, we derive the constraints on the model parameters by combining the observations on solar system and cosmological scales.

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I. INTRODUCTION

The current cosmic acceleration [1] can be elucidated within general relativity (GR) by introducing dark energy [2]. Some prominent candidates for dark energy are the cosmological constant [3], a dynamically evolving scalar field quintessence [4], a phantom field [5], a quintom field [6], etc. Alternatively, the accelerated expansion of the Universe can also be explained through modified gravity (MG) theories [7]. On large scales, we do not have very strict experiments to verify GR, and then infrared (IR) modification of gravity is the direction that is supposed to be worth a try [8]. Weinberg's theorem states that any Lorentz invariant spin-2 field theory must reduce to GR at a low-energy limit [9], and thus any MG theories must involve extra degree(s) of freedom. The scalar degree of freedom universally exists in fundamental physics (such as compactified extra dimensions [10], string theory, and brane world [11]). Since the Higgs boson in the Standard Model of particles was found [12], we know that scalar particles really exist in nature. Moreover, scalar fields are also widely used in cosmology. Quintessence scalar field can replace the cosmological constant and drive cosmic acceleration at late times [4]. The inflation is a short period of rapid expansion in the very early Universe, which could also be caused by a scalar field [13,14]. These scalar fields may couple to matter fields, which slightly violates GR and could be detected as the continuous improvement of experimental accuracy.

Most MG theories involve scalar field, and the simplest one is the so-called scalar-tensor gravity [15–19]. The fundamental building blocks of scalar-tensor theories are the tensor gravitational field and scalar field. Moreover, scalar-tensor theories can be justified by the low-energy limit of string theory or supergravity [20,21]. Scalar-tensor theories are usually expressed either in the Jordan frame or in the Einstein frame, which are related to each other by a conformal rescaling [22]. In the Einstein frame, a key ingredient of scalar-tensor theories is the conformal coupling of light scalar field with matter fields, which usually implies the existence of a new long-range fifth force. However, at present, fifth forces have not been detected in either solar system or laboratory experiments, which means that the strength of the fifth force should be much weaker than that of the gravitational force [23,24]. Therefore, we need the screening mechanisms, which can suppress the fifth force and allow MG theories to evade the tight gravitational tests in the solar system and the laboratory.

Examples of such screened models abound. The chameleon mechanism [25–29] operates a thin-shell shielding scalar field, which acquires a large mass in dense environments and suppresses its ability to mediate a fifth force. The symmetron mechanism [30–35] relies on the scalar field with the $\mathbb{Z}_2$ symmetry breaking potential. In high density regions, the $\mathbb{Z}_2$ symmetry is unbroken and the fifth force is absent, whereas in low density regions, the $\mathbb{Z}_2$ symmetry is spontaneously broken and the fifth force is present. The dilaton mechanism [20,36–38] is similar to the symmetron. The coupling between dilaton and matter is negligible in dense regions, while in low density regions the dilaton mediates a gravitational-strength fifth force. These screening mechanisms can be described by the same formalism [39], which is defined by a potential $V(\phi)$ and a coupling function $A(\phi)$ in scalar-tensor theory in the

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Einstein frame. Such scalar-tensor gravity with a screening mechanism is often called screened modified gravity (SMG) [39,40,42]. A basic requirement of SMG is that the effective potential must have a minimum [39], which can naturally be understood as a stable vacuum. This requirement can roughly constrain the shapes of two dynamical functions $V(\phi)$ and $A(\phi)$.

In this paper, we focus on a generic SMG with arbitrary potential $V(\phi)$ and coupling function $A(\phi)$, and we calculate the parametrized post-Newtonian (PPN) parameters γ and β in the case of a static spherically symmetric source. Moreover, SMG contains a scalar degree of freedom, whose potential can naturally provide the vacuum energy to drive the cosmic acceleration at late times. These two analyses allow us to investigate the theoretical framework on solar system and cosmological scales to derive the combined constraints on model parameters.

In the literature [43,44], the PPN parameters of a generic scalar-tensor theory were calculated under the assumption of a point source surrounded by vacuum. This assumption is generally not appropriate to solve the massive scalar field since the exterior scalar field of an extended source behaves quite different from that of a point source, and screening mechanisms can show up due to nonlinear effects of a scalar field [44]. So, these results are not applicable to SMG, whose scalar field is always massive and can be screened in dense bodies.

In this paper, we solve the massive scalar field in the Einstein frame in the case of an extended source surrounded by a homogeneous background. Making use of this scalar solution and the PPN formalism [45,46], in the Einstein frame, we solve the massless metric field in the weak field limit around the flat Minkowski background and the vacuum expectation value (VEV) of the scalar field (scalar background). Then, we transform them to the Jordan frame and calculate the PPN parameters γ and β and the effective gravitational constant $G_{\rm eff}$. It turns out that these parameters (γ, β, $G_{\rm eff}$) depend not only on the distance r between the source object and the test mass but also on the screened parameter ϵ.

Moreover, SMG contains a scalar degree of freedom, and the bare potential VEV of the scalar field can play the role of dark energy to accelerate the expansion of the Universe. Further analysis shows that a generic SMG can converge back to GR with a cosmological constant in the limiting case $\epsilon \to 0$. In particular, we focus on three specific theories of SMG (chameleon, symmetron, and dilaton models) and use our formulas to calculate their PPN parameters and cosmological constant. We find that our expressions of the PPN parameters for these three models can reduce to previous results derived by other authors in the appropriate cases. Finally, we combine solar system and cosmological constraints on these three models.

This paper is organized as follows. In Sec. II, we display the action and field equations for a generic SMG and solve the scalar field equation in the Einstein frame. In Sec. III, we derive the post-Newtonian metric field equations in the Einstein frame. In Sec. IV, we solve the post-Newtonian metric field equations and calculate the PPN parameters and cosmological constant for a generic SMG. In Sec. V, we discuss chameleon, symmetron, and dilaton models and constrain them by the current observations. Finally, we conclude our results in Sec. VI.

Throughout this paper, the metric convention is chosen as $(-,+,+,+)$ and Greek indices $(\mu,\nu,\cdots)$ run over 0,1,2,3. We set the units to $c=\hbar=1$, and therefore, the reduced Planck mass is $M_{\rm Pl}=\sqrt{1/8\pi G}$, where G is the Newtonian gravitational constant.

II. ACTION FUNCTIONAL AND FIELD EQUATIONS

A general scalar-tensor theory with two arbitrary functions is given by the following action in the Einstein frame [15–19]:

$$S_{\rm E}=\int d^4x\sqrt{-g_{\rm E}}\left[\frac{M_{\rm Pl}^2}{2}R_{\rm E}-\frac{1}{2}(\nabla_{\rm E}\phi)^2-V(\phi)\right] + S_m[A^2(\phi)g^{\rm E}_{\mu\nu},\psi_m^{(i)}], \quad (1)$$

where $g_{\rm E}$ is the determinant of the Einstein frame metric $g^{\rm E}_{\mu\nu}$, $R_{\rm E}$ is the Ricci scalar, $\psi_m^{(i)}$ are various matter fields labeled by i, $V(\phi)$ is a bare potential characterizing the scalar self-interaction, and $A(\phi)$ is a conformal coupling function. In the Einstein frame, the scalar field ϕ interacts directly with matter fields $\psi_m^{(i)}$ through the conformal coupling function $A(\phi)$. In the Jordan frame, the matter fields $\psi_m^{(i)}$ couple to the Jordan frame metric $g^{\rm J}_{\mu\nu}$ through a conformal rescaling of the Einstein frame metric $g^{\rm E}_{\mu\nu}$ as [22]

$$g^{\rm J}_{\mu\nu}=A^2(\phi)g^{\rm E}_{\mu\nu}, \quad (2)$$

where the coupling function $A(\phi)$ is usually different for different matter fields $\psi_m^{(i)}$. For simplicity, from now on, we assume that all matter fields couple in the same way to the scalar field with a universal coupling function $A(\phi)$.

The variation of the action (1) with respect to the metric field and the scalar field yields the metric field equation of motion (EOM) and the scalar field EOM:

$$R^{\rm E}_{\mu\nu}=8\pi G[S^{\rm E}_{\mu\nu}+\partial_\mu\phi\partial_\nu\phi+V(\phi)g^{\rm E}_{\mu\nu}], \quad (3)$$

$$\Box\phi=\frac{{\rm d}V(\phi)}{{\rm d}\phi}-T^{\rm E}\frac{{\rm d}A(\phi)}{A(\phi){\rm d}\phi}, \quad (4)$$

with

$$S^{\rm E}_{\mu\nu}\equiv T^{\rm E}_{\mu\nu}-\frac{1}{2}g^{\rm E}_{\mu\nu}T^{\rm E}, \quad (5)$$

where $T^{\rm E}_{\mu\nu} \equiv (-2/\sqrt{-g_{\rm E}})\delta S_m/\delta g^{\mu\nu}_{\rm E}$ is the energy-momentum tensor of matter in the Einstein frame, $T^{\rm E}$ is the trace of the energy-momentum tensor $T^{\mu\nu}_{\rm E}$, and $\Box \equiv g^{\mu\nu}_{\rm E}\nabla_\mu\nabla_\nu$. The scalar field EOM (4) can be rewritten as follows (Klein-Gordon equation):

$$\Box\phi = \frac{dV_{\rm eff}}{d\phi}, \tag{6}$$

with the effective potential

$$V_{\rm eff}(\phi) \equiv V(\phi) + \rho[A(\phi) - 1], \tag{7}$$

where the matter is assumed to be nonrelativistic. Here, ρ is defined as the conserved energy density in the Einstein frame; i.e., ρ is independent of ϕ. The density ρ is related to the Einstein frame and Jordan frame matter densities by [47]

$$\rho = \frac{\rho_{\rm E}}{A} = A^3\rho_{\rm J}. \tag{8}$$

The scalar field is governed by the effective potential $V_{\rm eff}(\phi)$, and the shape of the effective potential determines the behavior of the scalar field. For a general scalar-tensor theory with two arbitrary functions $V(\phi)$ and $A(\phi)$, the shape of the effective potential $V_{\rm eff}(\phi)$ is usually arbitrary, and this scalar field generally does not have screening properties. For suitably chosen functions $V(\phi)$ and $A(\phi)$, the effective potential $V_{\rm eff}(\phi)$ can have a minimum; i.e., the scalar field has a physical vacuum. Around this minimum (physical vacuum), the scalar field acquires an effective mass which increases as the ambient density increases, and the scalar field can be screened in high density environments. This kind of scalar-tensor gravity with a screening mechanism is often called screened modified gravity (SMG) [39,40,42], which can generate the screening effect to suppress the fifth force in high density environments and pass the solar system tests. There are many SMG models in the market, including the chameleon, symmetron, and dilaton models [39], in which the functions $V(\phi)$ and $A(\phi)$ are chosen as the specific forms.

