PHYSICAL REVIEW D **93,** 123508 (2016)

Small changes to the inflaton potential can result in large changes in observables

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(Received 4 January 2016; published 6 June 2016)

We show that a tiny correction to the inflaton potential can make critical changes in the inflationary observables for some types of inflation models.

DOI: 10.1103/PhysRevD.93.123508

I. INTRODUCTION

Inflation [1,2] is a key concept of modern cosmology, which provides a simple and compelling solution to the main problems of old big bang cosmology. Also, the quantum fluctuations of the inflaton (typically a dynamically rolling scalar field controlling the duration of inflation) are regarded as the most plausible seeds of the structures we observe at the present universe [3,4]. Generically, a realization of an observationally consistent inflation requires slow-roll, i.e. an inflaton whose effective mass-square parameter is small as compared to the square of the expansion rate during inflation, and the duration of inflation responsible for our visible universe should be around 50–60 e-folds, depending on the thermal history after inflation [5]. In addition, thanks to various precise observations, the inflationary observables are being constrained more and more tightly such that many models have been being ruled out or disfavored (see for example Ref. [6]).

Inflaton can be a nontrivial trajectory in a multidimensional field space, but a majority of models are contained into the single-field scenario. Conventionally, in a single-field scenario, there is *a* simple form of the effective inflaton potential. However, it should be noted that the inflaton couples to other fields, an imprescindible step in order to reheat the universe to recover the standard hot universe necessary for a successful big bang nucleosynthesis and later cosmology. Such a coupling(s) is indeed effective subleading contribution(s) to the inflaton potential. These contributions might be smaller than the leading potential by several (or many) orders of magnitude, and one may naively expect that such tiny corrections can be ignored. This may be true in some cases, but may not be always the case.

As one example of critical corrections to the inflaton potential, one may recall the so called η-problem [7,8] associated with the mass of inflaton. The point of the problem is that generically the inflaton can have a gravitational (Planck-suppressed) quadratic interaction to the potential energy of the universe with an order one numerical coupling constant, and such an interaction as a correction to the inflaton potential results in a Hubble scale mass for the inflaton, invalidating the usual slow-roll approximation which seems necessary for a nearly scale invariant power spectrum of the density perturbation in the present universe. This η-problem may be circumvented or removed for example by using a specific type of potential [9,10], introducing a symmetry [11–14], realizing inflation in a multidimensional field space [15–24], or introducing extra-dimensions [25]. However, irrespective of the η-problem, there can be other type of critical corrections which may be from higher order loop corrections, non-perturbative effects, or their combinations induced by interactions of the inflaton to other field-contents in a full theory. Generically, it is difficult to see how those corrections would look like and if a solution to η-problem can remove them too, unless an explicit calculation in a full theory is done but that is likely to be highly nontrivial. So, it may be better to take a phenomenological approach to the issue.

In this paper, we show that, a tiny correction which might arise from higher order loop (and/or nonperturbative) effects can make a critical impact on the dynamics of inflaton, even if it is several orders of magnitude smaller than the leading order inflaton potential, altering critically the predictions obtained by analyzing only the leading order inflaton potential.

II. MODELS

For a potential V of a single-field slow-roll inflation scenario, the spectral behavior of the density power spectrum originated from the quantum fluctuations of the inflaton is determined by the first two slow-roll parameters defined as

$$\epsilon \equiv \frac{1}{2}\left(\frac{M_{\rm P} V'}{V}\right)^2, \qquad \eta \equiv \frac{M_{\rm P}^2 V''}{V} \tag{1}$$

where "′" denotes derivative with respect to the inflaton field ϕ, which is treated as a real scalar field. They also determine the duration of inflation, i.e., the number of e-foldings which is defined as

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$$N_e \equiv \int_{t_*}^{t_e} H dt \simeq \frac{1}{M_\mathrm{P}} \int_{\phi_*}^{\phi_e} \frac{d\phi}{-\mathrm{Sign}(V')\sqrt{2\epsilon}} \tag{2}$$

where the subscripts "$_*$" and "$_e$" represent the time or field value at the horizon exit of a given cosmological scale and at the end of inflation, respectively. In order to be consistent with observations, any inflation model should have ϵ and $|\eta|$ less than about $\mathcal{O}(10^{-2})$ for the scales probed by CMB observations (e.g., WMAP [26] or Planck [27]) with e-foldings upper-bounded as $N_e \lesssim 60$ [5]. It is easy to see that, if $|\eta|$ varies slowly around $\mathcal{O}(10^{-2})$ for most of the e-foldings of inflation, it is necessary to have $\epsilon \sim \mathcal{O}(10^{-3}\text{–}10^{-2})$ in order not to have too many e-foldings. In other words, only if $|\eta|$ varies rapidly, ϵ can be smaller than $|\eta|$ by several (or many) orders of magnitude. Although some large field scenarios share such a feature (see for example [28,29]), this is mostly the case of small-field inflation scenarios where the excursion of the inflaton is limited to be at most Planckian.

