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Disruption of cosmic string wakes by Gaussian fluctuations

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We study the stability of cosmic string wakes against the disruption by the dominant Gaussian fluctuations which are present in cosmological models. We find that for a string tension given by $G\mu = 10^{-7}$ wakes remain locally stable until a redshift of $z = 6$, and for a value of $G\mu = 10^{-14}$ they are stable beyond a redshift of $z = 20$. We study a global stability criterion which shows that wakes created by strings at times after $t_{\rm eq}$ are identifiable up to the present time, independent of the value of $G\mu$. Taking into account our criteria it is possible to develop strategies to search for the distinctive position space signals in cosmological maps which are induced by wakes.

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I. INTRODUCTION

Cosmic strings exist as solutions of the field equations in many particle physics models beyond the Standard Model. A sufficient criterion is that the vacuum manifold $\mathcal{M}$ of the model (the space of field configurations which minimize the potential energy density) has nonvanishing first homotopy group $\Pi_1(\mathcal{M}) \neq 1$. Roughly speaking the condition is that the vacuum manifold has the topology of a circle. A simple causality argument [1] leads to the important conclusion that in models which admit cosmic string solutions, a network of such strings inevitably forms during the symmetry breaking phase transition in the early universe and survives to the present time (see [2] for reviews of the role of cosmic strings in cosmology). Cosmic strings carry energy and hence induce gravitational effects which can lead to signatures in cosmological observations. The strength of these effects is proportional to the string tension μ which in turn is given (up to a numerical constant) by η^2, where η is the scale of symmetry breaking at which the strings are formed. Hence, searching for cosmic strings in cosmological observations is a way to probe particle physics beyond the *Standard Model* which is complementary to accelerator searches (which can only probe new physics at low energy scales) [3].

Based on analytical arguments [2] it is expected that the distribution of cosmic strings will take on a "scaling solution" according to which the statistical properties of the distribution of strings are independent of time if all lengths are scaled to the Hubble radius $H^{-1}(t)$ [where $H(t)$ is the cosmic expansion rate at time t]. The distribution of strings consists of a network of infinite strings with mean curvature radius and separation $c_1 t$ (where c_1 is a constant of order 1 whose precise value needs to be determined in numerical simulations) [5] and a set of string loops which are the remnants of intersections of long string segments. Numerical simulations [6] have confirmed that the distribution of strings takes on a scaling solution.

String loops oscillate and gradually decay by emitting gravitational radiation. Long string segments moving through the plasma of the early universe will lead to nonlinear overdensities in the plane behind the moving string. These are called string *wakes* [7]. Wakes are formed because the geometry of space perpendicular to a long string segment is conical with deficit angle

$$\alpha = 8\pi G\mu, \tag{1}$$

where G is Newton's gravitational constant [8]. A string moving through the plasma with a velocity v perpendicular to the tangent vector of the string will lead to a velocity perturbation

$$\delta v = 4\pi G\mu v\gamma(v) \tag{2}$$

from both sides towards the plane behind the moving string [where $\gamma(v)$ is the relativistic gamma factor associated with the velocity v]. In turn, this leads to a thin region behind the string with twice the background density, the *wake*. The dimensions of the wake behind a string at time t_i are

$$c_1 t_i \times v\gamma(v)t_i \times 4\pi G\mu v\gamma(v)t_i, \tag{3}$$

where the dimensions are length along the string, depth of the wake in direction of string motion, and mean thickness of the wake, respectively. We will denote these dimensions by ψ_1, ψ_2 and ψ_3 respectively, when using comoving coordinates.