The following two conditions (9a) and (9b) guarantee that the effective potential $V_{\rm eff}(\phi)$ has a minimum. Differentiation of the effective potential with respect to ϕ is zero at $\phi = \phi_{\rm min}(\rho)$, i.e.,

$$\left.\frac{dV_{\rm eff}}{d\phi}\right|_{\phi_{\rm min}} = 0, \tag{9a}$$

and the value of $\phi_{\rm min}(\rho)$ decreases as the ambient density increases. The effective mass $m_{\rm eff}(\rho)$ of the scalar field at the minimum is defined as

$$m^2_{\rm eff} \equiv \left.\frac{d^2V_{\rm eff}}{d\phi^2}\right|_{\phi_{\rm min}}, \tag{9b}$$

which should be a positive and monotonically increasing function of the ambient density.

Let us consider a static spherically symmetric and constant density source object, which is embedded in a homogeneous background. Then, the scalar field EOM (6) simplifies to

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} = m^2_{\rm m}(\rho)[\phi - \phi_{\rm m}(\rho)], \tag{10}$$

with

$$\rho(r) = \begin{cases} \rho_0 & \text{for } r < R \\ \rho_\infty & \text{for } r > R \end{cases}, \tag{11}$$

where R is the radius of the source object, ρ_0 is the density of the source object, and ρ_∞ is the background matter density. For the solar system, in general, ρ_∞ is the cosmological matter density or galactic matter density [39,48], which corresponds to the cosmological background or galactic background, respectively.

Equation (10) is a second-order differential equation, and the boundary conditions are required as follows [25]:

$$\begin{aligned} &\frac{d\phi}{dr} = 0 \quad \text{at } r = 0, \\ &\phi \to \phi_\infty \quad \text{as } r \to \infty, \end{aligned} \tag{12}$$

where ϕ_∞ is the scalar field VEV (scalar background), depending on the background matter density ρ_∞. The first condition guarantees that the scalar field is nonsingular at the origin [25], and the second one implies that the scalar field asymptotically converges to the scalar background. Moreover, ϕ and $d\phi/dr$ are, of course, continuous at the surface of the source object. By solving Eq. (10) directly, we get the exact solution

$$\phi(r < R) = \phi_0 + \frac{A}{r}\sinh(m_0 r), \tag{13a}$$

$$\phi(r > R) = \phi_\infty + \frac{B}{r}e^{-m_\infty r}, \tag{13b}$$

with

$$A = \frac{(\phi_\infty - \phi_0)(1 + m_\infty R)}{m_0\cosh(m_0 R) + m_\infty \sinh(m_0 R)}, \tag{14a}$$

$$B = -e^{m_\infty R}(\phi_\infty - \phi_0)\frac{m_0 R - \tanh(m_0 R)}{m_0 + m_\infty \tanh(m_0 R)}, \tag{14b}$$

where ϕ_0 and ϕ_∞ are, respectively, the positions of the minimum of $V_{\rm eff}$ inside and far outside the source object, and m_0 and m_∞ are, respectively, the effective masses of the scalar field at ϕ_0 and ϕ_∞. All these quantities can be obtained by two given functions $V(\phi)$ and $A(\phi)$.

The scalar field is screened on solar system scales (high density), which requires that the typical scale of the solar system R is much larger than the fifth force range m_0^{-1}. In addition, the scalar field works on cosmological scales (low density), which requires that m_∞^{-1} is close to the Hubble scale. So, the conditions $m_0 R \gg 1$ and $m_\infty R \ll 1$ can always be satisfied on solar system scales. In this paper, we only consider the exterior solution of the scalar field. Using these two relations, the exterior scalar field (13b) is reduced to

$$\phi(r) = \phi_\infty - \epsilon M_{\mathrm{Pl}} \frac{GM_{\mathrm{E}}}{r} e^{-m_\infty r}, \tag{15}$$

with

$$\epsilon \equiv \frac{\phi_\infty - \phi_0}{M_{\mathrm{Pl}} \Phi_{\mathrm{E}}}, \tag{16}$$

where M_{E} is the mass of the source object in the Einstein frame, $\Phi_{\mathrm{E}} \equiv GM_{\mathrm{E}}/R$ is the Newtonian potential at the surface of the source object in the Einstein frame, and the parameter ϵ depends on background matter density ρ_∞ and the physical properties (density ρ_0 and radius R) of the source object. Obviously, the screening effect is very strong for $\epsilon \ll 1$ and quite weak for $\epsilon \gtrsim 1$, so ϵ is always called the screened parameter or the thin-shell parameter in the literature [25].

This completes the solution of the scalar field EOM in the Einstein frame. In the next section, we use the scalar field solution to derive the post-Newtonian metric field equations in the Einstein frame.

III. POST-NEWTONIAN APPROXIMATION IN THE EINSTEIN FRAME

In order to solve the metric field EOM (3), we make use of the PPN formalism introduced in [45,46]. In this formalism, the gravitational field of the source is weak $GM/r \ll 1$, and the typical velocity $\vec{v}$ of the source matter is small $v^2 \sim GM/r \ll 1$. Thus, we can use the perturbative expansion method to solve the field equations, and all dynamical quantities can be expanded to $\mathcal{O}(n) \propto v^{2n}$ (note that other authors use the convention $\mathcal{O}(n) \propto v^n$).

In this section, we consider a static spherically symmetric source and assume that the source object is constituted by a perfect fluid which obeys the post-Newtonian hydrodynamics. We start from this assumption and expand the metric field EOM to $\mathcal{O}(n) \propto v^{2n}$ in the weak field limit around the flat Minkowski background and the scalar background (scalar field VEV). The resulting equations can then be solved subsequently for each order of magnitude in the next section.

For the metric field $g_{\mu\nu}$ in the weak field, it can be expanded around the flat Minkowski background as follows:

$$\begin{aligned} g_{\mu\nu} &= \eta_{\mu\nu} + h_{\mu\nu} \\ &= \eta_{\mu\nu} + h_{\mu\nu}^{(1)} + h_{\mu\nu}^{(2)} + \mathcal{O}(3). \end{aligned} \tag{17}$$

This metric can also be written in the spherically symmetric and isotropic coordinates $(t_{\mathrm{E}}, r, \theta, \varphi)$ in the Einstein frame,

$$\begin{aligned} ds_{\mathrm{E}}^2 = &- [1 - h_{\mathrm{E}00}^{(1)}(r) - h_{\mathrm{E}00}^{(2)}(r)] dt_{\mathrm{E}}^2 \\ &+ [1 + h_{\mathrm{E}rr}^{(1)}(r)](dr^2 + r^2 d\Omega^2), \end{aligned} \tag{18}$$

where t_{E} and r are the time and radial coordinates in the Einstein frame, respectively. Each term $h_{\mathrm{E}\mu\nu}^{(n)}$ is of order $\mathcal{O}(n)$. The term $d\Omega^2$ is defined by $d\Omega^2 \equiv dr^2 + \sin^2\theta d\varphi^2$, and the flat Minkowski background is $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, r^2, r^2 \sin^2\theta)$.

For the scalar field ϕ, the exterior solution (15) is a following expansion in the weak field limit around the scalar background,

$$\phi(r) = \phi_\infty + \phi^{(1)}(r), \tag{19}$$

where $\phi^{(1)}$ is of order $\mathcal{O}(1)$, given by

$$\phi^{(1)}(r) = -\epsilon M_{\mathrm{Pl}} \frac{GM_{\mathrm{E}}}{r} e^{-m_\infty r}, \tag{20}$$

$\phi_\infty \equiv \phi_{\mathrm{VEV}}$ is the scalar background (scalar field VEV), which depends on the background matter density ρ_∞. Note that, the term $\phi^{(2)}$ naturally does not exist in our expression of the scalar field ϕ (15), which is different from the results derived in other method [43,44]. Then, the bare potential $V(\phi)$ and the coupling function $A(\phi)$ can be expanded in Taylor's series around the scalar background,

$$V(\phi) = V_{\mathrm{VEV}} + V_1(\phi - \phi_\infty) + V_2(\phi - \phi_\infty)^2 + \mathcal{O}(3), \tag{21a}$$

$$A(\phi) = A_{\mathrm{VEV}} + A_1(\phi - \phi_\infty) + A_2(\phi - \phi_\infty)^2 + \mathcal{O}(3), \tag{21b}$$

where $A_{\mathrm{VEV}} \equiv A(\phi_{\mathrm{VEV}})$ is the coupling function VEV, and $V_{\mathrm{VEV}} \equiv V(\phi_{\mathrm{VEV}})$ is the bare potential VEV, which acts as the effective cosmological constant to accelerate the expansion of the late Universe.

The energy-momentum tensor is given by that of a perfect fluid [45,46]

$$T^{\mu\nu} = (\rho + \rho\Pi + p) u^\mu u^\nu + p g^{\mu\nu}, \tag{22}$$

and the tensor $S^{\rm E}_{\mu\nu}$ (5) is expanded in the form

$$S^{\rm E}_{00} = \frac{1}{2}\rho_{\rm E}(1+\Pi_{\rm E}+2v^2_{\rm E}-h^{(1)}_{\rm E00})+\frac{3}{2}p_{\rm E}+\mathcal{O}(3), \tag{23a}$$

$$S^{\rm E}_{rr} = \frac{1}{2}\rho_{\rm E}+\mathcal{O}(2), \tag{23b}$$

$$S^{\rm E}_{\theta\theta} = \frac{1}{2}\rho_{\rm E}r^2+\mathcal{O}(2), \tag{23c}$$

$$S^{\rm E}_{\varphi\varphi} = \frac{1}{2}\rho_{\rm E}r^2\sin^2\theta+\mathcal{O}(2), \tag{23d}$$

where ρ is density of rest mass, p is pressure, Π is internal energy per unit rest mass, u^μ is four-velocity, and the index E indicates that a quantity is defined in the Einstein frame. For solar system tests, we typically have $p \ll \rho$, $\Pi \ll 1$, and $v \ll 1$. So, we neglect the effects of pressure, internal energy, and velocity in the following discussions.

