Question of Lorentz violation in muon decay

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Possibilities to test the Lorentz invariance of the weak interaction in muon decay are considered. We derive the direction-dependent muon-decay rate with a general Lorentz-violating addition to the W-boson propagator. We discuss measurements of the directional and boost dependence of the Michel parameters and of the muon lifetime as a function of absolute velocity. The total muon-decay rate in the Lorentz-violating standard model extension is addressed. Suggestions are made for dedicated (re)analyses of the pertinent data and for future experiments.

I. INTRODUCTION

Muon decay has historically been an important tool to establish the left-handed "$V - A$" Lorentz structure of the weak interaction in the development of the standard model (SM) of particle physics. Nowadays, it is used to search for new interactions that arise in SM extensions to energies above the electroweak scale [1–3]. In recent years, the limits on such contributions have been significantly improved [4]. In this article, we add yet another twist to muon decay: We propose muon decay as a precision laboratory to test the invariance of the weak interaction under Lorentz transformations (boosts and rotations). Scenarios that break Lorentz and CPT invariance occur in many proposals to unify the SM with general relativity [5,6], a central open issue in high-energy physics. While CPT invariance, in particular, has been tested in neutrino and neutral-meson oscillations, the available evidence for Lorentz invariance of weak decays is limited [7]. Further motivation for investigating the muon comes from experiments on the muon anomalous magnetic moment ("$g - 2$") [8] and muonic hydrogen [9], where at present puzzling deviations from the SM exist.

To explore Lorentz violation in weak decays and to guide and interpret the pertinent experiments, an effective field theory approach was developed [10–12] in which Lorentz violation is parametrized by a complex tensor $\chi^{\mu\nu}$ ($\chi^{\mu\nu}$ is CPT even or odd depending on its momentum dependence [10]). This approach includes a wide class of Lorentz-violating effects, in particular contributions from a modified W-boson propagator $\langle W^{\mu+} W^{\nu-} \rangle = -i(g^{\mu\nu} + \chi^{\mu\nu})/M_W^2$ or from a Lorentz-violating vertex $-i\gamma_\nu(g^{\mu\nu} + \chi^{\mu\nu})$ [13]; $g^{\mu\nu}$ is the Minkowski metric. Bounds on components of $\chi^{\mu\nu}$ were previously extracted from semileptonic allowed [14,15] and forbidden nuclear β decay [11] and pion decay [16,17], and from nonleptonic kaon decay [18]. These bounds were translated into limits on parameters of the standard model extension (SME) [19,20], which is the most general effective field theory for the breaking of Lorentz and CPT invariance.

In this article, we derive the muon-decay rate in our general framework. We show that muon decay offers many possibilities to search for Lorentz violation and we discuss some general issues for dedicated laboratory experiments. We give examples of Lorentz-violating observables and how they could be measured. From a measurement by the TWIST Collaboration [4] we extract bounds on components of $\chi^{\mu\nu}$. From available data on the lifetime of muons at rest and in flight, we constrain the boost dependence of the muon lifetime. We propose reanalyses of existing measurements and new muon-decay and muon $g - 2$ experiments. We argue that dedicated laboratory experiments with muons are preferred over observations of cosmic-ray muons. Finally, we summarize our conclusions.

II. MUON-DECAY RATE

When $\chi^{\mu\nu}$ is included in the W-boson propagator, the matrix element for the decay $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$, corresponding to the tree-level W-exchange diagram, reads ($\hbar = 1 = c$)

$$iM = \frac{G_F}{\sqrt{2}} (g^{\mu\nu} + \chi^{\mu\nu})[\bar{u}(k_1)\gamma_\mu(1 - \gamma_5)u(l)] \times [\bar{u}(p)\gamma_\nu(1 - \gamma_5)v(k_2)], \tag{1}$$

where G_F is the Fermi coupling constant, and l, p, k_1, and k_2 are the momenta of the muon, electron, muon neutrino, and electron antineutrino, respectively. For simplicity, we only consider the dominant momentum-independent part of $\chi^{\mu\nu}$. Although this implies $\chi^{\mu\nu} = \chi^{\nu\mu*}$ when $\chi^{\mu\nu}$ originates from the W-boson propagator, we keep contributions from the real-antisymmetric and the imaginary-symmetric parts of $\chi^{\mu\nu}$ for generality. Such contributions can result e.g. from a Lorentz-violating correction to the vertex [13].