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Cosmic string loops accrete matter in a roughly spherical way and give rise to density fluctuations which are hard to tell apart from fluctuations formed by other point sources. String wakes, on the other hand, give rise to signals with a clear geometrical signature, and have hence been the focus of a lot of recent work (see e.g. [9]). Long cosmic string segments produce line discontinuities in cosmic microwave background (CMB) temperature maps [10]. The contribution to the power spectrum of cosmological perturbations is scale invariant [11]. However, the fluctuations are active and incoherent [12] and hence do not lead to acoustic oscillations in the angular power spectrum of CMB anisotropies. At the present time, the angular CMB power spectrum in fact provides the most robust upper bounds on the string tension [13] (see the introduction of [14] for a more detailed discussion on string tension limits as well as [15] for earlier studies)

$$G\mu < 1.3 \times 10^{-7}. \tag{4}$$

Hence, it follows that cosmic strings are only a subdominant component to the power spectrum of perturbations. The dominant contribution must be due to almost Gaussian and almost adiabatic fluctuations such as those produced by inflation (or by alternatives to inflation such as *string gas cosmology* [16] or the *matter bounce* [17]).

Whereas overall cosmic strings are a subdominant component to structure formation, string wakes can nevertheless give rise to prominent signatures in position space maps. They give rise to a network of edges in CMB temperature maps across which the temperature jumps [10], rectangles in the sky with a specific CMB polarization signal (statistically equal E-mode and B-mode polarization with a polarization angle which is uniform over the rectangle and whose amplitude has a linear gradient [18]), and thin wedges of extra absorption or emission in 21 cm redshift maps [19] (see also [14,20]). These features are most prominent at high redshifts when string wakes are already nonlinear fluctuations but the Gaussian fluctuations are still in their linear regime. The cosmic string signals are also most easily visible in position space maps (e.g. with edge detection algorithms [21]), whereas the distinctive stringy features are washed out in power spectra (see e.g. [22]).

At early times, cosmic strings dominate the nonlinearities in the universe, the reason being that wakes are nonlinear perturbations beginning at the time they are formed, whereas Gaussian perturbations are linear at early times. At late times, however, the Gaussian fluctuations dominate the structure in the universe. Most of the nonlinearities at the present time are due to the Gaussian fluctuations. The question we wish to address in this paper is whether the string-induced inhomogeneities, which at early times are clearly visible, are still observable as coherent objects in position space maps at later times (in particular times after reionization). Concretely, we wish to study whether string wakes will remain coherent or whether they are disrupted by the Gaussian fluctuations. This analysis is a crucial preliminary step towards identifying string signals at low redshifts, e.g. in 21 cm redshift maps at redshifts comparable and smaller than the redshift or reionization, or in large-scale structure redshift surveys.

In this paper we study various stability criteria for string wakes. We study the stability of a wake to local disruption and find the redshift above which a cosmic string wake remains locally intact, as a function of $G\mu$. However, even if Gaussian fluctuations cause the wake to be locally disrupted, a global signal may remain. We study a specific criterion which can be used to search for the signals of primordial wakes. This analysis shows that signals of string wakes remain from a global perspective to the present time. Interestingly, the signals can be identified independently of the value of $G\mu$, and do not depend on whether the wakes are shock heated or diffuse (see [23] for a discussion of the difference between these two cases). Our various stability criteria will be relevant for developing robust observational strategies to search for string wakes.

In the following section we give a brief review of cosmic string wakes. In Sec. III we present a local stability condition based on a *displacement condition*. In Sec. IV we consider a local density contrast consideration. The resulting stability condition shows that wakes are locally disrupted by the Gaussian perturbations at a redshift lower than some critical redshift which depends on $G\mu$. In Sec. V we discuss a global stability condition which shows that wakes are visible up to the present time independent of the value of $G\mu$.

II. STRING WAKE REVIEW

Consider a string segment at time t_i moving with velocity v in the direction perpendicular to the string. This segment will produce an overdense region with twice the background density behind it whose dimensions are given by (3). Once formed, this wake will be stretched in the planar directions by the expansion of space, and it will grow in thickness by accreting matter from above and below. This accretion can be studied using the Zel'dovich approximation [24]. We will consider wakes produced at times $t_i > t_{\rm eq}$, where $t_{\rm eq}$ is the time of equal matter and radiation. Those produced earlier cannot grow until $t_{\rm eq}$ and they will hence be smaller.