By using these relations, the right-hand sides of the metric field EOM (3) can be expanded to the required order in the form

$$R^{\rm E}_{00} = 8\pi G\left[-V_{\rm VEV}+\frac{\rho_{\rm E}}{2}+V_{\rm VEV}h^{(1)}_{\rm E00}-V_1\phi^{(1)}-\frac{\rho_{\rm E}}{2}h^{(1)}_{\rm E00} + V_{\rm VEV}h^{(2)}_{\rm E00}+V_1\phi^{(1)}h^{(1)}_{\rm E00}-V_2(\phi^{(1)})^2\right]+\mathcal{O}(3), \tag{24a}$$

$$R^{\rm E}_{rr} = 8\pi G\left[V_{\rm VEV}+\frac{\rho_{\rm E}}{2}+V_{\rm VEV}h^{(1)}_{{\rm E}rr}+V_1\phi^{(1)}\right]+\mathcal{O}(2), \tag{24b}$$

$$R^{\rm E}_{\theta\theta} = 8\pi Gr^2\left[V_{\rm VEV}+\frac{\rho_{\rm E}}{2}+V_{\rm VEV}h^{(1)}_{{\rm E}rr}+V_1\phi^{(1)}\right]+\mathcal{O}(2), \tag{24c}$$

$$R^{\rm E}_{\varphi\varphi} = R^{\rm E}_{\theta\theta}\sin^2\theta+\mathcal{O}(2). \tag{24d}$$

The left-hand sides of the metric field EOM (3), i.e., the components of the Ricci tensor, are expanded to the same order in the form

$$R^{\rm E}_{00} = -\frac{1}{2}\nabla^2_r h^{(1)}_{\rm E00}-\frac{1}{2}\left(\nabla^2_r h^{(2)}_{\rm E00}-h^{(1)}_{{\rm E}rr}\nabla^2_r h^{(1)}_{\rm E00} +\frac{1}{2}(\partial_r h^{(1)}_{\rm E00})^2+\frac{1}{2}(\partial_r h^{(1)}_{\rm E00})(\partial_r h^{(1)}_{{\rm E}rr})\right)+\mathcal{O}(3), \tag{25a}$$

$$R^{\rm E}_{rr} = \frac{1}{2}\partial^2_r h^{(1)}_{\rm E00}-\partial^2_r h^{(1)}_{{\rm E}rr}-\frac{1}{r}\partial_r h^{(1)}_{{\rm E}rr}+\mathcal{O}(2), \tag{25b}$$

$$R^{\rm E}_{\theta\theta} = \frac{1}{2}r^2\left(\frac{1}{r}\partial_r h^{(1)}_{\rm E00}-\partial^2_r h^{(1)}_{{\rm E}rr}-\frac{3}{r}\partial_r h^{(1)}_{{\rm E}rr}\right)+\mathcal{O}(2), \tag{25c}$$

$$R^{\rm E}_{\varphi\varphi} = R^{\rm E}_{\theta\theta}\sin^2\theta+\mathcal{O}(2), \tag{25d}$$

where $\nabla^2_r \equiv \partial^2_r+2/r\partial_r$ is the flat space spherical coordinate Laplace operator. Obviously, Eqs. (24d) and (25d) are equivalent to Eqs. (24c) and (25c), so there are only three independent equations, which are solved to derive the PPN parameters in the following sections.

IV. STATIC SPHERICALLY SYMMETRIC SOLUTION

A. Metric in the Einstein frame

Since the metric gravitational field is always massless in SMG, similar to the previous work [43], we solve the metric field equations in the case of a point source, i.e., $\rho_{\rm E} = M_{\rm E}\delta(r)$. In the following calculation, we neglect dark energy $V_{\rm VEV}$ [3] and gravity $h^{(n)}_{\mu\nu}$ interaction terms $V_{\rm VEV}h^{(n)}_{\mu\nu}$ since the effect of this interaction is quite weak on solar system scales.

We consider the post-Newtonian metric field equations (24a) and (25a) up to first order and obtain the equation

$$\nabla^2_r h^{(1)}_{\rm E00} = 8\pi G(2V_{\rm VEV}-\rho_{\rm E}+2V_1\phi^{(1)}). \tag{26}$$

Using the scalar field (20), the solution is given by

$$h^{(1)}_{\rm E00}(r) = \frac{2GM_{\rm E}}{r}\left(1-\frac{V_1}{M_{\rm Pl}m^2_\infty}\epsilon e^{-m_\infty r}\right)+\frac{8\pi GV_{\rm VEV}}{3}r^2. \tag{27}$$

For the spatial components, up to first order, the post-Newtonian metric field equations (24b), (25b), (24c), and (25c) follow

$$\frac{1}{2}\partial^2_r h^{(1)}_{\rm E00}-\partial^2_r h^{(1)}_{{\rm E}rr}-\frac{1}{r}\partial_r h^{(1)}_{{\rm E}rr} = 8\pi G\left(V_{\rm VEV}+\frac{\rho_{\rm E}}{2}+V_1\phi^{(1)}\right), \tag{28a}$$

$$\frac{1}{r}\partial_r h^{(1)}_{\rm E00}-\partial^2_r h^{(1)}_{{\rm E}rr}-\frac{3}{r}\partial_r h^{(1)}_{{\rm E}rr} = 8\pi G(2V_{\rm VEV}+\rho_{\rm E}+2V_1\phi^{(1)}). \tag{28b}$$

Combining these two equations, and using Eq. (26), we have

$$\nabla^2_r h^{(1)}_{{\rm E}rr} = 8\pi G(-V_{\rm VEV}-\rho_{\rm E}-V_1\phi^{(1)}), \tag{29}$$

and the solution is also derived by applying the solution of scalar field in (20),

$$h_{\mathrm{E}rr}^{(1)}(r) = \frac{2GM_{\mathrm{E}}}{r}\left(1 + \frac{V_1}{2M_{\mathrm{Pl}}m_\infty^2}\epsilon e^{-m_\infty r}\right) - \frac{8\pi G V_{\mathrm{VEV}}}{6}r^2. \tag{30}$$

We now consider the post-Newtonian metric field equation [(24a) and (25a)]. Up to second order, we obtain the equation

$$\nabla_r^2 h_{\mathrm{E}00}^{(2)} + \frac{1}{2}\partial_r h_{\mathrm{E}00}^{(1)}\partial_r(h_{\mathrm{E}00}^{(1)} + h_{\mathrm{E}rr}^{(1)}) = 8\pi G[2V_1\phi^{(1)}(h_{\mathrm{E}rr}^{(1)} - h_{\mathrm{E}00}^{(1)}) + 2V_2(\phi^{(1)})^2], \tag{31}$$

where we have neglected the terms $\rho_{\mathrm{E}}h_{\mu\nu}^{(1)}$ and $\rho_{\mathrm{E}}\phi^{(1)}$, which correspond to the gravitational self-energies and do not affect the calculation of the PPN parameter β [43,45,46]. Using the metric fields (27) and (30) and the scalar field (20), the solution of Eq. (31) is given by

$$\begin{aligned} h_{\mathrm{E}00}^{(2)}(r) = &-\frac{2G^2M_{\mathrm{E}}^2}{r^2}\bigg[1 - \frac{5V_1}{4M_{\mathrm{Pl}}m_\infty^2}(1 - m_\infty r)\epsilon e^{-m_\infty r} \\ &- \frac{V_2}{2m_\infty^2}\left(1 - \frac{3V_1^2}{M_{\mathrm{Pl}}^2 m_\infty^2 V_2}\right)\epsilon^2 m_\infty r e^{-2m_\infty r} \\ &+ \frac{V_1^2}{4M_{\mathrm{Pl}}^2 m_\infty^4}(1 - m_\infty r)\epsilon^2 e^{-2m_\infty r} \\ &+ \frac{5V_1}{4M_{\mathrm{Pl}}m_\infty^2}\epsilon(m_\infty r)^2 \mathrm{Ei}(-m_\infty r) \\ &- \left(\frac{V_2}{m_\infty^2} - \frac{5V_1^2}{2M_{\mathrm{Pl}}^2 m_\infty^4}\right)\epsilon^2(m_\infty r)^2\mathrm{Ei}(-2m_\infty r)\bigg], \end{aligned} \tag{32}$$

where the function $\mathrm{Ei}(-x)$ is defined by the exponential integral

$$\mathrm{Ei}(-x) \equiv -\int_x^\infty da\frac{e^{-a}}{a}. \tag{33}$$

The quantity m_∞ is the effective mass of the scalar field at $\rho = \rho_\infty$. Using the relations (21) and (9b), this quantity can be written as

$$m_\infty^2 = 2(V_2 + \rho_\infty A_2). \tag{34}$$

B. Metric in the Jordan frame

SMG theories are usually expressed either in the Einstein frame or in the Jordan frame, and these two frames are related by a conformal rescaling [22]. The PPN parameters are defined in the Jordan frame [43,45,46], so we should transform to the Jordan frame to get the expressions of parameters γ and β.