From the matrix element in Eq. (1) and that for $\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$ we derive, to first order in Lorentz violation, the muon-decay rate

$$
\begin{aligned}
dW = \frac{G_F^2}{24\pi^4} \frac{d^3p}{2l^0 2p^0} [&q^2(L \cdot Q) + 2(L \cdot q)(Q \cdot q) \\
&+ 2\chi_{rs}^{\mu\nu}(2q^2 L_\mu Q_\nu + (L \cdot Q)q_\mu q_\nu \\
&- (q \cdot Q)q_\mu L_\nu - (L \cdot q)Q_\mu q_\nu) \\
&+ 2\chi_{ra}^{\mu\nu}(q^2 L_\mu Q_\nu - (q \cdot Q)q_\mu L_\nu - (L \cdot q)Q_\mu q_\nu) \\
&+ \chi_{ia}^{\mu\nu}\epsilon_{\mu\nu\varrho\sigma}((q \cdot Q)L^\varrho q^\sigma - (L \cdot q)q^\varrho Q^\sigma) \\
&- 2\chi_{is}^{\mu\nu} q_\mu \epsilon_{\nu\varrho\sigma\lambda}L^\varrho q^\sigma Q^\lambda],
\end{aligned} \tag{2}
$$

where we summed over the spins and integrated over the momenta of the (anti)neutrino. The subscripts r, i, and s, a

on the tensor $\chi^{\mu\nu}$ denote its real or imaginary and its symmetric or antisymmetric part, respectively. We defined the four-vectors $q = l - p$, $L = l \mp m_\mu s$, and $Q = p \mp m_e r$, where the upper (lower) sign applies for μ^- (μ^+) decay; m_μ and m_e are the muon and electron mass, respectively. The spin four-vector s of the muon is

$$
s = \left(\frac{\mathbf{l} \cdot \hat{\mathbf{s}}}{m_\mu}, \hat{\mathbf{s}} + \frac{(\mathbf{l} \cdot \hat{\mathbf{s}})\mathbf{l}}{m_\mu(l^0 + m_\mu)} \right), \tag{3}
$$

with $\hat{\mathbf{s}}$ a unit vector in the direction of the spin of the muon in its rest frame; the spin four-vector r of the $\beta^\mp$ particle (electron/positron) is defined analogously.

When we sum Eq. (2) over the spin of the β particle we obtain for the differential decay rate in the muon rest frame

$$
\begin{aligned}
\frac{dW}{dxd\Omega} = \frac{W_0}{\pi}x^2 \Big[&3(1-x) + \frac{2}{3}\varrho(4x-3) \mp (\hat{\mathbf{s}} \cdot \hat{\mathbf{p}})\xi \Big[1 - x + \frac{2\delta}{3}(4x-3)\Big] - (t_1 + \mathbf{v}_1 \cdot \hat{\mathbf{p}} \pm \mathbf{v}_2 \cdot \hat{\mathbf{s}} \pm \mathbf{v}_3 \cdot (\hat{\mathbf{s}} \times \hat{\mathbf{p}}) \\
&+ T_1^{ml}\hat{p}^m\hat{p}^l \pm T_2^{ml}\hat{p}^m\hat{s}^l \pm T_3^{ml}\hat{p}^m(\hat{\mathbf{s}} \times \hat{\mathbf{p}})^l)(1-x) - (z_1 + \mathbf{u}_1 \cdot \hat{\mathbf{p}} \pm \mathbf{u}_2 \cdot \hat{\mathbf{s}} \pm \mathbf{u}_3 \cdot (\hat{\mathbf{s}} \times \hat{\mathbf{p}}) \\
&+ H_1^{ml}\hat{p}^m\hat{p}^l \pm H_2^{ml}\hat{p}^m\hat{s}^l \pm H_3^{ml}\hat{p}^m(\hat{\mathbf{s}} \times \hat{\mathbf{p}})^l)(4x-3) \pm (\hat{\mathbf{s}} \cdot \hat{\mathbf{p}})[(t_2 + \mathbf{v}_4 \cdot \hat{\mathbf{p}})(1-x) + (z_2 + \mathbf{u}_4 \cdot \hat{\mathbf{p}})(4x-3)] \Big], \tag{4}
\end{aligned}
$$

where $W_0 = G_F^2 m_\mu^5/(192\pi^3)$ is the total SM decay rate, and $x = E/E_{\max}$ is the energy of the β particle relative to its maximum. We neglected terms proportional to m_e/m_μ, because the pertinent SM terms do not mimic Lorentz violation and Lorentz-violating terms proportional to m_e/m_μ are suppressed. The Lorentz-violating parameters in Eq. (4) are defined by

$$
t_1 = z_1 = z_2 = \frac{1}{2}\chi_{rs}^{00}, \qquad t_2 = \frac{5}{2}\chi_{rs}^{00}; \tag{5a}
$$

$$
v_1^l = \chi_{rs}^{0l} + 2\chi_{ra}^{0l} - 2\tilde{\chi}_{ia}^l, \qquad v_2^l = \frac{1}{2}\chi_{rs}^{0l} + \frac{7}{2}\chi_{ra}^{0l} + \frac{5}{4}\tilde{\chi}_{ia}^l,
$$

$$
v_3^l = \frac{3}{2}\chi_{ia}^{0l} + \frac{5}{2}\chi_{is}^{0l}, \qquad v_4^l = \frac{3}{2}\chi_{rs}^{0l} + \frac{3}{2}\chi_{ra}^{0l} - \frac{3}{4}\tilde{\chi}_{ia}^l; \tag{5b}
$$