The thickness of the wake at time $t > t_i$ is determined by computing the comoving distance $q_{nl}(t)$ of a shell of matter which is starting to collapse ("turning around") onto the wake, i.e. for which

$$\dot{h}(q_{nl}(t), t) = 0, \tag{5}$$

where the physical height is given by

$$h(q,t) = a(t)[q - \psi(q,t)], \tag{6}$$

where $a(t)$ is the cosmological scale factor and $\psi(q,t)$ is the comoving displacement induced by the gravity of the wake. A standard calculation (see e.g. [18,25]) yields

$$q_{nl}(t,t_i) = (z(t)+1)^{-1}\frac{24\pi}{5}v\gamma(v)G\mu(z(t_i)+1)^{1/2}t_0, \tag{7}$$

where $z(t)$ is the cosmological redshift at time t and t_0 is the present time. At the turnaround $\psi(q_{nl},t) = \frac{1}{2}q_{nl}$. After turnaround, the shell of baryonic matter virializes at a distance which is half of the turnaround radius, whereas the dark matter remains extended [26]. Hence, the physical height of the dark matter wake at time t is

$$h(t,t_i) = (z(t)+1)^{-1}q_{nl}(t,t_i). \tag{8}$$

This is also the displacement which a particle experiences due to the wake if this particle ends up at the edge of the wake. We also denote the wake thickness in comoving coordinates by

$$\psi_3(z) = \frac{24\pi}{5}10^{-7}(G\mu)_7 v\gamma(v)t_0\frac{\sqrt{1+z_i}}{(1+z)}, \tag{9}$$

where $(G\mu)_7$ is the value of $G\mu$ in units of 10^{-7}.

The result (8) shows that the thickest wakes are those produced at the earliest times, namely $t_i = t_{\rm eq}$. The thickness of a wake is obviously proportional to $G\mu$, and its comoving size grows linearly in the cosmological scale factor $a(t)$, as expected from linear cosmological perturbation theory.

III. DISPLACEMENT CONDITION

In this section we will obtain a stability condition which is based on displacements induced by primordial Gaussian fluctuations. For simplicity we will restrict the analysis of this section to the matter dominated period. In the next section we will extend the validity range to include dark energy in the evolution of the growth factor. The wake plane (formed by the ψ_1 and ψ_2 lengths) can be subdivided into pieces of area $(\psi_3)^2$, where ψ_3 is the thickness of the wake in comoving coordinates. We will compute the displacement (in a direction perpendicular to the wake plane) which is coherent on this scale. In order to do this we will integrate in time the fluctuation of the peculiar velocity field on the scale ψ_3.

If S_{ψ_3} is the induced physical displacement, then

$$S_{\psi_3}(t) < h(t,t_i) \tag{10}$$

is a local displacement condition for the stability of the wake. To compute S_{ψ_3} consider the continuity equation

$$\dot{\delta} + \frac{1}{a}\vec{\nabla}\vec{v} = 0, \tag{11}$$

where δ is the relative matter density contrast and $\vec{v}$ is the physical peculiar velocity field. Choosing a Fourier mode parallel to $\vec{v}$ and taking the modulus of the Fourier transform of the above equation we obtain a relation between the amplitudes of the velocity and density contrast fields in momentum space:

$$|v_k(z)| = \frac{faH}{k}|\delta_k(z)|, \tag{12}$$

where we used $\delta(z) = g(z)\delta(0)$, $g(z) = D(z)/D(0)$ and $D(z)$ is the growth factor [27]. For the matter dominated period, $g(z) = 1.29/(1+z)$ and the function $f(z) = \frac{a}{D(z)}\frac{dD(z)}{da}$ is approximately one.