When $\epsilon \ll |\eta|$, the spectral index given by $n_s = 1 - 6\epsilon + 2\eta$ is determined nearly only by η. However, although it might be tiny, ϵ affects critically the dynamics of the inflaton via the equation of motion. In this circumstance, if there is a correction to the inflaton potential, which is tiny in terms of its magnitude but has sizable derivatives, its impact on the predictions of V could be critical.

Keeping this possibility in mind, we assume that the inflaton potential is given by

$$V = V_\mathrm{B} + V_\mathrm{M} \tag{3}$$

where V_B and V_M are the leading order base potential and a subleading correction, respectively. As the examples of V_B, we consider a Coleman-Weinberg potential [30]

$$V_\mathrm{cw} = V_0 \left[1 + 4x^4 \left(\ln x - \frac{1}{4}\right)\right] \tag{4}$$

and a Hilltop potential of the form [31]

$$V_\mathrm{ht} = V_0 (1 - x^n)^2 \tag{5}$$

(modulo a completion term) with $x \equiv \phi/\phi_0$ and $\phi_0 \leq M_\mathrm{P} = 2.4 \times 10^{18}$ GeV for both potentials, and where only $n > 2$ is considered. For V_M, we consider

$$V_\mathrm{M} = \Lambda \left[\frac{\cos(\nu x)}{1 + x}\right] \tag{6}$$

with $\Lambda \lll V_0$ and $\nu > 1$. The specific form of V_M was chosen for a clear and clean illustration of our argument. Although it might arise from some heavy physics or nonperturbative effect, the origin of V_M is out of the scope of this paper.

As recently studied again in Ref. [32], in a small field regime, in a Coleman-Weinberg potential the e-foldings associated with the right value of the spectral index are too large to be consistent with observations, and $\epsilon_\mathrm{cw}(x_*^\mathrm{cw})$ is smaller than $|\eta_\mathrm{cw}(x_*^\mathrm{cw})|$ by many orders of magnitude. In the case of the Hilltop potential defined in Eq. (5) with $n > 2$, one finds

$$x_*^\mathrm{ht} \simeq \left[\frac{1}{2n(n-2)N_\mathrm{e,ht}} \left(\frac{\phi_0}{M_\mathrm{P}}\right)^2\right]^{\frac{1}{n-2}} \tag{7}$$

where

$$N_{e,\mathrm{ht}} \simeq \frac{2(n-1)}{(1-n_s)(n-2)} \tag{8}$$

for $x_*^\mathrm{ht} \ll 1$. From Eq. (8), one finds that Hilltop potential can accommodate $N_e \lesssim 60$ only if $n = 4$ with $n_s \lesssim 0.95$ or $n > 4$ for a larger n_s [33]. Note that for $2 < n \lesssim 4$, x_*^ht is smaller than unity at least by several orders of magnitude. Again, this means that $\epsilon_\mathrm{ht}(x_*^\mathrm{ht})$ is smaller than $|\eta_\mathrm{ht}(x_*^\mathrm{ht})|$ by several orders of magnitude. In these cases, those tensions (or the plain inconsistency) with observations may be alleviated (or solved) in the presence of V_M, since, even if $V_\mathrm{M} \lll V_\mathrm{B}$, V_M may provide a sizable (or large) contribution to V' (equivalently to ϵ), reducing the number of e-foldings so as for those models to be viable. Concretely, when $\phi_* \lll \phi_0$ which would be the case for our examples of V_B, if $\nu x_* \ll 1$, one finds

$$\frac{M_\mathrm{P} V_M'}{V} \simeq -\frac{\Lambda}{V} \frac{M_\mathrm{P}}{\phi_0} \tag{9}$$

$$\frac{M_\mathrm{P}^2 V_M''}{V} \simeq -\frac{\Lambda}{V} \left(\frac{M_\mathrm{P}}{\phi_0}\right)^2 (\nu^2 - 2). \tag{10}$$

Hence, if

$$\frac{M_\mathrm{P} V_B'}{V} \lesssim -\frac{\Lambda}{V} \frac{M_\mathrm{P}}{\phi_0} \lesssim \mathcal{O}(10^{-3}\text{–}10^{-2}) \tag{11}$$

a sizable additional force acts on the inflaton, terminating inflation earlier. Note that ν is constrained not to be too large in order to avoid a too large contribution to η. Note also that, as x becomes larger, ϵ_B and $|\eta_B|$ defined in the way of Eq. (1) with V_B increase rapidly, dominating over the contributions coming from V_M. Hence, even if V_M is a oscillatory function, the oscillatory behavior would be negligible as inflaton evolves toward the endpoint of inflation, depending on ν.