In the weak field limit, the metric is written in the spherically symmetric and isotropic coordinates $(t_{\mathrm{J}}, \chi, \theta, \varphi)$ as follows:

$$\begin{aligned} ds_{\mathrm{J}}^2 = &-[1 - h_{\mathrm{J}00}^{(1)}(\chi) - h_{\mathrm{J}00}^{(2)}(\chi)]dt_{\mathrm{J}}^2 \\ &+ [1 + h_{\mathrm{J}\chi\chi}^{(1)}(\chi)](d\chi^2 + \chi^2 d\Omega^2), \end{aligned} \tag{35}$$

where $d\Omega^2 \equiv dr^2 + \sin^2\theta d\varphi^2$, and t_{J} and χ are the time and radial coordinates in the Jordan frame, respectively, which relate to the corresponding quantities in the Einstein frame through the relations (38d). This metric naturally satisfies the standard post-Newtonian gauge [45], and the PPN parameters γ and β are defined in the form [45,46]

$$h_{\mathrm{J}00}^{(1)}(\chi) \equiv \frac{2G_{\mathrm{eff}}(\chi)M_{\mathrm{J}}}{\chi}, \tag{36a}$$

$$h_{\mathrm{J}\chi\chi}^{(1)}(\chi) \equiv \gamma(\chi)\frac{2G_{\mathrm{eff}}(\chi)M_{\mathrm{J}}}{\chi}, \tag{36b}$$

$$h_{\mathrm{J}00}^{(2)}(\chi) \equiv -\beta(\chi)\frac{4G_{\mathrm{eff}}^2(\chi)M_{\mathrm{J}}^2}{2\chi^2}, \tag{36c}$$

where G_{eff} is the effective gravitational "constant," and M_{J} is the mass of the source object in the Jordan frame, which relates to the mass in the Einstein frame through the relation (39). As mentioned above, in this paper, we neglect the effects of the pressure p, internal energy Π, and velocity v of the source object, which may contribute additional PPN parameters [43,45,46].

Using the relations (18) and (21b), the conformal rescaling (2) turns into

$$\begin{aligned} ds_{\mathrm{J}}^2 &= A^2(\phi)ds_{\mathrm{E}}^2 \\ &= -\bigg[1 - h_{\mathrm{E}00}^{(1)} + \frac{2A_1}{A_{\mathrm{VEV}}}\phi^{(1)} - h_{\mathrm{E}00}^{(2)} - \frac{2A_1}{A_{\mathrm{VEV}}}h_{\mathrm{E}00}^{(1)}\phi^{(1)} \\ &\quad + \left(\frac{2A_2}{A_{\mathrm{VEV}}} + \frac{A_1^2}{A_{\mathrm{VEV}}^2}\right)(\phi^{(1)})^2\bigg]A_{\mathrm{VEV}}^2 dt_{\mathrm{E}}^2 \\ &\quad + \left(1 + h_{\mathrm{E}rr}^{(1)} + \frac{2A_1}{A_{\mathrm{VEV}}}\phi^{(1)}\right)A_{\mathrm{VEV}}^2(dr^2 + r^2 d\Omega^2). \end{aligned} \tag{37}$$

Comparing this relation (37) with the Jordan frame metric in (35), we obtain the relations

$$h_{\mathrm{J}00}^{(1)} = h_{\mathrm{E}00}^{(1)} - \frac{2A_1}{A_{\mathrm{VEV}}}\phi^{(1)}, \tag{38a}$$

$$h_{\mathrm{J}\chi\chi}^{(1)} = h_{\mathrm{E}rr}^{(1)} + \frac{2A_1}{A_{\mathrm{VEV}}}\phi^{(1)}, \tag{38b}$$

$$h_{\mathrm{J}00}^{(2)} = h_{\mathrm{E}00}^{(2)} + \frac{2A_1}{A_{\mathrm{VEV}}}h_{\mathrm{E}00}^{(1)}\phi^{(1)} - \left(\frac{2A_2}{A_{\mathrm{VEV}}} + \frac{A_1^2}{A_{\mathrm{VEV}}^2}\right)(\phi^{(1)})^2, \tag{38c}$$

with

$$t_J = A_{VEV} t_E,$$
$$\chi = A_{VEV} r. \tag{38d}$$

Using the relations in (38d) and (8), the masses in these two frames are related by

$$M_J = \frac{M_E}{A_{VEV}}, \tag{39}$$

which follows the relation $M_J \chi = M_E r$.

Using the scalar field (20) and the metric fields (27), (30), and (32), from the relations (38), we obtain the components of the Jordan frame metric:

$$h^{(1)}_{J00}(r) = \frac{2GM_E}{r} + \left(\frac{A_1 M_{Pl}}{A_{VEV}} - \frac{V_1}{M_{Pl} m_\infty^2}\right)\epsilon \frac{2GM_E}{r} e^{-m_\infty r} + \frac{8\pi G V_{VEV}}{3} r^2, \tag{40a}$$

$$h^{(1)}_{J\chi\chi}(r) = \frac{2GM_E}{r} - \left(\frac{A_1 M_{Pl}}{A_{VEV}} - \frac{V_1}{2M_{Pl} m_\infty^2}\right)\epsilon \frac{2GM_E}{r} e^{-m_\infty r} - \frac{8\pi G V_{VEV}}{6} r^2, \tag{40b}$$

$$\begin{aligned} h^{(2)}_{J00}(r) = -\frac{2G^2 M_E^2}{r^2}\Bigg[&1 + \frac{2A_1 M_{Pl}}{A_{VEV}}\epsilon e^{-m_\infty r} - \frac{5V_1}{4M_{Pl} m_\infty^2}(1 - m_\infty r)\epsilon e^{-m_\infty r} + M_{Pl}^2\left(\frac{A_1^2}{2A_{VEV}^2} + \frac{A_2}{A_{VEV}}\right)\epsilon^2 e^{-2m_\infty r} \\ &- \frac{2V_1 A_1}{m_\infty^2 A_{VEV}}\epsilon^2 e^{-2m_\infty r} + \frac{V_1^2}{4M_{Pl}^2 m_\infty^4}(1 - m_\infty r)\epsilon^2 e^{-2m_\infty r} - \frac{V_2}{2m_\infty^2}\left(1 - \frac{3V_1^2}{M_{Pl}^2 m_\infty^2 V_2}\right)\epsilon^2 m_\infty r e^{-2m_\infty r} \\ &+ \frac{5V_1}{4M_{Pl} m_\infty^2}\epsilon (m_\infty r)^2 \mathrm{Ei}(-m_\infty r) - \left(\frac{V_2}{m_\infty^2} - \frac{5V_1^2}{2M_{Pl}^2 m_\infty^4}\right)\epsilon^2 (m_\infty r)^2 \mathrm{Ei}(-2m_\infty r)\Bigg]. \end{aligned} \tag{40c}$$

Note that the Jordan frame metrics contain the form of a Yukawa potential, which is controlled by the screened parameter.

C. PPN parameters γ and β and effective gravitational constant G_{eff}

Now, let us calculate the PPN parameters γ, and β and the effective gravitational constant G_{eff} as given in the Jordan frame metric (40). In this subsection, we neglect the cosmological constant V_{VEV} in the metric since its effect is very weak on solar system scales. In next subsection, we discuss its effect separately on cosmological scales.

Using the relations (38d) and (39), from the relations (36) and (40), we can identify the PPN parameters $\gamma(r,\epsilon)$, $\beta(r,\epsilon)$, and the effective gravitational constant $G_{\mathrm{eff}}(r,\epsilon)$ in the following form:

$$\gamma(r,\epsilon) = 1 - \frac{\left(\frac{2A_1 M_{Pl}}{A_{VEV}} - \frac{3V_1}{2M_{Pl} m_\infty^2}\right)\epsilon e^{-m_\infty r}}{1 + \left(\frac{A_1 M_{Pl}}{A_{VEV}} - \frac{V_1}{M_{Pl} m_\infty^2}\right)\epsilon e^{-m_\infty r}}, \tag{41a}$$

$$\begin{aligned} \beta(r,\epsilon) = 1 - \frac{1}{\left[1 + \left(\frac{A_1 M_{Pl}}{A_{VEV}} - \frac{V_1}{M_{Pl} m_\infty^2}\right)\epsilon e^{-m_\infty r}\right]^2}\Bigg\{&-\frac{3V_1}{4M_{Pl} m_\infty^2}\left(1 + \frac{5}{3}m_\infty r\right)\epsilon e^{-m_\infty r} + M_{Pl}^2\left(\frac{A_1^2}{2A_{VEV}^2} - \frac{A_2}{A_{VEV}}\right)\epsilon^2 e^{-2m_\infty r} \\ &+ \frac{3V_1^2}{4M_{Pl}^2 m_\infty^4}\left(1 + \frac{1}{3}m_\infty r\right)\epsilon^2 e^{-2m_\infty r} + \frac{V_2}{2m_\infty^2}\left(1 - \frac{3V_1^2}{M_{Pl}^2 m_\infty^2 V_2}\right)\epsilon^2 (m_\infty r) e^{-2m_\infty r} \\ &- \frac{5V_1}{4M_{Pl} m_\infty^2}\epsilon (m_\infty r)^2 \mathrm{Ei}(-m_\infty r) + \left(\frac{V_2}{m_\infty^2} - \frac{5V_1^2}{2M_{Pl}^2 m_\infty^4}\right)\epsilon^2 (m_\infty r)^2 \mathrm{Ei}(-2m_\infty r)\Bigg\}, \end{aligned} \tag{41b}$$

$$G_{\mathrm{eff}}(r,\epsilon) = G A_{VEV}^2\left[1 + \left(\frac{A_1 M_{Pl}}{A_{VEV}} - \frac{V_1}{M_{Pl} m_\infty^2}\right)\epsilon e^{-m_\infty r}\right]. \tag{41c}$$

This is one of the main results of this article. The Taylor coefficients $(V_{VEV}, V_1, V_2; A_{VEV}, A_1, A_2)$, the screened parameter ϵ, and the effective mass m_∞ can all be obtained from two arbitrary functions $V(\phi)$ and $A(\phi)$. Obviously, the PPN parameters and the effective gravitational constant depend not only on the distance r between the source object and the test mass but also on the screened parameter ϵ. The screened parameter depends on background matter density ρ_∞ and the physical properties (density ρ_0 and radius R) of the source object. That is to say, there are different PPN parameters and effective gravitational constants for different sources in SMG theories. Therefore, the observational constraints in the solar system, including the Cassini constraint and the perihelion shift of Mercury constraint, etc., are applicable only to the Sun but not to other sources in SMG theories.