$$
u_1^l = -\frac{1}{2}\tilde{\chi}_{ia}^l, \qquad u_2^l = \frac{1}{2}\chi_{rs}^{0l} + \frac{1}{2}\chi_{ra}^{0l} + \frac{1}{4}\tilde{\chi}_{ia}^l,
$$

$$
u_3^l = \frac{1}{2}\chi_{ia}^{0l} + \frac{1}{2}\chi_{is}^{0l}, \qquad u_4^l = \frac{1}{2}\chi_{rs}^{0l} + \frac{1}{2}\chi_{ra}^{0l} - \frac{1}{4}\tilde{\chi}_{ia}^l; \tag{5c}
$$

$$
T_1^{ml} = -\frac{3}{2}\chi_{rs}^{ml}, \qquad T_2^{ml} = \frac{7}{2}\chi_{rs}^{ml} + \frac{1}{2}\chi_{ra}^{ml}, \qquad T_3^{ml} = \frac{3}{2}\chi_{is}^{ml}; \tag{5d}
$$

$$
H_1^{ml} = -\frac{1}{2}\chi_{rs}^{ml}, \qquad H_2^{ml} = \frac{1}{2}\chi_{ra}^{ml}, \qquad H_3^{ml} = \frac{1}{2}\chi_{is}^{ml}, \tag{5e}
$$

where $\tilde{\chi}^l = \epsilon^{lmk}\chi^{mk}$. The first line of Eq. (4) gives the SM decay rate written in the conventional way [2]. For easy comparison, we inserted by hand the three standard Michel

parameters ϱ, ξ, and δ, which parametrize the energy and angular distribution of the β particles in polarized muon decay [21], and which are used to test the $V - A$ structure of the weak interaction. In the SM [and in our framework for Lorentz violation, *cf.* Eq. (1)] the currents have $V - A$ structure, in which case the values of the Michel parameters are $\varrho = 3/4$, $\xi = 1$, and $\delta = 3/4$. The TWIST Collaboration has in recent years put strong limits on non-$(V - A)$ contributions to the Michel parameters [4]. The next lines of Eq. (4), with the parameters defined in Eqs. (5), give Lorentz-violating, frame-dependent contributions to muon decay.

III. BOUNDS FROM THE MICHEL PARAMETERS

Equation (4) offers many possible tests of Lorentz invariance in muon decay. For example, the dependence of the decay rate on the β direction can be studied. In general, it is profitable to measure over extended periods of time and record the data with "time stamps." One can then search for signals that oscillate with periods of one or one-half sidereal day due to the rotation of Earth with respect to the standard Sun-centered inertial reference frame [10,20]. This strategy requires reanalyses of, typically statistics-limited, existing data [4] or new dedicated experiments. Another option is to compare experiments performed at different velocities, i.e. with different values for the Lorentz boost factor γ, because at higher γ values the Lorentz-violating signals are enhanced by a factor γ^2, so that with an equal number of events more stringent limits can be set. We discuss three examples in more detail.

(*i*) To illustrate the rotational dependence of muon decay, consider the decay rate of unpolarized muons depending on the direction of the outgoing β particles. If one measures the number of events in two detector halves, each spanning 2π of solid angle, one can determine an asymmetry, to first order in Lorentz violation, given by

$$\mathcal{A} = \frac{N_+ - N_-}{N_+ + N_-} = -\frac{1}{6}\mathbf{v}_1 \cdot \hat{\mathbf{n}}, \tag{6}$$

where $N_\pm$ is the number of particles emitted in the hemisphere with its axis in the $\pm\hat{\mathbf{n}}$ direction, while the "preferred direction" $\mathbf{v}_1$ in the muon rest frame is defined in Eq. (5a). When the laboratory $\hat{z}$ axis is perpendicular to Earth's surface, $\hat{x}$ points south, and $\hat{y}$ points east, the relation between $\mathbf{v}_1$ in the laboratory frame and $\mathbf{V}_1$ in the Sun-centered frame is given by [10]

$$\mathbf{v_1} = \begin{pmatrix} v_1^x \\ v_1^y \\ v_1^z \end{pmatrix}$$

$$= \begin{pmatrix} \cos\zeta\cos\Omega t V_1^x + \cos\zeta\sin\Omega t V_1^y - \sin\zeta V_1^z \\ -\sin\Omega t V_1^x + \cos\Omega t V_1^y \\ \sin\zeta\cos\Omega t V_1^x + \sin\zeta\sin\Omega t V_1^y + \cos\zeta V_1^z \end{pmatrix}, \tag{7}$$