The contribution to the standard deviation of the peculiar velocity field on a scale $L = \frac{2\pi}{k}$ at redshift z is denoted by $\Delta_v(k,z)$ and from the above equation we obtain

$$\Delta_v(k,z) = aH\left(\frac{L}{2\pi}\right)\Delta(k,z), \tag{13}$$

where

$$\Delta(k,z) \equiv \sqrt{\frac{k^3}{2\pi^2}P(k,z)} \tag{14}$$

is the dimensionless contribution to the standard deviation of the matter density fluctuations δ on a length scale corresponding to k, given the dimensional power spectrum $P(k,z)$ at redshift z. The induced physical displacement S_{ψ_3} is given by

$$S_L(z) = a\int_{z_i}^{z} a^{-1}(t')\Delta_v(k,z(t'))dt' \tag{15}$$

evaluated at $k = k_3$ where $k_3 = 2\pi/\psi_3$ is the wave number associated with the comoving thickness $\psi_3(z)$ of the wake. The integral will be dominated by the upper limit of integration, therefore

$$S_{\psi_3} = a\frac{\psi_3}{2\pi}\Delta(\psi_3(z),z), \tag{16}$$

and the displacement condition (10) becomes

$$\Delta(k_3(z),z) < 2\pi. \tag{17}$$

When the above equation holds, the coherent displacement in a region perpendicular to the wake plane will be smaller than the wake thickness ψ_3. This displacement condition agrees to within 1 order of magnitude with the “Local Delta Condition” of the next section and gives a physical interpretation to it. The above condition is valid during the matter dominated period, but in the next section this restriction will be extended to include the dark energy period.

IV. LOCAL DELTA CONDITION

Another criterium for the stability of a wake can be obtained by demanding that the rms Gaussian mass fluctuation Δ on the scale $k_3(z)$ of the wake thickness be smaller than unity, i.e.

$$\Delta(k_3(z), z) < 1. \tag{18}$$

We call (18) the Local Delta Condition, which is stronger than (17). If this condition is satisfied then the wake is locally stable. This condition can be justified by noticing that the matter density contrast δ in a volume within the wake fluctuates around 1 inside the wake and around zero outside, so if the standard deviation σ of δ is of order 1 the wake matter signal will be lost.

The late time power spectrum is obtained by multiplying the primordial power spectrum by the square of a *transfer function* T which comes from the nontrivial evolution of fluctuations on sub-Hubble scales. Specifically, for scales which enter the Hubble radius before the redshift z_{eq} of equal matter and radiation the fluctuations in matter on sub-Hubble scales grow only logarithmically since the universe is dominated by a smooth radiation fluid at these times and on these scales.

The late time power spectrum for a model with Gaussian fluctuations with fixed spectral index is obtained from ([28], page 184)

$$P(k,z) = 2\pi^2 {\delta_H}^2 \frac{k^n}{{H_0}^{n+3}} T^2(k) g^2(z), \tag{19}$$

where we use the expression given by [27] in the growth factor, which now includes dark energy, $T(k)$ is the transfer function, n is the scalar spectral index, and δ_H is the amplitude of Δ evaluated for a Fourier mode that corresponds to the Hubble scale. We choose a normalization that gives $\sigma_8 = 0.83$, where σ_8 is the rms fluctuation smoothed on a scale 8 Mpc/h using a top-hat window function. We use $n = 0.97$ and $\Omega_\Lambda = 0.7$. At this point, we will switch from natural units to units used conventionally in cosmology, namely Mpc for lengths and seconds for time. In these units $c = 9.6 \times 10^{-15}$ Mpc/s, and the expression on the right-hand side of (19) has to be multiplied by c^{n+3}. We will also use $v\gamma(v) = c/\sqrt{3}$, $z_i = 1000$ and $t_0 = 4.35 \times 10^{17}$ s.