Note that for the compact objects (such as the Sun, the Earth, and the Moon) the screening effect is very strong, and the fifth force is much weaker than the gravitational force. However, for galaxies and galaxy clusters, their densities are very low, the screening effect becomes weak, and the fifth force becomes comparable with the gravitational force. The extra fifth force may manifestly change the behavior of the circular velocity for the test objects in the outskirts of the galactic halo [29] and be involved to explain their observed cored density distribution [41]. The scalar field may be screened in the interior of the cluster, while its outer region can still be affected by the fifth force. The potential governing the dynamics of the matter fields can differ significantly from the lensing potential, which leads to a difference between the mass of the halo obtained from dynamical measurements (e.g., velocity dispersion) and that obtained from gravitational lensing [34,42]. So, we expect that the model parameter space of SMG would be further depressed if observations at galactic scales were included. This issue will be addressed in our future study.

In the solar system, the distance r is always much less than the Compton wavelength m_∞^{-1}, which roughly is of cosmological scales, i.e., $m_\infty r \ll 1$ is satisfied. At the same time, the screening effect is very strong for the Sun (dense body) and the screened parameter $\epsilon \ll 1$. In the case of $x \ll 1$, the asymptotic behavior of the exponential integral function $\mathrm{Ei}(-x)$ is

$$\mathrm{Ei}(-x) \simeq \ln x + \gamma_{\mathrm{EM}} - x + \frac{x^2}{4} + \mathcal{O}(x^3), \tag{42}$$

where $\gamma_{\mathrm{EM}} = 0.57721\cdots$ is the Euler-Mascheroni constant. Therefore, in the case $m_\infty r \ll 1$, the terms involving $(m_\infty r)^2 \mathrm{Ei}(-m_\infty r)$ fall off proportional to $(m_\infty r)^2 \ln(-m_\infty r)$, and the terms involving $(m_\infty r)\exp(-m_\infty r)$ fall off proportional to $m_\infty r$. All these terms may be neglected. Thus, the PPN parameters and the effective gravitational constant are simplified as

$$\gamma(\epsilon) = 1 - \left(\frac{2A_1 M_{\mathrm{Pl}}}{A_{\mathrm{VEV}}} - \frac{3V_1}{2M_{\mathrm{Pl}} m_\infty^2}\right)\epsilon + \left(\frac{2A_1^2 M_{\mathrm{Pl}}^2}{A_{\mathrm{VEV}}^2} - \frac{7V_1 A_1}{2m_\infty^2 A_{\mathrm{VEV}}} + \frac{3V_1^2}{2M_{\mathrm{Pl}}^2 m_\infty^4}\right)\epsilon^2, \tag{43a}$$

$$\beta(\epsilon) = 1 + \frac{3V_1}{4M_{\mathrm{Pl}} m_\infty^2}\epsilon + \left[\left(\frac{A_2}{A_{\mathrm{VEV}}} - \frac{A_1^2}{2A_{\mathrm{VEV}}^2}\right) M_{\mathrm{Pl}}^2 - \frac{3V_1 A_1}{2m_\infty^2 A_{\mathrm{VEV}}} + \frac{3V_1^2}{4M_{\mathrm{Pl}}^2 m_\infty^4}\right]\epsilon^2, \tag{43b}$$

$$G_{\mathrm{eff}}(\epsilon) = G A_{\mathrm{VEV}}^2 \left[1 + \left(\frac{A_1 M_{\mathrm{Pl}}}{A_{\mathrm{VEV}}} - \frac{V_1}{M_{\mathrm{Pl}} m_\infty^2}\right)\epsilon\right]. \tag{43c}$$

These relations are applicable to the solar system (or other solar systems), in which the screening effect is very strong $\epsilon \ll 1$ and the PPN parameters γ and β are both close to unity. Comparing the effective gravitational constant (43c) with the PPN parameter γ (43a), we find the approximate relation

$$G_{\mathrm{eff}}(\epsilon) \simeq G A_{\mathrm{VEV}}^2 \left[1 - \frac{\gamma(\epsilon) - 1}{2}\right]. \tag{44}$$

In fact, in the case $\epsilon \ll 1$, a general relation like this can be obtained from the relations (41c) and (41a),

$$G_{\mathrm{eff}}(r, \epsilon) \simeq G A_{\mathrm{VEV}}^2 \left[1 - \frac{\gamma(r, \epsilon) - 1}{2}\right], \tag{45}$$

which is applicable to the generic SMG.

Let us consider a general coupling function $A(\phi)$ in the form

$$A(\phi) = 1 + \sum_{n=1}^{+\infty} a_n \left(\frac{\phi - \phi_\star}{M_{\mathrm{Pl}}}\right)^n, \tag{46}$$

where a_n and $\phi_\star$ are free parameters. Using the screened parameter (16), the coupling function VEV can be expressed as

$$A_{\mathrm{VEV}} \sim 1 + \sum_{n=1}^{+\infty} a_n (\Phi_{\mathrm{E}} \epsilon)^n, \tag{47}$$

where Φ_{E} is the Newtonian potential at the surface of the source object in the Einstein frame. For the compact objects (such as the Sun, the Earth, and the Moon), Φ_{E} is always much less than unity, and the screening effect is very strong $\epsilon \ll 1$, which follows that $|A_{\mathrm{VEV}} - 1| \ll 1$. Using this result and the Cassini constraint $|\gamma_{\mathrm{obs}} - 1| \lesssim 2.3 \times 10^{-5}$ [49], from the relation (44), we have

$$\frac{|G_{\mathrm{eff}}(\epsilon_{\mathrm{Sun}}) - G|}{G} \simeq \frac{|\gamma_{\mathrm{Sun}} - 1|}{2} \lesssim 1.1 \times 10^{-5}, \tag{48}$$

which is applicable to any generic SMG. This result implies that the effective gravitational constant $G_{\mathrm{eff}}(\epsilon_{\mathrm{Sun}})$ is approximately equal to the Newtonian gravitational constant G within 10^{-5} accuracy in the solar system.

For the limiting case with $\epsilon \to 0$, from the relations (43) and (47), we obtain $\gamma \to 1$, $\beta \to 1$, and $G_{\mathrm{eff}} \to G$. These imply that SMG converges back to GR in this limiting case because of the PPN parameters $\gamma = \beta = 1$ in GR [45,46].

D. Effective cosmological constant

SMG contains a scalar degree of freedom, whose potential can naturally provide the vacuum energy required to drive cosmic acceleration at late times. More precisely, SMG requires that the effective potential of the scalar field has a minimum, which can be understood as a stable vacuum. Around this minimum (physical vacuum), the bare potential has a VEV, which can play the role of cosmological constant (or, equivalently, dark energy). In this subsection, we discuss this issue for the generic SMG.

Considering the metric of SMG around the dense object (such as white dwarf, neutron star, and black hole), the screened parameter is $\epsilon \to 0$. In this limiting case, from the relation (47), we have $A_{\rm VEV} \to 1$. Using this, and the relations in (38) and (39), we derive

$$g^{\rm J}_{\mu\nu} \to g^{\rm E}_{\mu\nu}, \tag{49a}$$

$$t_{\rm J} \to t_{\rm E}, \qquad \chi \to r, \qquad M_{\rm J} \to M_{\rm E}, \tag{49b}$$

which imply that the Einstein and Jordan frame converge to the same frame in this limit. Furthermore, in this limit, from the Jordan frame metric (40) or the Einstein frame metric (27) and (30), we find that these two frame metrics both converge to

$$ds^2 \simeq -\left(1 - \frac{2GM}{r} - \frac{\Lambda}{3} r^2\right) dt^2 + \left(1 + \frac{2GM}{r} - \frac{\Lambda}{6} r^2\right)(dr^2 + r^2 d\Omega^2), \tag{50}$$

with

$$\Lambda \equiv 8\pi G V_{\rm VEV}. \tag{51}$$

This is the isotropic form of the Schwarzschild-(A)de Sitter metric [50] in the weak field limit. Using the coordinate transformation

$$r \simeq \tilde{r}\left(1 - \frac{GM}{\tilde{r}} + \frac{\Lambda}{12}\tilde{r}^2\right), \tag{52}$$

we obtain the standard form of the Schwarzschild-(A)de Sitter metric in the weak field limit,

$$ds^2 \simeq -\left(1 - \frac{2GM}{\tilde{r}} - \frac{\Lambda}{3}\tilde{r}^2\right) dt^2 + \left(1 + \frac{2GM}{\tilde{r}} + \frac{\Lambda}{3}\tilde{r}^2\right) d\tilde{r}^2 + \tilde{r}^2 d\Omega^2. \tag{53}$$

It is easy to identify the cosmological constant Λ, and we can see that SMG converges back to GR with a cosmological constant in the limit $\epsilon \to 0$. Thus, the density of the effective cosmological constant (or effective "dark energy") is given by

$$\rho_\Lambda = V_{\rm VEV} = V[\phi_{\rm VEV}(\rho_m)], \tag{54}$$

which can be constrained by various cosmological observations. In addition, in order to consist with current observations, the dark energy density should be nearly equal to a constant, and the evolution with the redshift should be slow, which is beyond the scope of the present work. In this paper, we only consider the current energy density of the effect "dark energy" (labeled by the subscript "0") and constrain the parameters of some specific SMG models, including chameleon, symmetron, and dilaton.

V. SOLAR SYSTEM AND COSMOLOGICAL CONSTRAINTS

There are different experimental constraints on the PPN parameters γ and β. Currently, the high accuracy experimental constraints mainly come from the solar system tests. The most stringent constraint on γ in the solar system comes from the measurements of the Cassini spacecraft, which measured the Shapiro time delay of a radio signal sent from and to the Cassini spacecraft while close to conjunction with the Sun and got $\gamma_{\rm obs} - 1 = (2.1 \pm 2.3) \times 10^{-5}$ at the 1σ confidence level [49].

The most stringent constraint on β comes from measurements of the perihelion shift of Mercury, which depends on the combination $|2\gamma - \beta - 1|$ of the PPN parameters and the solar quadrupole moment J_2. The latest inversions of helioseismology data give $J_2 = (2.2 \pm 0.1) \times 10^{-7}$ [51]. Adopting the Cassini bound on γ, these analyses yield a bound on $\beta_{\rm obs} - 1 = (-4.1 \pm 7.8) \times 10^{-5}$ [46].