where ζ is the colatitude of the site of the experiment on Earth and $\Omega \simeq 2\pi/(23\text{h}56\text{m})$ is the angular rotation frequency of Earth. Equation (7) shows that observables like $\mathcal{A}$ will oscillate with a period of one sidereal day. In Fig. 1, three possible scenarios for $\mathbf{v}_1 \cdot \hat{\mathbf{n}}$ are illustrated. The green line shows no oscillation, since the axis of the detector halves is parallel to Earth's axis. The red line is for the case where this axis points east, i.e. perpendicular to Earth's axis, in which case the offset of the oscillation is zero. This can help to identify systematic effects, since any constant
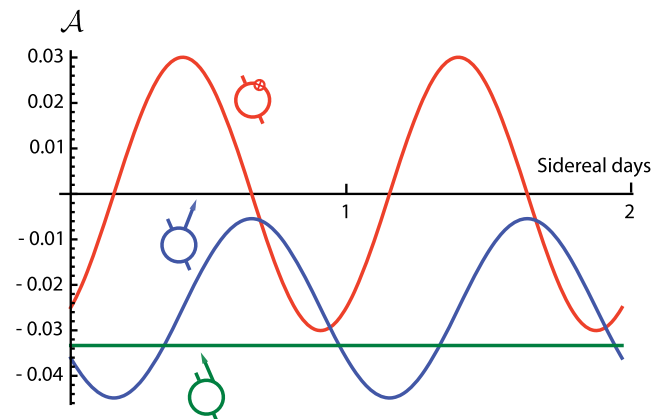

FIG. 1. The asymmetry $\mathcal{A}$ of Eq. (6) as a function of sidereal time, for $V_1^x = 0.1$, $V_1^y = 0.15$, $V_1^z = 0.2$, and $\zeta = 41°$. Its amplitude and offset are determined by the direction of the β particle, as depicted relative to Earth's axis. Present limits indicate $\mathcal{A} < \mathcal{O}(10^{-4})$.

offset is not due to the $\mathbf{v}_1 \cdot \hat{\mathbf{n}}$ term. For the blue line, where $\hat{\mathbf{n}}$ is perpendicular to Earth's surface, there is an oscillation as well as an offset.

(*ii*) When we integrate Eq. (4) over the energy of the β particle, all terms proportional to $4x - 3$ disappear. Defining the muon-polarization direction as the z axis and integrating over the azimuthal angle ϕ of the β momentum, we find

$$\frac{dW}{d\cos\theta} = \frac{W_0}{6}\left[3 - t_1 \mp v_2^z \mp \cos\theta(\xi - t_2 \pm v_1^z + T_2^{zz}) - \cos^2\theta(T_1^{zz} \mp v_4^z) - \frac{1}{2}\sin^2\theta(T_1^{xx} + T_1^{yy})\right], \tag{8}$$

where θ is the angle between the polarization axis and the β momentum, with the muon at rest in the laboratory frame. Thus, when one determines the Michel parameter ξ by fitting the θ dependence of the decay rate, a term with $\cos^2\theta$ dependence has to be included. Since the Lorentz-violating coefficients of the θ-dependent terms vary over the course of a sidereal day, one has to express them in terms of $X^{\mu\nu}$, by which we denote $\chi^{\mu\nu}$ in the Sun-centered frame, as in Eq. (7), and integrate over the relevant measurement periods. Some observables that depend on parameters with two spacelike indices, i.e. T_i^{ml} and H_i^{ml} in Eqs. (5c) and (5e), will oscillate in addition with a period of half a sidereal day.

(*iii*) The TWIST value of the Michel parameter $\varrho_{\text{exp}} = 0.74977(26)$ for μ^+ decay [4] can already be used to derive a bound on $\chi^{\mu\nu}$. The decay rate as a function of positron energy, without selecting a particular direction for the positrons, follows from Eq. (4) as

$$\frac{dW}{dx} = 4W_0 x^2\left[3(1-x)(1 + \mathbf{n}_1 \cdot \hat{\mathbf{s}}) + \left(\frac{2}{3}\varrho - \frac{1}{3}\chi_{rs}^{00} + \mathbf{n}_2 \cdot \hat{\mathbf{s}}\right)(4x - 3)\right], \tag{9}$$

with $n_1^l = \chi_{ra}^{0l} + \frac{1}{2}\tilde{\chi}_{ia}^l$ and $n_2^l = \frac{1}{3}(\chi_{rs}^{0l} + \chi_{ra}^{0l} + \tilde{\chi}_{ia}^l)$. When the difference between the measured value and the SM prediction is attributed to $\chi^{\mu\nu}$, we get $\varrho_{\text{exp}} = \varrho_{\text{SM}} - \chi_{rs}^{00}/2 + (3\mathbf{n}_2/2 - \varrho_{\text{SM}}\mathbf{n}_1) \cdot \hat{\mathbf{s}}$, which results in the 95% confidence limit (C.L.)