III. NUMERICAL ANALYSIS

In this section, we present the results of our numerical analysis showing the impact of $V_{\rm M}$ in Eq. (6) on the predictions of $V_{\rm B} = V_{\rm cw}$ and $V_{\rm ht}$.

In Fig. 1, the case of Coleman-Weinberg base potential is shown for various combinations of $(\Lambda/V_0, \nu)$. As shown in the top left panel of the figure, even if Λ is extremely small as compared to V_0, it can make a significant change in the dynamics of ϕ, terminating inflation much earlier. The evolution of the inflaton is shown in the top middle panel where we plot several combinations of $(\Lambda/V_0, \nu)$ represented as colored lines to show the dependence of the inflaton dynamics on the parameters Λ and ν. It can be seen there that the change of ϵ is more critical than that of η. The time dependence of ϵ is shown in the top right panel of the figure, and our choice of Λ increases ϵ by a factor about 192 while η increases by a factor of 2.2 at t_* corresponding to $N_e = 60$, but $\epsilon(t_*)/\epsilon_{\rm cw}(t_*^{\rm cw}) \simeq 2.68$ and $\eta(t_*)/\eta_{\rm cw}(t_*^{\rm cw}) \simeq 0.54$. The early termination of inflation requires the inflaton to be pushed back toward the origin for a given amount of e-foldings, resulting in a smaller η. Hence, it becomes possible to obtain the observed amount of n_s for the right amount of e-foldings, as shown in the bottom left panel of the figure. As shown in the bottom middle and right panels, there are not significant changes in the running and the running of the running of the spectral index, although the running is pushed slightly to a more negative value. These weak impacts on the spectral runnings are because the contributions of $V_{\rm M}$ to higher derivatives of V are, at most, comparable to those of the $V_{\rm B}$ we are considering.

In Fig. 2, one can find a similar impact of $V_{\rm M}$ on $V_{\rm B} = V_{\rm ht}$ with $n = 3$. In this case, our choice of Λ increased ϵ by a factor about 5×10^6 and η by a factor about 13 at t_* associated with $N_e = 50$, but $\epsilon(t_*)/\epsilon_{\rm ht}(t_*^{\rm ht}) \simeq 3.16$ and $\eta(t_*)/\eta_{\rm ht}(t_*^{\rm ht}) \simeq 0.38$. As expected, the impact on the running of the spectral index is stronger than the case of $V_{\rm cw}$, but still the change is of a factor less than 2. Although we do not present the case here, we found that, Λ/V_0 in $n = 4$ case had to be slightly increased relative to the case of $n = 3$ in order to obtain a similar value of n_s for the same amount of e-foldings. This behavior is due to the fact that, as n increases, $V_{\rm ht}$ becomes flatter toward the origin and allows more e-foldings for a given value of ϕ.

It should be noted that in order to get consistency with observations, the contribution of $V_{\rm M}$ to ϵ should fall within a certain range. This means that from Eq. (9) mostly Λ is constrained as long as $\nu x_* \ll 1$ for cosmological scales of interest. However, as can be seen from the chosen parameter sets in the bottom-left panel of Fig. 1, once it falls into the required ballpark, Λ can vary by a factor of $\mathcal{O}(0.1\text{–}10)$ within the uncertainty of observational data. Also, it should be noted that, even if the slope of the inflaton potential is significantly affected by the presence of $V_{\rm M}$, ϵ is still much smaller than $\mathcal{O}(10^{-2})$ by several orders of magnitude as can be seen from the top-right and bottom-left panels of Fig. 1 and 2. Hence, the tensor-to-scalar ratio is still too small to be measured in near future experiments.