The transfer function T from [29] (page 60) is used to obtain an analytic expression for $\Delta(k_3)$, which together with the approximation $(k_3(z))^{-0.0145} \approx 1$ gives

$$\Delta(k_3(z), z) = 0.607 \ln(1 + 22.7 k_3(z)) g(z). \tag{20}$$

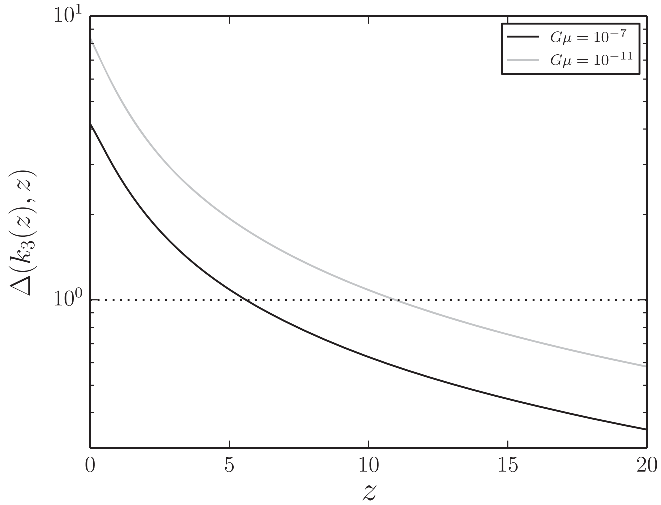

FIG. 1. Plot of $\Delta(k_3(z), z)$ (vertical axis) as a function of redshift z (horizontal axis) for $G\mu = 10^{-7}$ (black line) and $G\mu = 10^{-11}$ (gray line).

This computation of Delta can now be applied to either condition (17) or (18). For example, using the Local Delta Condition (18), we find that the disruption redshift, the redshift when $\Delta(k_3(z), z) = 1$ depends only logarithmically on the wake thickness and hence on the value of $G\mu$. We see that wakes are stable to fairly late times.

In Fig. 1 we plot the value of $\Delta(k_3(z), z)$ (vertical axis) as a function of redshift (horizontal axis) for the values $(G\mu)_7 = 1$ (black line) and $(G\mu)_7 = 10^{-4}$ (gray line). The dashed horizontal line is $\Delta = 1$. We see that the wake is locally stable for z above approximately 6 in the case of $G\mu = 10^{-7}$ and for z above approximately 11 in the case of $G\mu = 10^{-11}$.

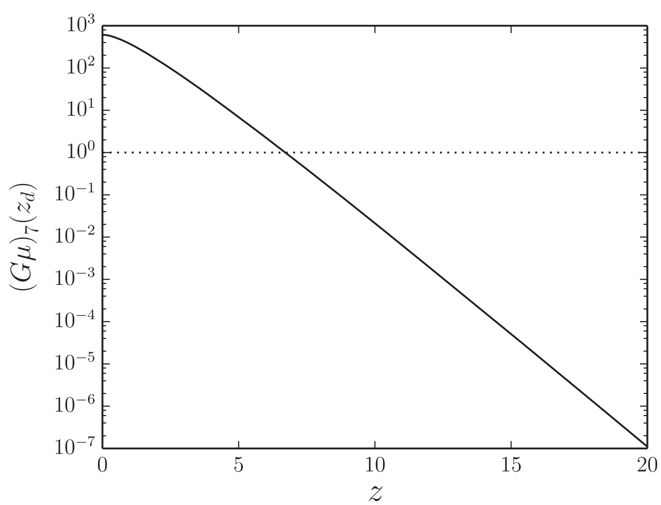

FIG. 2. The value of $G\mu$ (vertical axis in units of 10^{-7}) above which the local Delta wake stability condition is satisfied as a function of redshift z (horizontal axis).