$$-5.6 \times 10^{-4} < X_{rs}^{00} + \sin\zeta\cos\phi\left(X_{rs}^{0Z} - \frac{1}{2}X_{ra}^{0Z} + \frac{1}{4}\tilde{X}_{ia}^Z\right) < 1.5 \times 10^{-3}, \tag{10}$$

where $\zeta \simeq 41°$ is the colatitude of Vancouver, and $\phi \simeq 52°$ is the angle that the μ^+ momentum, when entering the TWIST solenoid, makes with the north-south direction (anticlockwise) in the plane parallel to the surface of Earth, so $\sin\zeta\cos\phi = 0.40$ (the μ^+ spin points opposite to its

TABLE I. Muon lifetimes in μs at rest, τ, and in flight at $\gamma \simeq 29.3$, τ', and the relative difference $\Delta = (\tau'/\gamma - \tau)/\tau$. Listed in the rows are the values for μ^+, μ^-, and their average (μ). The numbers between parentheses in the entries are the total errors.

	τ (μs)	Ref.	τ'/γ (μs)	Ref.	$10^4\Delta$
μ^+	$2.1969803(22)_{\text{tot}}$	[24]	$2.1966(20)_{\text{tot}}$	[27]	$-1.7(9.1)_{\text{tot}}$
μ^-	$2.196998(31)_{\text{tot}}$	[25]	$2.1948(10)_{\text{tot}}$	[27]	$-10.0(4.6)_{\text{tot}}$
μ	$2.1969804(22)_{\text{tot}}$		$2.19516(89)_{\text{tot}}$		$-8.3(4.1)_{\text{tot}}$

momentum). Eq. (10) indicates that with dedicated analyses, along the lines of examples *(i)* and *(ii)*, and realistic statistics muon decay can improve on the existing bounds of order $\leq \mathcal{O}(10^{-4})$ [20].

IV. BOOST DEPENDENCE OF THE MUON-DECAY RATE

Strong bounds on Lorentz violation can be obtained by comparing muon-lifetime measurements at different absolute muon velocities, i.e. different Lorentz boost factors γ. The data that are available, unfortunately, are only for the total muon decay rate W, i.e. the muon lifetime. For a matrix element of the form of Eq. (1), two pure $V - A$ currents contracted by $g^{\mu\nu} + \chi^{\mu\nu}$, a general argument [22,23], summarized in the Appendix, shows that the total decay rate of unpolarized muons depends neither on $\chi_s^{\mu\nu}$ nor on $\chi_a^{\mu\nu}$, while the total decay rate of polarized muons depends only on $\chi_a^{\mu\nu}$. These conclusions are borne out by an explicit calculation starting from Eq. (4). A residual dependence on the muon polarization or on the electron/positron direction, however, could introduce a dependence on $\chi^{\mu\nu}$ (see next section).

The available data for the $\mu^\pm$ lifetime that we used are collected in Table I. The most precise measurement of the μ^+ lifetime at rest, $\tau = 1/W$, comes from the MuLan experiment [24]. The μ^- lifetime at rest was derived from the MuCap experiment [25] by correcting for the muon capture rate on hydrogen, for which we used the theoretical value $\Gamma(\mu^- p \to \nu_\mu n) = 718(7)$ s^{-1} [26], obtained in chiral perturbation theory, the low-energy effective field theory of QCD. We also list in Table I the averaged μ^+ and μ^- lifetimes as μ, which is relevant when assuming *CPT* invariance. The muon lifetime in flight, τ', was obtained from data published by the CERN $g - 2$ Collaboration [27,28]. In this experiment, which was performed at the muon "magic momentum," corresponding to $\gamma \simeq 29.3$, the muons are kept in a circular orbit (the effects of acceleration on the lifetime are claimed to be negligible, cf. Refs. [29,30]). The arrival times and energies of the β particles were recorded together with the magnetic-field strength. From these, the dilated lifetime τ' and $\gamma = 29.327(4)$ were obtained [27].

The error in τ' is dominated by statistics. The main systematic error is due to unknown gain variations in the electron detectors, which result in time-dependent

variations in the detection efficiencies. Muon losses, caused e.g. by muon scraping on the ring, and a background of stored protons contribute significantly less (for μ^-, the effect of stored antiprotons was negligible). The average of the μ^+ and μ^- lifetimes and its errors is calculated by weighing with the inverse of the square of the error.

To determine bounds on Lorentz violation, we compare the lifetime for muons in flight to the one obtained for muons at rest. When Lorentz invariance holds, the muon lifetime at rest is calculated from $\tau = \tau'/\gamma$, therefore $\Delta = (\tau'/\gamma - \tau)/\tau$ is the relevant dimensionless quantity. By using the values for τ' and γ of the $g - 2$ experiment, as given in Ref. [27], together with the values for τ in Table I, we calculated the values for Δ for μ^+, μ^-, and for the average of μ^+ and μ^-, respectively, as listed in the last column of Table I. All results are consistent with zero, although there is some mild stress for the negative muon, where Δ deviates from zero by 2.2σ. To test *CPT* invariance of the μ^+ and μ^- lifetimes, we consider the ratio $R = 2(\tau_{\mu^+} - \tau_{\mu^-})/(\tau_{\mu^+} + \tau_{\mu^-})$. For muon decay at rest and in flight, we find $R = -0.8(1.4) \times 10^{-5}$ and $R = 8.2(10) \times 10^{-4}$, respectively.