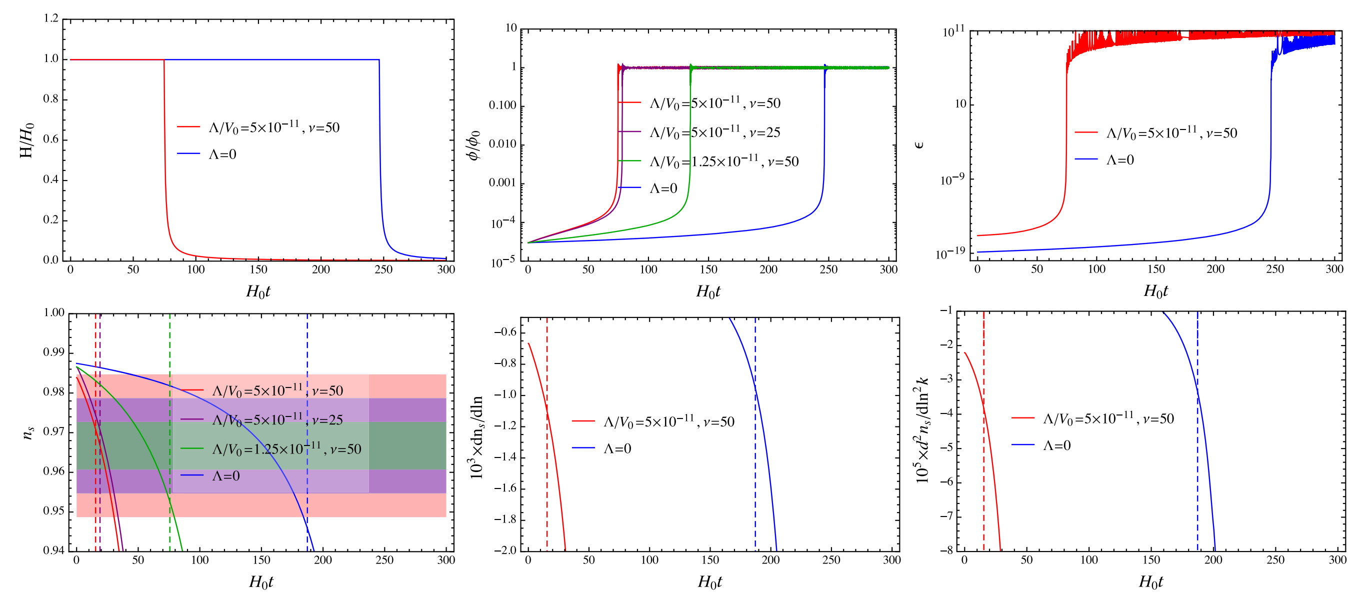

FIG. 1. The impact of $V_{\rm M}$ for $V_{\rm B} = V_{\rm cw}$ with $\phi_0 = M_{\rm GUT}$. The cosmic time was normalized by $H_0 \equiv V_0^{1/2}/\sqrt{3}M_{\rm P}$. Colors indicate different combinations of $(\Lambda/V_0, \nu)$. Top: Evolution of the Hubble paramere (left), inflaton (middle), and the first slow-roll parameter (right) as functions of time. Bottom: Solid lines are spectral index (left), the running of the spectral index in unit of 10^{-3} (middle), and the running of the running in unit of 10^{-5} (right) as functions of time. Dashed lines correspond to t_* associated with $N_e = 60$ with the same color scheme as solid lines. Shaded regions in the left one of bottom panels are 1-, 2-, and 3-σ uncertainties of n_s as by Planck data [27].