In Fig. 2 we plot (the solid black line) the value of $(G\mu)_7$ (vertical axis) for which the stability condition of a wake ceases to be satisfied at redshift z_d (horizontal axis). From this plot it follows that at $z_d = 20$ all wakes $(G\mu) \geq 10^{-14}$ are stable. The dashed horizontal line is $(G\mu)_7 = 1$, and we see that it intersects the solid black line (which gives the value of $G\mu$ below which the wake is disrupted) at $z \approx 6$, confirming the result of Fig. 1. To obtain the value of $(G\mu)_7(z_d)$ such that the wake will be disrupted at redshift z_d [when the equality of (18) is satisfied] we use

$$k_3(z_d) = 113(1+z_d)/(G\mu)_7 \tag{21}$$

in (20) to obtain

$$(G\mu)_7(z_d) = \frac{2565(1+z_d)}{e^{1/0.607g(z)} - 1}. \tag{22}$$

V. GLOBAL SIGMA CONDITION

The local Delta stability condition studied in the previous section is a very strict condition. It is demanding that no section of the wake gets moved on a scale of the wake thickness. A less restrictive condition is to demand that the wake remains visible if we probe space with a filter which has the shape of the three-dimensional extended wake, i.e. which has two large dimensions given by the length and depth of the wake, respectively, and one small dimension given by the wake thickness. We call the resulting condition the "Global Sigma Condition."

The variance of δ_w for a nonisotropic window function $\tilde{W}_w$ is given by

$$\sigma_w^2 = \frac{g^2(z)}{(2\pi)^3}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dk_1 dk_2 dk_3 P(\|\vec{k}\|)\tilde{W}_w^2(\vec{k}, z), \tag{23}$$

where $g(z)$ is the growth factor and P is the power spectrum at the present time. Note that we are working in terms of comoving momenta. The Global Sigma Condition then is

$$\sigma_w < 1 \tag{24}$$

when we consider a window function whose two large dimensions are given by the planar size of the wake which is fixed in comoving coordinates.

The first guess would be to choose the small dimension to be given by the wake thickness which is increasing in comoving coordinates. Before making this choice, however, let us choose the thickness of the window to be fixed in comoving coordinates, and present a rough analytical analysis. The integral (23) is essentially cut off by the radial planar size that corresponds to the comoving momentum k_r, and the orthogonal size that correspond to k_3, with $k_3 \gg k_r$. We then obtain

$$\sigma_w^2 \sim \frac{g^2(z)}{2\pi^2}\int_0^{k_3} dk_3 \int_0^{k_r} k_r P\left(\sqrt{k_r^2 + k_3^2}\right)$$
$$\sim \frac{g^2(z)}{4\pi^2} k_r^2 \int_0^{k_3} dk_3 P(k_3). \tag{25}$$

For a roughly scale-invariant power spectrum of Gaussian fluctuations, the final integral is dominated by scales which enter the Hubble radius at around $t_{\rm eq}$ where the power spectrum turns over (i.e. changes from scaling as k^{-3} for large values of k to scaling as k for small values). Let us denote this value of k as k_{to}. Then (25) yields

$$\sigma_w^2 \sim \frac{g^2(z)}{4\pi^2}\left(\frac{k_r}{k_{to}}\right)^2 \Delta(k_{to})^2, \tag{26}$$

where $\Delta(k)^2$ is given by (14). Note that the result is independent of k_z as long as $k_z \gg k_{to}$.

Our result (26) lets us draw important conclusions. Most importantly, the global delta criterium (24) is independent of the thickness of the wake, and hence independent of the string tension $G\mu$. The equation (26) also shows that wakes with larger planar extent, i.e. those laid down later, are easier to identify than smaller wakes. The dependence on k_r is linear. This prediction can be used as a consistency check on the numerical analysis.

Another nice feature about our result is that it tells us that we can choose a window function with a width greater than what we expect the local displacements of the wake to be.

We now turn to the quantitative evaluation of the condition. First, the comoving planar dimensions of the wake can be read off from (3). They are

$$\psi_1 = \frac{c_1 t_0}{\sqrt{1+z_i}}, \tag{27}$$

$$\psi_2 = \frac{v\gamma t_0}{\sqrt{1+z_i}}. \tag{28}$$

The wake thickness in comoving coordinates depends on z and is given by Eq. (9).