V. INTERPRETATION

The measurements and analyses of the muon lifetime at rest and in flight, discussed in the previous section, were designed to be sensitive only to the total unpolarized muon decay rate. In order to properly investigate the presence of Lorentz violation according to Eq. (4), details of the analyses from which the total lifetimes were extracted are required, since these analyses involve taking averages over the muon direction and spin. For instance, it becomes relevant that the muons in some measurements may have been polarized, even on average. This could have been due to, for instance, a small residual polarization of the muons used in the analyzed MuLan and muon $g - 2$ data sets, or to a polarization component along the magnetic field in the muon $g - 2$ data sets, which does not average out.

Such effects can be important, in particular, for the muon lifetime in flight, since the related Lorentz-violating contributions are enhanced by boost factors. In the muon $g - 2$ experiments, the arrival-time distribution of the β particles is modulated due to the muon-spin precession relative to its momentum. The analysis as reported in Ref. [27] was such that the result for τ' is predominantly sensitive to the exponential decay rather than this modulation. Furthermore, the fit to the exponent of the decay curve is sensitive to the decay rate, independent of the direction or energy of the outgoing β particles, even though the detectors are only sensitive to part of this parameter space. Also most effects of the muon polarization are removed, because it precesses around the magnetic field and the muons are unpolarized on average, hence the net transverse component of the polarization vanishes. However, any polarization component parallel to the magnetic field does

not average out and may thus persist as a residual vertical polarization. When averaged over a sidereal day, a possible effect due to this residual polarization is further reduced as only the component along Earth's axis remains. Taking such an effect into account, the decay rate is given by

$$W = \frac{1}{\gamma} W_0 (1 \mp \gamma \cos \zeta \mathcal{P}_{\parallel} N_1^Z), \qquad (11)$$

where $\mathcal{P}_{\parallel}$ is the residual polarization parallel to the magnetic field oriented vertically, ζ is the colatitude of the experiment, and $\mathbf{N}_1$ is $\mathbf{n}_1$ defined below Eq. (9) in the Sun-centered frame. Because the velocity of the muon is perpendicular to the magnetic-field directions, only one factor of γ appears in Eq. (11). Effects of incomplete rotation cycles of the muon, the muon spin, and incomplete sidereal days are estimated to be suppressed by several orders of magnitude relative to the Lorentz-violating effect in Eq. (11).

From the available information, we cannot assess whether such residual sensitivities would result in limits for the components of $\chi_{\mu\nu}$ that can compete with the existing bounds. Therefore, our findings should motivate more complete reanalyses of the existing data that differentiate the directions of the β particles and consider in detail residual polarizations of the muons. Moreover, dedicated new experiments are called for.

VI. TOTAL MUON-DECAY RATE IN THE SME

Since $\chi^{\mu\nu}$ does not contribute to the total decay rate, we briefly consider two tensors from the minimal standard model extension (mSME), *i.e.* the power-counting renormalizable part of the full SME [31]. The first one is the tensor $c^{\mu\nu}$ that originates from the *CPT*-even part of the mSME Lagrangian, viz.

$$\mathcal{L} = c_{\mu\nu}[i\bar{\ell}\gamma^\mu\partial^\nu\ell + i\bar{\nu}\gamma^\mu\partial^\nu\nu + \bar{\ell}_L W^{\nu+}\gamma^\mu\nu_L$$
$$+ \bar{\nu}_L W^{\nu-}\gamma^\mu\ell_L], \qquad (12)$$

where ℓ is the charged-lepton field, in our case the muon, and ν is the corresponding neutrino. The second one is the vector b^μ that comes from a *CPT*-odd Lorentz-violating interaction involving the vectors a_L^μ and a_R^μ. Taking the flavor-diagonal part of the SME Lagrangian [19] and redefining the muon and muon-neutrino field by $\psi \rightarrow [1 - \frac{i}{2}(a_L^\mu + a_R^\mu)x_\mu]\psi$, the relevant part of the Lagrangian reads, to first order in Lorentz violation,

$$\mathcal{L} = \bar{\ell}(i\partial\!\!\!/ - m - \gamma_5 b\!\!\!/)\ell + \bar{\nu}_L(i\partial\!\!\!/ - b\!\!\!/)\nu_L + \bar{\ell}_L W\!\!\!\!/^+ \nu_L$$
$$+ \bar{\nu}_L W\!\!\!\!/^- \ell_L, \qquad (13)$$

where $b^\mu = \frac{1}{2}(a_L^\mu - a_R^\mu)$ multiplies a dimension-three operator. (Since in the absence of interaction terms with the W

boson, the neutrino field could be redefined such that the Lorentz-violating parameter would disappear from the neutrino part of the Lagrangian, it is unobservable when only neutrinos are detected. The vector b^μ therefore cannot be constrained by neutrino oscillations or time-of-flight measurements on neutrinos. In Ref. [32], this point was discussed for the equivalent electron-neutrino parameters.)