A number of advanced experiments or space missions are under development or have been proposed, which could lead to significant improvements in values of the PPN parameters. The Gaia satellite was launched from Europe's Spaceport in 2013, which is located around the L2 Lagrange point of the Sun-Earth system. The Gaia satellite is a high-precision astrometric orbiting telescope; it could measure light deflection and is expected to improve the constraint on γ to the 10^{-6} level [52]. The BepiColombo is a mission to explore the planet Mercury, which is scheduled for launch in 2017. An eight-year mission could yield further improvements by factors of 2–5 in β [53,54].

For the cosmological constraints, we need the current values of the dark energy density ρ_{Λ_0} and the cosmological matter density ρ_{m_0} or, equivalently, the current values of the density parameters Ω_{Λ_0} and Ω_{m_0} and the Hubble constant H_0. The latest results come from the observations of the Planck satellite, the best-fit values of these parameters are $\Omega_{\Lambda_0} = 0.683$, $\Omega_{m_0} = 0.317$, and $H_0 = 67.3\ {\rm km \cdot s^{-1}\, Mpc^{-1}}$ [55].

In this section, we focus on three specific theories of SMG (chameleon, symmetron, and dilaton models). By investigating these models on solar system and cosmological scales, we derive the combined constraints on model parameters.

A. Chameleons

1. The original chameleon

In order that a certain massive scalar-tensor gravity can satisfy the solar system experiments, the chameleon model was introduced as a screening mechanism by Khoury and Weltman [25–27]. The original chameleon model is characterized by the Ratra-Peebles runaway potential and an exponential coupling function

$$V(\phi) = \frac{M^{4+\alpha}}{\phi^\alpha}, \tag{55a}$$

$$A(\phi) = \exp\left(\frac{\xi\phi}{M_{\rm Pl}}\right), \tag{55b}$$

where M is a constant with the dimension of mass, ξ is a positive coupling constant, and $\alpha \sim \mathcal{O}(1)$ is a positive constant index.

The chameleon effective potential has a minimum. Using the relations in (9a) and (9b), we obtain the chameleon field value and the effective mass of the chameleon at this minimum,

$$\phi_{\rm min}(\rho) \simeq \left(\frac{\alpha M_{\rm Pl} M^{4+\alpha}}{\xi\rho}\right)^{\frac{1}{\alpha+1}}, \tag{56a}$$

$$m^2_{\rm eff}(\rho) \simeq (\alpha+1)\frac{\xi\rho}{M_{\rm Pl}\phi_{\rm min}}. \tag{56b}$$

We find that for the higher ambient density ρ, the value of $\phi_{\rm min}$ is smaller and the effective mass $m_{\rm eff}$ is larger. The Ratra-Peebles runaway potential $V(\phi)$ and the exponential coupling function $A(\phi)$ are expanded in Taylor's series at the chameleon VEV $\phi_\infty \equiv \phi_{\rm min}(\rho_\infty)$ as follows:

$$V(\phi) = \frac{\xi\rho_\infty\phi_\infty}{\alpha M_{\rm Pl}} - \frac{\xi\rho_\infty}{M_{\rm Pl}}(\phi-\phi_\infty) + \frac{(\alpha+1)\xi\rho_\infty}{2M_{\rm Pl}\phi_\infty}(\phi-\phi_\infty)^2 + \cdots, \tag{57a}$$

$$A(\phi) = e^{\frac{\xi\phi_\infty}{M_{\rm Pl}}} + \frac{\xi}{M_{\rm Pl}}e^{\frac{\xi\phi_\infty}{M_{\rm Pl}}}(\phi-\phi_\infty) + \frac{\xi^2}{2M^2_{\rm Pl}}e^{\frac{\xi\phi_\infty}{M_{\rm Pl}}}(\phi-\phi_\infty)^2 + \cdots. \tag{57b}$$

From these formulas, we obtain the expansion coefficients

$$V_{\rm VEV} = \frac{\xi\rho_\infty\phi_\infty}{\alpha M_{\rm Pl}}, \qquad V_1 = -\frac{\xi\rho_\infty}{M_{\rm Pl}}, \qquad V_2 = \frac{(\alpha+1)\xi\rho_\infty}{2M_{\rm Pl}\phi_\infty}, \tag{58a}$$

$$A_{\rm VEV} = e^{\frac{\xi\phi_\infty}{M_{\rm Pl}}}, \qquad A_1 = \frac{\xi e^{\frac{\xi\phi_\infty}{M_{\rm Pl}}}}{M_{\rm Pl}}, \qquad A_2 = \frac{\xi^2 e^{\frac{\xi\phi_\infty}{M_{\rm Pl}}}}{2M^2_{\rm Pl}}, \tag{58b}$$

where ρ_∞ is the background matter density of the solar system. If considering the cosmological background, ρ_∞ is the cosmological matter density ρ_{m_0}. However, if considering the galactic background, ρ_∞ is the galactic matter density $\rho_{gal} \simeq 10^5\rho_{m_0}$.

Using these coefficients and the relations in (56) and (16), from the relations in (43), we obtain the expressions of parameters $(\gamma, \beta, G_{\rm eff})$ as

$$\gamma - 1 = -\frac{2\xi\phi_\infty}{M_{\rm Pl}\Phi}, \tag{59a}$$

$$\beta - 1 = -\frac{3}{4(\alpha+1)}\left(\frac{\phi_\infty}{M_{\rm Pl}}\right)^2\frac{1}{\Phi}, \tag{59b}$$

$$\frac{G_{\rm eff}}{G} - 1 = \frac{\xi\phi_\infty}{M_{\rm Pl}\Phi}, \tag{59c}$$

where Φ is the Newtonian potential at the surface of the source object, and for the Sun we have $\Phi \simeq 2.12\times10^{-6}$. These results are consistent with the previous ones in the literature [48], where only γ parameter was obtained. From these formulas, we can also get the relations between these parameters,

$$\beta - 1 = -\frac{3\Phi(\gamma-1)^2}{16\xi^2(\alpha+1)}, \quad \frac{G_{\rm eff}}{G} - 1 = -\frac{\gamma-1}{2}, \quad A_{\rm VEV} - 1 = -\frac{\Phi(\gamma-1)}{2}. \tag{60}$$

Obviously, $|\beta-1| \ll |\gamma-1|$. Using the Cassini constraint $|\gamma_{\rm obs}-1| \lesssim 2.3\times10^{-5}$, we obtain the constraint on the model parameters,

$$\frac{\xi\phi_\infty}{M_{\rm Pl}} = \xi\left(\frac{\alpha M^{4+\alpha}}{\xi M^\alpha_{\rm Pl}\rho_\infty}\right)^{\frac{1}{\alpha+1}} \lesssim 2.4\times10^{-11}. \tag{61}$$

In addition, the bounds on the other parameters are also derived,

$$|\beta-1| \lesssim 10^{-16} \quad \text{for } \xi \sim O(1), \quad \left|\frac{G_{\rm eff}}{G} - 1\right| \lesssim 1.1\times10^{-5}, \quad |A_{\rm VEV}-1| \lesssim 2.4\times10^{-11}, \tag{62}$$

which strongly indicate that the PPN parameter $\beta = 1$, the effective gravitational constant $G_{\rm eff} \simeq G$, and the exponential coupling function VEV $A_{\rm VEV} = 1$ for chameleon.

Unfortunately, for the original chameleon, it is impossible to explain cosmic acceleration and to pass the solar system experiments at the same time. For the current Universe, the cosmological observations give the density ratio $\rho_{\Lambda_0}/\rho_{m_0} = 2.15$. However, in the theoretical side, from the relations $V_{\rm VEV}$ (58a) and (54), we get the ratio between them,

$$\frac{\rho_{\Lambda_0}}{\rho_{m_0}} = \frac{\xi\phi_\infty(\rho_{m_0})}{\alpha M_{\rm Pl}} = 2.15, \tag{63}$$

where the density $\rho_\infty = \rho_{m_0}$, corresponding to the cosmological matter density. Using the relation in (56a), Eq. (63) turns into

$$\log M = \frac{\alpha \log m_{\rm Pl} + \log \rho_{\Lambda_0}}{4+\alpha} + \frac{\alpha}{4+\alpha}\log\frac{\alpha\rho_{\Lambda_0}}{\sqrt{8\pi}\xi\rho_{m_0}}, \tag{64}$$

where $m_{\rm Pl} \simeq 1.22\times 10^{19}$ GeV is the Planck mass, and $\rho_{\Lambda_0} \simeq 2.51\times 10^{-47}$ GeV^4 is the dark energy density. In the case with $\xi \sim \mathcal{O}(1)$ and $\alpha \sim \mathcal{O}(1)$, the relation (64) is reduced to

$$\log M({\rm GeV}) = \frac{19\alpha - 47}{4+\alpha}, \tag{65}$$

which is the same relation as found in [48,56]. This implies that the influences of the coupling constant ξ and the cosmological matter density ρ_{m_0} are much weaker than that of parameter α.

From the solar system constraint (61), we obtain its equivalent form

$$\frac{\xi\phi_\infty(\rho_{m_0})}{\alpha M_{\rm Pl}} \lesssim 2.4\times 10^{-11}\cdot\frac{1}{\alpha}\left(\frac{\rho_\infty}{\rho_{m_0}}\right)^{\frac{1}{\alpha+1}}. \tag{66}$$

Obviously, in the cases with either the cosmological background ($\rho_\infty = \rho_{m_0}$) or the Milky Way galaxy background ($\rho_\infty = \rho_{gal}$), the solar system constraint (66) is always incompatible with the cosmological relation (63) for $\alpha \sim \mathcal{O}(1)$. In other words, the original chameleon cannot explain cosmic acceleration and pass solar system constraints at the same time, which is consistent with conclusion found in [48].

2. The exponential chameleon

The original chameleon is ruled out by the combined constraints of the solar system and cosmology. However, the idea of chameleon can be resurrected by modifying the potential in the form,

$$V(\phi) = M^4 \exp\left(\frac{M^\alpha}{\phi^\alpha}\right). \tag{67}$$

This chameleon model is called the exponential chameleon and proposed in [28].