Considering a wake region V centered at the origin of coordinate space in the form of a parallelepiped of volume $V_w = \psi_1 \times \psi_2 \times \psi_3$ the wake window function in real space becomes

$$W_w(X,Y,Z) = \begin{cases} \frac{1}{V_w} & \text{if } (X,Y,Z) \in V \\ 0 & \text{if } (X,Y,Z) \notin V \end{cases} \tag{29}$$

and the Fourier transform of the above quantity is

$$\tilde{W}_w(k_1,k_2,k_3,z) = \frac{1}{V_w(z)}\left[\frac{2\sin(k_1\psi_1/2)}{k_1}\right]\left[\frac{2\sin(k_2\psi_2/2)}{k_2}\right]$$
$$\times\left[\frac{2\sin(k_3\psi_3(z)/2)}{k_3}\right]. \tag{30}$$

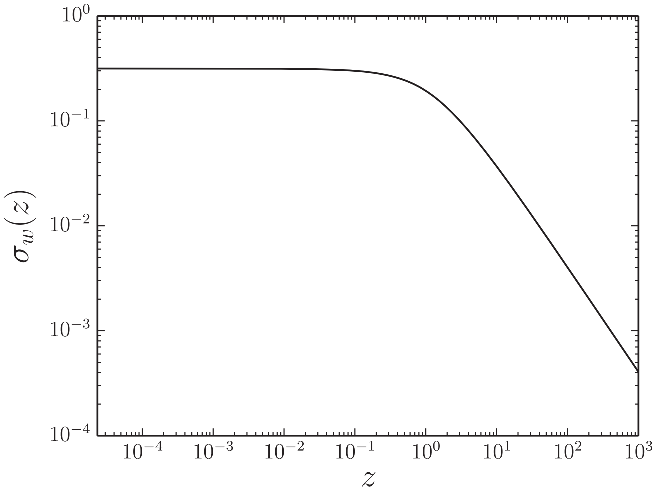

FIG. 3. The rms value of the density contrast of the Gaussian perturbations in an anisotropic region which corresponds to the size of a wake produced at $t_{\rm eq}$ (vertical axis) as a function of redshift (horizontal axis). Note that the density fluctuations remain smaller than one.

The variance of δ_w is given by (23). Replacing (30) into (23) results in

$$\sigma_w^2(z) = \frac{1}{(2\pi)^3}\left(\frac{g(z)}{V_w(z)}\right)^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dk_1 dk_2 dk_3 P(\|\vec{k}\|) \times \left(\left[\frac{2\sin(k_1\psi_1/2)}{k_1}\right]\left[\frac{2\sin(k_2\psi_2/2)}{k_2}\right] \times \left[\frac{2\sin(k_3\psi_3(z)/2)}{k_3}\right]\right)^2 \tag{31}$$

Note that in the integrand above the $2\sin(k_3\psi_3(z)/2)/k_3$ term together with the ψ_3 term of V_w approaches 1 as $G\mu \to 0$. Since for large k_3 the power spectrum also goes to zero, we can take this last term as 1 and hence for small $G\mu \sigma_w$ is independent of $G\mu$. This confirms our expectation from Eq. (26). But this does not mean string wakes are visible at arbitrarily low string tension. A wake should not be disrupted in order for it to be seen. In this sense the global delta condition is a necessary but not a sufficient reason for detection. Though very low $G\mu$ wakes may not be disrupted, they are not necessarily detectable, since cosmic string wake signals are proportional to the string tension (see introduction of Ref. [14] for a more detailed discussion of this point). We explicitly verified the independence of σ_w on $G\mu$ by evaluating the above integral numerically for several values from $G\mu = 0$ to 10^{-7}. It was assumed that $v\gamma(v) = c/\sqrt{3}$ and $z_i = 1000$. We find that $\sigma_w(0) = 0.32$. Note that the entire z dependence for $\sigma_w(z)$ is given by the $g(z)$ factor in front of the integral. Until the time when dark energy becomes important we have $g(z) \propto 1/(z+1)$ and

$$\sigma_w(z) = 0.32 g(z). \tag{32}$$

The plot of $\sigma_w(z)$ is shown in Fig. 3. We conclude that even if the wake is locally disrupted, the overall density pattern remains manifest. Good strategies for cosmic string searches need to take this result into account.