Since Eqs. (12) and (13) consist of free-fermion operators for external particles, some care has to be taken in the calculation of the muon-decay rate. Following the procedures developed in Refs. [23,33], one finds that the total muon-decay rate is given by $W = W_0(1 - 21c_{00}/5)$ and $W = W_0(1 \mp 4b^0/m_\mu)$ for $c^{\mu\nu}$ and b^μ, respectively. Since we work to first order in Lorentz violation the effects of $c^{\mu\nu}$ and b^μ can be added perturbatively. When boosted to a frame wherein the muons are moving, the resulting decay rate becomes

$$W = \frac{1}{\gamma} W_0 \left[1 - \frac{21\gamma^2}{5} \left(\frac{c^{\mu\nu} p_\mu p_\nu}{(p^0)^2} \right) \mp \frac{4\gamma b^\alpha p_\alpha}{p^0 m_\mu} \right], \qquad (14)$$

where $c^{\mu\nu}$ and b^μ are defined in the laboratory frame and p is the muon momentum in this frame. Averaging this over a rotation of the muon around the ring and over a full sidereal day we find in the Sun-centered frame

$$\int_0^{2\pi} \frac{d\phi}{2\pi} \int_0^{2\pi/\Omega} \frac{\Omega dt}{2\pi} W'$$
$$\simeq \frac{1}{\gamma} W_0 \left[1 - 5.8\gamma^2 (c^{TT} - 0.1c^{ZZ}) \mp \frac{4\gamma b^T}{m_\mu} \right], \qquad (15)$$

where Ω is the angular rotation frequency of Earth, and we used that the colatitude of CERN is about $43.8°$ and that $\gamma^2 \gg 1$. We neglected effects of incomplete rotation cycles of the muon, the muon spin, and effects of incomplete sidereal days. These effects are estimated to be several orders of magnitude smaller than the Lorentz-violating effect in Eq. (15).

In Fig. 2, we plot the joint 95% C.L. region of $c^{TT} - 0.1c^{ZZ}$ and b^T for a fit to the data points of Table I. The most likely point in this parameter space is given by

$$c^{TT} - 0.1c^{ZZ} = -1.2(1.1) \times 10^{-7},$$
$$b^T/m_\mu = -3.6(4.5) \times 10^{-6}, \qquad (16)$$

with 1σ errors. It is represented by the dot at the center of the ellipse in Fig. 2. To our knowledge, these are the first values for the b^T coefficient of the second-generation leptons. The space components of b^μ have been bounded to a level of 10^{-23}–10^{-24} GeV by analyzing the spin-precession frequency of muons [34]. The values in Eq. (16) are an order of magnitude larger than the bounds on the neutrino coefficients derived in Ref. [32], but these bounds are for first-generation coefficients.
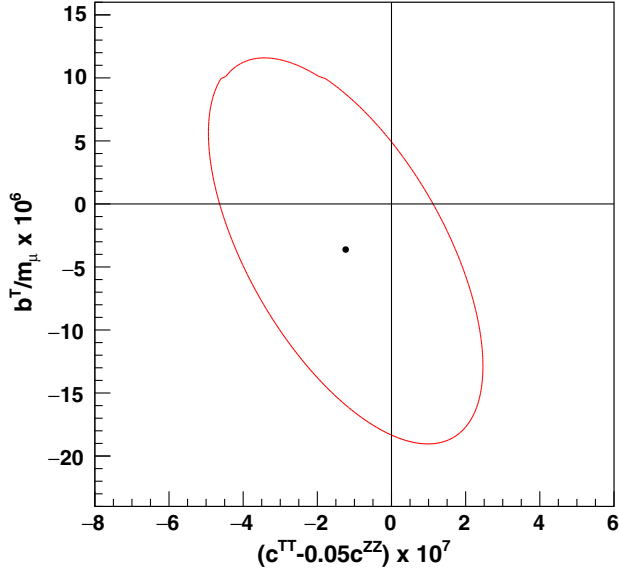
FIG. 2.   Limits on *CPT*-even and *CPT*-odd Lorentz-violating couplings. The ellipse shows the joint 95% C.L. region. The most likely point corresponds to Eq. (16).

VII. COSMIC-RAY MUONS

Because large γ factors are advantageous, cosmic-ray muons, which can have energies up to at least 10^4 GeV, are obvious candidates to search for Lorentz violation. In Ref. [35], it was pointed out that the rate of the flavor-violating muon-decay mode $\mu \to e + \gamma$ could be enormously enhanced when Lorentz invariance is violated. Strong bounds on Lorentz violation for this decay mode were subsequently obtained in Ref. [36]. Therefore, it is of interest to discuss here whether cosmic-ray muons could also be used to put strong limits on Lorentz violation for the ordinary weak decay of the muon.