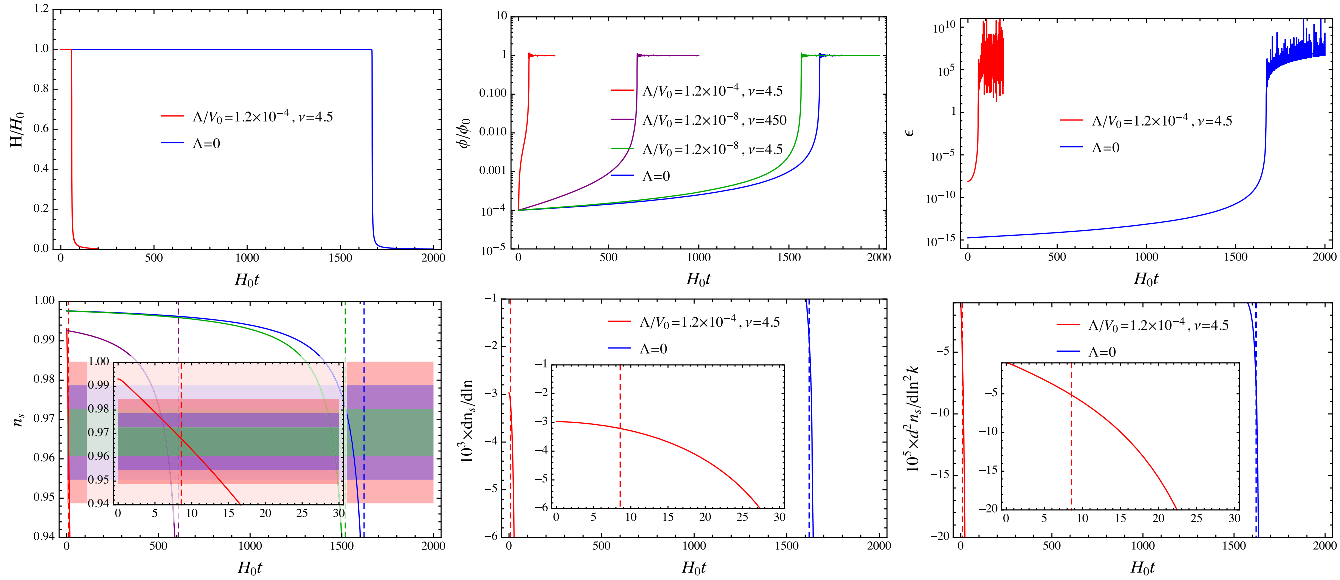

FIG. 2. The impact of $V_{\rm M}$ for $V_{\rm B} = V_{\rm ht}$ with $n = 3$ and $\phi_0 = M_{\rm P}$, presented in the same way as Fig. 1. The color scheme of the bottom-left panel is the same as the top-middle panel. In the bottom panels, dashed lines correspond to t_* associated with $N_e = 50$ with the same color scheme as solid lines. The small plot inside the bottom panels are the magnification for a clearer view of the red lines.

In all of these cases including the case of Coleman-Weinberg potential, the magnitude of the running is of $\mathcal{O}(1-4)\times 10^{-3}$ for $N_e \sim 40$–60. Such largish spectral running seems to be a characteristic of small-field inflation models which have rapidly varying potentials. A fact that implies large higher derivatives of the potential, and make them distinguishable from their large field competitors [34,35].

There could be a Hubble-induced mass term in both $V_{\rm cw}$ and $V_{\rm ht}$ in the sense of supergravity. However, we found that, as long as the effective mass-square of the quadratic term is less than the square of the expansion rate by about $\mathcal{O}(10^{-2})$ (modulo the negative sign assumed) in order to match observations, it makes only minor changes leading to a slightly smaller n_s but does not modify our findings, since the contribution of the mass term to the slope of potential at the relevant flat region is subleading relative to the contribution of Λ. However, note that, when $\Lambda = 0$, the term can become the main contribution to the derivative of the inflaton potential and can reduce the e-foldings by a large amount, although changes in the observables do not seem significant (or go to wrong direction).

From these examples, it is clear that, even if it might be extremely small, a correction to the base potential can make critical changes in the inflationary observables predicted from the base potential only. This can be a generic situation for base potentials which have rapidly varying ϵ and η. The specific choice $V_{\rm M}$ might be questioned, but its form was designed for a clear illustration of our argument. The main point we want to deliver is that, irrespective of its magnitude relative to the base potential, if a correction can give a sizable contribution at least to the first slow-roll parameter, it can produce significant change in the dynamics of the inflaton such that the predictions of inflationary observables can be critically altered.

IV. CONCLUSIONS

In this paper, we showed that a miserably tiny correction to the inflaton potential can make a significant change in the predictions of inflationary observables. If the inflaton potential is such that its slope and curvature vary rapidly in a monotonic way, a tiny correction to the potential can give a sizable contribution to the slope and curvature in the very flat region of the potential. Precisely the region that determines the inflationary observables. In this case, mostly because of the extra force acting on it, the inflaton evolves more rapidly, terminating inflation earlier. Hence, the inflaton should be pushed to a flatter region (where the curvature is also smaller) in order to have a given amount of e-foldings. These effects make significant changes in the inflationary observables, especially the spectral index of the density power spectrum. Therefore the status of some models, e.g. the Coleman Weinberg model, should be revised under this new light.

Typically, only the leading order term of the inflaton potential is considered for the analysis of inflationary observables. However, our findings imply that in order to predict the observables correctly it is necessary to take into account the possible corrections to the potential (probably to a level at least several orders of magnitude smaller than the leading order potential), although its impact depends on the specific forms of the base potential and that of the corrections.

All in all, subleading contributions to the potential do exist and their impact in inflation scenarios (in particular small field inflation models) cannot be underestimated (unless previously studied in depth).

ACKNOWLEDGMENTS

The authors acknowledge support from the MEC and FEDER (EC) Grants No. SEV-2014-0398 and No. FPA2014-54459 and the Generalitat Valenciana under Grant No. PROME-TEOII/2013/017. G. B. acknowledges partial support from the European Union FP7 ITN INVISIBLES (Marie Curie Actions, PITN-GA-2011-289442).

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