Note that the $\sigma < 1$ conditions (both the global and local ones) are good provided the fraction of additional matter (due to the wake) that is within a region of the window function is of order 1. In this case δ will fluctuate around 1 inside the wake and around 0 outside, so $\sigma < 1$ will be a good conditional to distinguish between the presence and absence of a wake in a given region of space.

VI. DISCUSSION AND CONCLUSIONS

We have studied the disruption of a cosmic string wake by the gravitational effects of the Gaussian fluctuations which dominate the current spectrum of cosmological perturbations. At large redshifts the wakes are stable whereas at smaller redshifts they are locally disrupted. The crossover redshift depends on the string tension $G\mu$. For $G\mu = 10^{-9}$ the crossover redshift is $z \simeq 11$. At redshifts greater than $z = 20$, wakes are stable down to tensions of $G\mu = 10^{-14}$. To arrive at this result we investigated both a local density contrast criterion and a displacement criterion.

As an example, let us evaluate the possibility of seeing a $G\mu = 10^{-9}$ cosmic string wake in a particular slice of the 21 cm maps from the square kilometre array (SKA). Just above its local disruption of $z \simeq 11$, such a generically oriented wake has a projected wake thickness $\Delta z_{\rm wake}$, 2 orders of magnitude smaller than the SKA redshift resolution of $\Delta z_{\rm SKA} = 10^{-4}$. The planar size of the wake is $N = 10^5$ times greater than the SKA angular resolution of 10^{-7} radians. Since the wake is not disrupted there is a slight overdensity over the entire $0.01\ {\rm rad} \times 0.01\ {\rm rad}$ region in redshift space. Consider those N^2 pixels that contain the wake as N^2 measurements in a no-wake theory. Knowing that the wake is undisrupted allows us to calculate the χ^2 between a no-wake theory and a theory with a wake for these pixels [30]. We find that $\chi^2 = N^2 \times (\Delta z_{\rm wake}/\Delta z_{\rm SKA})^2 = 10^6$. Such a large χ^2 results because we have assumed that all our pixels contain the wake. Obviously we have not addressed how to choose such candidate pixels, however here we wish only to show that a wake is visible in the scenario where our pixels do contain a wake.

The physical difference between the Local Delta Condition and the Global Sigma Condition is due to the fact that in the local criteria, the relevant scale of the problem is the wake thickness, and this scale is proportional to the string tension. On the other hand, as discussed above, the relevant scale for the global criteria is the planar dimension of the wake which is independent of the string tension.

Even if a string wake is locally disrupted by Gaussian fluctuations, it could possibly be identified using a Global Sigma Condition. We have computed the rms density contrast due to the Gaussian fluctuations for an anisotropic window function whose planar dimensions correspond to those of a wake, and whose thickness is much smaller than the scale where the density power spectrum turns over, and shown that the result is smaller than 1 for all redshifts. Hence, if we smooth the density field with such a window function, then the wake will be visible even if it is locally disrupted. This global condition is independent of the value of $G\mu$. We are looking for the dark matter component, so we do not have to consider (baryonic) diffuse wake corrections to the wake thickness.

Our work has implications for search strategies to find string signals. Local features of wakes (e.g. discontinuity lines in CMB polarization maps or sharp edges in three dimensional 21 cm redshift surveys) will only be visible for redshifts higher than the crossover redshift determined by our local criteria. In contrast, searches for string signals using global signals (e.g. statistical analyses of maps obtained by smearing the original maps by an anisotropic window function of the shape of the expected wake signal) will be promising even at very low redshifts. We are currently studying this question.

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