When dealing with such ultrahigh energies, one has to address the question up to which energy the theoretical framework is valid. Frames that move relatively slow with respect to Earth are called concordant frames [37]. In these frames, all Lorentz-violating parameters are expected to be small. However, in frames that are highly boosted with respect to concordant frames, the Lorentz-violating parameters can become so large that they cause problems with stability and causality in the theory. For very large boost factors the muon-decay rate could even become negative. When we denote the dimensionless Lorentz-violating effect in the muon rest frame by A, then for large γ the tensor $\chi^{\mu\nu}$ with two Lorentz indices scales schematically as $A \propto \gamma^2 a$, where a is the Lorentz-violating effect in a concordant frame. A large γ factor gives better bounds on a when we have a bound for A. However, the theory can only be trusted up to some $A = A_{max}$. If we take as a guideline $A_{max} = 10^{-2}$ [37], the decay rate has to be determined with

subpercent precision to get reliable bounds. This kind of precision is very hard to achieve for ultrahigh-energy cosmic-ray muons. Therefore, results for the boost dependence of the decay rate of such high-energy muons are hard to relate to Lorentz-violating coefficients in an effective field theory approach.

This is less of a problem for the analysis in Ref. [36], because $\mu \to e + \nu + \nu$ is the main allowed decay mode in the SM, while $\mu \to e + \gamma$ is a forbidden process that gets enhanced with respect to the SM decay mode, even without a large boost factor. Moreover, the amplitude for $\mu \to e + \gamma$ does not interfere with a SM amplitude, because it is not a correction to an already existing SM process. Although it therefore depends quadratically on a Lorentz-violating parameter, the enhancement with γ will also be squared, resulting in a scaling with γ^4. For $\mu \to e + \gamma$ Lorentz violation could thus become detectable for values of the boost factor γ for which the Lorentz-violating effect is still sufficiently small.

VIII. CONCLUSION

We derived the most general Lorentz-violating muon-decay rate in the context of our theoretical framework [10,11]. Our main result, given by Eqs. (4) and (5), offers a wealth of possible precision tests of Lorentz invariance of the weak interaction in muon decay. From the measurement of the Michel parameter ϱ we derived bounds of order 10^{-3} on X_{rs}^{00}. Similar bounds on $X^{\mu\nu}$ could be obtained from the measurements of other Michel parameters. However, this requires reanalyses of existing data [4] or dedicated new measurements of the Michel parameters with higher statistics. We gave examples of the types of analyses that are required for Lorentz-violating observables. We compared the lifetime of muons at rest to the lifetime derived from measurements on muons in flight and derived bounds of order 10^{-6}–10^{-7} on specific parameters in the mSME.

The available data from refereed publications are consistent with Lorentz invariance. We advocated reanalyses of the data in terms of $X^{\mu\nu}$ that take into account the dependence of the muon-decay rate on the directions of the β particles and the polarization of the muons. Analyses in terms of nonminimal SME operators, along the lines of Refs. [38,39], would then also be of interest. Ultrahigh-energy cosmic-ray muons are not a good substitute for the experiments that we suggest, since the decay rates would have to be measured with subpercent precision to get reliable bounds on Lorentz violation. We suggest a dedicated analysis of the muon lifetime in the planned new muon $g - 2$ experiment [40]. Based on the $\mathcal{O}(10^{12})$ expected muons, an order of magnitude improvement on the dilated lifetime appears feasible. Since, apart from the required precision, the main challenge in future experiments is to obtain competitive statistics, the muon-beam facilities under

consideration for a new generation of collider experiments are most relevant [41].

APPENDIX: TOTAL MUON-DECAY RATE

Consider the square of the matrix element $[\bar{u}_a \mathcal{O}_\mu u_b][\bar{u}_c \mathcal{O}_\nu u_d]$, where $\mathcal{O}_\mu = \gamma_\mu(1 - \gamma_5)$ is a pure $V - A$ current. After Fierz transforming and contracting with $g^{\mu\nu}, \chi_s^{\mu\nu}$, and $\chi_a^{\mu\nu}$, it follows that if one does not observe the momenta and spins of particles a and c, the symmetric part $\chi_s^{\mu\nu}$ does not enter the expression for the decay rate. If, in addition, the momentum and spin of particle d and the spin of particle b are unobserved, the terms with the antisymmetric part $\chi_a^{\mu\nu}$ in the decay rate also vanish. Applying this to muon decay, it means that the total unpolarized decay rate cannot depend on $\chi^{\mu\nu}$, since it cannot depend on $\chi_s^{\mu\nu}$, nor on $\chi_a^{\mu\nu}$. It also implies that the total decay rate of polarized muons can only depend on $\chi_a^{\mu\nu}$.

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