We consider the case with $\phi/M \gg 1$. Using the relation (56a) and considering the cosmological matter density $\rho = \rho_{m_0} \simeq 1.17\times 10^{-47}$ GeV^4, we get that

$$M \gg 1.69\times 10^{-13}\ {\rm eV}. \tag{68}$$

In this case, the exponential potential (67) is reduced to

$$V(\phi) = M^4 + \frac{M^{4+\alpha}}{\phi^\alpha}, \tag{69}$$

which is equivalent to the Ratra-Peebles runaway potential plus a cosmological constant (71). Therefore, all calculations of the exponential chameleon are the same as the calculations of the original chameleon, except for the effective dark energy density. The dark energy density of the exponential chameleon is given by

$$\rho_{\Lambda_0} = M^4 + \frac{\xi\phi_\infty(\rho_{m_0})}{\alpha M_{\rm Pl}}\rho_{m_0}. \tag{70}$$

Taking into account the solar system constraint (66), the cosmological relation (70) is simplified to

$$M = \rho_{\Lambda_0}^{1/4} \simeq 0.002\ {\rm eV}, \tag{71}$$

which is consistent with the relation (68). Using this, the solar system constraint on the parameters ξ and α becomes

$$(19.5 - \log\xi)\alpha - \log\alpha \gtrsim 10.6 - \log\frac{\rho_\infty}{\rho_{\Lambda_0}}. \tag{72}$$

In Fig. 1, we plot the constraints on the model parameters α and ξ by considering the cosmological background or the galactic background. In both cases, we find that the constraint on ξ is much looser than that on α. For the strong coupling with $\xi \gtrsim 1$, we have $\alpha \gtrsim 0.547$ in the case with the cosmological background and $\alpha \gtrsim 0.257$ in the case with the galactic background. Even in the limit case with $\xi \gtrsim 10^{-10}$, the constraint on α is slightly looser, which is $\alpha \gtrsim 0.355$ in the case with the cosmological background and $\alpha \gtrsim 0.163$ in the case with the galactic background.

B. Symmetron

The symmetron models are characterized by a $\mathbb{Z}_2$ symmetry breaking potential (a Mexican hat potential) and a quadratic coupling function [30–34],

$$V(\phi) = \mathbb{V}_0 - \frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4}\phi^4, \tag{73a}$$

$$A(\phi) = 1 + \frac{\phi^2}{2M^2}, \tag{73b}$$

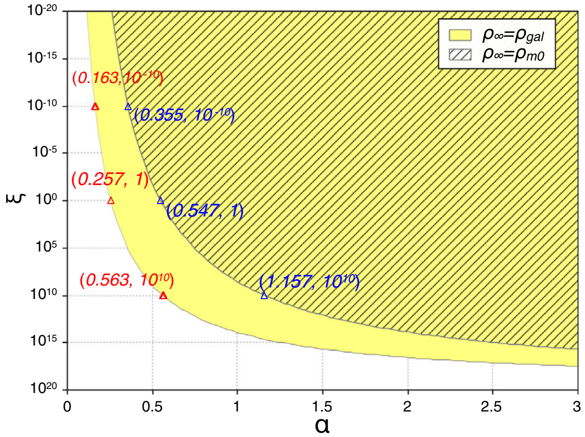

FIG. 1. In the parameter space of exponential chameleon models, the shadow region is allowed by the Cassini experiment, if assuming the cosmological background, i.e., $\rho_\infty = \rho_{m_0}$. While the yellow region is allowed if assuming the galactic background, i.e., $\rho_\infty = \rho_{gal}$.

where μ and M are mass scales, λ is a positive dimensionless coupling constant, and $\mathbb{V}_0$ is the vacuum energy of the bare potential $V(\phi)$. The effective potential $V_{\rm eff}$ of symmetron has a minimum. Using the relations (9a) and (9b), we obtain the field value and the effective mass of the symmetron at this minimum,

$$\phi_{\rm min}(\rho) = \begin{cases} 0 & \text{for } \rho > \rho_{\rm SSB} \\ \pm \frac{\mu}{\sqrt{\lambda}}\left(1 - \frac{\rho}{\rho_{\rm SSB}}\right)^{\frac{1}{2}} & \text{for } \rho < \rho_{\rm SSB} \end{cases}, \tag{74a}$$

$$m^2_{\rm eff}(\rho) = \begin{cases} \mu^2\left(\frac{\rho}{\rho_{\rm SSB}} - 1\right) & \text{for } \rho > \rho_{\rm SSB} \\ 2\mu^2\left(1 - \frac{\rho}{\rho_{\rm SSB}}\right) & \text{for } \rho < \rho_{\rm SSB} \end{cases} \tag{74b}$$

where $\rho_{\rm SSB} \equiv M^2\mu^2$ is the critical matter density of spontaneous symmetry breaking (SSB). In high density regions, where $\rho > \rho_{\rm SSB}$, the effective potential has a minimum at $\phi_{\rm min} = 0$, and the $\mathbb{Z}_2$ symmetry $\phi \to -\phi$ is ensured. However, in low density regions, where $\rho < \rho_{\rm SSB}$, the $\mathbb{Z}_2$ symmetry $\phi \to -\phi$ is spontaneously broken. In this case, the effective potential has two of the same minima, and the field settles at one of them. Note that for either the positive VEV of scalar field or the negative one the physical results are same since the scalar field VEV always exists as its square form in the PPN parameters and effective gravitational constant [see Eq. (78)]. Without loss of generality, we choose the positive scalar field VEV,

$$\phi_\infty = \frac{\mu}{\sqrt{\lambda}}\left(1 - \frac{\rho_\infty}{\rho_{\rm SSB}}\right)^{\frac{1}{2}} \quad \text{for } \rho_\infty < \rho_{\rm SSB}, \tag{75}$$

where ρ_∞ is the background matter density of the solar system. Similar to the chameleon models, we have $\rho_\infty = \rho_{m_0}$ if considering the cosmological matter density as background, while $\rho_\infty = \rho_{gal}$ if setting the galactic matter density as background.

The $\mathbb{Z}_2$ symmetry breaking potential $V(\phi)$ and the quadratic coupling function $A(\phi)$ are expanded in Taylor's series at this VEV,

$$V(\phi) = \mathbb{V}_0 - \frac{\rho^2_{\rm SSB} - \rho^2_\infty}{4\lambda M^4} - \frac{\rho_\infty \phi_\infty}{M^2}(\phi - \phi_\infty) + \left(\mu^2 - \frac{3\rho_\infty}{2M^2}\right)(\phi - \phi_\infty)^2 + \cdots, \tag{76a}$$

$$A(\phi) = 1 + \frac{\phi^2_\infty}{2M^2} + \frac{\phi_\infty}{M^2}(\phi - \phi_\infty) + \frac{1}{2M^2}(\phi - \phi_\infty)^2 + \cdots. \tag{76b}$$

The expansion coefficients are obtained directly,

$$V_{\rm VEV} = \mathbb{V}_0 - \frac{\rho^2_{\rm SSB} - \rho^2_\infty}{4\lambda M^4}, \qquad V_1 = -\frac{\rho_\infty\phi_\infty}{M^2}, \qquad V_2 = \mu^2 - \frac{3\rho_\infty}{2M^2}, \tag{77a}$$

$$A_{\rm VEV} = 1 + \frac{\phi^2_\infty}{2M^2}, \qquad A_1 = \frac{\phi_\infty}{M^2}, \qquad A_2 = \frac{1}{2M^2}. \tag{77b}$$

Using these coefficients and the relations, we get the expressions of γ, β, and $G_{\rm eff}$ as follows:

$$\gamma - 1 = -\frac{2\phi^2_\infty}{M^2\Phi}, \tag{78a}$$

$$\beta - 1 = \frac{1}{2}\left(\frac{\phi_\infty}{M\Phi}\right)^2, \tag{78b}$$

$$\frac{G_{\rm eff}}{G} - 1 = \frac{\phi^2_\infty}{M^2\Phi}. \tag{78c}$$

These results are consistent with the previous ones in the literature [30], where only the γ parameter was obtained. From these formulas, we can also get the useful relations between these parameters,

$$\gamma - 1 = -4\Phi(\beta - 1), \qquad \frac{G_{\rm eff}}{G} - 1 = -\frac{\gamma - 1}{2}, \qquad A_{\rm VEV} - 1 = \Phi^2(\beta - 1). \tag{79}$$

Obviously, for the symmetron, we always have $|\gamma - 1| \ll |\beta - 1|$, so the constraints on the model parameters are mainly from the measurements of β instead of γ. Using the perihelion shift of the Mercury constraint $|\beta_{\rm obs} - 1| \lesssim 7.8 \times 10^{-5}$ and the relations in (78), we obtain the following bound on the symmetron parameters:

$$\frac{\rho_{\rm SSB}-\rho_\infty}{\lambda M^4} \lesssim 7.0\times 10^{-16}. \tag{80}$$

Using this bound, we also get the constraints on γ, $G_{\rm eff}$, and $A_{\rm VEV}$ for the symmetron models,

$$\begin{aligned} |\gamma-1| &\lesssim 6.6\times 10^{-10}, \\ \left|\frac{G_{\rm eff}}{G}-1\right| &\lesssim 3.3\times 10^{-10}, \\ |A_{\rm VEV}-1| &\lesssim 3.5\times 10^{-16}, \end{aligned} \tag{81}$$

which strongly indicate that the PPN parameter $\gamma=1$, the effective gravitational constant $G_{\rm eff}=G$, and the quadratic coupling function VEV $A_{\rm VEV}=1$ for symmetron.

From the formula $V_{\rm VEV}$ in (77a) and the relation in (54), we obtain the energy density of effective dark energy in symmetron models,

$$\begin{aligned} \rho_{\Lambda_0} &= \mathbb{V}_0 - \frac{\rho^2_{\rm SSB}-\rho^2_{m_0}}{4\lambda M^4}, \\ &= \mathbb{V}_0 - \frac{\mu^4}{4\lambda} + \frac{\rho^2_{m_0}}{4\lambda M^4}, \end{aligned} \tag{82}$$

where we have used $\rho_\infty=\rho_{m_0}$ for the cosmological background. In order to get the accelerated expansion of the Universe ($\rho_{\Lambda_0}>0$), we need

$$\mathbb{V}_0 > \frac{\mu^4}{4\lambda} - \frac{\rho^2_{m_0}}{4\lambda M^4} > 0, \tag{83}$$

i.e., the vacuum energy $\mathbb{V}_0$ of the bare potential must be positive. Now, let us consider two specific cases of $\mathbb{V}_0$ function.

Case 1: $\mathbb{V}_0=0$

This is the original symmetron model suggested in the literature [30,31]. From the relation (82), we can see that the original symmetron has a negative cosmological constant and cannot drive cosmic acceleration at late times, which is consistent with the conclusion in the previous work [31].

Case 2: $\mathbb{V}_0=\mu^4/4\lambda$

This kind of model is proposed in [35]. In this case, the density of symmetron dark energy is

$$\rho_{\Lambda_0} = \frac{\rho^2_{m_0}}{4\lambda M^4}, \tag{84}$$

and the solar system constraint (80) becomes

$$0 < \frac{\rho_{\rm SSB}-\rho_\infty}{\rho_{m_0}} \lesssim 8.1\times 10^{-17}. \tag{85}$$

From these relations, we evaluate the model parameters of symmetron as follows:

$$\lambda M^4 = \frac{\rho_{m_0}}{4}\left(\frac{\rho_{m_0}}{\rho_{\Lambda_0}}\right) \simeq 1.4\times 10^{-48}\ \text{GeV}^4, \tag{86}$$

and

$$M^2\mu^2 = \begin{cases} \rho_{m_0} \simeq 1.2\times 10^{-47}\ \text{GeV}^4 & \text{for CB} \\ \rho_{gal} \simeq 10^{-42}\ \text{GeV}^4 & \text{for GB} \end{cases}, \tag{87}$$

where $\rho_\infty=\rho_{m_0}$ and $\rho_\infty=\rho_{gal}$, corresponding to the cosmological background (CB) and the galactic background (GB), respectively.

C. Dilaton

The dilaton model, inspired by string theory in the large string coupling limit, has an exponentially runaway potential and a quadratic coupling function [20,36–38],

$$V(\phi) = \mathcal{V}_0 \exp\left(-\frac{\phi}{M_{\rm Pl}}\right), \tag{88a}$$

$$A(\phi) = 1 + \frac{(\phi-\phi_\star)^2}{2M^2}, \tag{88b}$$

where $\mathcal{V}_0$ is a constant with the dimension of energy density, M labels the energy scale of the theory, and $\phi_\star$ is approximately the value of ϕ today.

The dilaton effective potential $V_{\rm eff}$ also has a minimum. Using the relations (9a) and (9b), we obtain the dilaton field value and the effective mass of the dilaton at this minimum,

$$\phi_{\rm min}(\rho) \simeq \phi_\star + \frac{M^2\mathcal{V}_0}{M_{\rm Pl}\rho} e^{-\frac{\phi_\star}{M_{\rm Pl}}}, \tag{89a}$$

$$m^2_{\rm eff}(\rho) \simeq \frac{\rho}{M^2} + \frac{\mathcal{V}_0}{M^2_{\rm Pl}} e^{-\frac{\phi_\star}{M_{\rm Pl}}}. \tag{89b}$$

The exponentially runaway potential $V(\phi)$ and the quadratic coupling function $A(\phi)$ can be expanded in Taylor's series at the dilaton VEV $\phi_\infty \equiv \phi_{\rm min}(\rho_\infty)$,

$$\begin{aligned} V(\phi) &= \mathcal{V}_0 e^{-\frac{\phi_\infty}{M_{\rm Pl}}} - \frac{\mathcal{V}_0}{M_{\rm Pl}} e^{-\frac{\phi_\infty}{M_{\rm Pl}}}(\phi-\phi_\infty) \\ &\quad + \frac{\mathcal{V}_0}{2M^2_{\rm Pl}} e^{-\frac{\phi_\infty}{M_{\rm Pl}}}(\phi-\phi_\infty)^2 + \cdots, \end{aligned} \tag{90a}$$

$$\begin{aligned} A(\phi) &= 1 + \frac{(\phi_\infty-\phi_\star)^2}{2M^2} + \frac{\phi_\infty-\phi_\star}{M^2}(\phi-\phi_\infty) \\ &\quad + \frac{1}{2M^2}(\phi-\phi_\infty)^2 + \cdots. \end{aligned} \tag{90b}$$

So, the expansion coefficients are derived directly,

$$V_{\rm VEV}=\mathcal{V}_0 e^{-\frac{\phi_\infty}{M_{\rm Pl}}},\quad V_1=-\frac{\mathcal{V}_0 e^{-\frac{\phi_\infty}{M_{\rm Pl}}}}{M_{\rm Pl}},\quad V_2=\frac{\mathcal{V}_0 e^{-\frac{\phi_\infty}{M_{\rm Pl}}}}{2M_{\rm Pl}^2},\tag{91a}$$

$$A_{\rm VEV}=1+\frac{(\phi_\infty-\phi_\star)^2}{2M^2},\qquad A_1=\frac{\phi_\infty-\phi_\star}{M^2},\qquad A_2=\frac{1}{2M^2},\tag{91b}$$

where ρ_∞ is the background matter density of the solar system.

Using these coefficients and the relations in (89), (16), and (43), we obtain the PPN parameters and effective gravitational constant,

$$\gamma-1=-\frac{2(\phi_\infty-\phi_\star)^2}{M^2\Phi},\tag{92a}$$

$$\beta-1=\frac{1}{2}\left(\frac{\phi_\infty-\phi_\star}{M\Phi}\right)^2,\tag{92b}$$

$$\frac{G_{\rm eff}}{G}-1=\frac{(\phi_\infty-\phi_\star)^2}{M^2\Phi}.\tag{92c}$$

The useful relations between them are also derived directly,

$$\gamma-1=-4\Phi(\beta-1),\qquad \frac{G_{\rm eff}}{G}-1=-\frac{\gamma-1}{2},\qquad A_{\rm VEV}-1=\Phi^2(\beta-1).\tag{93}$$

Note that these relations are exactly same with the ones in symmetron model. Therefore, among the solar system tests, the perihelion shift of the Mercury constraint $|\beta_{\rm obs}-1|\lesssim 7.8\times10^{-5}$ follows the most stringent constraint on the model parameter, which is

$$\frac{M\mathcal{V}_0}{M_{\rm Pl}\rho_\infty}e^{-\phi_\star/M_{\rm Pl}}\lesssim 2.6\times10^{-8}.\tag{94}$$

The bounds of the other parameters in the dilaton models are

$$|\gamma-1|\lesssim 6.6\times10^{-10},\qquad \left|\frac{G_{\rm eff}}{G}-1\right|\lesssim 3.3\times10^{-10},\qquad |A_{\rm VEV}-1|\lesssim 3.5\times10^{-16}.\tag{95}$$

Using the relations in Eqs. (91a) and (54), we obtain the density of dilaton dark energy,

$$\rho_{\Lambda_0}=\mathcal{V}_0 e^{-\phi_\infty/M_{\rm Pl}}\simeq\mathcal{V}_0 e^{-\phi_\star/M_{\rm Pl}}\simeq 2.51\times10^{-47}\ {\rm GeV}^4,\tag{96}$$

where $\rho_\infty=\rho_{m_0}$ on cosmological scales. This is consistent with the relation found in [36]. Taking into account this relation, the solar system constraint (94) turns into

$$\frac{M\rho_{\Lambda_0}}{M_{\rm Pl}\rho_\infty}\lesssim 2.6\times10^{-8},\tag{97}$$

that is,

$$\frac{M}{M_{\rm Pl}}\lesssim 1.2\times10^{-8}\quad \text{for } \rho_\infty=\rho_{m_0},\tag{98a}$$

$$\frac{M}{M_{\rm Pl}}\lesssim 1.2\times10^{-3}\quad \text{for } \rho_\infty=\rho_{gal},\tag{98b}$$

where $\rho_\infty=\rho_{m_0}$ and $\rho_\infty=\rho_{gal}$ correspond to the cosmological background and galactic background, respectively.

VI. CONCLUSIONS

Screened modified gravity (SMG) is a kind of scalar-tensor theory with screening mechanisms, which can generate a screening effect to suppress the fifth force and pass the solar system tests. In this paper, we calculated the PPN parameters γ and β for SMG with a general potential V and coupling function A in the case of a static spherically symmetric source. In addition, we discussed the effective cosmological constant in the generic SMG. These two analyses allow us to constrain the model parameters by combining the observations on solar system and cosmological scales.

The PPN parameters were typically calculated under the assumption of a point source surrounded by a vacuum [43,44], but this assumption is generally not appropriate to solve the massive scalar field. In order to overcome this defect and calculate the PPN parameters for the generic SMG, in which the scalar field is always massive, we solved the scalar field in the Einstein frame in the case of an extended source surrounded by a homogeneous background, which is the more realistic case for the source as the Sun or the Earth. Then, we solved the massless metric field in the Einstein frame. By transforming the results to the Jordan frame through a conformal rescaling of the metric, we obtained the PPN parameters γ and β and the effective gravitational constant $G_{\rm eff}$ for the general SMG models.

We found that the parameters (γ, β, $G_{\rm eff}$) depend not only on the distance between the source object and the test mass but also on the screened parameter ϵ, which is determined by the physical properties of the source object. Moreover, SMG contains a scalar degree of freedom, whose effective potential has a minimum (physical vacuum), and the bare potential has a VEV at this minimum. The bare potential

VEV can naturally play the role of dark energy to accelerate the expansion of the Universe at late times. So, as anticipated, the SMG could not only pass the strict solar tests but also account for the accelerated expansion of the Universe.

We applied our results to three specific cases of SMG theories (chameleon, symmetron, and dilaton models), and calculated their PPN parameters and effective cosmological constant. By investigating the current experiments on solar system and cosmological scales, we derived the combined parameter constraints on these three models. Consistent with all the previous works, we found the following results for these SMG models: The original chameleon cannot explain cosmic acceleration and pass solar system constraints at the same time, but this difficulty is overcome in the exponential chameleon. The original symmetron ($\mathbb{V}_0 = 0$) has a negative cosmological constant and cannot drive cosmic acceleration. However, the modified symmetron with $\mathbb{V}_0 = \mu^4/4\lambda$ can realize it. The dilaton is a fine model for both passing solar system tests and accelerating the expansion of the Universe in the late stage. For each of these healthy models (the exponential chameleon, the modified symmetron, and the dilaton), we obtained the constraints on the model parameters, respectively.

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